Prospects of inclusive reactions using ~~quantum computers~~ real time calculations

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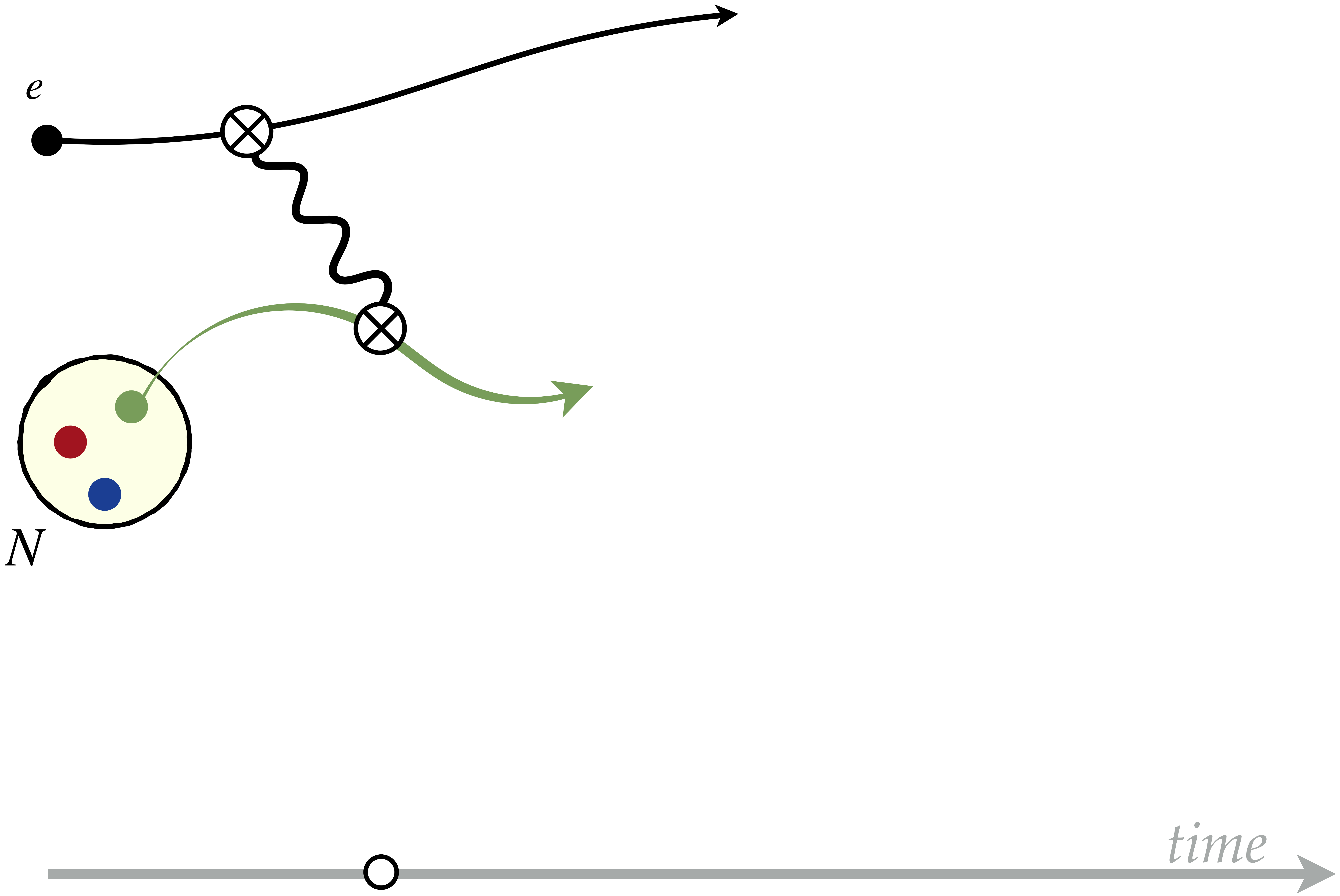
Carrillo, Guerrero, Sturzu, RB (to appear)
RB, Guerrero, Hansen, Sturzu (2020)

Inclusive reactions *just a subclass*

- Virtual Compton scattering: PDFs, GPDs,…



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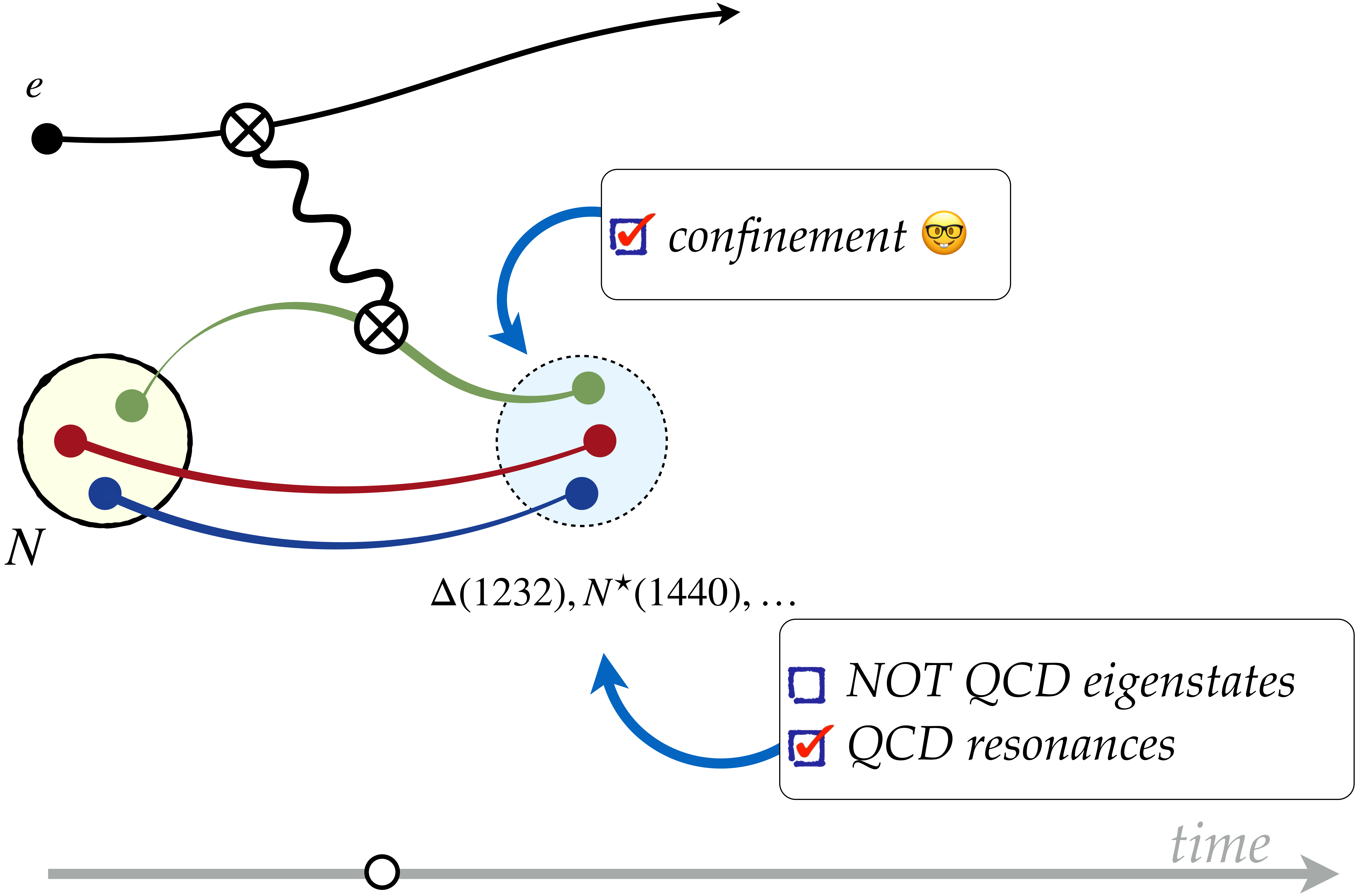

- [x] *confinement* 🤓

- [ ] *NOT QCD eigenstates*
- [x] *QCD resonances*

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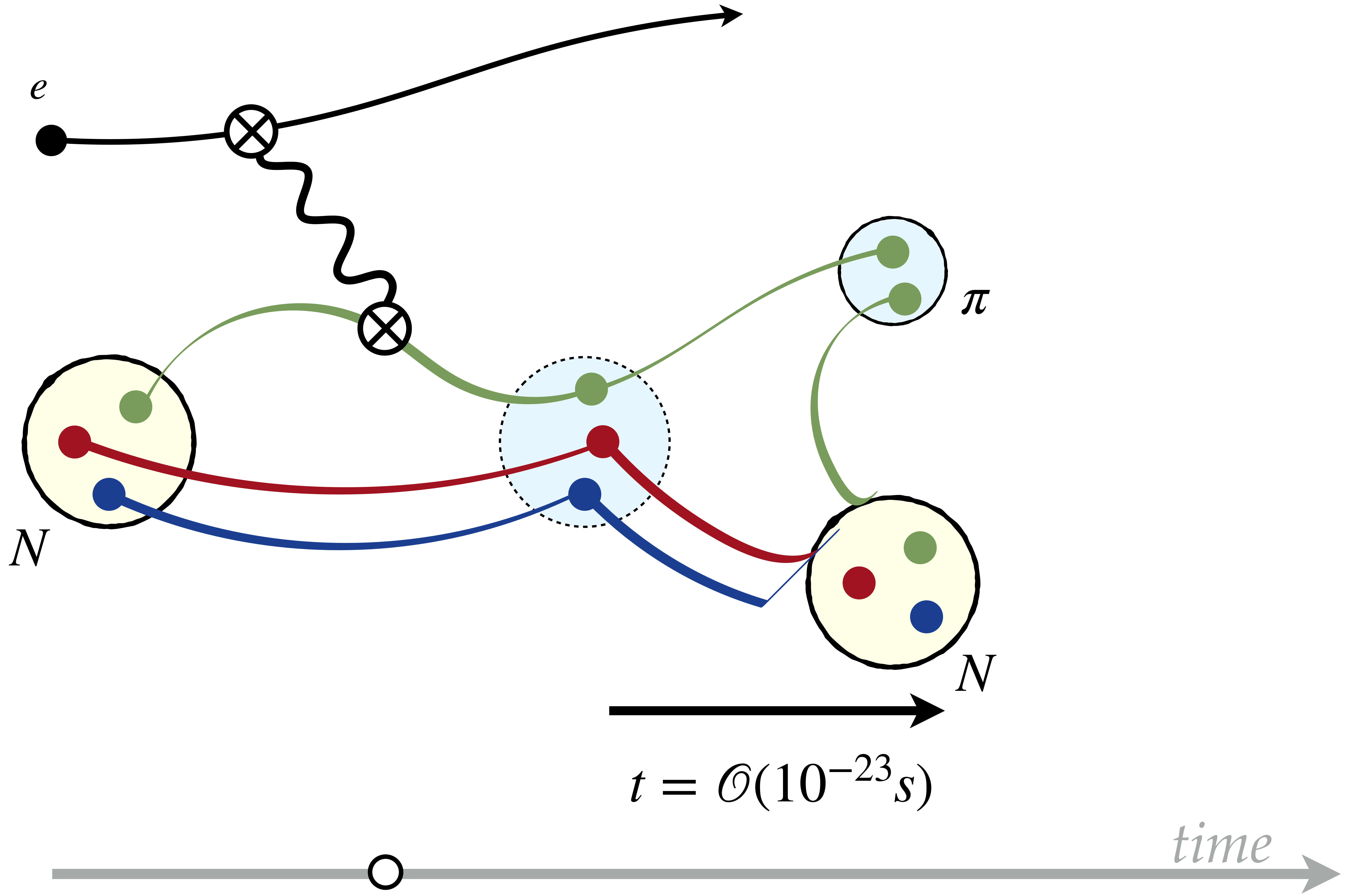

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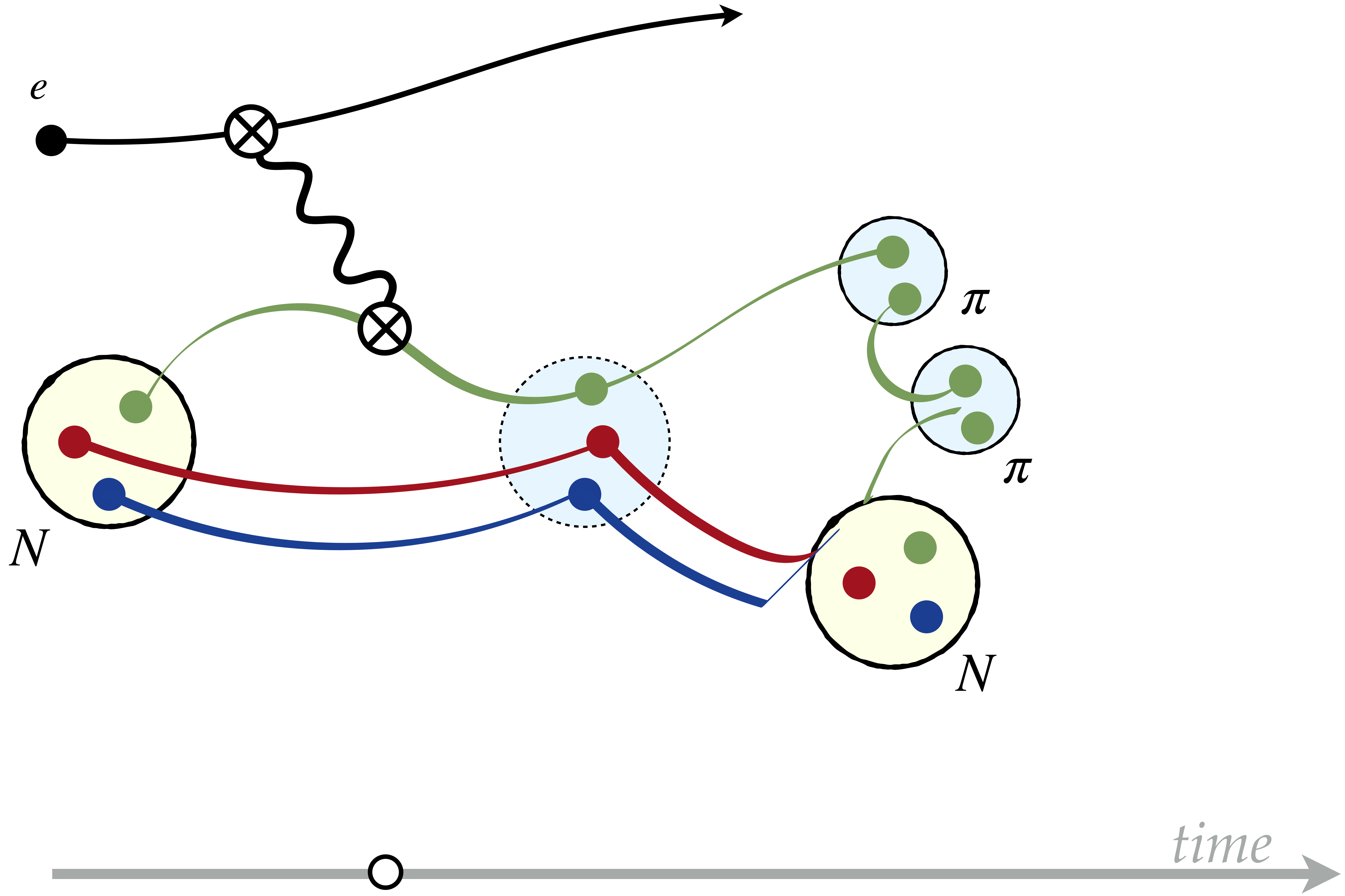

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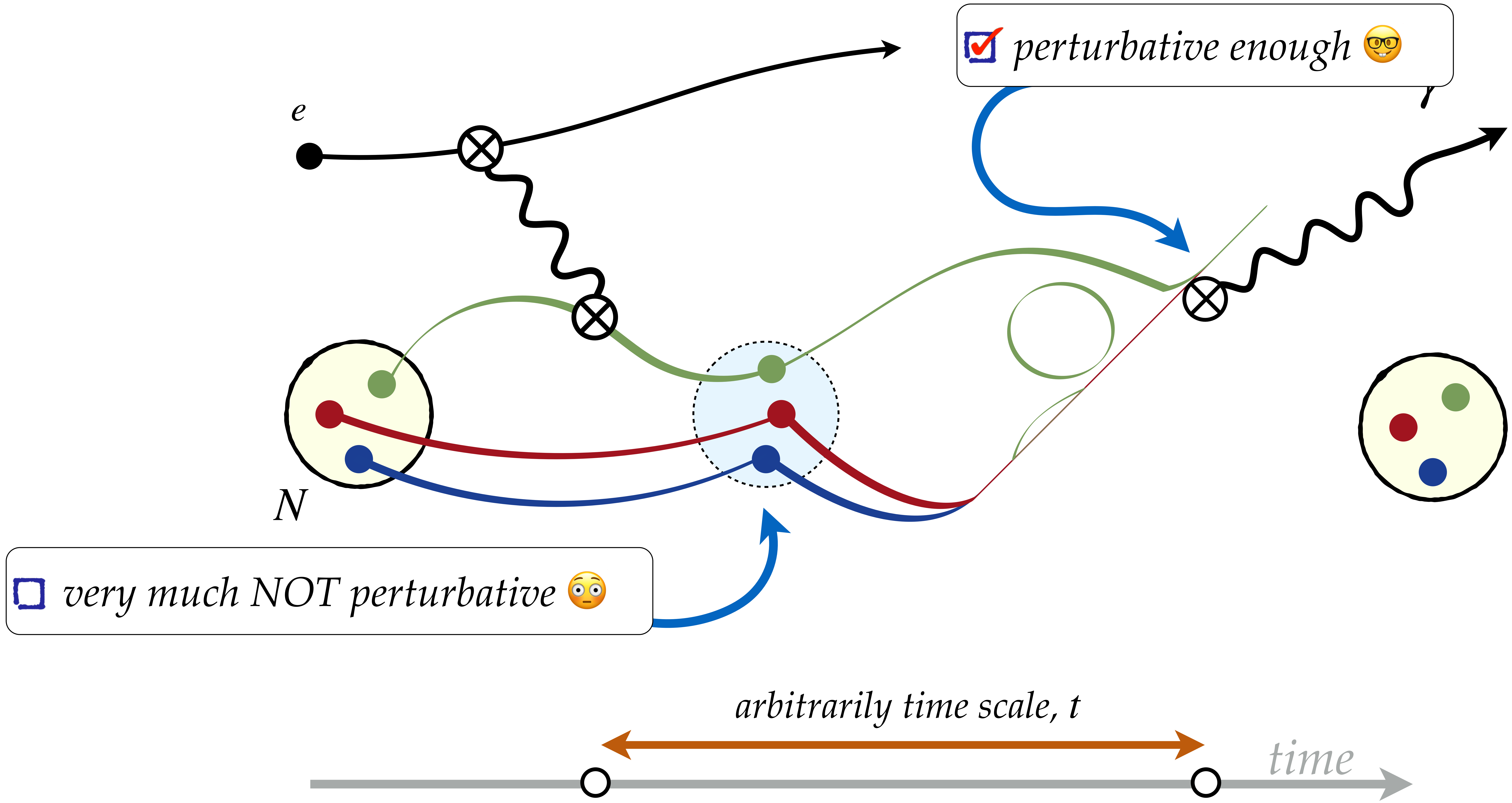


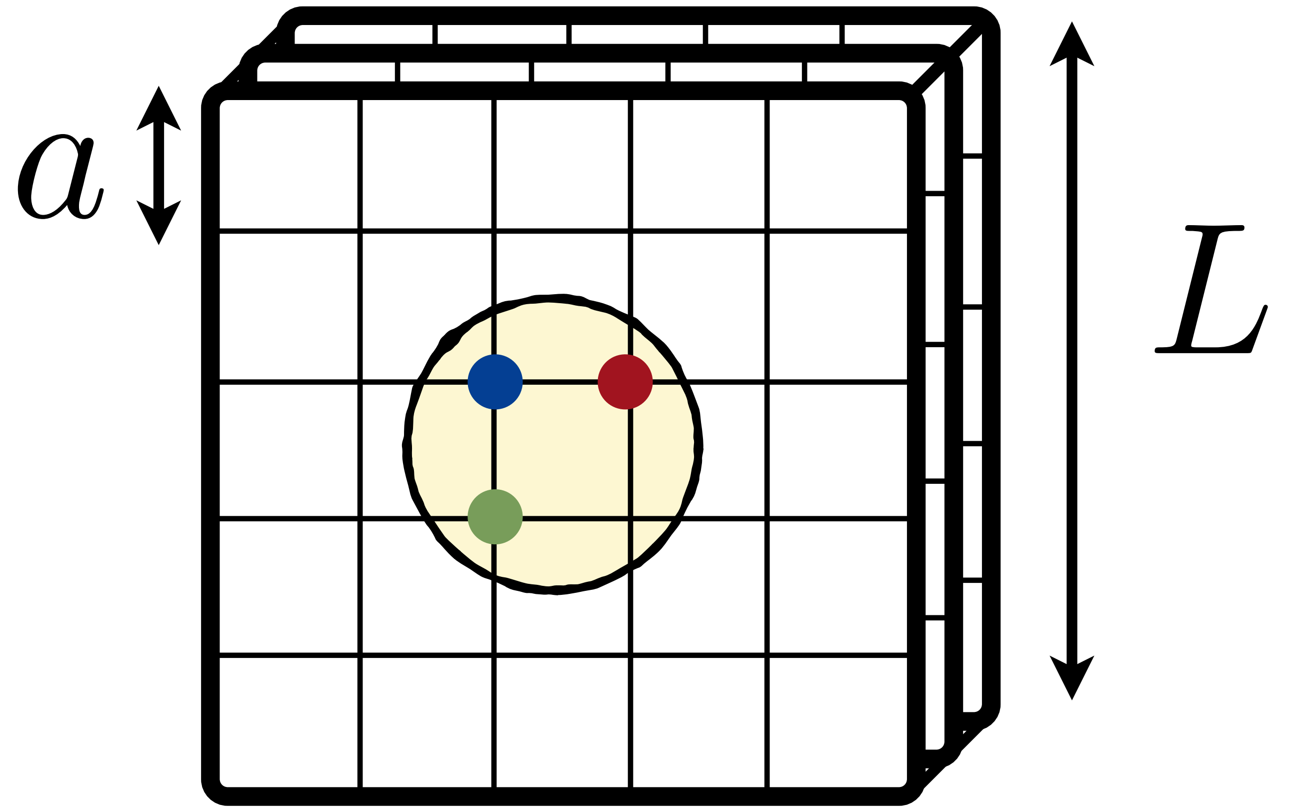
Inclusive reactions *just a subclass*

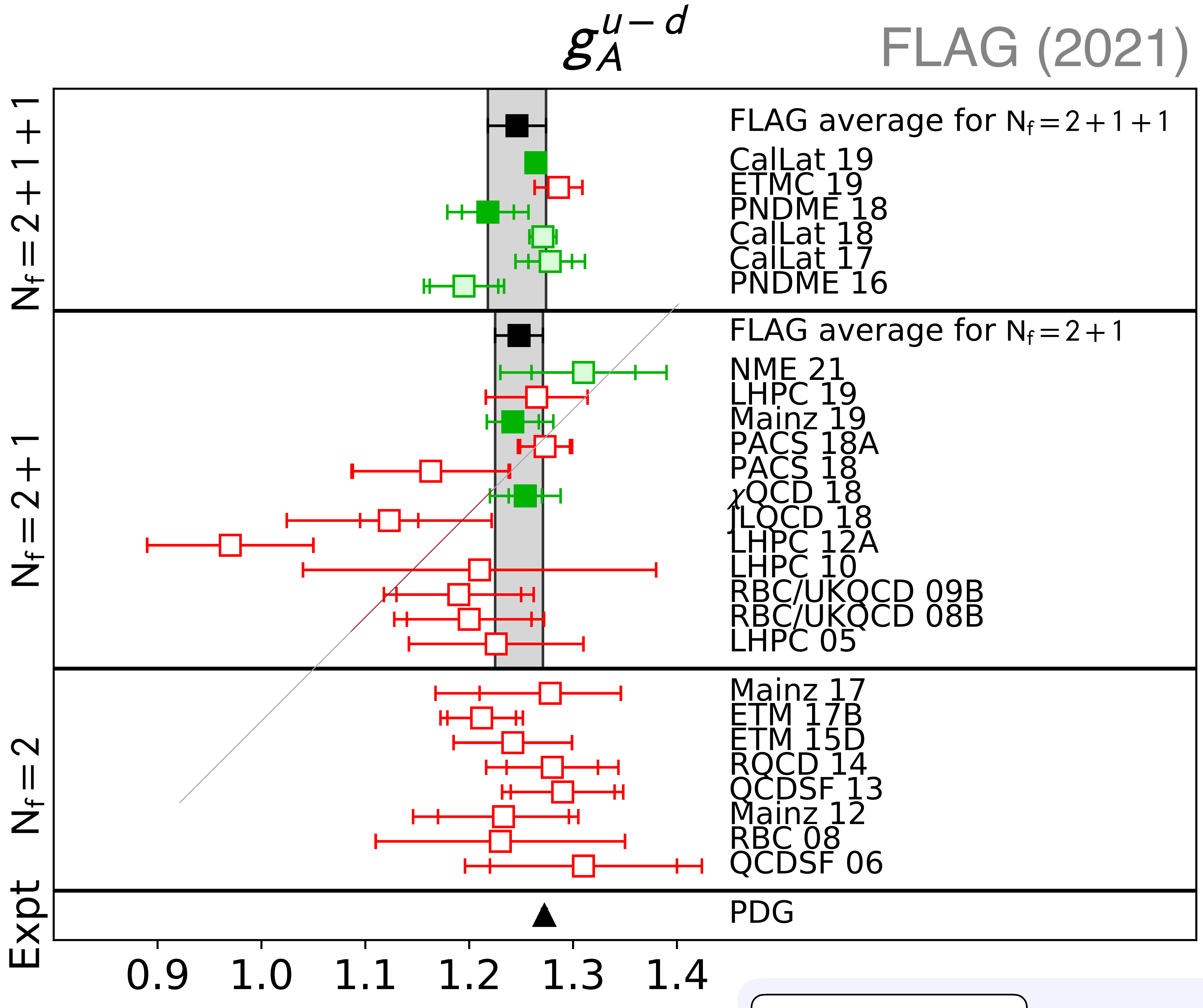
- Virtual Compton scattering: PDFs, GPDs,…
- inclusive neutrino-nucleus scattering,
- double beta decay
- Glueball structure,
- Radiative corrections in weak decays
- …

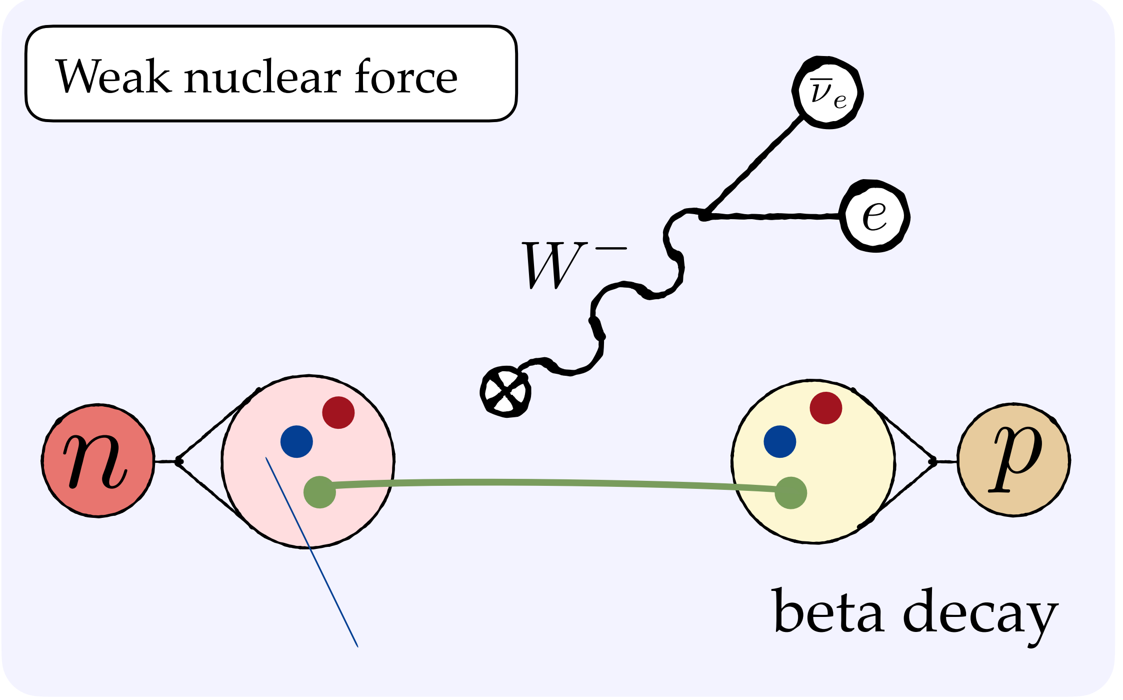
All can be defined as: $\mathscr{T} \sim \int d^4x\, e^{ix\cdot q} \langle n_f |\, T\left[\mathscr{J}_{2,M}(t)\,\mathscr{J}_1(0)\right] | n_i\rangle_\infty$

lattice QCD *only current non-perturbative QCD tool*

- Euclidean spacetime: $t_M \to -it_E$
  - Monte Carlo sampling
- finite volume: $L \sim 1 - 10$ fm
- lattice spacing: $a \sim 0.03 - 0.1$ fm
- quark masses: $m_q \to m_q^{\text{phys}}$







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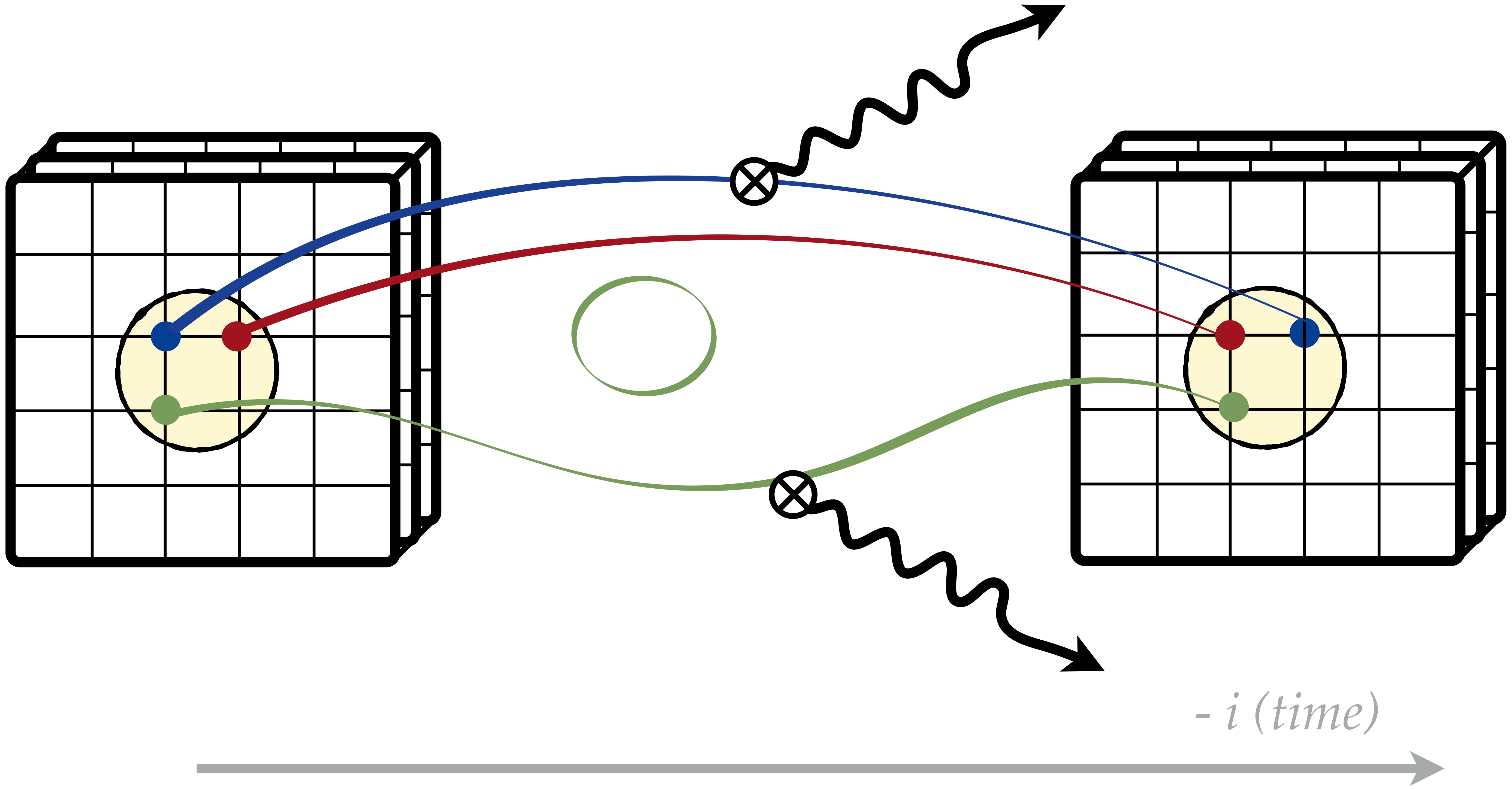
strongly correlated issues:
"time evolution operator $\sim e^{-t\hat{H}_L}$ *depends on both the time-signature and size of the volume"*



~~lattice QCD~~ quantum computers, tensor networks, ...

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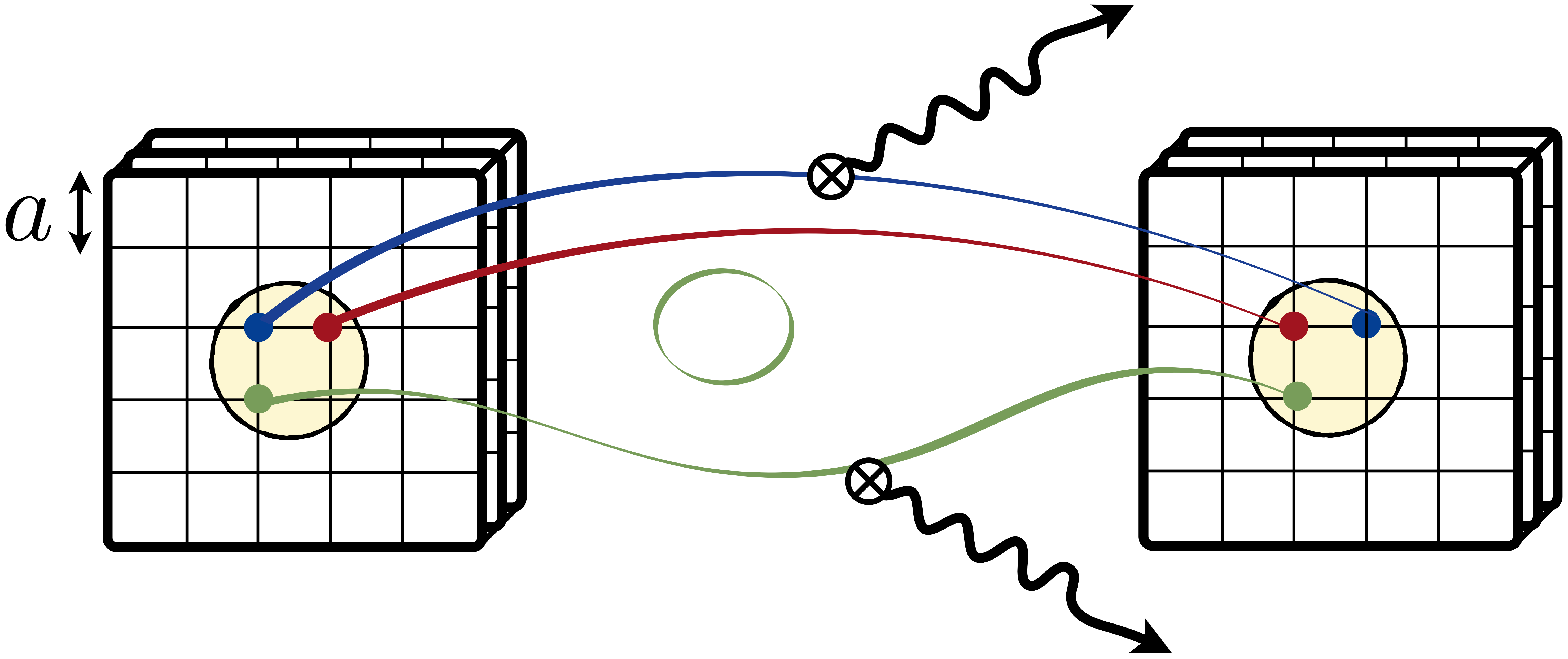
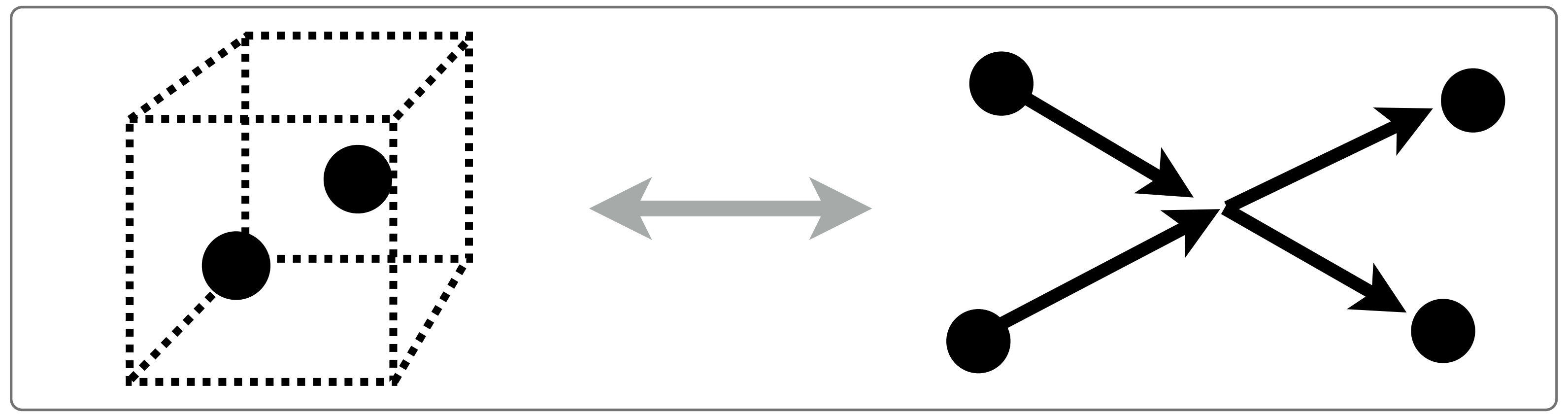
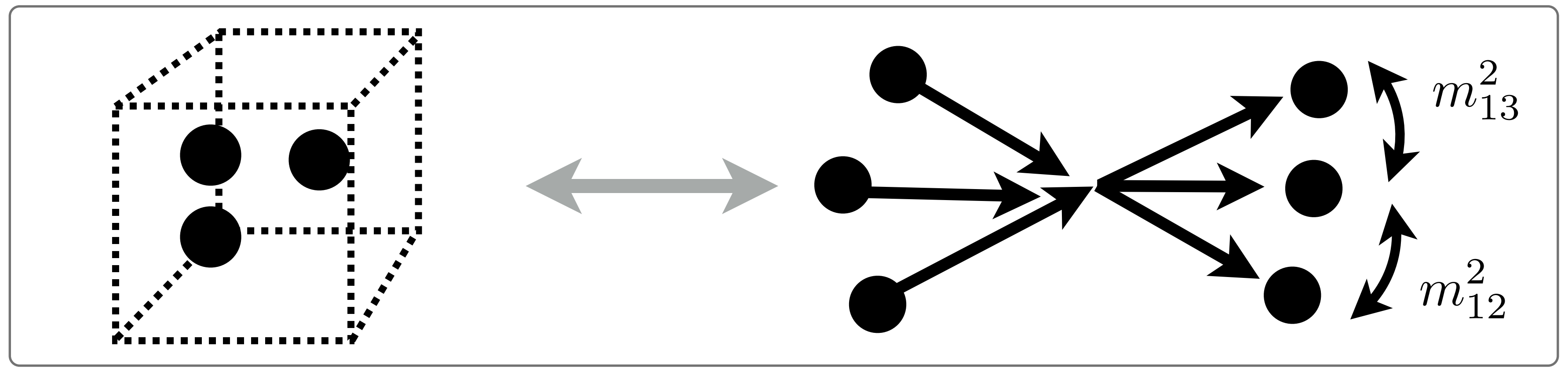
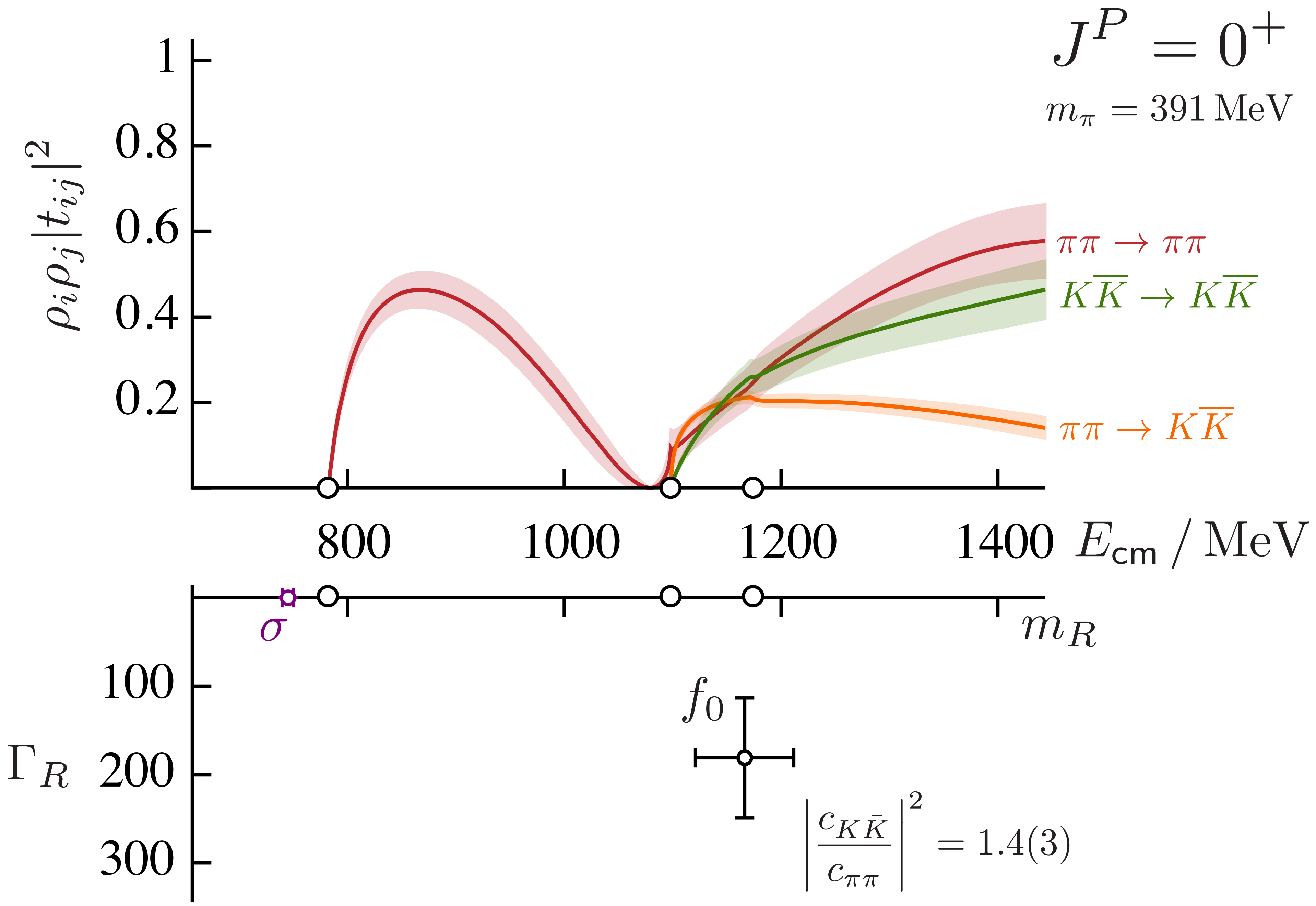


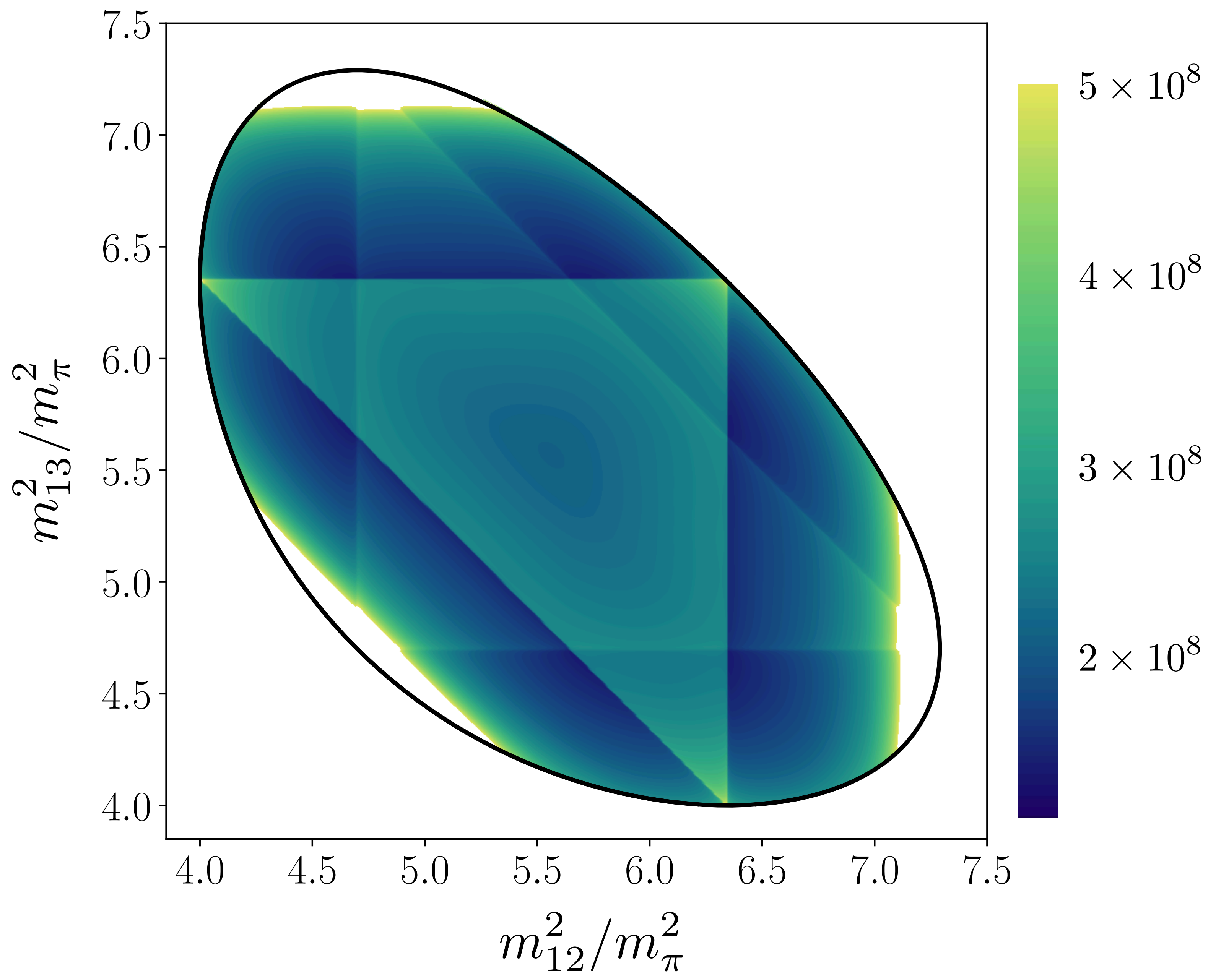
Exclusive vs. inclusive reactions

- If exclusive and *interesting*
  - After developing increasingly complex formalism…
  - Lattice QCD **will always win**







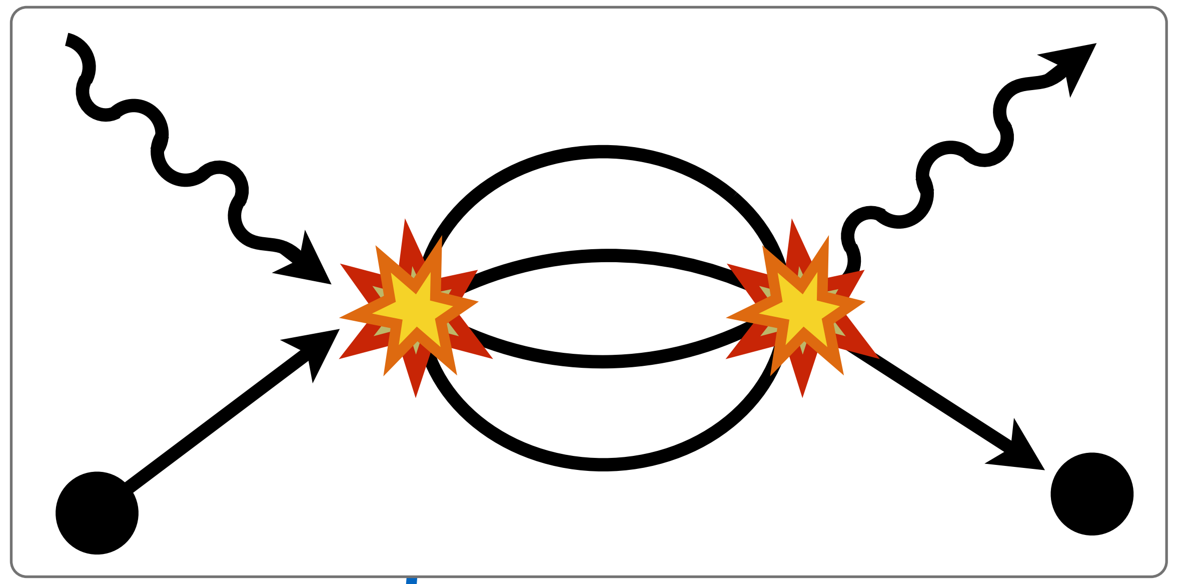


RB, Dudek, Edwards, Wilson (2018)

Hansen, RB, Edwards, Thomas, & Wilson (2020)

Exclusive vs. inclusive reactions

- If exclusive and *interesting*
  - After developing increasingly complex formalism…
  - Lattice QCD **will always win**
- Inclusive reactions, QC methods *may* be needed and worth investigating.



Infinite-volume reactions

- complex functions,
- kinematic singularities,
- due to intermediate on-shell states.

Hamiltonian frameworks

Four-point functions in a finite, Minkowski spacetime

$$\mathcal{T} \sim \int_0^T d^4x\, e^{it(\omega+i\epsilon)} \langle n_f | \mathcal{J}(t)\, \mathcal{J}(0) | n_i \rangle_\infty$$

[only considering one time ordering, introduced ϵ as a regulator]

$$= \sum_n \int_0^T d^4x\, e^{it(E_f+\omega-E_n+i\epsilon)} \langle n_f | \mathcal{J}(0) | n \rangle \langle n | \mathcal{J}(0) | n_i \rangle_\infty$$

[inserting a complete set of discrete finite-volume states]

$$\approx \sum_n i \frac{\langle n_f | \mathcal{J}(0) | n \rangle \langle n | \mathcal{J}(0) | n_i \rangle_\infty}{(E_f + \omega - E_n + i\epsilon)}$$

[assuming $\epsilon T \gg 1$]

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[assuming $\epsilon / E \ll 1$]

Hamiltonian frameworks

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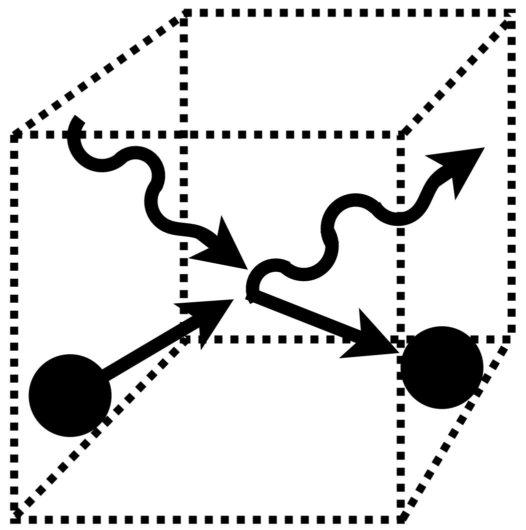
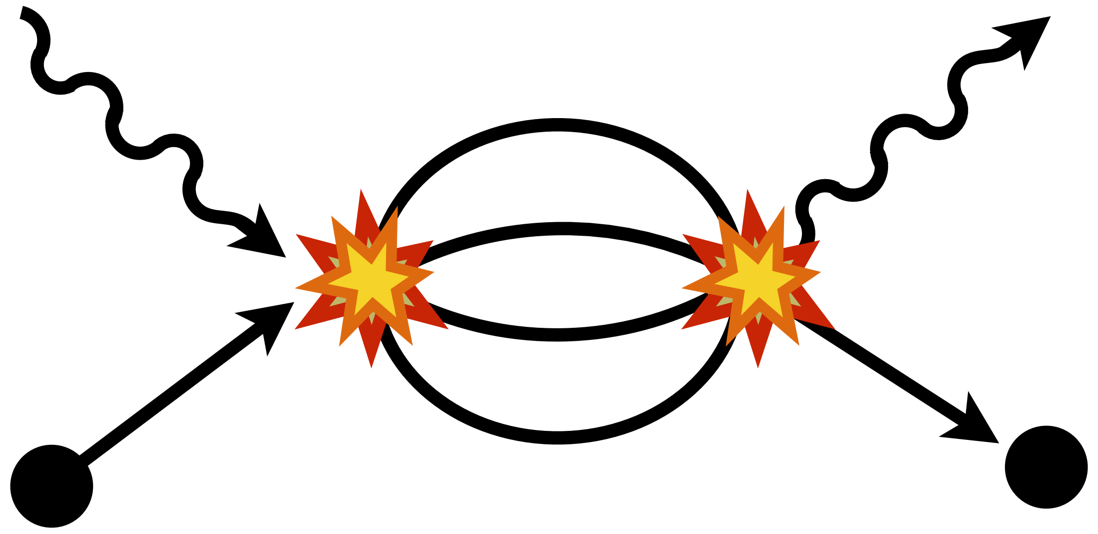
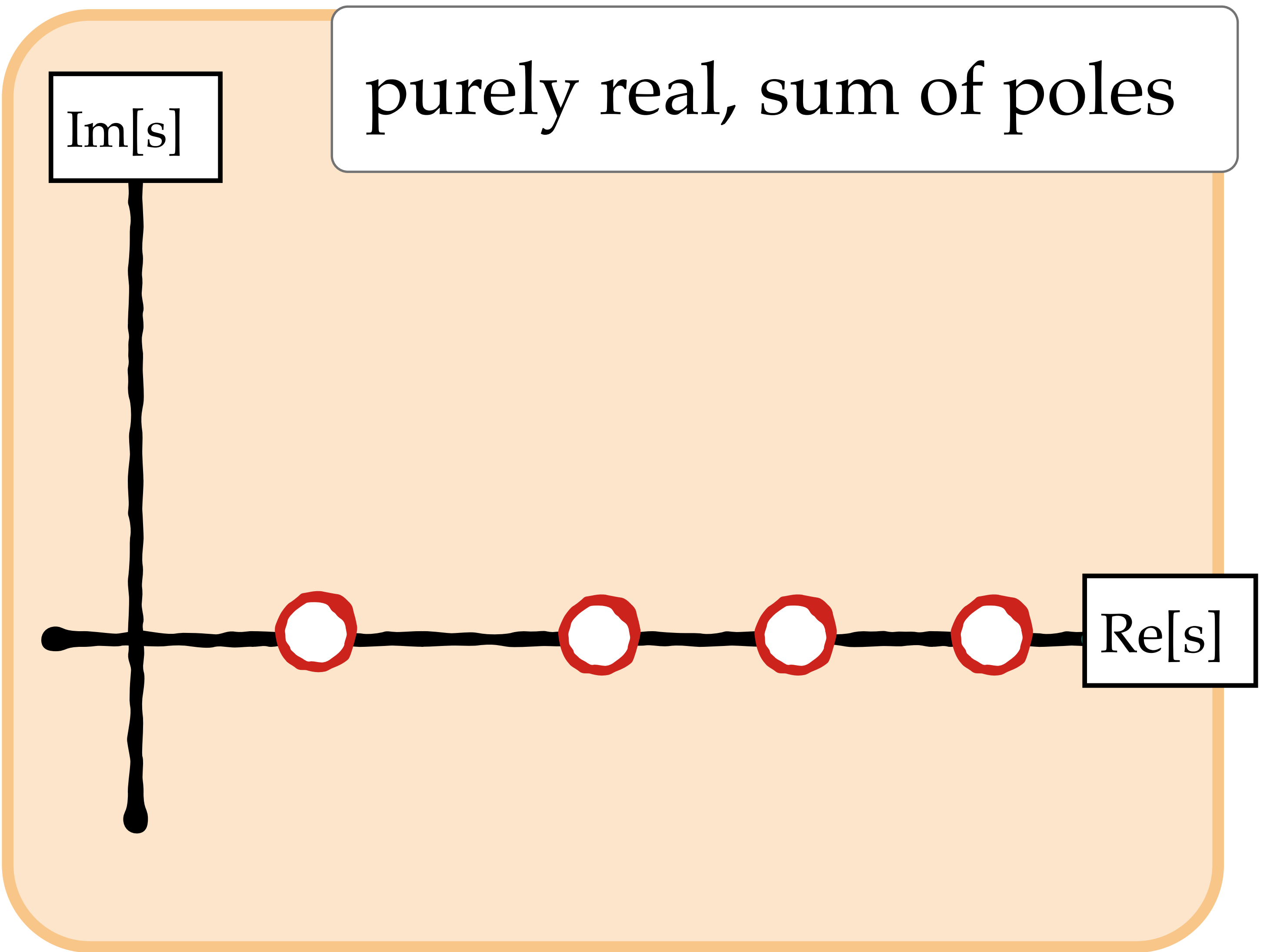
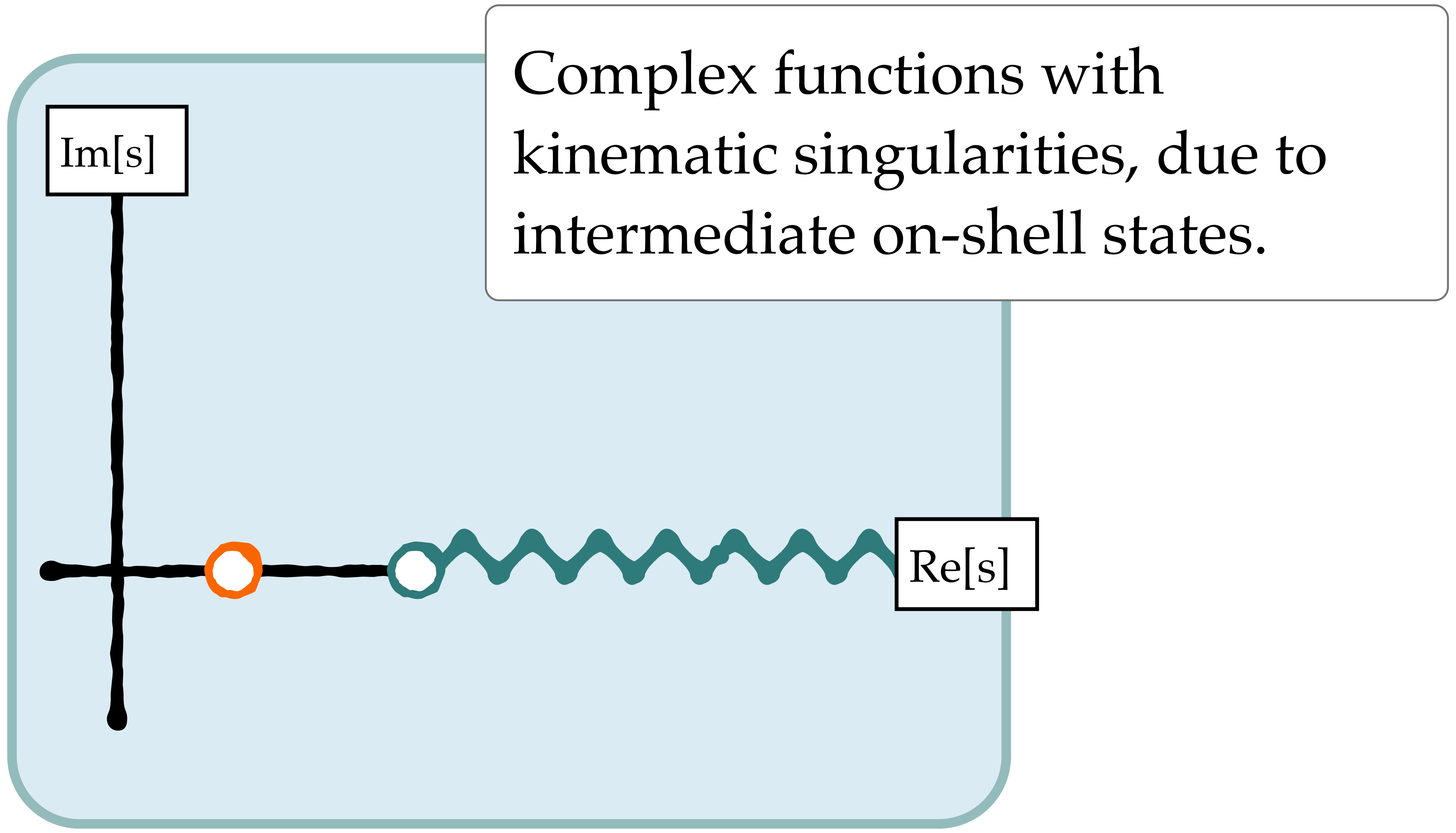
sum over real-valued poles…
clearly unphysical 😬









Assessing feasibility *in 1+1D*

- (hopefully) interesting and not obviously straightforward.
- Extrapolation is not even obviously well defined.
- To explore and test new ideas, we can use existing formalism:
  - **exact** relationship between finite- and infinite-volume amplitude,
  - derived using principles of scattering theory.
- For simplicity: assume scalar currents and current hadrons, and $(2m)^2 < s < (3m)^2$

$$\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q_f^2)\,\frac{1}{F^{-1}(P,L) + \mathcal{M}(s)}\,\mathcal{H}(s, Q_i^2)$$

RB, Davoudi, Hansen, Schindler, Baroni (2019)

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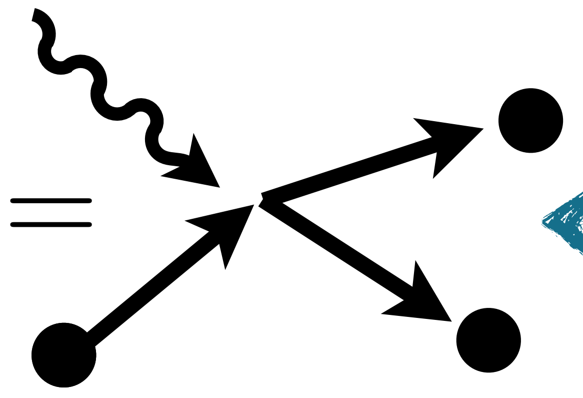
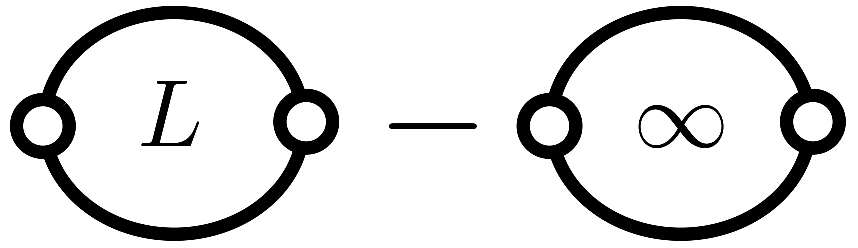
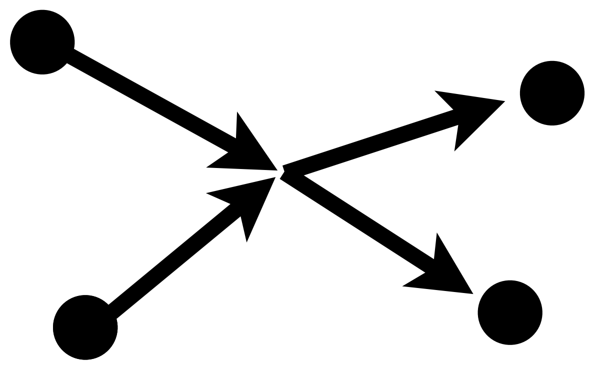
$i\mathcal{H}(s, Q_f^2) =$ 

$iF(P,L) =$ 

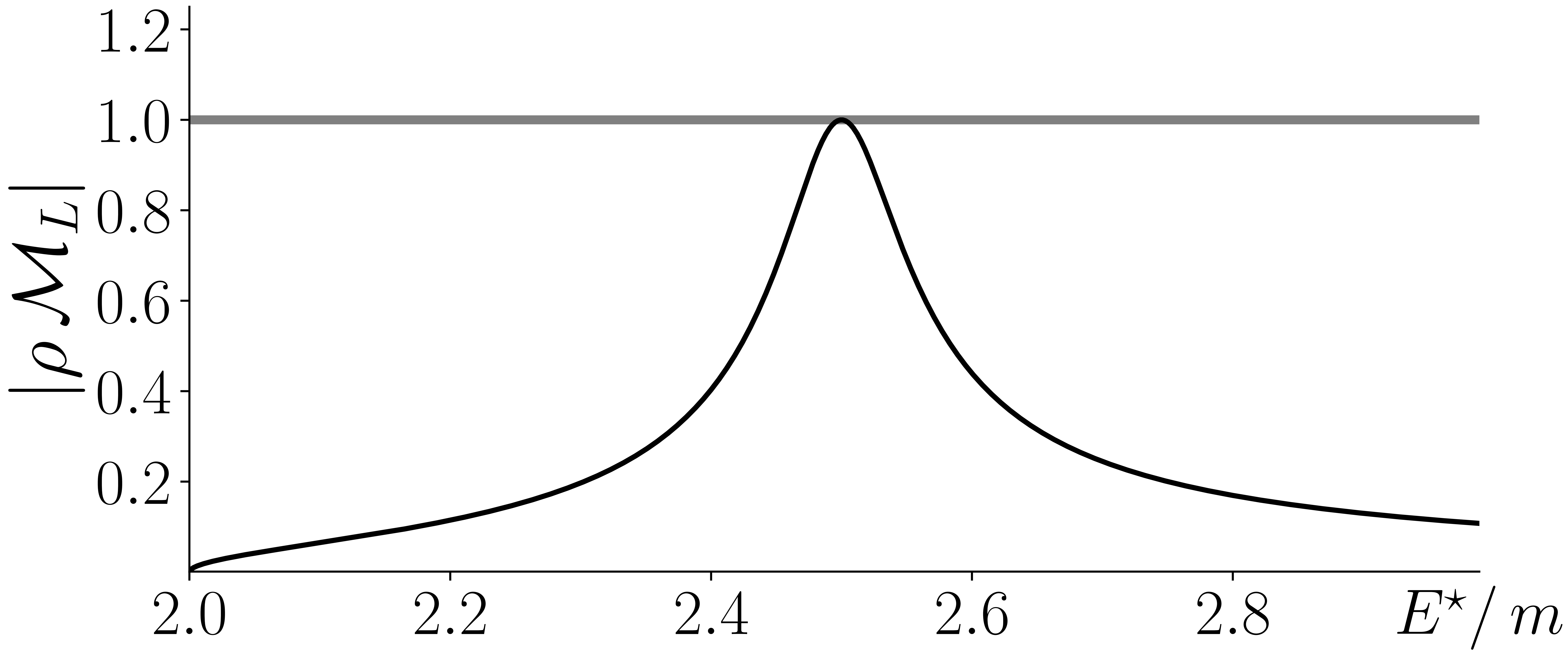
$i\mathcal{M}(s) =$ 

RB, Davoudi, Hansen, Schindler, Baroni (2019)

How bad are these effects? *real bad!*

In a finite-volume, the spectrum and all observables depend on the total momentum P = $2\pi d/L$, where d is discrete.

$$\mathcal{M}_L(s) = \mathcal{M}(s) - \mathcal{M}(s)\,\frac{1}{F^{-1}(P,L) + \mathcal{M}(s)}\,\mathcal{M}(s)$$



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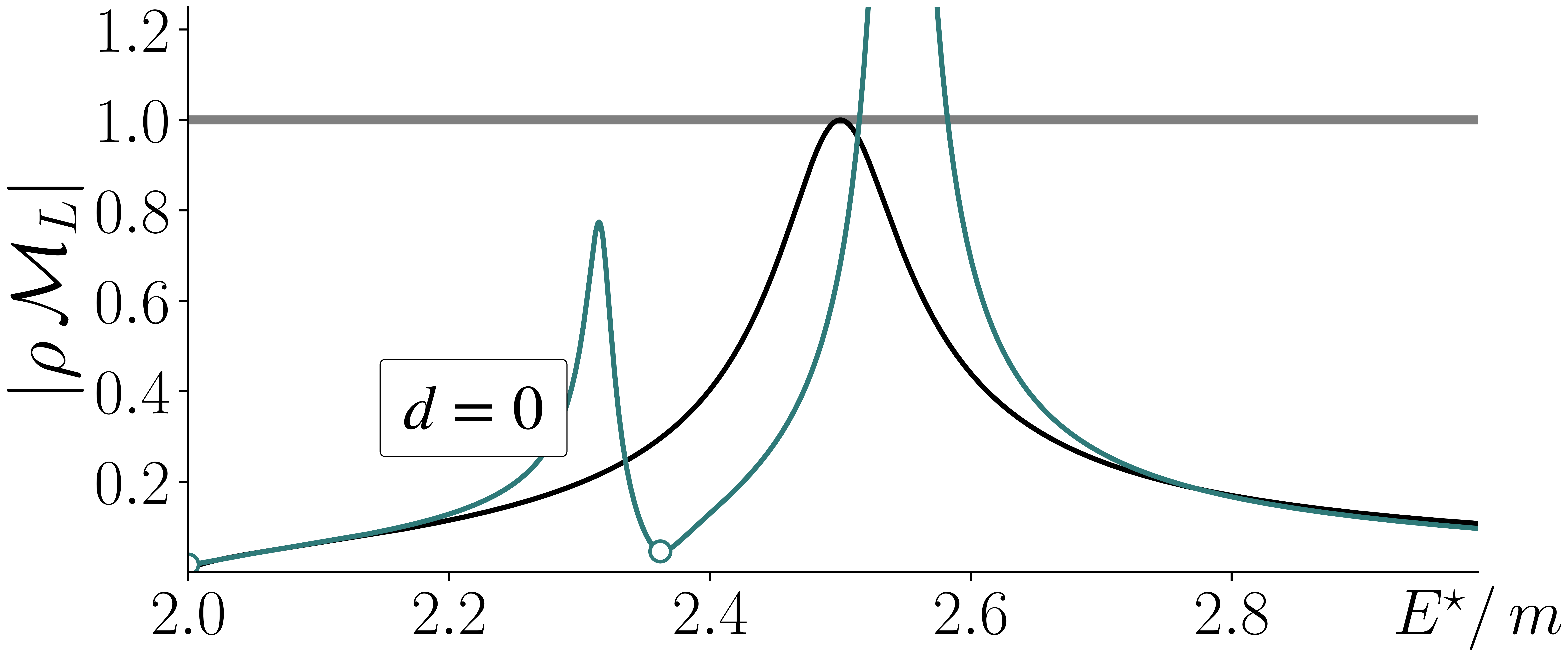

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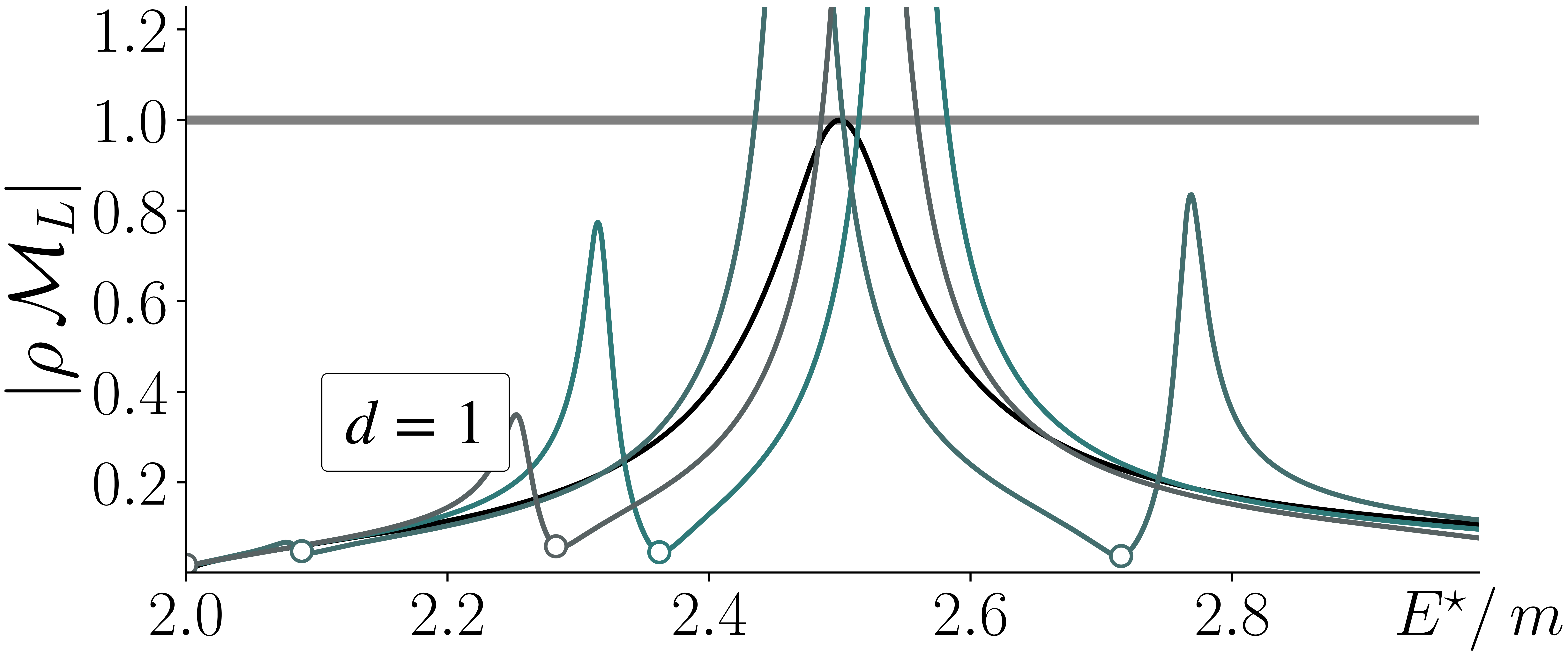
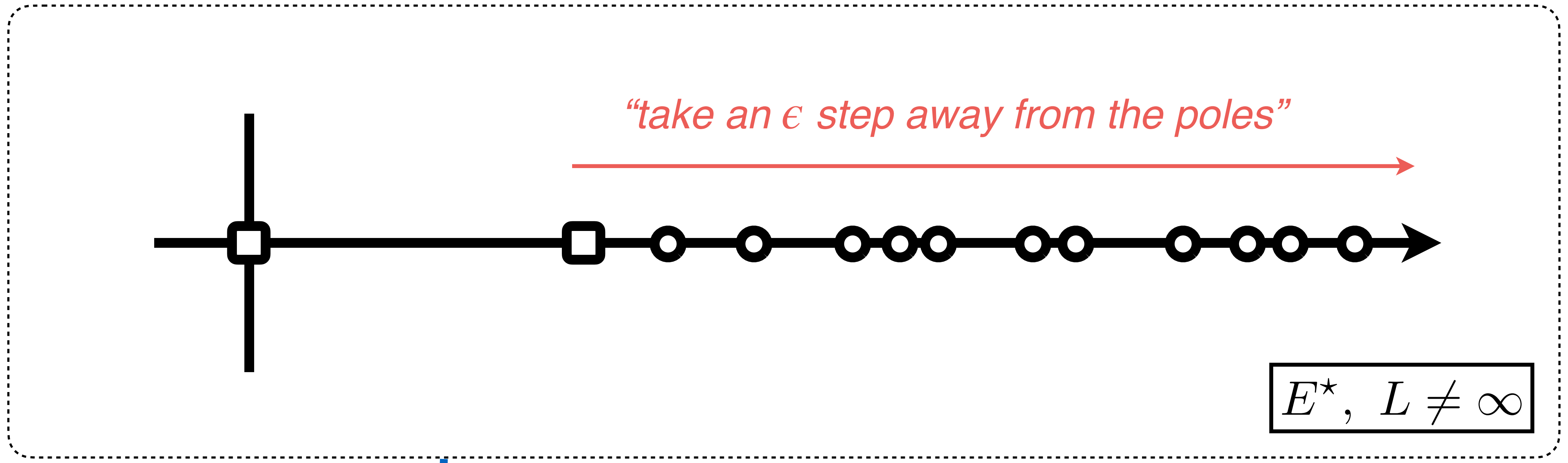


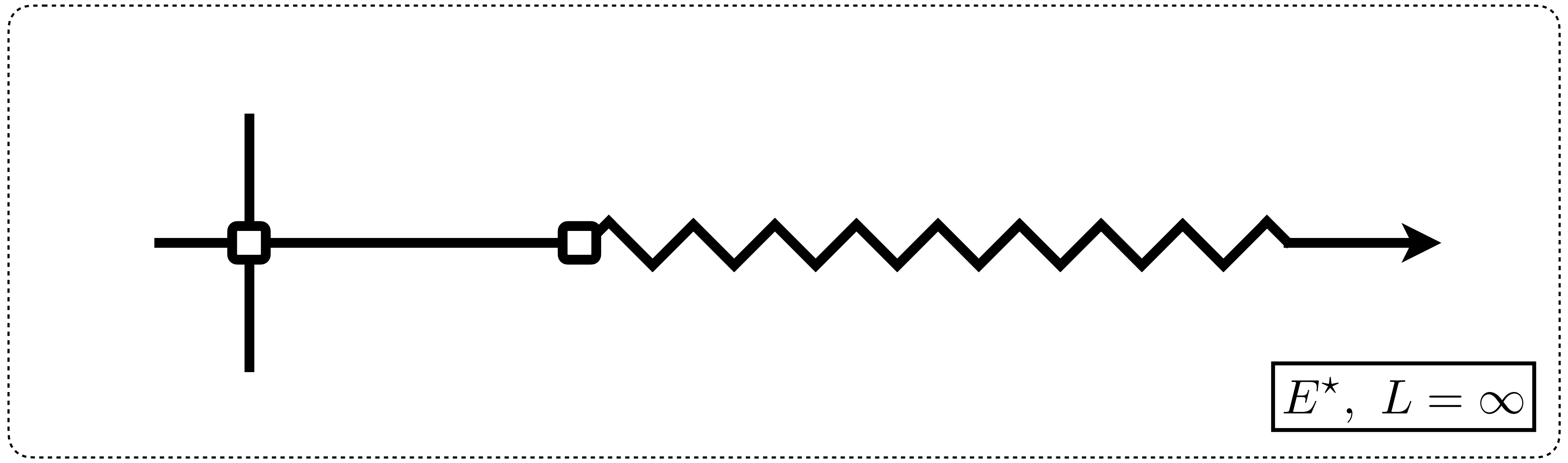
Constructing reliably estimators

- ☑ Determine time-dependent matrix elements [easier said than done 😅]
- ☑ Introduce an $i\epsilon$ by hand

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} d\tau\, e^{iq_0 t - \epsilon|t|} \langle n_f |\, T[\mathcal{J}_2(t)\, \mathcal{J}_1(0)]\, | n_i \rangle_L$$



For large enough L and small enough ϵ



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☑ Determine time-dependent matrix elements [easier said than done 😅]

☑ Introduce an $i\epsilon$ by hand [makes sense 🙂]

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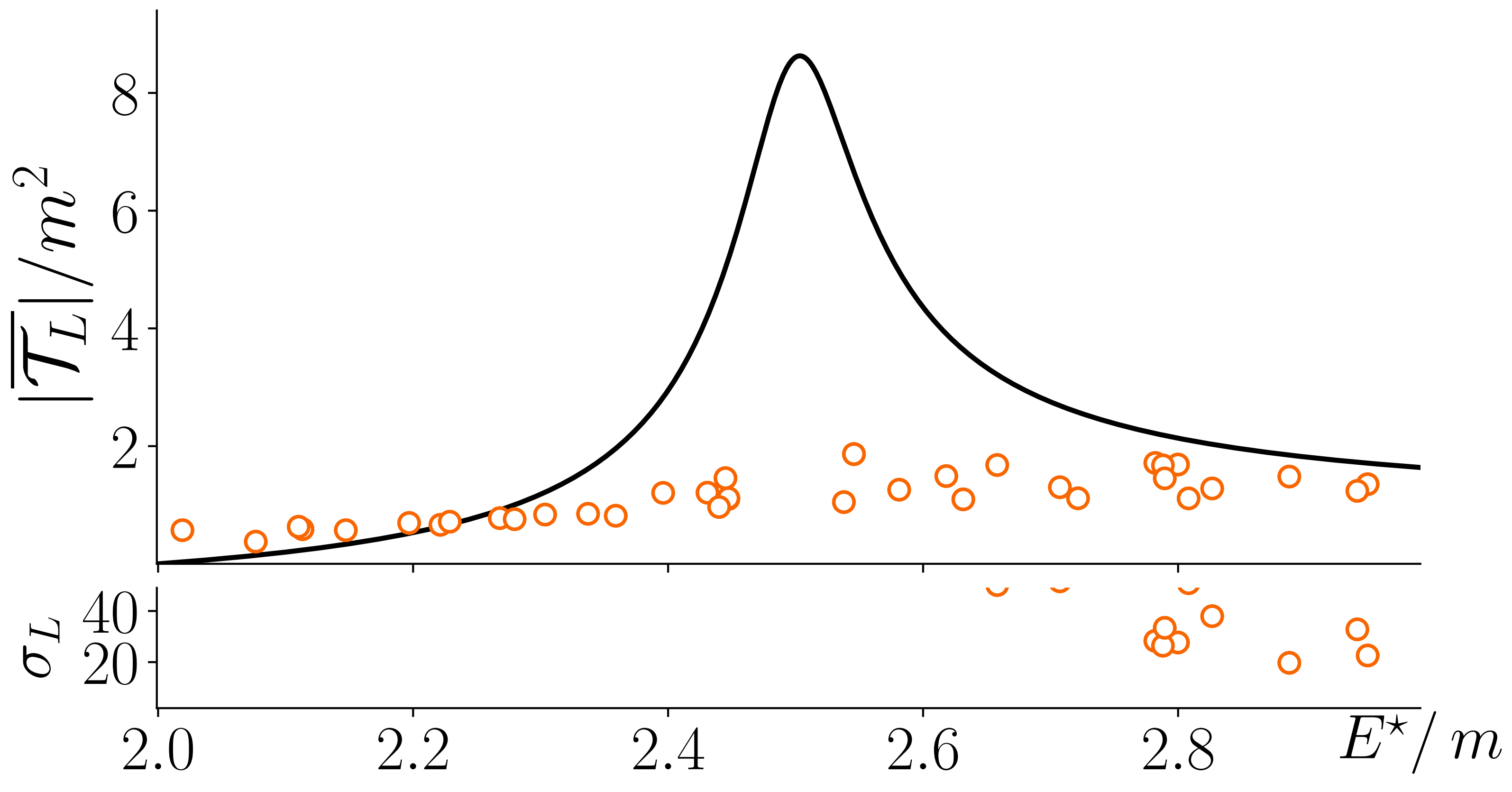
☑ Binning / wave packets [makes sense 🙂]

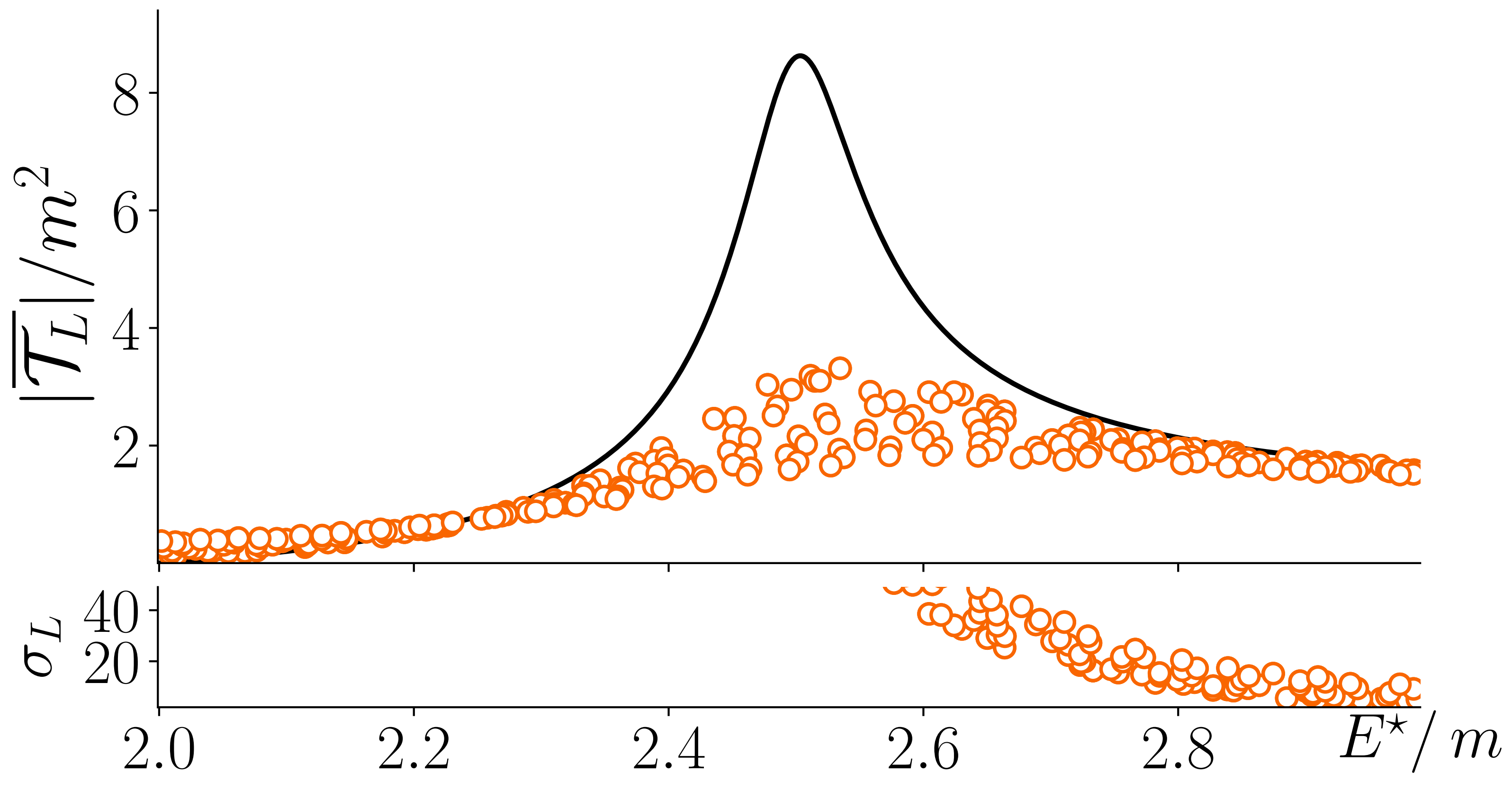
☑ Exploit symmetry: [uh? 🤔]

- ☑ Physical amplitudes only depend on Lorentz scalars.
- ☑ Boost average

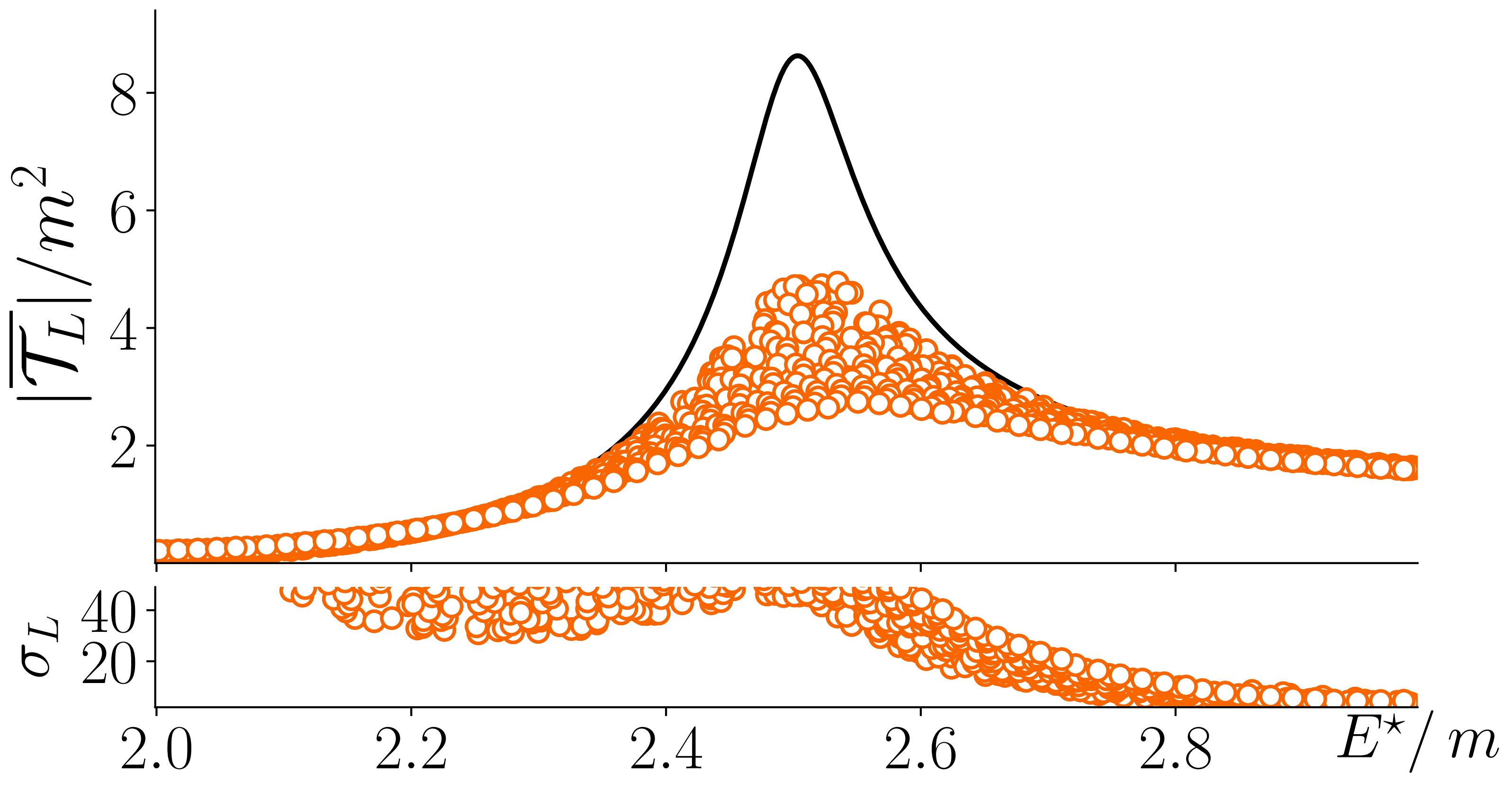
Toy model investigation for $\mathscr{T}$



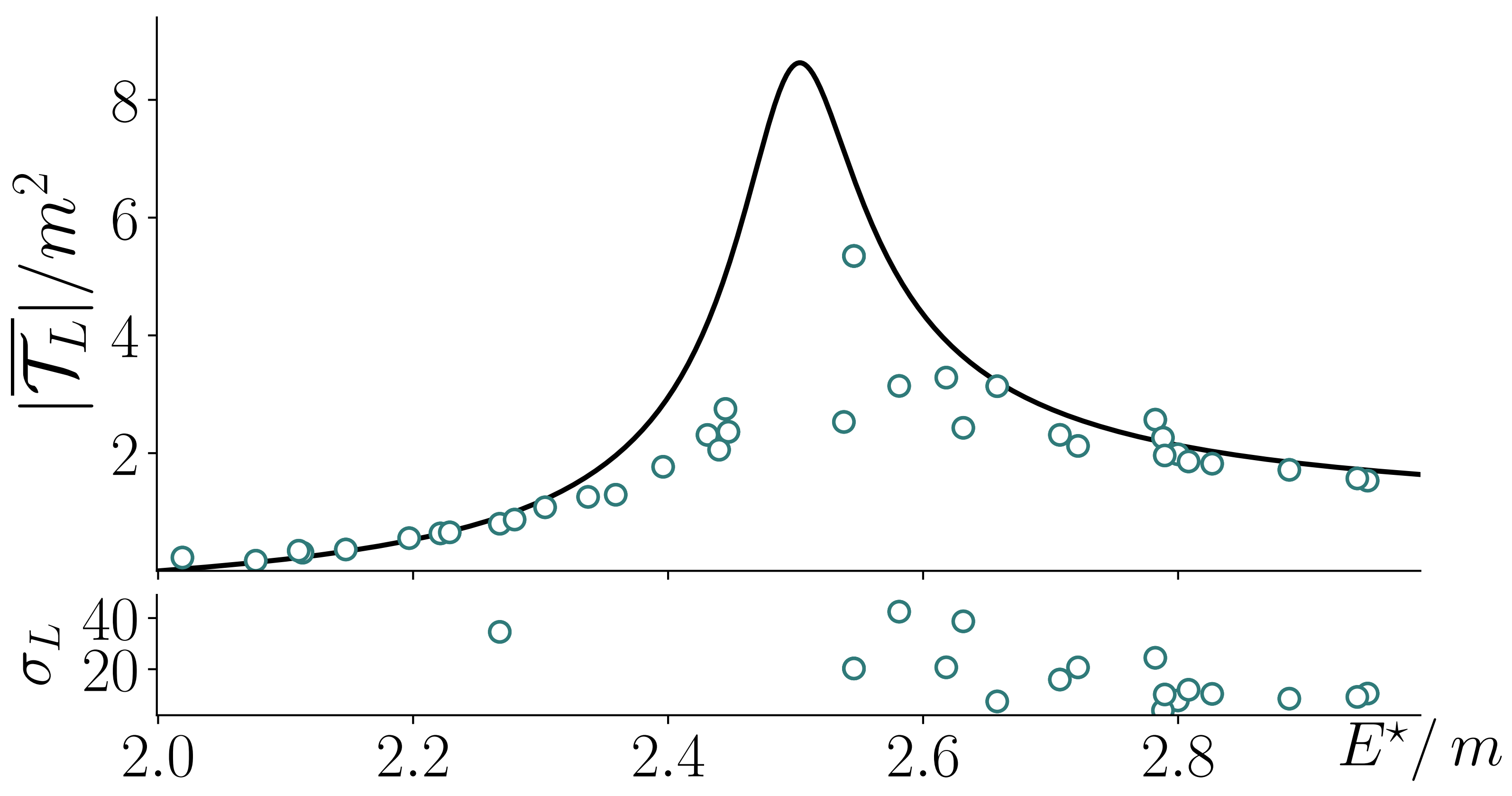
$mL = 20,\ \epsilon L = 4$



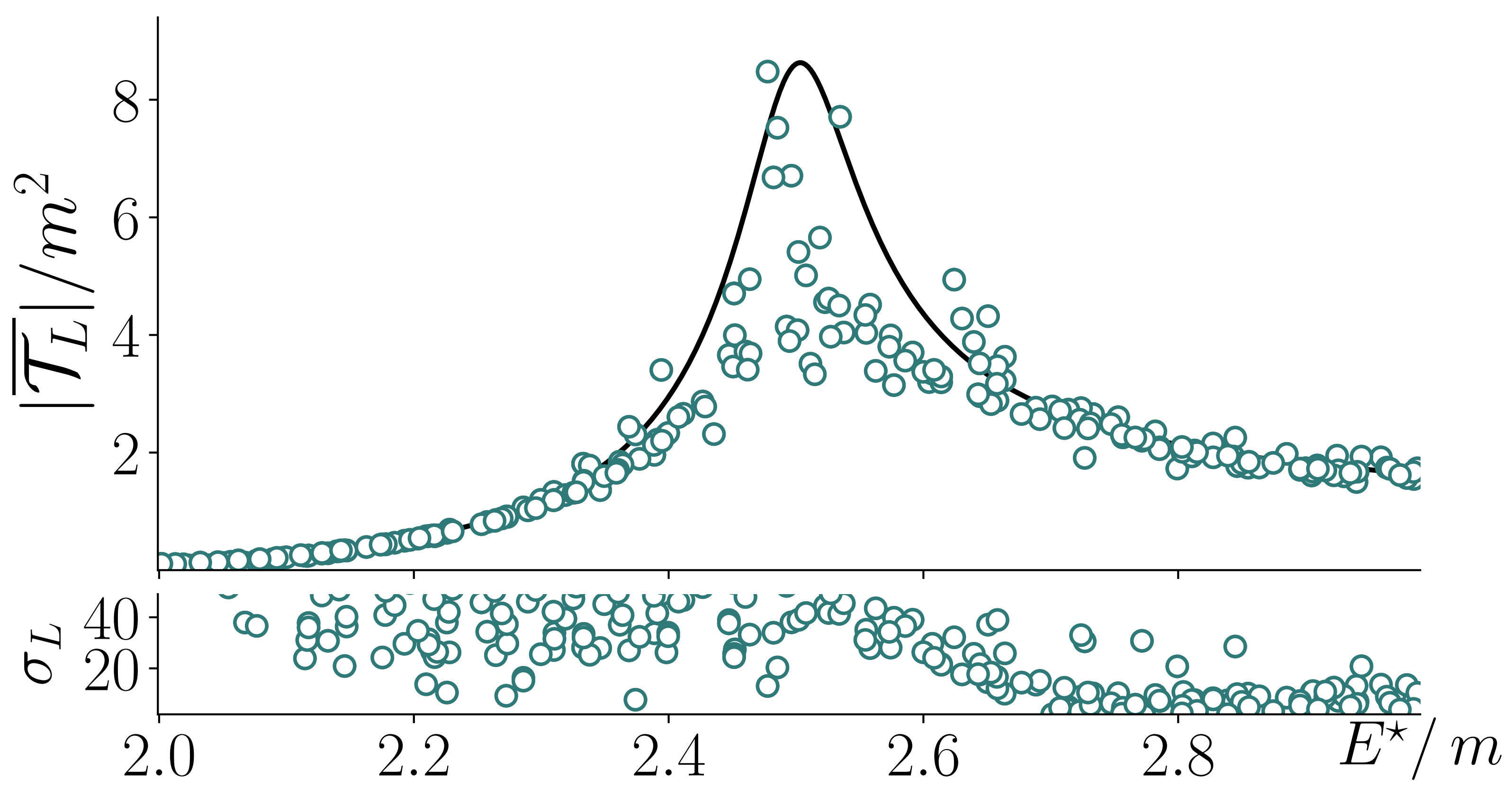
$mL = 50,\ \epsilon L = 4$



$mL = 100,\ \epsilon L = 4$



$mL = 20,\ \epsilon L = 1$



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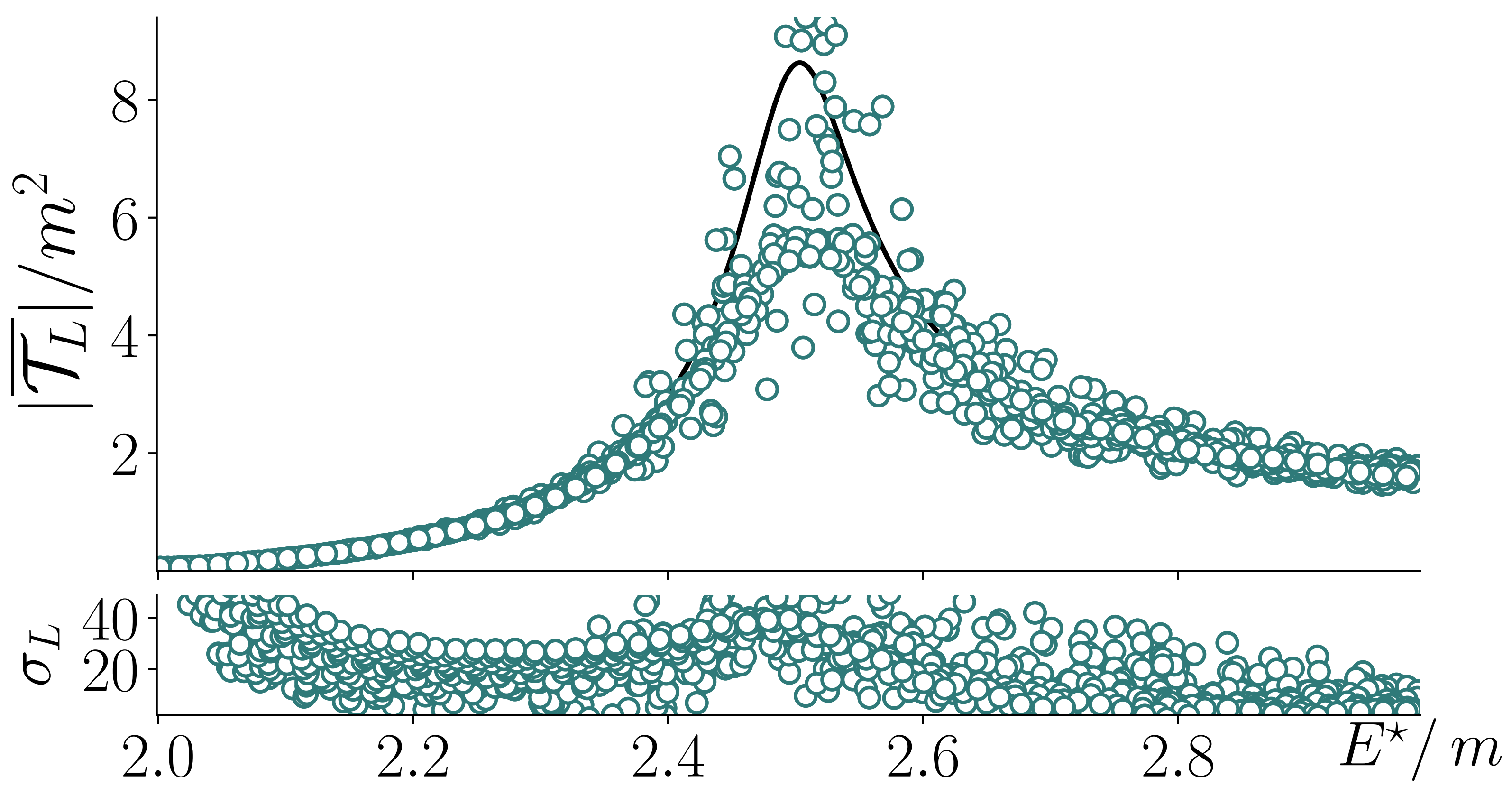


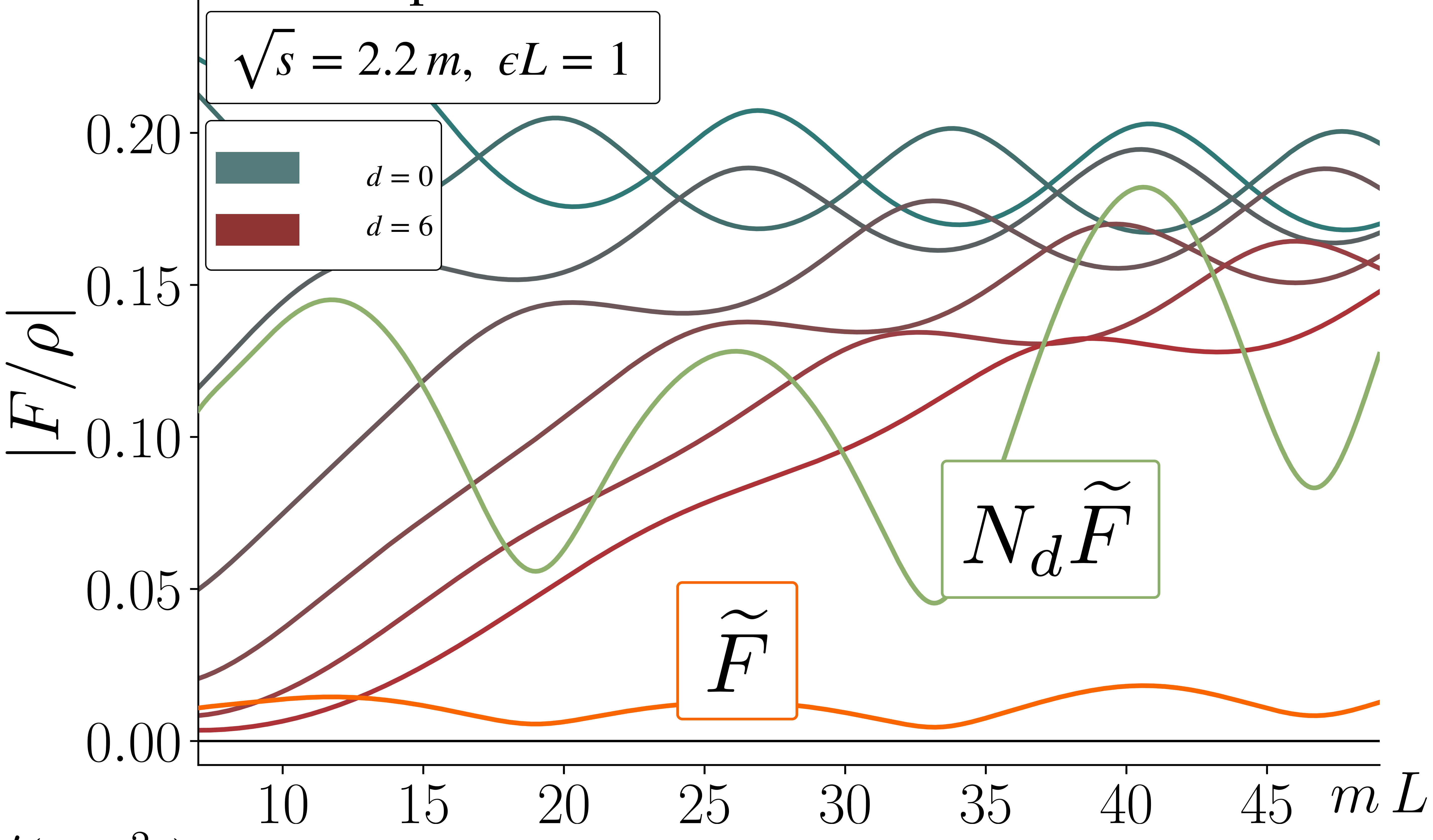
$mL = 100,\ \epsilon L = 1$

Boost averaging

The volume effects are encoded in $F(P, L)$, which is not a Lorentz scalar.

Asymptotic behavior $F \sim e^{-L\epsilon\alpha_0}(-1)^{\boldsymbol{d}}$

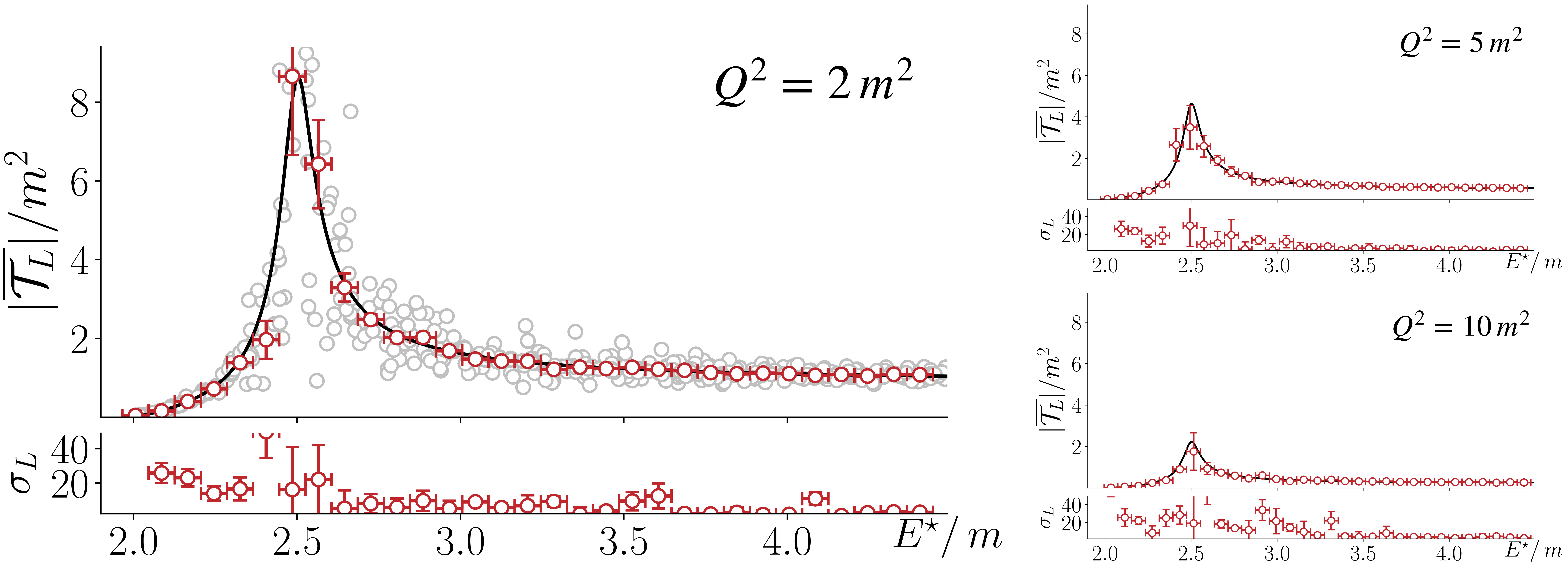
Averaging over different boosts, volume effects are expected to reduce.



$$\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2)\,\frac{1}{F^{-1}(P, L) + \mathcal{M}(s)}\,\mathcal{H}'(s, Q^2_{if})$$

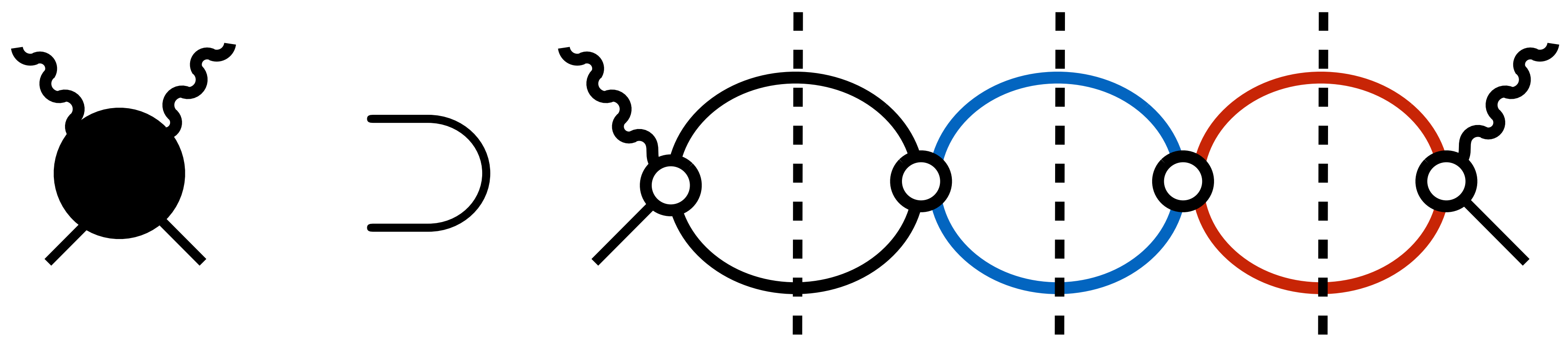
Following the recipe

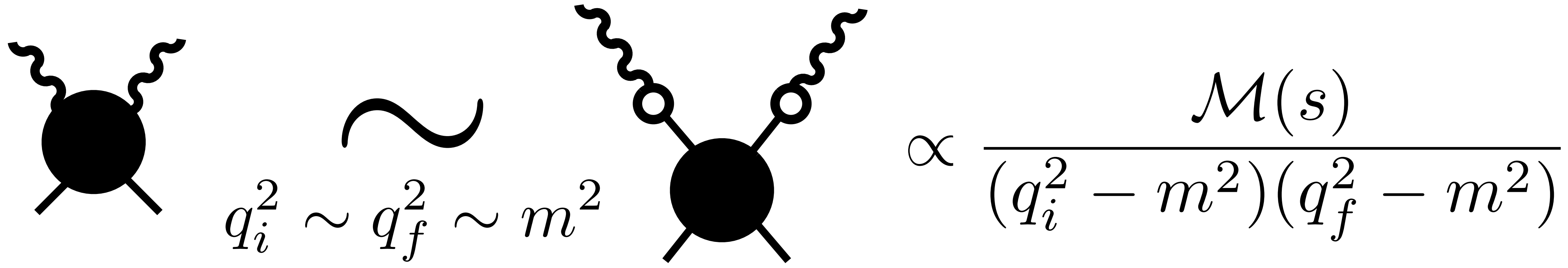
By averaging over $mL = [20,25,30]$ boost with $d \leq mL$, and binning in energy and virtualities.

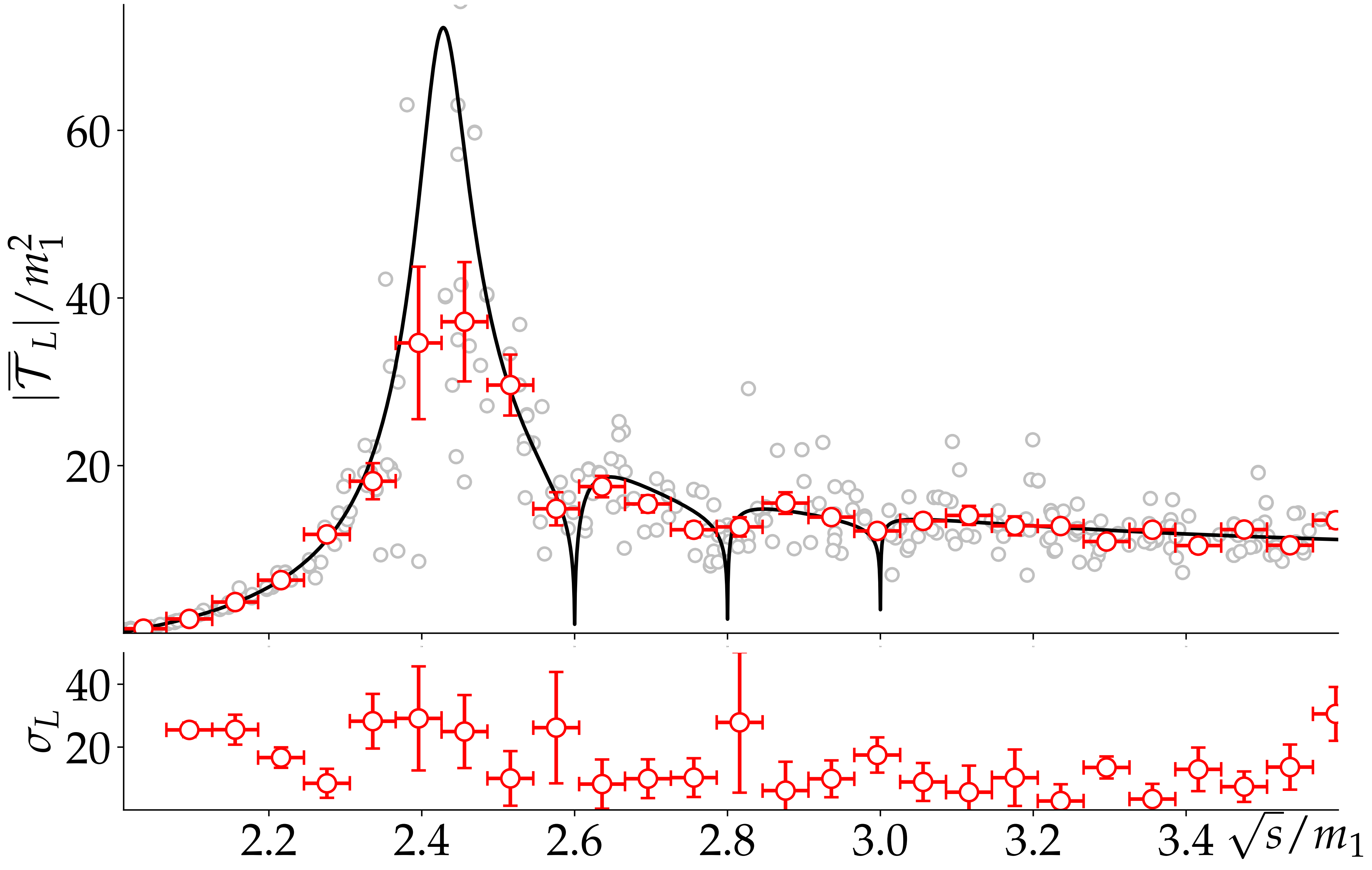


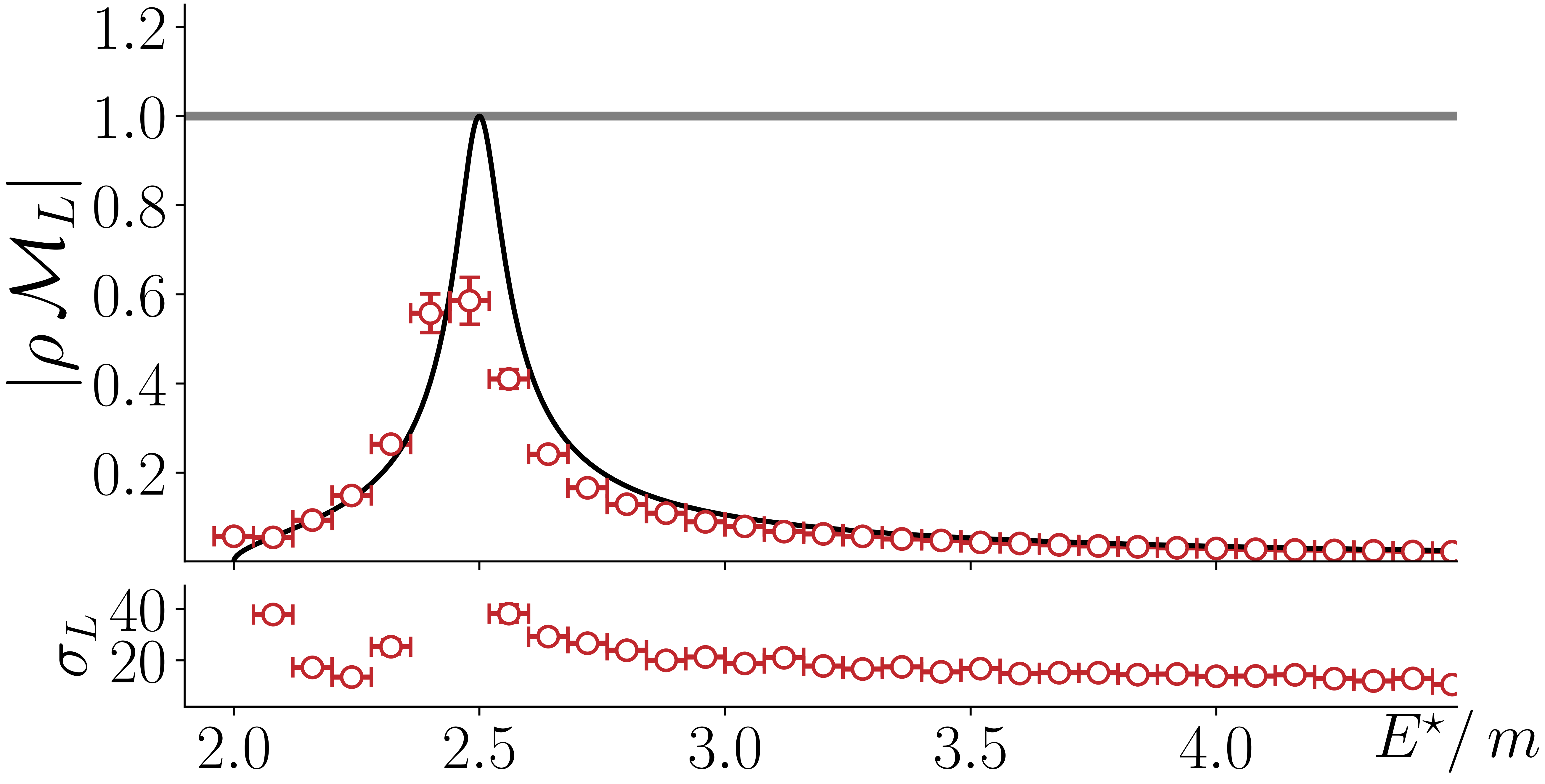
Extensions *other amplitudes*

- ☑ Checks on arbitrary number of channels
- ☑ Purely hadronic amplitudes using LSZ





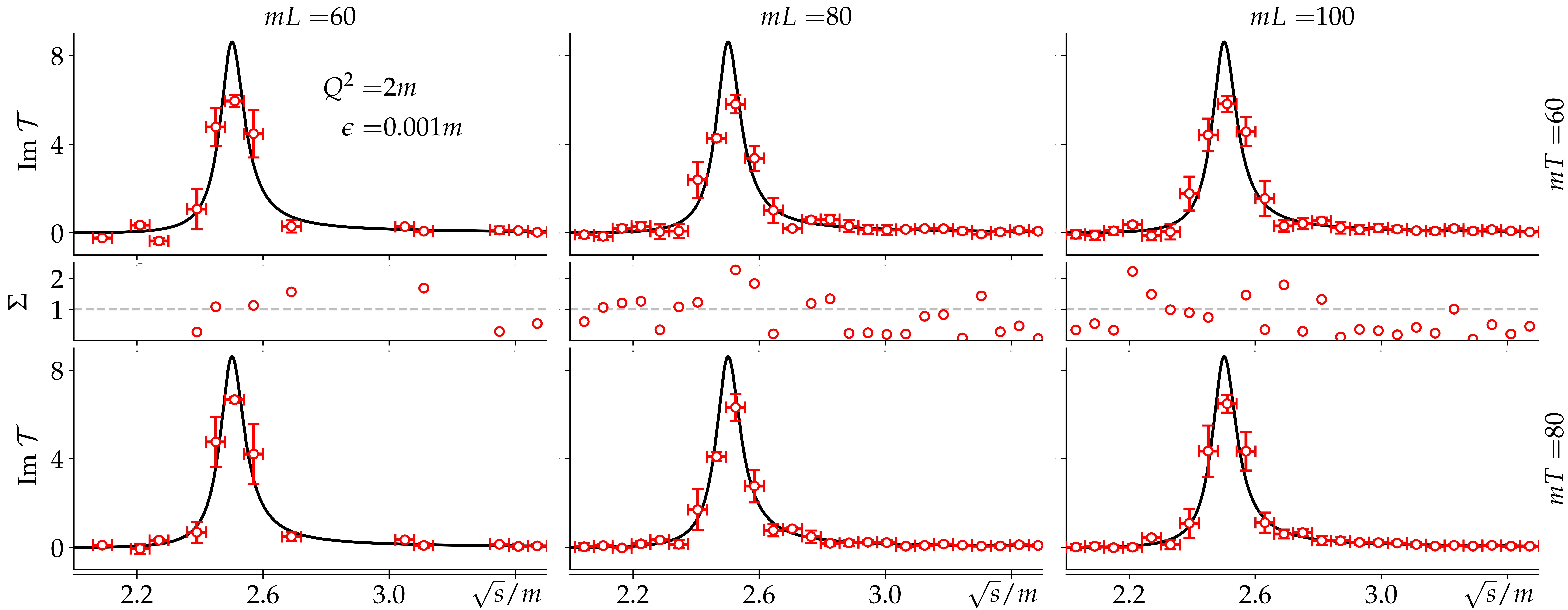




Testing finite-time dependence

- Using spectral decomposition, we can estimate the order of magnitude of time (T) neede

$$\mathcal{T} \sim \int_0^T d^4x\, e^{it(\omega+i\epsilon)} \langle n_f | \mathcal{J}(t)\, \mathcal{J}(0) | n_i \rangle_\infty \;=\; \sum_n \int_0^T d^4x\, e^{it(E_f+\omega-E_n+i\epsilon)} \langle n_f | \mathcal{J}(0) | n \rangle \langle n | \mathcal{J}(0) | n_i \rangle_\infty$$



Take-home message

- [x] Inclusive scattering observables are not out of the question
- [x] No sophisticated formalism is needed
    - [ ] existing formalism serves as diagnostic tool
- [ ] Does life get harder or easier in $3 + 1$D?
- [ ] Test on toy theory [quantum simulation vs. lattice]

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William & Mary

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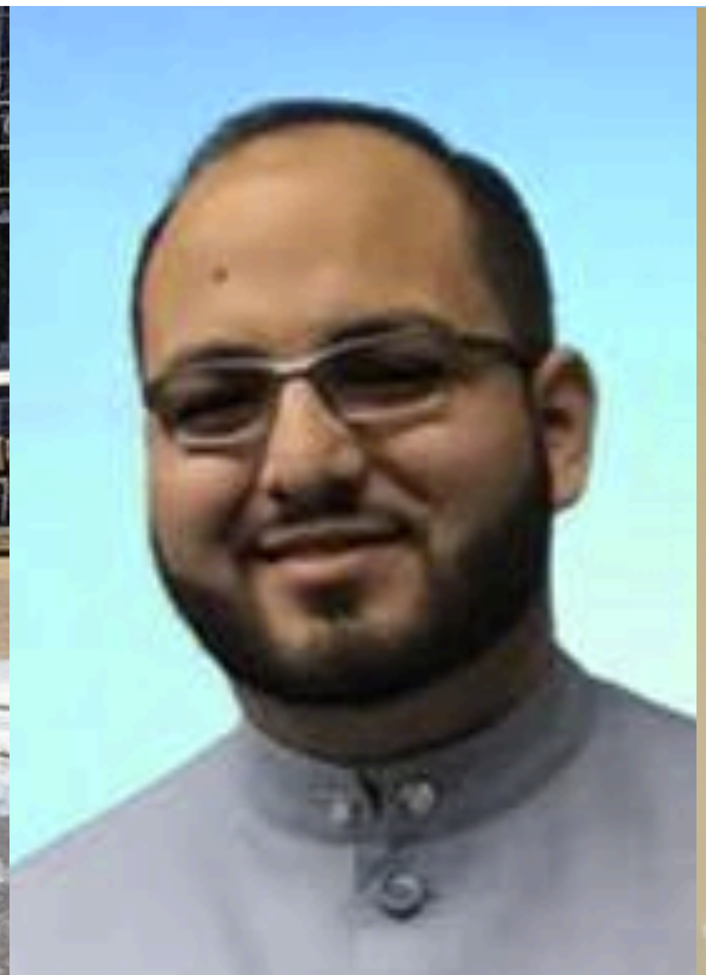

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