UNPOLARIZED PROTON PDF AT NNLO. LAMET APPROACH

ARXIV:2212.12569

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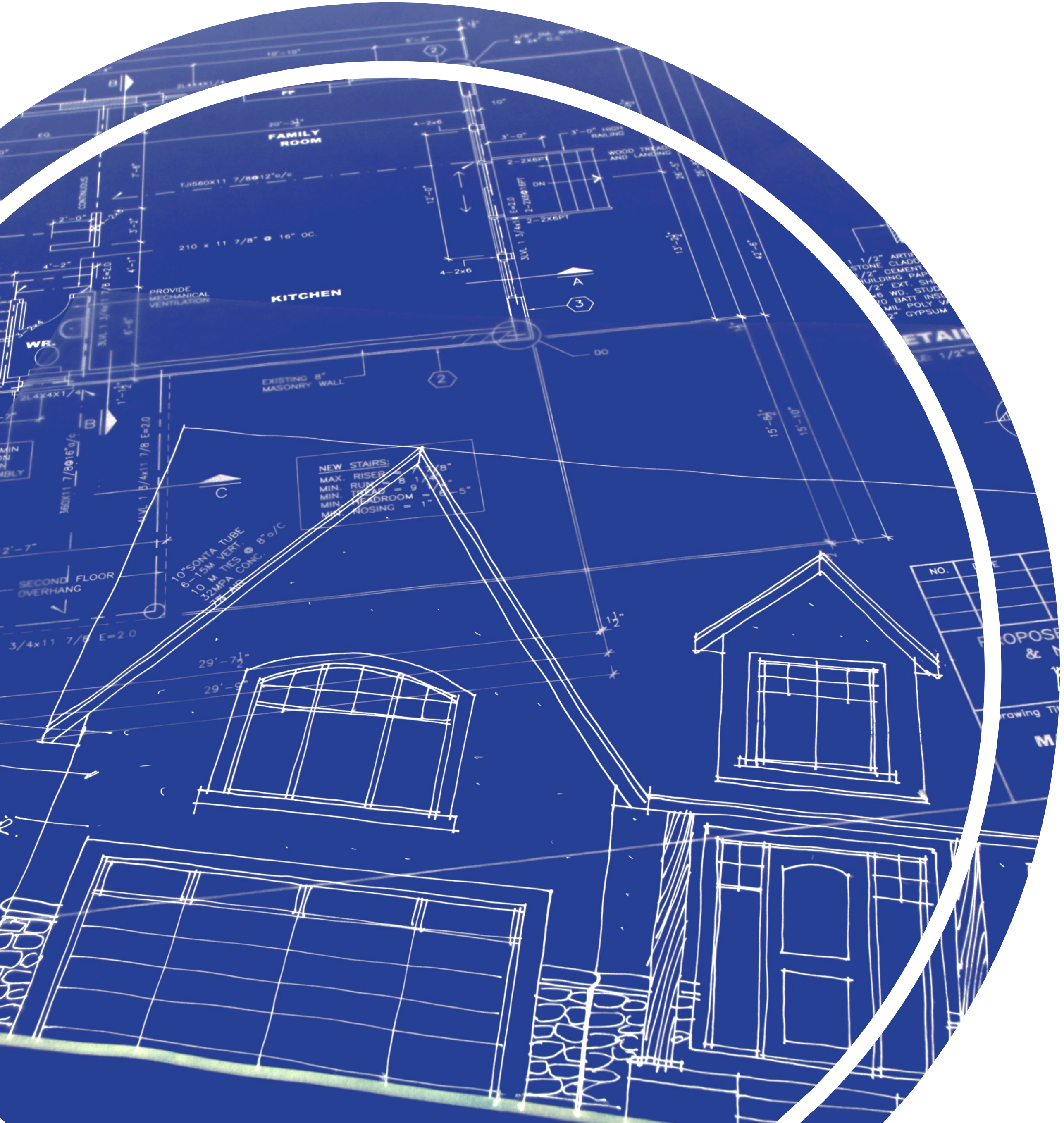
30th March 2023

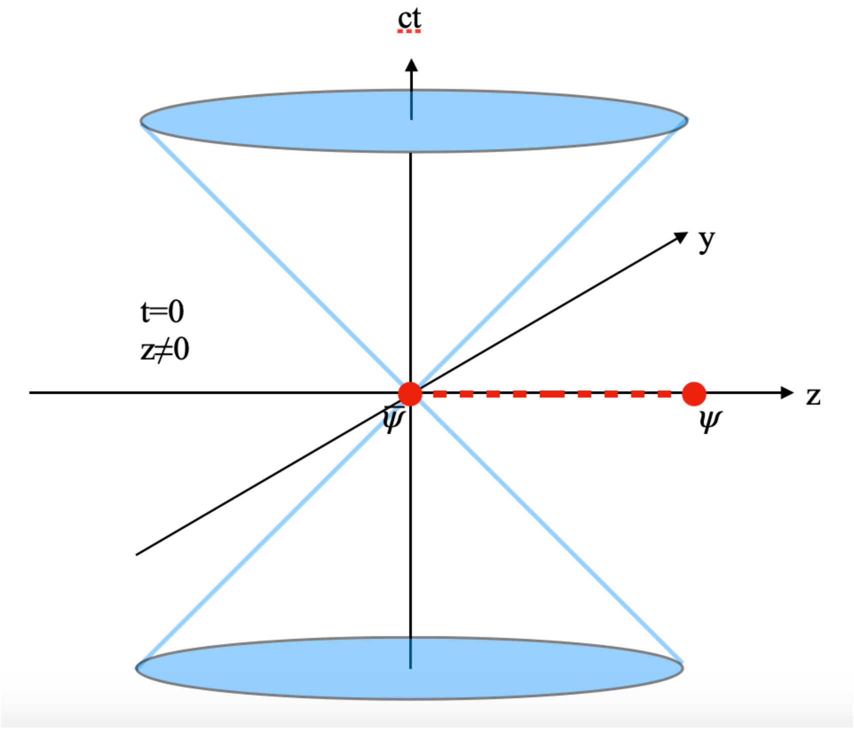


OUTLINE

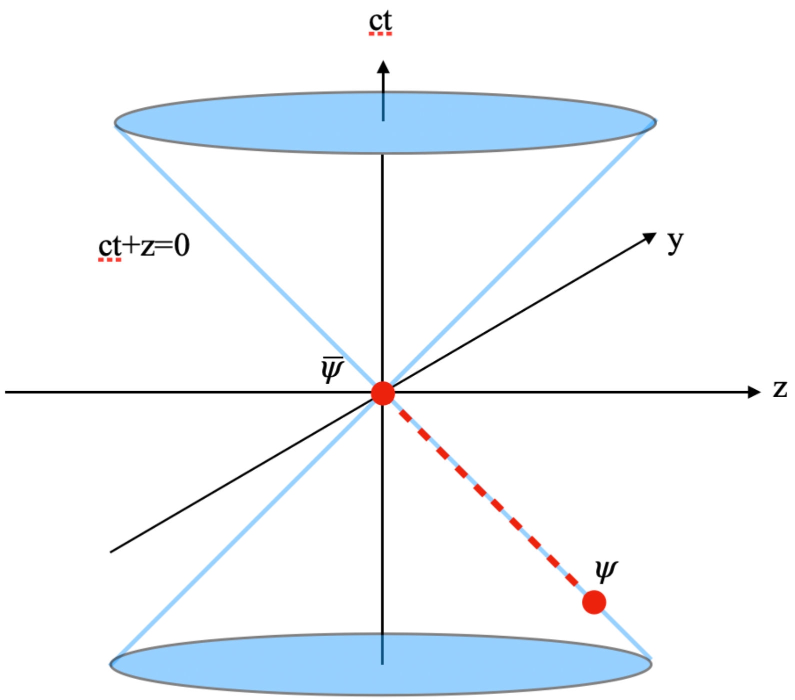
- Brief summary of large-momentum effective theory (LaMET).
- Parton distribution functions (PDFs).
- Calculation:
  - Renormalization.
  - Extrapolation to infinite distance.
  - Matching to the lightcone.
- Results.
- Conclusion.

LARGE-MOMENTUM EFFECTIVE THEORY

Quasi matrix element



Matrix element

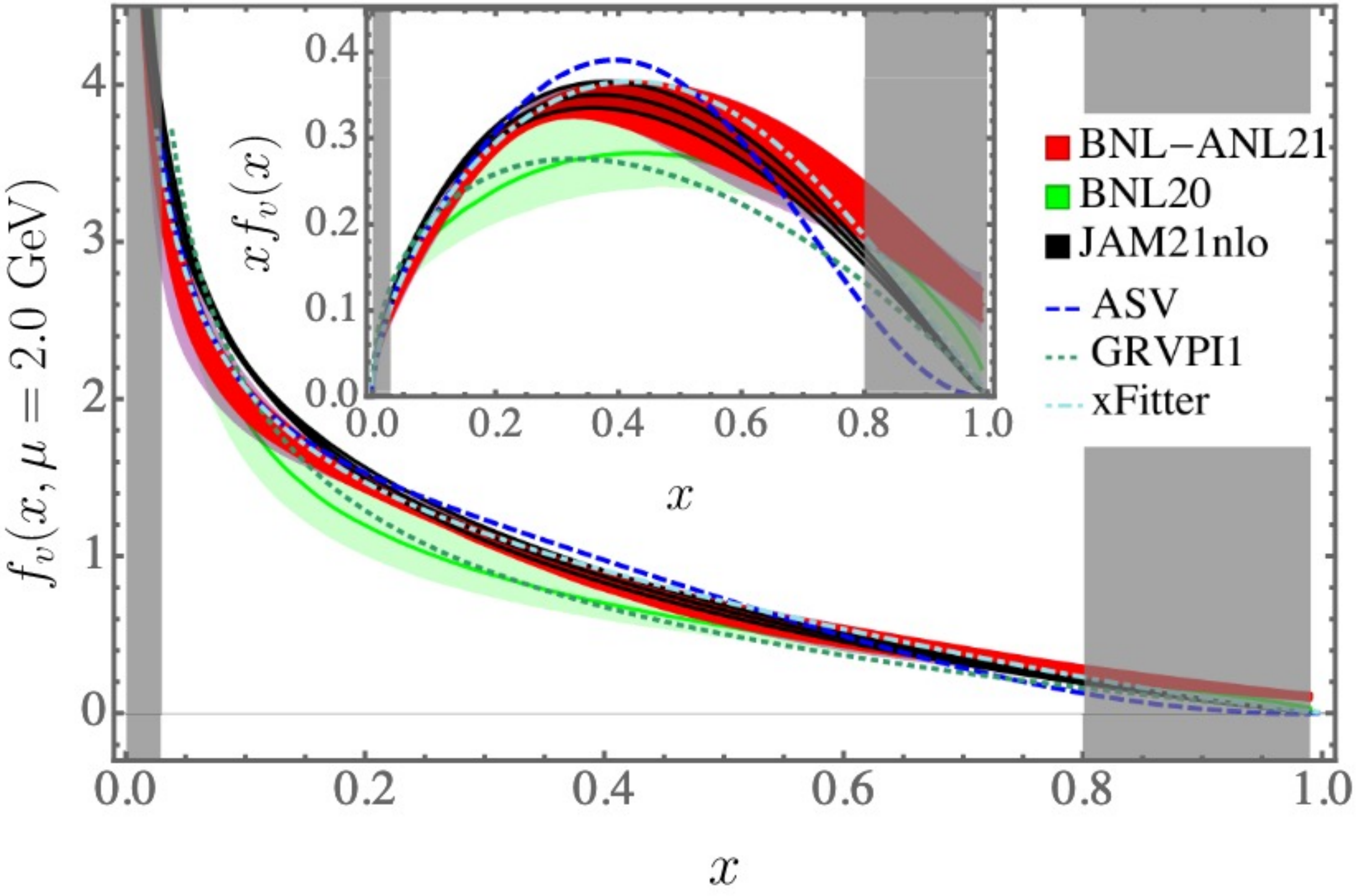


Lorentz boost

$\Lambda(\infty)$

See, also, Yong Zhao's talk from Altarelli Prize Ceremony. 3/27 14:00.

UNPOLARIZED PDF



- Unpolarized pdf:

$$f(x) = \begin{cases} f_u(x) - f_d(x) & x > 0 \\ f_{\bar{d}}(x) - f_{\bar{u}}(x) & x < 0 \end{cases}$$

- This was computed for the pion in 2021[1] using the method of LaMET.
- We apply the same method to the proton[2].

[1]X. Gao et. al. Phys. Rev. Lett. **128**, 142003.
[2]X. Gao et al. arXiv:2212.12569.

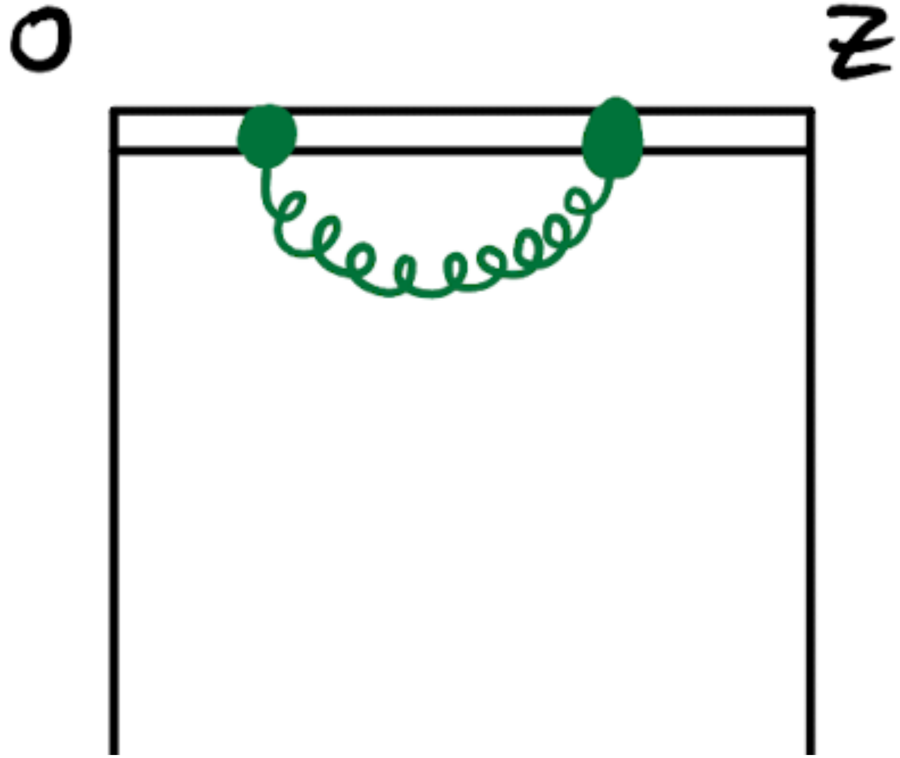
UNPOLARIZED PROTON PDF

- I applied the method of LaMET to compute the proton PDF.
- Process involves:

$$\tilde{h}_B(z,P_z) \xrightarrow{\text{Renormalize}} \tilde{h}_R(z,P_z) \xrightarrow{\text{Fourier Transform}} \tilde{f}(x,P_z) \xrightarrow{\text{Lightcone Matching}} f(x,\mu)$$

- The method of a deep-neural-network was used independently and the results were compared.
  - *See Andrew Hanlon's talk: Thursday 3/30 15:00-15:20.*

RENORMALIZATION



- Self energy of the Wilson line → linear divergence. **Independent of external state.**
- Handle the short-distance (UV) and long-distance (IR) behaviour separately.
- Use the hybrid renormalization scheme[1] in the same way as was computed in the pion case[2].

[1]X. Ji et. al. Nucl. Phys. B. **964**, 115311.
[2]X. Gao et. al. PRL. **128**, 142003.

HYBRID RENORMALIZATION

- In lattice regularization, we renormalize a bare operator O_Γ^B via

$$O_\Gamma^B(z,a) = e^{-\delta m(a)|z|} Z_O(a) O_\Gamma^R(z)$$

Linear div. Logarithmic div.

- The $\delta m(a)$ term includes the linear divergence as well as the renormalon ambiguity. Can be computed from the interquark potential[1,2]:

$$\delta m(a) = \frac{m_{-1}(a)}{a} + m_0.$$

- The m_0 term is scheme-dependent.

[1]Bazavov et. al. Phys. Rev. D **90**, 094503
[2]Bazavov et. al. Phys. Rev. D **97**, 014510

HYBRID RENORMALIZATION

- The data at short distances ($z \lesssim 0.2$ fm) must agree with the operator product expansion (OPE):

$$\underbrace{h(z, P_z)}_{\text{Lattice data}} = \sum_{n=0}^{\infty} \frac{(-izP_z)^n}{n!} \underbrace{C_n(z^2\mu^2)}_{\text{Wilson coefficients}} \underbrace{\langle x^n \rangle(\mu)}_{\text{Mellin moments}} + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

- At $P_z = 0$, only $n = 0$ terms contribute:

$$h(z, 0) = C_0(z^2\mu^2) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

HYBRID RENORMALIZATION

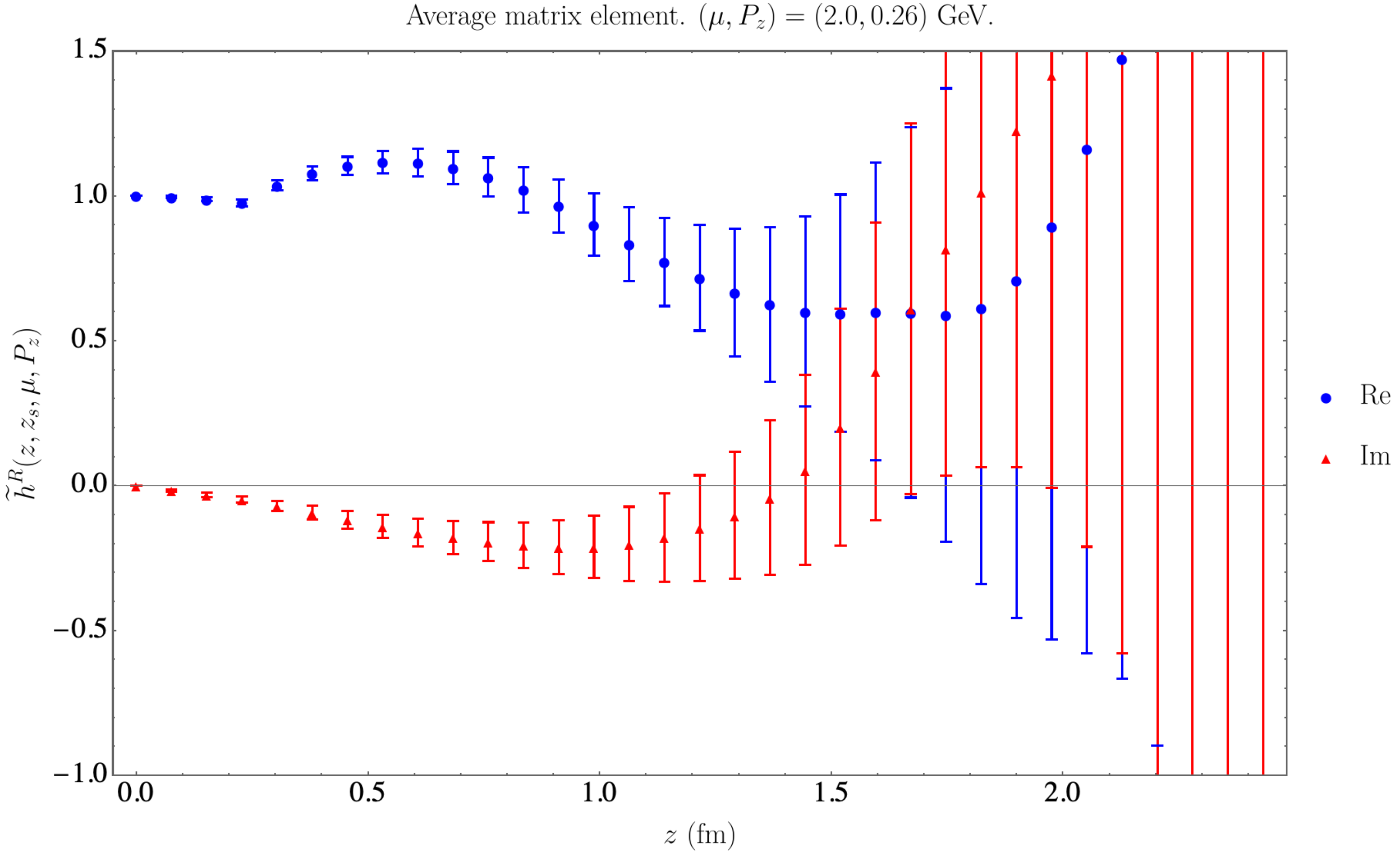
- Accounting for the scheme-conversion, linear divergence and using the OPE at $P_z = 0$:

$$e^{(\overline{m}_0+\delta m(a))z}\,\underbrace{\tilde{h}^B(z, P_z = 0)}_{\text{Lattice data}} \propto \Big(\underbrace{C_0(z^2\mu^2)}_{\text{Twist-2, } P_z = 0 \text{ Wilson coefficient}} - \underbrace{\Lambda z^2}_{\text{Higher twist effects}}\Big)$$

- $C_0(z^2\mu^2)$ can be computed “by hand”.
- The terms $(\overline{m}_0, \Lambda)$ are fitting parameters.

- Can fit the data to $P_z = 0$ Wilson coefficients since δm and the fitting parameters are independent of the external states.
- $a\delta m(a) = 0.1597(16)$[1]

[1]Refs. [69-73] in 2212.12569v1



HYBRID RENORMALIZATION

- Define $\lambda = zP_z$, $N = \frac{h^B(0,0,a)}{h^B(0,P_z,a)}$ and $\delta m' = \delta m + \overline{m}_0$.
- Ratio scheme at short distances. Remove self-energy at large distances.
- $\tilde{h}^R(z, z_s, \mu, P_z) = N \frac{\tilde{h}^B(z,P_z,a)}{\tilde{h}^B(z,0,a)} \frac{C_0(z^2\mu^2) - \Lambda z^2}{C_0(z^2\mu^2)} \theta(z_s - z) + N e^{\delta m'(z - z_s)} \frac{\tilde{h}^B(z,P_z,a)}{\tilde{h}^B(z_s,0,a)} \frac{C_0(z_s^2\mu^2) - \Lambda z_s^2}{C_0(z_s^2\mu^2)} \theta(z - z_s)$

LARGE-λ EXTRAPOLATION

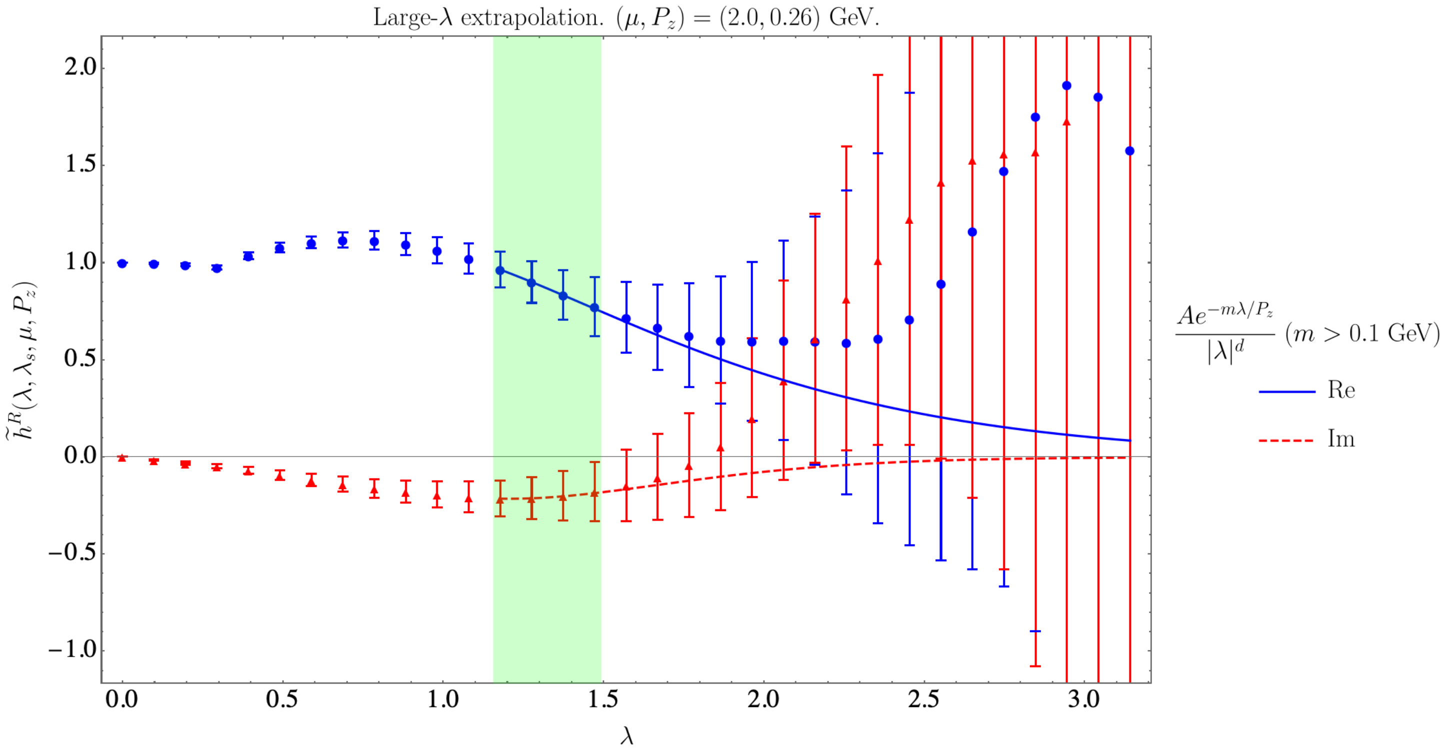
- To transform to momentum space, we require an extrapolation of $\tilde{h}^R$ to $\lambda \to \infty$.

$$\tilde{f}(x, P_z) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \tilde{h}^R(\lambda, P_z)$$

- $\tilde{h}^R(\lambda, P_z) \to \frac{Ae^{-m\lambda/P_z}}{|\lambda|^d}$ as $\lambda \to \infty$ for both $\Re$ and $\Im$[1]. Fitting parameters are A, m and d.
- Constrain $m > 0.1$ GeV to ensure suppression at large-λ.

[1]App. B2 of arXiv:2112.02208

LARGE-λ EXTRAPOLATION



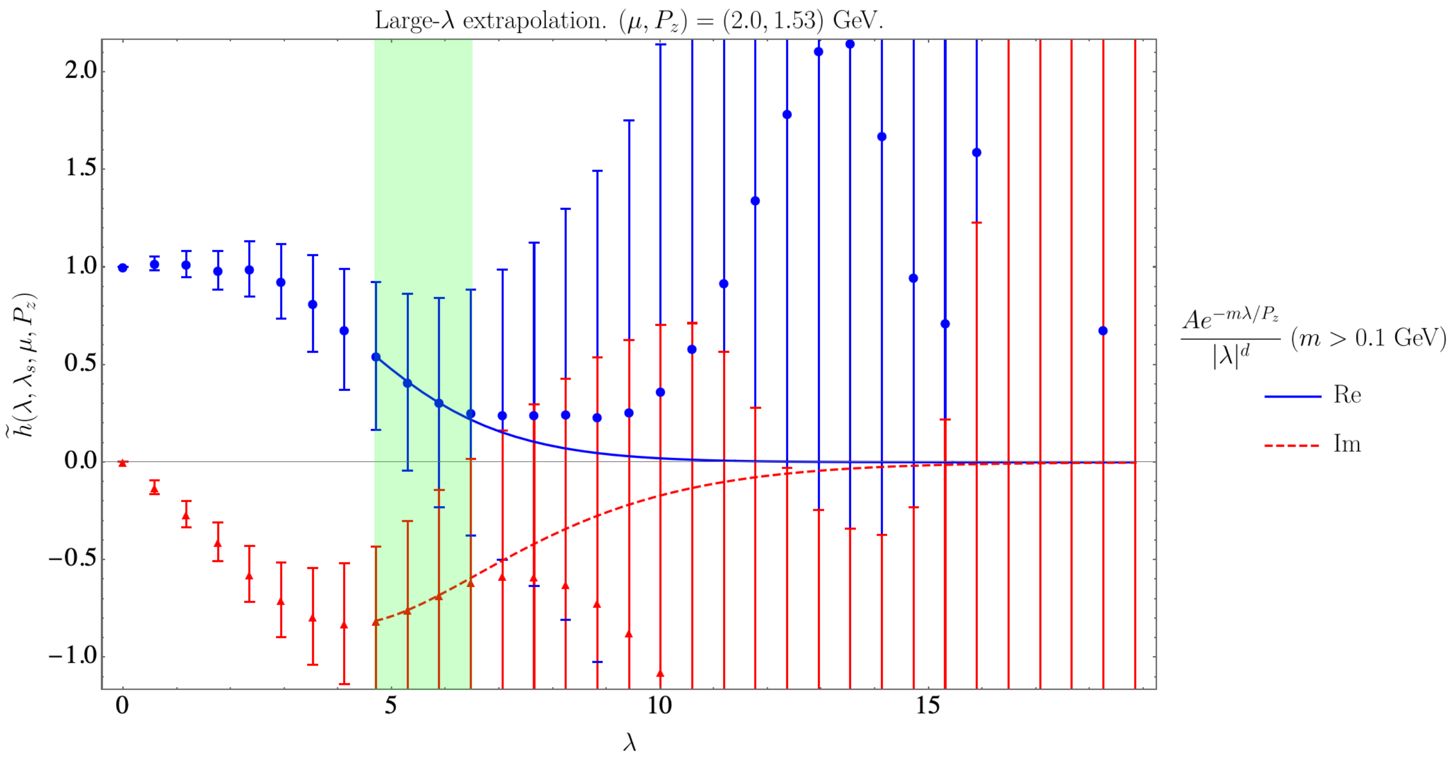
- Real and imaginary parts are each fitted separately to the extrapolation model derived from HQET.

$$\frac{Ae^{-m\lambda/P_z}}{|\lambda|^d}$$

- Green shaded area are the points used for curve-fitting.

- Choice of z-values is delicate.

LARGE-λ EXTRAPOLATION



- Larger momentum.
- Noisier signal but larger momentum necessary for LaMET.
- $(\mu, P_z) = (2.0, 1.53)\ GeV$

LIGHTCONE MATCHING

- The quasi-PDF is related to the lightcone PDF by

$$\tilde{f}(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} \mathcal{C}\left(\frac{x}{y}, \frac{\mu}{yP_z}, |y|\lambda_s\right) f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2(1-x)P_z^2}\right)$$

- We introduce the shorthand

$$\tilde{f} = \mathcal{C} \otimes f + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2(1-x)P_z^2}\right)$$

LIGHTCONE MATCHING

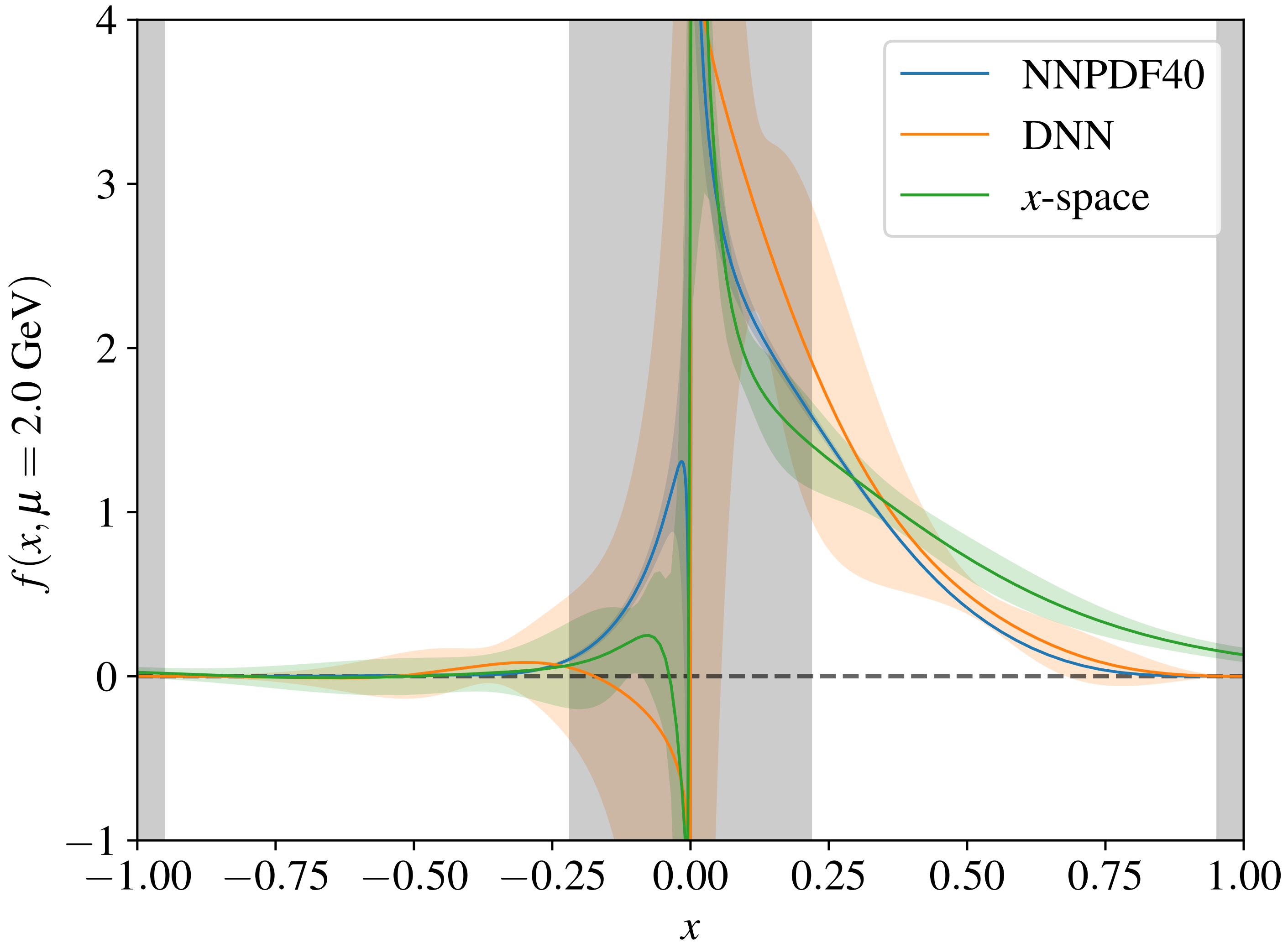
- The matching kernel can be expanded in α_s:

$$\mathcal{C}\left(\frac{x}{y}, \frac{\mu}{yP_z}, |y|\lambda_s\right) = \delta\left(\frac{x}{y} - 1\right) + \sum_{n=1}^{\infty} \alpha_s^n \mathcal{C}^{(n)}\left(\frac{x}{y}, \frac{\mu}{yP_z}, |y|\lambda_s\right)$$

and the inverse as

$$\mathcal{C}^{-1}\left(\frac{x}{y}, \frac{\mu}{yP_z}, |z|\lambda_s\right) = \delta\left(\frac{x}{y} - 1\right) - \alpha_s \mathcal{C}^{(1)}\left(\frac{x}{y}, \frac{\mu}{yP_z}, |y|\lambda_s\right)$$

$$+\alpha_s^2 \mathcal{C}^{(1)}\left(\frac{x}{z}, \frac{\mu}{zP_z}, |z|\lambda_s\right) \otimes \mathcal{C}^{(1)}\left(\frac{z}{y}, \frac{\mu}{yP_z}, |y|\lambda_s\right) - \alpha_s^2 \mathcal{C}^{(2)}\left(\frac{x}{y}, \frac{\mu}{yP_z}, |z|\lambda_s\right) + \mathcal{O}(\alpha_s^3)$$

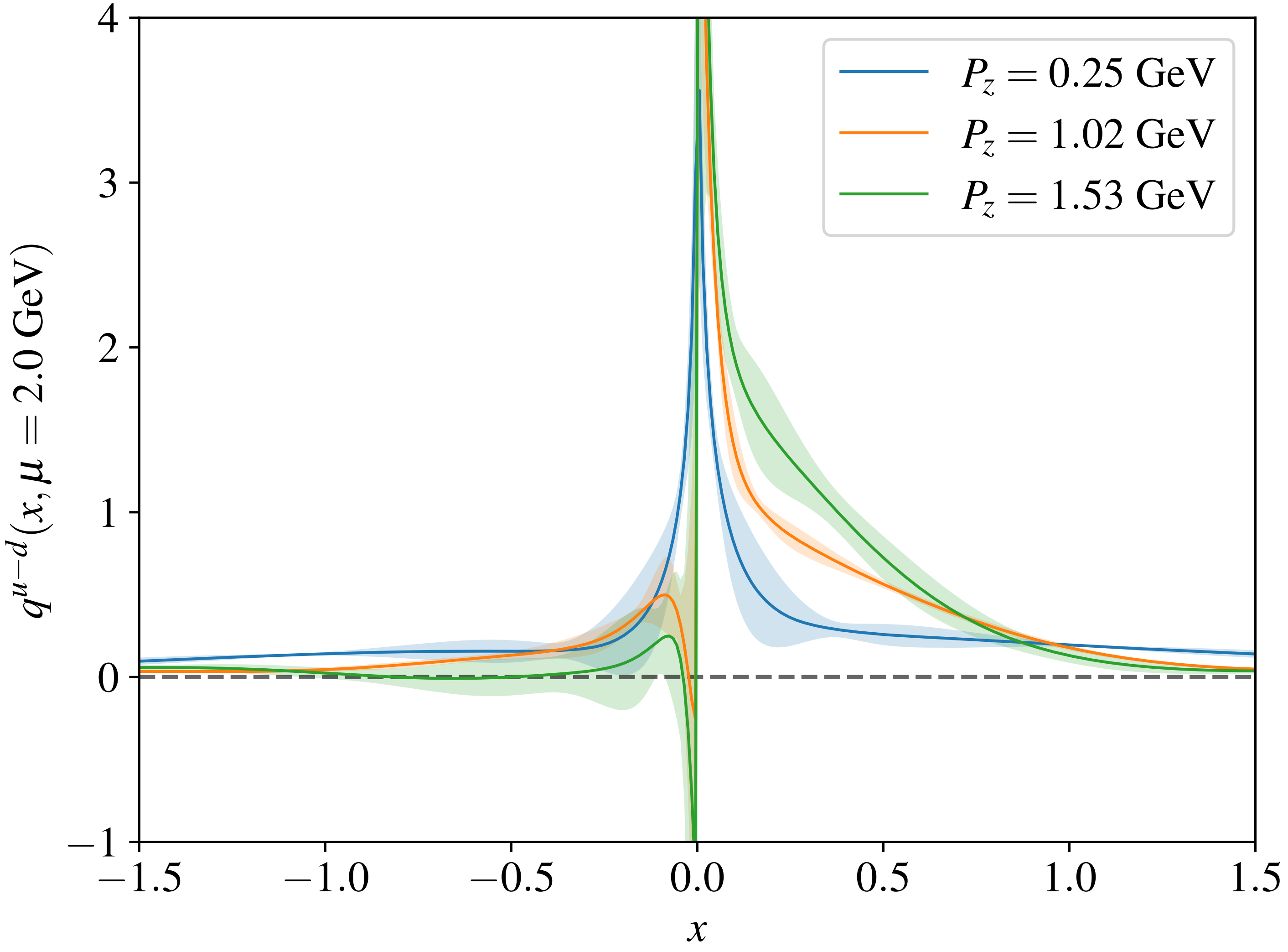


RESULTS

- Comparison of PDF as computed with LaMET, DNN and global analysis from NNPDF4.0[1].
- The gray regions are those in which the LaMET expansion breaks down.
- $f(x,\mu) = \mathcal{C}^{-1}\left(\frac{x}{y}\right) \otimes \tilde{f}(y) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2(1-x)P_z^2}\right)$.

[1]Ball et. al. Eur. Phys J. C **82**, 428 (2022)

THANK YOU.



(LC)PDF FOR DIFFERENT P_Z

- No clear convergence.
- Source of disagreement between DNN and NNPDF40?
- Could use the computation of the Mellin moments to determine the end-point behaviour like in arXiv:2209.09332?