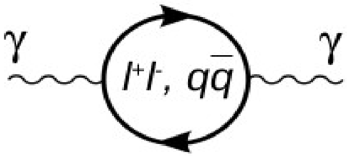
The W-boson mass

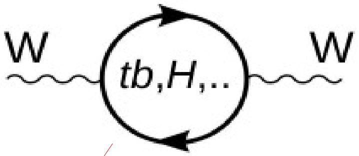
M.Boonekamp

DIS 2023
MSU, March. 30, 2023

Prediction of m_W in the SM – a snapshot

$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-4\frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2}\frac{1}{1-\Delta r}}\right)$$

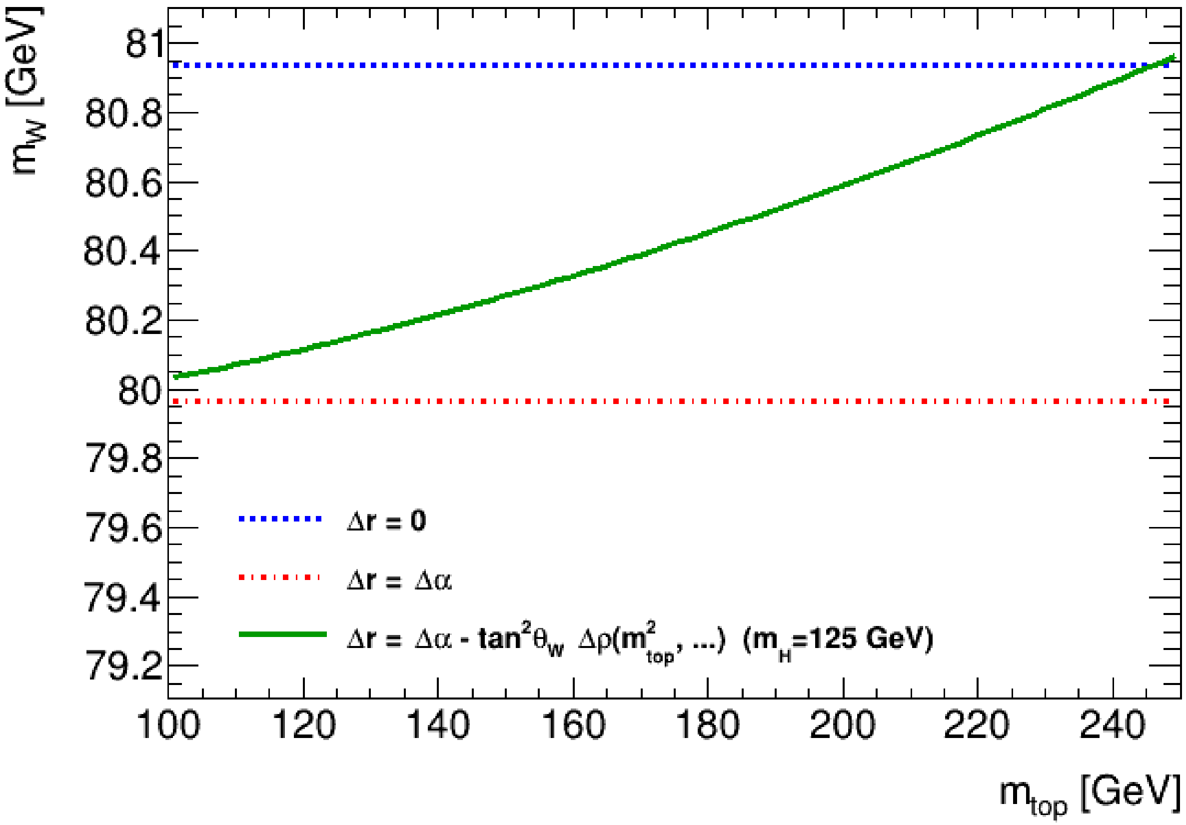




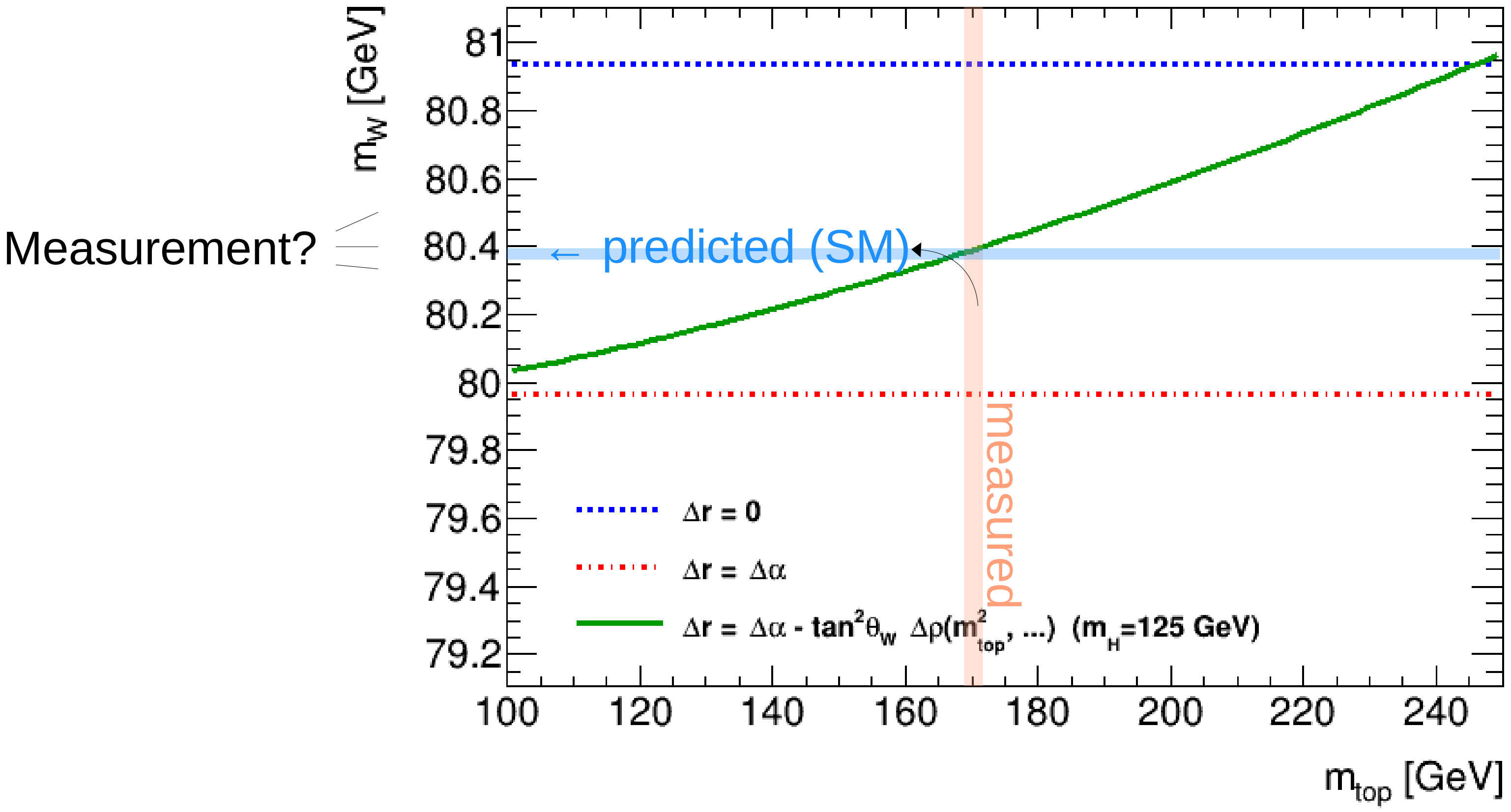
$$\Delta r = \Delta\alpha - \tan^2\theta_W \Delta\rho = \sim 0.059 - \frac{3G_\mu m_W^2}{8\sqrt{2}\pi^2}\left[\frac{m_{top}^2}{m_W^2}\cot^2\theta_W - \left(\ln\frac{m_H^2}{m_W^2} - \frac{5}{6}\right) + \ldots\right]$$

$\alpha(0) \sim 1/137.. \rightarrow \alpha(m_Z) \sim 1/128.9$

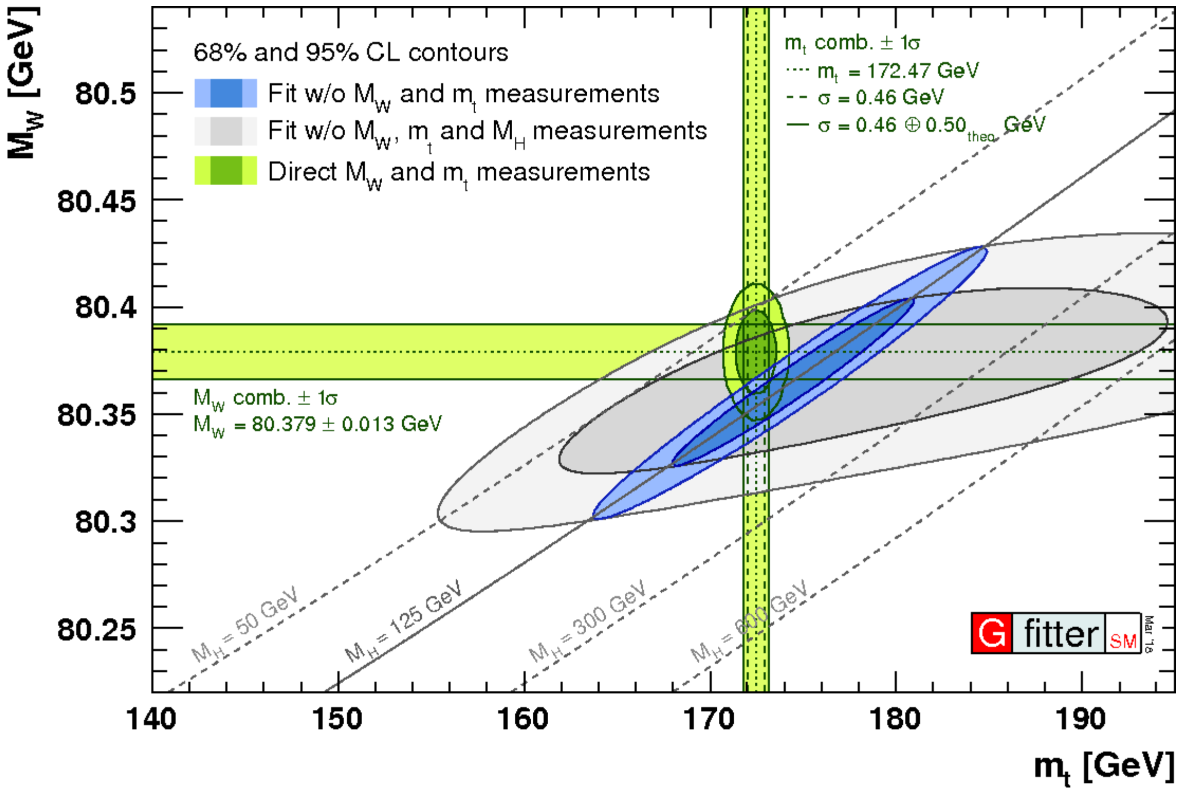
Prediction of m_W in the SM – a snapshot



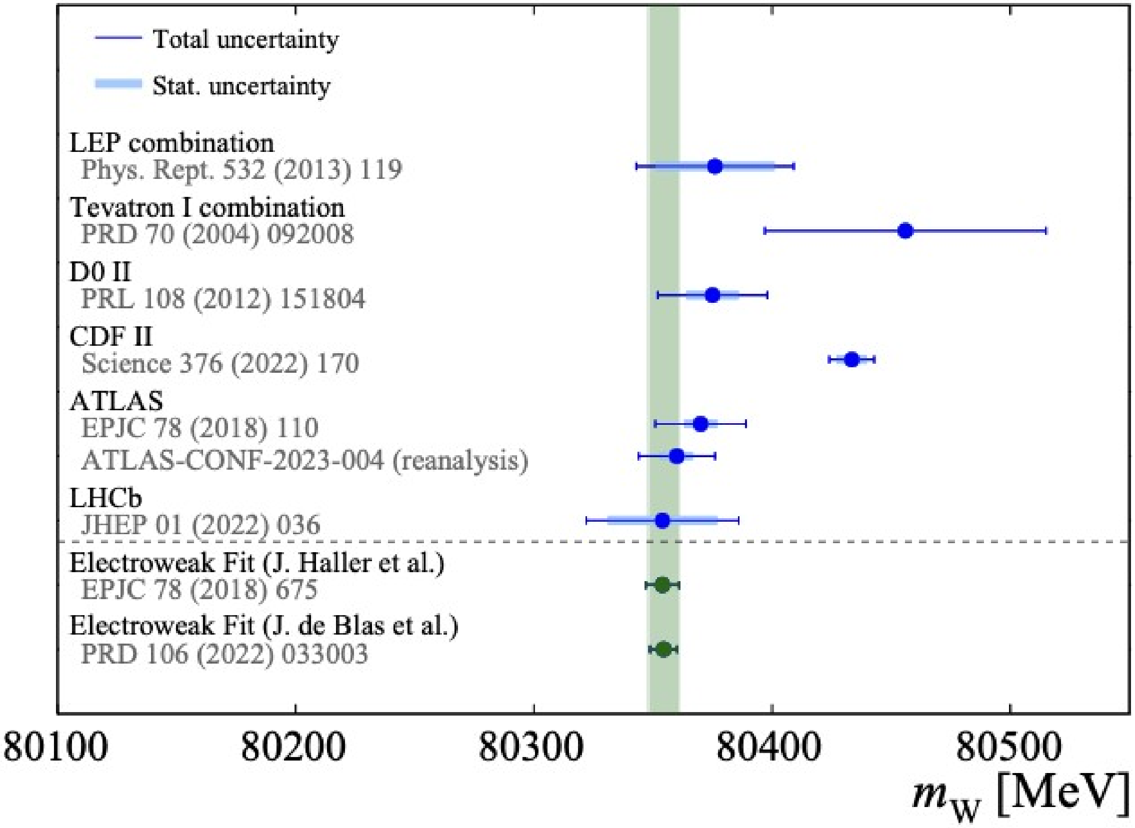
Prediction of m_W in the SM – a snapshot



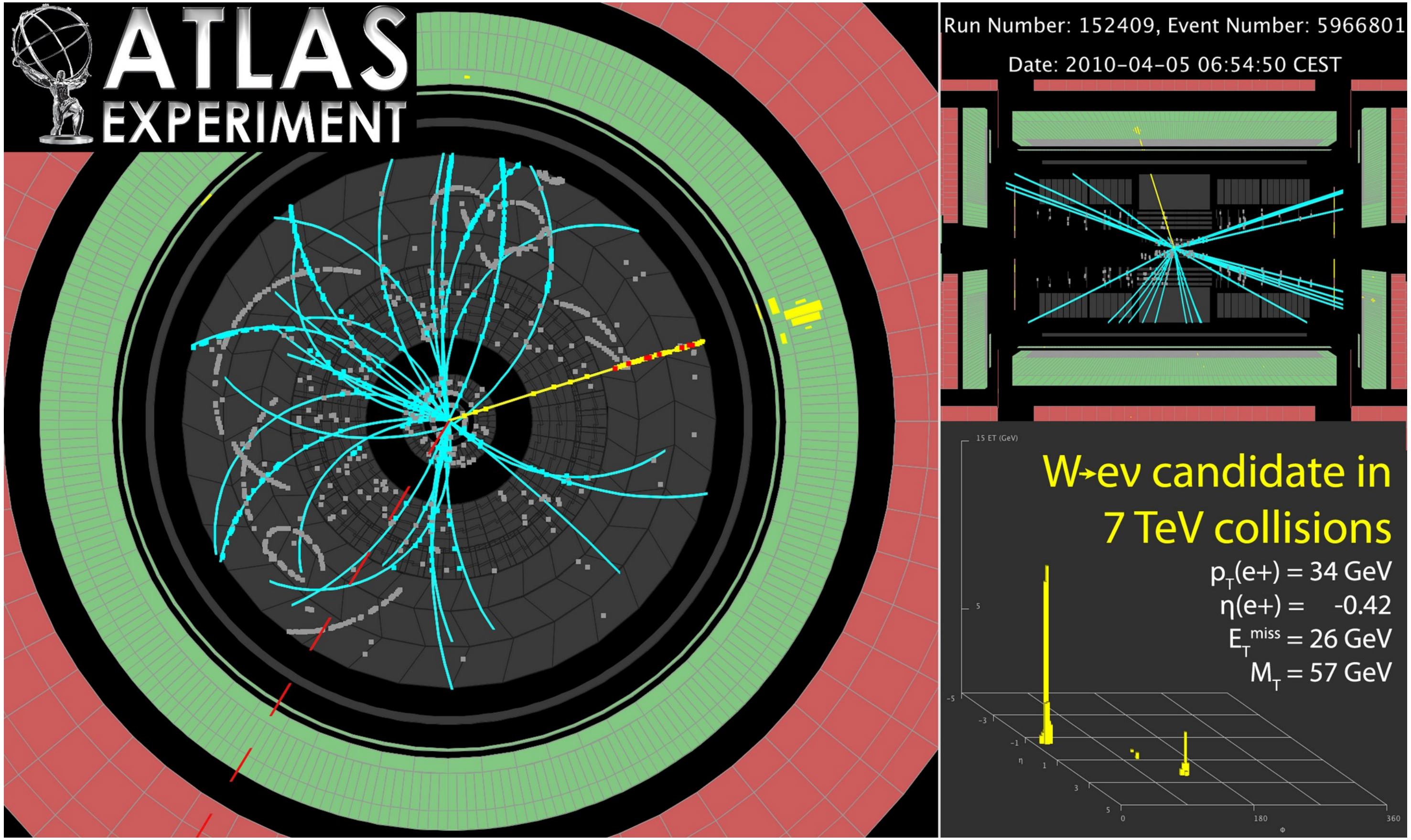
Prediction of m_W in the SM – a snapshot



Measurements



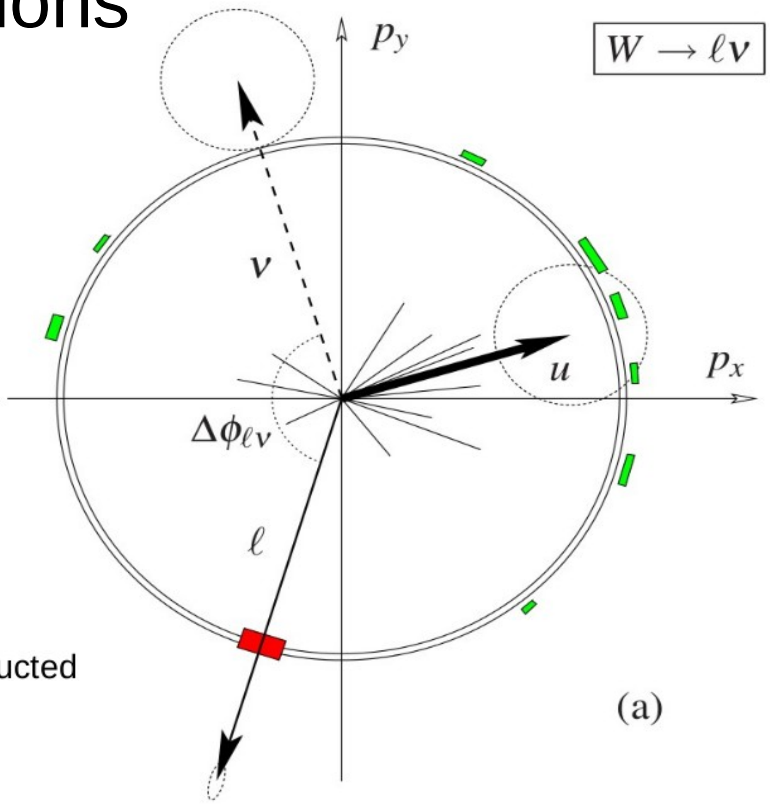
The W boson mass in proton collisions



The W boson mass in proton collisions

- Incomplete kinematics (missing neutrino!)
  → no invariant mass
  → rely on measured quantities, and exploit momentum conservation in the transverse plane

- Event representation :



- Main signature :
  single electron or muon $\boxed{\overrightarrow{p_T}^{l}}$ $\vec{p}_T^{\,l}$

- Recoil : sum of "everything else" reconstructed in the calorimeters; a measure of $p_T^{W,Z}$

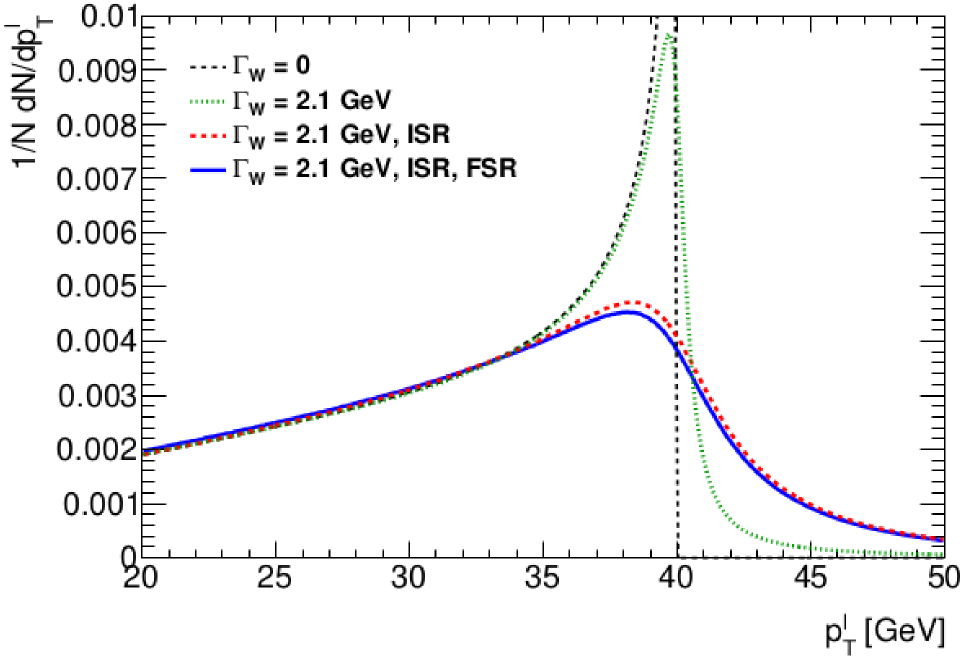
  $$\boxed{\vec{u}_T = \sum_i \vec{E}_{T,i}}$$

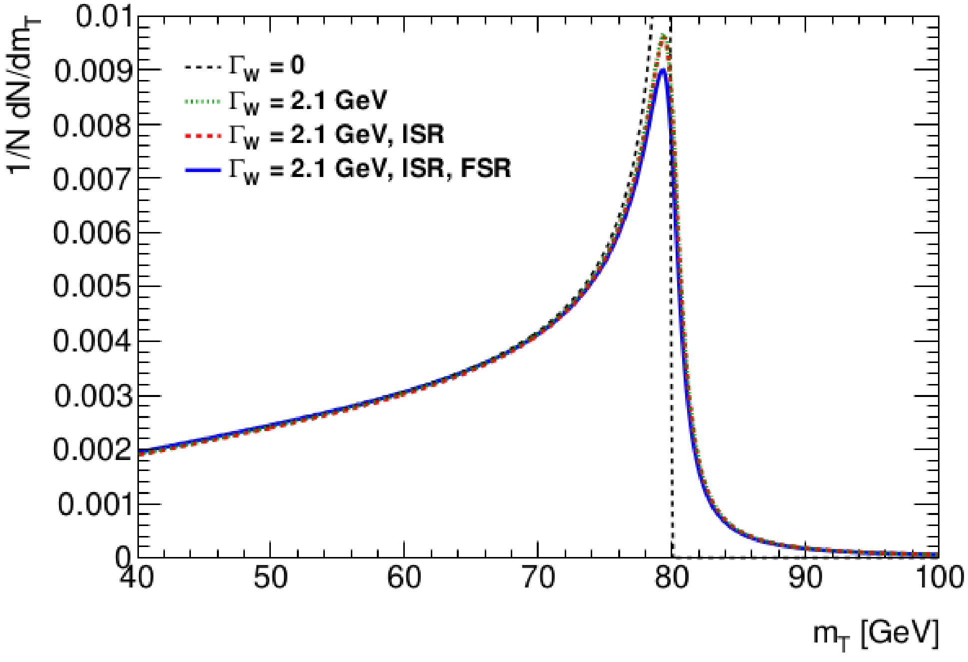
- Derived quantities : $\boxed{\vec{p}_T^{\,miss} = -\left(\vec{p}_T^{\,\ell} + \vec{u}_T\right)}$ $\boxed{m_T = \sqrt{2 p_T^{\ell} p_T^{miss} (1 - \cos\Delta\phi)}}$

The W boson mass in proton collisions

- Physics corrections : W width; QCD and QED ISR and FSR, PDFs, ...

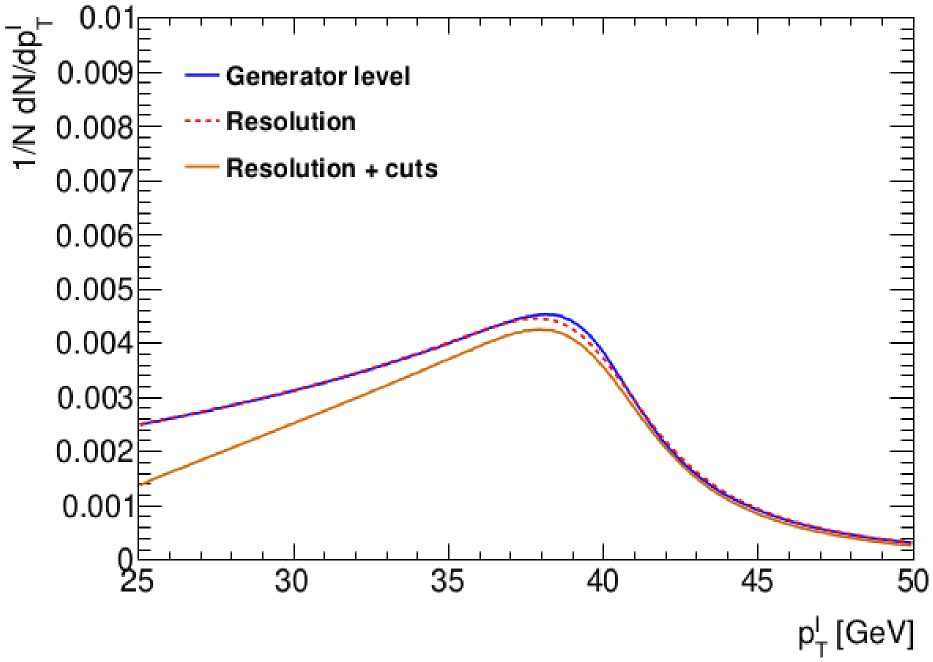
  → all carry uncertainties to be quantified!

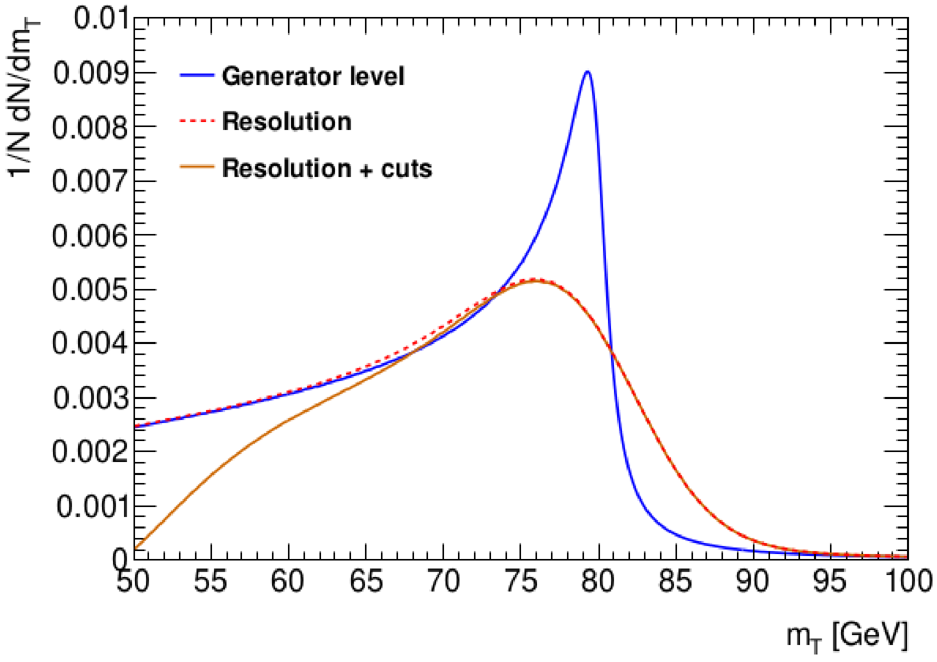




The W boson mass in proton collisions

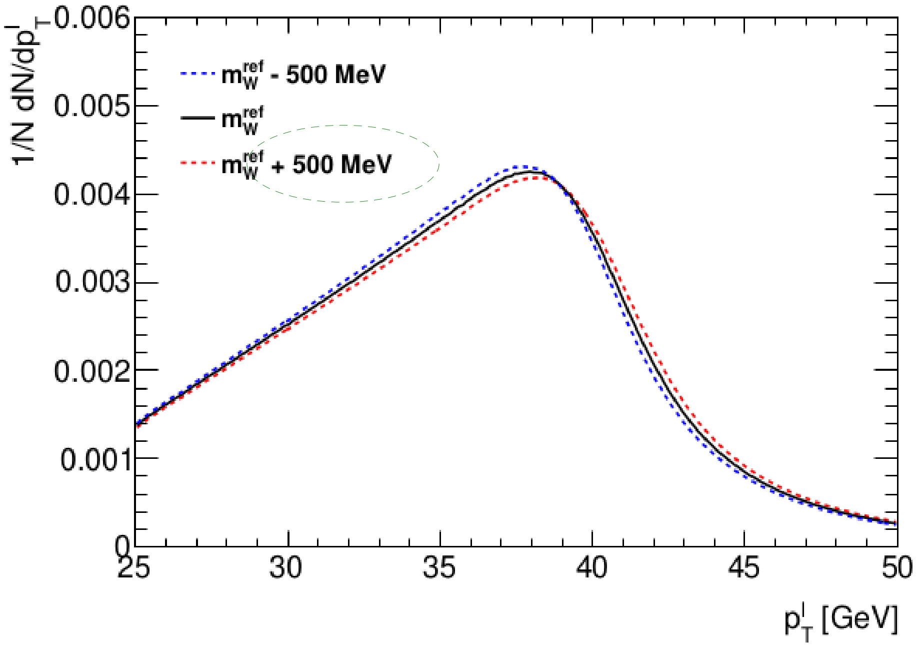
- Detector effects, also with uncertainties :
  - Lepton calibration and resolution; Missing E_T resolution ~ 5 – 15 GeV
  - Efficiencies and acceptance ~15% (with non-trivial kinematic dependence!)

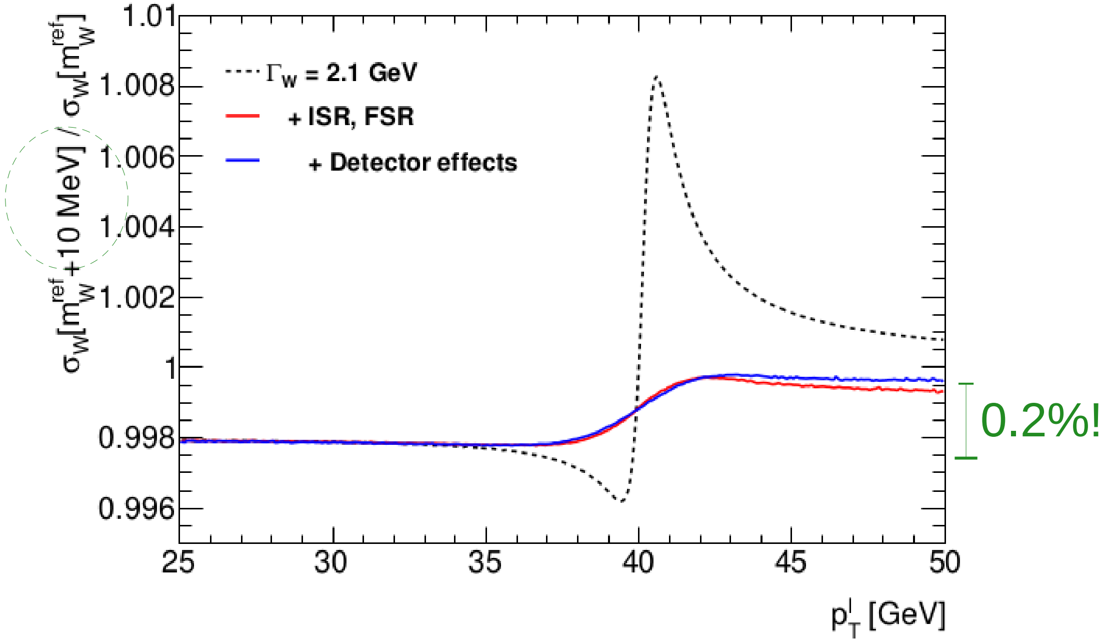




The W boson mass in proton collisions

- Mass measurement : produce models (“templates”) of the final state distributions for different mass hypotheses; compare to data



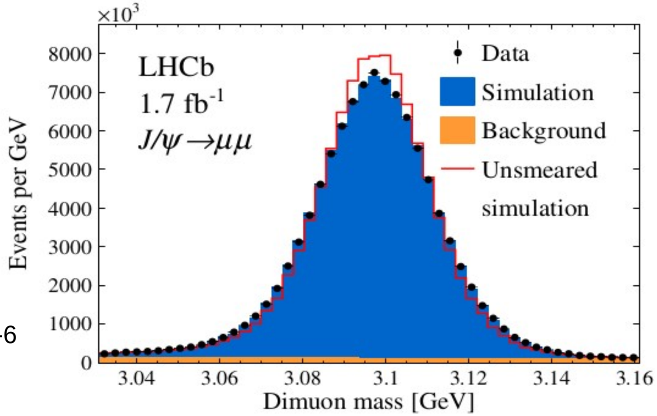


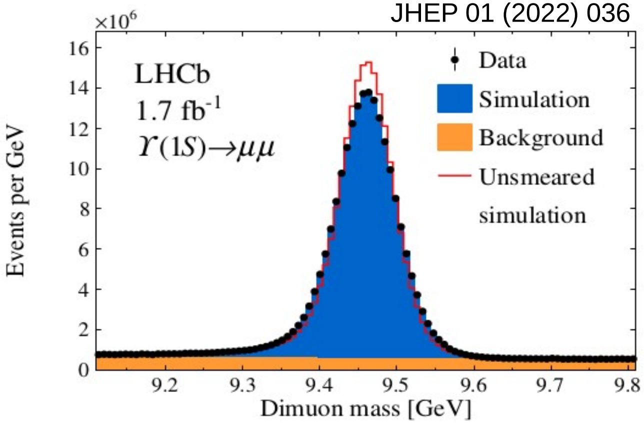
Two slides on calibration

- Leptons calibration from “perfectly known” resonances

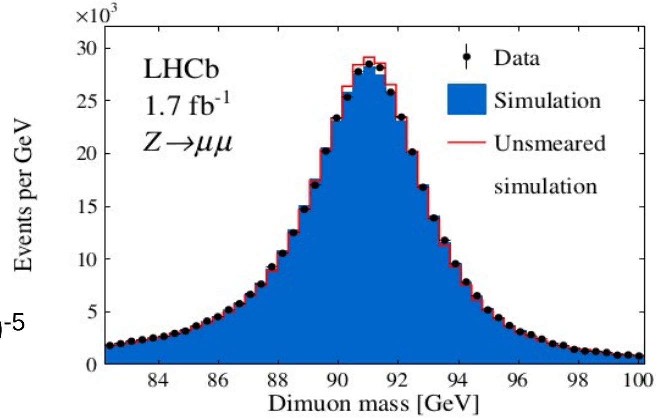
JHEP 01 (2022) 036

$\delta m_{J/psi} / m_{J/psi} \sim 10^{-6}$



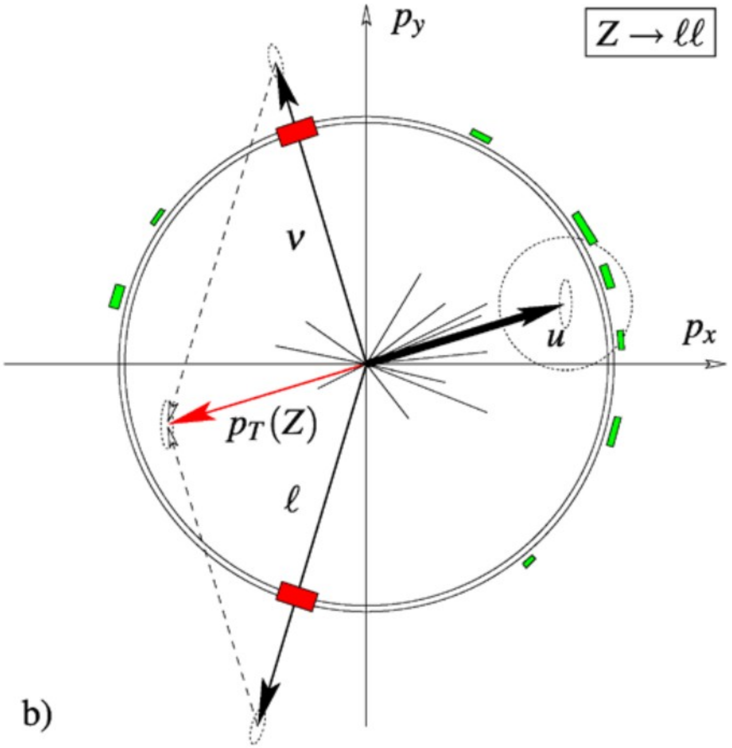


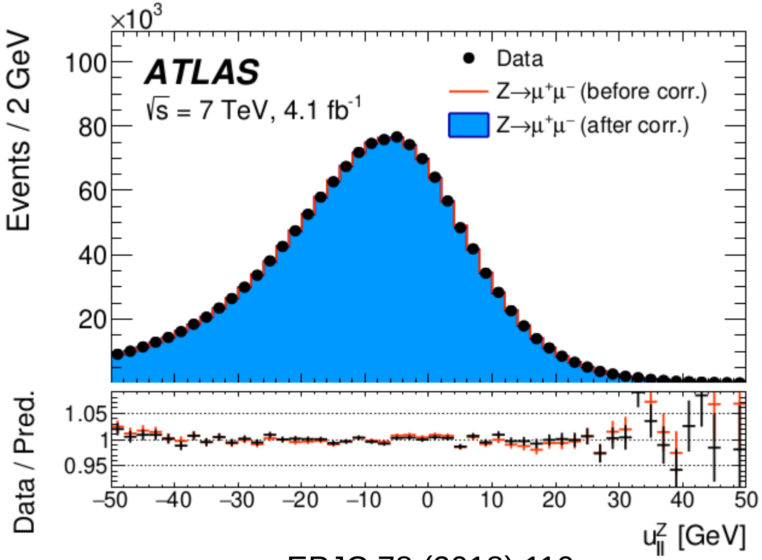
$\delta m_Z / m_Z \sim 2.10^{-5}$



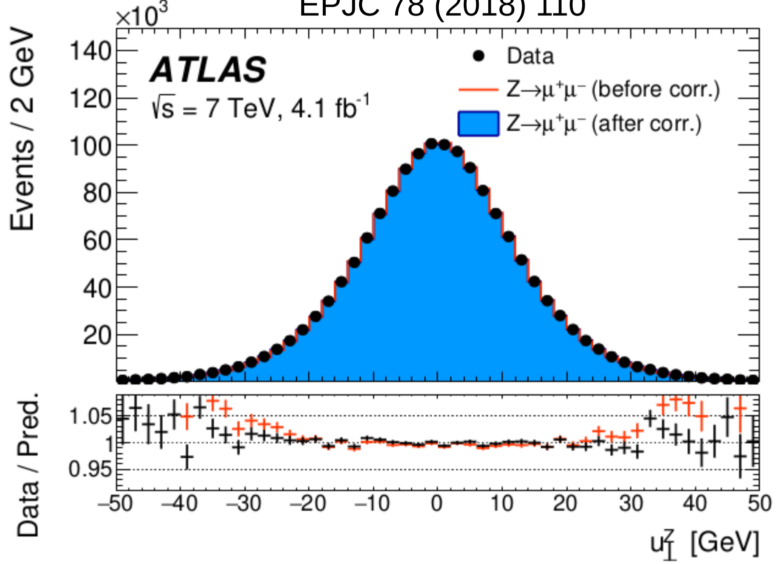
Two slides on calibration

- Recoil response & resolution calibrated using over-constrained kinematics in Z events





EPJC 78 (2018) 110



Vector-boson production at the LHC

- The magic formula, true to all orders in QCD:

$$\frac{d^5\sigma}{dp_1\,dp_2}=\frac{d^3\sigma}{dm\,dy\,dp_T}\left[\left(1+\cos^2\theta\right)+\sum_i A_i\left(p_T,y\right)f_i\left(\theta,\phi\right)\right]$$

production decay

  - Not implemented in this way in generators (which evaluate matrix elements and PDFs) but useful to factor the different QCD modelling aspects, and describe each component using the most appropriate tool
  - Also holds in presence of FSR QED radiation; small deviations expected when due to ISR, IFI

Vector-boson production at the LHC

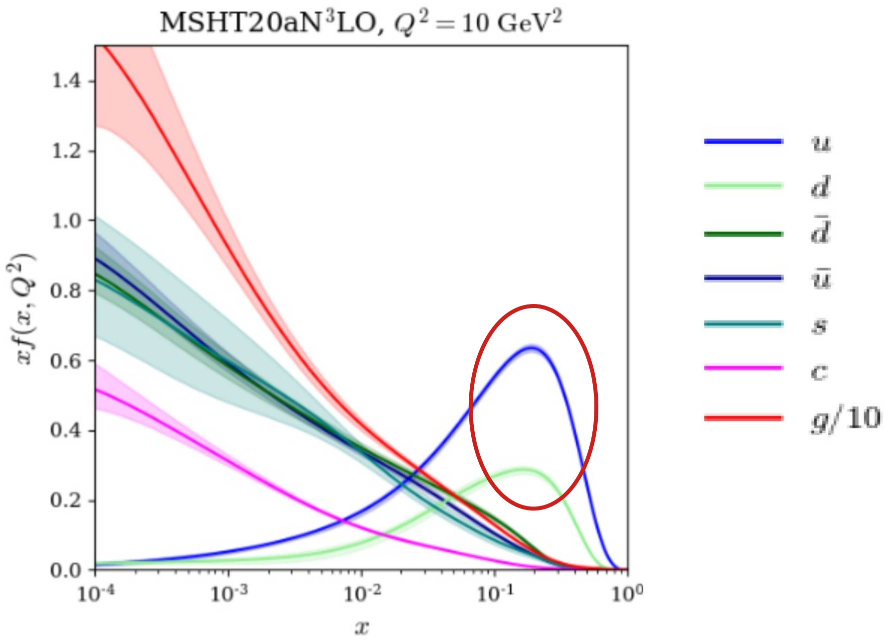
- Rewritten like this for our purpose (integrating over mass) :

$$\frac{d^4\sigma}{dp_1\,dp_2}=\left[\frac{d^2\sigma(y)}{dy}\right]\left[\frac{1}{\sigma(y)}\frac{d\sigma(p_T|y)}{p_T}\right]\left[(1+\cos^2\theta)+\sum_i A_i(p_T,y)f_i(\theta,\phi)\right]$$

production | decay

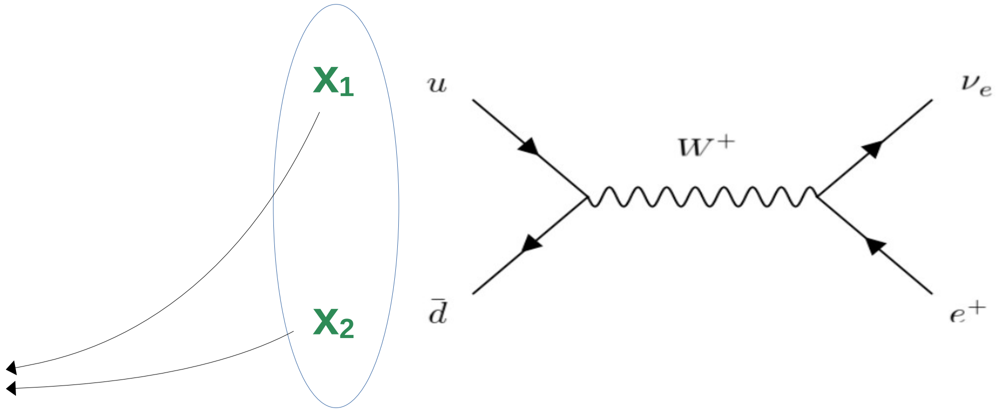
- First factor : p_T-integrated rapidity distribution → fixed-order QCD (NNLO so far)
- Second factor : p_T distribution at given rapidity → parton showers or resummation
- Third factor : spin correlation → fixed-order QCD (NNLO so far)

Rapidity distribution and PDFs



$m_W \sim 80$ GeV $\quad$ $x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

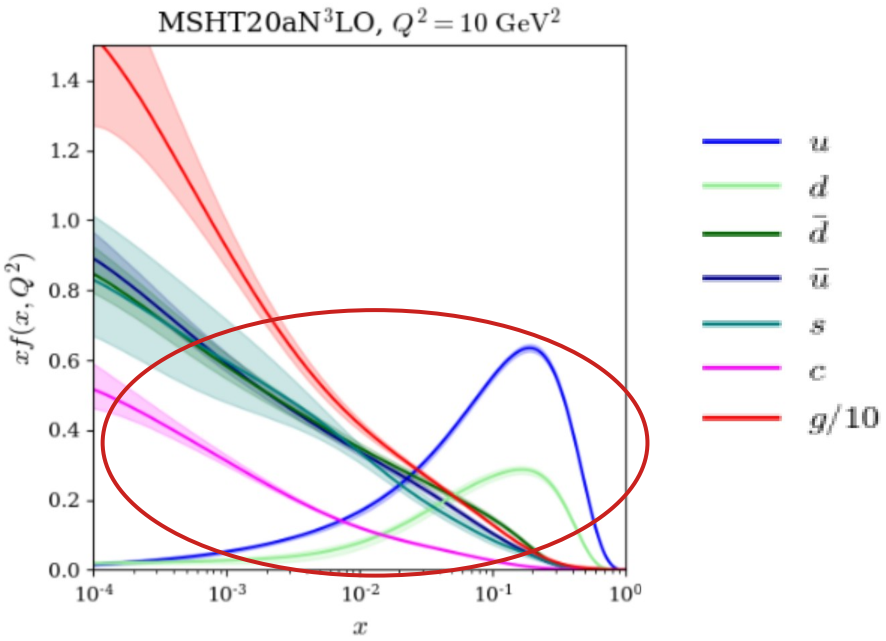
Tevatron	$\sqrt{s} \sim 2$TeV	$p\bar{p}$	$0<y<2$	$x_{1,2} \sim 10^{-2} - 10^{-1}$
ATLAS	$\sqrt{s} \sim 7$TeV	pp	$0<y<3$	$x_{1,2} \sim 10^{-3} - 10^{-1}$
LHCb	$\sqrt{s} \sim 13$TeV	pp	$y \sim 4$	$x_{1,2} \sim 10^{-4} - 10^{-1}$

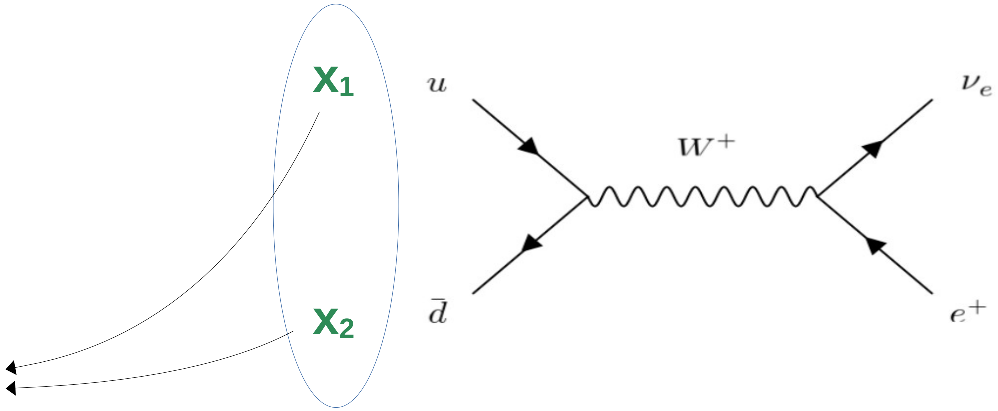


Rapidity distribution and PDFs

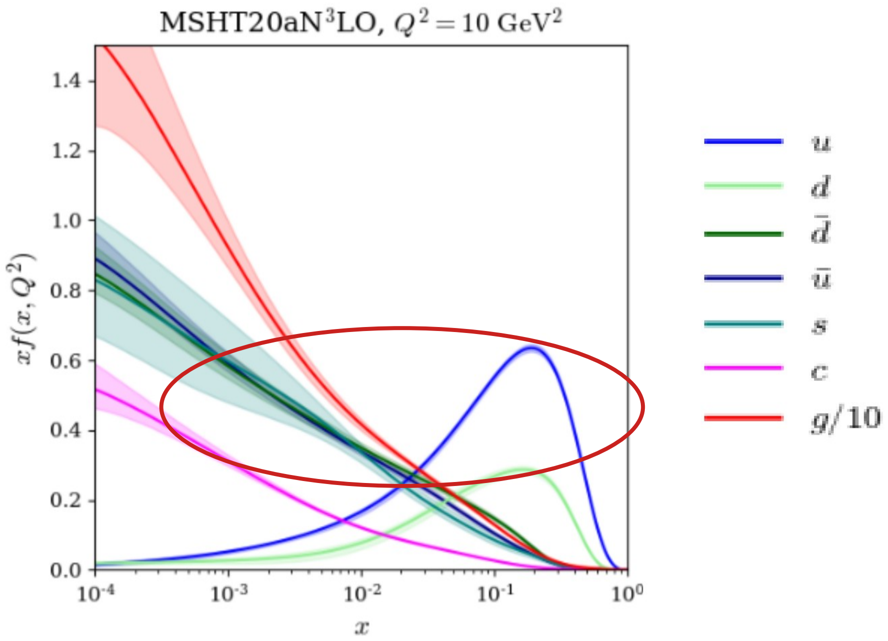
$m_W \sim 80$ GeV $\qquad x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

Tevatron	$\sqrt{s} \sim 2$TeV	$p\bar{p}$	$0<y<2$	$x_{1,2} \sim 10^{-2} – 10^{-1}$
ATLAS	$\sqrt{s} \sim 7$TeV	pp	$0<y<3$	$x_{1,2} \sim 10^{-3} – 10^{-1}$
LHCb	$\sqrt{s} \sim 13$TeV	pp	$y \sim 4$	$x_{1,2} \sim 10^{-4} – 10^{-1}$



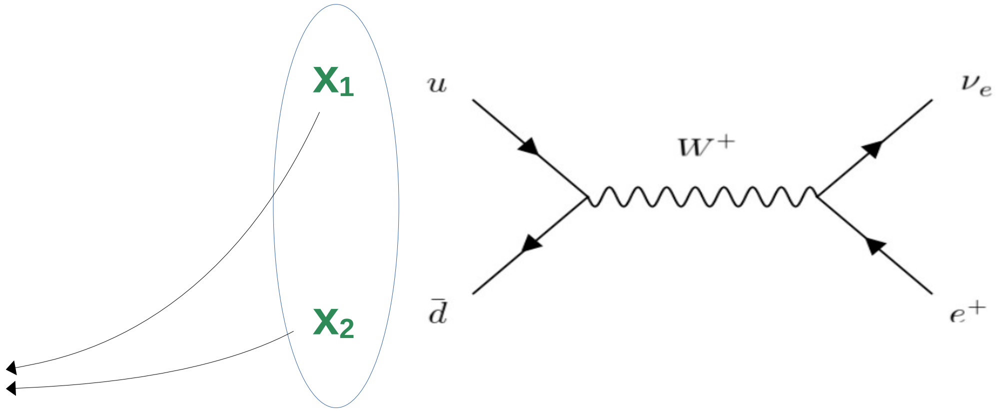


Rapidity distribution and PDFs



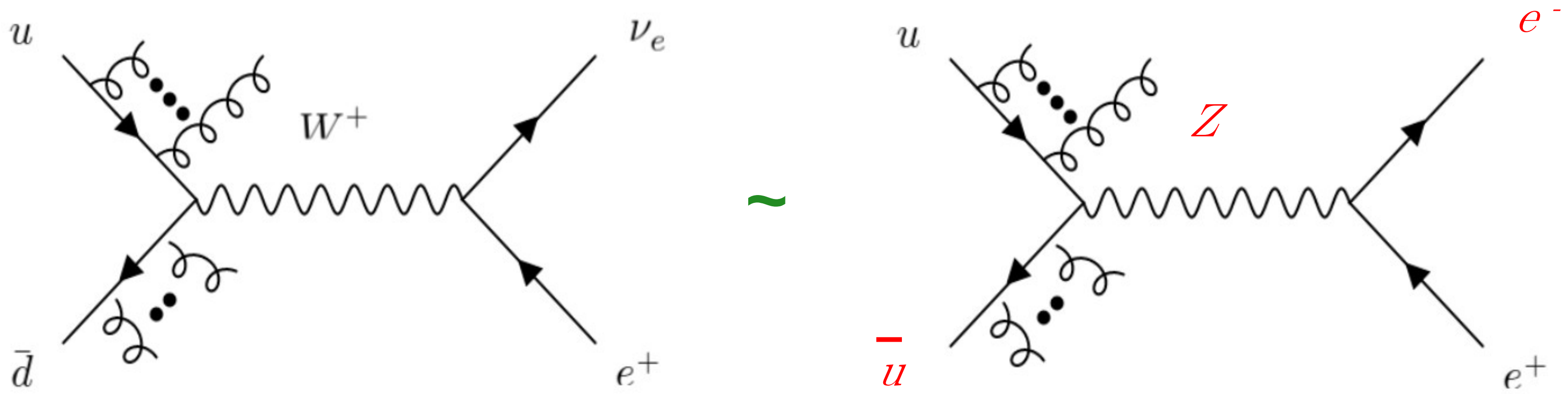
m_W ~ 80 GeV $\quad$ $x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

Tevatron	$\sqrt{s}$~ 2TeV	$p\bar{p}$	$0<y<2$	$x_{1,2} \sim 10^{-2} – 10^{-1}$
ATLAS	$\sqrt{s}$~ 7TeV	pp	$0<y<3$	$x_{1,2} \sim 10^{-3} – 10^{-1}$
LHCb	$\sqrt{s}$~13TeV	pp	y~4	$x_{1,2} \sim 10^{-4} – 10^{-1}$



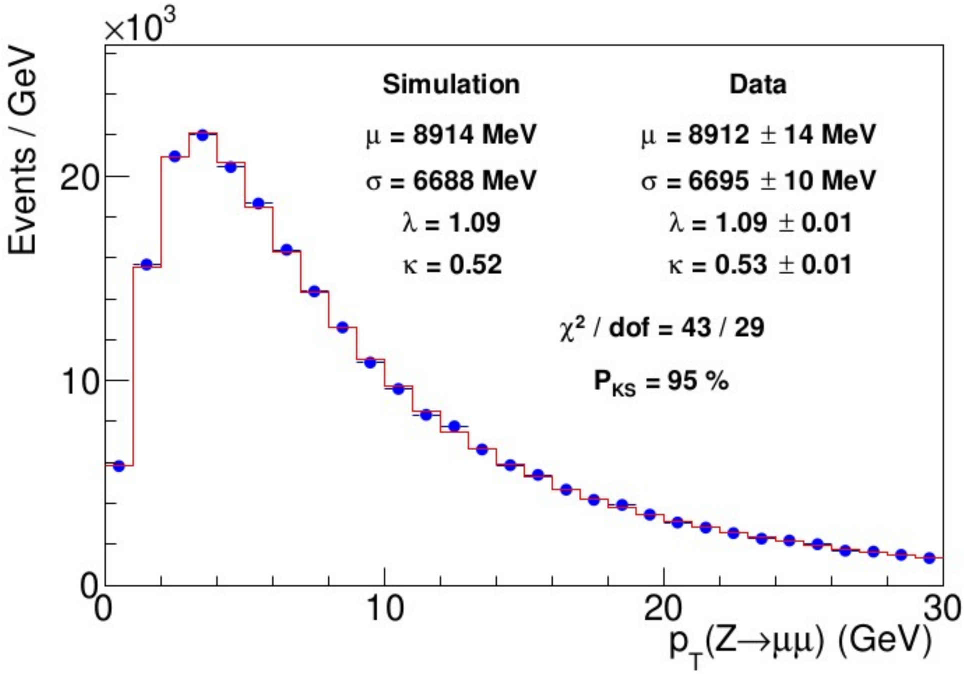
Transverse momentum distribution

- Initial state radiation involves large corrections, and is in part non-perturbative. W events are only partly measured (neutrino!)
- Approach : adjust model parameters using Z events, which are close to W's and can be measured precisely; extrapolate to W production

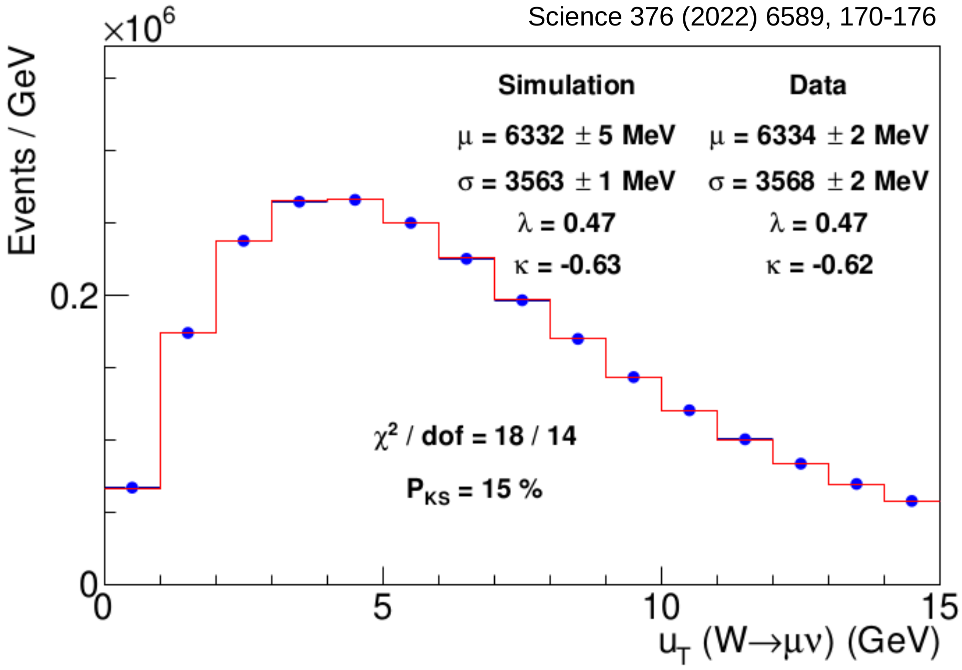


Transverse momentum distribution

- **Tevatron** : Z-based model tuning (Resbos); no extrapolation uncertainties, but validation with W events



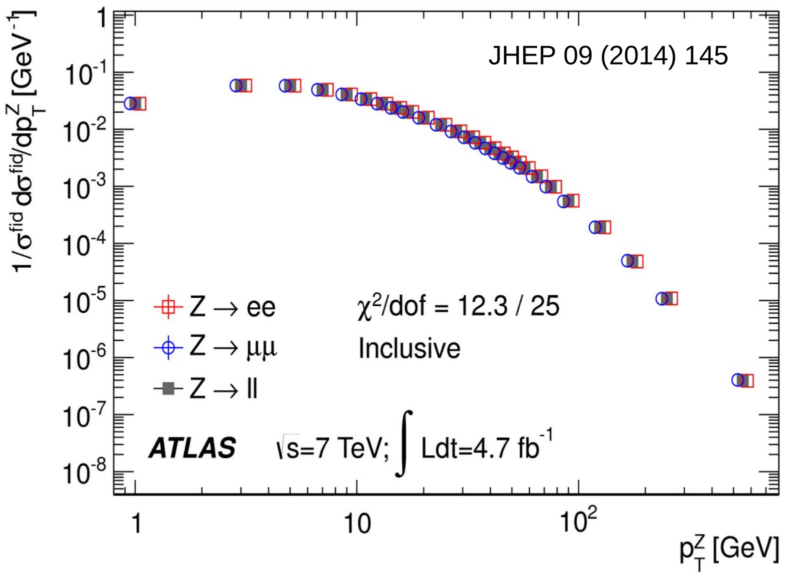
Science 376 (2022) 6589, 170-176

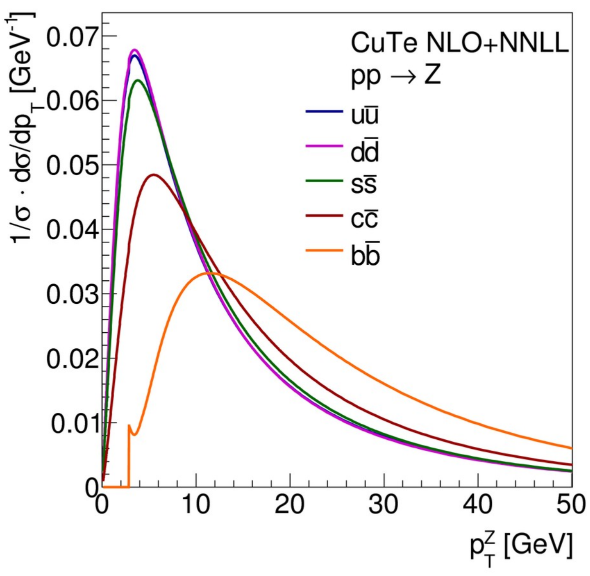


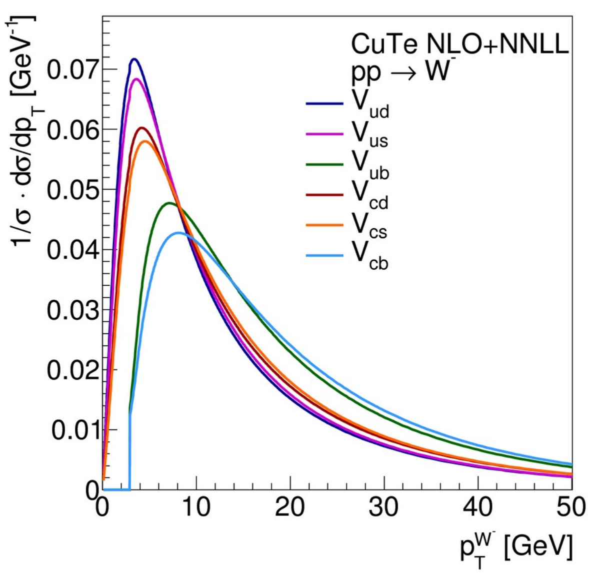
Transverse momentum distribution

- **ATLAS** : Z-based model tuning (Pythia) + Z→W extrapolation
- Corresponding uncertainties :
    - HQ mass treatment in showers and resummation
    - HQ PDFs

Measurement precision ~0.5%

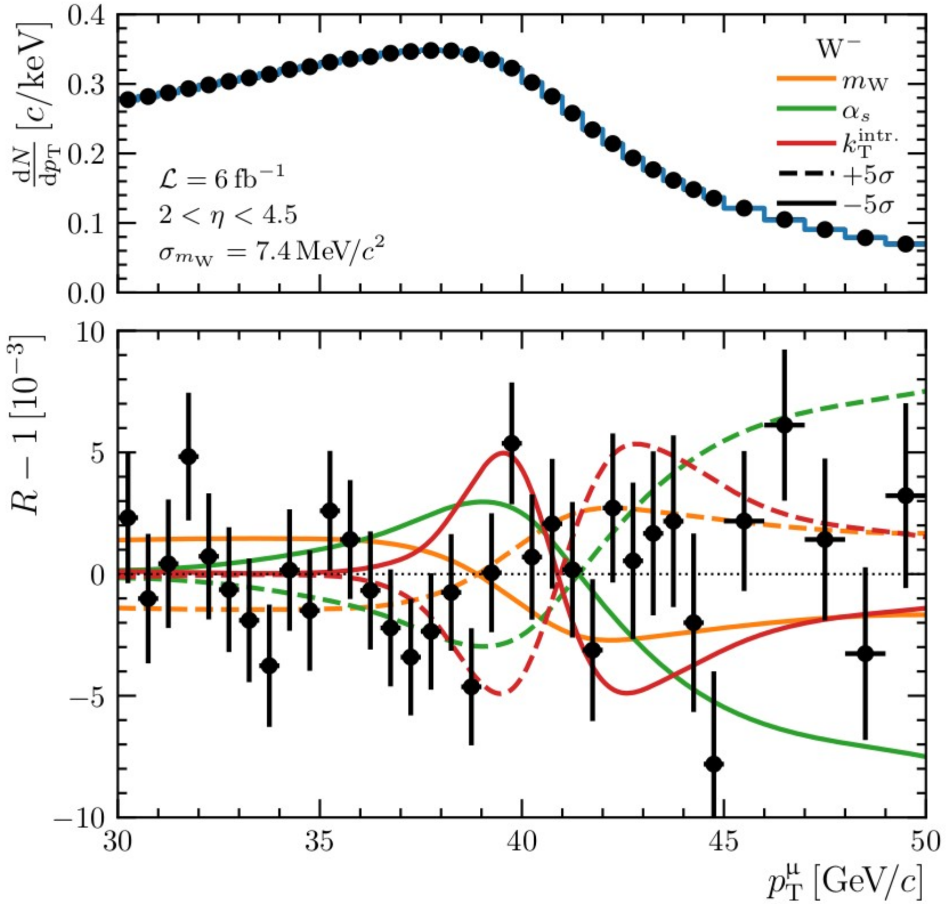






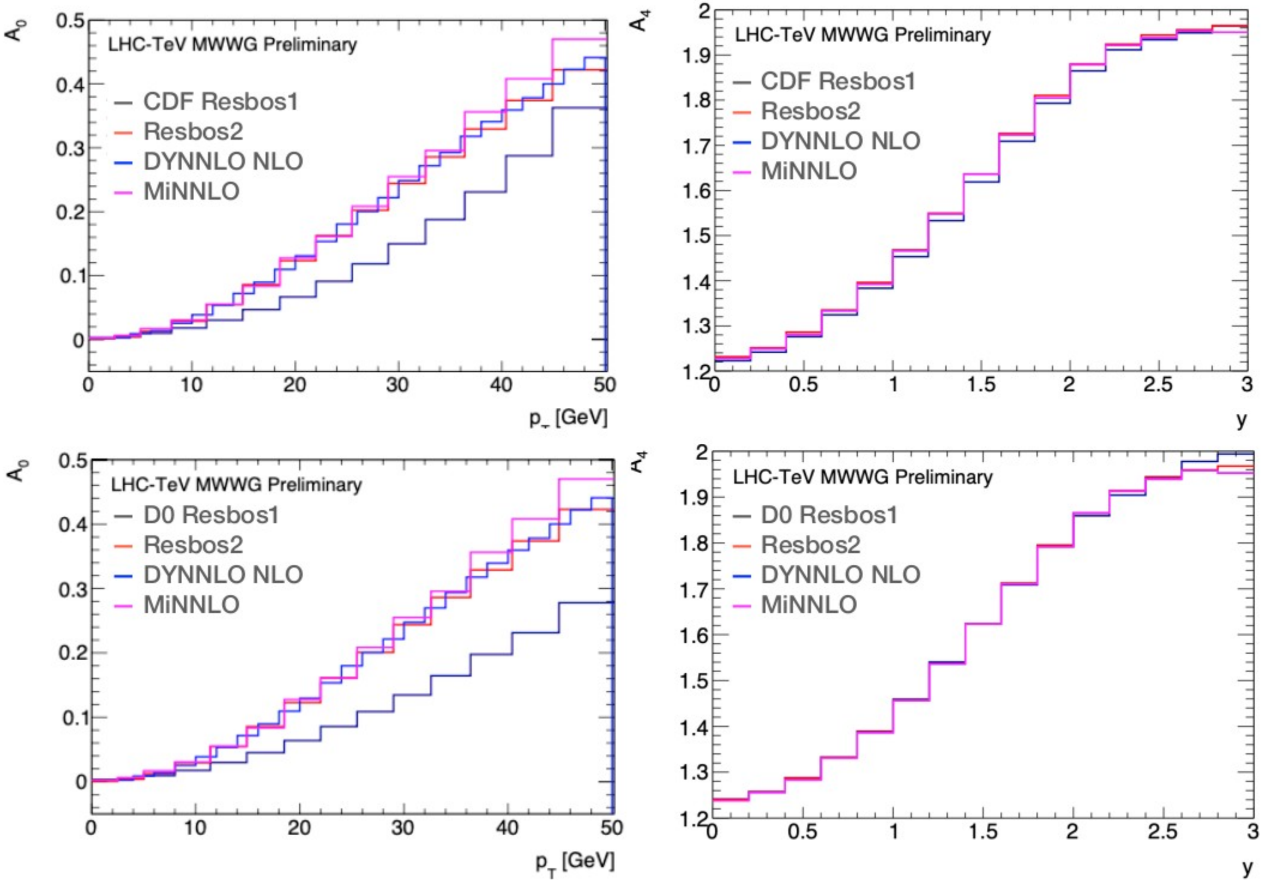
Transverse momentum distribution

- **LHCb** :
  - Z data
  - simultaneous fits to mW and pTW in W events
  - repeated for different theoretical models



Spin correlations

- Boson polarisation in legacy Resbos different from Resbos2 and other codes

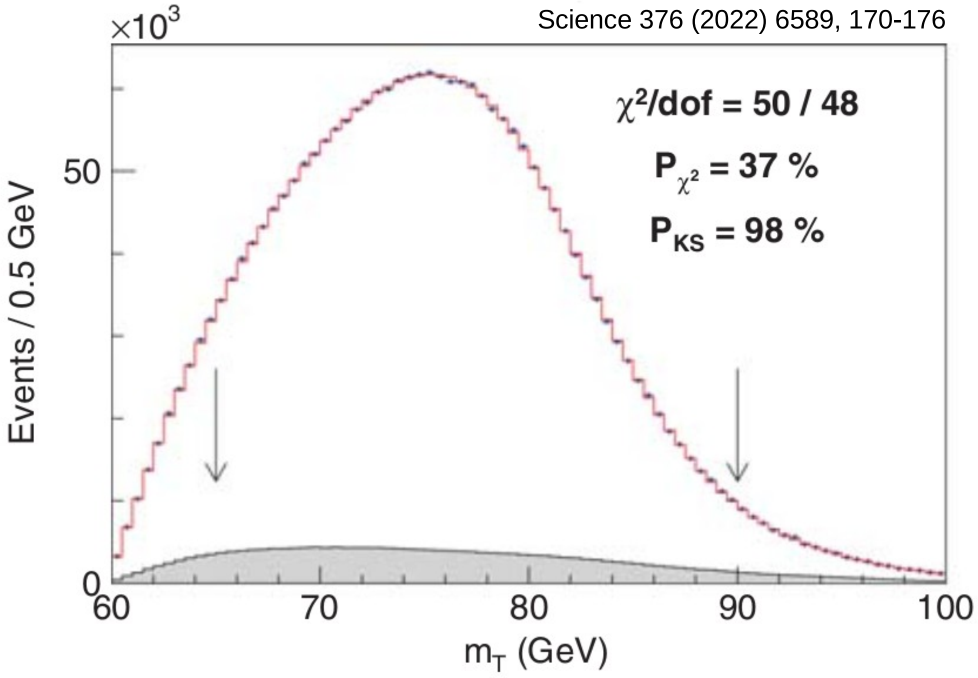


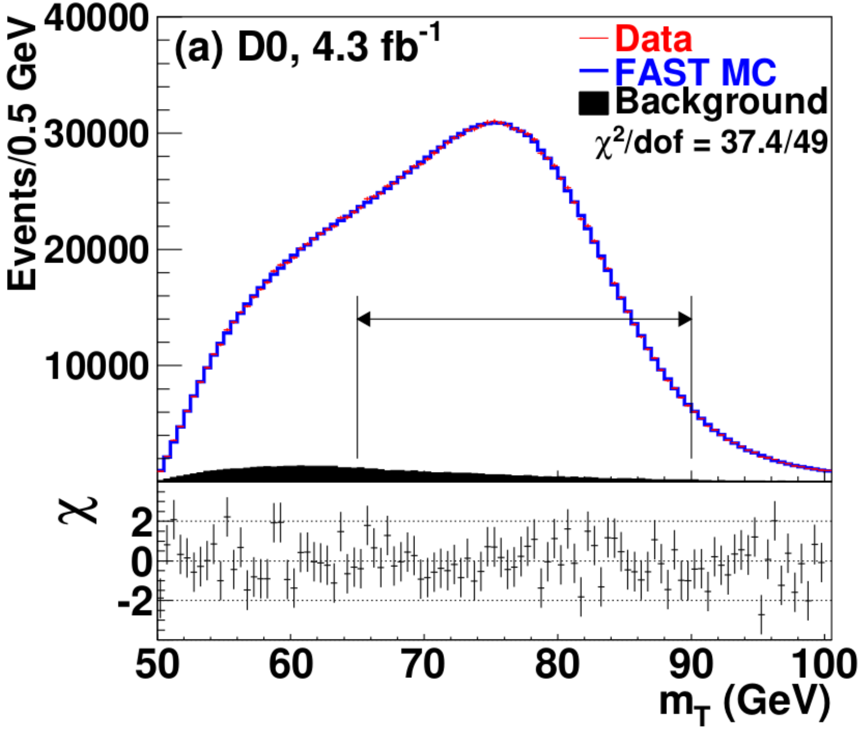
- NNLO matching in Resbos not fully differential
  ➜ affects D0
- Issue with Ai resummation,
  ➜ affects CDF/D0
  - Only unpolarised and A_4 resummed, leads to differences from fixed-order Ai
  - Differences visible comparing to DYNNLO or MiNNLOPS
- Motivates a correction of Tevatron measurements to a common QCD calculation

S.Amoroso, ICHEP 2022

After all is said and done...

- CDF, D0

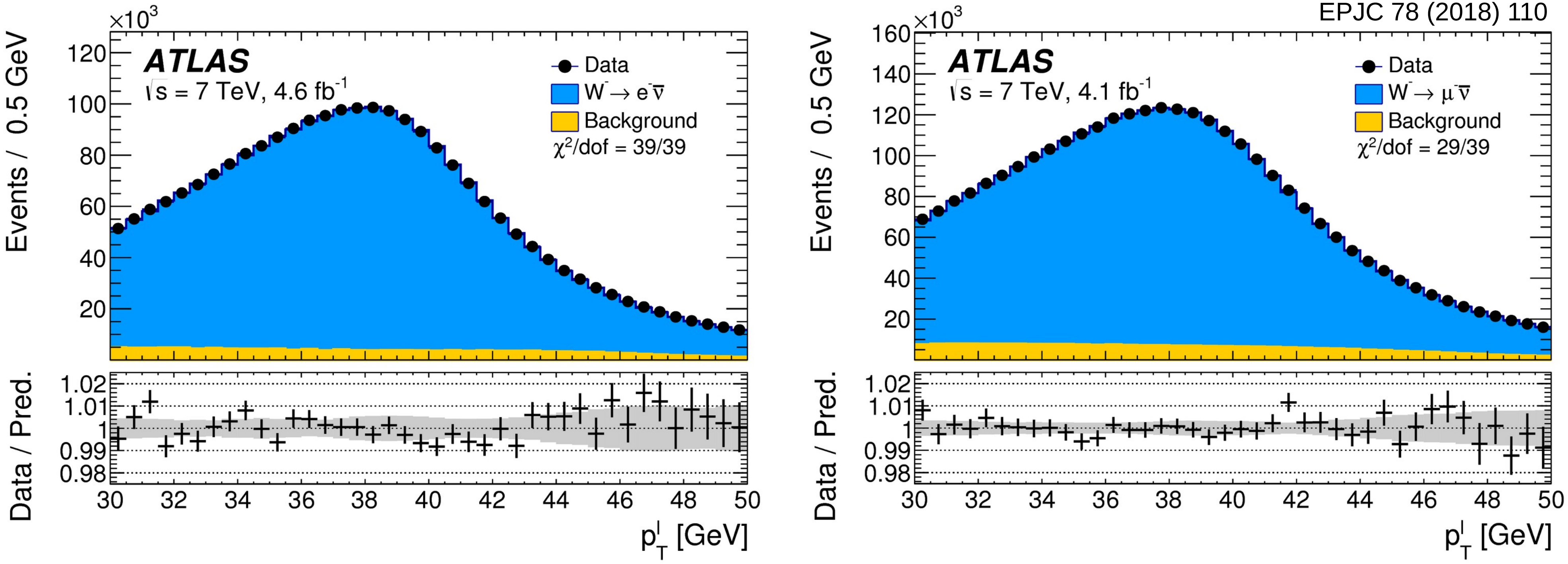




After all is said and done...

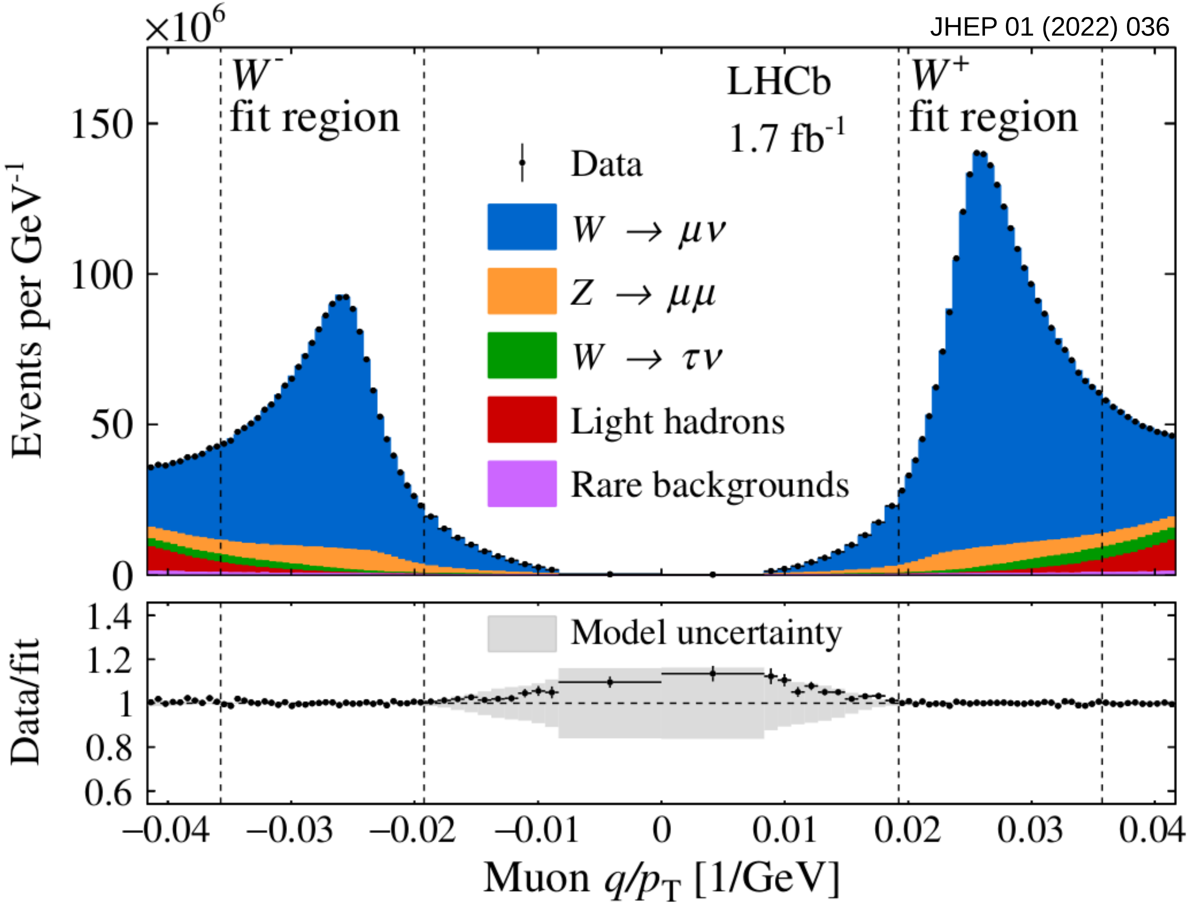
- ATLAS

EPJC 78 (2018) 110



After all is said and done...

- LHCb



Comments

- The W-boson mass measurement does typically **not** use state of the art theory… which sounds unfortunate, for such an important test
  - Bad reasons : tradition; sociology; disconnection from theory caused by the lengthy experimental procedures, ….
  - Better reasons : being based on detector-level distributions, the measurement requires a fully exclusive description of the final state (QCD and QED showers, underlying event). Exclusive tools are generally behind, in terms of perturbative accuracy
- Recent developments of relevance for the measurement : N^3LO / N^3LL QCD; mixed QCD/EW corrections.
  - When not using this, at least quote the corresponding uncertainties
  - Most often fixed-order results : difficult to exploit!
- The “dream tool” for this measurement would be a consistent interface between the exclusive MC generators and state-of-the-art perturbative accuracy. Huge challenge, but ultimately fundamental for this field.

Comments

- Recent ATLAS update and PDF uncertainties (ATLAS-CONF-2023-004)

PDF-Set	p_T^ℓ [MeV]	m_T [MeV]	combined [MeV]
CT10	$80355.6^{+15.8}_{-15.7}$	$80378.1^{+24.4}_{-24.8}$	$80355.8^{+15.7}_{-15.7}$
CT14	$80358.0^{+16.3}_{-16.3}$	$80388.8^{+25.2}_{-25.5}$	$80358.4^{+16.3}_{-16.3}$
CT18	$80360.1^{+16.3}_{-16.3}$	$80382.2^{+25.3}_{-25.3}$	$80360.4^{+16.3}_{-16.3}$
MMHT2014	$80360.3^{+15.9}_{-15.9}$	$80386.2^{+23.9}_{-24.4}$	$80361.0^{+15.9}_{-15.9}$
MSHT20	$80358.9^{+13.0}_{-16.3}$	$80379.4^{+24.6}_{-25.1}$	$80356.3^{+14.6}_{-14.6}$
NNPDF3.1	$80344.7^{+15.6}_{-15.5}$	$80354.3^{+23.6}_{-23.7}$	$80345.0^{+15.5}_{-15.5}$
NNPDF4.0	$80342.2^{+15.3}_{-15.3}$	$80354.3^{+22.3}_{-22.4}$	$80342.9^{+15.3}_{-15.3}$

~15% improvement in uncertainty from using a profile likelihoo analysis

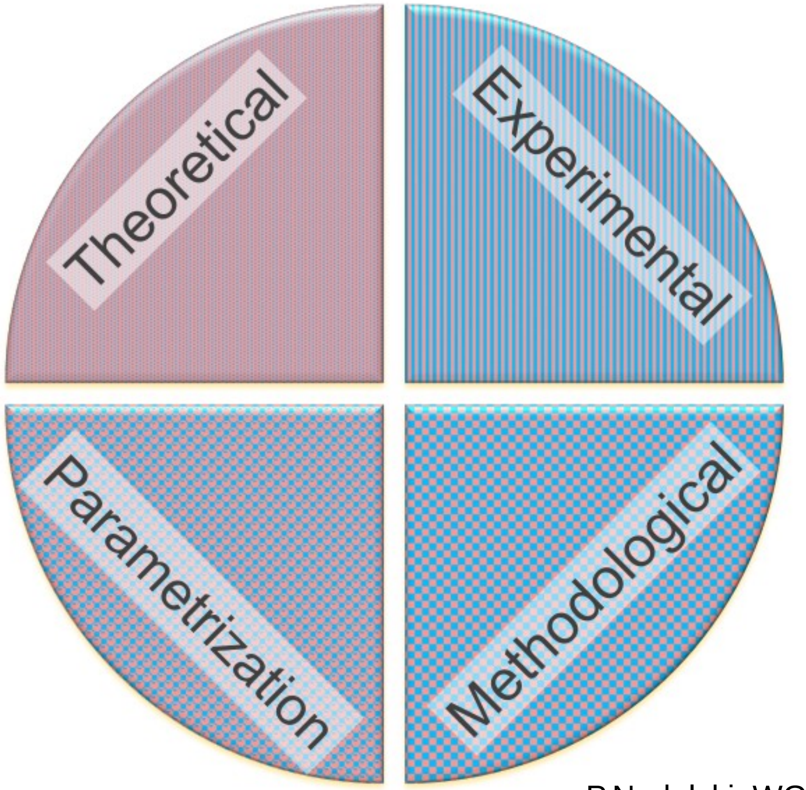
Large PDF dependence; eg NNPDF4.0 and CT18 differ by 18 MeV.

Estimated PDF uncertainties 3 → 9 MeV. What to do??

(current choice is to discard NNPDF due to worse description of W and Z distributions, and use CT18 as most conservative PDF set among the others. To be revisited for the publication)

Comments

- Recent ATLAS update and PDF uncertainties (ATLAS-CONF-2023-004)
  - Experiments WELCOME the ongoing inclusion of theoretical uncertainties in PDF fits.
  - Still, very difficult to understand the significance of differences between results obtained using different PDF sets
    - Very interesting discussion in WG1
    - better uncertainty decomposition required

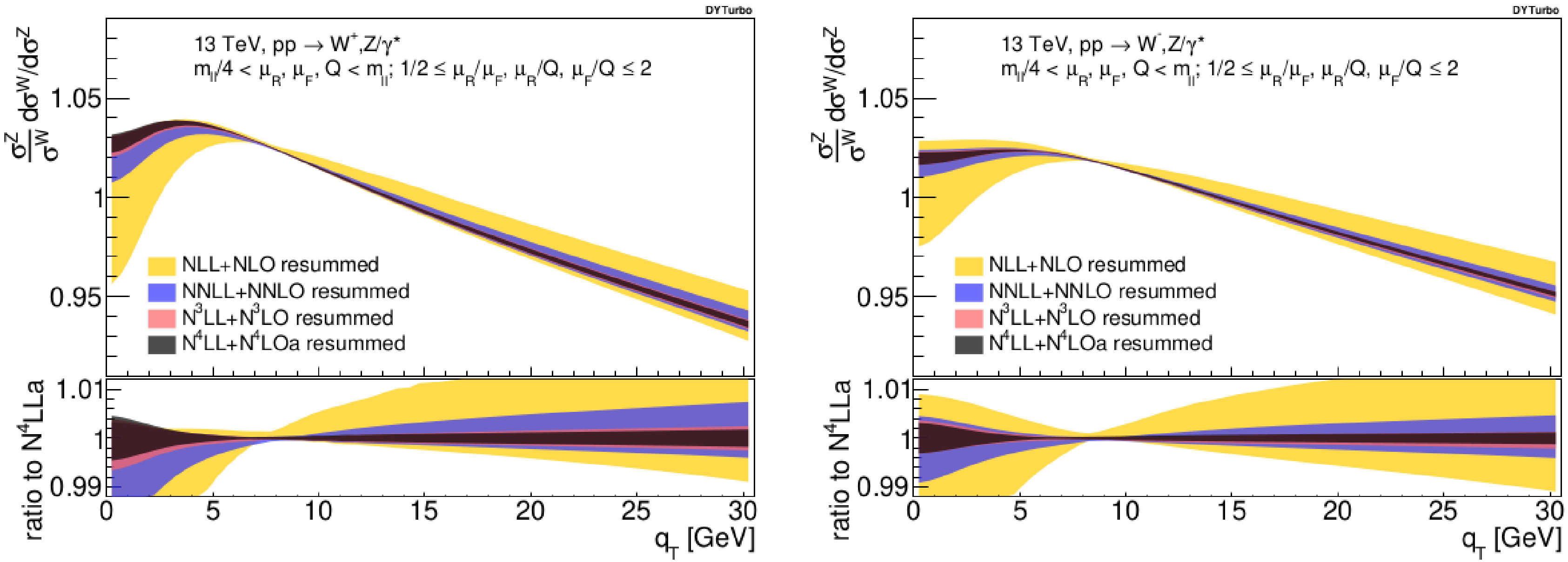


P.Nadolski, WG1

Comments

- Analytical resummation – now at approximate N4LO+N4LL

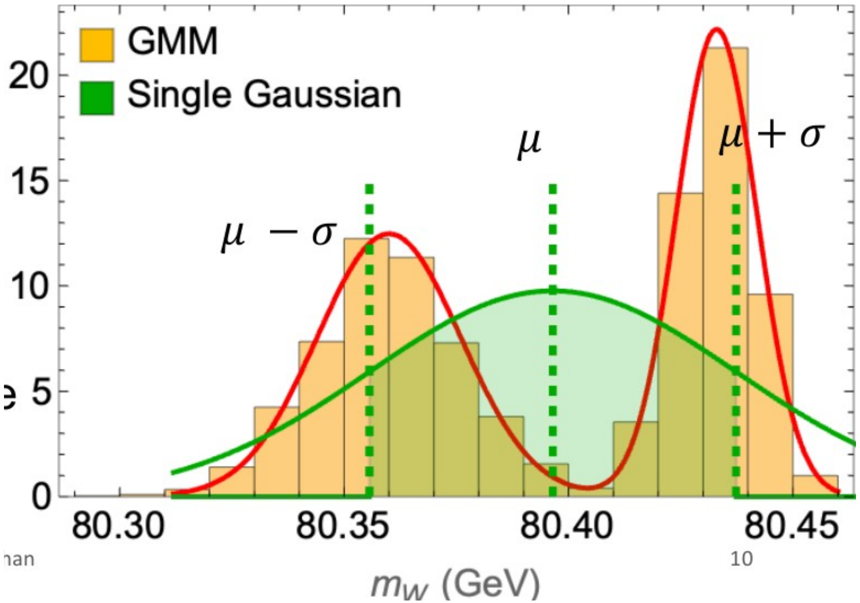
2303.12781



  - Essentially removing any uncertainty in the W/Z pT distribution ratio
  - However, analysis is not complete : flavour-dependent intrinsic kT; heavy-quark mass effects; process-dependent EWK effects… are not (yet) addressed

Tevatron/LHC Combination

- An essentially completed project, waiting to be published
- Addressed QCD and PDF corrections needed to “match” the available measurements; not on the scale of the presently observed discrepancy
- Final presentation of results still under discussion (difficult!)



Kirtimaan Mohan, WG1

Conclusions

- M_W is such an active field, all of a sudden!
- Uncertainty propagation for this measurement currently *almost* broken by the PDFs – we should improve, and the discussions this week were extremely helpful
- Theoretical improvements are of utmost importance to us. Major items needed to enable mW using the recent developments :
  - Mixed QCDxEW corrections including resummation in some form
  - QCD resummation : heavy-quark mass effects and further process-dependent corrections
- Measurement compatibility currently problematic; unsolved, but beyond QCD effects
- More results expected on a fairly short timescale (1-2 years).

Back up

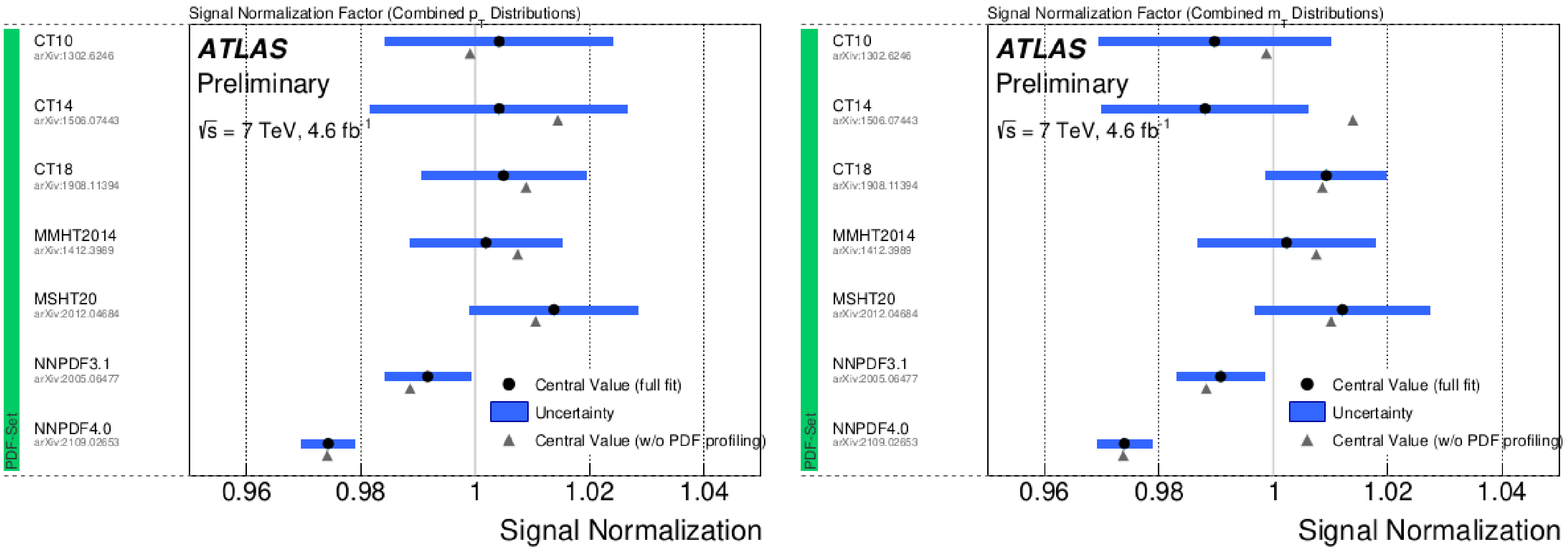


Figure 11: Overview of the global signal normalization factors obtained from the combined PLH fits using the p_{T}^{ℓ} (left) and the m_{T} (right) distributions with different PDF sets. The central values of the normalization factors without PDF profiling in the combined PLH fit are also indicated.

Combination strategy

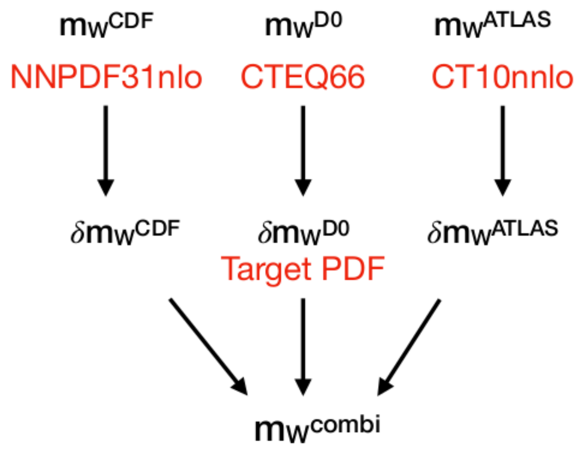
- PDFs main source of correction and uncertainty correlations
  - Other sources very small (EW corrections) or mostly decorrelated (p_T W/Z)

$$m_W^{new} = m_W^{ref} - \delta m_W^{QCD} - \delta m_W^{PDF}$$

published value — Improved predictions — PDF extrapolation

δm_W^{PDF} correction to reference PDF

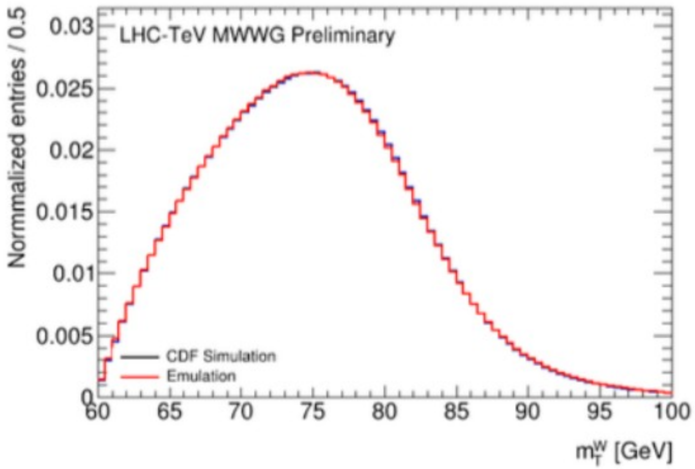
δm_W^{QCD} correction to QCD modelling beyond quoted uncertainties

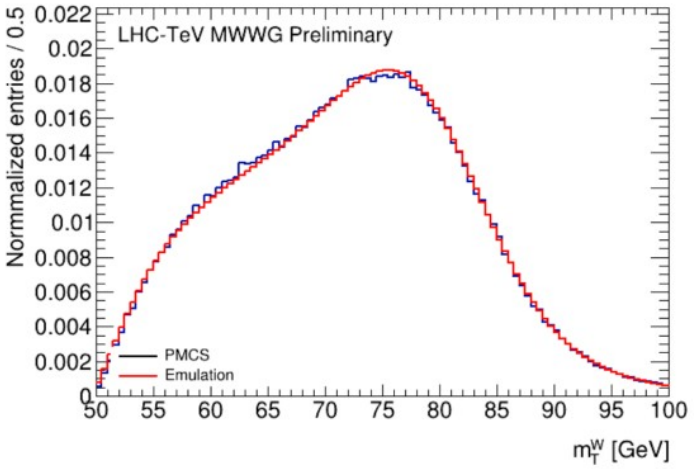


- Correction applied in a two-step procedure:
  1. Correct all measurements to a common PDF/QCD
  2. Combine them properly including correlations

Measurement emulation

- Original analyses and detector simulations cannot be easily reproduced
  - Exception is LHCb for which the analysis will be rerun
- Use **parametrized detector response,** following published information
  - Leptons : η- and p_T-dependent energy/momentum scale as well as resolution and efficiencies
  - Recoil response: include “lepton removal” effects, dependence on boson p_T and event activity
  - Reproduces published distributions at the % level corresponding to **~1-2 MeV precision in $\delta m_W^{QCD,PDF}$**
- Event selection and fit ranges from publications





Event generators

- Fully **reproduced the event generation chain** from the original measurements

  D0: Resbos CP (NNLO+NNLL) generated with CTEQ66 (NLO)

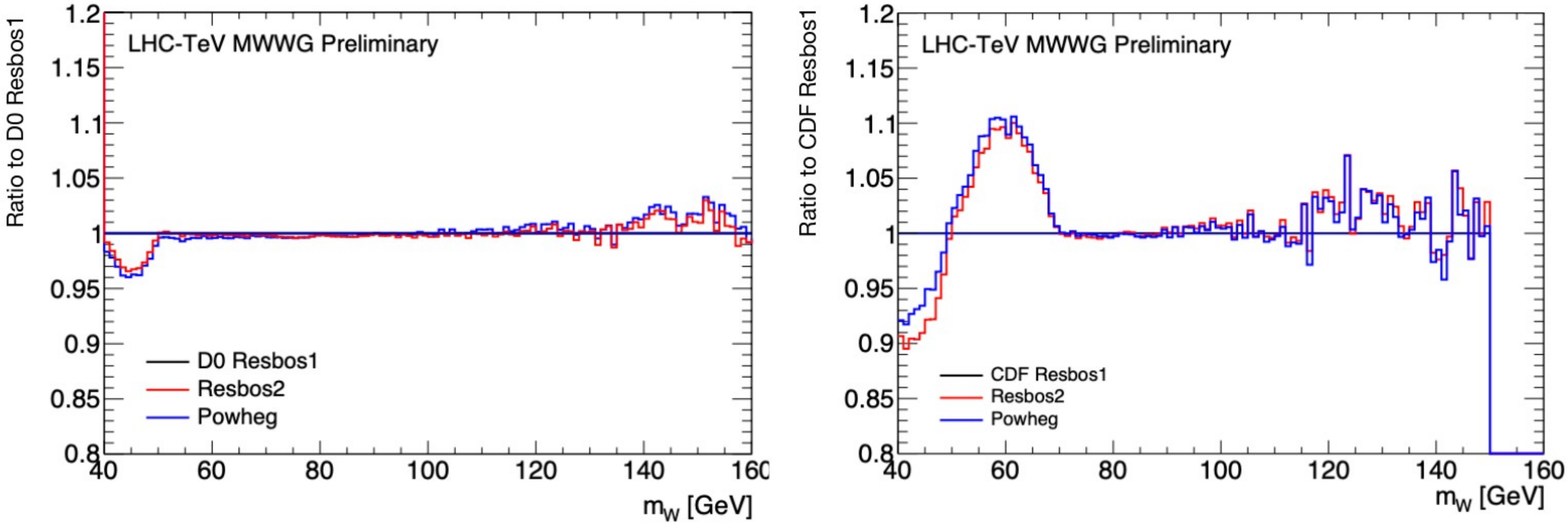
  CDF: Resbos C (NLO+NNLL) generated with CTEQ6M (NLO)

  ATLAS: Powheg+Pythia8 (NLO+PS); y_W + Ai at NNLO with CT10 (NNLO)

  LHCb: Powheg+Pythia8 (NLO+PS); Ai at NNLO,
  as PDF the average of NNPDF3.1,CT18,MSHT20 (NLO)

- Variety of predictions used to validate the PDF shifts and estimate the possible need of QCD correction to published m_W
  - Powheg (NLO+PS), MiNNLOPS (NNLO+PS), DYNNLO (NLO/NNLO F.O.)
  - In addition, updated integration grids from the Resbos authors (dubbed here Resbos2) at NLO+NNLL and NNLO+NNLL with improved treatment of spin correlations [*2205.02788*]

Lineshape



- Invariant mass distribution shows trends wrt modern generators
  - Visible **cut of m_W<150 GeV** in the CDF Resbos sample, small bias on m_W
  - Structures at **low invariant masses** (m_W<50 GeV for D0, m_W< 70 GeV for CDF) and **small overall slope** through the full mass range, negligible impact on m_W

Spin correlations in W-boson decay

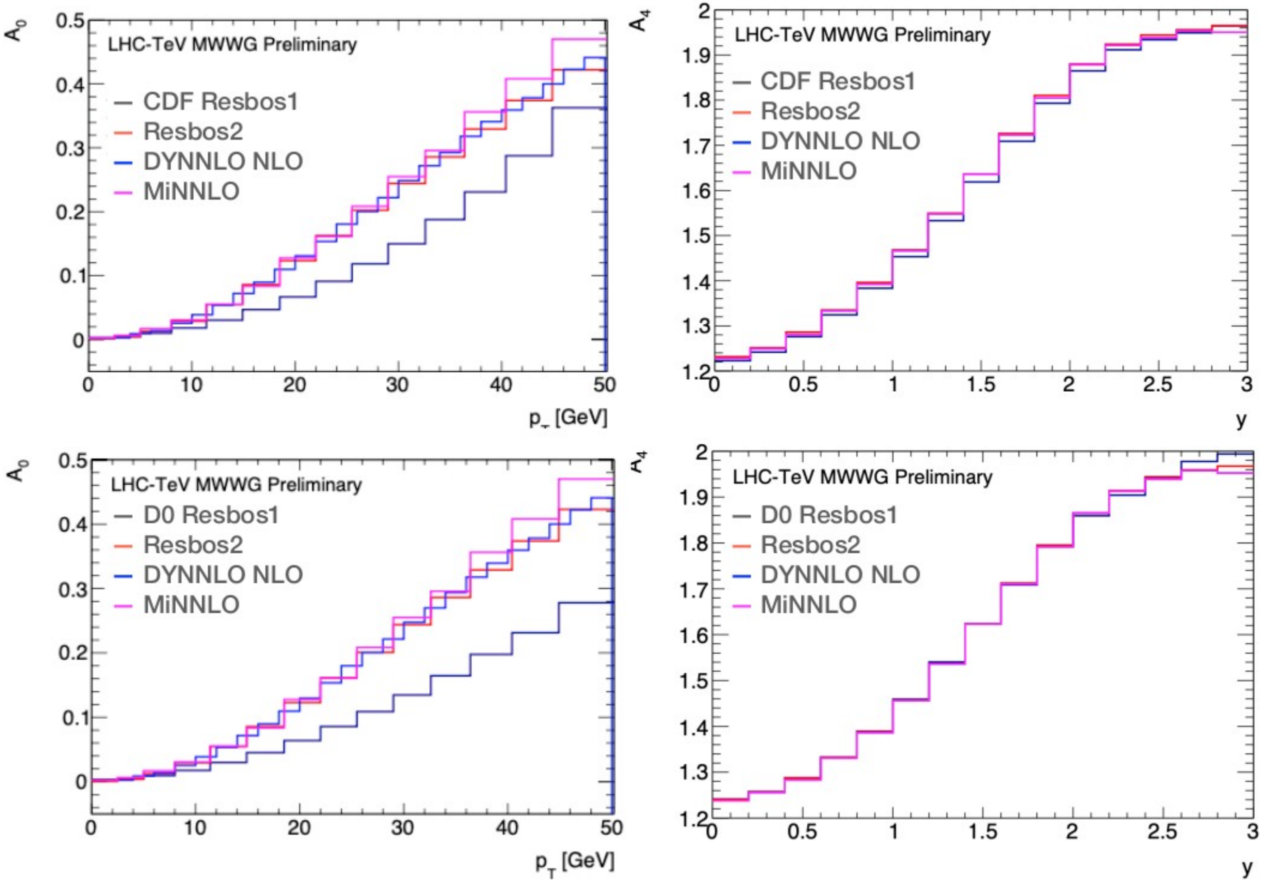
- The cross-section for the production of a spin-1 resonance can be expanded to all-orders in QCD into an **angular coefficients decomposition**:

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{dm\,dp_{\mathrm{T}}\,dy}\Big[\ (1+\cos^2\theta) \quad &+ \quad \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi \\
&+ \quad \frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi \\
&+ \quad A_4\cos\theta + A_5\sin^2\theta\sin 2\phi \\
&+ \quad A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\ \Big],
\end{aligned}
$$

- A_4 the only term at LO QCD; $A_{5,6,7}$ start at $O(\alpha_S^2)$ and remain small
- Measured to high precision in Z events at the LHC [*JHEP* 08(2016) 159, 2203.01602] and well described by fixed-order calculations (known to $O(\alpha_S^3)$ [*JHEP* 11 (2017) 003])

Spin correlations in W-boson decay

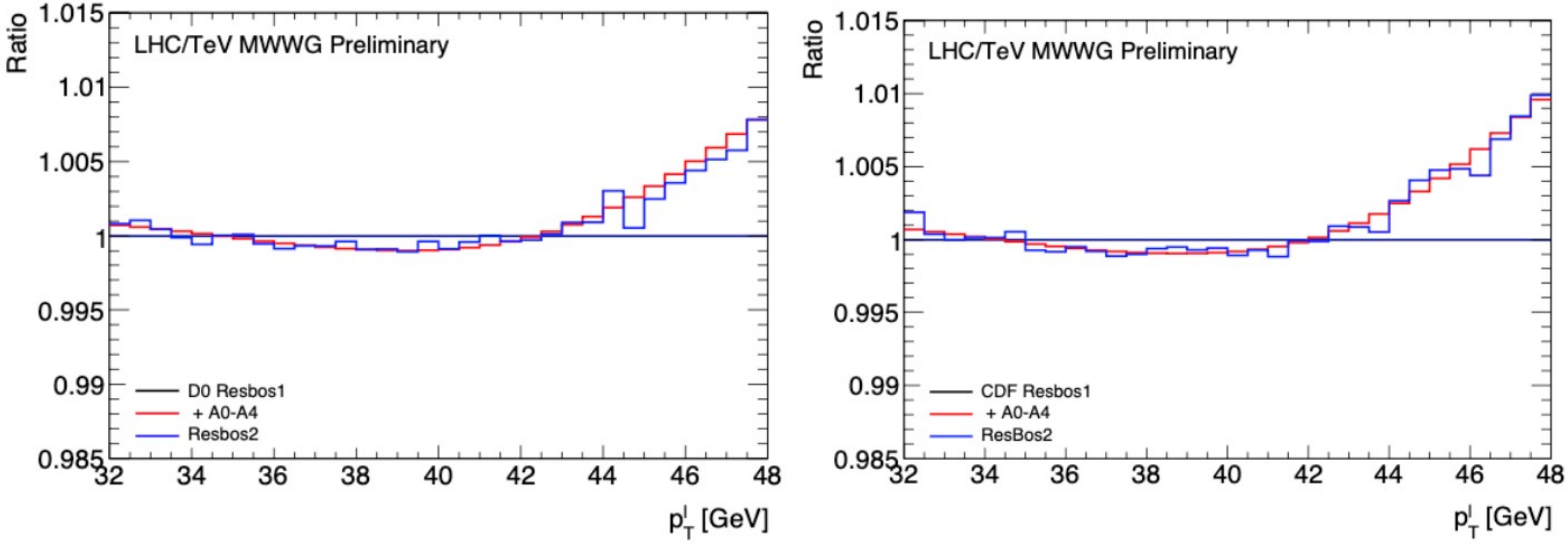
- Boson polarisation in legacy Resbos different from Resbos2 and other codes



- NNLO matching in Resbos not fully differential
  ➜ affects D0
- Issue with Ai resummation,
  ➜ affects CDF/D0
  - Only unpolarised and A_4 resummed, leads to differences from fixed-order Ai
  - Differences visible comparing to DYNNLO or MiNNLOPS
- Motivates a correction of Tevatron measurements to a common QCD calculation

Spin correlations in W-boson decay

- Impact of change in Ai to the new Resbos well-reproduced by reweighting $A_{0\text{-}4}$
  - Effect of up to 1% on detector-level distributions
  - Distributions become harder, m_W in data expected to decrease



- Change in the full phase-space Ai modifies the fiducial $p_T^{W/Z}$ distribution
  - Overestimate δm_W as measurements tune their p_T^W model to data
  - To gauge an uncertainty, change evaluated also constraining the p_T^W distribution

Impact of generator updates

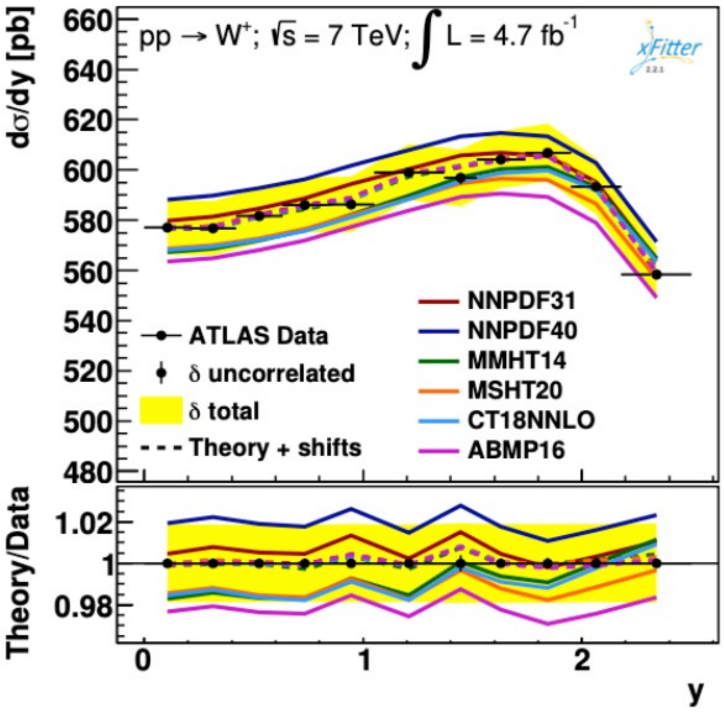
- δm_W^{QCD} reweighing the D0 Resbos-CP NNLO+NNLL predictions to the newer Resbos2 at NLO+NNLL
  - Negligible effect of correcting y_W and m_{Inv}
  - Ai-reweighting dominated by A_0 coefficient
  - δm_W about -10 MeV depending on distribution and p_T^W constraint
  - ~2 MeV uncertainty from systematics on the emulation

Correction	$\delta m_W^{\mathrm{QCD}}$ [MeV]					
	p_T^W-constrained			No constraint		
	p_T^ℓ	m_T	p_T^ν	p_T^ℓ	m_T	p_T^ν
Invariant mass	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1
Rapidity	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1
A_0	7.6	10.0	15.8	16.0	12.6	19.5
A_1	-2.4	-1.9	-1.8	-1.2	-1.6	-1.4
A_2	-3.0	-2.6	2.9	-4.2	-3.0	2.3
A_3	2.9	1.6	-0.5	3.5	1.8	-0.2
A_4	2.4	-0.1	-0.5	0.1	-0.7	-1.0
$A_0 - A_4$	7.6	7.0	16.0	14.1	9.1	18.9
Total	7.6	7.0	16.0	14.1	9.1	18.9
ResBos2	7.3±1.1	8.4±1.0	16.6±1.2	13.9±1.1	10.3±1.0	19.8±1.2
Non-closure	-0.3±1.1	1.4±1.0	0.6±1.2	-0.2±1.1	1.2±1.0	0.9±1.2

Choice of PDF sets

- Performed a benchmarking of PDF sets against Tevatron and LHC cross-section measurements
  - Considering measurements of W and Z cross-sections from Tevatron and LHC
  - Theory predictions at NNLO QCD x NLO EW

PDF set	Chi2/ndf	PDF set	Chi2/ndf
Cteq66	231/126	**CT18NNLO**	163/126
CT10	179/126	**CT18ANNLO**	170/126
NNPDF31	200/126	**MSHT20**	270/126
NNPDF40	195/126	**ABMP16**	236/126



- Modern NNLO PDFs provide the best description, no set gives a χ^2/ndf~1
- Decision on the final PDF will consider χ^2 and uncertainty of the combination itself

Combination – status

- Analysis completed :
  - Generator corrections and PDF extrapolations finalized for all experiments
  - Results available for a variety of PDF sets : ABMP16, CT14, CT18, MMHT2014, MSHT20, NNPDF3.1 and NNPDF4.0
    - Important messages on the PDF dependence of the measurement
  - Compatibility quantified for the full combination, and for relevant subsets of measurements : LHC only; Tevatron only; “All – 1”
  - Final recommendation : ?

→ currently under review by all collaborations