Generalized Parton Distributions through Universal Moment Parameterization (GUMP): Gluons at non zero skewness

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Outline

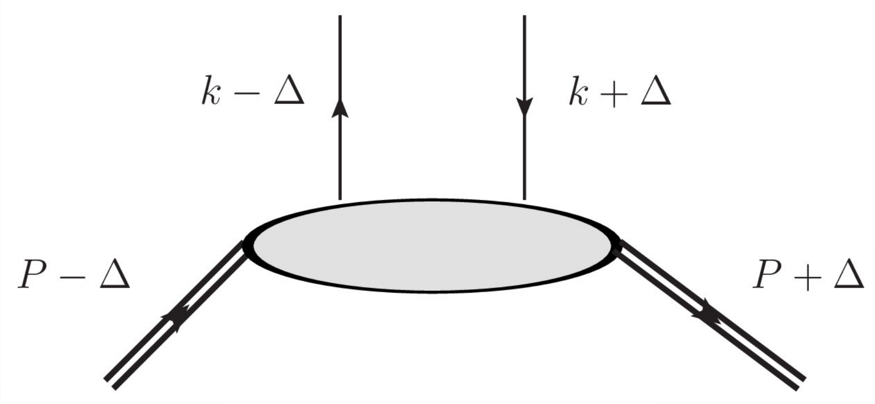
- GPD Review
- GUMP Program
  - Conformal moment parameterization
  - Global analysis
  - Prior results
- Meson Production
  - Light vector mesons – gluons in collinear factorization
  - Heavy vector mesons – better gluon constraints
- Preliminary ρ^0 Production Fit
- Moving Forward
- Conclusions

GPDs

- GPDs generalize the well known PDFs to encode full 3 dimensional information on the quarks and gluons within hadrons

$$f(x) \to F(x, \xi, t)$$

$$x \sim \text{parton momentum fraction}, \ \xi \sim \text{longitudinal momentum transfer},$$
$$t = \Delta^2 \sim \text{momentum transfer squared}$$



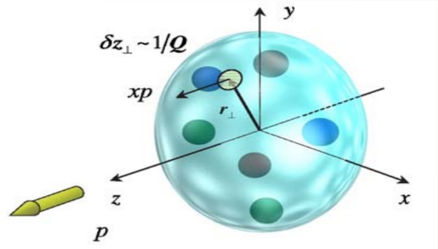
GPDs

- Polarization of the hadron and its parton constituents connects GPDs to the distribution of angular momentum within hadrons (*X. Ji* 1997)
  - Ji sum rule

$$J_i = \frac{1}{2}\int_0^1 \mathrm{d}x\, x\,[H_i(x,\xi) + E_i(x,\xi)]$$

- Related via a Fourier transform to the impact parameter distribution of partons (*M. Burkardt* 2003)

$$\rho(x, r_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot r_\perp} H(x, 0, \Delta_\perp^2)$$



- Related to bulk properties of hadron states encoded in form factors

$$\int \mathrm{d}x\, xH_i(x,\xi,t) = A_i(t) + (2\xi)^2 C_i(t), \quad \int \mathrm{d}x\, xE_i(x,\xi,t) = B_i(t) - (2\xi)^2 C_i(t)$$

GUMP program: Moment Parameterization

- Parameterize GPDs by directly parameterizing their conformal moments

$$F_i(x,\xi,t)=\sum_{j=0}^{\infty}(-1)^j p_{i,j}(x,\xi)\mathcal{F}_{i,j}(\xi,t)$$

(D. Mueller and A. Schafer 2006)

- Expansion based on eigenfunctions of evolution – Gegenbauer polynomials

$$(-1)^j p_j(x,\xi)=\xi^{-j-1}\frac{2^j\Gamma(\frac{5}{2}+j)}{\Gamma(\frac{3}{2})\Gamma(j+3)}\left[1-\left(\frac{x}{\xi}\right)^2\right]C_j^{3/2}\left(\frac{x}{\xi}\right)$$

conformal wave function

$$\int_{-1}^{1}\frac{\mathrm{d}x'}{|\xi|}\mathcal{K}\left(\frac{x}{\xi},\frac{x'}{\xi}\right)C_j^{3/2}\left(\frac{x}{\xi}\right)=\gamma_j C_j^{3/2}\left(\frac{x}{\xi}\right)$$

GPD evolution kernel

GUMP program: Moment Parameterization

- Conformal moment parameterization has nice features for fitting GPDs
- Simple evolution implementation – conformal moments are multiplicatively renormalized at LO
  - Follows from using eigenfunctions of evolution kernel
- Polynomiality condition (*X. Ji* 1998) automatically enforced on conformal moments

$$F_{i,n}(\xi,t)=\int_{-1}^{1}\mathrm{d}x\,x^{n-1}F(x,\xi,t)=\sum_{k=0,\ \mathrm{even}}^{n}\xi^k F_{i,n,k}(t)$$

$$\downarrow$$

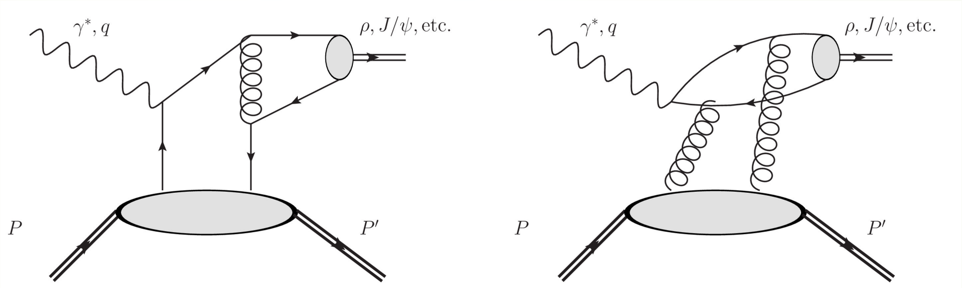
$$\mathcal{F}_{i,j}(\xi,t)=\sum_{k=0,\ \mathrm{even}}^{j+1}\xi^k\mathcal{F}_{i,j,k}(t)$$

GUMP program: Road to full global GPD analysis

- Applied in GUMP program for global analysis of u and d quark GPDs at non zero skewness with LO scale evolution
- Included lattice calculations, global PDF fit input, form factors, and DVCS data
  - JAM (2022) PDF global analysis results
  - Globally extracted form factors (*Z. Ye et al* 2018)
  - Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)
  - DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)
- See Yuxun Guo's talk today at 2:20 pm for full story!

Adding in gluons

- DVCS at LO is only sensitive to gluon GPDs through scale evolution
- Using Deeply Virtual Meson Production (DVMP) gives a direct probe of gluons at LO

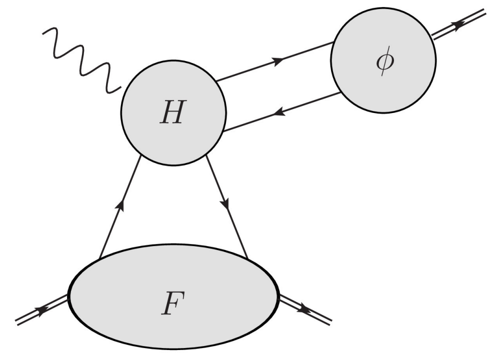


- Light vector mesons have similar sensitivity to quarks and gluons
  - KM framework applied to produce simultaneous fits of DVCS and DVMP for ρ^0 and ϕ meson production with data from HERA (*M. Meskauskas and D. Muller 2011)*
- Add heavy vector meson to obtain better constraints on gluon GPDs – use J/ψ production!

DVMP

- For light mesons the high Q^2 data range from eg. HERA is much larger than the other scales in the process so we can use the collinear factorization theorem (*J. C. Collins, L. Frankfurt, M. Strikman* 1997)

$$\mathcal{M} = \sum_i \underbrace{F_i(x,\xi,t)}_{\text{GPD}} \otimes_x \underbrace{H_i(x,\xi,z)}_{\text{Hard scattering}} \otimes_z \underbrace{\phi^V(z)}_{\text{Distribution amplitude}}$$



- GPDs now come in with transition form factors (TFFs) instead of usual Compton form factors (CFFs)
  - Still have inverse problem from convolution with hard factor
  - Need distribution amplitudes constrained by other input

Deeply Virtual J/ψ Production

- Charm quark contribution for nucleon target is negligible – direct probe of gluons
- Complementary with GUMP work on quark GPDs, but mostly sensitive to small-x_B region whereas JLab data combined with HERA gives better constraint at moderate x_B
- Caveat: mass of the J/ψ is too large for usual collinear factorization

$$M_{J/\psi}^2/Q^2_{\text{max bin}} \approx 9/20 \to \text{corrections of order } 1/2$$

- Need to take heavy mass corrections into account – non-relativistic (NR) QCD!

Non-relativistic model approach

- Encoding the J/ψ formation into NR matrix elements

$$\Gamma[J/\psi \to l^+l^-] \approx \frac{2e_c^2\pi\alpha_{EM}^2}{3m_c^2}\langle\mathcal{O}_1\rangle_{J/\psi}\left(1-\frac{8\alpha_s}{3\pi}\right)^2$$

- Maintain the form of the factorization theorem for the process – still sensitive to leading twist GPDs (*D. Y. Ivanov et al* 2004)

$$\mathcal{M} = \left(\frac{\langle\mathcal{O}_1\rangle_V}{m_c}\right)^{1/2}\sum_i F_i(x,\xi,t)\otimes_x H_i(x,\xi)$$

- Systematically improvable with relativistic, higher twist, and NLO QCD corrections
- Bridge between electroproduction and photoproduction regimes

Preliminary fits for ρ^0 production

- Building off quark GPDs in GUMP, work with two light quark flavors u and d, collinear factorization theorem and LO evolution in Q^2
- Skewness terms are taken as proportional to the forward terms

$$F_{i,j} = F_{i,j,0}(t) + \xi^2 R_{\xi^2} F_{i,j,0}(t) + \xi^4 R_{\xi^4} F_{i,j,0}(t) \ldots$$

- GPDs are given by their moments taken as

$$F_{i,j,0} = N_i \underbrace{B(j+1-\alpha_i, 1+\beta_i)}_{\text{Euler Beta Function}} \underbrace{\frac{j+1-\alpha_i}{j+1-\alpha_i(t)}}_{\text{Regge trajectory}} \beta(t)$$

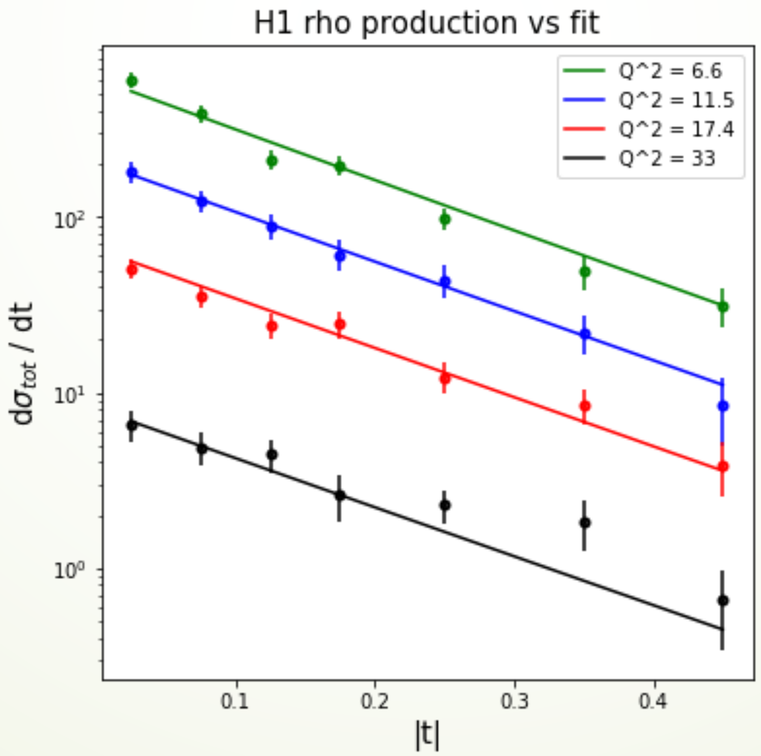
$$\beta(t) = e^{-b|t|}$$

$$\alpha(t) = \alpha + \alpha' t$$

- Start by only considering contribution from GPD H_g which dominates at HERA kinematics

Preliminary fits for ρ^0 production

- Fit to t-dependent cross section data from H1 (2010) with $2.5 \leq Q^2 \leq 60\ GeV^2$, $|t| < 3\ GeV^2$, $10^{-4} < x_B < 10^{-2}$
- Four parameter fit of $\{N_g, \alpha_g, \beta_g, b\}$ yields $\chi^2/dof \approx 0.8$



This fit used only gluon forward parameters, not implemented as part of a global analysis!

Small vs Moderate x_B

- The majority of the input beyond DVMP is more sensitive to moderate to large x_B GPDs
  - Eg. Residual trajectory $\beta(t)$ term doesn't match well onto larger x_B input
- Using a single term in the ansatz for each GPD worked well for constraining the quarks, but most of the data sensitive to gluons is at small x_B
  - May need extra terms in the ansatz

$$F_{i,j,0} = \sum_l N_{i,l} B(j+1-\alpha_{i,l}, 1+\beta_{i,l}) \frac{j+1-\alpha_{i,l}}{j+1-\alpha_{i,l}(t)} \beta_l(t)$$

- Need to be able to combine small x_B into full global analysis to take advantage of future EIC data!

Future Improvements/Additions

- Implement ρ^0 and J/ψ electroproduction fits within global analysis
  - Sequential forward → off-forward data and eventually simultaneous
- Add J/ψ photoproduction – more data sensitive to gluons!
- Add quark flavors – implement ϕ electroproduction
- Implement NLO evolution
  - Calculated for DVCS (*Kumericki et al* 2007) and DVMP for light mesons (*Mueller et al* 2013)
- Implement t-integrated cross sections – very high time cost currently but lots of data available

Conclusions

- Developing the GUMP program to include gluon GPDs in global analysis through DVMP data
- Need NR formalism to make use of J/ψ production
- Likely need more sophisticated ansatz to simultaneously handle small and moderate-large x_B
- Several directions for future improvements available