Target mass corrections in lepton-nucleus DIS

UNIVERSITÉ Grenoble Alpes

Ingo Schienbein
UGA/LPSC Grenoble

Laboratoire de Physique Subatomique et de Cosmologie

Based on R. Ruiz et al, arXiv:2301.07715

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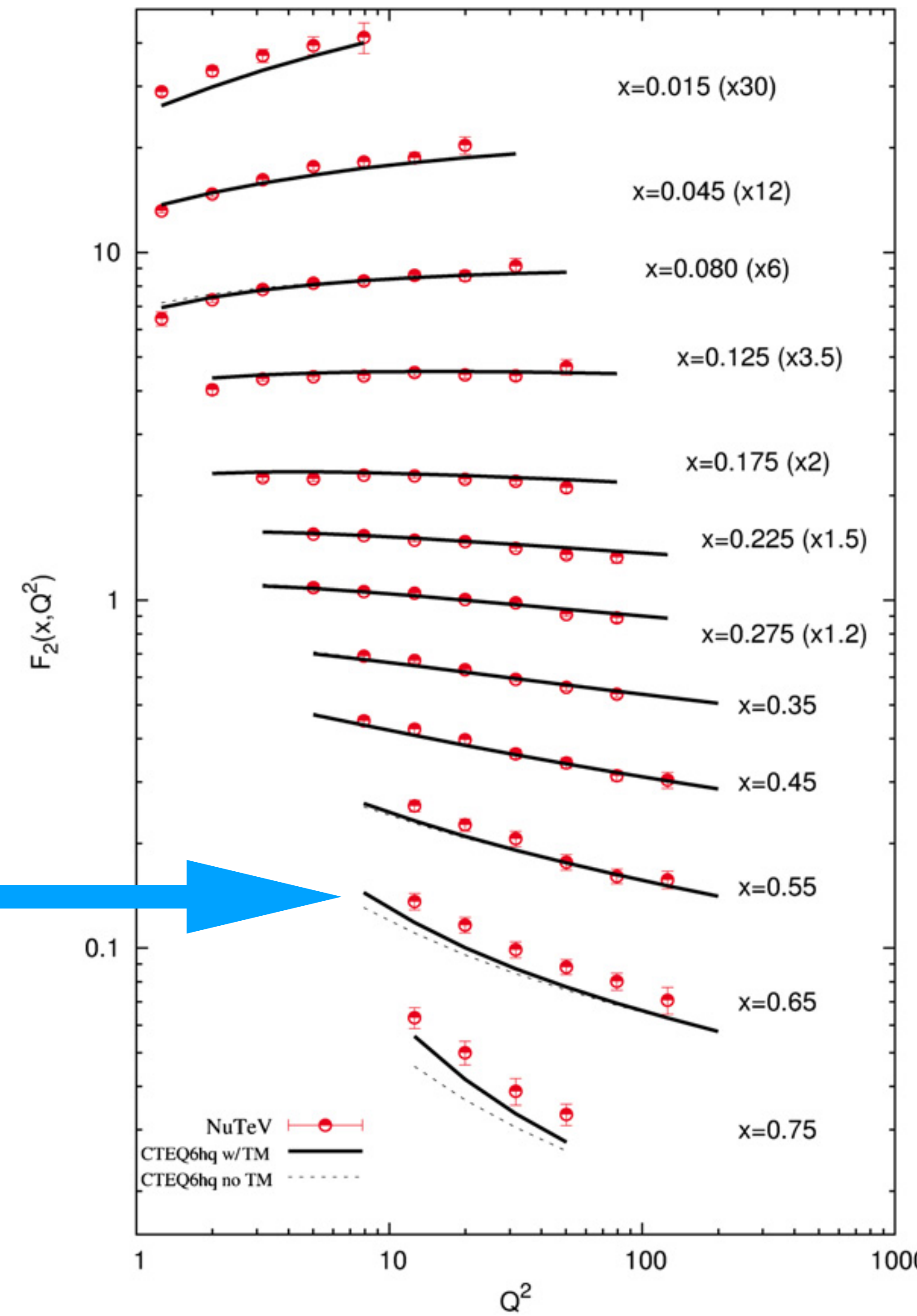
Introduction

- Deeply-inelastic scattering (DIS)
  - Key process for studying the structure of hadrons
  - Backbone of global analyses of parton distribution functions (PDFs)
  - DIS off nucleons and nuclei will be at the forefront again with high precision studies at the EIC, future neutrino facilities, …
- Target mass corrections (TMC) to DIS structure functions
  - Improve description of **high-x/low-Q** DIS data
  - More precise data in this region will provide **important tests of QCD**
  - Timely to review again with particular **focus on nuclear targets**

Figure from **IS et al, arXiv:0709.1775**



Two major theoretical approaches to DIS

- **Operator Product Expansion (OPE)**
  - Georgi, Politzer '76: LO QCD
  - Barbieri et al '76: LO+including quark masses
  - De Rujula, Georgi, Politzer '77: NLO QCD
- **Parton model**
  - Ellis, Furmanski, Petronzio '83: LO+partonic transverse momentum ($k_T \neq 0$)
    Agreement of non-collinear parton approach with OPE at LO shown!
  - Aivazis, Collins, Olness, Tung '93: TMCs in collinear parton model ($k_T = 0$)
- **Theory not carved in stone!**
  - OPE proven only for simple scalar models
  - Non-collinear parton model not covered by QCD factorisation theorems
  - Threshold problem: TMC corrected structure functions do not vanish for $x \to 1$.

Master formula

$$F_1^{A,\mathrm{TMC}}(x_N, Q^2) = \left(\frac{x_N}{\xi_N r_N}\right) F_1^{A,(0)}(\xi_N, Q^2) + \left(\frac{M_N^2 x_N^2}{Q^2 r_N^2}\right) h_2^A(\xi_N, Q^2) + \left(\frac{2M_N^4 x_N^3}{Q^4 r_N^3}\right) g_2^A(\xi_N, Q^2)$$

$$F_2^{A,\mathrm{TMC}}(x_N, Q^2) = \left(\frac{x_N^2}{\xi_N^2 r_N^3}\right) F_2^{A,(0)}(\xi_N, Q^2) + \left(\frac{6M_N^2 x_N^3}{Q^2 r_N^4}\right) h_2^A(\xi_N, Q^2) + \left(\frac{12M_N^4 x_N^4}{Q^4 r_N^5}\right) g_2^A(\xi_N, Q^2)$$

$$F_3^{A,\mathrm{TMC}}(x_N, Q^2) = \left(\frac{x_N}{\xi_N r_N^2}\right) F_3^{A,(0)}(\xi_N, Q^2) + \left(\frac{2M_N^2 x_N^2}{Q^2 r_N^3}\right) h_3^A(\xi_N, Q^2) + 0$$

IS et al, A review of TMC, arXiv:0709.1775

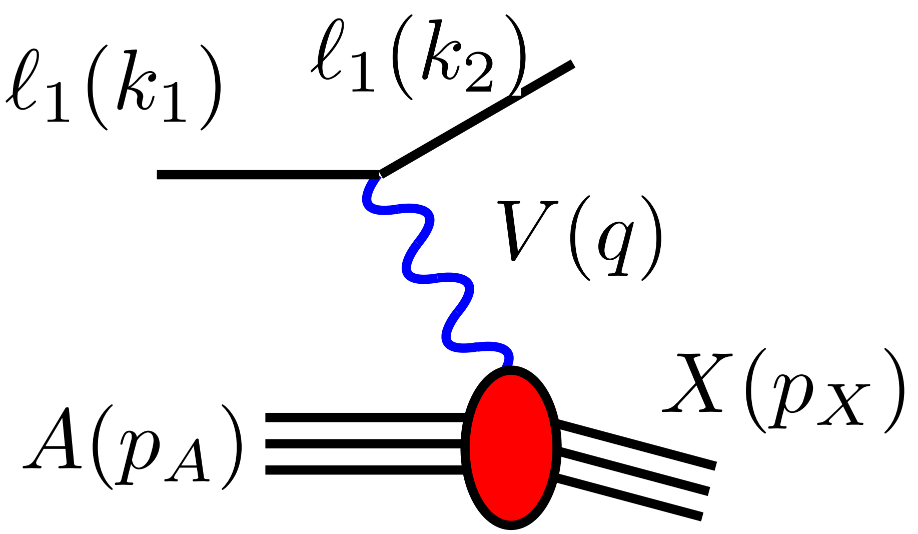
- **Nucleon** mass M_N
- **Nachtmann variable** $\xi_N = 2x_N/(1+r_N), r_N = \sqrt{1 + 4x_N^2 M_N^2/Q^2}$
- $F_i^{A,(0)}$ standard parton model structure functions with $M_N = 0$
- h_i^A, g_i^A convolution integrals over $F_i^{A,(0)}$
- **Modular**, easy to use (organising the rather complicated expressions in the OPE literature)
- Valid to any order in α_s, **quark masses included** in $F_i^{A,(0)}$, valid for **nucleons and nuclei!**

New review 2301.07715

- Reconsider TMC from OPE with **particular focus on nuclear case**
    - Attention to notation exhibiting kinematics
    - Consider conditions of light cone dominance for nuclei
    - Consider spin of target nucleus which can be different from spin-1/2 of nucleons
    - Present derivation of TMCs from OPE in **much** greater detail
    - Prove validity of TMC master equation **for nuclei**
      (Why does the nucleon mass M_N appear and not M_A?)
- Consider full nuclear target:
    - No use of nucleonic degrees of freedom
    - Proper theoretical definition of nuclear structure functions and PDFs as they are intuitively used in the literature
- Parametrization of TMC accurate at the sub-percent level.
  Useful to calculate TMC to structure functions with any PDF set

Kinematics of lepton-nucleus DIS

Nucleus A		**Nucleon N**
$M_A = A\,M_N$		$M_N = M_A/A$
$p_A = A\,p_N$		$p_N = p_A/A$
$x_A = \frac{Q^2}{2p_A\cdot q} \equiv x_N/A$ $x_A \in [0,1]$		$x_N = \frac{Q^2}{2p_N\cdot q} \equiv A\,x_A$ $x_N \in [0,A]$
$W_A^2 = (p_A+q)^2$		$W_N^2 = (p_N+q)^2$
$\nu_A = (q\cdot p_A)/M_A \equiv \nu_N$		$\nu_N = (q\cdot p_N)/M_N \equiv \nu_A$
$y_A = \nu_A/E \equiv y_N$		$y_N = \nu_N/E \equiv y_A$

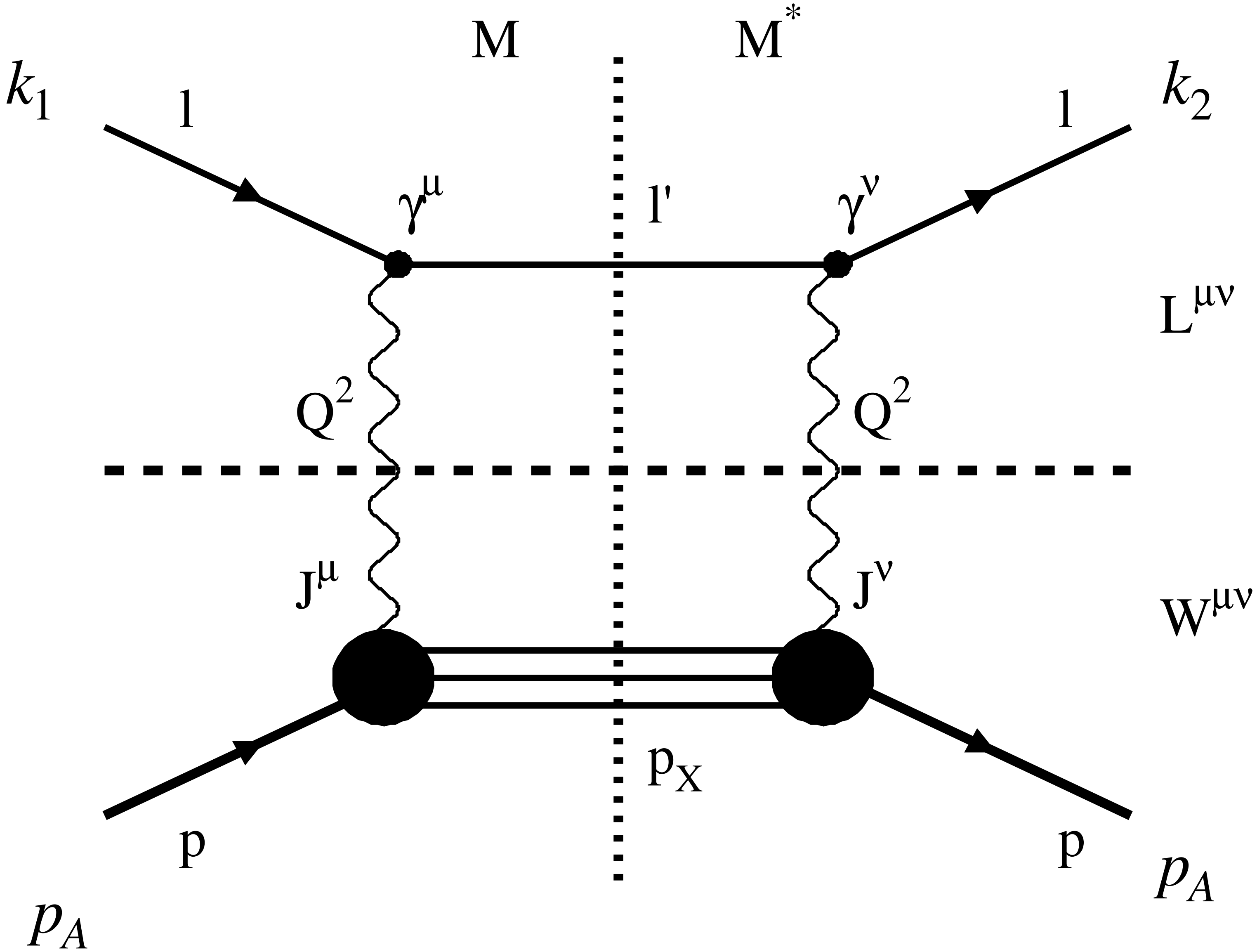


$$q = k_1 - k_2$$

$$p_A^2 = M_A^2$$

Cross section

$$d\sigma \sim L^{\mu\nu}\ \tilde{W}^{A}_{\mu\nu}$$



Leptonic tensor

calculable in pert. theory

Hadronic tensor

not calculabe in pert. theory

Hadronic tensor

Most general form in terms of structure functions:

$$\begin{aligned}
\tilde{W}^{A}_{\mu\nu}(p_A, q) &\equiv \frac{1}{4\pi} \sum\!\!\!\!\!\!\!\int d^4z \; e^{iq\cdot z} \langle A(p_A)|\, J_\mu(z) |X(p_X)\rangle \langle X(p_X)| J_\nu(0)\, |A(p_A)\rangle \\
&= -g_{\mu\nu}\tilde{W}_1 + \frac{p_{A\mu}p_{A\nu}}{M_A^2}\tilde{W}_2 - i\epsilon_{\mu\nu\rho\sigma}\frac{p_A^\rho q^\sigma}{M_A^2}\tilde{W}_3 \\
&+ \frac{q_\mu q_\nu}{M_A^2}\tilde{W}_4 + \frac{p_{A\mu}q_\nu + p_{A\nu}q_\mu}{M_A^2}\tilde{W}_5 + \frac{p_{A\mu}q_\nu - p_{A\nu}q_\mu}{M_A^2}\tilde{W}_6 \;.
\end{aligned}$$

Modern notation:

$$\begin{aligned}
&\left\{\tilde{F}_1,\ \tilde{F}_2,\ \tilde{F}_3,\ \tilde{F}_4,\ \tilde{F}_{5,6}\right\} \\
&\qquad = \left\{\tilde{W}_1,\ \frac{Q^2}{2x_A M_A^2}\tilde{W}_2,\ \frac{Q^2}{x_A M_A^2}\tilde{W}_3,\ \frac{Q^2}{2M_A^2}\tilde{W}_4,\ \frac{Q^2}{2x_A M_A^2}\tilde{W}_{5,6}\right\}
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\end{aligned}
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$d\sigma_{|W_4} \propto m_l^2$ $\quad$ $d\sigma_{|W_5} \propto m_l^2$ $\quad$ $d\sigma_{|W_6} = 0$

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&= -g_{\mu\nu}\tilde{W}_1 + \frac{p_{A\mu}p_{A\nu}}{M_A^2}\tilde{W}_2 - i\epsilon_{\mu\nu\rho\sigma}\frac{p_A^\rho q^\sigma}{M_A^2}\tilde{W}_3 \\
&+ \frac{q_\mu q_\nu}{M_A^2}\cancel{\tilde{W}_4} + \frac{p_{A\mu}q_\nu + p_{A\nu}q_\mu}{M_A^2}\cancel{\tilde{W}_5} + \frac{p_{A\mu}q_\nu - p_{A\nu}q_\mu}{M_A^2}\cancel{\tilde{W}_6} \; .
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$$

Master formula from OPE

$$\tilde{F}_1^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A}\right)\tilde{F}_1^{A,(0)}(\xi_A) + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^2}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^3}\right)\tilde{g}_2^A(\xi_A)\,,$$

$$\tilde{F}_2^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A^2}{\xi_A^2 r_A^3}\right)\tilde{F}_2^{A,(0)}(\xi_A) + \left(\frac{6M_A^2 x_A^3}{Q^2 r_A^4}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{12M_A^4 x_A^4}{Q^4 r_A^5}\right)\tilde{g}_2^A(\xi_A)\,,$$

$$\tilde{F}_3^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_3^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_3^A(\xi_A)\,,$$

- **Nucleus** mass M_N
- **Nachtmann variable** $\xi_A = 2x_A/(1+r_A), r_A = \sqrt{1+4x_A^2M_A^2/Q^2}$
- $\tilde{F}_i^{A,(0)}(x_A, Q^2)$ standard parton model structure functions with $M_A = 0$
- $\tilde{h}_i^A, \tilde{g}_i^A$ convolution integrals over $\tilde{F}_i^{A,(0)}$
- Same structure as master formula for nucleons. Detailed derivation.

From nuclear to averaged nucleon kinematics

Nucleus A		Nucleon N
$M_A = A\, M_N$		$M_N = M_A/A$
$p_A = A\, p_N$		$p_N = p_A/A$
$x_A = \frac{Q^2}{2p_A\cdot q} \equiv x_N/A$ $x_A \in [0,1]$		$x_N = \frac{Q^2}{2p_N\cdot q} \equiv A\, x_A$ $x_N \in [0,A]$
$W_A^2 = (p_A + q)^2$		$W_N^2 = (p_N + q)^2$
$\nu_A = (q\cdot p_A)/M_A \equiv \nu_N$		$\nu_N = (q\cdot p_N)/M_N \equiv \nu_A$
$y_A = \nu_A/E \equiv y_N$		$y_N = \nu_N/E \equiv y_A$

Nachtmann Variable & Hadronic Mass		
$r_A = \sqrt{1+\frac{4x_A^2 M_A^2}{Q^2}} \equiv r_N$		$r_N = \sqrt{1+\frac{4x_N^2 M_N^2}{Q^2}} \equiv r_A$
$\xi_A = R_M x_A \equiv \xi_N/A$ $\xi_A \in [0,1]$		$\xi_N = R_M x_N \equiv A\,\xi_A$ $\xi_N \in [0,A]$
Since $r_A = r_N \equiv r$, then $R_M = \frac{2}{1+r_{A,N}}$		
Also, $\xi_A/x_A = \xi_N/x_N = R_M = \frac{2}{1+r}$		
We define $\varepsilon = (xM/Q)$.		

The M_A-terms are always Accompanied by x_A factors:

$$\frac{M_A^{2j}\, x_A^{2j}}{Q^{2j}} = \frac{(M_N^{2j}\, A^{2j})\, x_A^{2j}}{Q^{2j}} = \frac{M_N^{2j}\, x_N^{2j}}{Q^{2j}}$$

Rescaled structure functions:

$$A\, W_{\mu\nu}^A(p_N, q) := \tilde{W}_{\mu\nu}^A(p_A, q)$$

$$F_2^A(x_N, Q^2) := \tilde{F}_2^A(x_A, Q^2)$$

$$x_N F_{1,3}^A(x_N, Q^2) := x_A \tilde{F}_{1,3}^A(x_A, Q^2)$$

From nuclear to averaged nucleon kinematics

One easily finds for the convolution integrals:

$$\tilde{h}_2^A(\xi_A) = \int_{\xi_A}^1 du_A \frac{\tilde{F}_2^{A(0)}(u_A)}{u_A^2} = A\int_{\xi_N}^A du_N \frac{F_2^{A(0)}(u_N)}{u_N^2} =: Ah_2^A(\xi_N)$$

$$\tilde{h}_3^A(\xi_A) = \int_{\xi_A}^1 du_A \frac{\tilde{F}_3^{A(0)}(u_A)}{u_A} = A\int_{\xi_N}^A du_N \frac{F_3^{A(0)}(u_N)}{u_N} =: Ah_3^A(\xi_N)$$

$$\tilde{g}_2^A(\xi_A) = \int_{\xi_A}^1 du_A \tilde{h}_2^A(u_A) = \frac{1}{A}\int_{\xi_N}^A du_N Ah_2^A(u_N) =: g_2^A(\xi_N)$$

With these expressions and $x_N = Ax_A$ **,** $\xi_N = A\xi_A$**,** $r_A = r_N$**,** $M_A = AM_N$

$$F_1^{A,\mathrm{TMC}}(x_N, Q^2) = \left(\frac{x_N}{\xi_N r_N}\right) F_1^{A,(0)}(\xi_N, Q^2) + \left(\frac{M_N^2 x_N^2}{Q^2 r_N^2}\right) h_2^A(\xi_N, Q^2) + \left(\frac{2M_N^4 x_N^3}{Q^4 r_N^3}\right) g_2^A(\xi_N, Q^2)$$

$$F_2^{A,\mathrm{TMC}}(x_N, Q^2) = \left(\frac{x_N^2}{\xi_N^2 r_N^3}\right) F_2^{A,(0)}(\xi_N, Q^2) + \left(\frac{6M_N^2 x_N^3}{Q^2 r_N^4}\right) h_2^A(\xi_N, Q^2) + \left(\frac{12M_N^4 x_N^4}{Q^4 r_N^5}\right) g_2^A(\xi_N, Q^2)$$

$$F_3^{A,\mathrm{TMC}}(x_N, Q^2) = \left(\frac{x_N}{\xi_N r_N^2}\right) F_3^{A,(0)}(\xi_N, Q^2) + \left(\frac{2M_N^2 x_N^2}{Q^2 r_N^3}\right) h_3^A(\xi_N, Q^2) + 0$$

Discussion

- Everything defined in terms of a nuclear state, QCD operators and a kinematics rescaling

- No use of nucleonic degrees of freedom was made

- Similarly, nuclear PDFs are introduced in the variable x_A.

  - DGLAP and Sum rules in x_A

- Then, one can define the rescaling to the variable x_N:
  $f_i^A(x_N)dx_N := \tilde{f}_i^A(x_A)dx_A$

  - DGLAP and Sum rules in x_N

- Again **no use of (bound) nucleon PDFs**, **just nuclear PDFs** (which is what is determined by data!)

- The rescaling at the hadronic level and the parsonic level are fully consistent

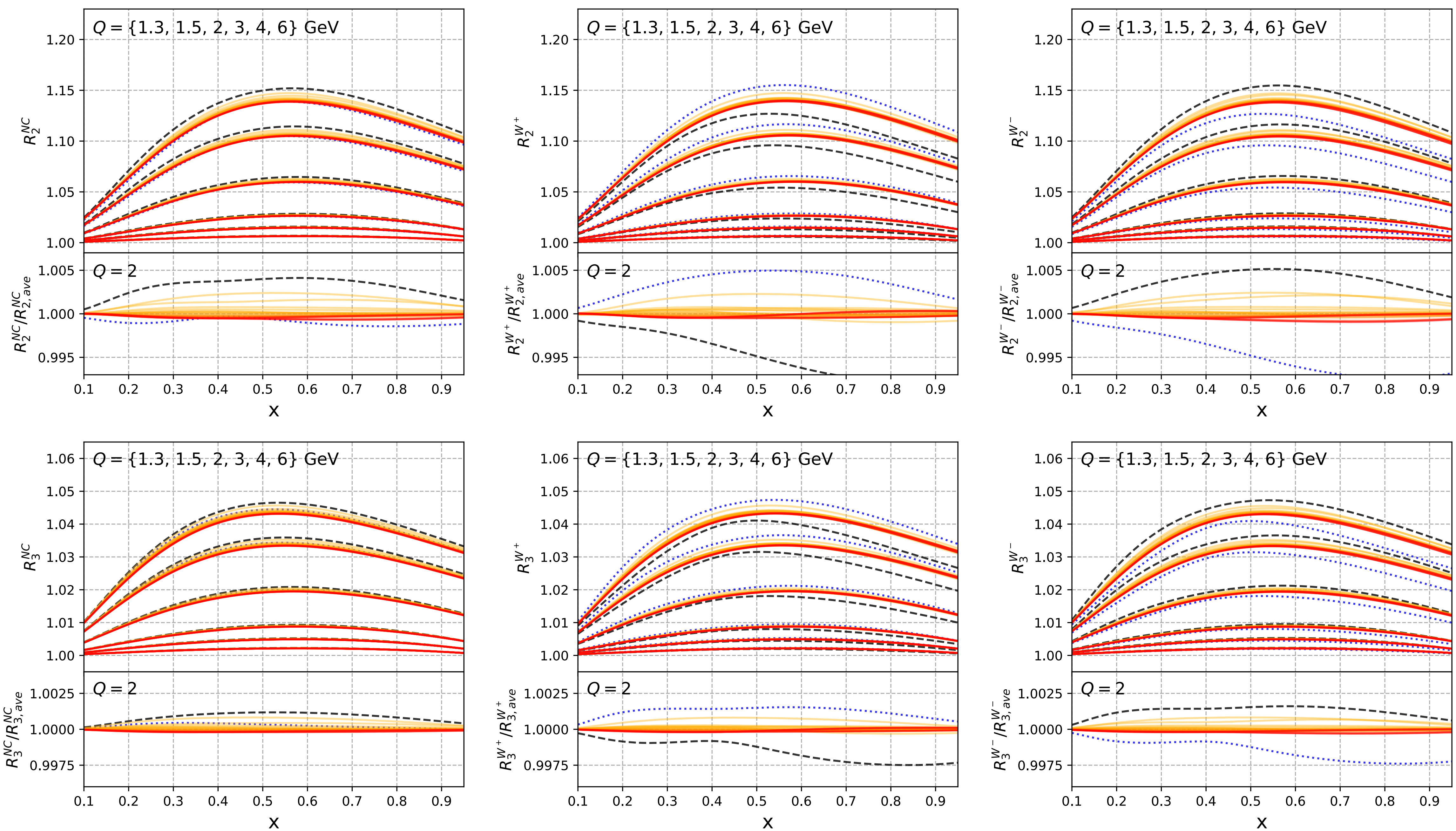
Which terms are included?

$F_i^{(0)}$

$F_i^{A,\mathrm{TMC}}$ **Twist=2**	1	$\frac{M_N^2}{Q^2}$	$\left(\frac{M_N^2}{Q^2}\right)^2$	...
$F_i^{A,\mathrm{TMC}}$ **Twist=4**	$\frac{Q_0^2}{Q^2}$	$\frac{Q_0^2}{Q^2}\frac{M_N^2}{Q^2}$	$\frac{Q_0^2}{Q^2}\left(\frac{M_N^2}{Q^2}\right)^2$	...
$F_i^{A,\mathrm{TMC}}$ **Twist=6**	$\left(\frac{Q_0^2}{Q^2}\right)^2$	$\left(\frac{Q_0^2}{Q^2}\right)^2\frac{M_N^2}{Q^2}$	$\left(\frac{Q_0^2}{Q^2}\right)^2\left(\frac{M_N^2}{Q^2}\right)^2$	...
...	...	...	...	...

- Master formula resums leading twist TMC to all orders in $(M_N^2/Q^2)^n$
- Higher twist terms $(Q_0^2/Q^2)^n$ not included where Q_0 is a hadronic scale (To be modelled separately)

$$F_i^{\mathrm{TMC}}(x_N, Q^2)/F_i^{\mathrm{TMC,leading}}(x_N, Q^2)$$



- Ratios very insensitive to nuclear A except A=1 (dashed, dotted)
- Simple parametrization of full TMC for everybody to use in numerical calculations (see 2301.07715 for details)

Summary

- New review of TMC from OPE with **particular focus on nuclear case [2301.07715]**
  - Attention to notation exhibiting kinematics
  - Consider conditions of light cone dominance for nuclei
  - Consider spin of target nucleus which can be different from spin-1/2 of nucleons
  - Present derivation of TMCs from OPE in **much** greater detail
  - Prove validity of TMC master equation **for nuclei**
- Consider full nuclear target:
  - No use of nucleonic degrees of freedom
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- Parametrization of TMC accurate at the sub-percent level.
  Useful to calculate TMC to structure functions with any PDF set

“Yesterday’s sensation is today’s calibration and tomorrow’s backup slide”

–Richard Feynman (modified)

Light cone dominance of nuclear DIS

$$\tilde{W}^{A}_{\mu\nu}(p_A, q) = \frac{1}{4\pi} \int d^4z \, e^{iq\cdot z} \langle A | J^{\dagger}_{XA\mu}(z) \, J_{XA\nu}(0) | A \rangle$$

DIS limit:

$$Q^2 \to \infty, \quad \nu_A \to \infty, \quad \text{such that} \quad \frac{Q^2}{\nu_A} = 2 M_A x_A \quad \text{is fixed}$$

Dominant contribution to Fourier integral: $0 \leq z^2 \leq \mathrm{const}/Q^2$

What does the DIS limit mean in practice?

Nucleon case (see textbook by Muta): $Q^2 \sim p_N \cdot q \gtrsim M_N^2$

Nuclear case (naively): $Q^2 \sim p_A \cdot q \gtrsim M_A^2$ **would suggest very large** Q^2

We argue instead: $\boxed{Q^2 \sim \nu_A \gtrsim \Lambda^2_{\mathrm{had}} \gg \Lambda^2_{\mathrm{QCD}}}$

Forward Compton scattering amplitude

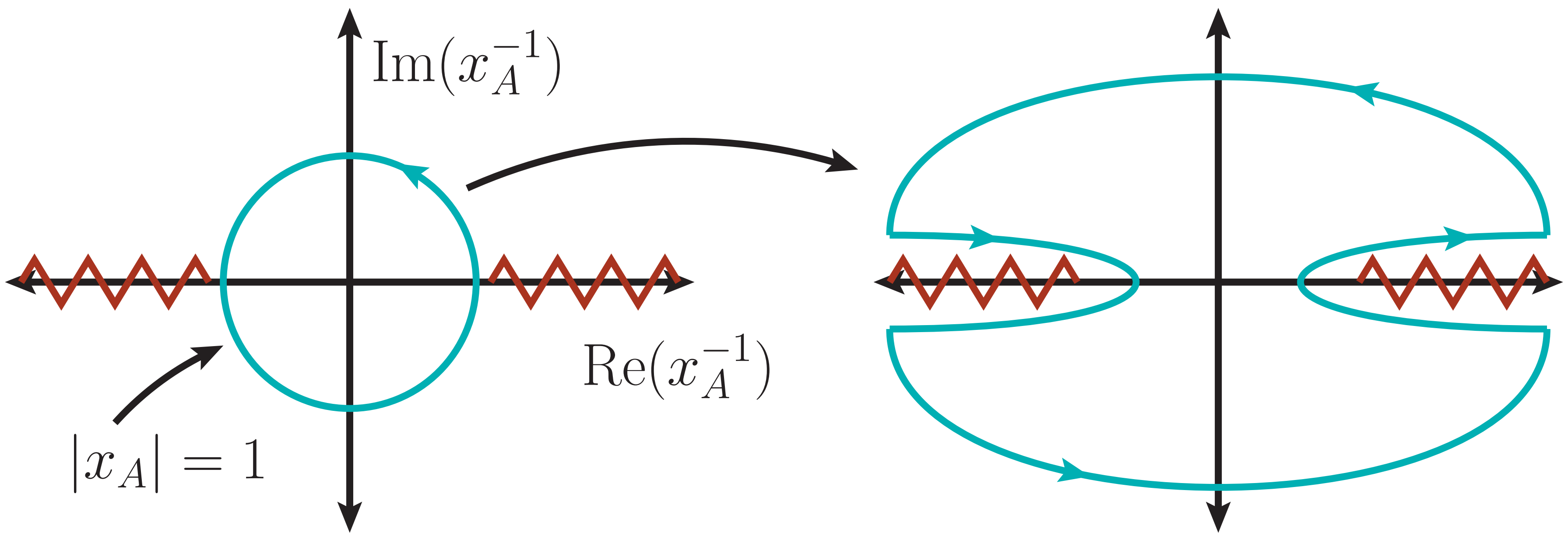
$$
\begin{aligned}
\tilde{T}^A_{\mu\nu}(p_A, q) \;\equiv & \int d^4z \; e^{iq\cdot z} \; \langle A|\mathcal{T} J^\dagger_{A\mu}(z) \; J_{A\nu}(0)|A\rangle \; , \\
& = - g_{\mu\nu}\Delta\tilde{T}^A_1 + \frac{p_{A\mu}p_{A\nu}}{M_A^2}\Delta\tilde{T}^A_2 - i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha_A q^\beta}{M_A^2}\Delta\tilde{T}^A_3 \\
& \quad + \frac{q_\mu q_\nu}{M_A^2}\Delta\tilde{T}^A_4 + \frac{(p_{A\mu}q_\nu \pm p_{A\nu}q_\mu)}{M_A^2}\Delta\tilde{T}^A_{5,6} \; ,
\end{aligned}
$$

For comparison

$$
\begin{aligned}
\tilde{W}^A_{\mu\nu}(p_A, q) \quad & \equiv \frac{1}{4\pi}\sum\!\!\!\!\!\!\int d^4z \; e^{iq\cdot z}\langle A(p_A)|\, J_\mu(z)|X(p_X)\rangle\langle X(p_X)|J_\nu(0)\,|A(p_A)\rangle \\
& = -g_{\mu\nu}\tilde{W}_1 + \frac{p_{A\mu}p_{A\nu}}{M_A^2}\tilde{W}_2 - i\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho_A q^\sigma}{M_A^2}\tilde{W}_3 \\
& + \frac{q_\mu q_\nu}{M_A^2}\tilde{W}_4 + \frac{p_{A\mu}q_\nu + p_{A\nu}q_\mu}{M_A^2}\tilde{W}_5 + \frac{p_{A\mu}q_\nu - p_{A\nu}q_\mu}{M_A^2}\tilde{W}_6 \; .
\end{aligned}
$$

Relation between W and T

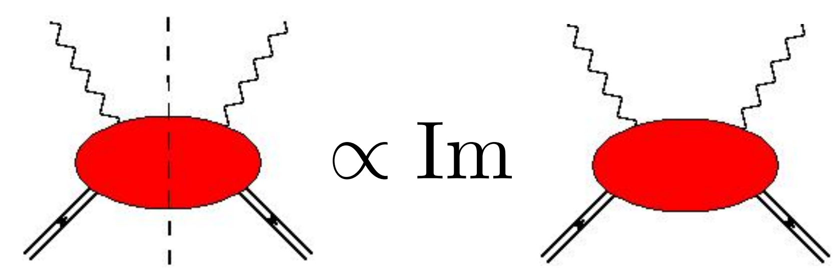
Analytical structure of $\tilde{T}^A$ in the complex ω_A plane:



- $\tilde{T}^A$ converges in the region $|\omega_A| < 1$ where $\omega_A = x_A^{-1}$ (short distance region)
- The discontinuity in the physical DIS region $|\omega_A| > 1$ is related to $\tilde{W}^A$ (after analytic continuation)

$$\tilde{T}^A_{\mu\nu}(p_A, q)\Big|_{(1/x_A)-i\varepsilon}^{(1/x_A)+i\varepsilon} = 4\pi\ \tilde{W}^A_{\mu\nu}(p_A, q), \quad \text{for} \quad x_A > 0\ ,$$

$$\tilde{T}^A_{\mu\nu}(p_A, q)\Big|_{(1/x_A)+i\varepsilon}^{(1/x_A)-i\varepsilon} = 4\pi\ \left[\tilde{W}^A_{\mu\nu}(p_A, -q)\right]^\dagger, \quad \text{for} \quad x_A < 0$$



$$\tilde{W}^A_{\mu\nu} \propto \mathrm{Im}\tilde{T}_{\mu\nu}$$

Relation between W and T

$$\tilde{T}^A_{\mu\nu}(p_A, q)\Big|^{(1/x_A)+i\varepsilon}_{(1/x_A)-i\varepsilon} = 4\pi\ \tilde{W}^A_{\mu\nu}(p_A, q), \quad \text{for} \quad x_A > 0\ ,$$

$$\tilde{T}^A_{\mu\nu}(p_A, q)\Big|^{(1/x_A)-i\varepsilon}_{(1/x_A)+i\varepsilon} = 4\pi\ \left[\tilde{W}^A_{\mu\nu}(p_A, -q)\right]^\dagger, \quad \text{for} \quad x_A < 0$$

An important consequence is the following link between individual $\Delta\tilde{T}^A_i$ **and the Mellin moments of the structure functions**

$$\Delta\tilde{T}^A_i \sim \sum_N \tilde{F}^A_i(N, Q^2)x_A^{-N}$$

Operator Product Expansion (OPE)

There are two different expansions:

a) short distance expansion

$$A(x)B(0) \underbrace{\simeq}_{x_\mu \to 0} \sum_i C_i(x) O_i(x/2)$$

b) light cone expansion

$$A(x/2)B(-x/2) \underbrace{\simeq}_{x^2 \to 0} \sum_{j,i} C_i^{(j)}(x) x^{\mu_1} \cdots x^{\mu_j} O^{(j,i)}_{\mu_1 \cdots \mu_j}(0)$$

Light cone dominance of DIS hadronic tensor

Wilson coefficients

local ops. of definite spin j (symmetric traceless tensors of rank j)

$$C_i^{(j)} \underbrace{\propto}_{x^2 \to 0} (\sqrt{x^2})^{d_{j,i} - j - d_A - d_B}$$

Light cone ops. with lowest twist dominate!

twist = dimension - spin

Short distance expansion of $\tilde{T}^A_{\mu\nu}$

$$\lim_{z\to 0} T^A_{\mu\nu}(p_A,q) \overset{\text{OPE}}{=} -2i \sum_{k,\iota} \boxed{c^{\tau=2,\iota}_{\mu\nu\mu_1\ldots\mu_k}(q)}\,\boxed{\langle A(p_A)|\mathcal{O}^{\mu_1\ldots\mu_k}_{\iota,\tau=2}|A(p_A)\rangle} + \mathcal{O}(\tau>2)$$

Local operators

$$\boxed{\langle A|\mathcal{O}^{\mu_1\ldots\mu_{2k}}_{\iota,\tau=2}|A\rangle = A^{2k}_{\tau=2} \times \tilde{\Pi}^{\mu_1\ldots\mu_{2k}}}$$

where

$\tilde{\Pi}^{\mu_1\cdots\mu_{2k}}$: **Traceless, symmetric rank-2k tensor**

$$\tilde{\Pi}^{\mu_1\ldots\mu_{2k}} = \sum_{j=0}^{k}(-1)^j \frac{(2k-j)!}{2^j(2k)!}\,\eta(j,2k-2j)\;\underbrace{\{g\ldots g\}}_{j\ g^{\mu_n\mu_m}\,'s}\;\underbrace{\{p_A\ldots p_A\}}_{(2k-2j)\ p_A^{\mu_n}\,'s}\,(p_A^2)^j$$

$$\boxed{\begin{aligned} c^{\tau=2,\iota}_{\mu\nu\mu_1,\ldots,\mu_{2k}}(q) = \Big[&-2g_{\mu\nu}q_{\mu_1}q_{\mu_2}C^{2k}_1 + g_{\mu\mu_1}g_{\nu\mu_2}Q^2C^{2k}_2 - i\epsilon_{\mu\nu\alpha\beta}g^{\alpha}_{\mu_1}q^{\beta}q_{\mu_2}C^{2k}_3 \\ &+4\frac{q_\mu q_\nu}{Q^2}q_{\mu_1}q_{\mu_2}C^{2k}_4 + 2(g_{\mu\mu_1}q_\nu q_{\mu_2} \pm g_{\nu\mu_1}q_\mu q_{\mu_2})C^{2k}_{5,6}\Big] \times \frac{2^{2k}}{(Q^2)^{2k}} \times \left(\prod_{m=3}^{2k} q_{\mu_m}\right). \end{aligned}}$$

Short distance expansion of $\tilde{T}^{A}_{\mu\nu}$

Evaluating the contractions of Lorentz indices gives:

$$\int_0^1 dx_A\, x_A^{N-2}\tilde{F}_2^{A,\mathrm{TMC}}(x_A,Q^2) = \sum_{j=0}^{\infty}\left(\frac{M_A^2}{Q^2}\right)^j \frac{(N+j)!}{j!\,(N-2)!}\;\frac{C_2^{N+2j}\,A_{\tau=2}^{N+2j}}{(N+2j)(N+2j-1)}$$

Similar for the other structure functions

The master equations in x-space are then obtained by inverse Mellin transformation