Analysis of the higher twist GTMD F_{31} for proton in the light-front quark-diquark model.

Harleen Dahiya

In Collaboration with

Shubham Sharma

Dr. B. R. Ambedkar National Institute of Technology, Jalandhar, India

dahiyah@nitj.ac.in

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Outline

1. *Internal Structure of the Hadrons*
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   - Generalized Parton Distributions (GPDs)
   - Transverse Momentum-Dependent Parton Distributions (TMDs)
   - Wigner Distributions (WDs)
   - Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)
2. *Light-Front Quark-Diquark Model*
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Internal Structure of the Hadrons

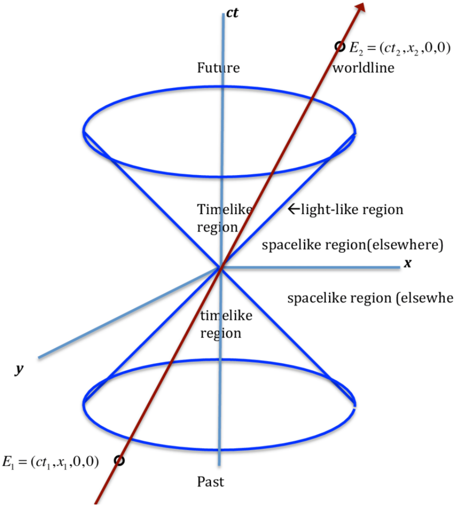
- Quantum Chromodynamics (QCD) provides a fundamental description of hadronic structure and dynamics in terms of their elementary quark and gluon degrees of freedom.
- Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.

Quantum chromodynamics (QCD): Theory of Strong Interactions

- At high energies, (α_s is small), QCD can be used perturbatively.
- At low energies, (α_s becomes large), one has to use other methods such as effective Lagrangian models to describe physics.
- New experimental tools are continually being developed to probe the nonperturbative structure of the theory, for example the hard diffractive reactions, semi-inclusive reactions, deeply virtual Compton scattering etc..
- Many fundamental questions have not been resolved. The most challenging nonperturbative problem in QCD is to determine the structure and spectrum of hadrons in terms of their quark and gluon degrees of freedom.

From Special Theory of Relativity:

- Space and time independently are not invariant quantities.
- Rather space-time is an invariant object.



Instant form v/s Front form

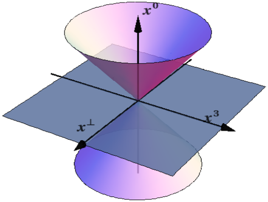


Figure 1: The instant form

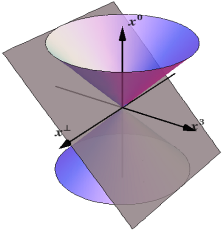


Figure 2: The front form

- All measurements are made at fixed t i.e. at $x^0 = 0$.

- All measurements are made at fixed light-cone time x^+ i.e. at $x^+ = x^0 + x^3 = 0$.

- Energy-momentum dispersion relation:

In the instant form,
$p^0 = \sqrt{\vec{p}^2 + m^2}.$

In the front form,
$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}.$

No square-root for the Hamiltonian in light front form.
Therefore, simplifes the dynamical structure.

- Instant-form vacuum is infinitely complex.
- Light-front vacuum is simple, as all the massive fluctuations in the ground state are absent.

Light-front provides the wavefunctions (LFWFs) required to describe the structure and dynamics of hadrons in terms of their constituents (quarks and gluons).

- S. J. Brodsky, G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).

Why Light Front?

- Ideal Framework to describe theoretically the hadronic structure. It can overcome many obstacles and has many advantages:
  - Simple vacuum structure ∼ vacuum expectation value is zero.
  - A dynamical system is characterized by ten fundamental quantities: energy, momentum, angular momentum and boost.
    ∼ seven out of which are kinematical. It allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents.
  - Dispersion Relation (for ON shell particles)

    $$k^- = \frac{(k\perp)^2+m^2}{k^+}$$

    ∼ no square root factor.

Light-Front Coordinates

- A generic four Vector x^μ in light-cone coordinates is describe as $x^\mu = (x^-, x^+, x_\perp)$.
- $x^+ = x^0 + x^3$ is called as light-front time.
- $x^- = x^0 - x^3$ is called as light-front longitudinal space variable.
- $x^\perp = (x^1, x^2)$ is the transverse variable.
- Similarly, we can define the longitudinal momentum $k^+ = k^0 + k^3$ and light-front energy $k^- = k^0 - k^3$.

Light-Front in QCD

- Light Front QCD (LFQCD) is an *ab initio* approach to study the strongly interacting system. It is similar to perturbative and lattice QCD and is directly connected to the QCD Lagrangian.
- It is a Hamiltonian method, formulated in Minkowinski space rather than Euclidean space.
- The theory is quantized at fixed light-cone time $\tau = t + z/c$ rather than ordinary time t.

PDFs, GPDs and TMDs

Parton Distribution Functions (PDFs)
Generalized Parton Distributions (GPDs)
Transverse Momentum-Dependent Parton Distributions (TMDs)

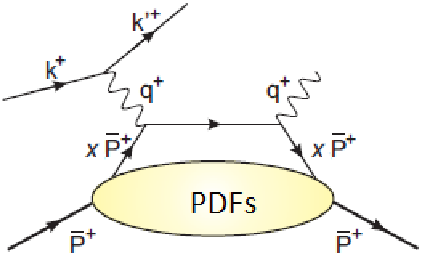
Parton Distribution Functions (PDFs)

To understand the structure of the hadron in terms of quarks and gluons, different categories of parton distributions are present.

- PDFs were introduced by Feynman in 1969.
- PDFs are the basic ingredient to understand the internal hadron structure.
- From parton densities one can extract the distribution of longitudinal momentum carried by the quarks, antiquarks and gluons and their polarizations.
- PDF $f(x)$ imparts information about the probability of finding a parton carrying a longitudinal momentum fraction x inside the hadron.

- Partonic structure is probed in scattering processes such as Deep Inelastic Scattering (DIS).
- The quark-quark correlation to evaluate proton PDFs are defined as

$$F^{[\gamma^+]}(x) = \frac{1}{2}\int \frac{dz^-}{4\pi} e^{ik^+z^-/2} \langle P|\bar{\Psi}(0)\gamma^+\Psi(z^-)|P\rangle\Big|_{z^+=\mathbf{z}_\perp=0}.$$



How partons are distributed in the plane transverse to the direction in which the hadron is moving, or how important their orbital angular momentum is in making up the total spin of a nucleon?

This missing information is compensated in Generalized Parton Distributions (GPDs). The GPDs are physical observables which can provide deep insight about the internal structure of the nucleon and more generally, in non-perturbative QCD.

Generalized Parton Distributions (GPDs) I

- GPDs provide a 3-D picture of the partonic nucleon structure. From 3-D we mean that GPDs encode information on the distribution of partons both in the transverse plane and longitudinal direction.
- Generalized Parton Distributions can be accessed through deep exclusive processes such as DVCS or DVMP. DVCS reaction $\gamma^* + p \rightarrow \gamma + p$ has extraordinary senstivity to fundamental features of the proton's structure.
- GPDs are much richer in content about the hadron structure than ordinary parton distributions.
- GPDs allows us to access partonic configurations with a given longitudinal momentum fraction, but also at specific location (transverse) inside the hadron.
- GPDs depends on three variables x, ζ, t.

Generalized Parton Distributions (GPDs) II

- x is the fraction of momentum transfer.
- ζ gives the longitudinal momentum transfer.
- t is the square of the momentum transfer in the process.

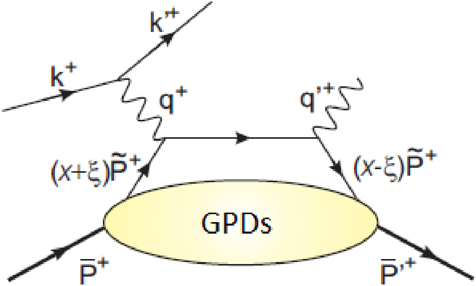
- Several experiments such as H1 collaboration, ZEUS collaboration and fixed target experiments at HERMES have finished taking data on DVCS. In the forward limit of zero momentum transfer, the GPDs reduce to ordinary parton distributions.

- One can define the correlation to evaluate unpolarized GPD in proton $\mathrm{F}^{[\Gamma]}(x, \zeta = 0, t)$ as

$$F^{[\Gamma]}(x, 0, t) = \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ixP^+ z^-/2} \langle P_f | \bar{\Psi}(0) \Gamma \Psi(z) | P_i \rangle \Big|_{z^+ = \mathbf{z}_\perp = 0}.$$

Generalized Parton Distributions (GPDs) III

- The GPDs explain through various exclusive processes such as Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP).



Transverse Momentum-Dependent Parton Distributions (TMDs) I

To get the information of hadron structure in momentum space, transverse momentum-dependent parton distributions (TMDs) were introduced.

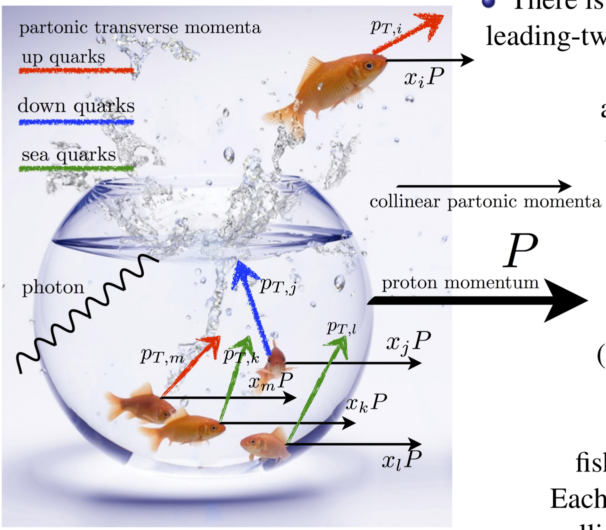
- TMDs describe the probability to find a parton with longitudinal momentum fraction x and transverse momentum with respect to the direction of the parent hadron momentum in a hadron.
- TMDs $f(x, \vec{k}_\perp)$, are function of longitudinal momentum fraction carried by the active quark $x = \frac{k^+}{P^+}$ and the quark transverse momentum $\vec{k}_\perp$.
- TMDs are also of particular importance because they give rise to single spin asymmetries (SSAs).
- TMDs represent three-dimensional hadron picture in *momentum space*.

Transverse Momentum-Dependent Parton Distributions (TMDs) II

- They can be measured in a variety of reactions in lepton-proton and proton-proton collisions as semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan production where a final-state particle is observed with a transverse momentum.

- The quark-quark correlation to evaluate quark TMDs in proton is given by

$$\Phi^{[\Gamma]}(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\mathbf{z}_\perp}{(2\pi)^2} e^{ik.z/2} \langle P|\bar{\Psi}(0)\Gamma\Psi(z)|P\rangle \Big|_{z^+=0}.$$



-Image courtesy: A. Signori

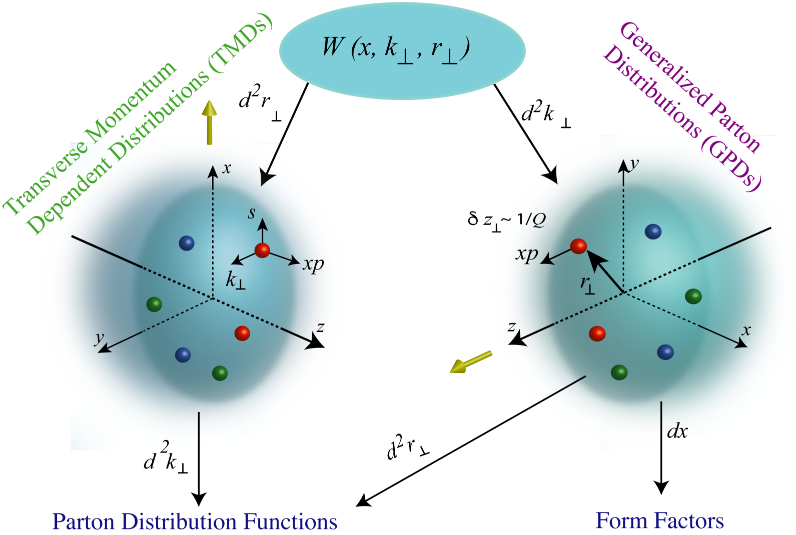
- There is one quark TMD at leading-twist in case of kaon, while 8 quark and gluon TMDs at the leading twist in case of nucleon (spin-1/2).
- In the figure, partons (quarks and gluons) are like fishes confined inside a fishbowl (the proton). Each parton has its own collinear and transverse velocity, indicated by black and colored arrow respectively.

Wigner Distributions (WDs) I

- To understand the hadron structure more precisely, *the joint position and mo* (quantum analog to the classical phase-space distributions) Wigner distributions were introduced.
- Wigner distributions were first introduced by E. Wigner in 1932.

  -E. Wigner Phys. Rev. 70, 749 (1932)
- These distributions are the *quasi-probabilistic distributions*.
- No experiments yet.

Wigner Distributions



- In QCD, Wigner distributions were first introduced by Xiangdong Ji

  -X. -d. Ji, Phys. Rev. Lett. 91, 062001 (2003).

$$\rho^{[\Gamma]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} W^{[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{k}_\perp, x),$$

where $W^{[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{k}_\perp, x)$ in the proton state at fixed light-cone time $z^+ = 0$ is defined as

$$W^{[\Gamma]}(\vec{\Delta}_\perp, \vec{p}_\perp, x) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ip\cdot z} \langle P_f | \bar{\psi}(-z/2) \Gamma \mathcal{W}_{[-\frac{z}{2}, \frac{z}{2}]} \psi(z/2) | P_i \rangle.$$

Here, Γ indicates the Dirac γ-matrix, specifically for twist-2

- γ^+ : *corresponding to unpolarized quark,*
- $\gamma^+\gamma_5$: *corresponding to longitudinally-polarized quark,*
- $i\sigma^{j+}\gamma_5$: *corresponding to transversely-polarized parton*, where $j = 1$ or 2, depending upon the polarization direction of quark.

Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)

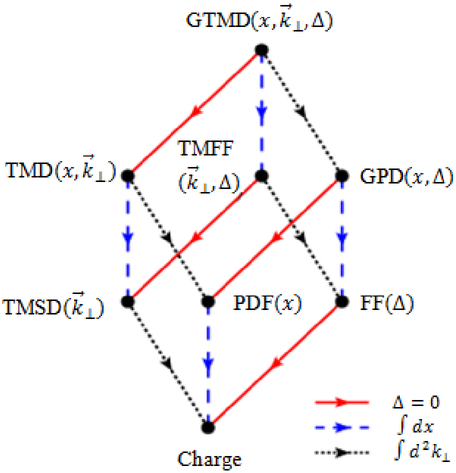
- Leading-twist GTMDs are the function of six variables $(x, \zeta, \mathbf{k}_\perp^2, \mathbf{k}_\perp.\mathbf{\Delta}_\perp, \mathbf{\Delta}_\perp^2)$.
- The GTMDs are accessible through double Drell-Yan processes.

GTMDs

Quark polarization (columns); Nucleon polarization (rows)

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_x \bar{G}_{12} + \Delta_x \bar{G}_{13}\right)$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_y \bar{G}_{12} + \Delta_y \bar{G}_{13}\right)$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

-C. Lorce



- GTMDs are known as *mother distributions*.
- One can obtain GPDs and TMDs from GTMDs under certain kinematic limits.

Outline

Light-Front Quark-Diquark Model I

- In this model the proton is described as an aggregate of an active quark and a diquark spectator of definite mass.
- The proton has spin-flavor $SU(4)$ structure and it has been expressed as a made up of isoscalar-scalar diquark singlet $|u\ S^0\rangle$, isoscalar-vector diquark $|u\ A^0\rangle$ and isovector-vector diquark $|d\ A^1\rangle$ states

$$|P;\pm\rangle = C_S|u\ S^0\rangle^\pm + C_V|u\ A^0\rangle^\pm + C_{VV}|d\ A^1\rangle^\pm.$$

  Here, the scalar and vector diquark has been denoted by S and A respectively. Their isospin has been represented by the superscripts on them.
- The light-cone convention $z^\pm = z^0 \pm z^3$ has been used.
- The frame is picked such that the proton's transverse momentum does not exist i.e., $P \equiv (P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp)$.

Light-Front Quark-Diquark Model II

- The momentum of the smacked quark (p) and diquark (P_X) are

$$p \equiv \left(xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\right),$$
$$P_X \equiv \left((1-x)P^+, P_X^-, -\mathbf{p}_\perp\right).$$

- The Fock-state expansion in the case of two particle for $J^z = \pm 1/2$ for the scalar diquark can be expressed as

$$|u\, S\rangle^\pm = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\left[\psi_+^{\pm(\nu)}(x,\mathbf{p}_\perp)\left| +\frac{1}{2}\ s; xP^+, \mathbf{p}_\perp\right\rangle + \psi_-^{\pm(\nu)}(x,\mathbf{p}_\perp)\left| -\frac{1}{2}\ s; xP^+, \mathbf{p}_\perp\right\rangle\right],$$

where, flavour index is $\nu = u, d$.

Light-Front Quark-Diquark Model III

- $|\lambda_q \ \lambda_S; xP^+, \mathbf{p}_\perp\rangle$ represents the state of two particle having helicity of struck quark as λ_q and helicity of a scalar diquark as λ_S.
- The LFWFs for the scalar diquark are expressed as

$$\begin{aligned}
\psi_{+}^{+(\nu)}(x, \mathbf{p}_\perp) &= N_S \ \varphi_1^{(\nu)}(x, \mathbf{p}_\perp), \\
\psi_{-}^{+(\nu)}(x, \mathbf{p}_\perp) &= N_S \Big(- \frac{p^1 + ip^2}{xM} \Big) \varphi_2^{(\nu)}(x, \mathbf{p}_\perp), \\
\psi_{+}^{-(\nu)}(x, \mathbf{p}_\perp) &= N_S \Big(\frac{p^1 - ip^2}{xM} \Big) \varphi_2^{(\nu)}(x, \mathbf{p}_\perp), \\
\psi_{-}^{-(\nu)}(x, \mathbf{p}_\perp) &= N_S \ \varphi_1^{(\nu)}(x, \mathbf{p}_\perp).
\end{aligned}$$

Here $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ are LFWFs and N_S is the normalization constant.

Light-Front Quark-Diquark Model IV

- Similarly, Fock-state expansion in the case of two particle for the vector diquark is given as

$$
\begin{aligned}
|\nu\, A\rangle^{\pm} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \Bigg[& \psi^{\pm(\nu)}_{++}(x,\mathbf{p}_\perp) \left| +\frac{1}{2}\ \ +1; xP^+, \mathbf{p}_\perp \right\rangle \\
& + \psi^{\pm(\nu)}_{-+}(x,\mathbf{p}_\perp) \left| -\frac{1}{2}\ \ +1; xP^+, \mathbf{p}_\perp \right\rangle + \psi^{\pm(\nu)}_{+0}(x,\mathbf{p}_\perp) \left| +\frac{1}{2}\ 0; xP^+, \mathbf{p}_\perp \right\rangle \\
& + \psi^{\pm(\nu)}_{-0}(x,\mathbf{p}_\perp) \left| -\frac{1}{2}\ 0; xP^+, \mathbf{p}_\perp \right\rangle + \psi^{\pm(\nu)}_{+-}(x,\mathbf{p}_\perp) \left| +\frac{1}{2}\ \ -1; xP^+, \mathbf{p}_\perp \right\rangle \\
& + \psi^{\pm(\nu)}_{--}(x,\mathbf{p}_\perp) \left| -\frac{1}{2}\ \ -1; xP^+, \mathbf{p}_\perp \right\rangle \Bigg].
\end{aligned}
$$

Here $|\lambda_q\ \lambda_D; xP^+, \mathbf{p}_\perp\rangle$ is the state of two-particle with helicity of quark being $\lambda_q = \pm\frac{1}{2}$ and helicity of vector diquark being $\lambda_D = \pm 1, 0$ (triplet).

Light-Front Quark-Diquark Model V

- The LFWFs for the vector diquark for the case when $J^z = +1/2$ are given as

$$\psi_{+\,+}^{+(\nu)}(x,\mathbf{p}_\perp) = N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1 - ip^2}{xM}\right)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),$$
$$\psi_{-\,+}^{+(\nu)}(x,\mathbf{p}_\perp) = N_1^{(\nu)}\sqrt{\frac{2}{3}}\varphi_1^{(\nu)}(x,\mathbf{p}_\perp),$$
$$\psi_{+\,0}^{+(\nu)}(x,\mathbf{p}_\perp) = -N_0^{(\nu)}\sqrt{\frac{1}{3}}\varphi_1^{(\nu)}(x,\mathbf{p}_\perp),$$
$$\psi_{-\,0}^{+(\nu)}(x,\mathbf{p}_\perp) = N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1 + ip^2}{xM}\right)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),$$
$$\psi_{+\,-}^{+(\nu)}(x,\mathbf{p}_\perp) = 0,$$
$$\psi_{-\,-}^{+(\nu)}(x,\mathbf{p}_\perp) = 0,$$

Light-Front Quark-Diquark Model VI

- The LFWFs for the vector diquark for the case when $J^z = -1/2$ are given as

$$\psi^{-(\nu)}_{+\,+}(x, \mathbf{p}_\perp) = 0,$$
$$\psi^{-(\nu)}_{-\,+}(x, \mathbf{p}_\perp) = 0,$$
$$\psi^{-(\nu)}_{+\,0}(x, \mathbf{p}_\perp) = N_0^{(\nu)} \sqrt{\frac{1}{3}}\left(\frac{p^1 - ip^2}{xM}\right)\varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi^{-(\nu)}_{-\,0}(x, \mathbf{p}_\perp) = N_0^{(\nu)} \sqrt{\frac{1}{3}}\varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi^{-(\nu)}_{+\,-}(x, \mathbf{p}_\perp) = -N_1^{(\nu)} \sqrt{\frac{2}{3}}\varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi^{-(\nu)}_{-\,-}(x, \mathbf{p}_\perp) = N_1^{(\nu)} \sqrt{\frac{2}{3}}\left(\frac{p^1 + ip^2}{xM}\right)\varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$

where N_0, N_1 are the normalization constants.

Light-Front Quark-Diquark Model VII

- Generic ansatz of LFWFs $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ is being adopted from the soft-wall AdS/QCD prediction and the parameters a_i^ν, b_i^ν and δ^ν are established as

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} x^{a_i^\nu} (1-x)^{b_i^\nu} \exp\Big[-\delta^\nu \frac{\mathbf{p}_\perp^2}{2\kappa^2} \frac{\log(1/x)}{(1-x)^2} \Big].$$

Outline

Input Parameters I

- The parameters a_i^ν and b_i^ν have been fitted at model scale $\mu_0 = 0.8$ GeV using the Dirac and Pauli data of form factors.

ν	a_1^ν	b_1^ν	a_2^ν	b_2^ν	δ^ν
u	0.280	0.1716	0.84	0.2284	1.0
d	0.5850	0.7000	0.9434	0.64	1.0

Table 1: Values of model parameters corresponding to up and down quarks.

ν	N_S	N_0^ν	N_1^ν
u	2.0191	3.2050	0.9895
d	2.0191	5.9423	1.1616

Table 2: Values of normalization constants N_i^2 corresponding to both up and down quarks.

Input Parameters II

- The AdS/QCD scale parameter κ is chosen to be 0.4 GeV.
- Constituent quark mass (m) and the proton mass (M) are taken to be 0.055 GeV and 0.938 GeV sequentially.
- The coefficients C_i of scalar and vector diquarks are given as

$$
\begin{aligned}
C_S^2 &= 1.3872, \\
C_V^2 &= 0.6128, \\
C_{VV}^2 &= 1.
\end{aligned}
$$

Outline

GTMD Correlator I

GTMD Correlator

- The correlator $W^{\nu[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x; S)$ relates the GTMDs and in the Drell-Yan-West frame ($\Delta^+ = 0$) and fixed light-cone time $z^+ = 0$ is given by

$$W^{\nu[\Gamma]} = \frac{1}{2}\int \frac{dz^-}{(2\pi)}\frac{d^2 z_T}{(2\pi)^2} e^{ip.z}\langle P''; S|\bar{\psi}_i^\nu(-z/2)\Gamma\mathcal{W}_{[-z/2,z/2]}\psi_j^\nu(z/2)|P'; S\rangle\Big|_{z^+=0}.$$

- Proton's momentum is denoted by P and its heicity is λ.
- The spin components of proton are written as $S^+ = \lambda\frac{P^+}{M}$, $S^- = \lambda\frac{P^-}{M}$, and S_T.
- Light-cone gauge $A^+ = 0$ is selected and a symmetric frame is chosen where the momentum of the proton is $P \equiv (P^+, \frac{M^2}{P^+}, \mathbf{0})$, the momentum of virtual photon is $q \equiv (x_B P^+, \frac{Q^2}{x_B P^+}, \mathbf{0})$, where $x_B = \frac{Q^2}{2P.q}$ is the Bjorken variable and $Q^2 = -q^2$.
- The momentum of quark is given by $p \equiv (xP^+, \frac{p^2+|\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp)$ and that of diquark $P_X \equiv ((1-x)P^+, P_X^-, -\mathbf{p}_\perp)$. Here $x = p^+/P^+$ is the longitudinal momentum fraction carried by the struck quark.

GTMD Correlator II

- Helicity of struck quark is λ_q.
- The movement of Wilson line $\mathcal{W}_{[0,z]}$ is through the path $[0, 0, 0_\perp] \to [0, 1, 0_\perp] \to [0, 1, z_\perp] \to [0, z^-, z_\perp]$. Value of Wilson line is chosen to be 1.

GTMD Parameterization for proton at twist-4

$$W^{[\gamma^-]}_{\lambda\lambda'} = \frac{M}{2(P^+)^2}\,\bar{u}(p',\lambda')\Big[F_{3,1} + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{3,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{3,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{3,4}\Big]u(p,\lambda)\,,$$

$$W^{[\gamma^-\gamma_5]}_{\lambda\lambda'} = \frac{M}{2(P^+)^2}\,\bar{u}(p',\lambda')\Big[-\frac{i\varepsilon_T^{ij}k_T^i\Delta_T^j}{M^2}\,G_{3,1} + \frac{i\sigma^{i+}\gamma_5 k_T^i}{P^+}\,G_{3,2} + \frac{i\sigma^{i+}\gamma_5\Delta_T^i}{P^+}\,G_{3,3} + i\sigma^{+-}\,G_{3,4}\Big]u(p,\lambda)\,,$$

$$W^{[i\sigma^{j-}\gamma_5]}_{\lambda\lambda'} = \frac{M}{2(P^+)^2}\,\bar{u}(p',\lambda')\Big[-\frac{i\varepsilon_T^{ij}k_T^i}{M}\,H_{3,1} - \frac{i\varepsilon_T^{ij}\Delta_T^i}{M}\,H_{3,2} + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}\,H_{3,3} + \frac{k_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}\,H_{3,4} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}\,H_{3,5} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5\Delta_T^k}{M\,P^+}\,H_{3,6} + \frac{k_T^j\,i\sigma^{+-}\gamma_5}{M}\,H_{3,7} + \frac{\Delta_T^j\,i\sigma^{+-}\gamma_5}{M}\,H_{3,8}\Big]u(p,\lambda)\,.$$

Outline

Results

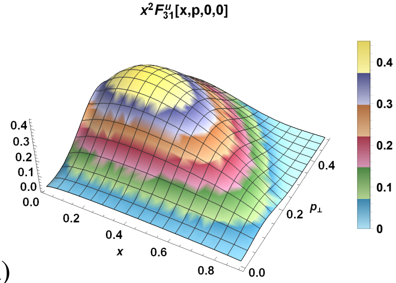
- For proton, the twist-4 GTMD $F^{\nu}_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for up quark is given as

$$F^u_{31} = \frac{1}{16\pi^3}\frac{1}{4x^2M^2}\Big(C_S^2 N_s^2 + \frac{1}{3}C_V^2\big(|N_0^u|^2 + |N_1^u|^2\big)\Big)$$
$$\Big[(4m^2 + 4p_\perp^2 - \Delta_\perp^2)|\varphi_1^u|^2 + (\frac{4m(1-x)\Delta_\perp^2}{xM})|\varphi_1^u||\varphi_2^u|$$
$$+\Big[(4m^2 + 4p_\perp^2 - \Delta_\perp^2)(p_\perp^2 - \frac{(1-x)^2}{4}\Delta_\perp^2)$$
$$+4(1-x)(p_\perp^2\Delta_\perp^2 - (p_\perp \cdot \Delta_\perp)^2)\Big]\frac{|\varphi_2^u|^2}{x^2M^2}\Big]$$

- For proton, the twist-4 GTMD $F^{\nu}_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for down quark is given as

$$F^d_{31} = \frac{1}{16\pi^3}\frac{1}{4x^2M^2}\left(\frac{1}{3}C^2_{VV}\Big(|N^d_0|^2 + |N^d_1|^2\Big)\right)$$
$$\Bigg[(4m^2 + 4p^2_\perp - \Delta^2_\perp)|\varphi^d_1|^2 + (\frac{4m(1-x)\Delta^2_\perp}{xM})|\varphi^d_1||\varphi^d_2|$$
$$+\Big[(4m^2 + 4p^2_\perp - \Delta^2_\perp)(p^2_\perp - \frac{(1-x)^2}{4}\Delta^2_\perp)$$
$$+4(1-x)(p^2_\perp\Delta^2_\perp - (p_\perp\cdot\Delta_\perp)^2)\Big]\frac{|\varphi^d_2|^2}{x^2M^2}\Bigg]$$

x and $p_\perp$ Dependence



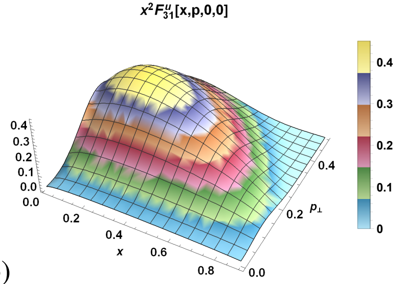
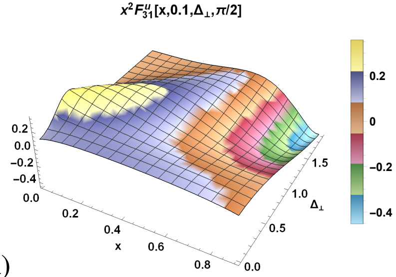
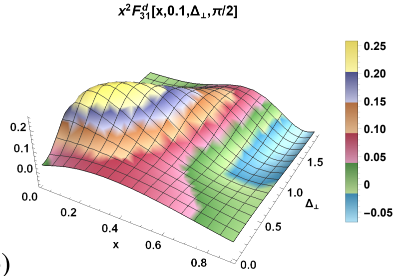


Figure 3: The GTMD $x^2 F_{31}^{\nu}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{p}_\perp$ at $\mathbf{\Delta}_\perp = \mathbf{0}$ (i.e., at TMD limit). The left and right column correspond to u and d quarks sequentially.

x and $\Delta_\perp$ Dependence



(a)



(b)

Figure 4: The GTMD $x^2 F^{\nu}_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{\Delta}_\perp$ at $\mathbf{p}_\perp = 0.1$ and $\theta = \frac{\pi}{2}$. The left and right column correspond to u and d quarks sequentially.

$p_\perp$ and $\Delta_\perp$ Dependence

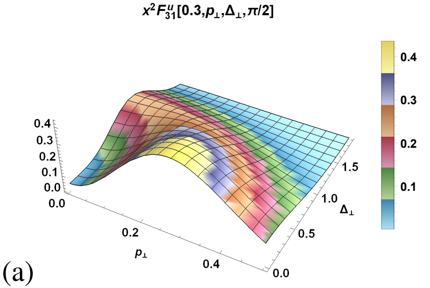


Figure 5: The GTMD $x^2 F_{31}^{\nu}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to $\mathbf{p}_\perp$ and $\mathbf{\Delta}_\perp$ at $x = 0.3$ and $\theta = \frac{\pi}{2}$. The left and right column correspond to u and d quarks sequentially.

Outline

Summary I

- The GTMD $x^2F_{31}^{\nu}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ remains positive for both u and d quarks.
- In plots of GTMD $x^2F_{31}^{\nu}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for both u and d quarks, it has been observed that the value diminishes
  - when $\mathbf{p}_\perp$ greater than 0.6 GeV
  - when $\mathbf{\Delta}_\perp$ greater than 1.9 GeV
- The GTMD F_{31} does not flip its sign on changing the quark flavour from u to d quarks.
- As the value of θ increases the value of GTMD increases.

Thank you!