

DDVCS as a window to the complete mapping of GPDs

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arXiv:2303.13668 [hep-ph]

- Starting point: **GPD**
- Double deeply virtual Compton scattering (**DDVCS**)
 - Goal & motivation
 - Formulation *à la* Kleiss & Stirling
 - Tests of our KS-based formulation
 - Observables and MC simulations
- Summary and conclusions

Partonic distribution

- **GPD** = Generalized Parton Distribution \approx “3D version of a PDF (Parton Distribution Function).” With x the fraction of the hadron’s longitudinal momentum carried by a quark:

$$\text{GPD}_f(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle N' | \bar{q}_f(-z/2) \gamma^+ \mathcal{W}[-z/2, z/2] q_f(z/2) | N \rangle \Big|_{z_\perp = z^+ = 0}$$

$$t = \Delta^2 = (p' - p)^2 \quad \xi = \frac{-\bar{q}\Delta}{2\bar{p}\bar{q}} \quad \bar{q} = \frac{q+q'}{2} \quad \bar{p} = \frac{p+p'}{2}$$

- **Importance:**

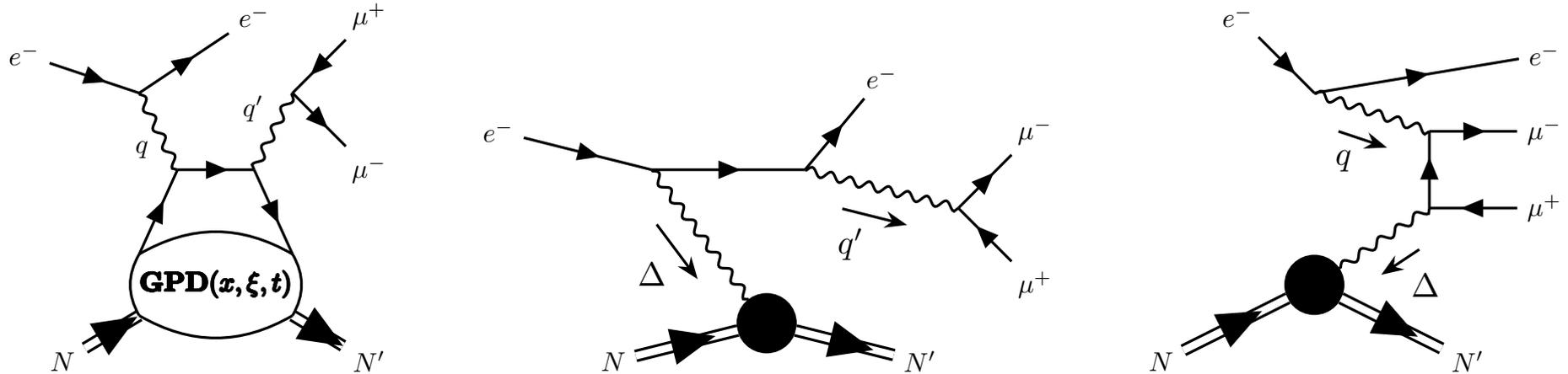
- Connected to QCD energy-momentum tensor, and so to spin. GPDs are a way to address the hadron’s **spin puzzle**
- **Tomography:** distribution of quarks in terms of the longitudinal momentum and in the transverse plane

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \Delta} \underbrace{H^q(x, 0, t = -\Delta^2)}$$

A particular GPD

Our goal

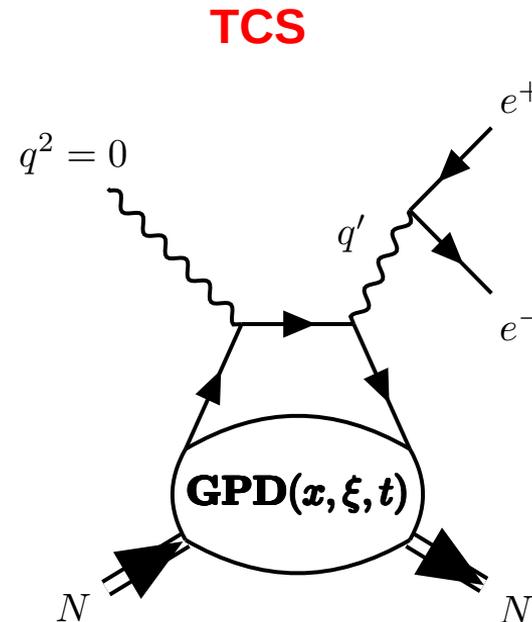
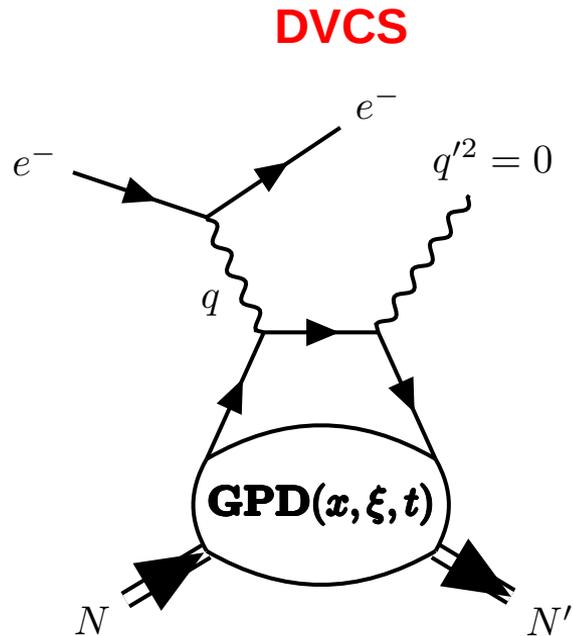
- **Goal:** phenomenology for JLab12, JLab20+ and EIC
- **What is DDVCS?** *Sub-process in the electroproduction of a lepton pair*



DDVCS + BH. Complementary crossed diagrams are not shown

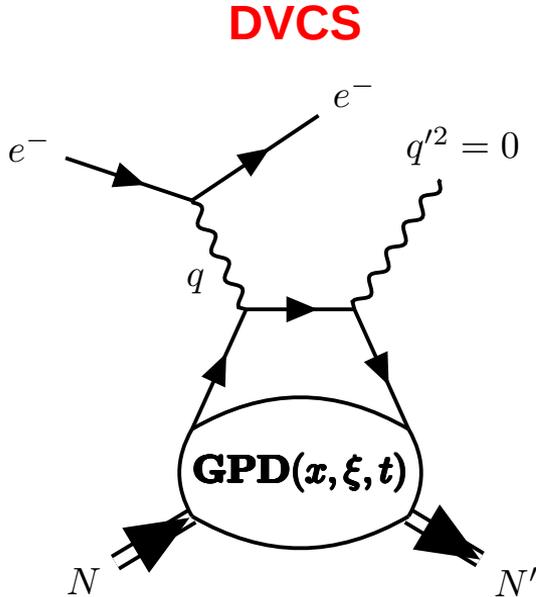
Why DDVCS?

- **Goal:** phenomenology for JLab12, JLab20+ and EIC
- **Problem:** currently, GPDs are accessible experimentally in processes such as deeply virtual (DVCS) and timelike Compton scattering (TCS)



Why DDVCS?

- **Goal:** phenomenology for JLab12, JLab20+ and EIC
- **Problem:** DVCS amplitude allows for measurement of the GPD with restriction to $x = \xi$. Similar situation happens with TCS for $x = -\xi$



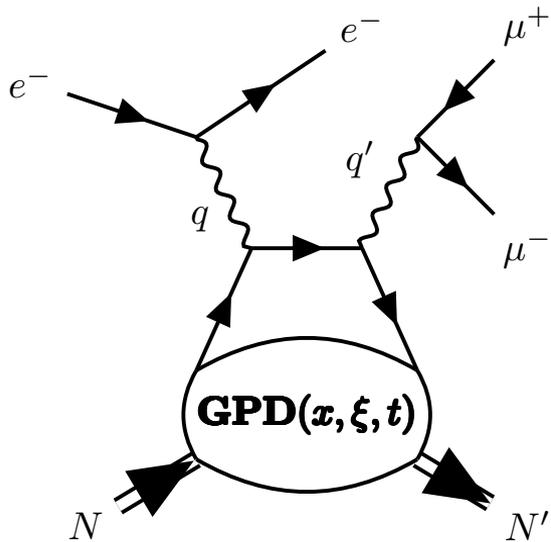
GPDs enter amplitude at LO via CFF:

$$\text{CFF}_{\text{DVCS}} \sim \text{PV} \left(\int_{-1}^1 dx \frac{1}{x-\xi} \text{GPD}(x, \xi, t) \right) - \int_{-1}^1 dx i\pi \delta(x - \xi) \text{GPD}(x, \xi, t) + \dots$$

Why DDVCS?

- **Problem:** DVCS amplitude allows for measurement of the GPD with restriction to $x = \xi$. Similar situation happens with TCS for $x = -\xi$
- **Solution by DDVCS:** the extra virtuality allows for the introduction of a new (generalized) Bjorken variable ρ so that we can access GPDs for $x = \rho \neq \xi$

DDVCS



GPDs enter amplitude at LO via CFF:

$$\text{CFF}_{\text{DDVCS}} \sim \text{PV} \left(\int_{-1}^1 dx \frac{1}{x-\rho} \text{GPD}(x, \xi, t) \right) - \int_{-1}^1 dx i\pi\delta(x-\rho) \text{GPD}(x, \xi, t) + \dots$$

$$\rho = \frac{-\bar{q}^2}{2\bar{p}\bar{q}} \quad \xi = \frac{-\bar{q}\Delta}{2\bar{p}\bar{q}}$$

A red arrow points from the ρ term in the CFF equation above to the ρ in this equation.

Original papers in DDVCS:

Belitsky & Muller, PRL 90, 022001 (2003)

Guidal & Vanderhaeghen, PRL 90, 012001 (2003)

Belitsky & Muller, PRD 68, 116005 (2003)

- In the view of new experiments, revisiting DDVCS is timely: **arXiv:2303.13668 [hep-ph] (our work)**
- **Rederivation of DDVCS' formulae** via Kleiss & Stirling's methods:
 - **Amplitudes as complex-numbers**
 - 2 scalars as building blocks:

$$s(a, b) = \bar{u}(a, +)u(b, -) = -s(b, a)$$

$$t(a, b) = \bar{u}(a, -)u(b, +) = [s(b, a)]^*$$

$$s(a, b) = (a^2 + ia^3) \sqrt{\frac{b^0 - b^1}{a^0 - a^1}} - (a \leftrightarrow b)$$

- **DDVCS sub-process amplitude:**

$$i\mathcal{M}_{\text{DDVCS}} = \frac{-ie^4}{(Q^2 - i0)(Q'^2 + i0)} \left(i\mathcal{M}_{\text{DDVCS}}^{(V)} + i\mathcal{M}_{\text{DDVCS}}^{(A)} \right)$$

- **Vector contribution:**

$$i\mathcal{M}_{\text{DDVCS}}^{(V)} = -\frac{1}{2} \left[f(s_\ell, \ell_-, \ell_+; s, k', k) - g(s_\ell, \ell_-, n^*, \ell_+)g(s, k', n, k) - g(s_\ell, \ell_-, n, \ell_+)g(s, k', n^*, k) \right] \\ \times \left[(\mathcal{H} + \mathcal{E})[Y_{s_2 s_1} g(+, r'_{s_2}, n, r_{s_1}) + Z_{s_2 s_1} g(-, r'_{-s_2}, n, r_{-s_1})] - \frac{\mathcal{E}}{M} \mathcal{J}_{s_2 s_1}^{(2)} \right]$$

- **Axial contribution:**

$$i\mathcal{M}_{\text{DDVCS}}^{(A)} = \frac{-i}{2} \epsilon_{\perp}^{\mu\nu} j_{\mu}(s_\ell, \ell_-, \ell_+) j_{\nu}(s, k', k) \left[\tilde{\mathcal{H}} \mathcal{J}_{s_2 s_1}^{(1,5)+} + \tilde{\mathcal{E}} \frac{\Delta^+}{2M} \mathcal{J}_{s_2 s_1}^{(2,5)+} \right]$$

- **f = contraction of 2 currents:**

$$f(\lambda, k_0, k_1; \lambda', k_2, k_3) = \bar{u}(k_0, \lambda) \gamma^\mu u(k_1, \lambda) \bar{u}(k_2, \lambda') \gamma_\mu u(k_3, \lambda')$$

$$= 2 [s(k_2, k_1) t(k_0, k_3) \delta_{\lambda-} \delta_{\lambda'+} + t(k_2, k_1) s(k_0, k_3) \delta_{\lambda+} \delta_{\lambda'-} + s(k_2, k_0) t(k_1, k_3) \delta_{\lambda+} \delta_{\lambda'+} + t(k_2, k_0) s(k_1, k_3) \delta_{\lambda-} \delta_{\lambda'-}]$$

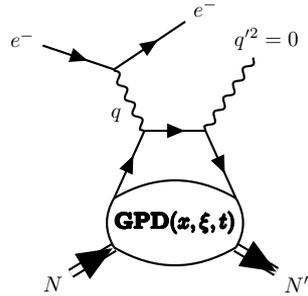
- **g = contraction of a current with a light-like vector:**

$$g(s, \ell, a, k) = \bar{u}(\ell, s) \not{\ell} u(k, s) = \delta_{s+} s(\ell, a) t(a, k) + \delta_{s-} t(\ell, a) s(a, k)$$

- DDVCS to DVCS**

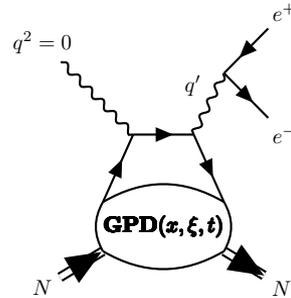
$$\int d\Omega_\ell \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q'^2 \rightarrow 0} \left(\underbrace{\frac{d^4\sigma}{dx_B dQ^2 d|t| d\phi}}_{\text{DVCS}} \right) \frac{\mathcal{N}}{Q'^2}$$

$$\mathcal{N} = \alpha_{\text{em}} / (3\pi)$$



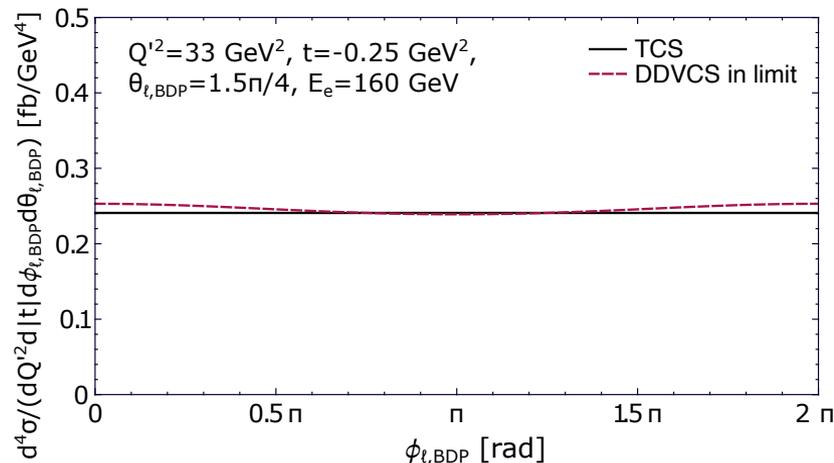
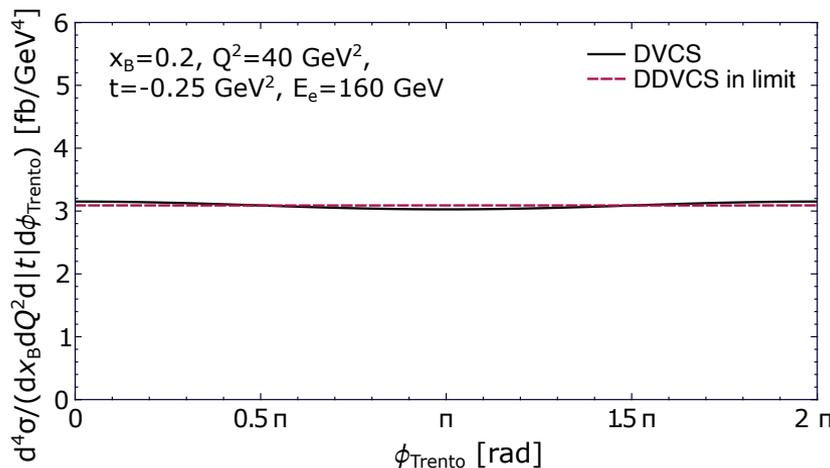
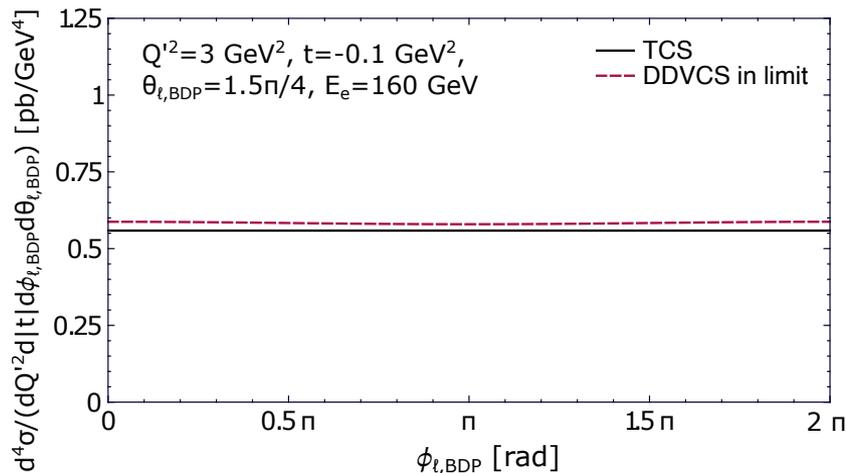
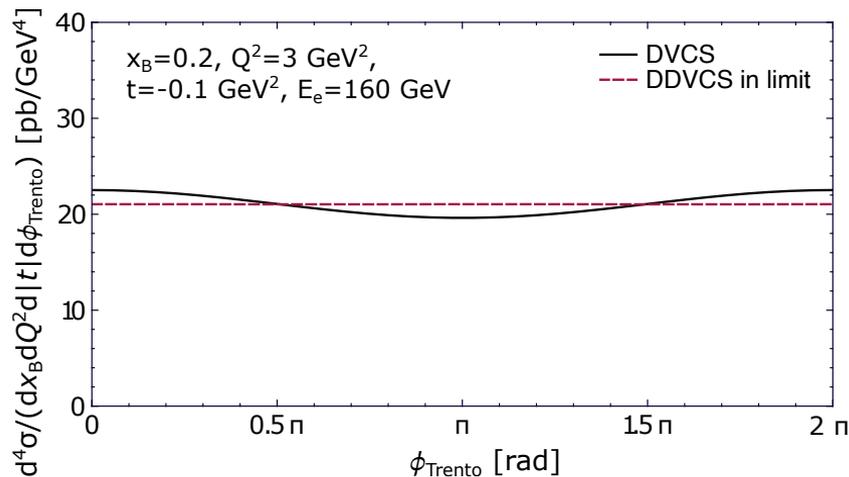
- DDVCS to TCS**

$$\int d\phi \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q^2 \rightarrow 0} \left(\underbrace{\frac{d^4\sigma}{dQ'^2 d|t| d\Omega_\ell}}_{\text{TCS}} \right) \frac{d^2\Gamma}{dx_B dQ^2}$$



$$\frac{d^2\Gamma}{dx_B dQ^2} = \frac{\alpha_{\text{em}}}{2\pi Q^2} \left(1 + \frac{(1-y)^2}{y} - \frac{2(1-y)Q_{\text{min}}^2}{yQ^2} \right) \frac{\nu}{Ex_B} \longleftarrow \text{EPA photon flux}$$

DVCS & TCS limits of DDVCS



GK model

Trento: PRD 70,
117504 (2004)

BPD: EPJC23, 675
(2002)

$$\sigma_{UU}(\phi_{\ell,\text{BDP}}) = \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} d\theta_{\ell,\text{BDP}} \sin \theta_{\ell,\text{BDP}} \left(\frac{d^7 \sigma^{\rightarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\text{BDP}}} + \frac{d^7 \sigma^{\leftarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\text{BDP}}} \right)$$

- Cosine component:

$$\sigma_{UU}^{\cos(n\phi_{\ell,\text{BDP}})}(\phi_{\ell,\text{BDP}}) = M_{UU}^{\cos(n\phi_{\ell,\text{BDP}})} \cos(n\phi_{\ell,\text{BDP}})$$

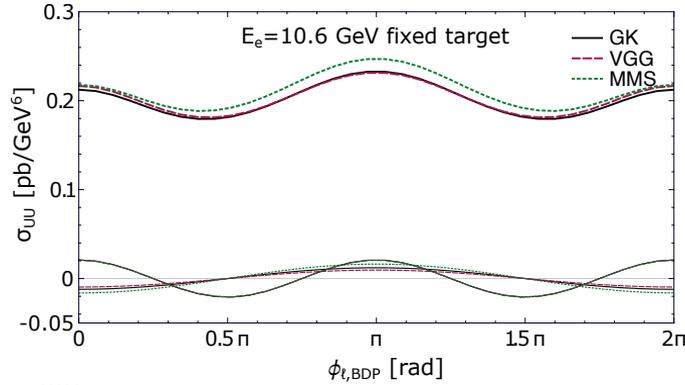
- Cosine moment:

$$M_{UU}^{\cos(n\phi_{\ell,\text{BDP}})} = \frac{1}{N} \int_0^{2\pi} d\phi_{\ell,\text{BDP}} \cos(n\phi_{\ell,\text{BDP}}) \sigma_{UU}(\phi_{\ell,\text{BDP}})$$

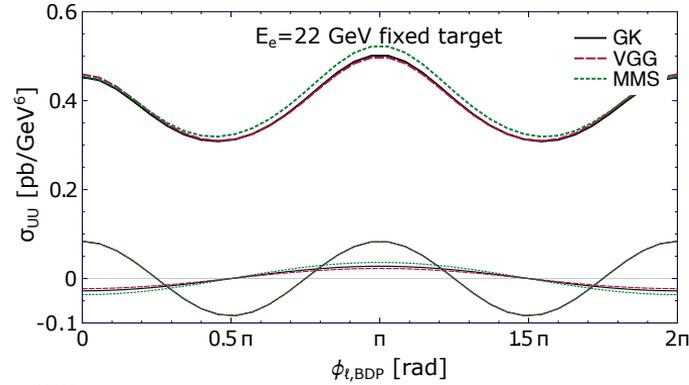
 $N = 2\pi$ for $n = 0$, $N = \pi$ for $n > 0$

Observables: cross-section

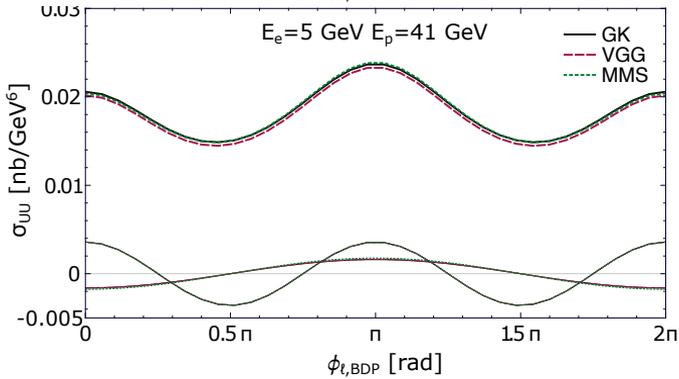
JLab12



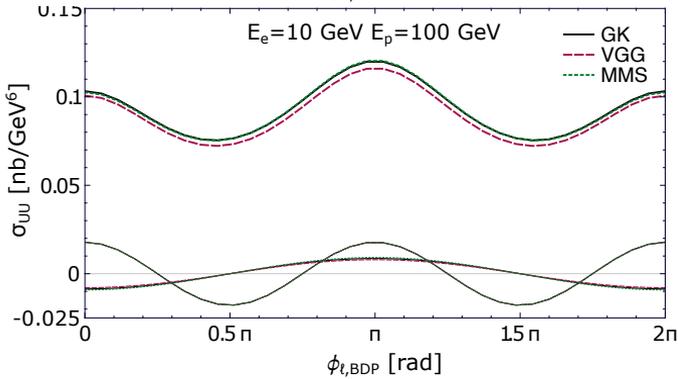
JLab20+



EIC 5x41



EIC 10x100



Experiment	Beam energies [GeV]	y	$ t $ [GeV ²]	Q^2 [GeV ²]	Q'^2 [GeV ²]
JLab12	$E_e = 10.6, E_p = M$	0.5	0.2	0.6	2.5
JLab20+	$E_e = 22, E_p = M$	0.3	0.2	0.6	2.5
EIC	$E_e = 5, E_p = 41$	0.15	0.1	0.6	2.5
EIC	$E_e = 10, E_p = 100$	0.15	0.1	0.6	2.5



$$\Delta\sigma_{LU}(\phi_{\ell,\text{BDP}}) = \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} d\theta_{\ell,\text{BDP}} \sin\theta_{\ell,\text{BDP}} \left(\frac{d^7\sigma^{\rightarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\text{BDP}}} - \frac{d^7\sigma^{\leftarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\text{BDP}}} \right)$$

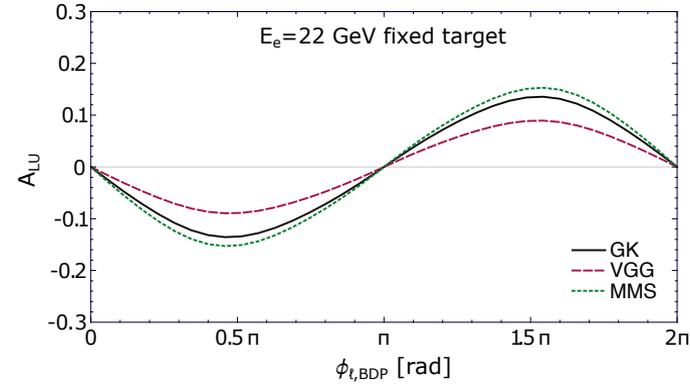
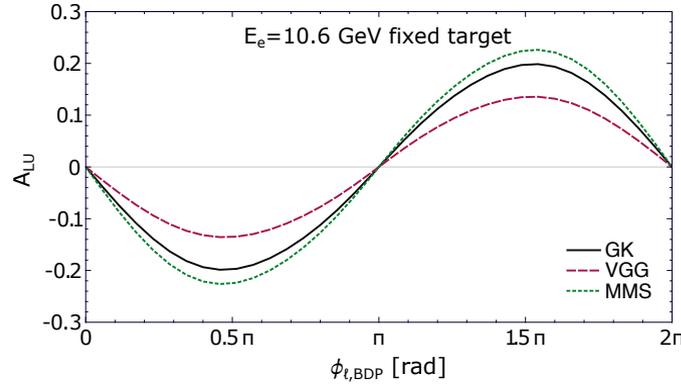
- Single beam-spin asymmetry for longitudinally polarized electrons:

$$A_{LU}(\phi_{\ell,\text{BDP}}) = \frac{\Delta\sigma_{LU}(\phi_{\ell,\text{BDP}})}{\sigma_{UU}(\phi_{\ell,\text{BDP}})}$$

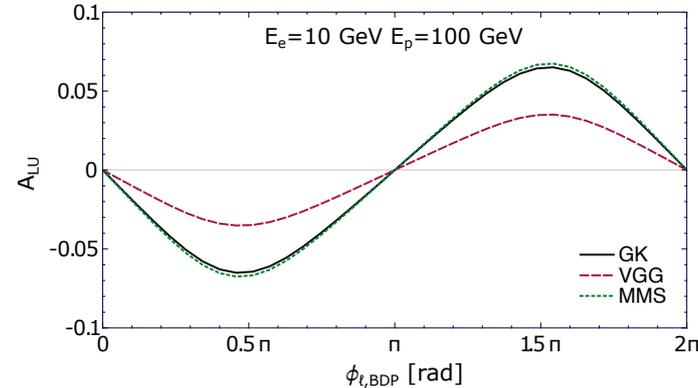
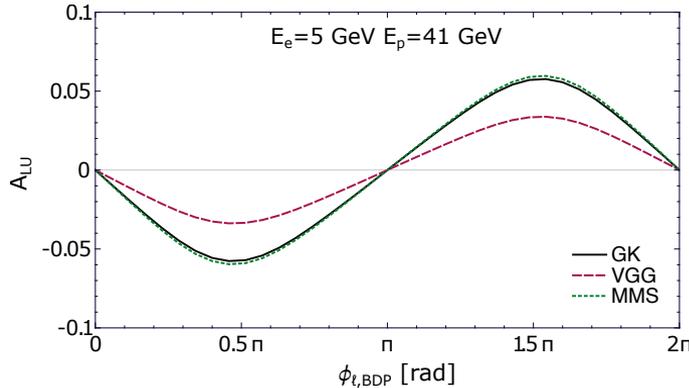
We consider $Q^2 < Q'^2$: our DDVCS is “more” timelike than spacelike

Observables: beam-spin asymmetry

Up to
15-20%
(JLab)

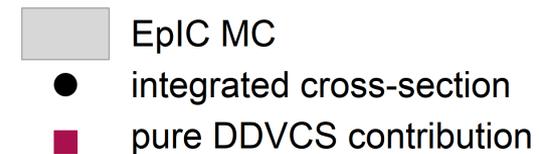
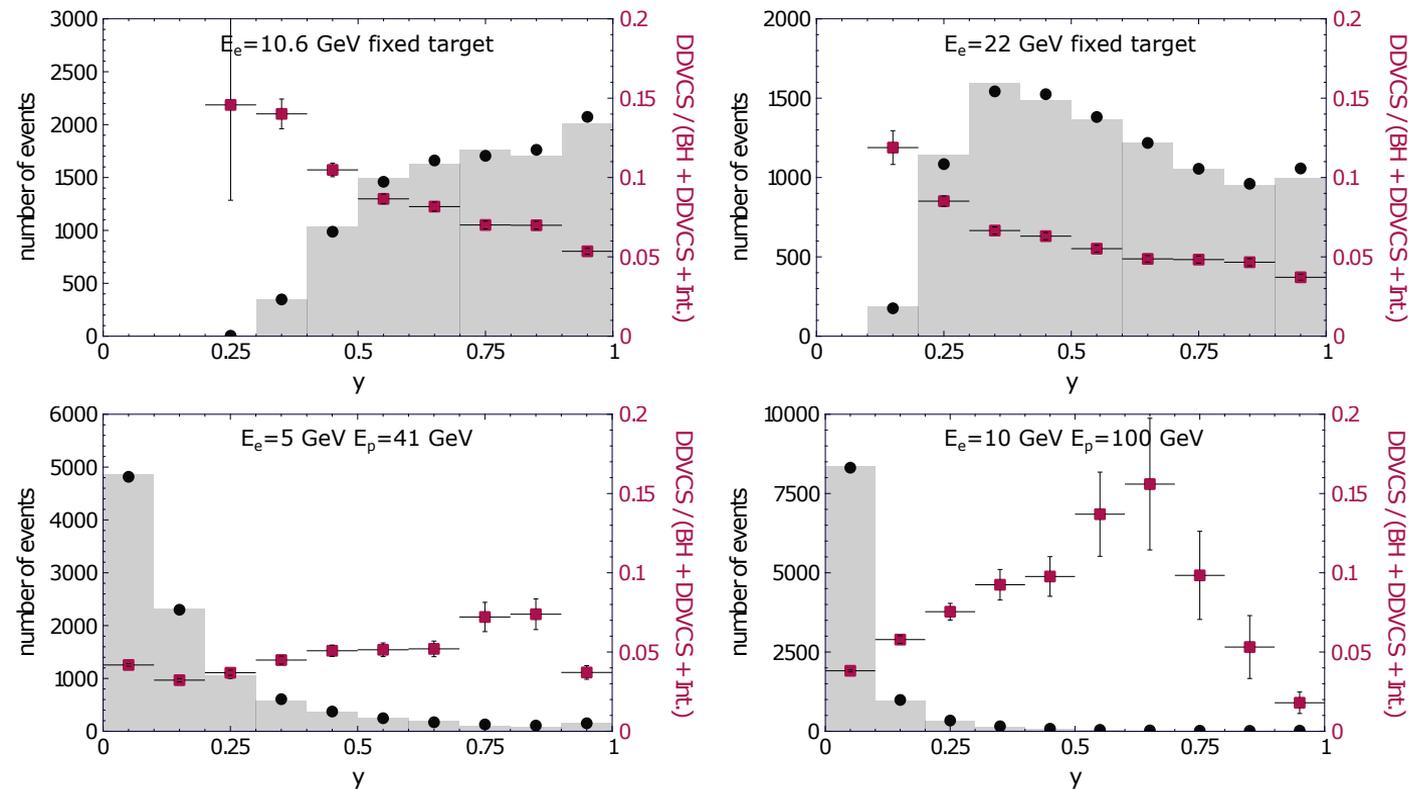


3-7%
(EIC)



Experiment	Beam energies [GeV]	y	$ t $ [GeV ²]	Q^2 [GeV ²]	Q'^2 [GeV ²]
JLab12	$E_e = 10.6, E_p = M$	0.5	0.2	0.6	2.5
JLab20+	$E_e = 22, E_p = M$	0.3	0.2	0.6	2.5
EIC	$E_e = 5, E_p = 41$	0.15	0.1	0.6	2.5
EIC	$E_e = 10, E_p = 100$	0.15	0.1	0.6	2.5

Monte Carlo study



Kinematic cuts:

- $Q^2 \in (0.15, 5) \text{ GeV}^2$
- $Q'^2 \in (2.25, 9) \text{ GeV}^2$
- JLab : $-t \in (0.1, 0.8) \text{ GeV}^2$
- EIC : $-t \in (0.01, 1) \text{ GeV}^2$
- $\phi, \phi_\ell \in (0.1, 2\pi - 0.1) \text{ rad}$
- $\theta_\ell \in (\pi/4, 3\pi/4) \text{ rad}$
- JLab : $y \in (0.1, 1)$
- EIC : $y \in (0.05, 1)$

Experiment	Beam energies [GeV]	Range of $ t $ [GeV ²]	$\sigma _{0 < y < 1}$ [pb]	$\mathcal{L}^{10k} _{0 < y < 1}$ [fb ⁻¹]	y_{\min}	$\sigma _{y_{\min} < y < 1} / \sigma _{0 < y < 1}$
JLab12	$E_e = 10.6, E_p = M$	(0.1, 0.8)	0.14	70	0.1	1
JLab20+	$E_e = 22, E_p = M$	(0.1, 0.8)	0.46	22	0.1	1
EIC	$E_e = 5, E_p = 41$	(0.05, 1)	3.9	2.6	0.05	0.73
EIC	$E_e = 10, E_p = 100$	(0.05, 1)	4.7	2.1	0.05	0.32

Neither acceptance nor detectors response are taken into account in this study



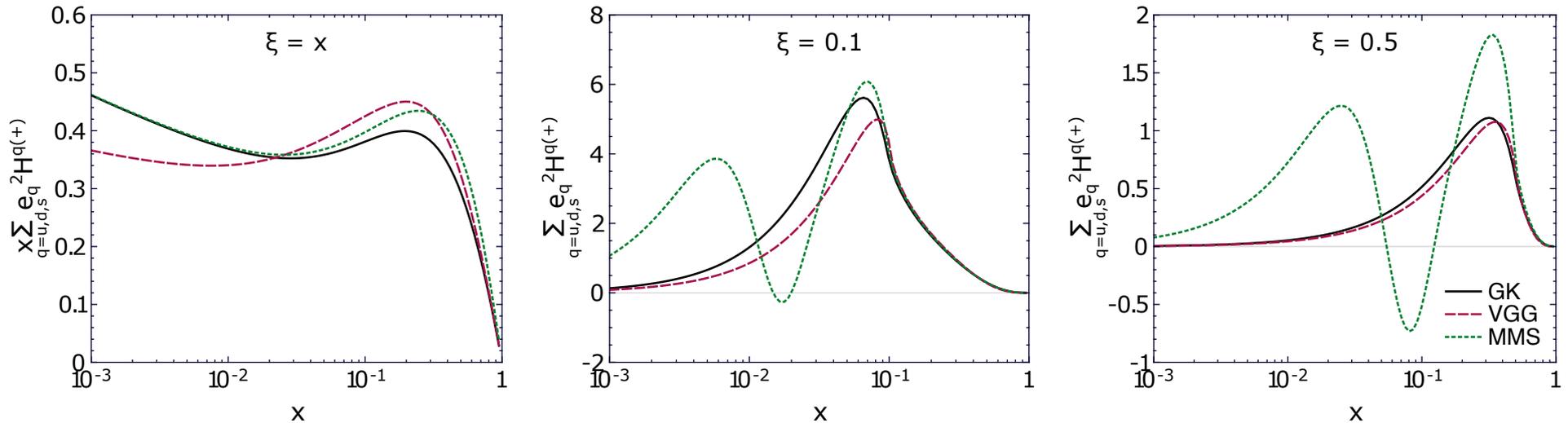
Summary and conclusions

- New analytical formulae for the electroproduction of a lepton pair have been derived.
- It is already implemented in  and  (LO + LT).
- Asymmetries are large enough for DDVCS to be measurable at both current, JLab12, and future, JLab20+ and EIC, experiments.
- Addressing GPD model dependence with cross-sections and asymmetries is possible.

Thank you!

Complementary slides

Models for the C-even part of GPD H



$\sum_q e_q^2 H^{q(+)}(x, \xi, t = -0.1 \text{ GeV}^2)$, where $q = u, d, s$ flavours. $H^{q(+)}(x, \xi, t) = H^q(x, \xi, t) - H^q(-x, \xi, t)$. The scale is chosen as $\mu_F^2 = 4 \text{ GeV}^2$

The solid black, dashed red and dotted green curves describe the GK, VGG and MMS GPD models, respectively.