Current status and future prospects of using Lattice-QCD inputs in the global analysis

Tie-Jiun Hou
University of South China
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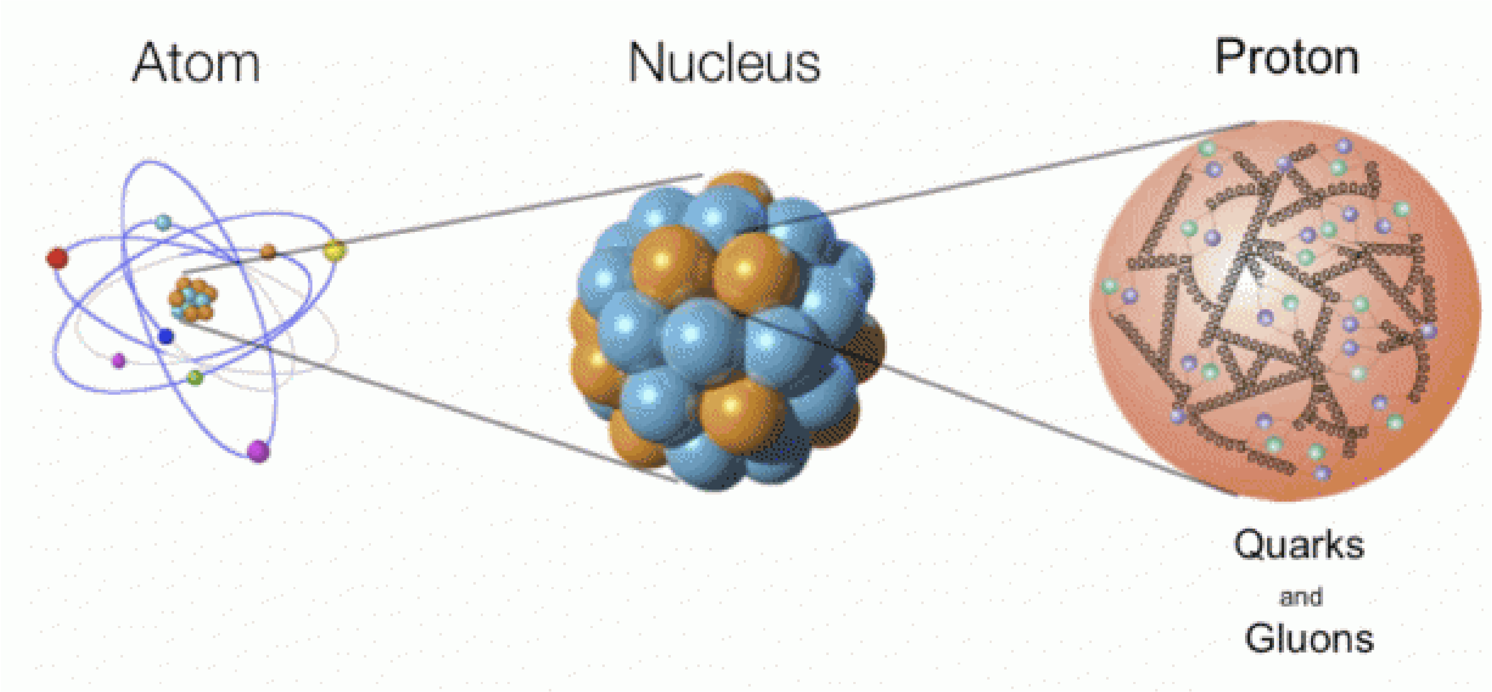
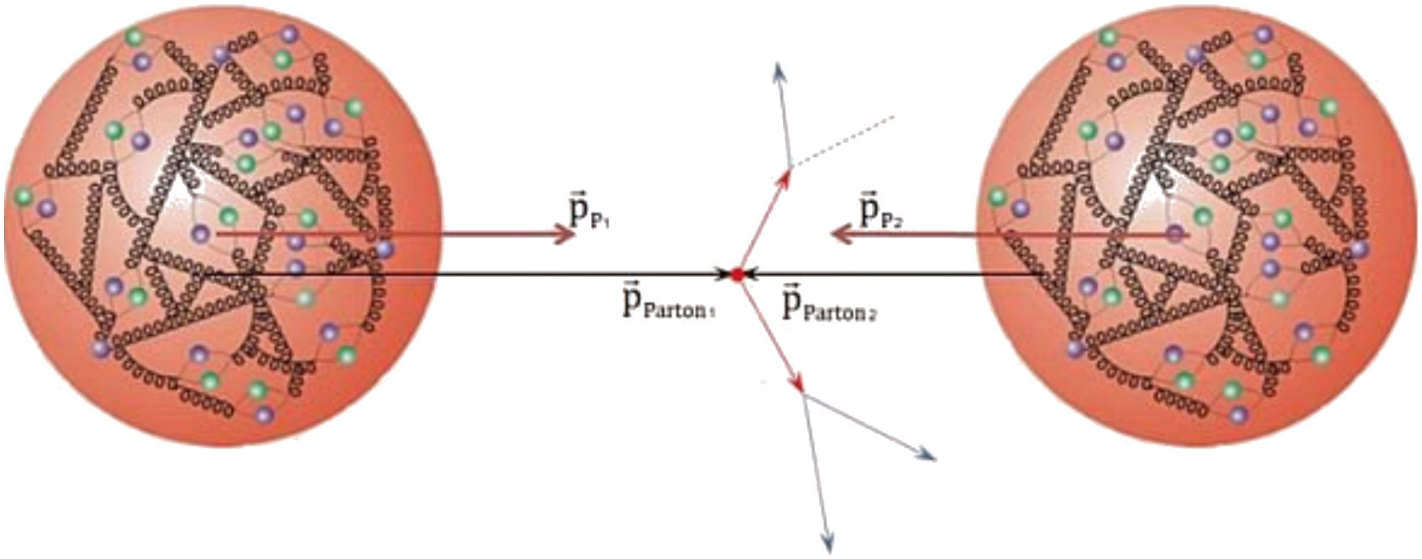
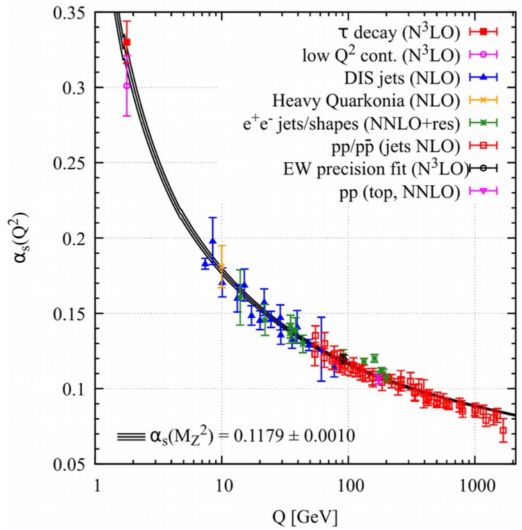
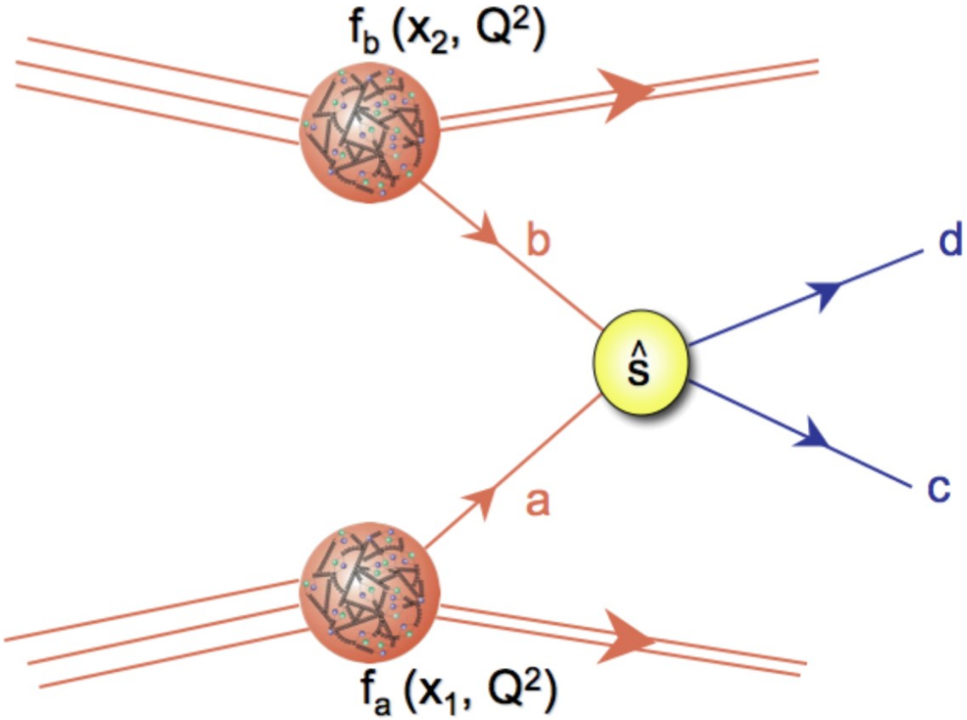
Long desire of knowing fundamental elements and structure of everything





- The long distance part of the hadron interaction suffers from non-perturbativity and IR singularity.

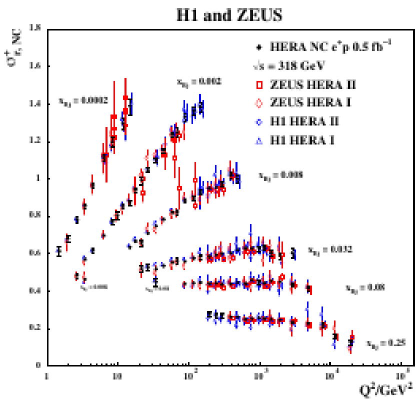


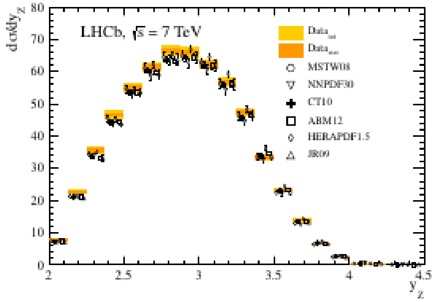


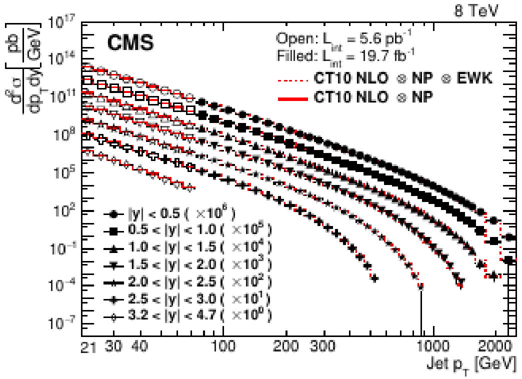
Because of confinement, the probe of proton structure needs **factorization** of short distance, pQCD calculable, parton scattering from long distance, nonperturbative, absorbing IR singular, Parton Distribution Functions (PDFs).
[Collins, Soper, Sterman, 1989]

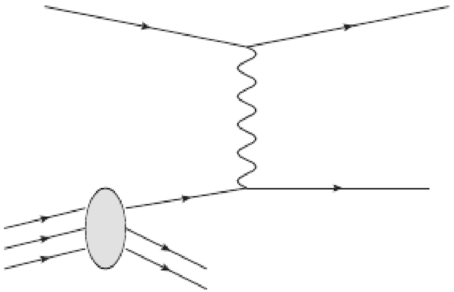
$$\sigma = f_a(x_1, Q^2) \otimes f_b(x_2, Q^2) \otimes \hat{\sigma}$$

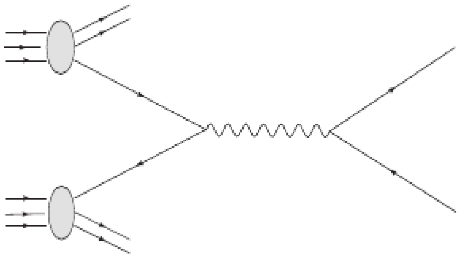
Universal

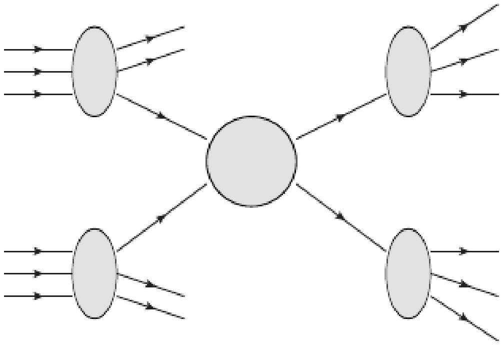


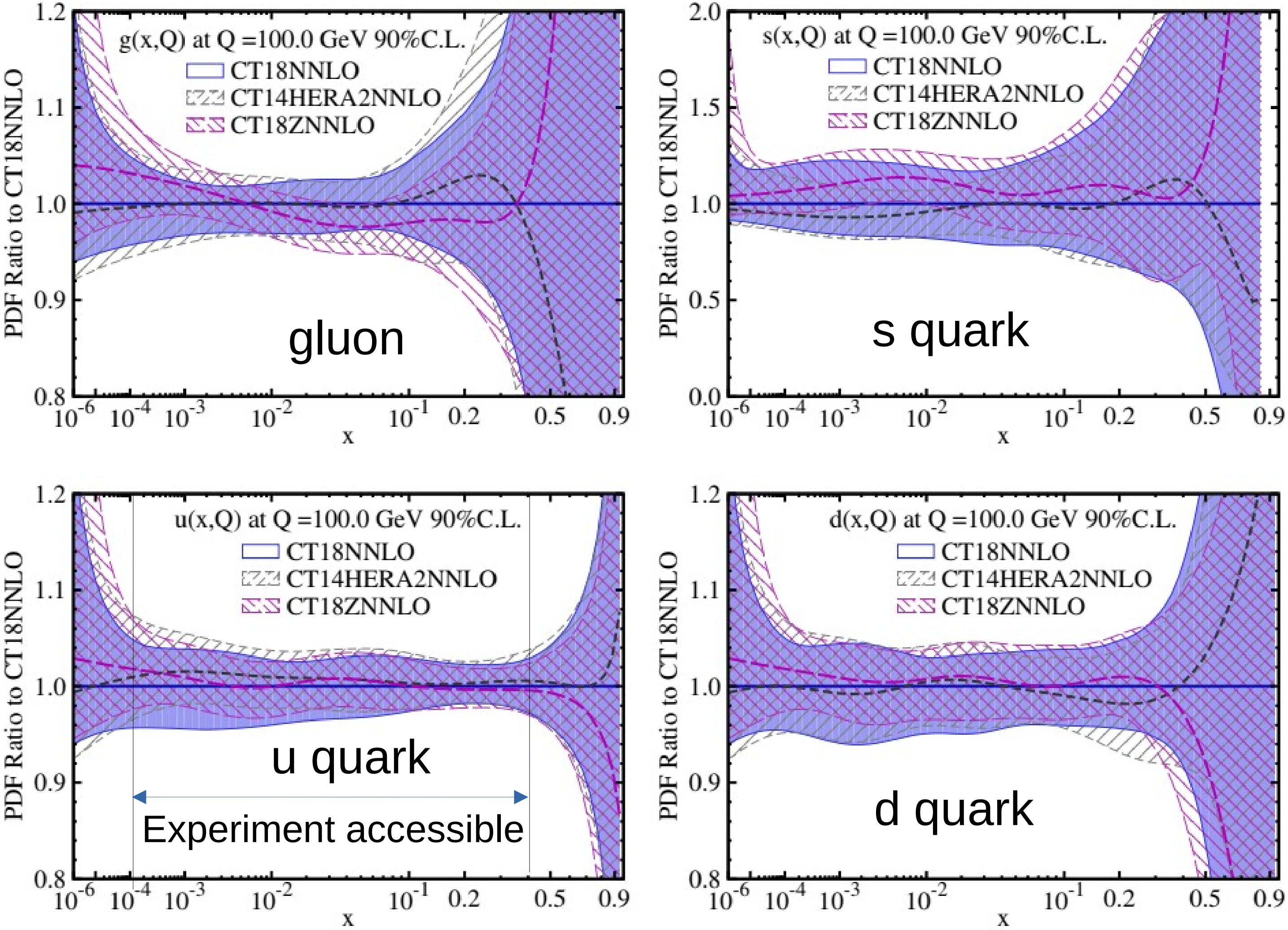










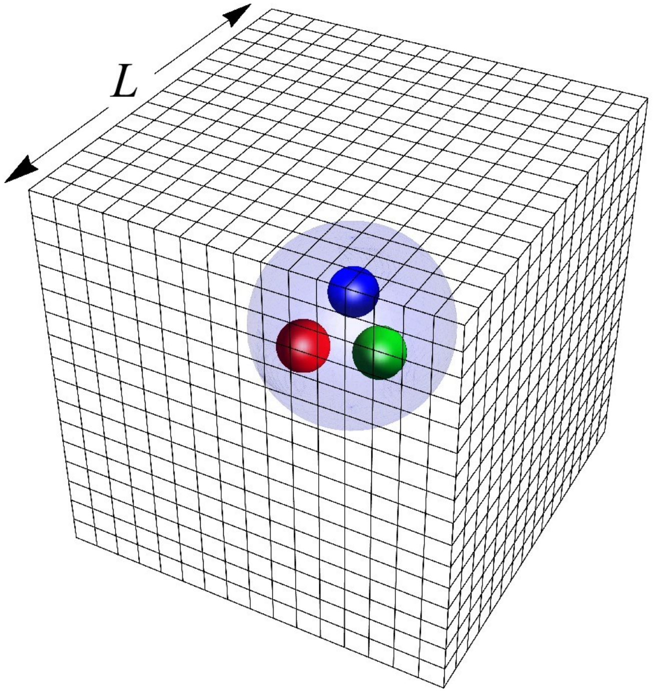


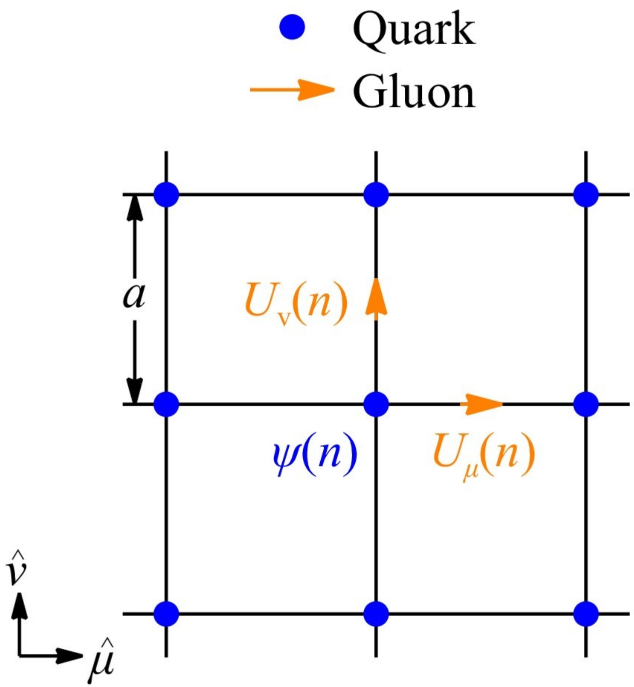
- PDFs are well determined in "middle-x" region:
- $10^{-4} \lesssim x \lesssim 0.4$.
- Region of x⟶1 and x⟶0 are not experimental accessible.
- Rather large uncertainty for strangeness PDF, especially in large x region.

[Hou et al, 1912.10053]

Do we have way out from first principle?

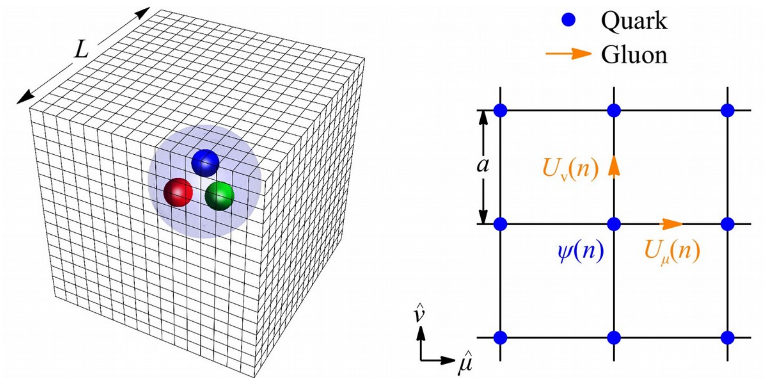
Yes! Lattice QCD!





- Lattice QCD handles strong interaction on 4D Euclidean lattice by first principle.
- Fruitful observable can be worked out by path-integral calculation in Lattice QCD method.

- Earlier lattice calculations rely on operator product expansion, only provide moments $\langle x^n \rangle$, where $\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$
- Ideally, x-dependent information of PDFs can be obtained by full moments.
- However, for higher moments, the operators mix with lower-dimension operators, only lower moments are reliable.



- The common feature of all the approaches to the lattice PDF calculations is that they rely to some extent on the factorization framework:

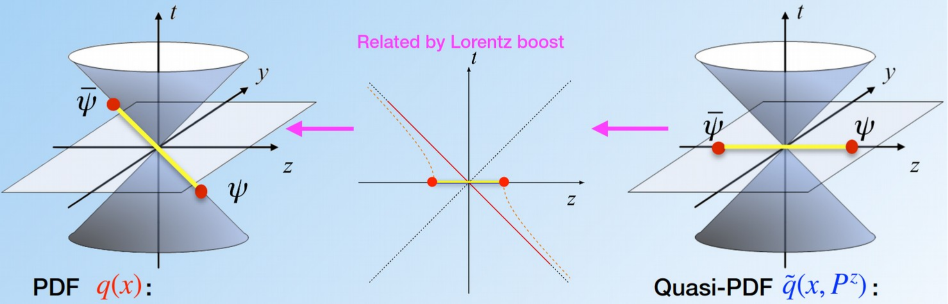
$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_R,\mu_F\right) \underbrace{q(y,\mu_F)}_{\text{PDF we want to extract}},$$

- Two classes of approaches:
  - ⋇ generalizations of light-cone functions; direct x-dependence,
  - ⋇ hadronic tensor; structure functions.

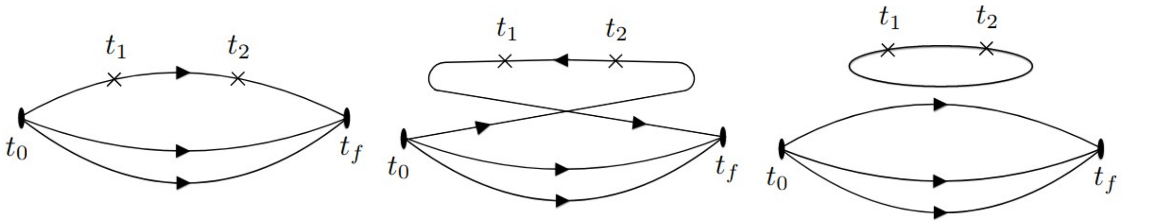
- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ, Γ' Dirac structure and objects $F(z)$.
  - ⋇ **hadronic tensor** - K.-F. Liu, S.-J. Dong, 1993
  - ⋇ **auxiliary scalar quark** - U. Aglietti et al., 1998
  - ⋇ **auxiliary heavy quark** - W. Detmold, C.-J. D. Lin, 2005
  - ⋇ **auxiliary light quark** - V. Braun, D. Müller, 2007

  $\Rightarrow$ structure functions
  - ⋇ **quasi-distributions** - X. Ji, 2013
  - ⋇ ”**good lattice cross sections**” - Y.-Q. Ma, J.-W. Qiu, 2014, 2017
  - ⋇ **pseudo-distributions** - A. Radyushkin, 2017
  - ⋇ ”**OPE without OPE**” - QCDSF, 2017

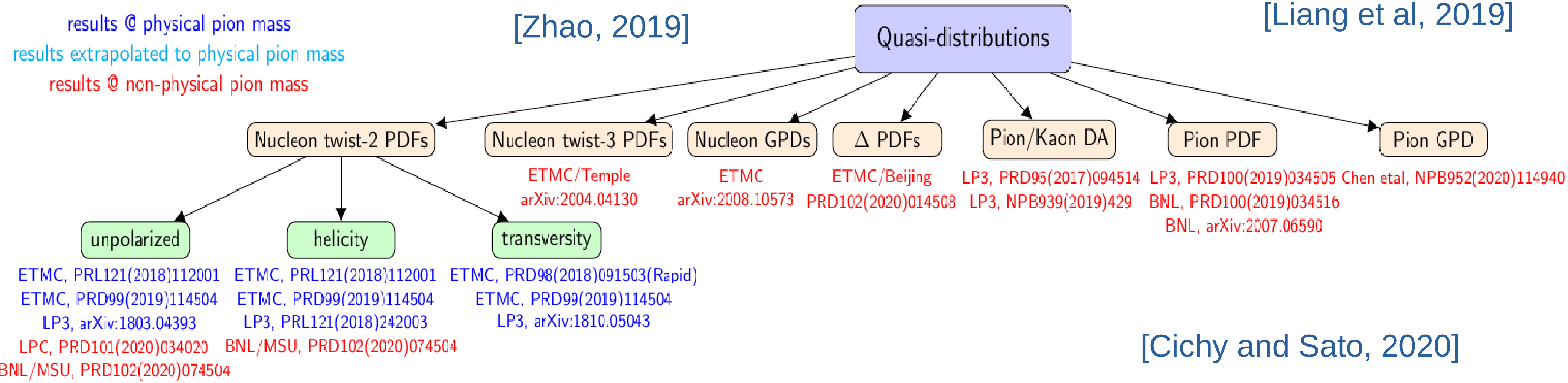
[Cichy and Sato, 2020]



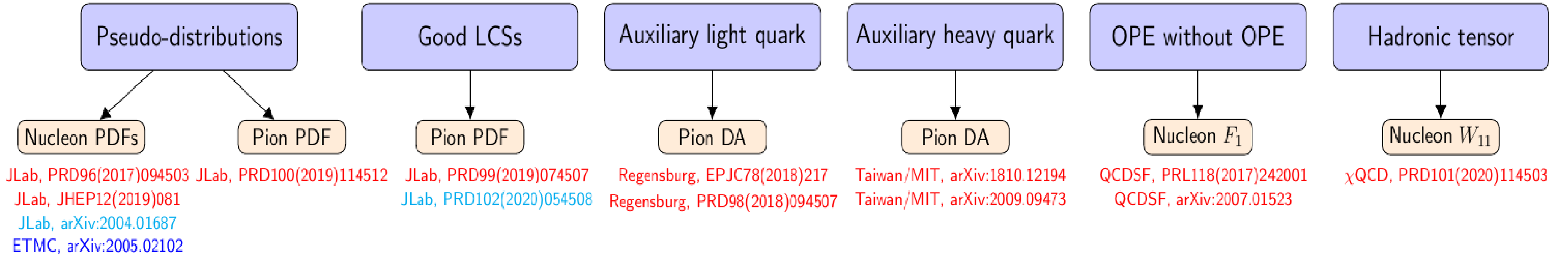
[Zhao, 2019]



[Liang et al, 2019]



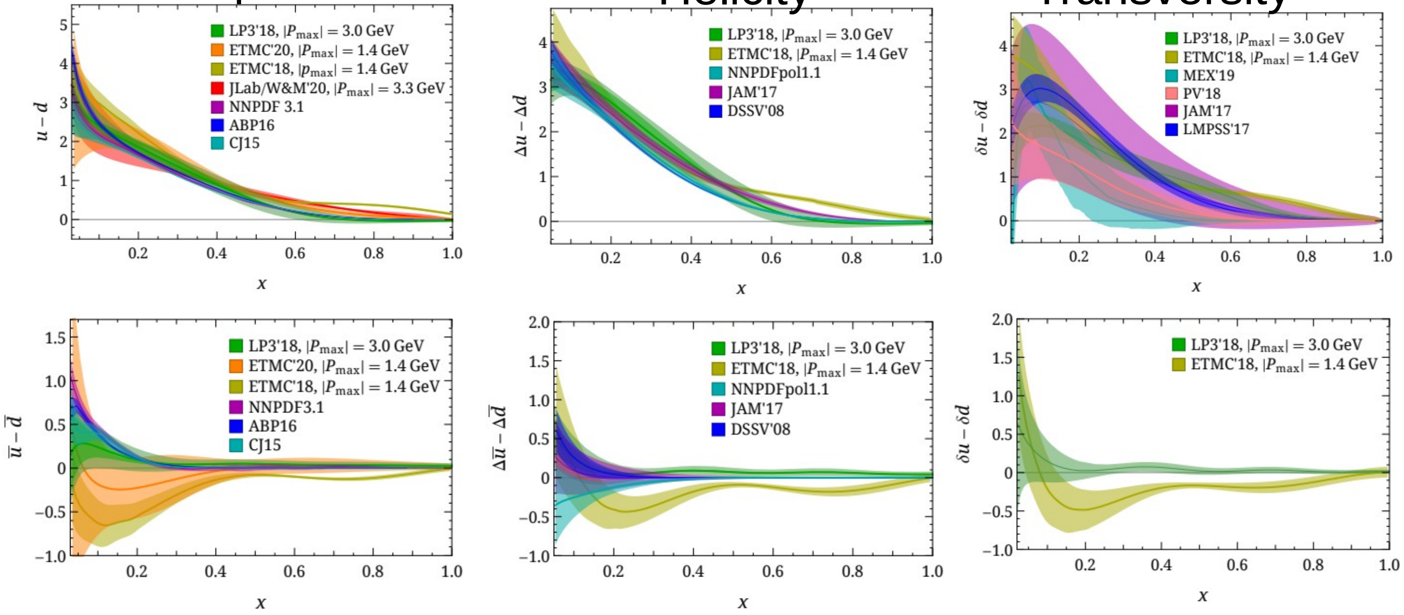
[Cichy and Sato, 2020]



Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES	Value
$\langle x \rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.171(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★	0.173(14)(07)
	ETMC 19	(Alexandrou *et al.*, 2020c)	2+1+1	■	★	○	★	★ **	0.178(16)
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★	0.180(25)($^{+14}_{-6}$)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.151(28)(29)
	ETMC 19	(Alexandrou *et al.*, 2020c)	2	■	★	○	★	★ **	0.189(23)
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★	0.195(07)(15)
$\langle x \rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.359(30)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.307(30)(18)
$\langle x \rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.188(19)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.160(27)(40)
$\langle x \rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.052(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.051(26)(5)
$\langle x \rangle_{g}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.427(92)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.482(69)(48)
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■	0.47(4)(11)

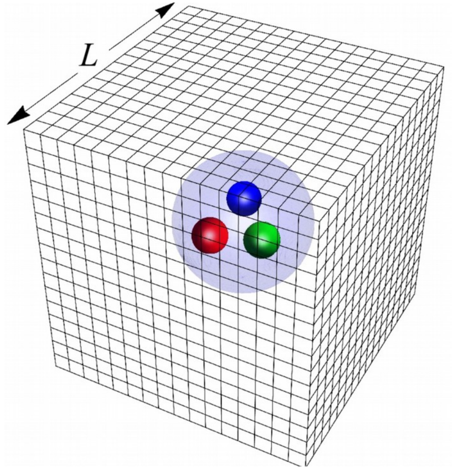
** No quenching effects are seen.

unpolarized Helicity Transversity

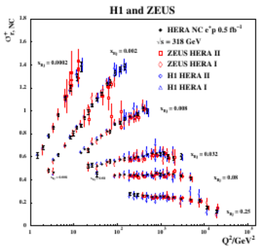


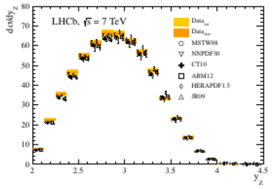
- Lots of benchmarks on moments and x-dependent PDFs between global analysis and Lattice calculation have been made. [Constantinou et al, 2006.08636, Constantinou, 2010.02445] And current developments have been discussed [Constantinou et al, 2202.07193, Del Debbio, 2211.00977]

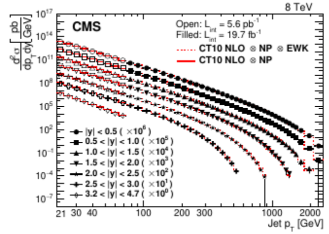
What about incorporating LQCD into global analysis?

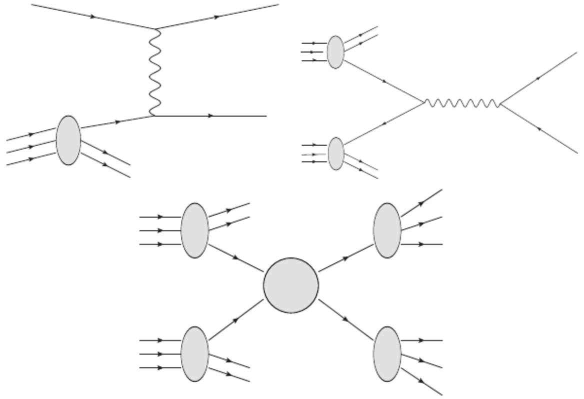


$$\sigma = f_a(x_1, Q^2) \otimes f_b(x_2, Q^2) \otimes \hat{\sigma}$$

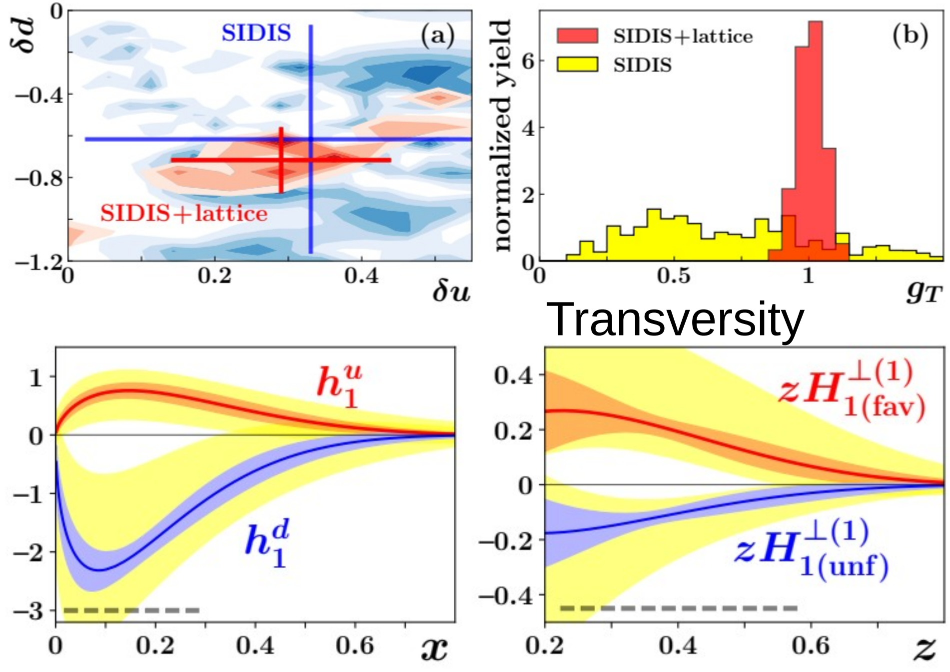




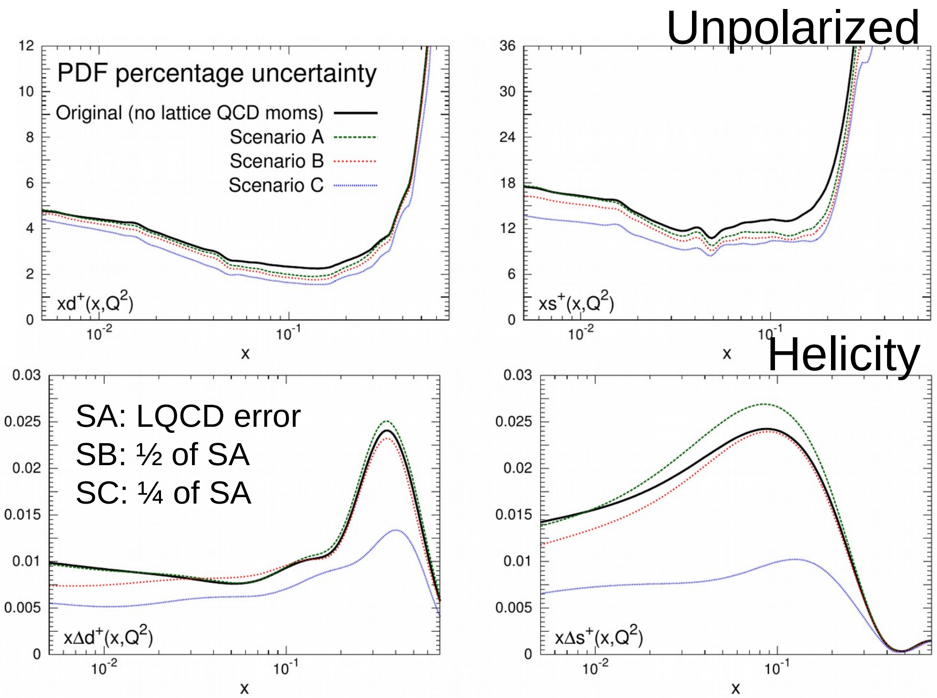




Global analysis with moments from LQCD

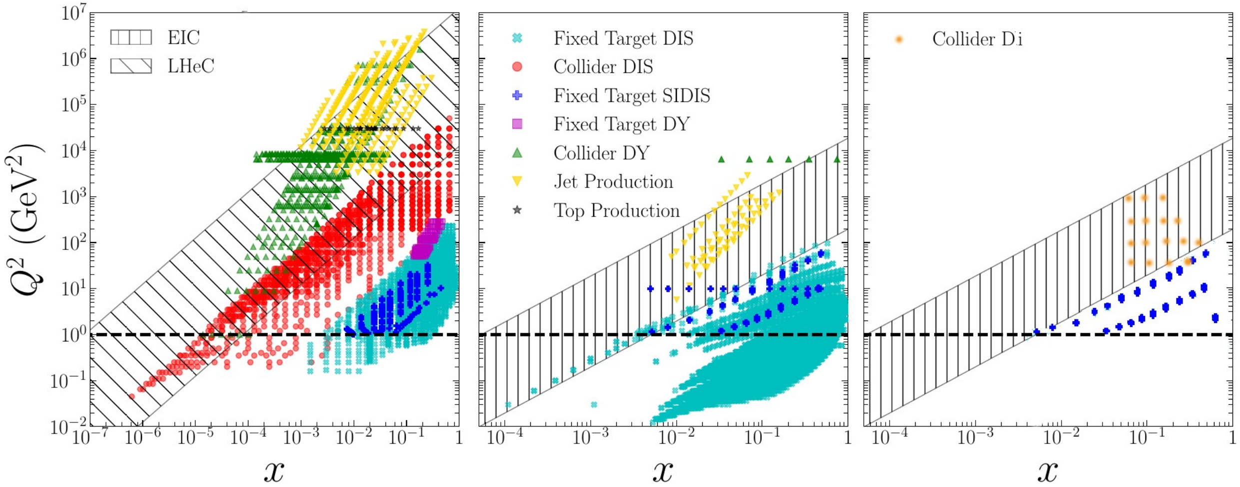


Global QCD analysis of the quark transversity distributions receiving constraint from the average lattice value of g_T [Lin et al, 1710.09858].



Potential impact of future lattice-QCD calculations in global unpolarized and polarized PDF fits [Lin et al, 1711.07916]

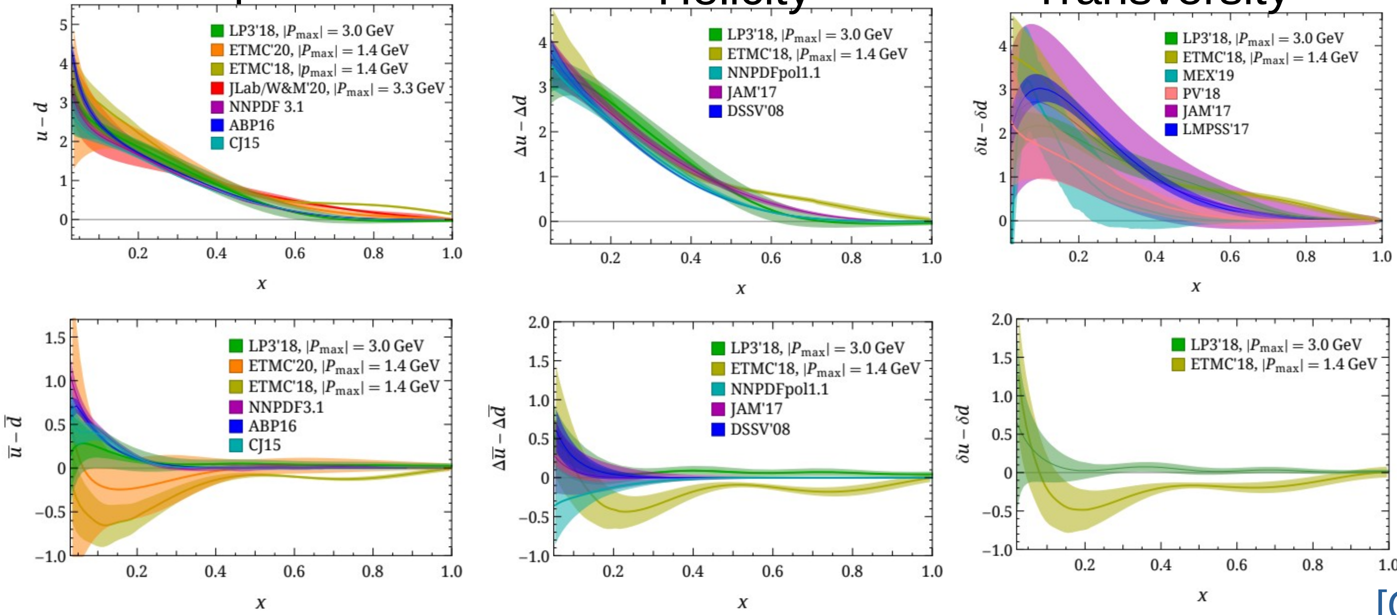
With the Lattice input in the format of moments, it is helpful on reducing PDFs uncertainty.



unpolarized

Helicity

Transversity



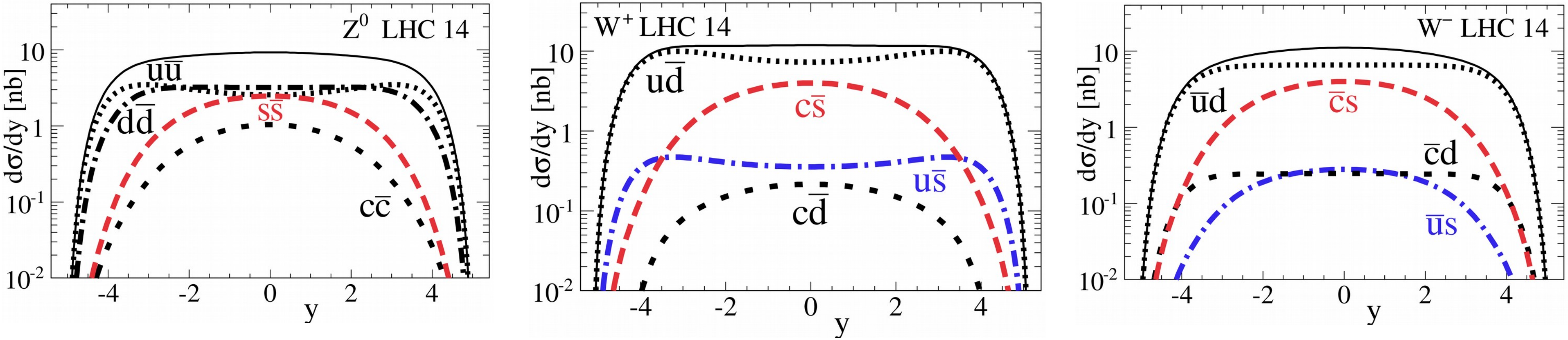
- Lots of efforts on including x-dependent lattice calculation in global analysis have been made. [1907.06037, 2009.05522, 2010.00548, 2010.03996, 2204.00543]
- Unpolarized collinear PDFs receive more experimental measurements in global analysis; while the helicity and transversity PDFs receive rather less experiments.

[Constantinou et al, 2006.08636]

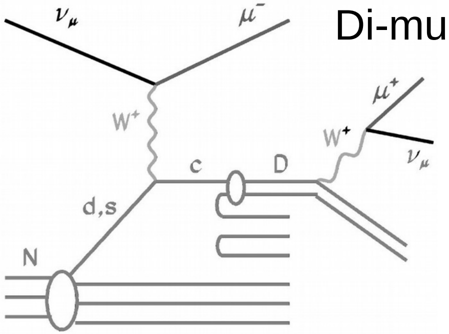
Can we further
reduce uncertainties of PDFs by
including Lattice input in global analysis?

- Flavor separation is one of the most challenging tasks in QCD global analysis, specially in the strangeness sector which plays an important role in precision electroweak physics, such as the determination of the W mass.

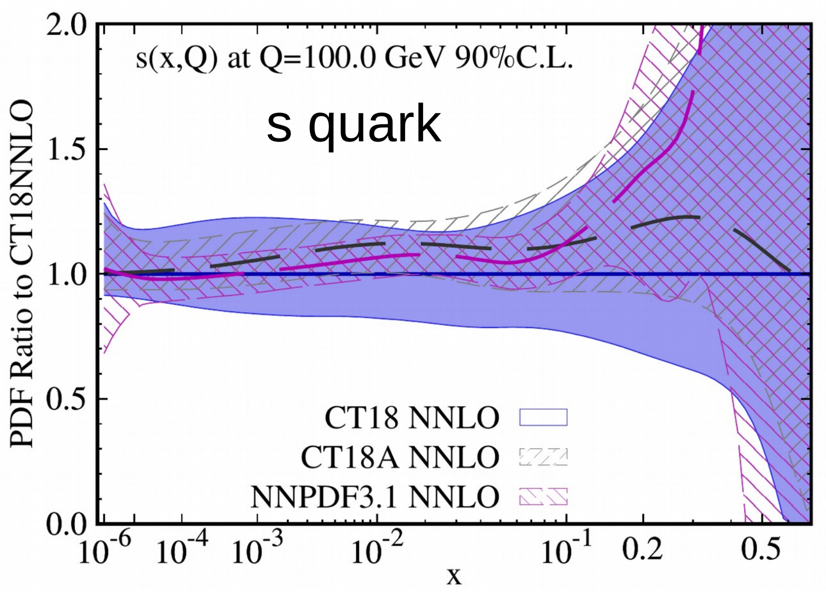
[Kusina et al, 1203.1290]



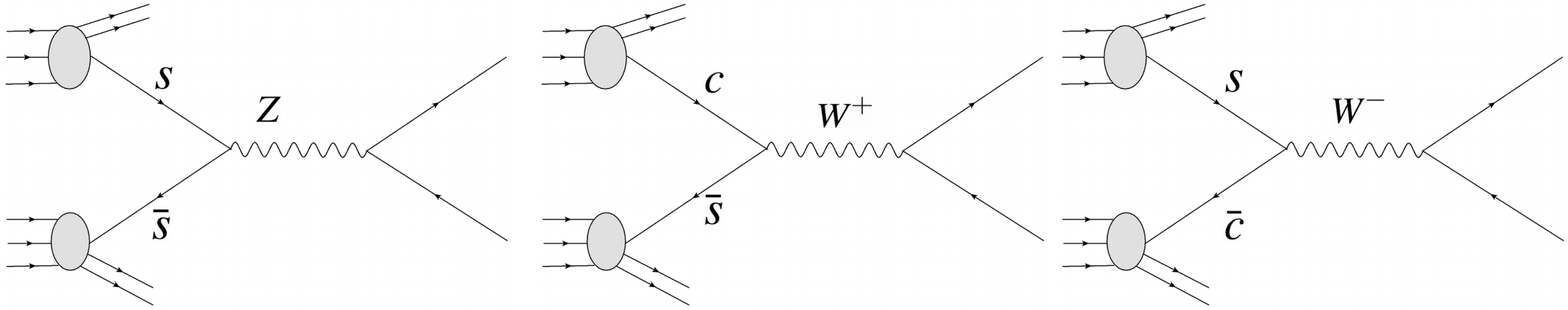
- In global analysis, only the SIDIS di-muon[Goncharov et al, hep-ex/0102049] data probe the strangeness directly: the neutrino process probe the strangeness PDF, while the anti-neutrino process probe the antistrangeness PDF.



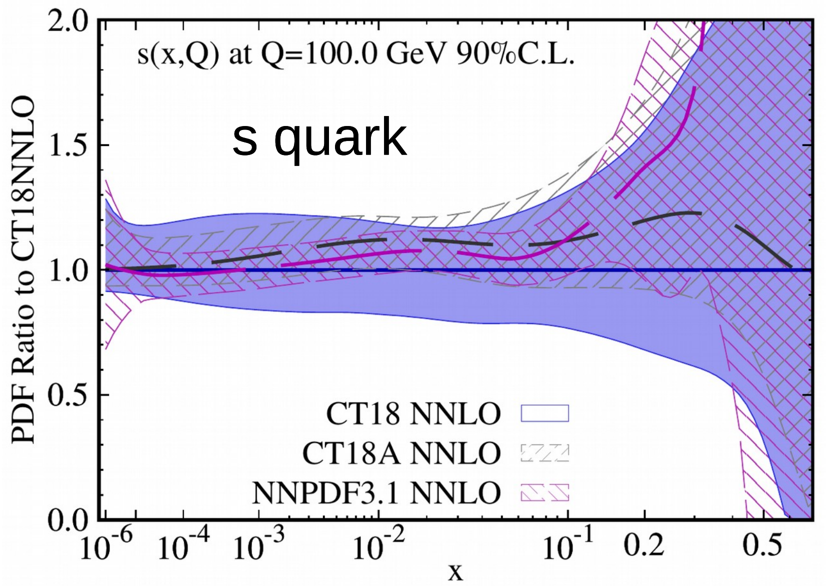
Di-muon

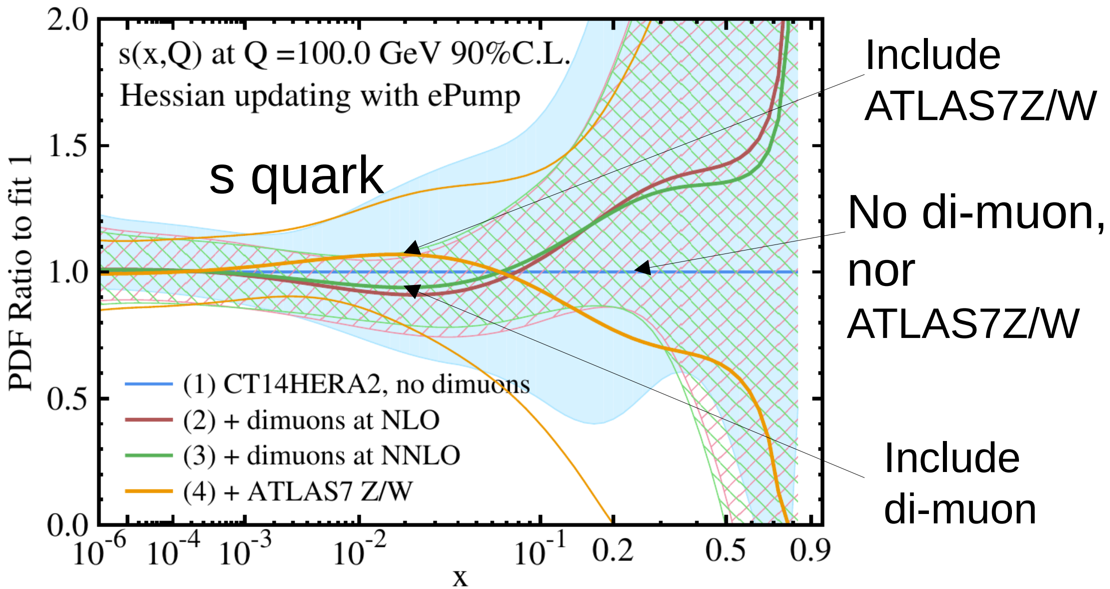


- In the CT18A global analysis with **ATLAS 7 TeV Z/W**[ATLAS, 1612.03016] data included, we observe significant enhancement of strangeness PDF as compared to CT18. This is also observed in MSHT[Bailey, 2012.04684] and NNPDF[Ball, 2109.02653].



Strangeness decomposition

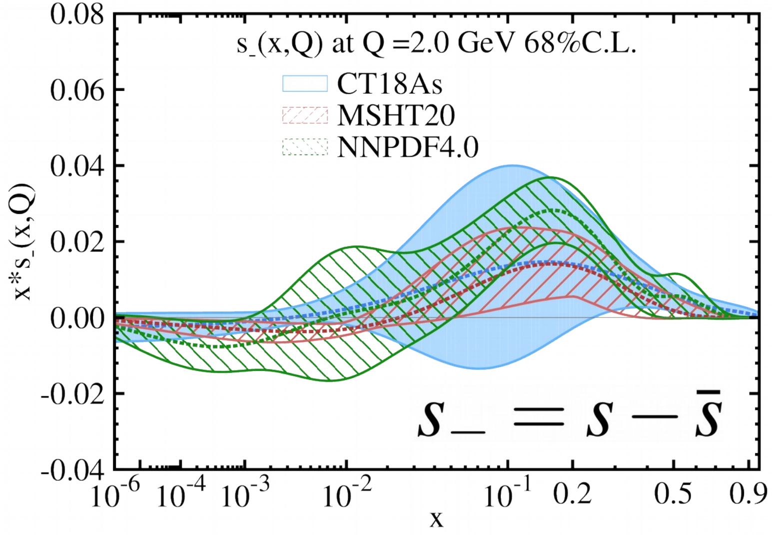




[Hou et al, 1912.10053]

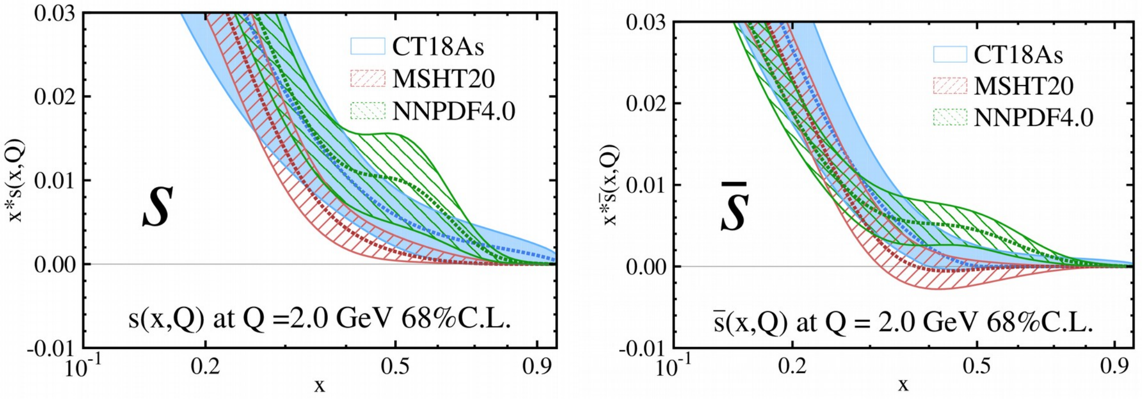
- Noticeable tensions between the SIDIS di-muon data and the precision ATLAS 7 TeV Z/W data were found in global analysis.
- In MSHT20[Bailey, 2012.04684], it was concluded that allowing $s \neq \bar{s}$ at the Q_0 scale can release some of these tensions.

CT18As: CT18A allowing $s \neq \bar{s}$ at Q_0 = 1.3 GeV



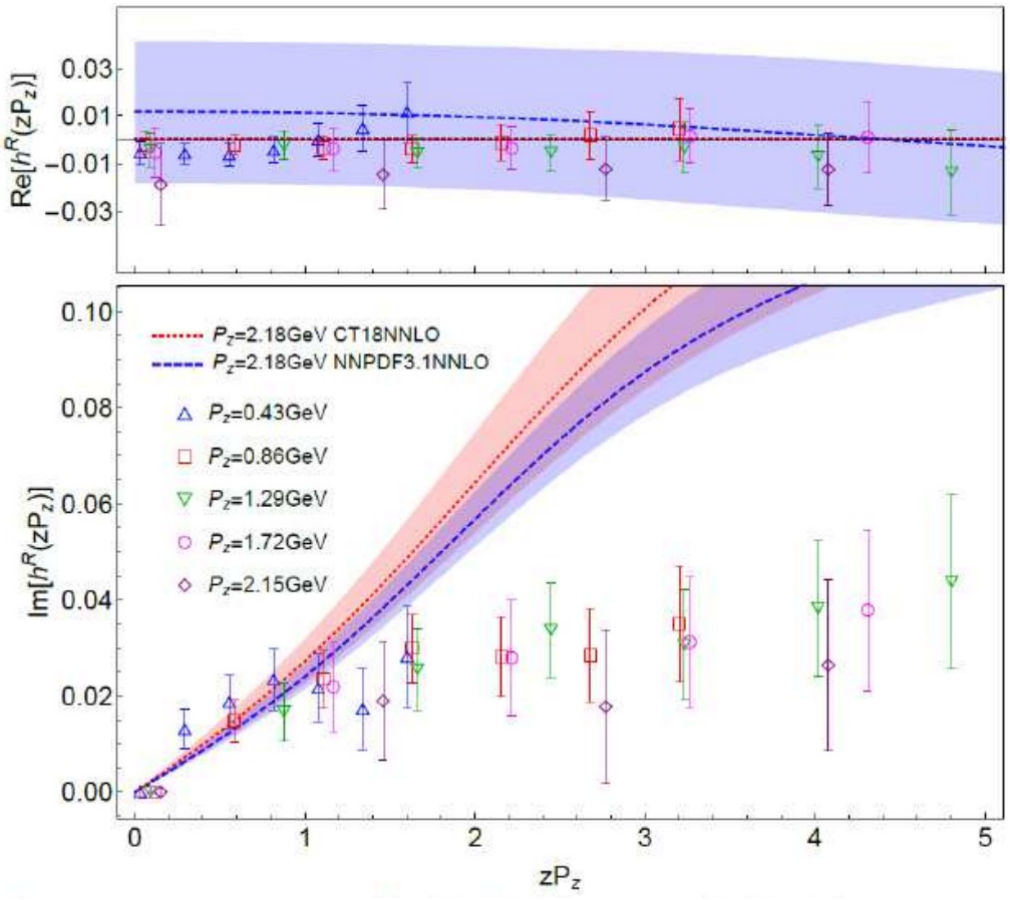
- Allowing strangeness not equal to antistrangeness, the CT18As, which is the CT18A with strangeness asymmetry, presents similar strangeness asymmetry as MSHT20 and NNPDF4.0.

- Both SIDIS di-muon and ATLAS 7 TeV Z/W data can constraint strangeness PDF.
- **Are there any other data for determining the strangeness asymmetry?**



[Tie-Jiun Hou, Huey-Wen Lin, Mengshi Yan, C.-P. Yuan, 2211.11064]

From quasi-PDF to PDF



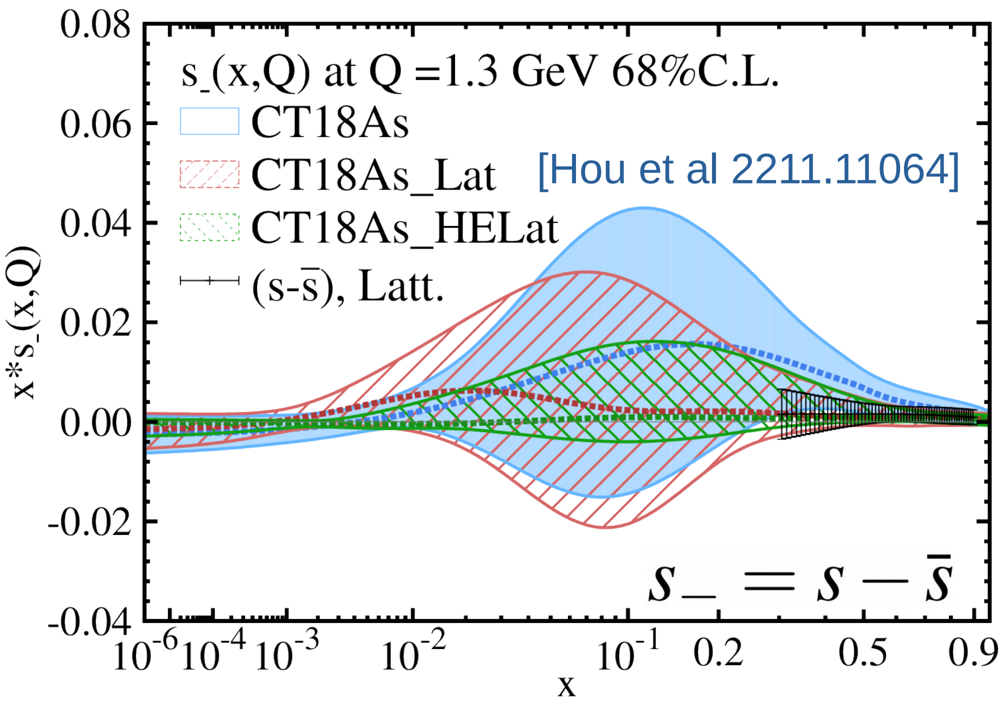
- Due to the large uncertainty in strangeness PDF from global analysis, lattice QCD calculation is able to provide more information.

$$\mathrm{Re}[h(z)] \propto \int dx\, (s(x) - \bar{s}(x)) \cos(xzP_z)$$

$$\mathrm{Im}[h(z)] \propto \int dx\, (s(x) + \bar{s}(x)) \sin(xzP_z)$$

- MSULat/quasi-PDF method
- Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum
- RI/MOM renormalization
- Extropolartion to M_pi = 140 MeV

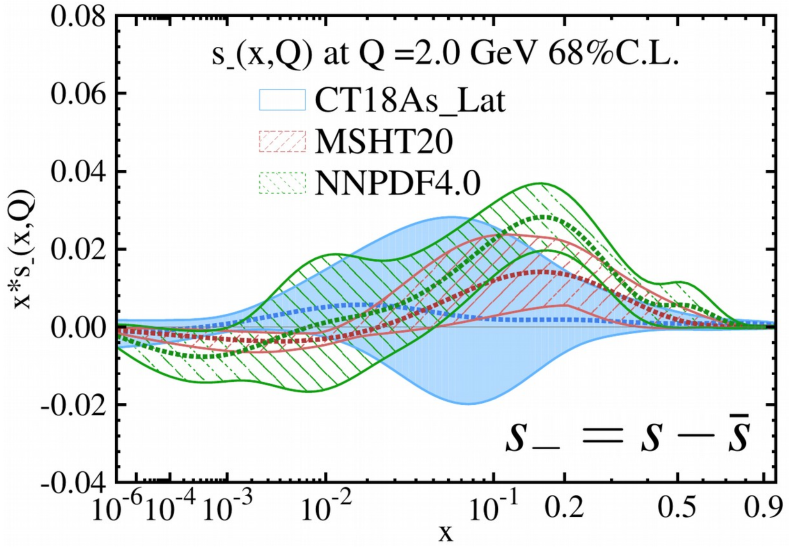
[Zhang et al, 2005.12015]



CT18As: CT18A with strangeness asymmetry at Q_0 = 1.3 GeV.

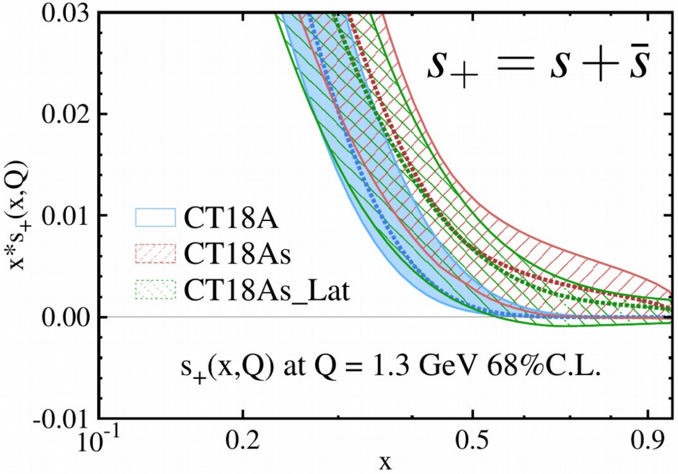
CT18As_Lat: PDFs with lattice input.

CT18As_HELat: PDFs with the lattice errors reduced by half.

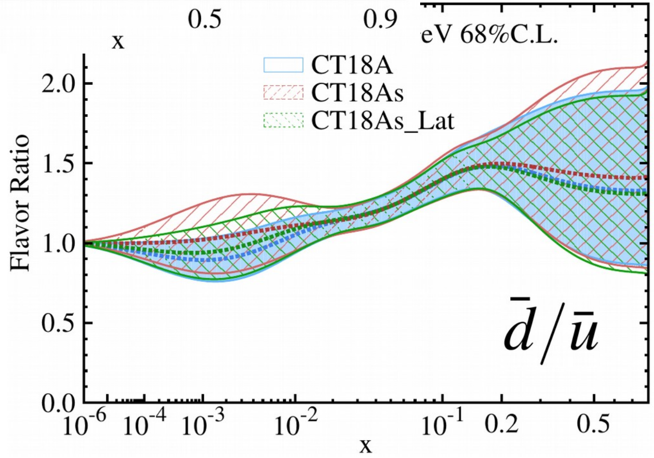


- Lattice QCD calculation provide prediction at 0.3 < x < 0.8, while the di-muon data constraint strangeness at 0.015 < x < 0.336.
- Lattice input improves the determination of strangeness asymmetry.
- LQCD can improve heavy flavor decomposition.

ATLAS7ZW	Z	W^+	W^-	R^2 ($R^2/131$)	reduced χ^2	total χ^2
CT18A	17.48	15.29	13.82	40.99 (0.31)	46.59	87.58
CT18As	15.78	15.72	11.96	32.13 (0.25)	43.46	75.59
CT18As_Lat	17.22	14.58	12.94	34.36 (0.26)	44.74	79.10
N_{pt}	12	11	11		34	34



- Tensions between ATLAS 7 TeV Z/W and SIDIS di-muon data are release by including strangeness asymmetry at Q_0 =1.3 GeV.
- Larger strangeness asymmetry at Q_0 scale (from CT18A to CT18As) would raise $\bar{d}/\bar{u}$ for $x > 0.2$ through sum rules, and thus reduce the χ^2 of SeaQuest(E906) data.



Experimental data set	$N_{\mathrm{pt},E}$	CT18A	CT18As	CT18As_Lat
E866 Drell-Yan process $\sigma_{pd}/(2\sigma_{pp})$ [61]	15	17.6	17.6	17.4
E906 Drell-Yan process $\sigma_{pd}/(2\sigma_{pp})$ [41]	6	5.5	4.5	5.0

Besides reducing uncertainties of PDFs and improving heavy flavor decomposition, what else can we gain from the collaboration between global analysis of PDFs and lattice calculations from first principle?

Gottfried sum rule

New Muon Collaboration (NMC PRL 66, 2712 (1991), PRD 50, R1 (1994)) first discover $\bar{u} \neq \bar{d}$, which violates the Gottfried sum rule.

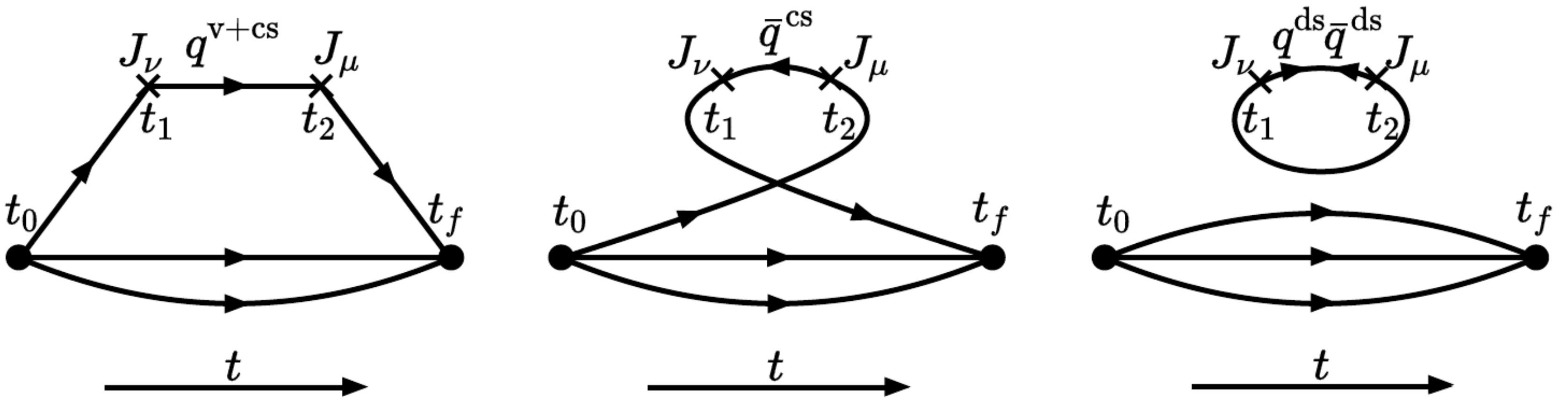
$$S_G = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,\left(\bar{d}(x) - \bar{u}(x)\right) + O(\alpha_s^2)$$

The following experiments like HERMES (PLB387, 419 (1996)) and E866 (PRD64, 052002 (2001)) also show preference of ū - đ flavor asymmetry.

Experiment	$\langle Q^2 \rangle$ (GeV^2)	$\int_0^1 [\bar{d}(x) - \bar{u}(x)]dx$
NMC/DIS	4.0	0.147 ± 0.039
HERMES/SIDIS	2.3	0.16 ± 0.03
FNAL E866/DY	54.0	0.118 ± 0.012

What is the origin of $\bar{u} \neq \bar{d}$?

- Euclidean path-integral formulation of the hadronic tensor predicts two kinds of sea partons: connected and disconnected



$$
\begin{aligned}
u &= u^{v+cs} + u^{ds}, \quad & d &= d^{v+cs} + d^{ds} \\
\bar{u} &= \bar{u}^{cs} + \bar{u}^{ds}, \quad & \bar{d} &= \bar{d}^{cs} + \bar{d}^{ds}
\end{aligned}
$$

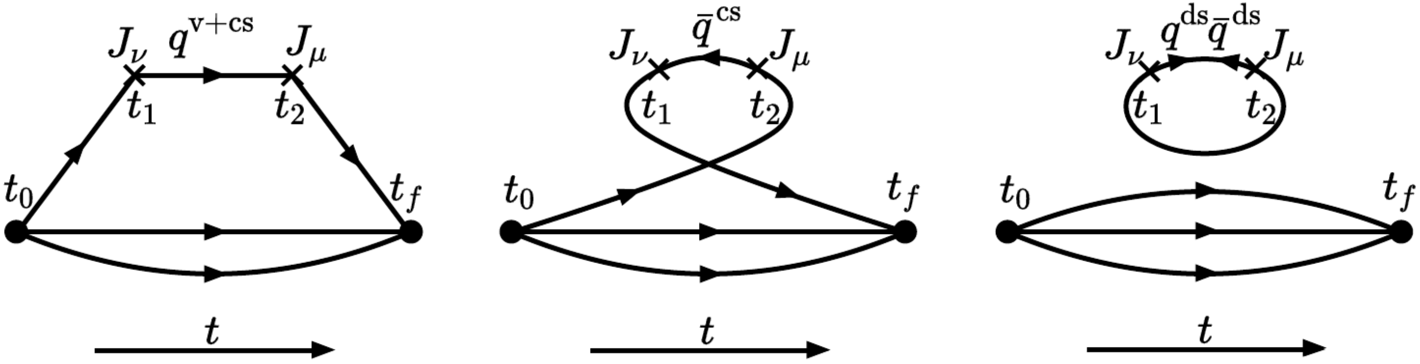
[Liu, 2007.15075]

Define $u^v \equiv u^{v+cs} - \bar{u}^{cs}$, which is equivalent to defining $u^{cs} \equiv \bar{u}^{cs}$.

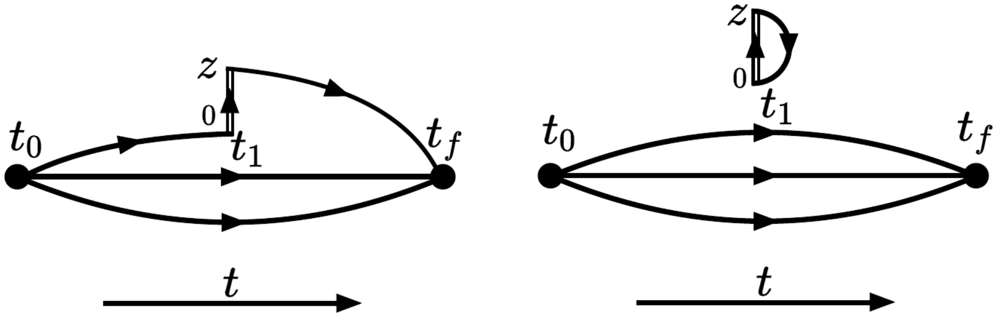
$$
\begin{aligned}
u - \bar{u} &\equiv (u^{v+cs} + u^{ds}) - (\bar{u}^{cs} + \bar{u}^{ds}) = u^v + (u^{ds} - \bar{u}^{ds}) \\
&\neq u^v, \quad \text{unless } u^{ds} = \bar{u}^{ds}
\end{aligned}
$$

Hadronic tensor in Euclidean path-integral formalism
versus
Quasi-PDF from Lattice QCD

Path-Integral Formalism



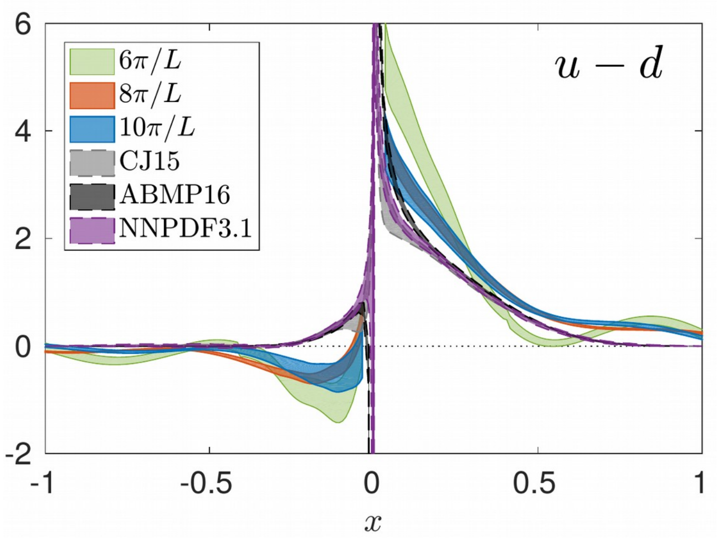
Quasi-PDF from Lattice QCD



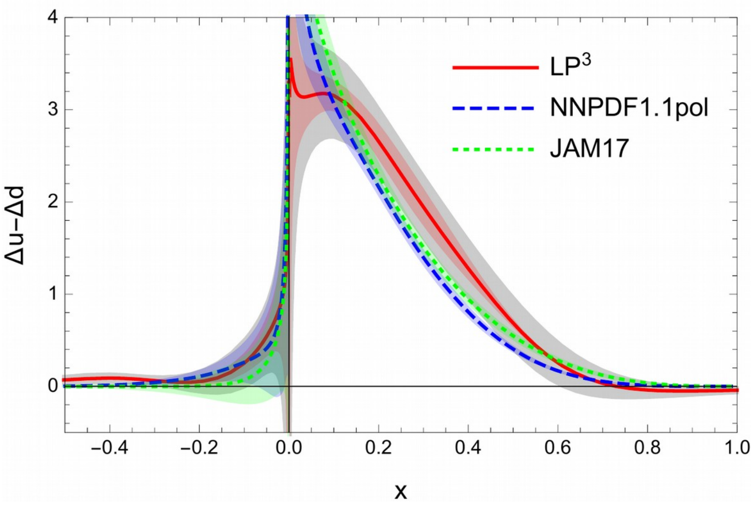
Connected Insertion(CI)   Disconnected Insertion(DI)

[Liu, 2007.15075]

Quasi PDF results from LP3 and ETMC connected insertion calculation



[Alexandrou et al, PRL, 1803.02685]



[LP3 – Lin et al, PRL, 1807.07431]

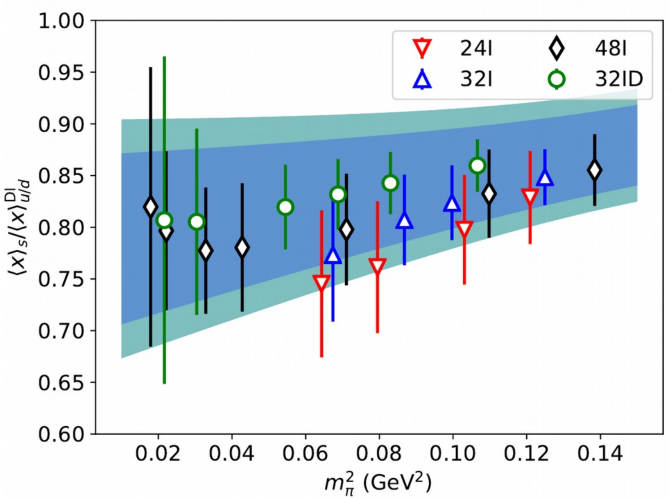
$$q(x > 0) = q^{v+cs}(x)$$

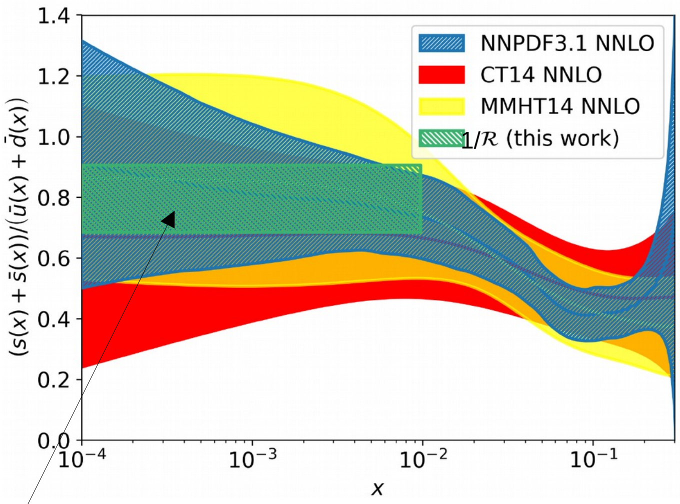
$$q(x < 0) = -\bar{q}^{cs}$$

Parton degrees of freedom are the same as in hadronic tensor - [Liu, 2007.15075]

Lattice input to global fitting of PDFs

With only one input from Lattice QCD, the ratio of moments between u, d and s in disconnected insertion(DI), the connected and disconnected sea are distinguishable in global analysis.





Lattice result from overlap on Nf = 2 + 1 DWF on 4 lattices, with one at physical pion mass [Liang et al, χQCD, 1901.07526]

$$\frac{1}{R} = \frac{\langle x\rangle_{s+\bar{s}}}{\langle x\rangle_{\bar{u}+\bar{d}}(DI)}(\text{at } 1.3 \text{ GeV}) = 0.822(69)(78)$$

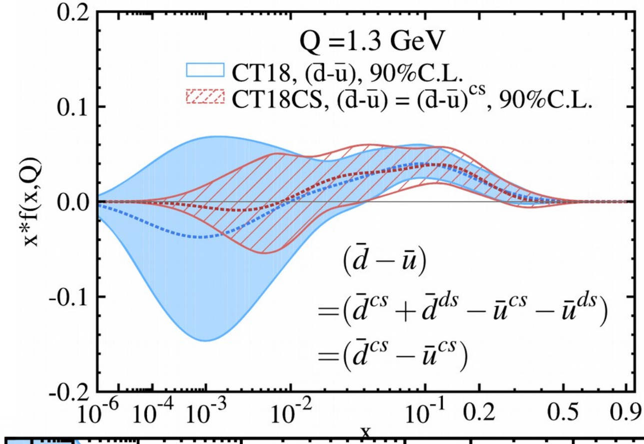
Connected and disconnected sea d.o.f. can be distinguished by assuming

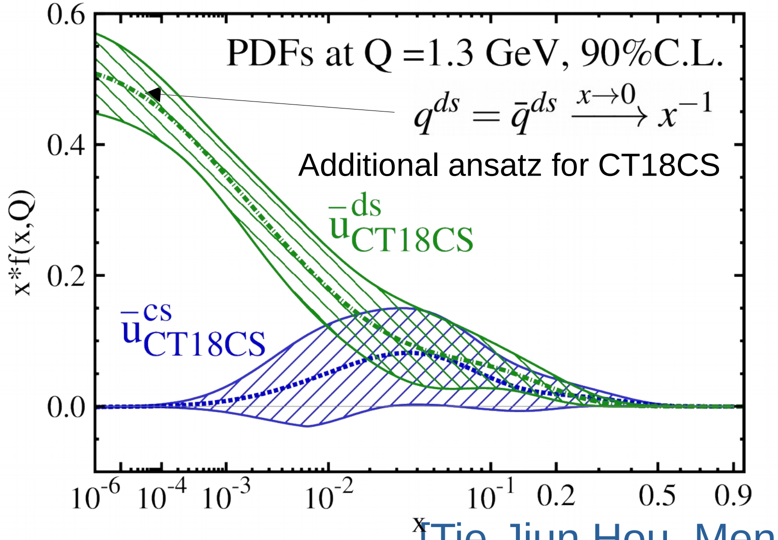
$$u^{ds} = \bar{u}^{ds} = d^{ds} = \bar{d}^{ds} = Rs = R\bar{s},$$

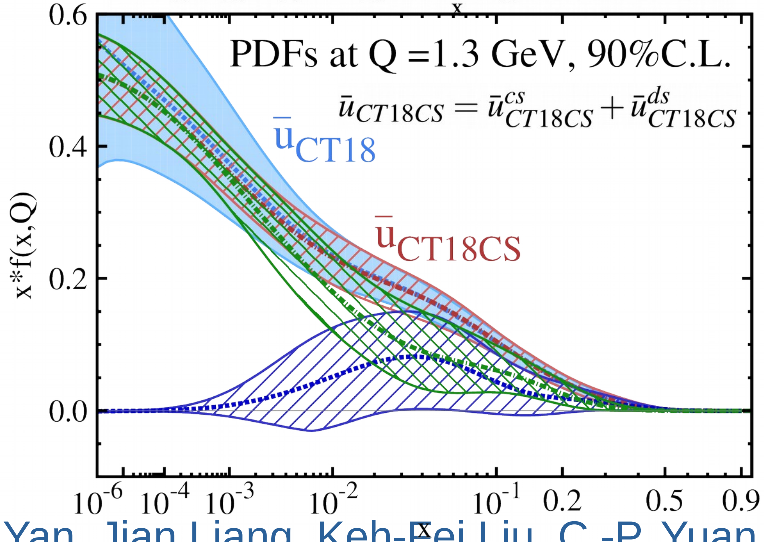
- Distinguish connected and disconnected flavor d.o.f. at $Q_0 = 1.3\ \mathrm{GeV}$ in global analysis.
- The difference between ū and đ come from the connected sea contribution.

CT18 CT18CS

$$
\begin{aligned}
g &= g \\
u^v &= u^v \\
d^v &= d^v \\
\bar{u} &= \bar{u}^{cs} + \bar{u}^{ds} = \bar{u}^{cs} + R s^{ds} \\
\bar{d} &= \bar{d}^{cs} + \bar{d}^{ds} = \bar{d}^{cs} + R s^{ds} \\
s &= \bar{s} = s^{ds}
\end{aligned}
$$







[Tie-Jiun Hou, Mengshi Yan, Jian Liang, Keh-Fei Liu, C.-P. Yuan, 2206.02431]

Direct comparison of all connected and disconnected parton moments between global analysis and lattice calculation instead of being limited to only u − d and s.

$$u^+ - d^+ = (u+\bar{u}) - (d+\bar{d}) = (u^{v+cs} + u^{ds} + \bar{u}^{cs} + \bar{u}^{ds}) - (d^{v+cs} + d^{ds} + \bar{d}^{cs} + \bar{d}^{ds})$$

$$\xrightarrow{CT18CS} (u^{v+cs} - d^{v+cs}) + (\bar{u}^{cs} - \bar{d}^{cs})$$

$$s^+ = s + \bar{s} = s^{ds} + \bar{s}^{ds} \xrightarrow{CT18CS} 2s^{ds}$$

	$Q = 2.0$ GeV CT18	Lattice	$Q = 1.3$ GeV CT18CS	CT18
$\langle x \rangle_{u^+-d^+}$	0.156(7)	$0.111-0.209^{N_f=2+1}$ $0.153-0.194^{N_f=2+1+1\dagger}$ $0.166-0.212^{N_f=2}$	0.173(7)	0.175(8)
$\langle x \rangle_{s^+}$	0.033(9)	$0.051(26)(5)^{\ddagger}$	0.027(8)	0.027(10)

† Prog. Part. Nucl. Phys., 121:103908, 2021. ‡ Phys. Rev. Lett., 121(21):212001, 2018

CT18 CT18CS

$$\begin{aligned} g &= g \\ u^v &= u^v \\ d^v &= d^v \\ \bar{u} &= \bar{u}^{cs} + \bar{u}^{ds} = \bar{u}^{cs} + Rs^{ds} \\ \bar{d} &= \bar{d}^{cs} + \bar{d}^{ds} = \bar{d}^{cs} + Rs^{ds} \\ s &= \bar{s} = s^{ds} \end{aligned}$$

PDF	$\langle x \rangle_{u^v}$	$\langle x \rangle_{d^v}$	$\langle x \rangle_g$	$\langle x \rangle_{\bar{u}}$	$\langle x \rangle_{\bar{d}}$	$\langle x \rangle_s$
CT18	0.325(5)	0.134(4)	0.385(10)	0.0284(22)	0.0361(27)	0.0134(52)
CT18CS	0.323(4)	0.136(3)	0.384(12)	0.0287(25)	0.0364(34)	0.0137(39)

PDF	$\langle x \rangle_{u^{v+cs}}$	$\langle x \rangle_{d^{v+cs}}$	$\langle x \rangle^*_{\bar{u}^{cs}}$	$\langle x \rangle^*_{\bar{d}^{cs}}$	$\langle x \rangle^\dagger_{u^{ds}}$
CT18CS	0.335(7)	0.155(8)	0.0120(64)	0.0197(70)	0.0167(49)

1.3 GeV

To be tested by lattice calculation

Complementarity between PDFs global analysis and Lattice QCD

- PDFs global analysis: Large amount of data to access hadron structure,
- Lattice QCD: Provides constraints on hadron structures not accessible experimentally,
- Reduction of uncertainties from LQCD input on helicity and transversity PDFs, which receive less constraint from experiments,
- Potential heavy flavor decomposition with the help of lattice calculation in global analysis,
- Incorporation of connected and disconnected sea d.o.f. within the global analysis would help on better understanding of the non-perturbative nature of hadron structure. Also one can directly compare lattice calculation with all separated connected and disconnected sea moments.

Thank you for your attention!