Approaching partons on a Euclidean Lattice

Guido Altarelli Award Acceptance

DIS2023: XXX International Workshop on Deep-Inelastic Scattering and Related Subjects

Michigan State University, MI, March 27-31, 2023

YONG ZHAO

MAR 27, 2023

Argonne
NATIONAL LABORATORY

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Xiangdong Ji, Iain Stewart, Feng Yuan, Swagato Mukherjee, William Detmold, Phiala Shanahan, Michael Wagman, Peter Petreczky, David C.-J. Lin, Jianhui Zhang, Huey-Wen Lin, Jiunn-Wei Chen, Yi-Bo Yang, Yoshitaka Hatta, Luchang Jin, … …

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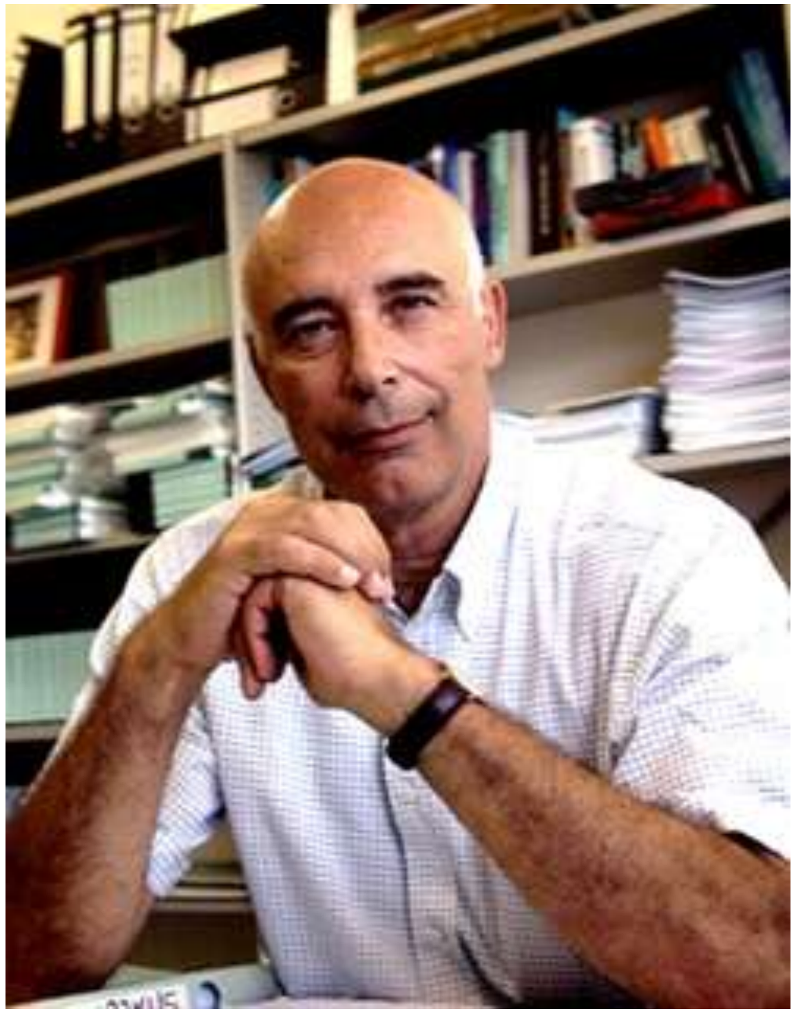
"A leading figure of modern particle physics who contributed to almost all the relevant aspects of the Standard Theory."

Maiani and Martinelli, Ann.Rev.Nucl.Part.Sci. 68 (2018)



Guido Altarelli (1941-2015)

- **Quantum chromodynamics**
  "Asymptotic freedom in parton language"
  Altarelli-Parisi equation
- **Precision electroweak physics**
- **Neutrino physics**
- **Polarized proton structure**
  "The anomalous gluon contribution ($\triangle G$) to polarized leptoproduction"
- **Grand unified theory**
- **Small-x physics**
- **… …**

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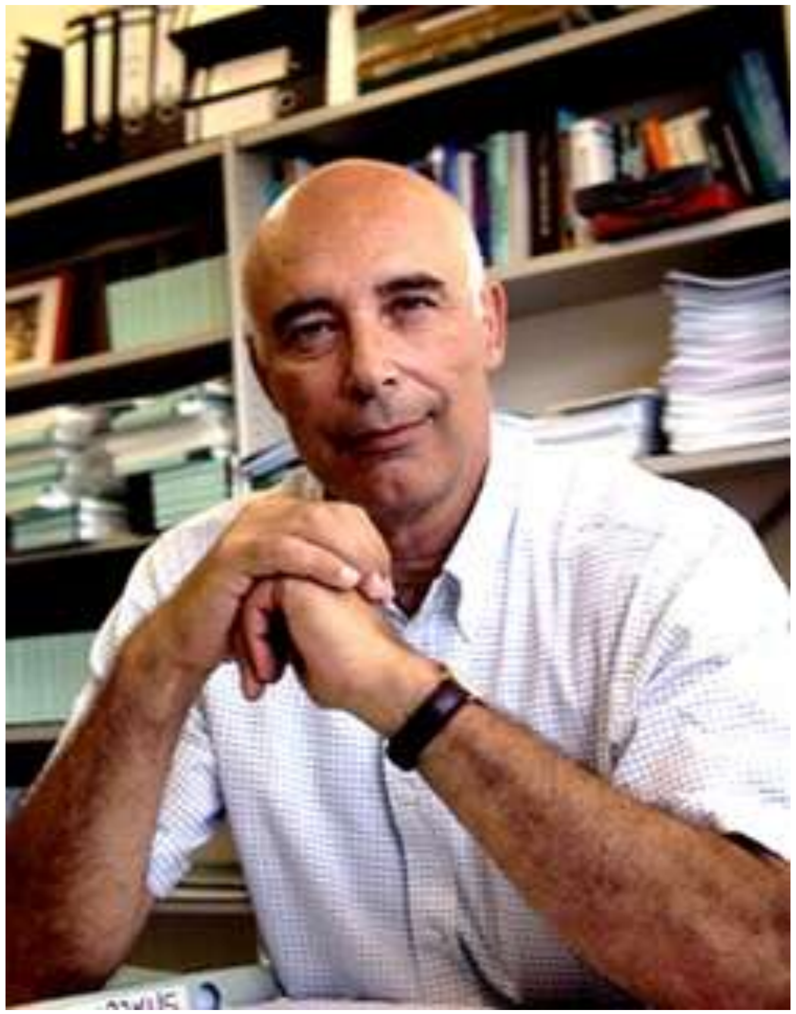

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- **… …**

My research trajectory started with inspirations from Altarelli's seminal works on parton physics.

"Asymptotic freedom in parton language"

G. Altarelli and G. Parisi, NPB **126** (1977).

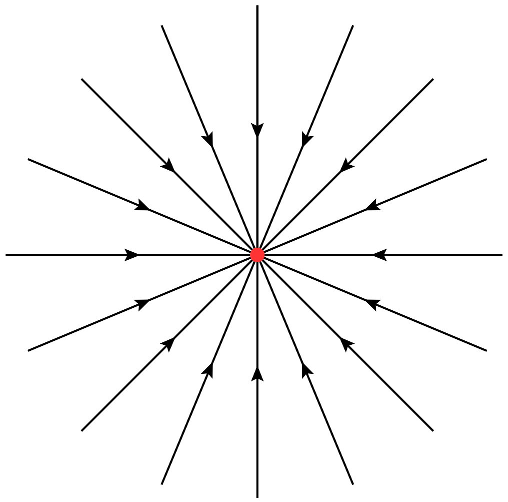
4. Calculation of logarithmic exponents. Spin averaged case

In this section we show that the functions $P(z)$ introduced in the previous section can be directly computed from the simple knowledge of the basic vertices of QCD. The method used is an extension of the von Weizsacker-Williams result in quantum electrodynamics [8]. In that case the equivalent number of photons inside an electron with fraction z of the electron momentum is evaluated to order α and contains a factor of $\ln E/m_e$, which plays the same role as $t = \ln Q^2/Q_0^2$ in our case.

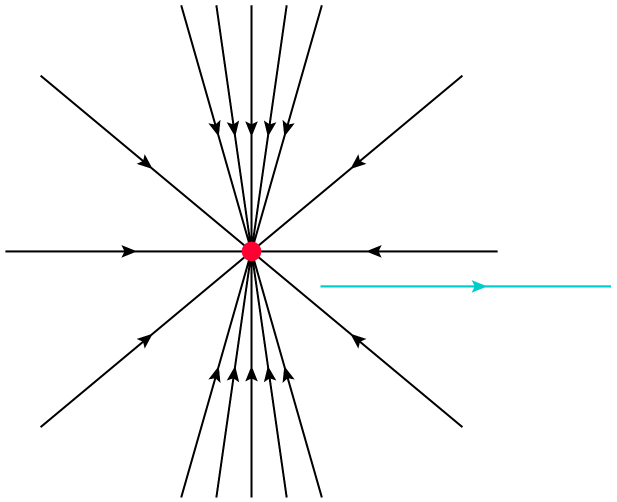
The Weizsacker-Williams approximation, or equivalent photon approximation, was used to justify the physical meaning of gluon spin in the **infinite momentum frame**:

- X. Ji, Y. Xu, and YZ, JHEP **08** (2012);
- X. Ji, J.-H. Zhang and YZ, PRL **111** (2013).

Static charge



Moving charge



$$\Delta G = \langle P_\infty | (\mathbf{E} \times \mathbf{A})^3 |_{\nabla\cdot\mathbf{A}=0} | P_\infty \rangle$$

The infinite momentum frame picture, where the parton model was originally introduced, motivated a new method to compute parton physics in lattice QCD!

Euclidean observable	Partonic observable
$\tilde{Q}(P^z, \Lambda_{\rm UV}) \equiv \langle P \,\vert\, \tilde{O}(\Lambda_{\rm UV}) \,\vert\, P \rangle$	$Q(\mu) \equiv \langle P_\infty \,\vert\, O(\mu) \,\vert\, P_\infty \rangle$
$\Lambda_{\rm UV}$: ultraviolet (UV) cutoff, $\sim \dfrac{2\pi}{a}$	μ: $\overline{\rm MS}$ scale. No P^z dependence.
$(P^z \ll \Lambda_{\rm UV})$ $\xrightarrow{\infty \text{ Lorentz boost}}$	$(P^z_\infty \gg \Lambda_{\rm UV})$

$$\tilde{Q}(P^z, \Lambda_{\rm UV}) = \boxed{Q(\mu)\ ?} + c_1 \frac{\Lambda_{\rm QCD}}{P^z} + c_2 \frac{\Lambda^2_{\rm QCD}}{P^2_z} + \ldots$$

✘ $\because$ $P^z \ll \Lambda_{\rm UV}$ and $P^z \gg \Lambda_{\rm UV}$ usually do not commute.

"*Large-momentum effective theory (LaMET)*":

a recipe for systematically controlled calculation of parton physics

- X. Ji, PRL **110** (2013); SCPMA **57** (2014).
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, Rev.Mod.Phys. **93** (2021).

Euclidean observable	**Partonic observable**
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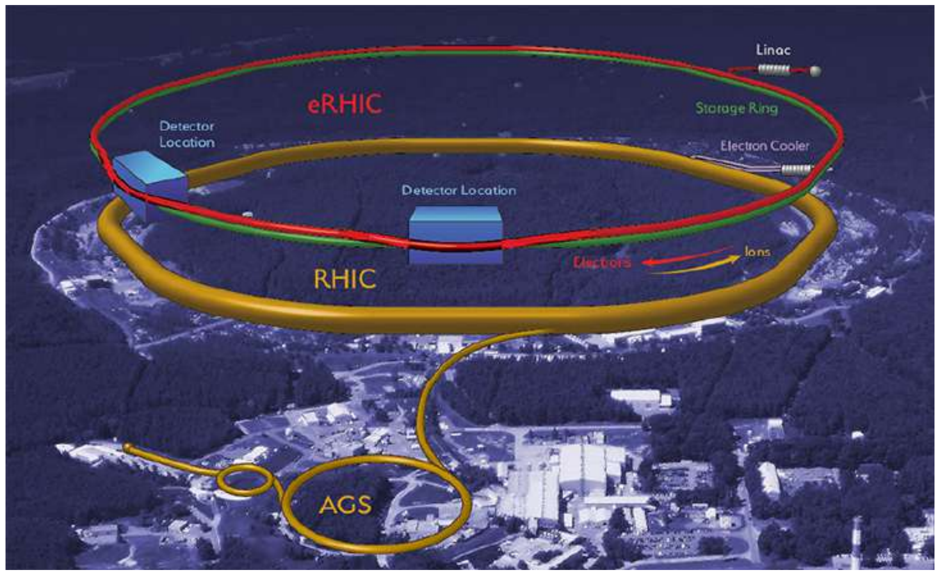
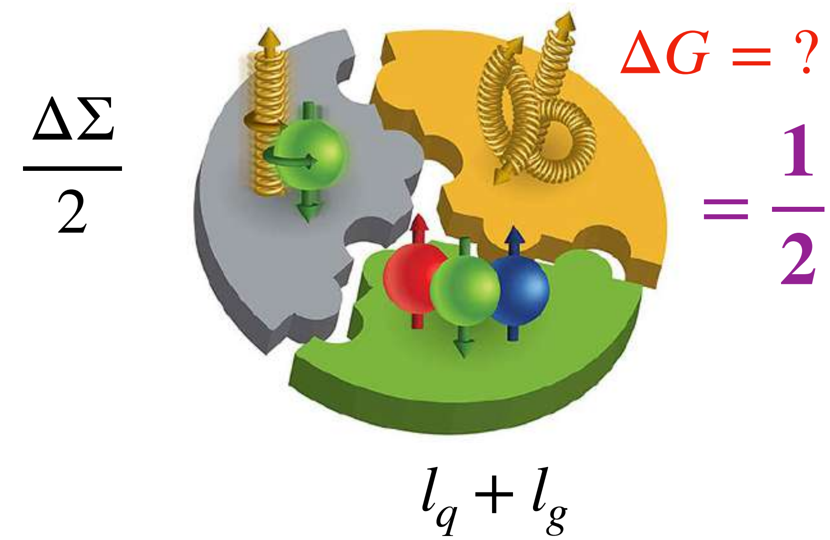
$(P^z \ll \Lambda_{\rm UV})$ $\xrightarrow{\infty \text{ Lorentz boost}}$	$(P^z_\infty \gg \Lambda_{\rm UV})$

$$\tilde{Q}(P^z, \Lambda_{\rm UV}) = \underbrace{C\left(\frac{\mu}{P^z}, \frac{\Lambda_{\rm UV}}{\mu}\right)}_{\text{Perturbative matching}} \otimes Q(\mu) + \underbrace{c_1 \frac{\Lambda_{\rm QCD}}{P^z} + c_2 \frac{\Lambda_{\rm QCD}^2}{P_z^2} + \ldots}_{\text{Power corrections}}$$

The gluon helicity ΔG

RHIC spin program and EIC



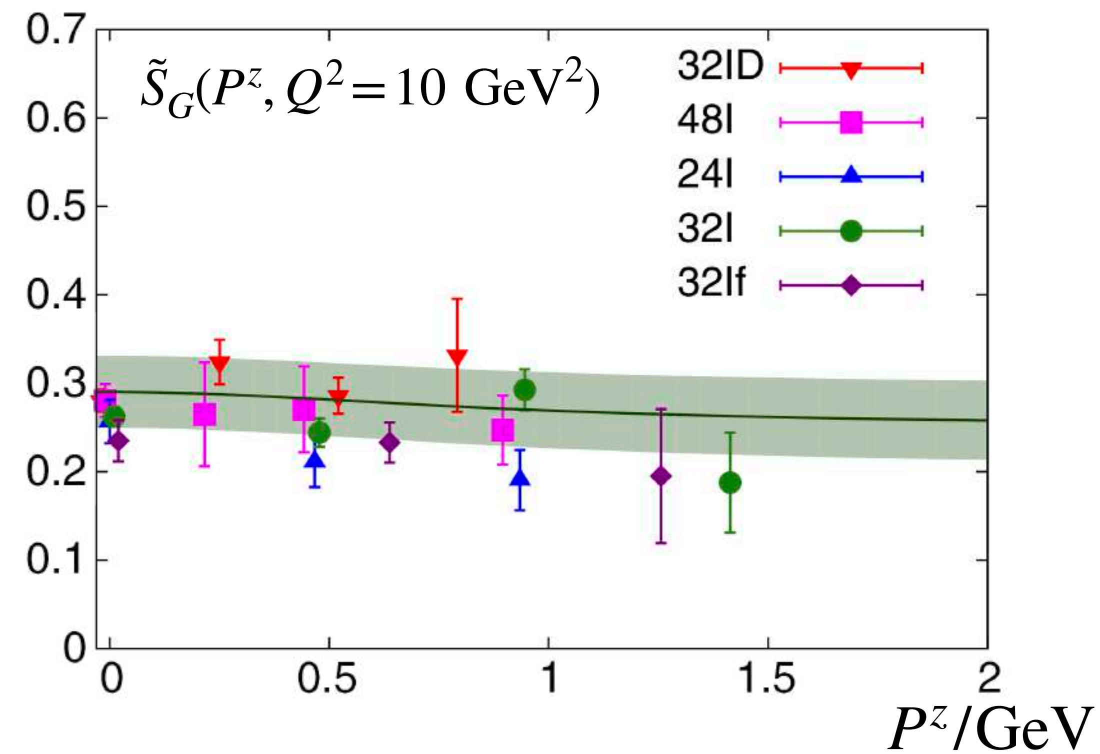


$$\tilde{S}_G(P^z, \mu) = \frac{\langle PS \,|\, (\mathbf{E} \times \mathbf{A})^3 \,|\, PS \rangle}{2S^z} \bigg|_{\nabla\cdot\mathbf{A}=0}$$

$$\tilde{S}_G(P^z, \mu) = C(P^z, \mu) \otimes (\Delta\Sigma, \Delta G) \; + \mathcal{O}(\Lambda^2_{\rm QCD}/P_z^2)$$

- X. Ji, J.-H. Zhang, and YZ, PRL **111** (2013);
- Y. Hatta, X. Ji and YZ, PRD **89** (2014);
- X. Ji, J.-H. Zhang, and YZ, PLB **743** (2015).

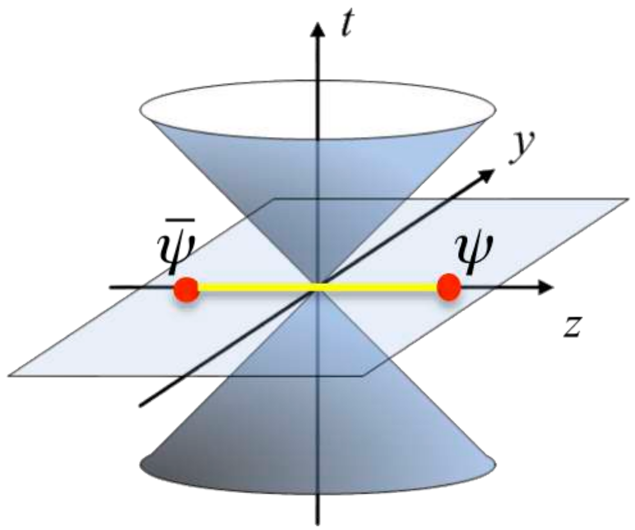
The first lattice result



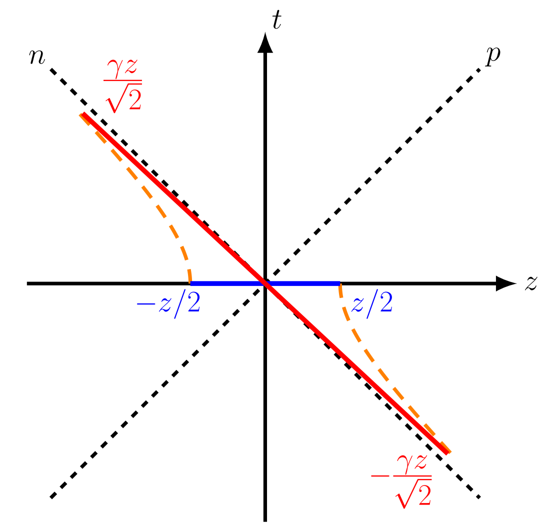
Y.-B. Yang, R. Sufian, YZ, et al. PRL **118** (2017)

Benchmark: lattice calculation of the PDFs

A quasi-PDF $\tilde{f}(x, P^z)$ to expand from

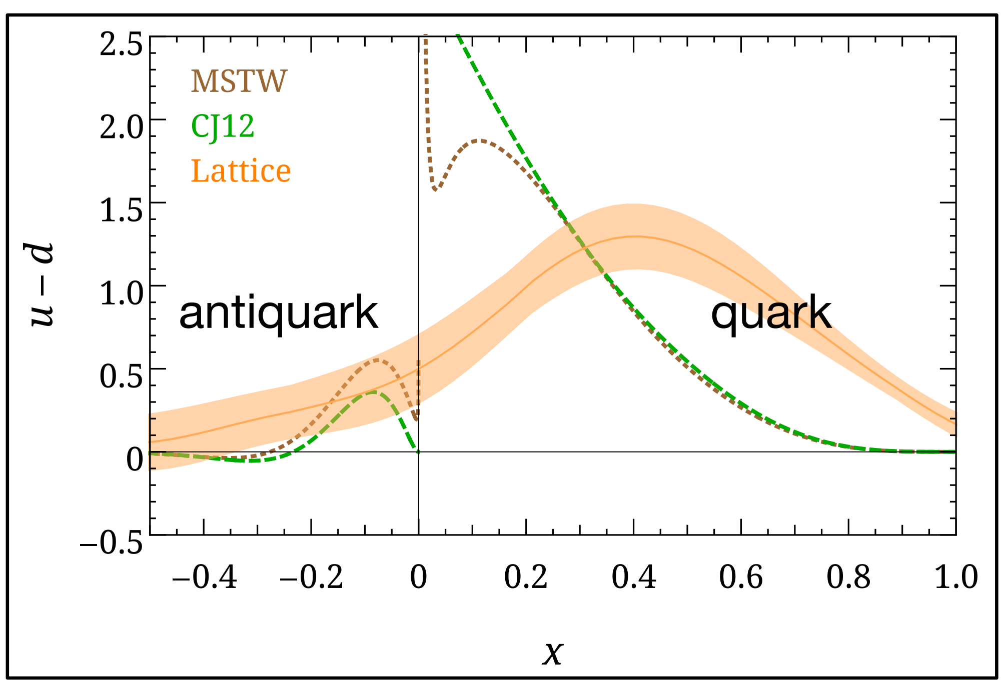


X. Ji, Phys. Rev. Lett. **110** (2013)



LaMET expansion: $\tilde{f}(y, P^z, \Lambda_{\mathrm{UV}}) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{P^z}, \frac{\Lambda_{\mathrm{UV}}}{\mu}\right) f(x, \mu) \ + \ \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$

X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD **90** (2014).



First exploratory lattice calculation

H.W. Lin et al., (LP3), PRD **91** (2015).

Renormalization and matching

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\mu}{\mathbf{y}P^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \ + \ \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

Altarelli-Parisi splitting kernel ← $\frac{1+\xi^2}{1-\xi} \ln \frac{\mu^2}{(yP^z)^2}$, $\quad \xi = \frac{x}{y}$

- Rigorous derivation of the exact form of matching formula.

T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD **98** (2018)

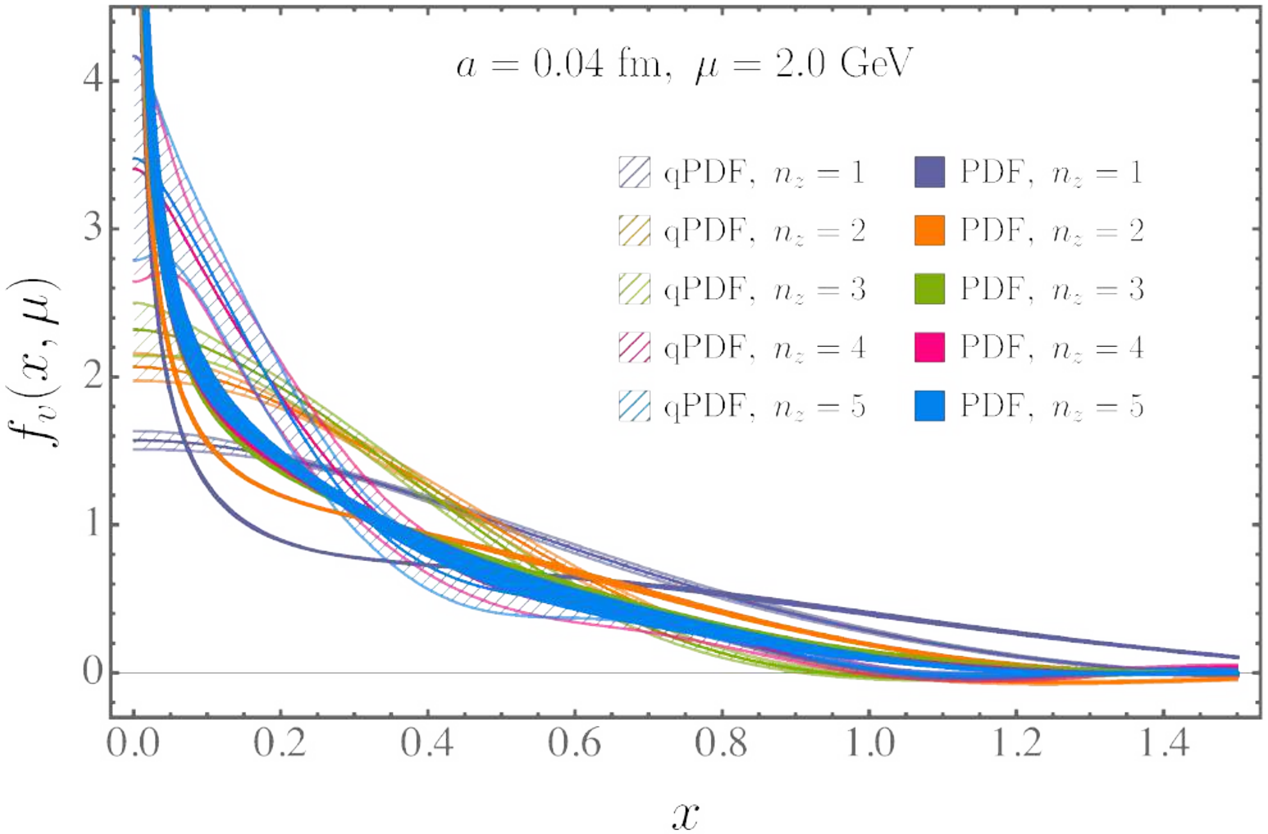
- Non-perturbative renormalization.
  - X. Ji, J.-H. Zhang and YZ, PRL **120** (2018);
  - Ishikawa, Ma, Qiu and Yoshida, PRD **96** (2017);
  - Green, Jansen and Steffens, PRL **121** (2018);
  - Constantinou and Panagopoulos, PRD **96** (2017);
  - I. Stewart and YZ, PRD **97** (2018);
  - X. Ji, YZ, et al., NPB **964** (2021).
- NNLO matching (for non-singlet quark case).
  - Chen, Zhu and Wang, PRL. **126** (2021);
  - Li, Ma and Qiu, PRL **126** (2021).
- Direct power expansion in parton momenta in *x*-space.

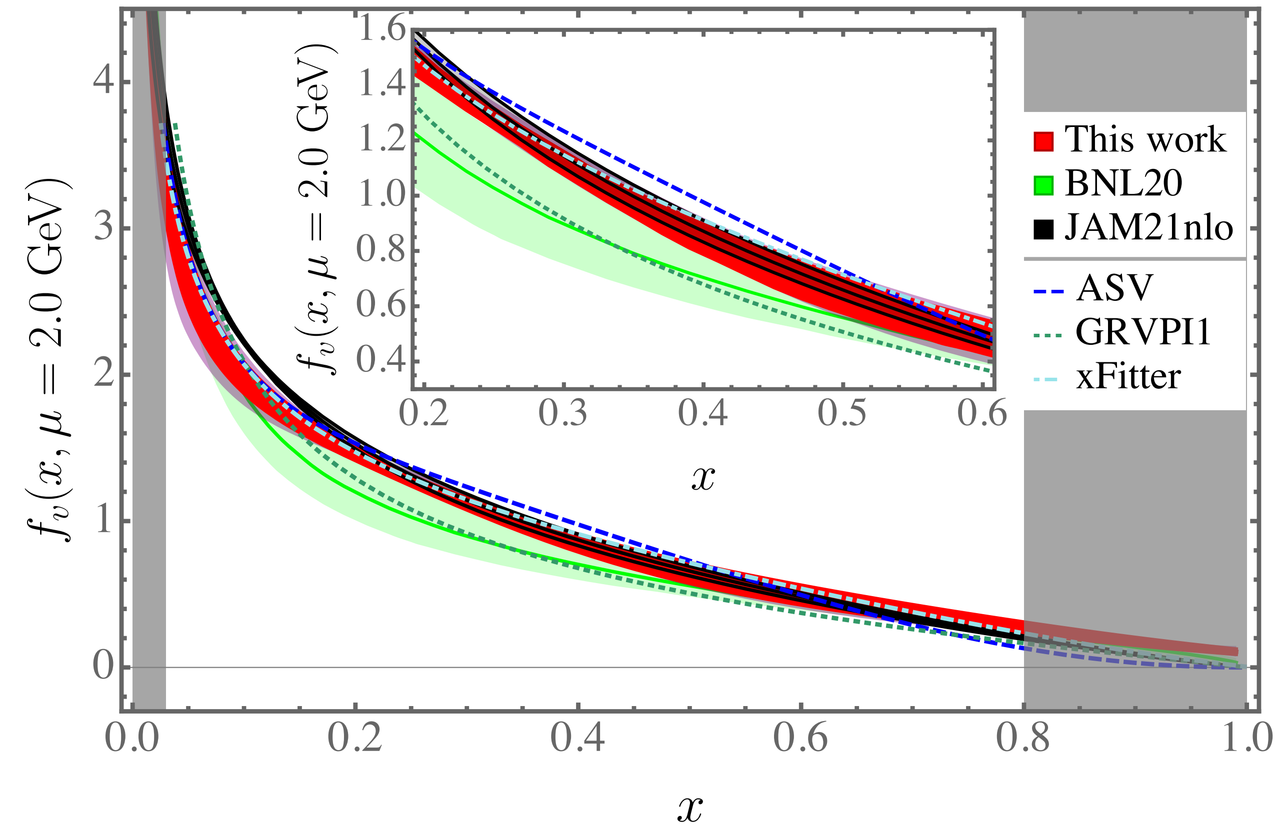
Reliable prediction within [$x_{\rm min}$, $x_{\rm max}$] at a given finite P^z !

State-of-the-art calculation of pion valence PDF

Gao, Hanlon, Mukherjee, Petreczky, Scior, Syritsyn and YZ, PRL128 (2022).

Super fine lattice spacing (*a*=0.04 fm and 0.06 fm), high momentum (P^z=2.42 GeV v.s. m_π=300 MeV), high statistics, first NNLO matching





- **JAM21nlo**, PRL **127** (2021);
- **xFitter (2020)**, PRD **102** (2020);
- **ASV**, PRL **105** (2010);
- **GRVPI1,** ZPC **53** (1992);
- **BNL20**, X. Gao, N. Karthik, YZ, et al., PRD **102** (2020).

Towards better systematic control

- **Lattice simulation:** larger P^z (excited states), spacing $a \to 0$ (renormalization), physical m_π, lattice size $L \to \infty$, etc.

- **Perturbative theory:** resummations at end-point regions.

  - x-space:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \ + \ \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Resummation of $\alpha_s \ln[\mu^2/(2xP^z)^2], \quad \alpha_s \ln(1-x)$

  - Coordinate space:

$$\tilde{h}(\lambda = zP^z, z^2\mu^2) = \sum_{n=0}^{\infty} C_n(z^2\mu^2)\frac{(-i\lambda)^n}{n!} a_n(\mu) \ + \ \mathcal{O}(z^2\Lambda_{\rm QCD}^2),$$

X. Gao, K. Lee, and YZ et al., PRD **103** (2021).

Resummation of $\alpha_s \ln[\mu^2 z^2], \quad \alpha_s^m \ln^n N$

- **Renormalons and power corrections:**

  Renormalon resummation improves determination of Wilson line mass correction and perturbative convergence.

J. Holligan, X. Ji, et al., submitted to journal.

Towards better systematic control

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Resummation o

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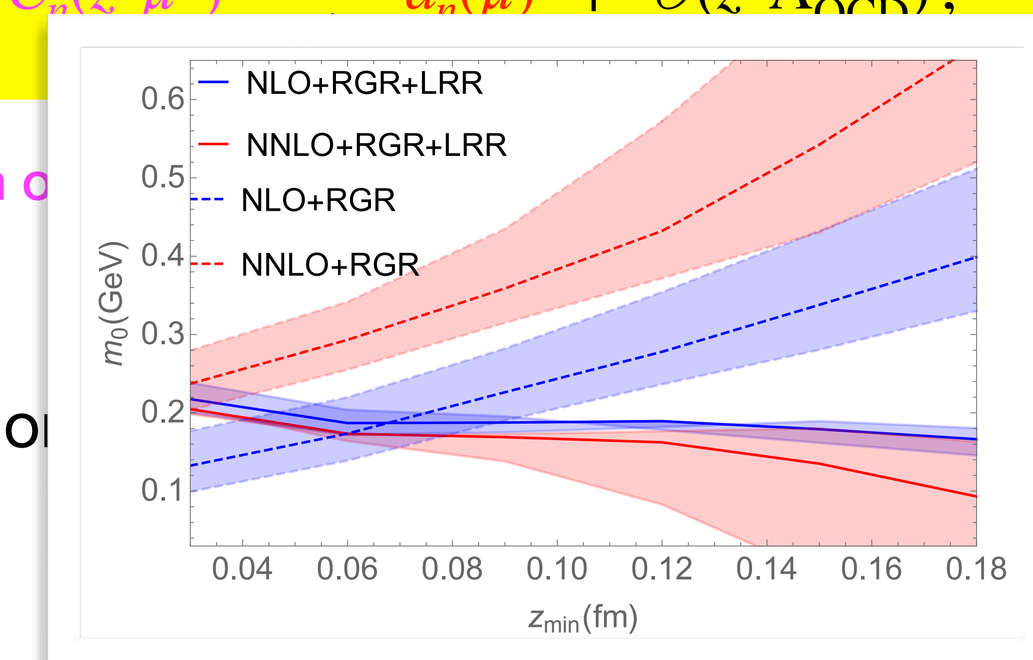
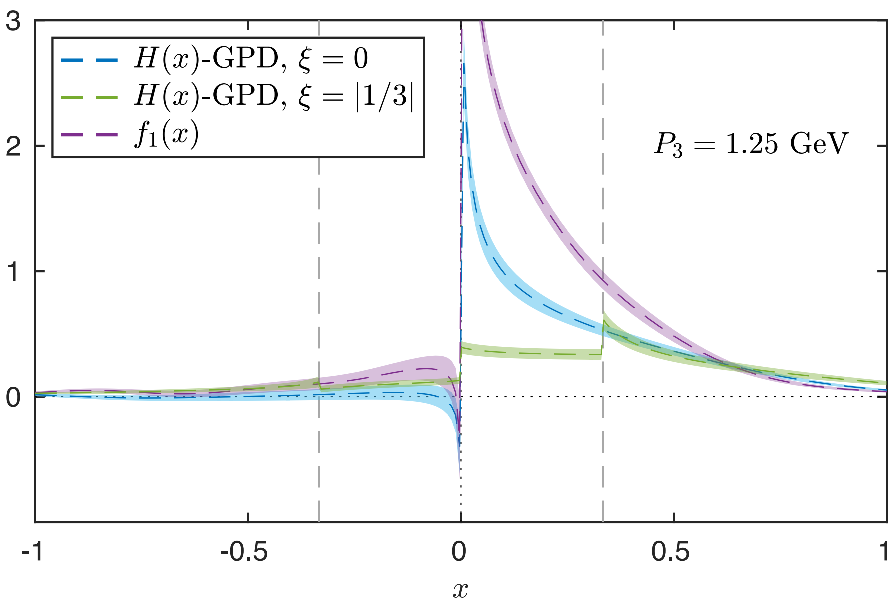
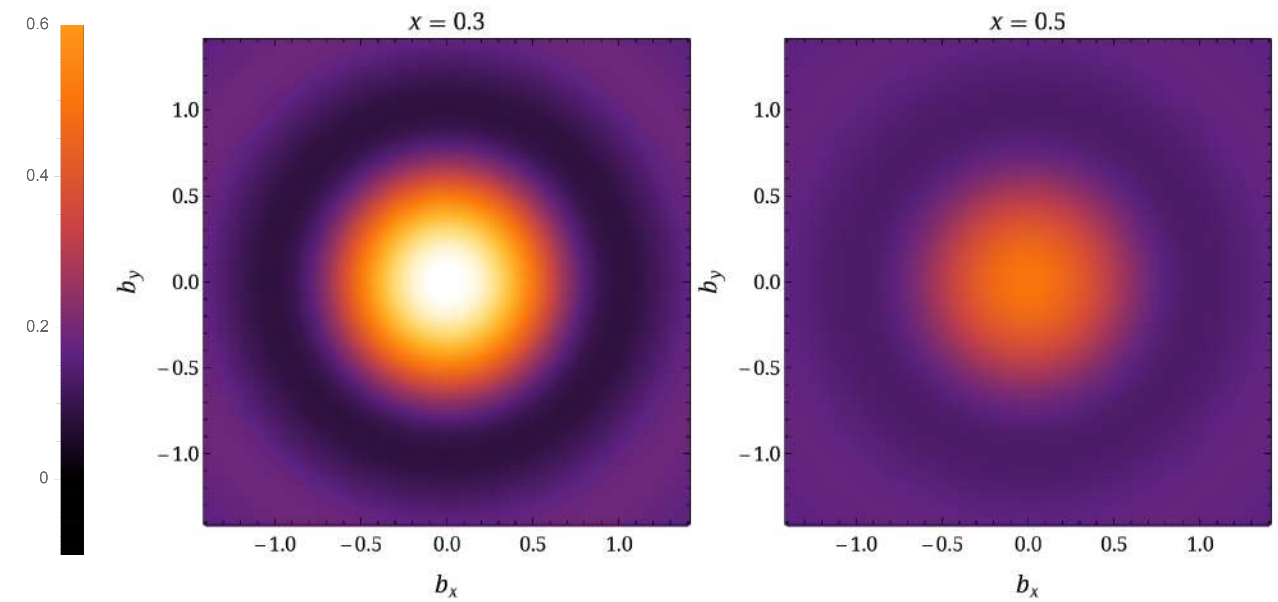


Generalized Parton Distributions (GPDs)

First lattice calculations at NLO:



C. Alexandrou et al. (ETMC), PRL **125** (2020).

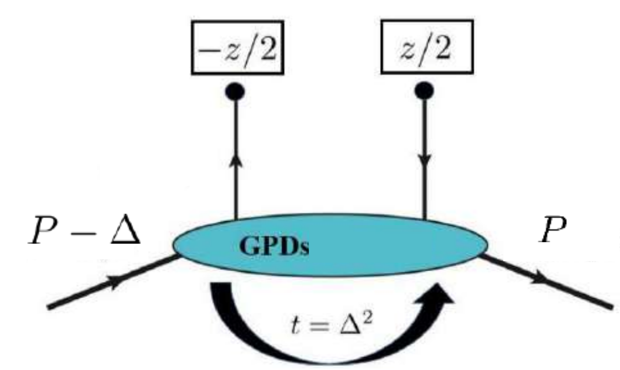


H.-W. Lin, PRL **127** (2021).

First attempt for twist-3 GPDs also made: S. Bhattacharya et al. (ETMC), PRD **102** (2020).

Recent advancement in extracting GPDs from less computationally expensive lattice matrix elements in the asymmetric frame.

Bhattacharya, Cichy, Constantinou, Dodson, Gao, Metz, Mukherjee, Scapellato, Steffens, and YZ, PRD **106** (2022).



See Dr. Xiang Gao and Martha Constantinou's parallel talks on Tue.

Lattice calculations of TMD physics

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x,\mathbf{b}_T,\mu,\tilde{P}^z)}{\sqrt{S_r^q(b_T,\mu)}} = C\left(\mu, x\tilde{P}^z\right)\ \exp\left[\frac{1}{2}K(\mu,b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f_{i/p}^{[s]}(x,\mathbf{b}_T,\mu,\zeta)\left\{1+\mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2},\frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

Reduced soft function ✓

X. Ji, Y.-S. Liu and Y. Liu, NPB 955 (2020), PLB 811 (2020).

- X. Ji, Sun, Xiong and Yuan, PRD **91** (2015);
- X. Ji, Jin, Yuan, Zhang and YZ, PRD **99** (2019);
- Ebert, Stewart, YZ, PRD **99** (2019).
- Ebert, Stewart, YZ, JHEP **09** (2019);
- X. Ji, Y.-S. Liu and Y. Liu, NPB 955 (2020), PLB 811 (2020).
- Vladimirov and Schäfer, PRD 101 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP **04** (2022).

✳ **Collins-Soper kernel $K(\mu, b_T)$;**

✳ **Flavor separation;**

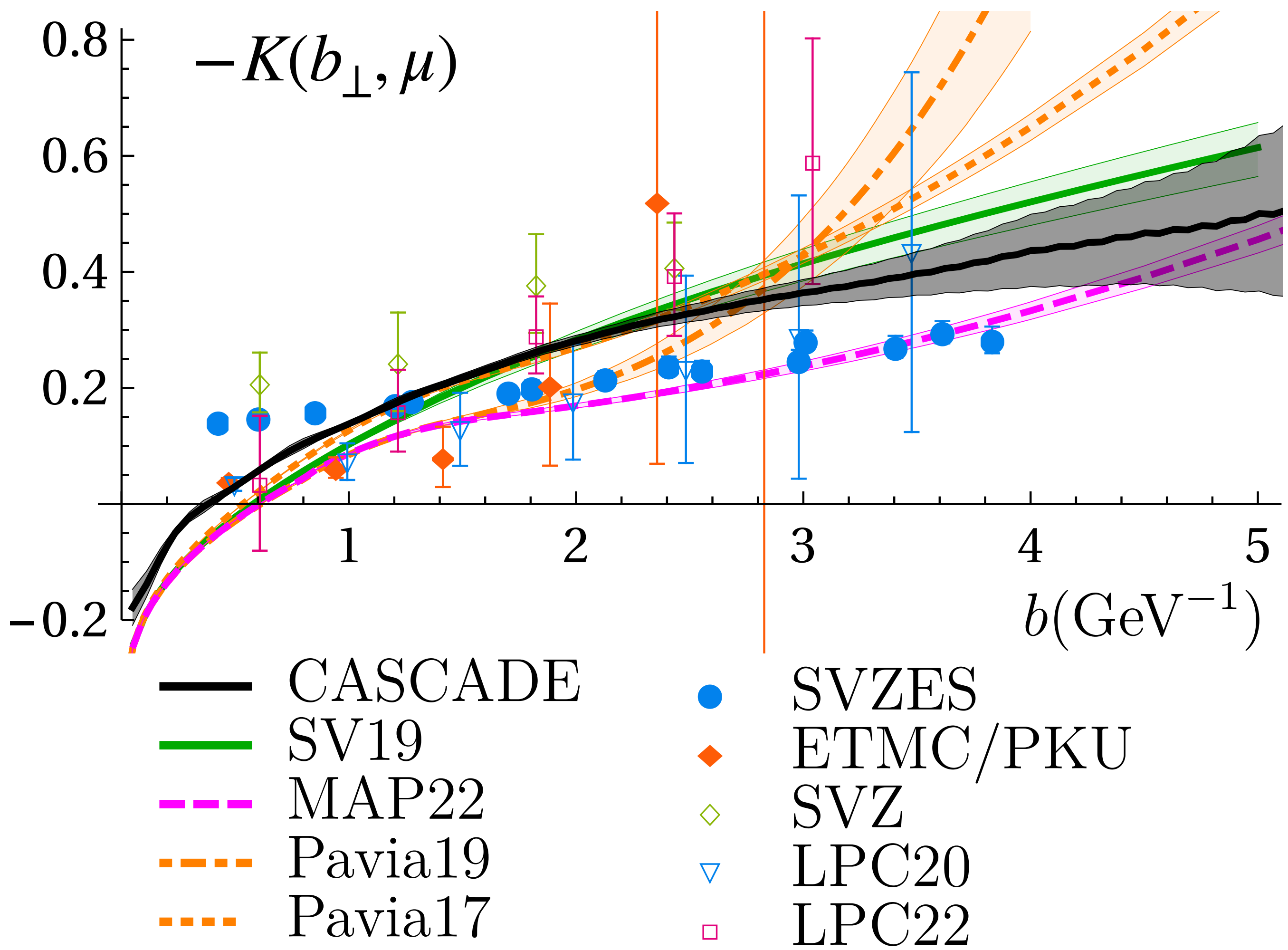
✳ **Spin-dependence, e.g., Sivers function;**

✳ **Full TMD kinematic dependence in $(x, \mathbf{b}_T)$.**

✳ **Twist-3 PDFs from small b_T expansion of TMDs.** Ji, Liu, Schäfer and Yuan, PRD **103** (2021).

Collins-Soper kernel for TMD evolution

Comparison between lattice results and global fits



MAP22: Bacchetta, Bertone, Bissolotti, et al., JHEP **10** (2022)
SV19: I. Scimemi and A. Vladimirov, JHEP **06** (2020)
Pavia19: A. Bacchetta et al., JHEP **07** (2020)
Pavia 17: A. Bacchetta et al., JHEP **06** (2017)
CASCADE: Martinez and Vladimirov, PRD **106** (2022)

Approach	Collaboration
Quasi beam functions	P. Shanahan, M. Wagman and YZ (SWZ21), Phys. Rev.D **104** (2021)
Quasi TMD wavefunctions	Q.-A. Zhang, et al. (LPC20), Phys.Rev.Lett. **125** (2020).
	Y. Li et al. (ETMC/PKU 21), Phys.Rev.Lett. **128** (2022).
	M.-H. Chu et al. (LPC22), arXiv: 2204.00200
Moments of quasi TMDs	Schäfer, Vladmirov et al. (SVZES21), JHEP **08** (2021)

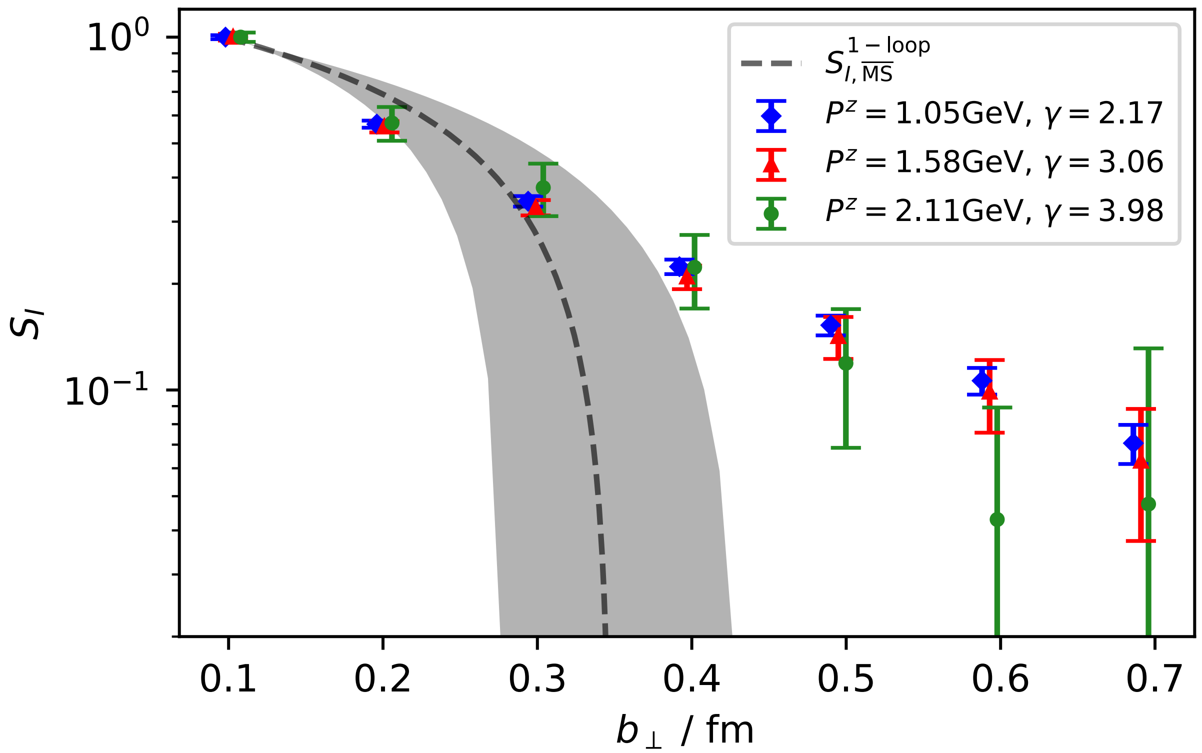
See also Dr. Yong Zhao's parallel talk on Thu.

Reduced soft factor for full TMD calculation

$$\langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P) \rangle \overset{P^z \gg m_\pi}{=} S_q^r(b_T, \mu) \int dx dx' \; H(x, x', \mu) \times \Phi^\dagger(x, b_T, P^z) \Phi(x', b_T, P^z)$$

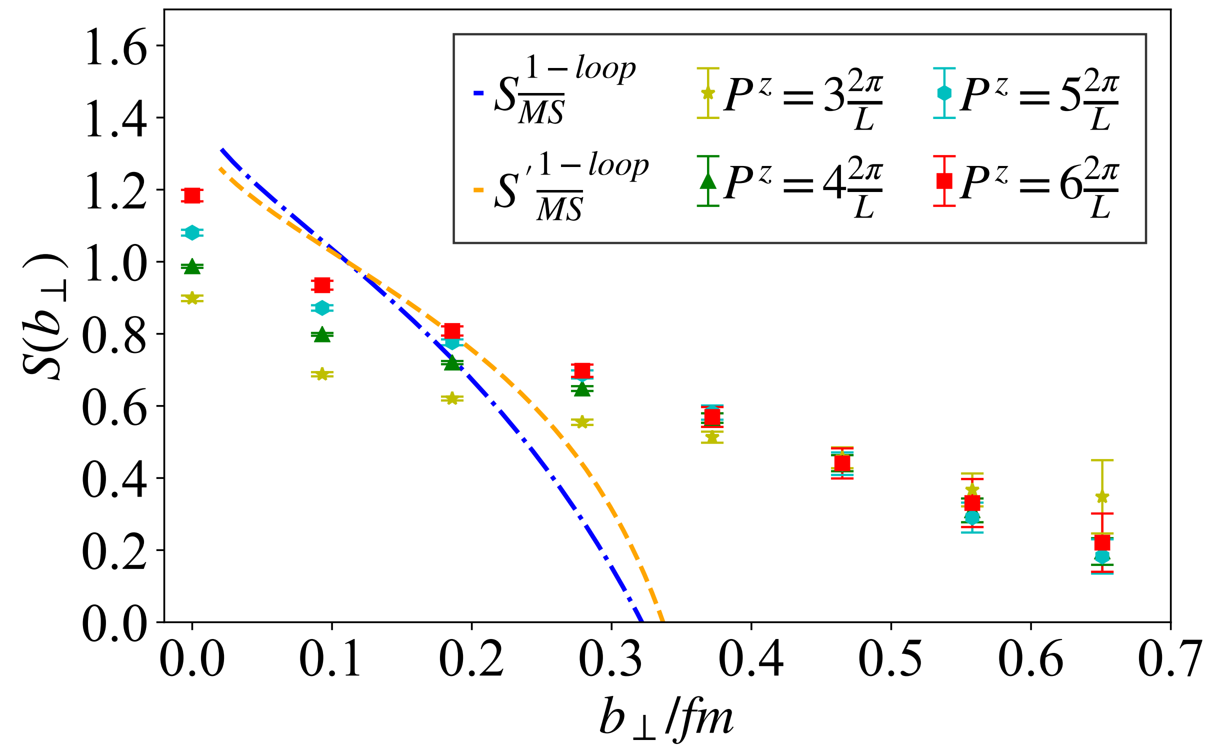
First lattice calculations at LO:

$a = 0.10$ fm, $m_\pi = 547$ MeV, $P^z_{\text{max}} = 2.11$ GeV



Q.-A. Zhang, et al. (LPC), PRL **125** (2020).

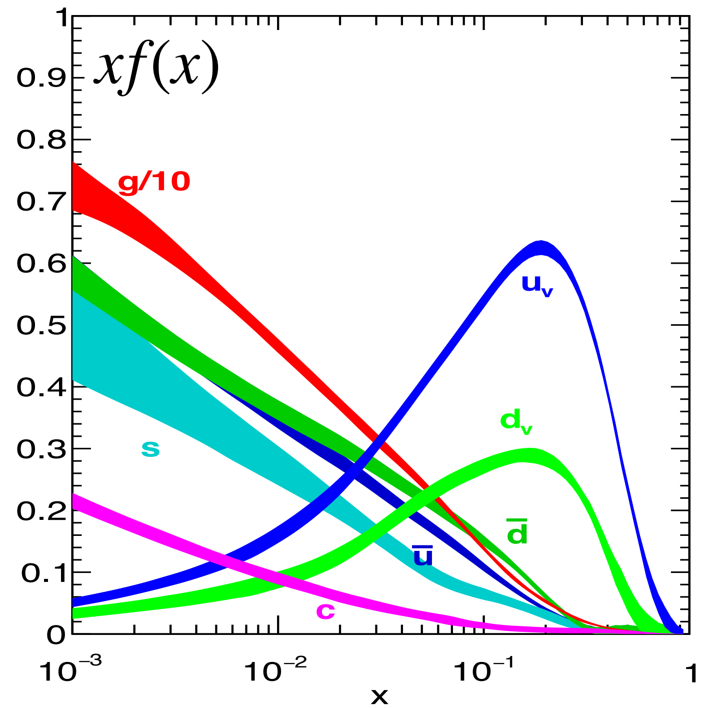
$a = 0.09$ fm, $m_\pi = 827$ MeV, $P^z_{\text{max}} = 3.3$ GeV



Y. Li et al. (ETMC/PKU), PRL **128** (2022).

3D Imaging of the Nucleon

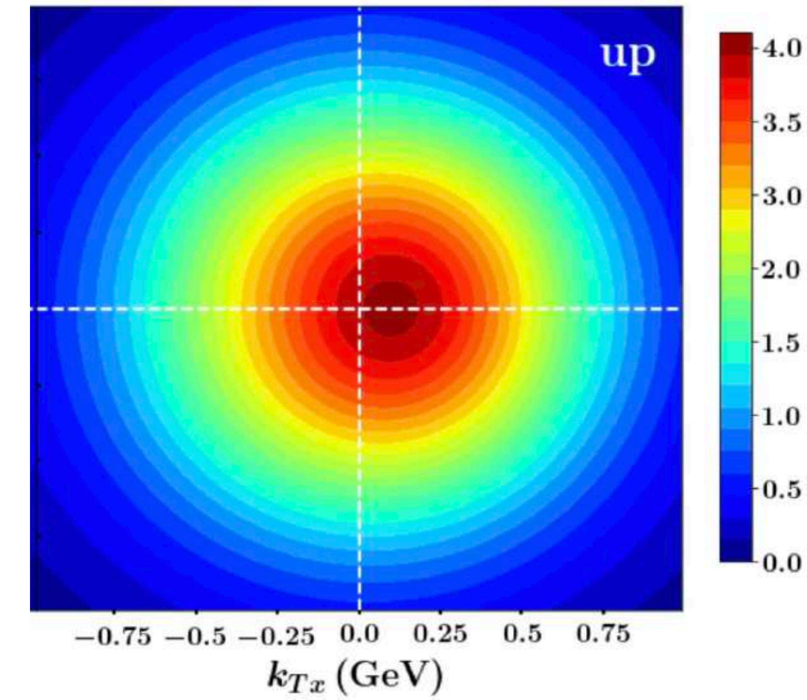
Parton Distribution Functions



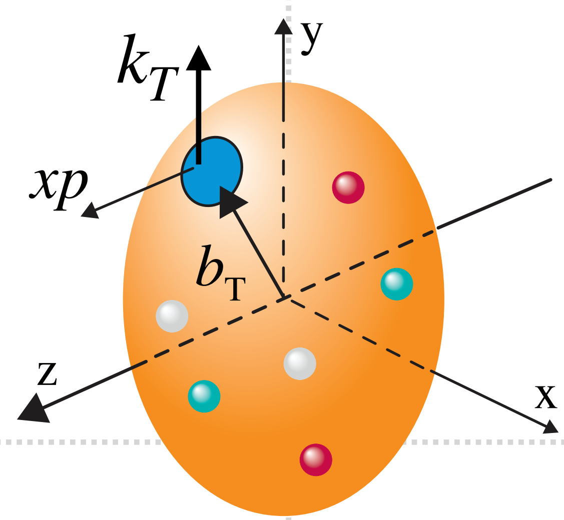
NNPDF, EPJ C77 (2017)

$\int d^2\vec{k}_T$

Transvers momentum distributions



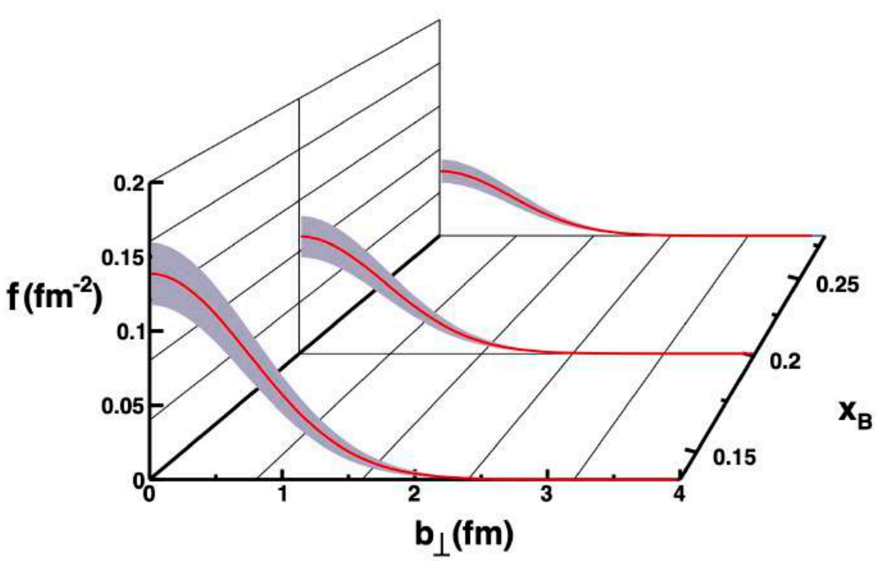
Cammarota, et al. (JAM), PRD **102** (2020).



$\int d^2\vec{b}_T$

$\int d^2\vec{b}_T$

Generalized parton distributions



W. Armstrong et al., arXiv: 1708.00888.

$\int d^2\vec{k}_T$

Wigner distributions/ Generalized TMDs

$$W(x, \vec{k}_T, \vec{b}_T)$$