

# Precision QCD simulations

Frank Krauss

Institute for Particle Physics Phenomenology  
Durham University

17.3.2023 – DIS – East Lansing



- introduction
- current precision
- better parton showers
- Future Experiments
- Outlook

instead of an introduction

## (executive summary)

## status: executive summary

- ✓ NNLO  $\otimes$  parton shower for colour singlet production  
(MINNLO: 1309.4634, 1407.2940, ..., 2208.12660; UNNNLOPs: 1405.4607, 1407.3773)
  - ✓ NNLO  $\otimes$  parton shower for heavy quarks  
(MINNLO: 2112.04168 ( $t\bar{t}$ ), 2302.01645 ( $b\bar{b}$ ))
  - ✓ MEPS@NLO: NLO multijet merging  
(SHERPA: 1207.5030; MADGRAPH: 1209.6215; PYTHIA: 1211.7278; HERWIG: 1705.06700 plus follow-ups & refinements)
  - ✓ all of the above including EW@NLO  
(explicit: 1511.08692, 1705.00598, ..., 2204.07652; Sudakov approximation: hep-ph/0010201, 2111.13453)
  - ✓ (N)NLO  $\otimes$  N<sup>1,2,3</sup>LL  $\otimes$  parton shower  
(GENEVA: 1211.7049, 1508.01475, 2102.08390, ...)
  - ▶ multijet merging with TMDs  
(2107.01224, 2208.02276 (not covered here))
  - ▶ improving parton showers  
((next-to leading) logarithmic accuracy (see below); amplitude evolution: 1802.08531, ... (not covered here))

## current precision

(where we are)

# fixed-order accuracy

(apologies for any omissions in active field with  $\approx 100$  publications/past 5 years)

- N<sup>3</sup>LO for single-boson production  
for DIS, and for VBF  $H$ -production in double DIS  
(1503.06056 ... 1802.00833, 1807.11501)  
(1803.09973; 1606.00840)
- NNLO for practically all  $2 \rightarrow 2$  (and some  $2 \rightarrow 3$ ) processes:
  - $jj$   
(1705.10271, 1905.09047, ...)
  - $Vj, \gamma j, Hj$   
(1408.5325, 1504.02131, 1504.07922, 1505.03893, 1705.04664, 1901.11041, 1905.13738, ...)
  - $tt$  & single top  
(1303.6254, 1511.00549; 1404.7116, ...)
  - $VV$  and  $\gamma\gamma$   
(1408.5243, 1504.01330, 1507.06257, 1604.08576, 1605.02716, 1708.02925, 1711.06631, ...)
  - $VBF$   
(1506.02660, 1802.02445, ...)
  - dijets in DIS  
(1804.05663, ...)
- virtual  $2 \rightarrow \geq 3$  amplitudes  
(1511.05409, 1511.09404, 1604.06631, 1712.02229, 1811.11699, ...)
- relative size argument:  $\alpha_s^2 \approx \alpha_W$ :  
must include NLO EW corrections for  $\mathcal{O}(1 - 10\%)$  accuracy  
⇒ automated in OPENLOOPS, RECOLA, aMC@NLO \_ MADGRAPH

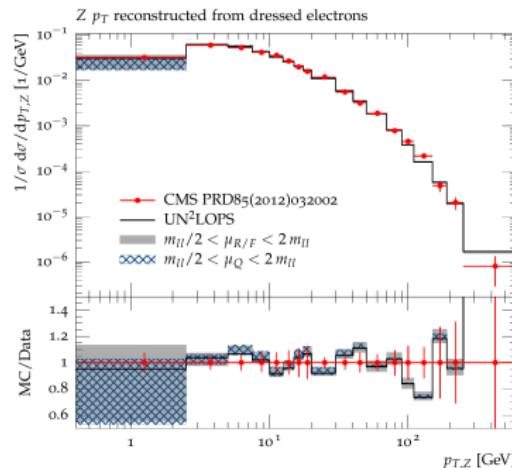
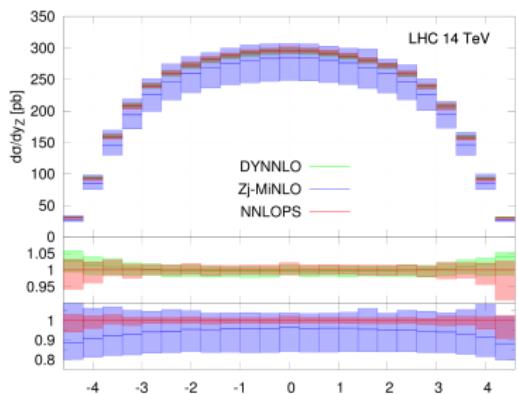
(1705.00598, 1704.05783, 1405.0301)

# SM precision simulation in a nutshell: Drell-Yan

- current “accuracy standard(s)”:
  - fixed-order:  $N^3\text{LO}$  for inclusive,  $\text{NNLO}$  for  $Vj$
  - matching:  $\text{NNLOPS}$  for inclusive  $V$
  - merging:  $\text{MEPS@NLO}$  for  $V + \leq 2$  jets at  $\text{NLO}$   $V + \geq 3$  jets at  $\text{LO}$
- dominating QCD effects:  $\mathcal{O}(10\text{-}30\%)$ 
  - low- $p_\perp$  region dominated by parton shower
  - high- $p_\perp$  region dominated by (multi-) jet topologies
  - higher accuracy in rate (and some shapes) through  $\text{NNLO}$  matching
- must add EW corrections for %-level precision
  - EW correction at large scales  $\mathcal{O}(10\%)$
  - QED FSR + EW for  $V$  line shapes at  $\mathcal{O}(1\%)$

# NNLOPs for $Z$ production: MINNLO & UNNLOPs

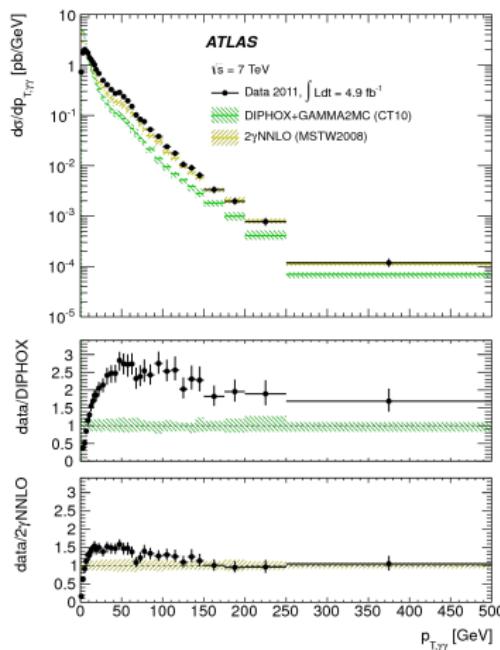
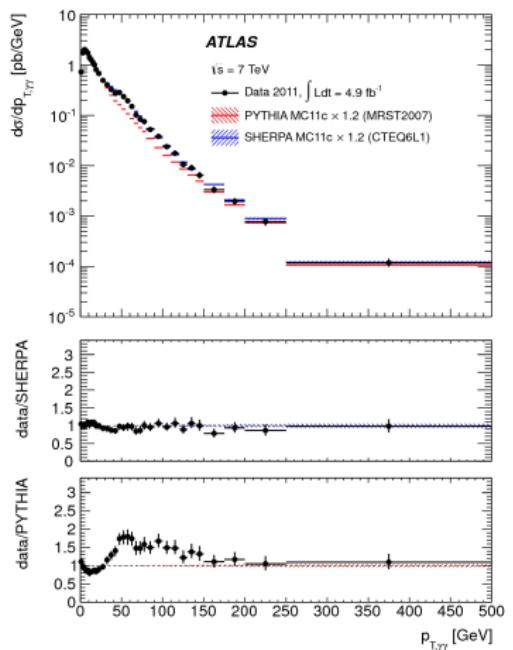
(1407.2904, 1405.3607)



- different logic of achieving NNLO precision
- available for  $H$ ,  $V$  production (both) and  $VV$  production (MINNLO)

# merging example: $p_{\perp,\gamma\gamma}$ in MEPS@LO vs. NNLO

(arXiv:1211.1913 [hep-ex])



# better parton showers?

(the story never gets old)

# parton showers, compact notation

- Sudakov form factor (**no-decay** probability)

$$\Delta_{ij,k}^{(\mathcal{K})}(t, t_0) = \exp \left[ - \int_{t_0}^t \frac{dt}{t} \frac{\alpha_s}{2\pi} \int dz \frac{d\phi}{2\pi} \underbrace{\mathcal{K}_{ij,k}(t, z, \phi)}_{\text{splitting kernel for}} \right]$$

$(ij) \rightarrow ij$  (spectator  $k$ )

- evolution parameter  $t$  defined by kinematics

generalised angle (HERWIG++) or transverse momentum (PYTHIA, SHERPA)

- will replace  $\frac{dt}{t} dz \frac{d\phi}{2\pi} \rightarrow d\Phi$  (subtle differences important for theoretical accuracy (LL vs. NLL)!)
- scale choice for strong coupling:  $\alpha_s(k_\perp^2)$  resums classes of higher logarithms
- regularisation through cut-off  $t_0$  scale for onset of non-perturbative effects (hadronization)!

# factorisation of amplitudes: colour coherence

- collinear:

$${}_n\langle 1, \dots, n | 1, \dots, n \rangle_n \xrightarrow{i \parallel j} \sum_{\lambda, \lambda' = \pm} {}_{n-1}\Big\langle 1, \dots, \cancel{\lambda}(ij), \dots, \cancel{\lambda}, \dots, n \Big| \frac{8\pi\alpha_s}{2p_i p_j} P_{(ij)i}^{\lambda\lambda'}(z) \Big| 1, \dots, \cancel{\lambda}(ij), \dots, \cancel{\lambda}, \dots, n \Big\rangle_{n-1}$$

with spin-dependent splitting function  $P_{(ij)i}^{\lambda\lambda'}(z)$

- soft:

$${}_n\langle 1, \dots, n | 1, \dots, n \rangle_n \xrightarrow{p_j \rightarrow 0} -8\pi\alpha_s \sum_{i, k \neq j} {}_{n-1}\langle 1, \dots, \cancel{\lambda}, \dots, n | \mathbf{T}_i \mathbf{T}_k w_{ik,j} | 1, \dots, \cancel{\lambda}, \dots, n \rangle_{n-1}$$

with colour-insertion operators  $\mathbf{T}_{i,k}$  & soft eikonal

$$w_{ik,j} = \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2} = \frac{1}{E_j^2} \frac{1 - \cos \theta_{ik}}{(1 - \cos \theta_{ij})(1 - \cos \theta_{jk})}$$

(obviously, frame-dependent when expressed by energies & angles)

# soft eikons, decomposed

- textbook decomposition (pink bible):  $W_{ik,j} = \tilde{W}_{ik,j}^i + \tilde{W}_{ki,j}^k$   
 with “radiator functions”  $\tilde{W}_{ik,j}^i$ : (identify “splitters” to combine with collinear terms)

$$\tilde{W}_{ik,j}^i = \frac{1}{2} \left( \frac{1 - \cos \theta_{ik}}{(1 - \cos \theta_{ij})(1 - \cos \theta_{jk})} + \frac{1}{1 - \cos \theta_{ij}} - \frac{1}{1 - \cos \theta_{jk}} \right)$$

- express  $\theta_{jk}$  for use in  $i$ -splitter term:

$$\cos \theta_{jk} = \cos \theta_{ij} \cos \theta_{ik} + \sin \theta_{ij} \sin \theta_{ik} \cos \phi_{jk}^i \dots$$

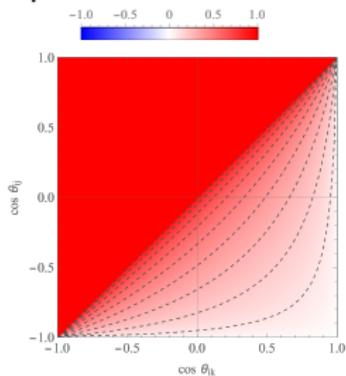
- ... and average over azimuth  $\phi_{jk}^i$ :

$$\frac{1}{2\pi} \int_0^{2\pi} d\phi_{jk}^i \tilde{W}_{ik,j}^i = \frac{\tilde{I}_{ik,j}^i}{1 - \cos \theta_j^i}, \quad \text{where} \quad \tilde{I}_{ik,j}^i = \begin{cases} 1 & \text{if } \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

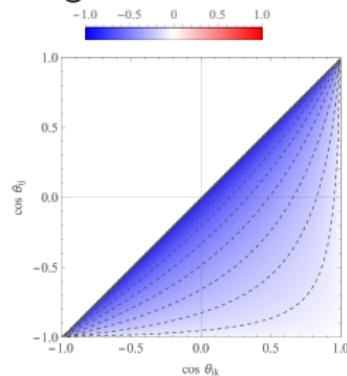
(this is the well-known source of angular ordering)

- azimuthally integrated radiator function (normalised to  $2\pi$ ):

positive contributions



negative contributions



- need to include azimuth modulation, if observables sensitive to it
- but: naive inclusion bound to fail (MC efficiency  $\rightarrow 0$ )

# a new approach: PANSCALES

(resolving problems)

# issues with existing/frequently used parton showers

- comparison with fixed-order results

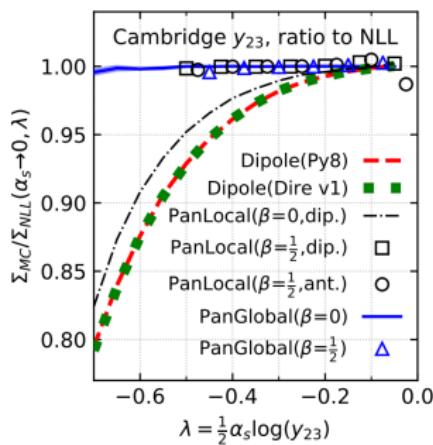
(PANSIMES: 1805.09327, 2205.02237)

# issues with existing/frequently used parton showers

- comparison with fixed-order results
- logarithmic accuracy in FS showering

(PANSCALES: 1805.09327, 2205.02237)

(PANSCALES: 2002.11114, 2011.10054)



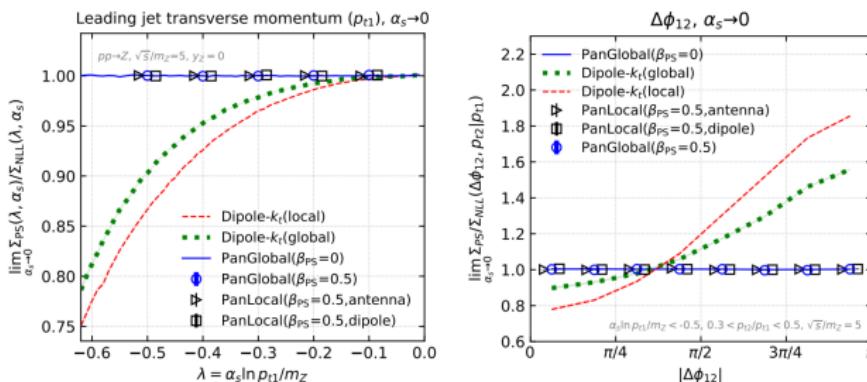
# issues with existing/frequently used parton showers

- comparison with fixed-order results
- logarithmic accuracy in FS showering
- logarithmic accuracy in IS showering

(PANSCALES: 1805.09327, 2205.02237)

(PANSCALES: 2002.11114, 2011.10054)

(PANSCALES: 2207.09467)



# issues with existing/frequently used parton showers

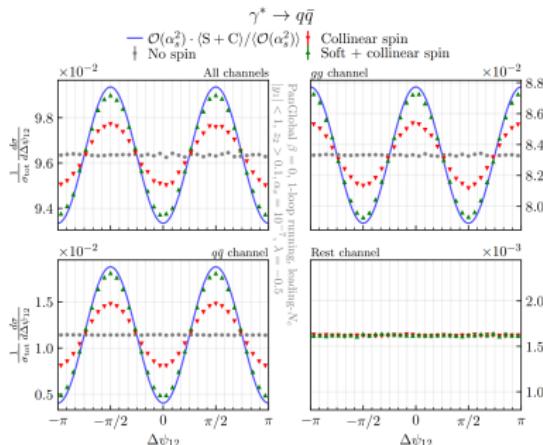
- comparison with fixed-order results
- logarithmic accuracy in FS showering
- logarithmic accuracy in IS showering
- spin correlations

(PANSCALES: 1805.09327, 2205.02237)

(PANSCALES: 2002.11114, 2011.10054)

(PANSCALES: 2207.09467)

(PANSCALES: 2103.16526, 2111.01161)



# issues with existing/frequently used parton showers

- comparison with fixed-order results (PANSCALES: 1805.09327, 2205.02237)
- logarithmic accuracy in FS showering (PANSCALES: 2002.11114, 2011.10054)
- logarithmic accuracy in IS showering (PANSCALES: 2207.09467)
- spin correlations (PANSCALES: 2103.16526, 2111.01161)
- ✓ PANSCALES encodes massless parton showers at NLL accuracy (1<sup>st</sup> matching in  $e^+e^-$ : 2301.09645)

# a new approach: ALARIC

(progress in SHERPA)

# soft eikonal, decomposed again

(2208.06057)

- define **positive definite** radiators:

(borrowing from Catani & Seymour, Nucl. Phys. B485 (1997) 291)

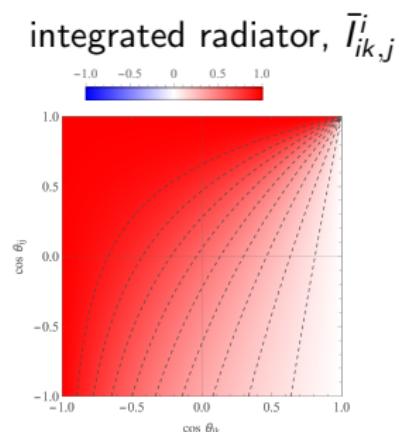
$$\bar{W}_{ik,j}^i = \frac{1 - \cos \theta_{ik}}{(1 - \cos \theta_{ij})(2 - \cos \theta_{ij} - \cos \theta_{jk})}$$

- same result after azimuth averaging,  
but  $\bar{l}_{ik,j}^i \rightarrow \bar{l}_{ik,j}^i$  with

$$\bar{l}_{ik,j}^i = \frac{1}{\sqrt{(\bar{A}_{ij,k}^i)^2 - (\bar{B}_{ij,k}^i)^2}}$$

where

$$\bar{A}_{ij,k}^i = \frac{2 - \cos \theta_j^i (1 + \cos \theta_k^i)}{1 - \cos \theta_k^i}, \quad \bar{B}_{ij,k}^i = \frac{\sqrt{(1 - \cos^2 \theta_j^i)(1 - \cos^2 \theta_k^i)}}{1 - \cos \theta_k^i}$$



# matching with collinear terms

- collinear limit of eikonal factors:

$$w_{ik,j} \xrightarrow{i||j} w_{ik,j}^{(\text{coll})}(z) = \frac{1}{2p_i p_j} \frac{2z}{1-z}, \quad \text{where} \quad z \xrightarrow{i||j} \frac{E_i}{E_i + E_j}$$

- compare with leading  $(1-z)$ -terms of splitting functions

( $1/z$  term in  $g \rightarrow gg$  captured with other "dipole")

$$P_{qq}(z) = C_F \left( \frac{2z}{1-z} + (1-z) \right),$$

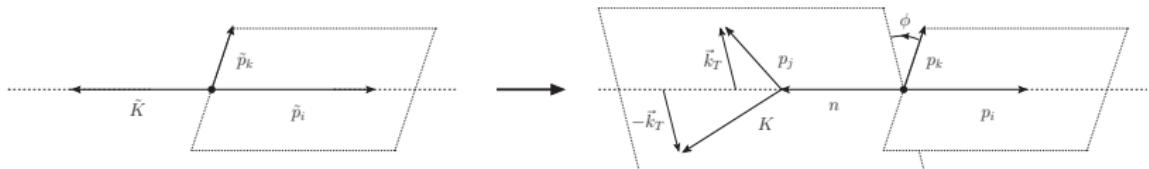
$$P_{gg}(z) = C_A \left( \frac{2z}{1-z} + z(1-z) \right),$$

$$P_{gq}(z) = T_R (1 - 2z(1-z)).$$

→ defines "collinear remnant"

# kinematics: birds-eye view

- kinematics as main obstacle to NLL accuracy in dipole showers:  
recoil of subsequent soft emissions may change “NLL history”
- construct new mapping  $\{\tilde{p}_I\} \longrightarrow \{p_I\}$   
(inspired by Catani & Seymour's treatment of identified hadrons)
- logic: disentangle colour spectator  $\tilde{p}_k$  and recoil partner  $\tilde{K}$   
(i.e. define a global recoil scheme, use spectator for eikonal/azimuth)

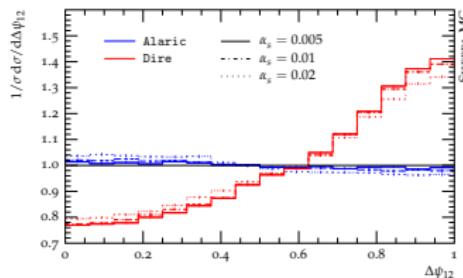


- basis to prove NLL accuracy analytically

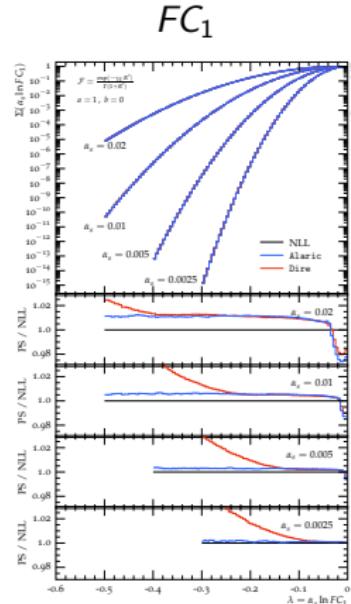
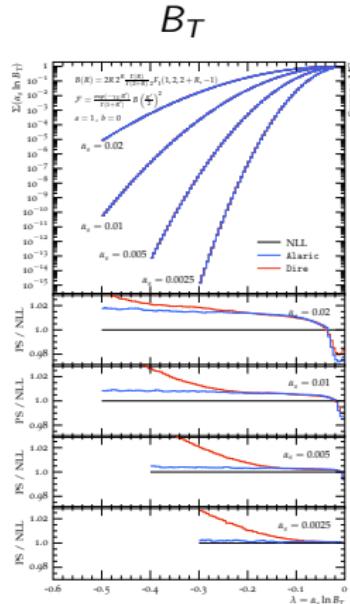
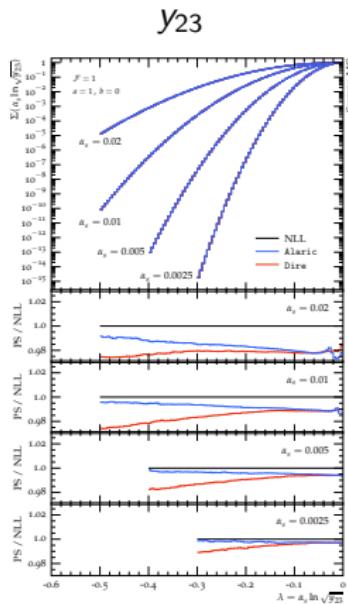
# set-up of numerical tests

- compare results in  $\alpha_S \rightarrow 0$  limit with NLL result
- set-up for checks
  - fixed  $\alpha_S$
  - leading colour  $C_A = 2C_F = 3$
  - all partons massless
- example: azimuth angle between two leading Lund-plane declusterings

(should be  $\Delta\Psi_{12} = 0$ )



# numerics: event shapes



# how about data?

(always nice to see practical impact, innit?)

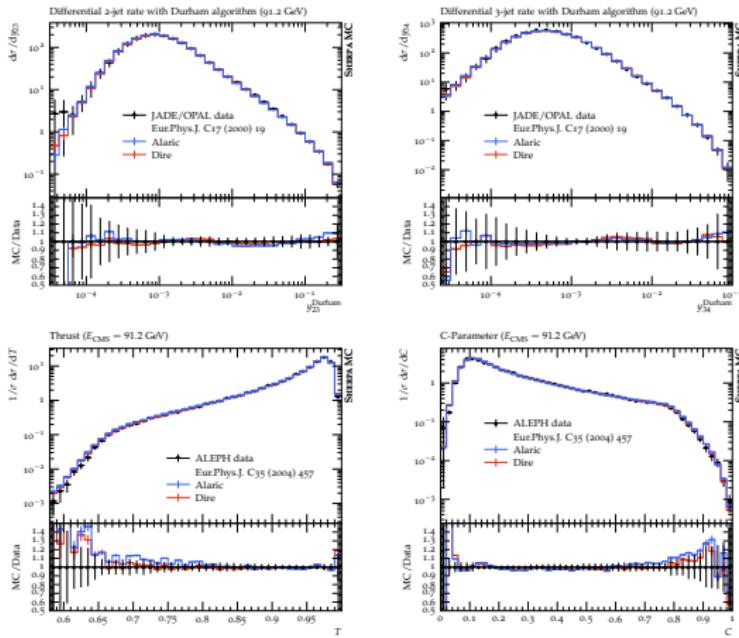
## set-up of data comparison

- compare hadron-level results with LEP data
- perturbative set-up
  - no higher orders (no matching or merging)
  - running two-loop  $\alpha_s$  with  $\alpha_s(M_Z) = 0.118$
  - use CMW scheme for soft eikonal parts
  - all partons massless, masses emulated through simplistic thresholds
  - leading colour  $C_A = N_c = 3$ ,  $C_F = \frac{N_c^2 - 1}{2N_c}$
- non-perturbative set-up
  - need to use PYTHIA hadronization

(ALARIC not yet ready for heavy hadron decays)

- default parameters of PYTHIA 6.4, but  
 $\text{PARJ}(21) = 0.3$ ,  $\text{PARJ}(41) = 0.4$ ,  $\text{PARJ}(42) = 0.36(\text{ALARIC})/0.45(\text{DIRE})$

# differential jet rates & event shapes



# MC4EIC?

(preparing for the future)

# precision QCD studies at HERA

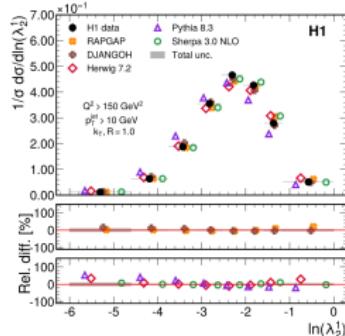
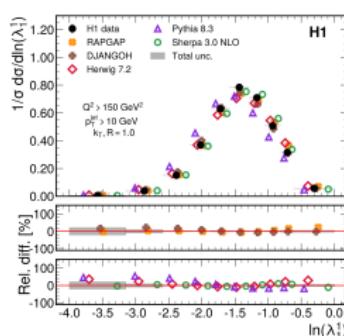
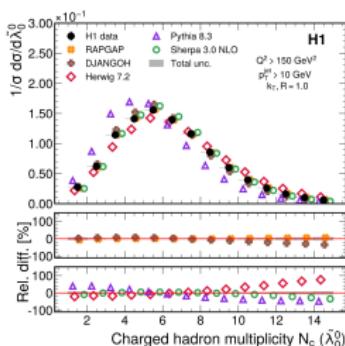
- “old” HERA data and analysis as boot-camp for EIC
- HERA = unique test-bed for (non-)perturbative QCD:
  - large- $Q^2$  DIS has no MPI → initial state showering “clean”
  - large- $Q^2$  DIS has no MPI → beam fragmentation “clean”
  - add HERA data to hadronization tunes?
- also: large photo-production cross section:
  - test hadronic structure of photon (relevant for EIC)
  - nota bene: last fits of photon-PDF are 20 years old
  - new fits urgently needed for EIC

(that is, if we want to treat collinear factorisation as limiting case for TMD's etc..)

# precision QCD studies at HERA

- incidentally, paper today by H1
- uses modern MC's (HERWIG 7, PYTHIA 8, SHERPA)

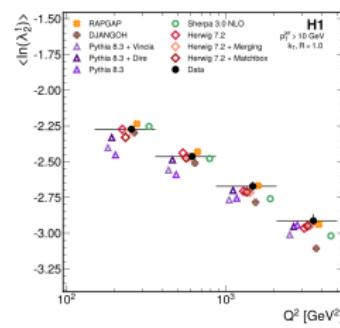
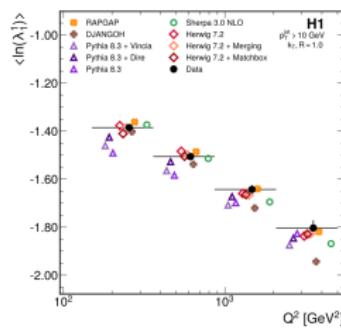
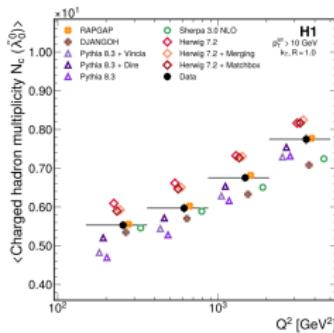
2303.13620



# precision QCD studies at HERA

- incidentally, paper today by H1
- uses modern MC's (HERWIG 7, PYTHIA 8, SHERPA)

2303.13620

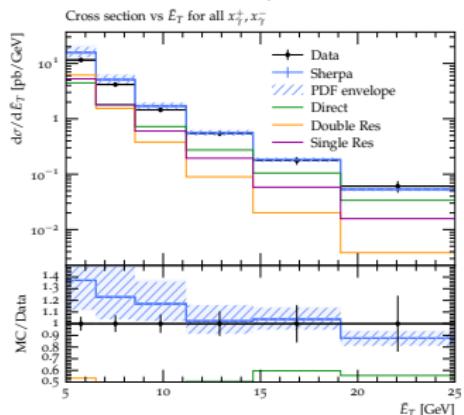


# resolved photon processes

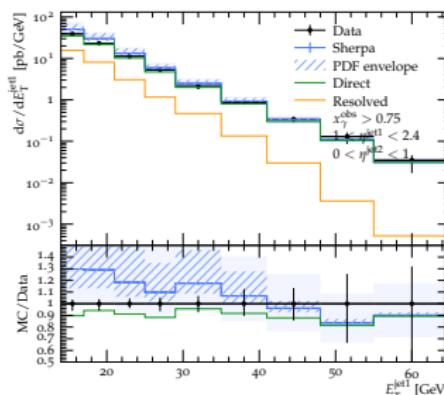
- “resolved” photons (i.e. with QCD structure/PDF) at LEP & HERA
- fits date from early 2000’s: can supplement with NLO MC machinery
- first steps (SHERPA) below

(for more details, P.Meinzingers talk tomorrow)

OPAL, hep-ex/0301013



ZEUS, hep-ex/0112029

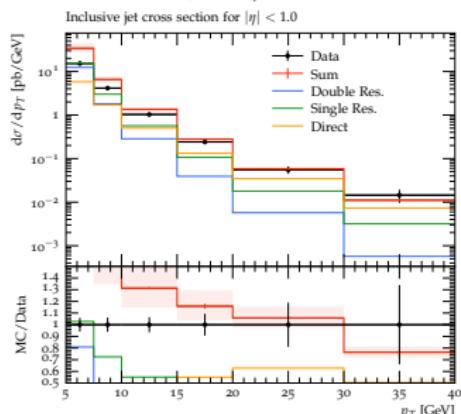


# resolved photon processes

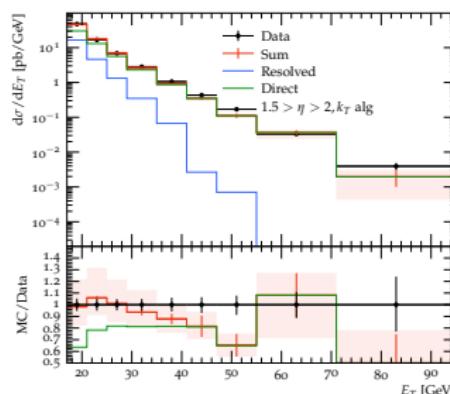
- “resolved” photons (i.e. with QCD structure/PDF) at LEP & HERA
- fits date from early 2000’s: can supplement with NLO MC machinery
- first steps towards NLO (SHERPA) below

(for more details, P.Meinzingers talk tomorrow)

OPAL, hep-ex/0706.4381



ZEUS, hep-ex/1205.6153



## summary & outlook

(the now and the future)

# summary

- current accuracy standard(s) well established & tested:
  - ✓ NNLO  $\otimes$  PS for inclusive observables
  - ✓ MEPS@NLO for multijet observables
- parton showers impediment to improved accuracy
  - ✓ prove NLL accuracy for  $\mathcal{O}(\alpha_s)$  kernels
  - ▶ embed  $\mathcal{O}(\alpha_s)$  kernels
    - (1705.00742, 1705.00982, 1805.03757)
      - (to allow seamless NNLO matching & merging a la MC@NLO)
    - ▶ consistent treatment of quark masses
      - (example: impact of quark masses on  $p_\perp^{(W)} / p_\perp^{(Z)}$ ?)
  - how about non-perturbative effects? (hadronization, MPI's, etc.)
  - higher twist (i.e. corrections of the form  $\Lambda_{QCD}/Q$ ): per-cent level for Drell-Yan?

# future collider concerns & suggestions

## ✓ simulations for LHC in healthy shape

(ongoing progress in hunt for ultimate perturbative accuracy, " getting more 'N's' into game")

## ✗ not quite true for EIC:

- very limited experience of DIS community with modern MC tools
- photo-production important part of cross section, but:
  - last fit for photon PDF from beginning 2000's
  - no systematic tune of MPI's in  $\gamma p$  collisions
  - no systematic tune of forward fragmentation