RCS1 transverse stability with resonator impedance

Single-turn and multi-turn wakefield case

D. Amorim, E. Métral

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Goal and scope of the study

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- **Many RF cavities** (O(200) for the first RCS) will be needed to provide the large acceleration gradient

- These cavities will create **high-order modes**
  - Resonances can be broad or very peaked, depending on cavity design, mode damping…
  - Resonance frequency will depend on the main RF frequency and cavity design
  - Resonance amplitude will depend on the cavity design and number of cavities

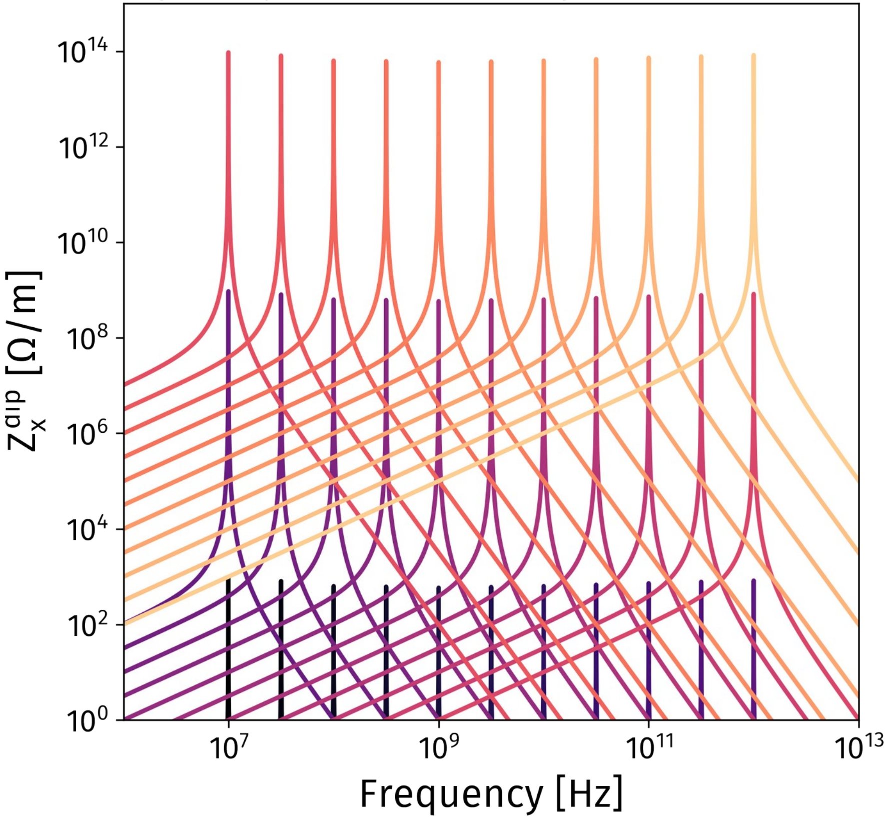
Goal and scope of the study

- Goal: obtain a first estimate of the **limits for resonator shunt impedance, resonance frequency and quality factor** in the transverse plane
- Since type and number of cavities are not fixed yet, **scan the parameter space** to find the limits in terms of transverse stability
- Investigate the **effect of single-turn and multi-turn wakefields** on the stability limits

Resonator impedance and wakefield

Resonator impedance model

Scan resonator
frequency and shunt impedance, Q=1000

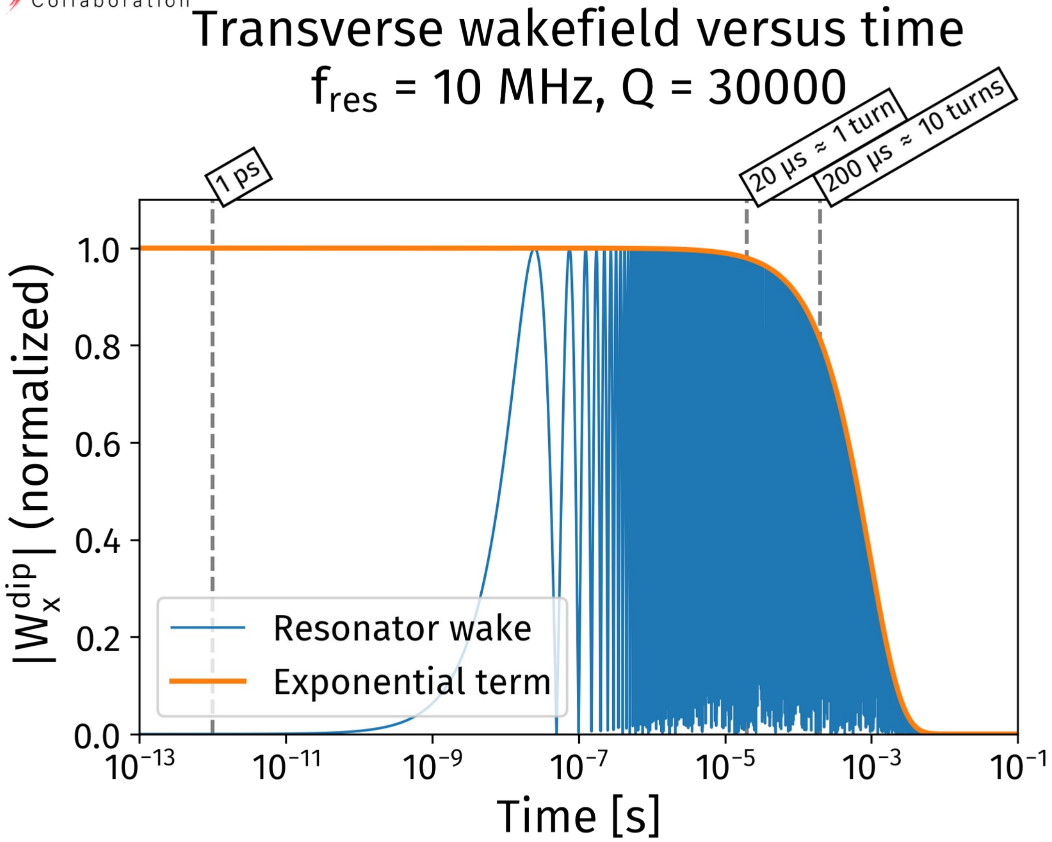


- Use a single horizontal dipolar resonator impedance/wakefield
- Scan its shunt impedance R_s, its resonance frequency f_{res} and its quality factor Q
- Note: Plot only shows a few shunt impedances, with Q=1000

Scan parameters

	Value
Resonator shunt impedance R_s	1 kΩ/m to 100 TΩ/m
Resonance frequency f_{res}	10 MHz to 1 THz
Quality factor Q	100, 300, 1000, 3000, 10000, 30000

Resonator impedance model



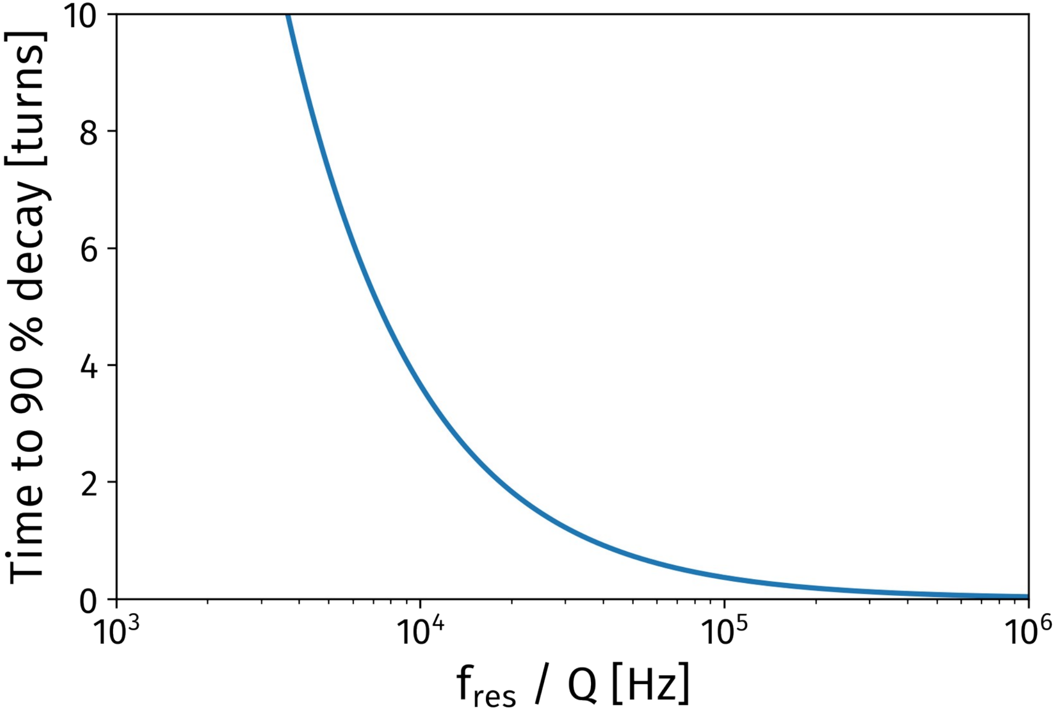
- For some (f_{res}, Q), the wakefield can extend well beyond one turn
- Example here with f_{res}=10 MHz.
- The wake can be written

$$W(t) = \frac{2\pi f_{res} R_s}{Q\sqrt{1-\frac{1}{4Q^2}}} \exp\left(-\frac{2\pi f_{res}}{2Q} t\right) \sin\left(2\pi f_{res}\sqrt{1-\frac{1}{4Q^2}}\, t\right)$$

- The exponential term (plotted in orange) dictates the wakefield decay

Resonator impedance model

Number of turns (1 turn = 20µs) to reach 90 % wakefield decay



- We can easily deduce the time t required to reach a 90 % wakefield decay:

$$t = -\frac{\ln(0.1)}{\pi}\left(\frac{f_{res}}{Q}\right)^{-1}$$

- With our machine parameters, **multiturn wakefield is required if $f_{res}/Q < 10^5$**

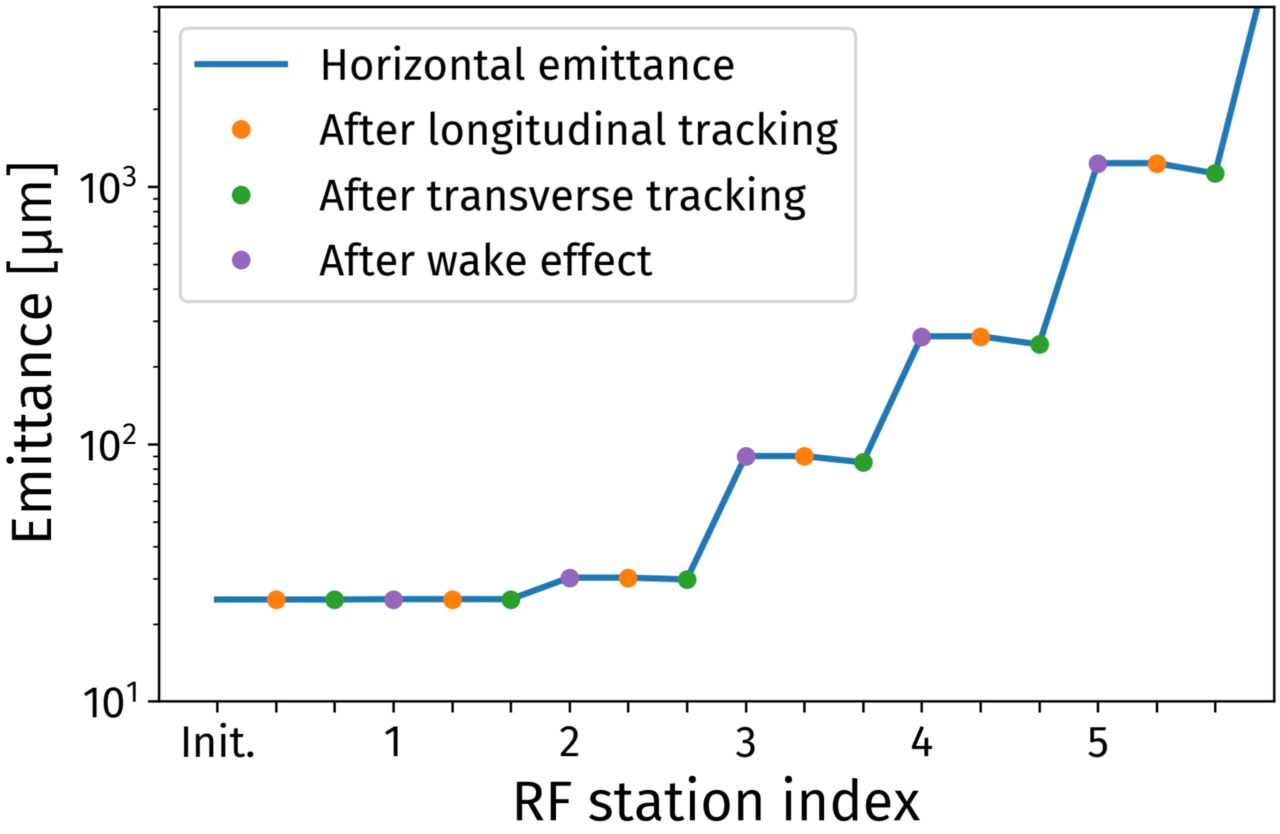
Transverse stability simulations parameters

Transverse stability simulation in the RCS 1

- Simulation including longitudinal map with **32 RF stations** + transverse map + transverse wakefield (single turn or multi-turn)
  - Wakefield and transverse map also divided in 32 stations
- Tracking over 100 turns, 5000 macroparticles with PyHEADTAIL
  - Injection energy: 63 GeV
  - Momentum increment: 14.2 GeV/c per turn (equally distributed in the 32 RF stations)
  - Note: only 17 turns are normally needed for the first acceleration stage (63 GeV to 313 GeV)
- Change the number of turns for the wakefield (1, 2, 3, 10 or 50 turns)

Instability growth can be very quick

- There are 32 RF stations per turn
- Between each station, there are three tracking elements: transverse, longitudinal and wakefield
- Here the **instability appears** already **after the second RF station**

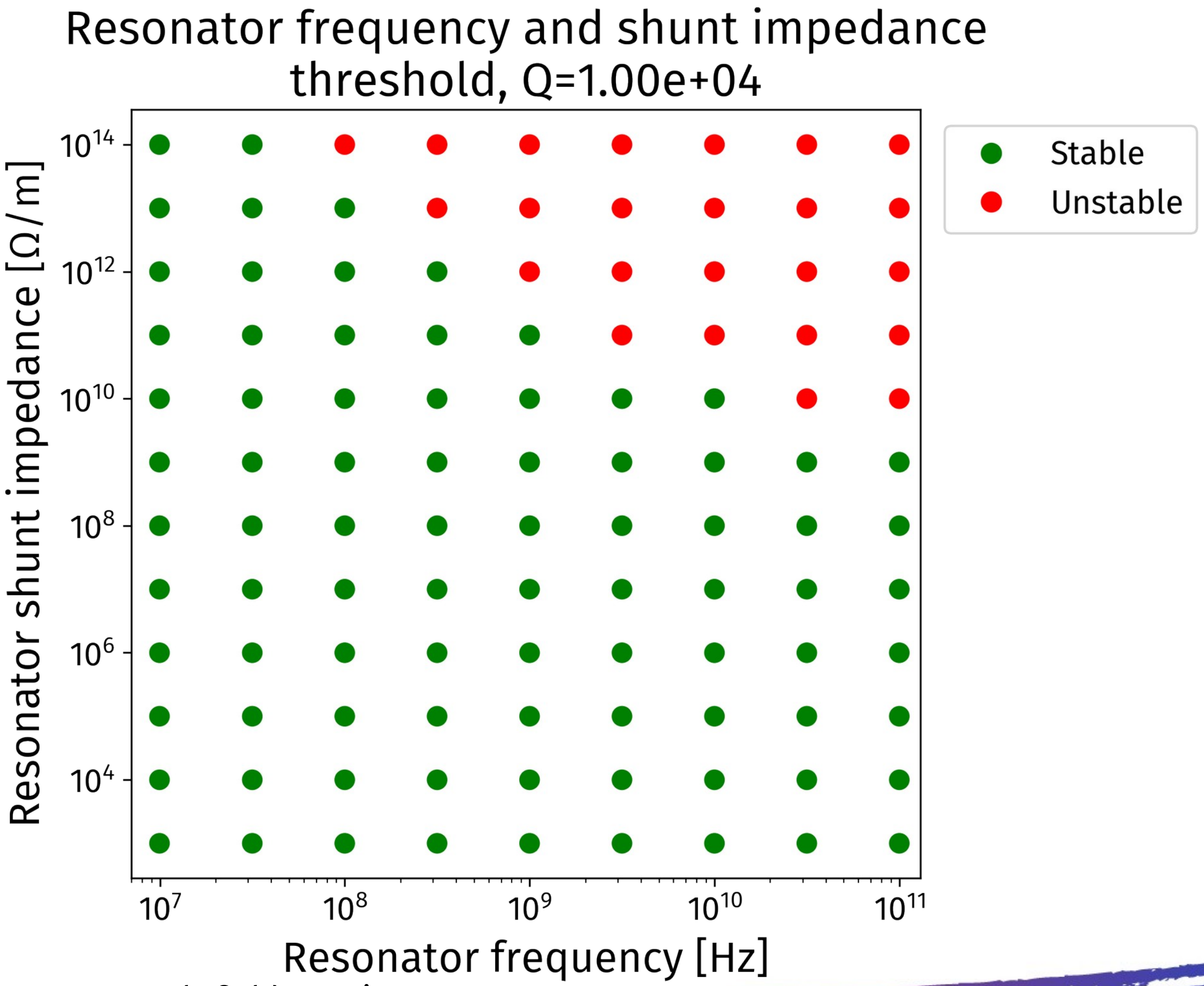


Transverse stability simulations results

Stability summary plots, single-turn wakefield

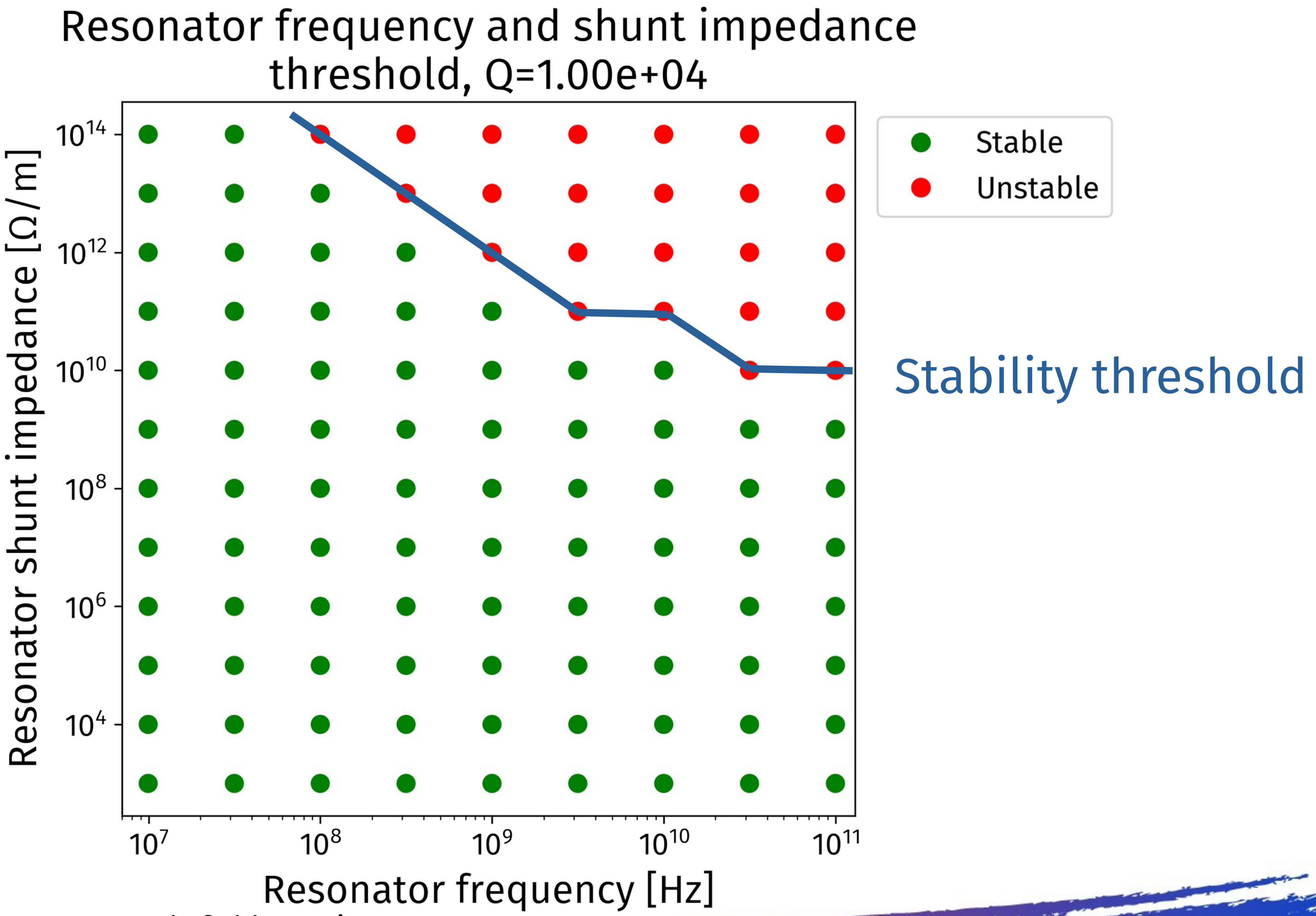
- Emittance is evaluated after 20 turns, slightly longer time than the 17 turns required for the acceleration in the RCS 1
- For each (R_s, f_{res}), indicate if there is **emittance growth (red dot) or not (green dot)**
  - Emittance growth = $\varepsilon_{turn\ 20}$ / $\varepsilon_{initial}$
  - Consider the beam unstable if emittance growth > 20 % (criterion to be refined)
- First show the results obtained with single-turn wakefields
- Focus on one high-Q case (Q=10 000)
  - Results for other Q factors reported in appendix

Q = 10000



Resonator frequency and shunt impedance threshold, Q=1.00e+04
Stable
Unstable
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^{4}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}

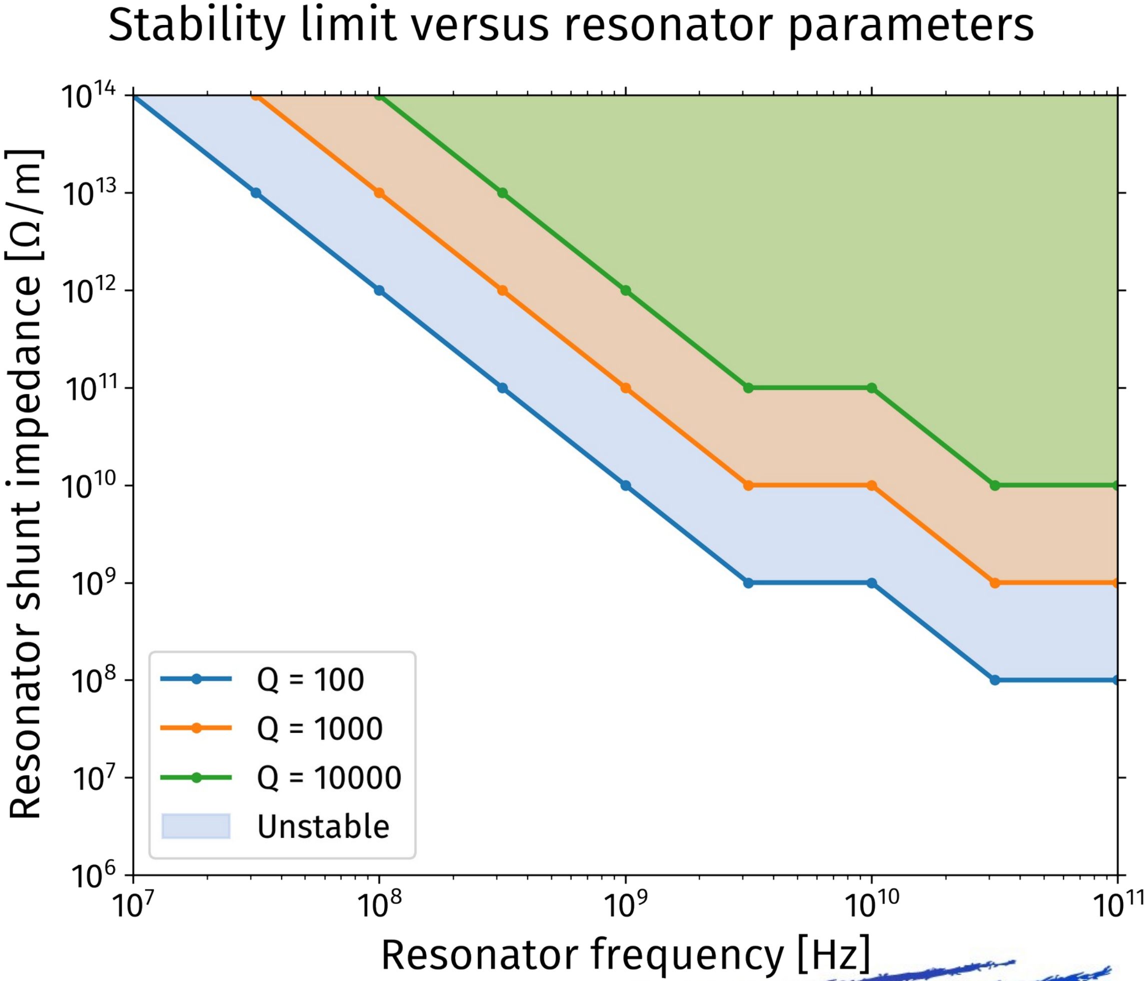
Q = 10000



Resonator frequency and shunt impedance threshold, Q=1.00e+04
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^{4}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Stable
Unstable
Stability threshold

Summary plot for Q=100/1000/10000

- Group the results for the different Q factor in one plot
- Line shows the first unstable simulation for a given Q factor, versus resonator shunt impedance and frequency
- Shaded area corresponds to the parameter space where the beam is unstable



Summary plot for Q=100/1000/10000

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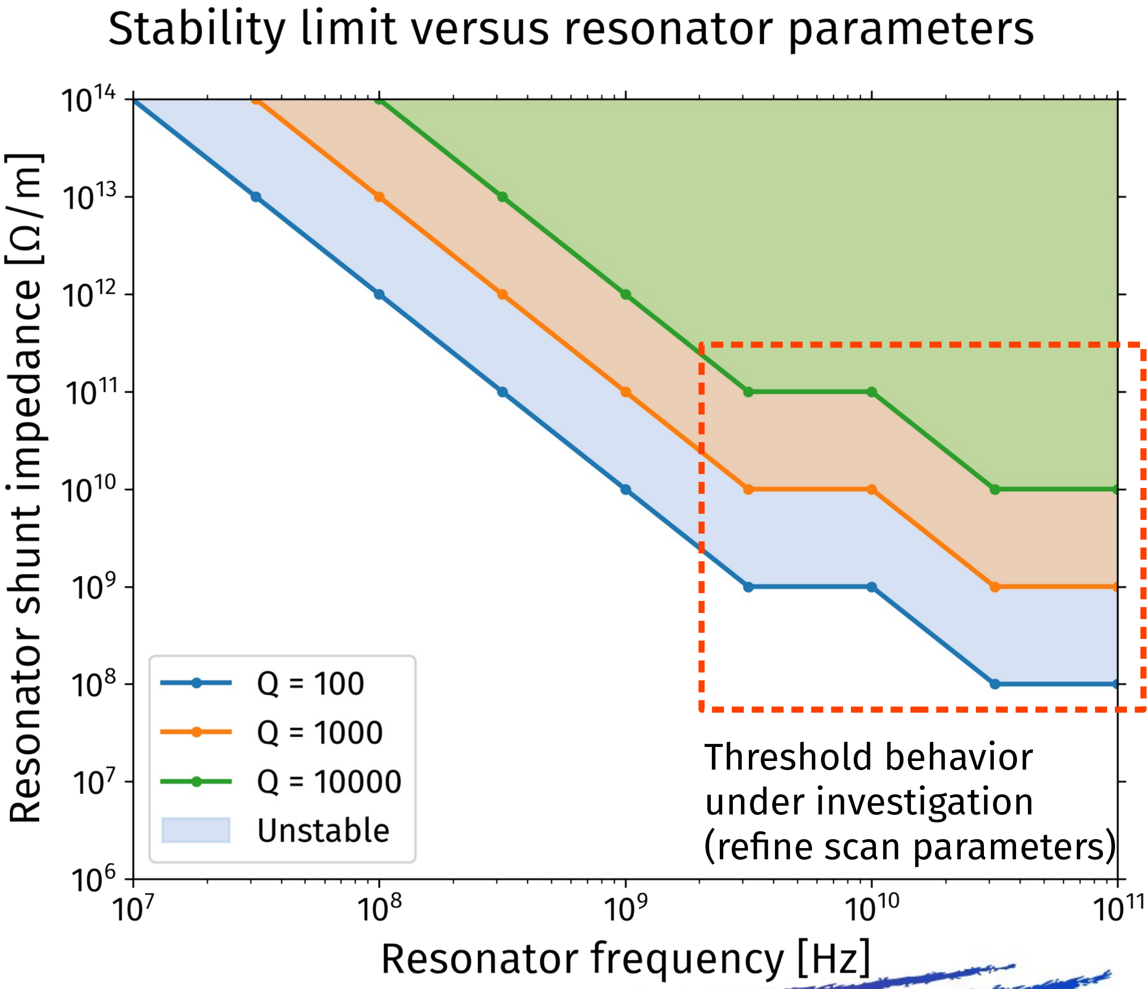
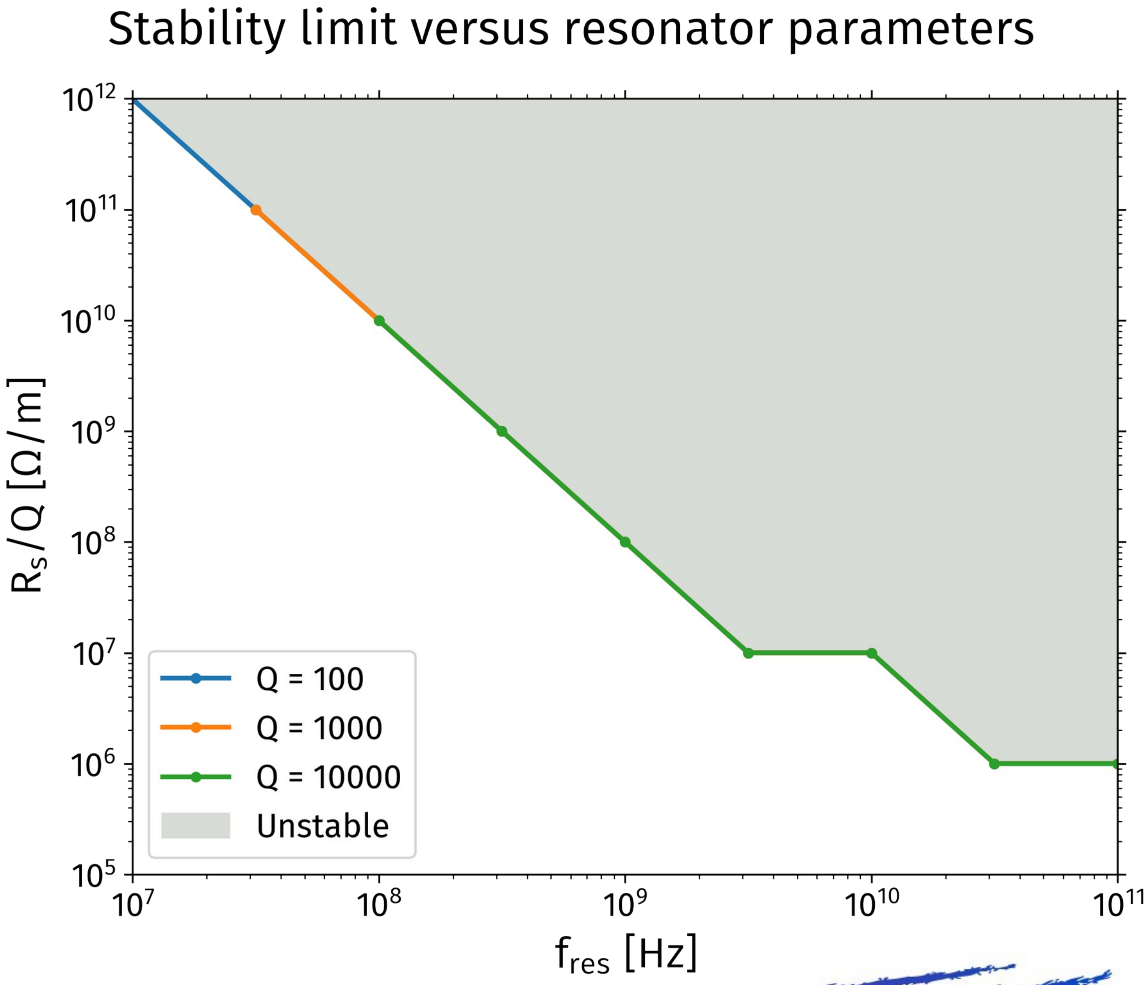


Summary plot for Q=100/1000/10000

- Now the **shunt impedance limit is divided by the resonator quality factor Q**
  - Provides a **limit on R_s/Q** for the whole ring

- This limit should be divided by the number of cavities to check if their design R_s/Q is within the limit
  - Example: at 100 MHz, R_s/Q = 10 GΩ/m. With 1000 cavities, R_s/Q limit is 10 MΩ/m per cavity



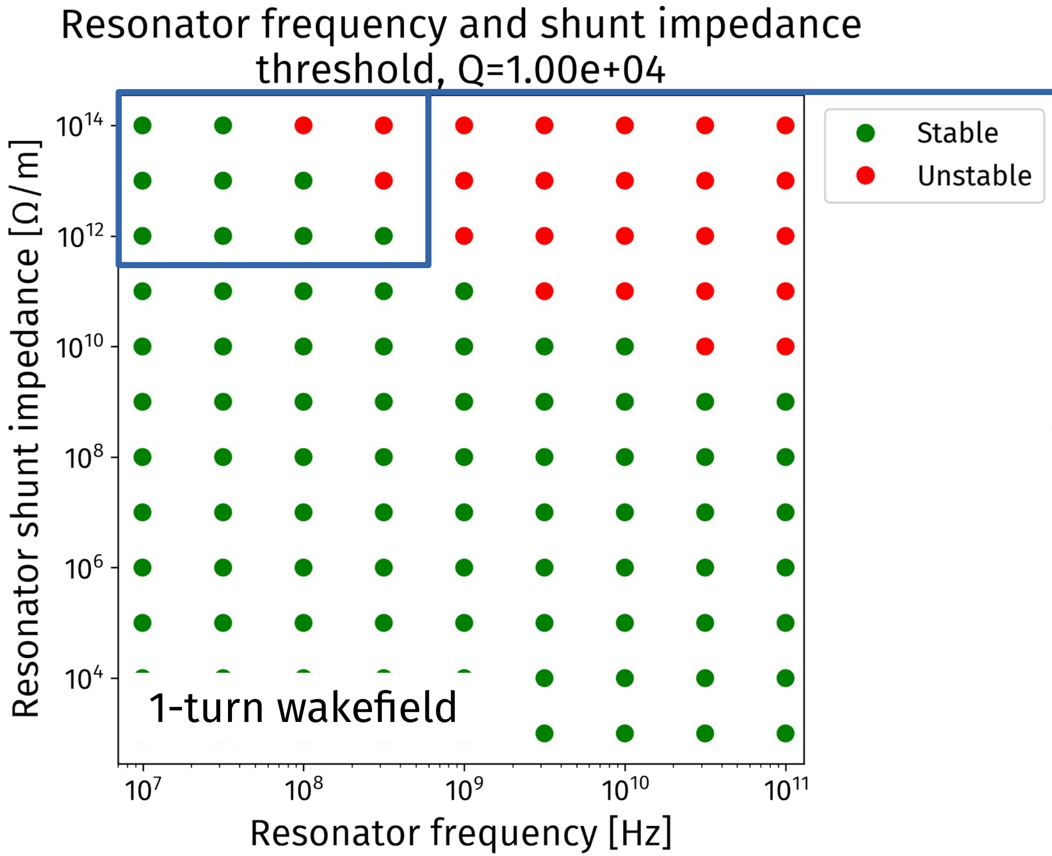
Stability summary plots, multi-turn wakefield

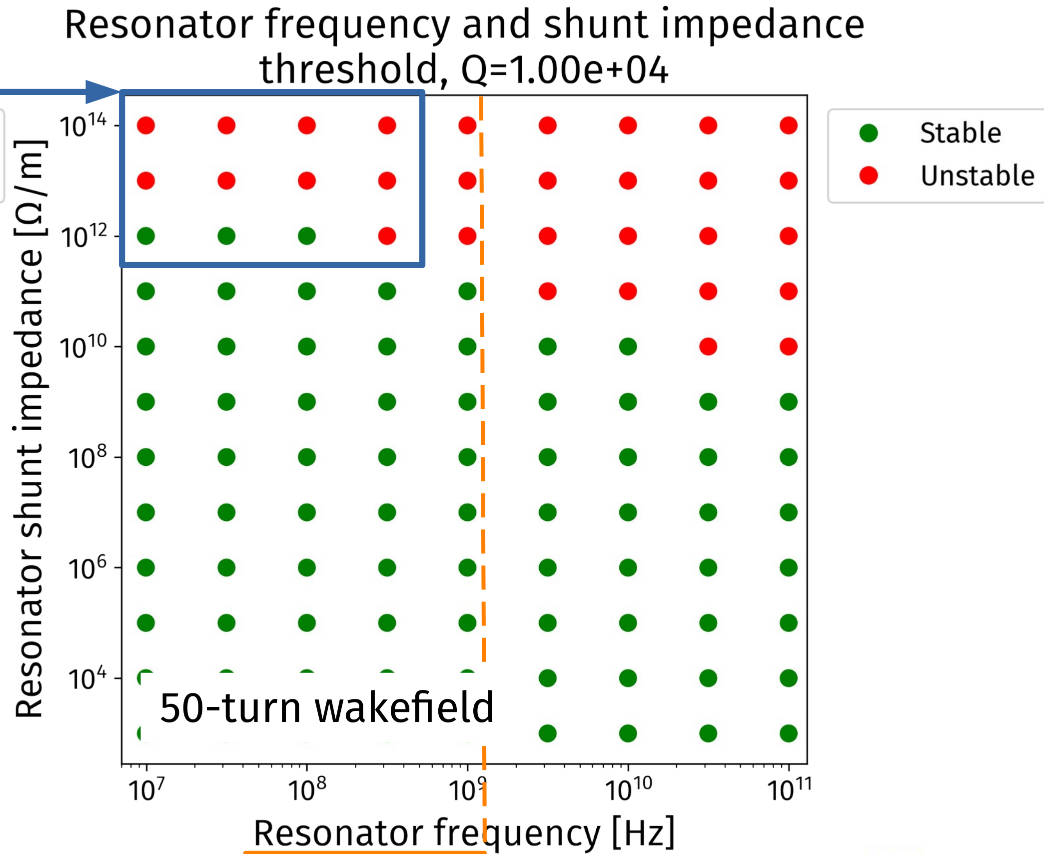
- Focus on one high-Q case (Q=10 000)
  - Results for different Q factor are reported in appendix

- Compare the 50-turn wakefield results to the single-turn wakefield case
  - Results for different number of wakefield turns and Q factor are reported in appendix

- In the plots, highlight the resonator frequency for which $f_{res}/Q < 10^5$
  - For Q=100, if $f_{res} < 10^7$ Hz, multi-turn wake should affect stability
  - For Q=10000, if $f_{res} < 10^9$ Hz, multi-turn wake should affect stability

Q = 10000, 50-turn wakefield





$f_{res}/Q < 10^5$

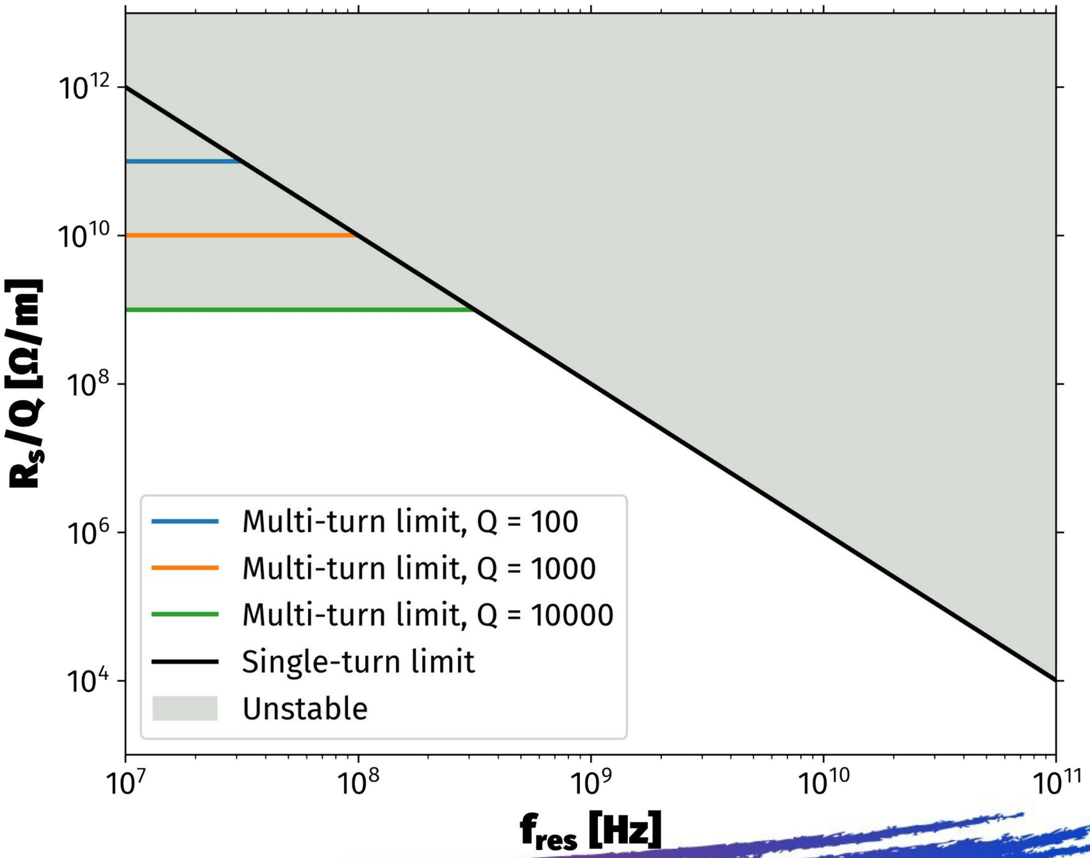
Summary of transverse stability simulation in the RCS 1

- Simulation including longitudinal map (**32 RF stations**) + transverse map + **transverse single-turn or multi-turn wakefield**
- Tracking over 100 turns, 5000 macroparticles with PyHEADTAIL
- In single-turn wakefield regime, the stability criterion depend on R_s /Q and f_{res}
- Multi-turn wakefield is required when **$f_{res}/Q < 10^5$** for the RCS1 case
- Effect is mostly visible for high-Q resonator (Q > 10000)
  - Below the $f_{res}/Q < 10^5$ criterion, simulations with high R_s become unstable
  - Above this criteron, we recover the single turn behavior studied previously

Summary of transverse stability simulation in the RCS 1, with multi-turn wakefield

- Summarize on one plot the single-turn and multi-turn wakefield stability limits
- The single-turn limit (black line) depends on the resonator frequency and the R_s/Q
- The multi-turn limit (color lines) depends on the resonator shunt impedance R_s

Stability limit versus resonator parameters



Summary of transverse stability simulation in the RCS 1, with multi-turn wakefield

$f_{res}/Q < 10^5$

Yes → Multi-turn regime: **$R_s < 10^{13}$ Ω/m**

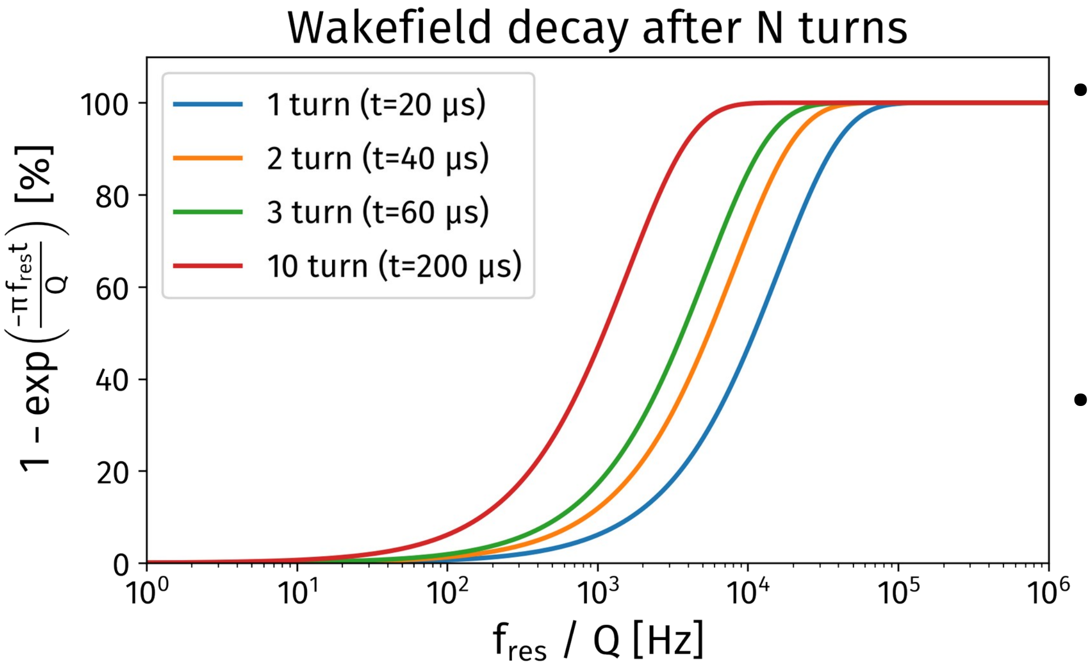
No → Single turn regime: **R_s / Q < $10^6 – 10^8$ Ω/m (depending on resonator frequency)**

Next steps

- Refine simulations with finer parameter space
  - In particular the resonator shunt impedance R_s
  - Convergence check with large number of macroparticles (convergence study) and wakefield slices
- Investigate in details the beam behavior during an instability
  - Dependence on number of RF stations and acceleration
  - Dependence on wake-field parameters…

Appendix : impedance and stability simulation parameters

Resonator impedance model

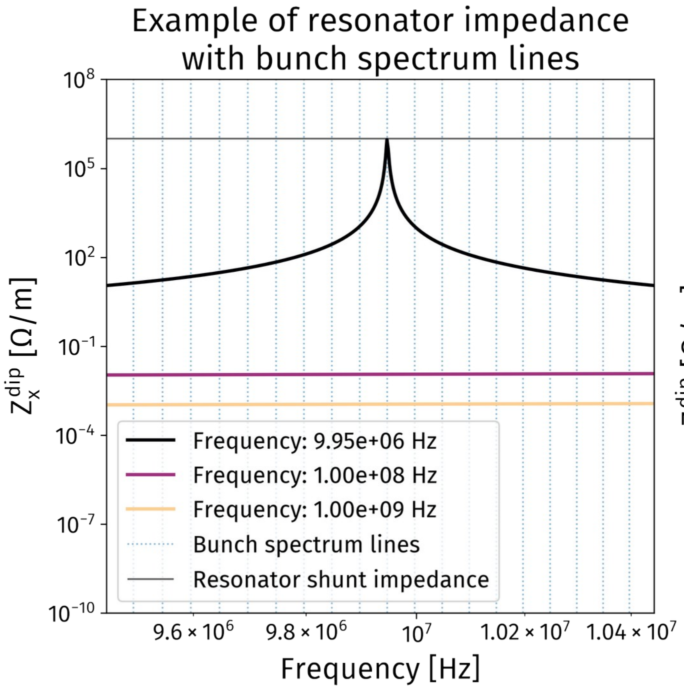


- We can plot the exponential term versus f_{res}/Q for a given time t (number of turns)

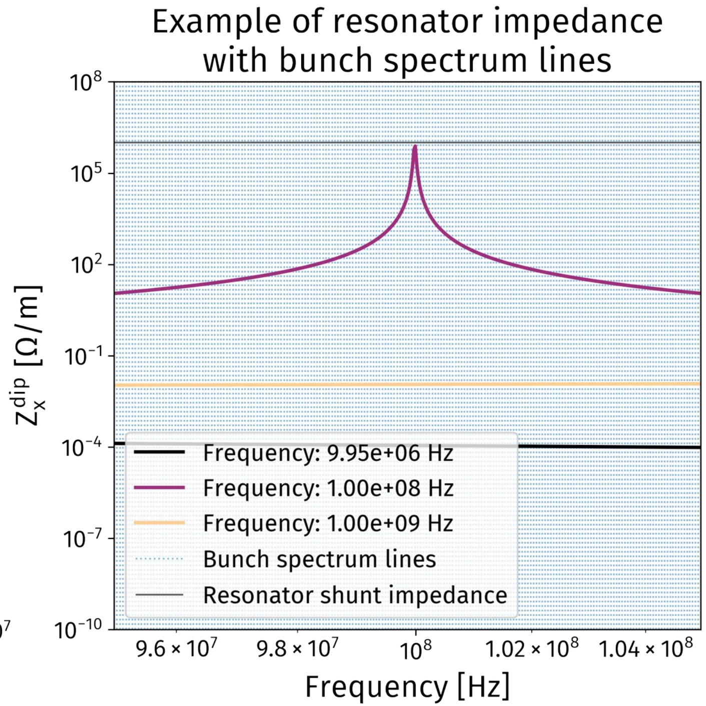
$$\exp\left(-\frac{2\pi f_{res}}{2Q} t\right)$$

- Shows by how much the wake decreased after N turns for a given (f_{res}, Q)

Resonator impedance model



f_{res} = 10 MHz
R_s = 1 MΩ/m
Q = 3000



f_{res} = 100 MHz
R_s = 1 MΩ/m
Q = 3000

- Resonator frequency is chosen to fall on a bunch spectrum line
- At high frequency (right plot), the resonance overlaps with many spectrum line
- Assumptions:
  - injection energy revolution frequency f_0
  - Q_x / Q_y = 0.26 / 0.26

Stability simulation parameters

Machine parameters

	Unit	Value
Circumference	m	5990
Beam momentum at injection	GeV/c	63.1
Momentum increase per turn	GeV/c	14.212
Rev. frequency	kHz	50
RF frequency	MHz	1300
Harmonic number		25957
RF voltage	MV	20 100
α_p		2.4e-3
Avg. beta x/y	m	50 / 50
Chromaticity Q'_x/Q'_y		0 / 0
Detuning from octupoles x/y	m^{-1}	0 / 0

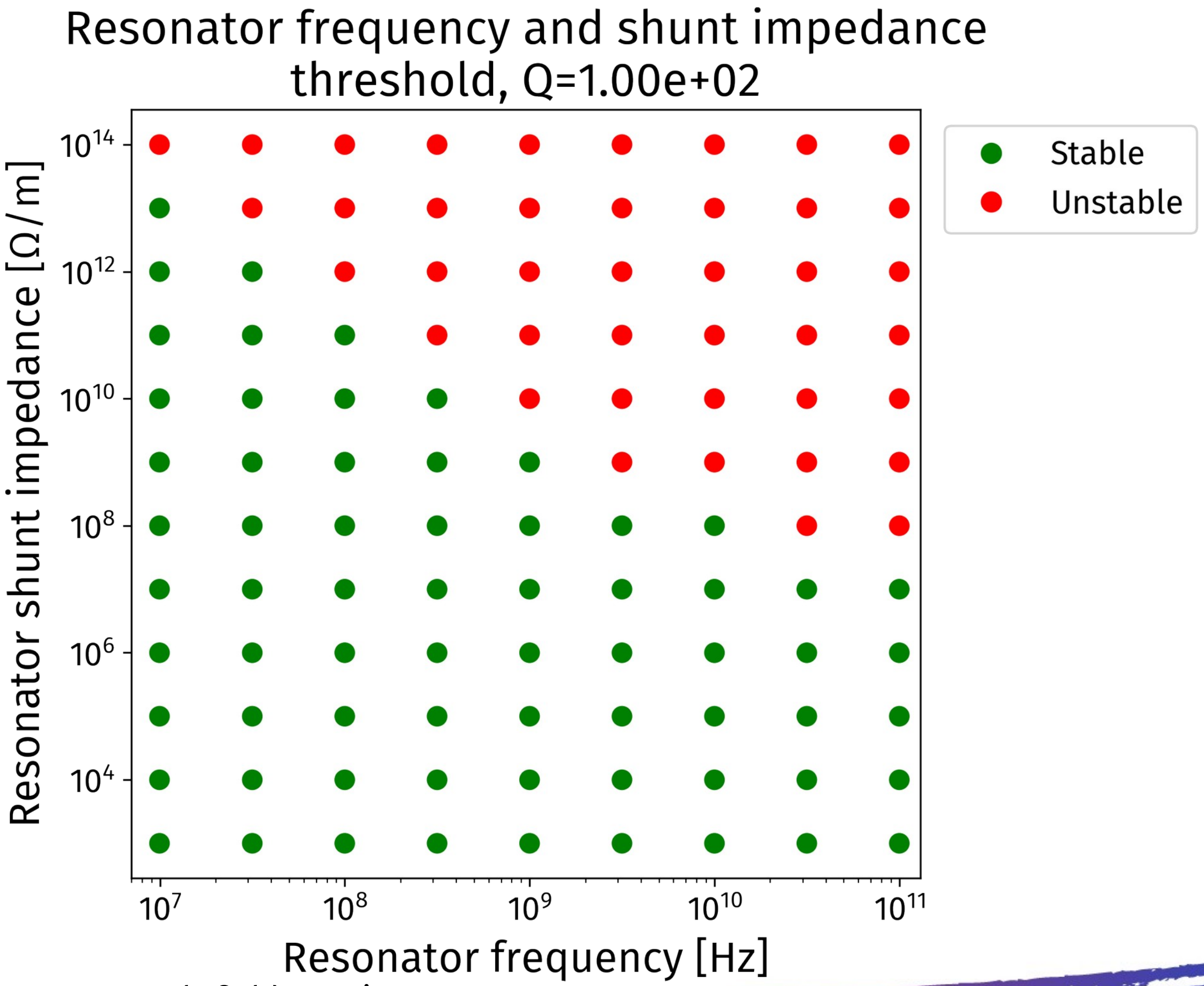
	Unit	Value
Synchrotron tune Q_s at injection		1.52
Synchrotron period	turns	0.66
Bunch length 1σ	**mm**	**25**
Bunch intensity	**Particles per bunch**	**2.6e12**
ε_x / ε_y	**µm rad**	**25**
# of macropaticles		5000

Scanned parameters

	Value
Resonator shunt impedance R_s	1 kΩ/m to 100 TΩ/m
Resonance frequency f_{res}	10 MHz to 1 THz
Quality factor Q	100, 300, 1000, 3000, 10 000, 30 000
Wakefield turns	1, 2, 3, 10, 50

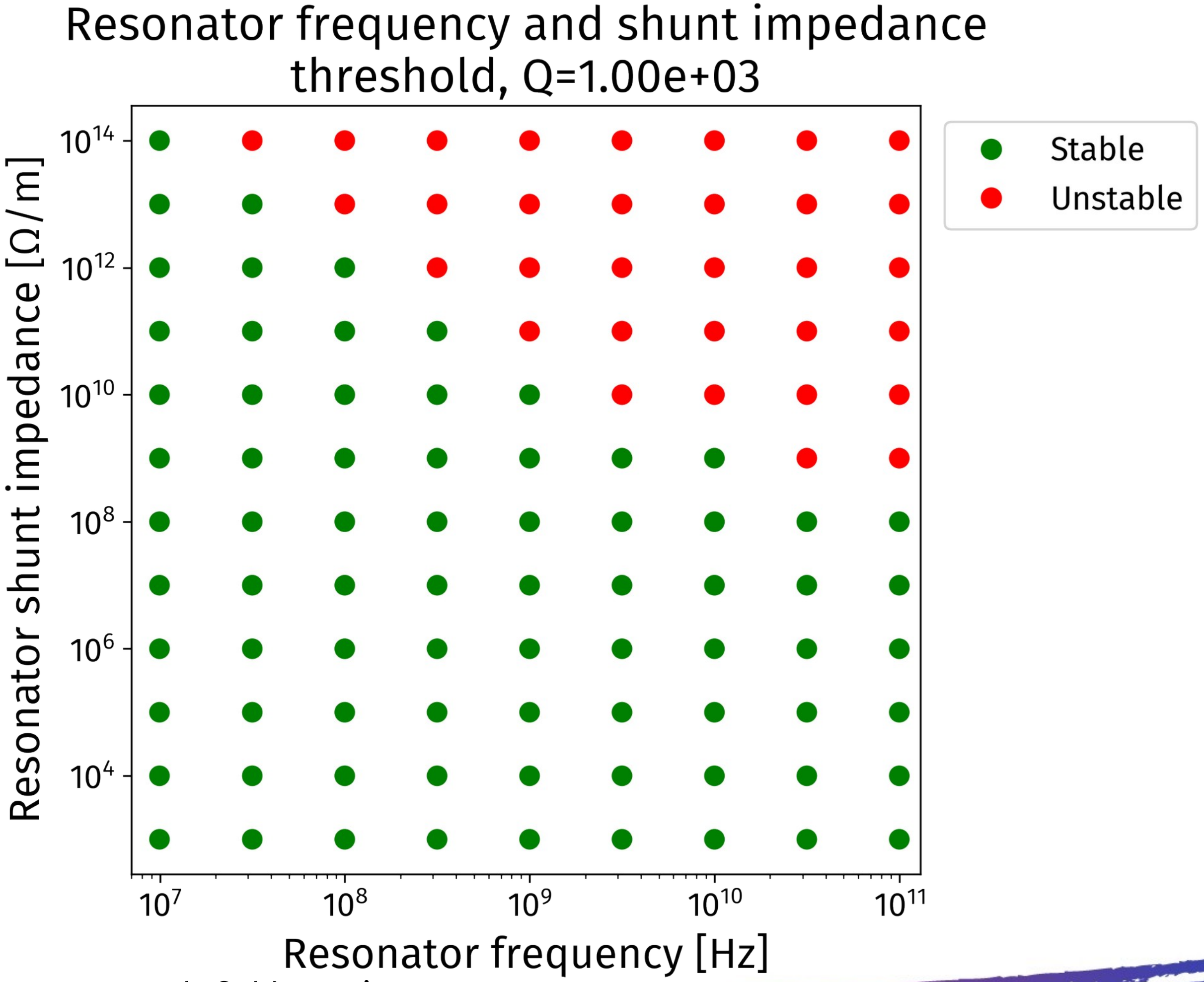
Appendix : stability results with single-turn wakefield

Q = 100



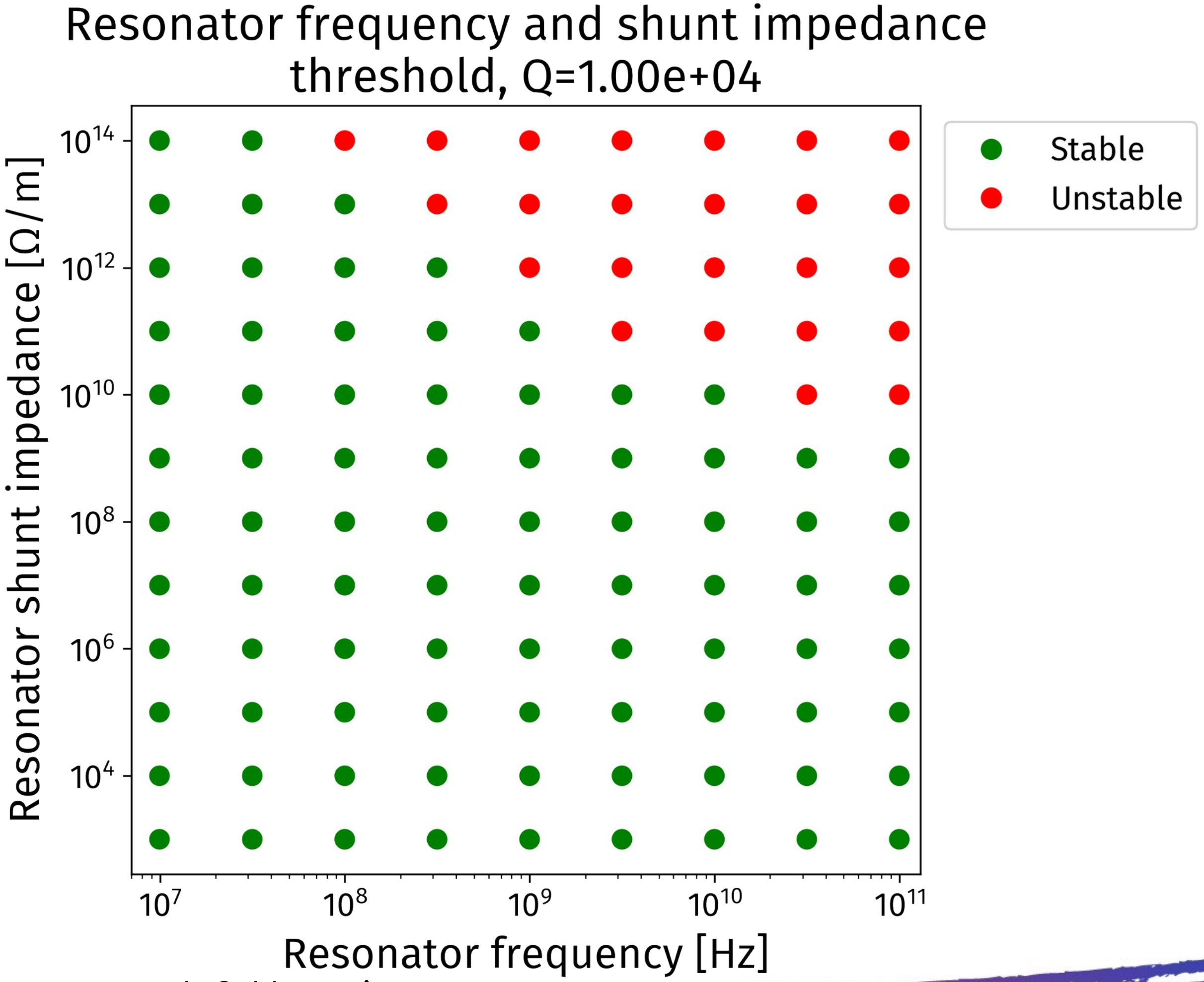
Resonator frequency and shunt impedance threshold, Q=1.00e+02
Stable
Unstable
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}

Q = 1000



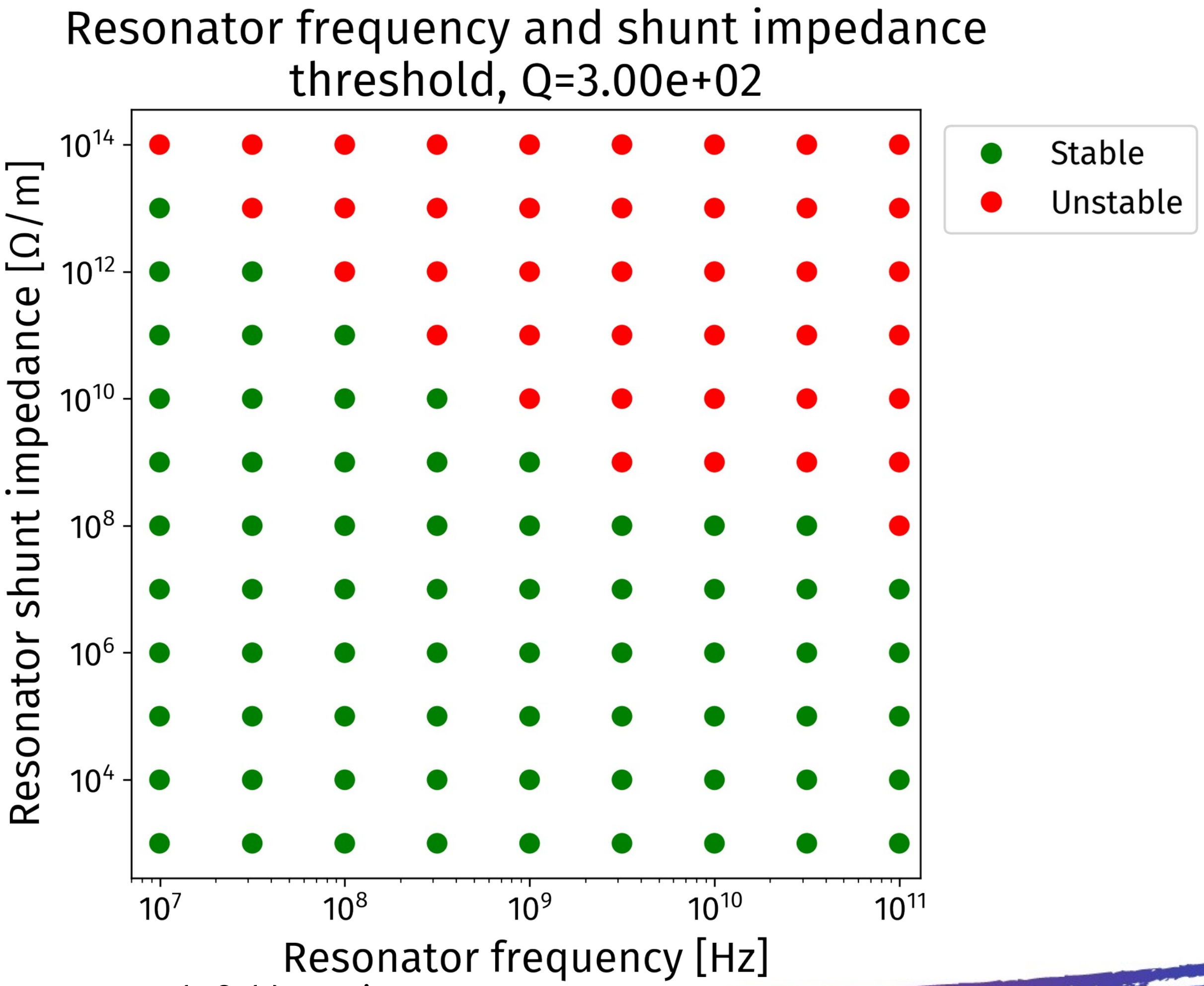
Resonator frequency and shunt impedance threshold, Q=1.00e+03
Resonator shunt impedance [Ω/m]
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Resonator frequency [Hz]
Stable
Unstable

Q = 10000



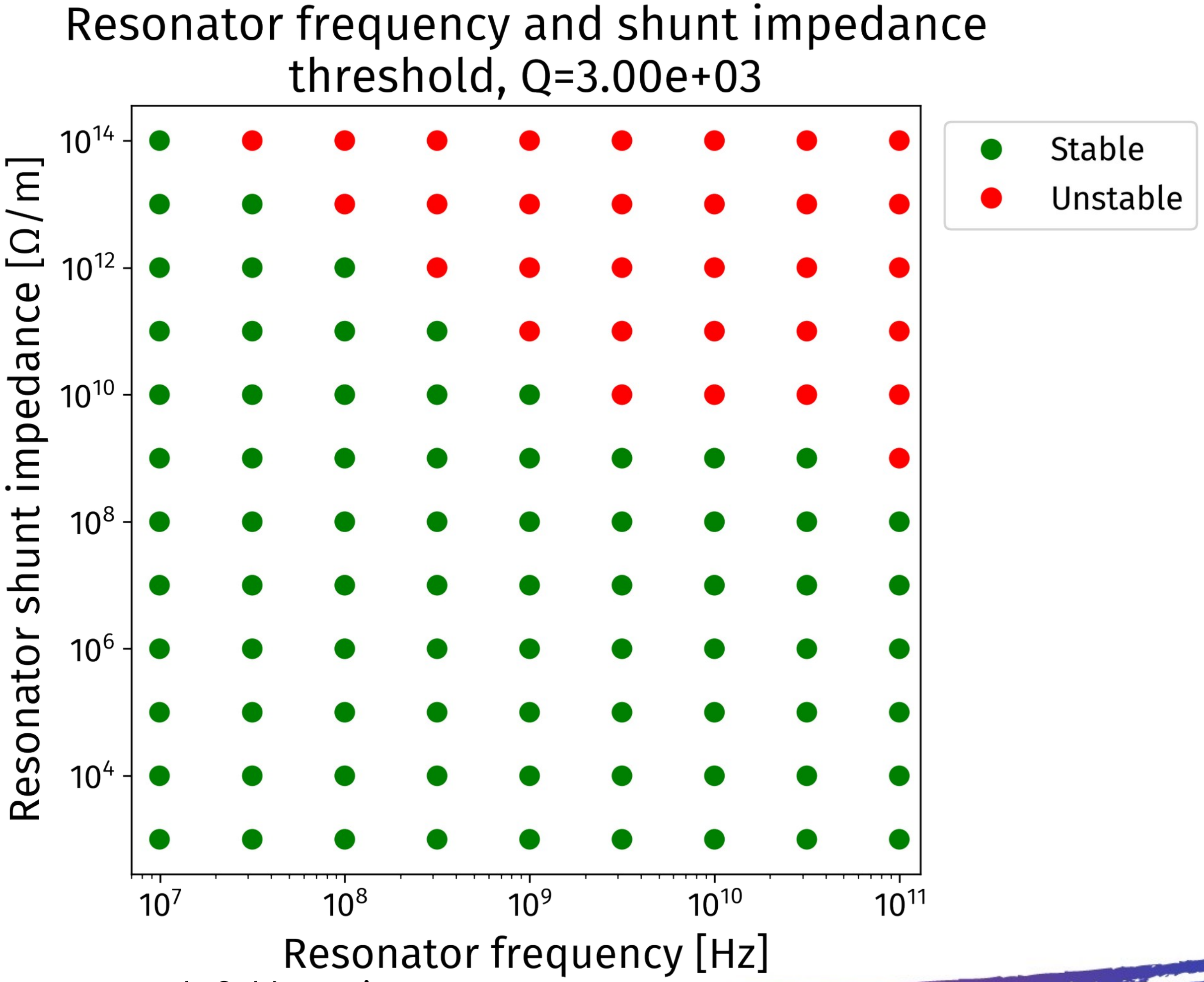
Resonator frequency and shunt impedance threshold, Q=1.00e+04
Resonator shunt impedance [Ω/m]
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Resonator frequency [Hz]
Stable
Unstable

Q = 300



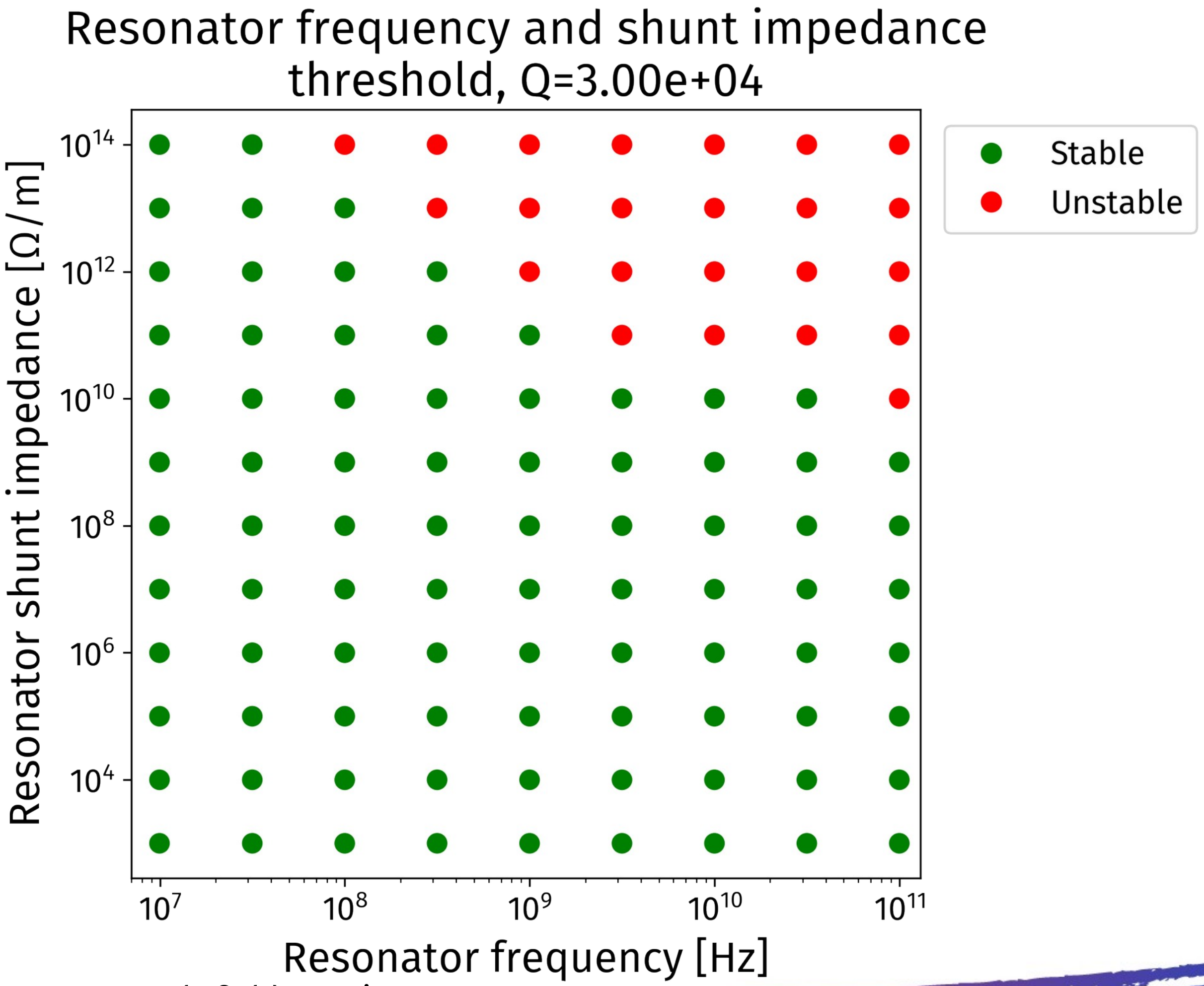
Resonator frequency and shunt impedance threshold, Q=3.00e+02
Resonator shunt impedance [Ω/m]
10^14
10^12
10^10
10^8
10^6
10^4
10^7
10^8
10^9
10^10
10^11
Resonator frequency [Hz]
Stable
Unstable

Q = 3000



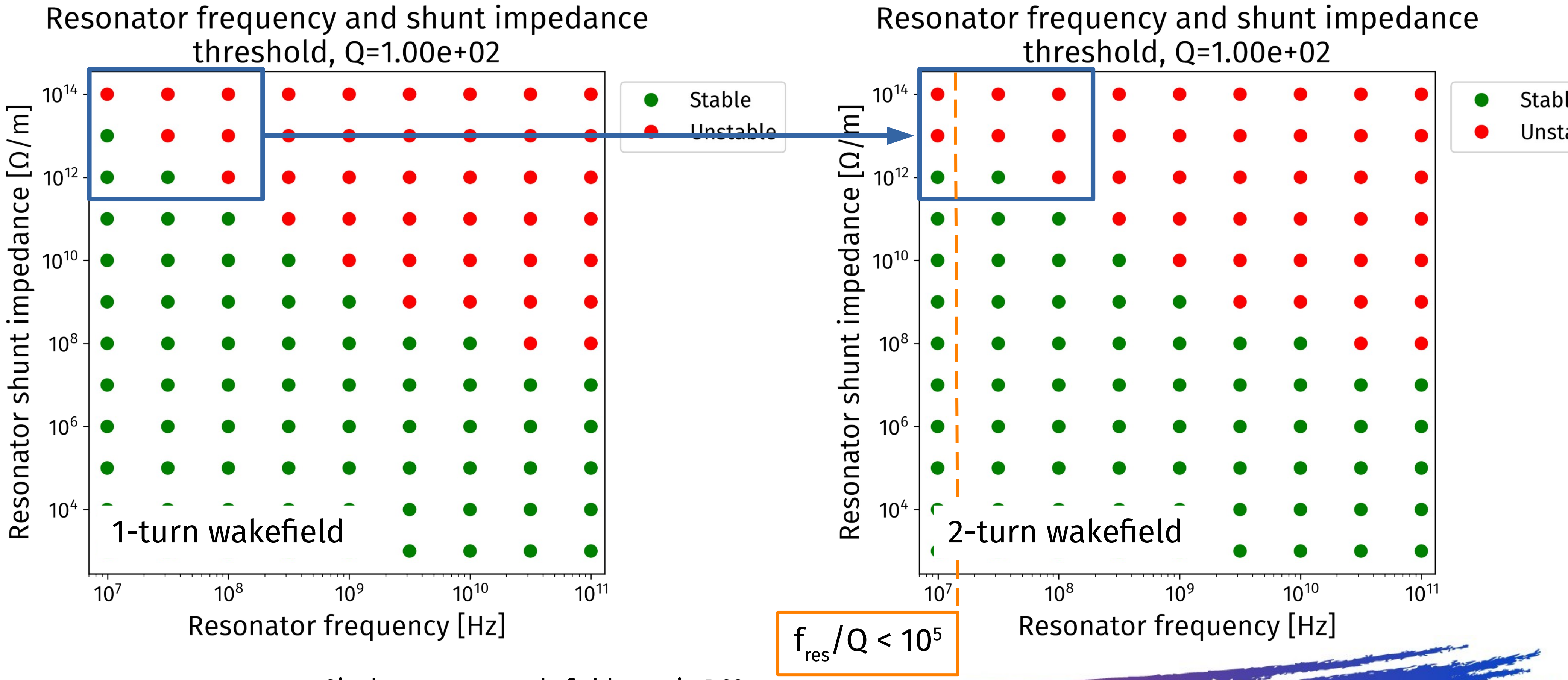
Resonator frequency and shunt impedance threshold, Q=3.00e+03
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Stable
Unstable

Q = 30000

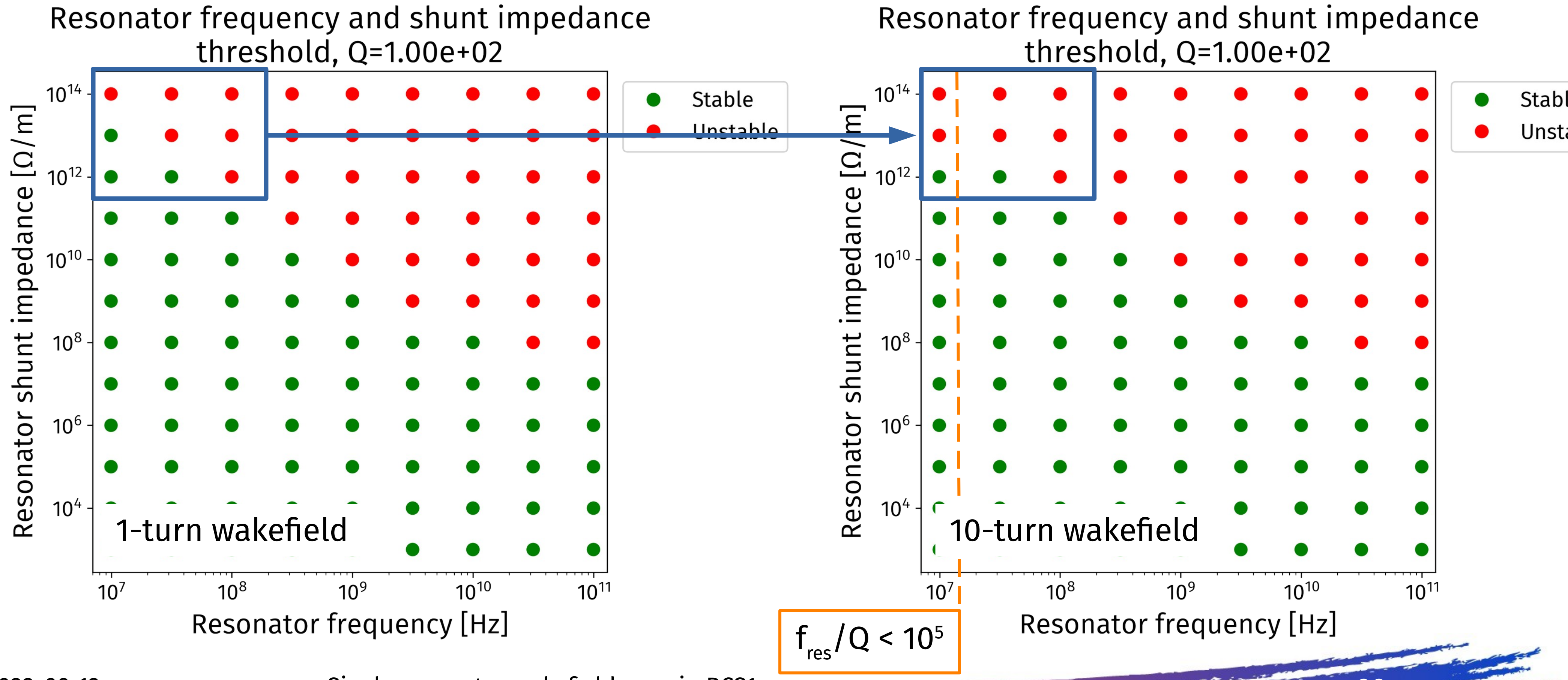

Resonator frequency and shunt impedance threshold, Q=3.00e+04
Resonator shunt impedance [Ω/m]
10^14
10^12
10^10
10^8
10^6
10^4
10^7
10^8
10^9
10^10
10^11
Resonator frequency [Hz]
Stable
Unstable

Appendix : stability results with multi-turn wakefield

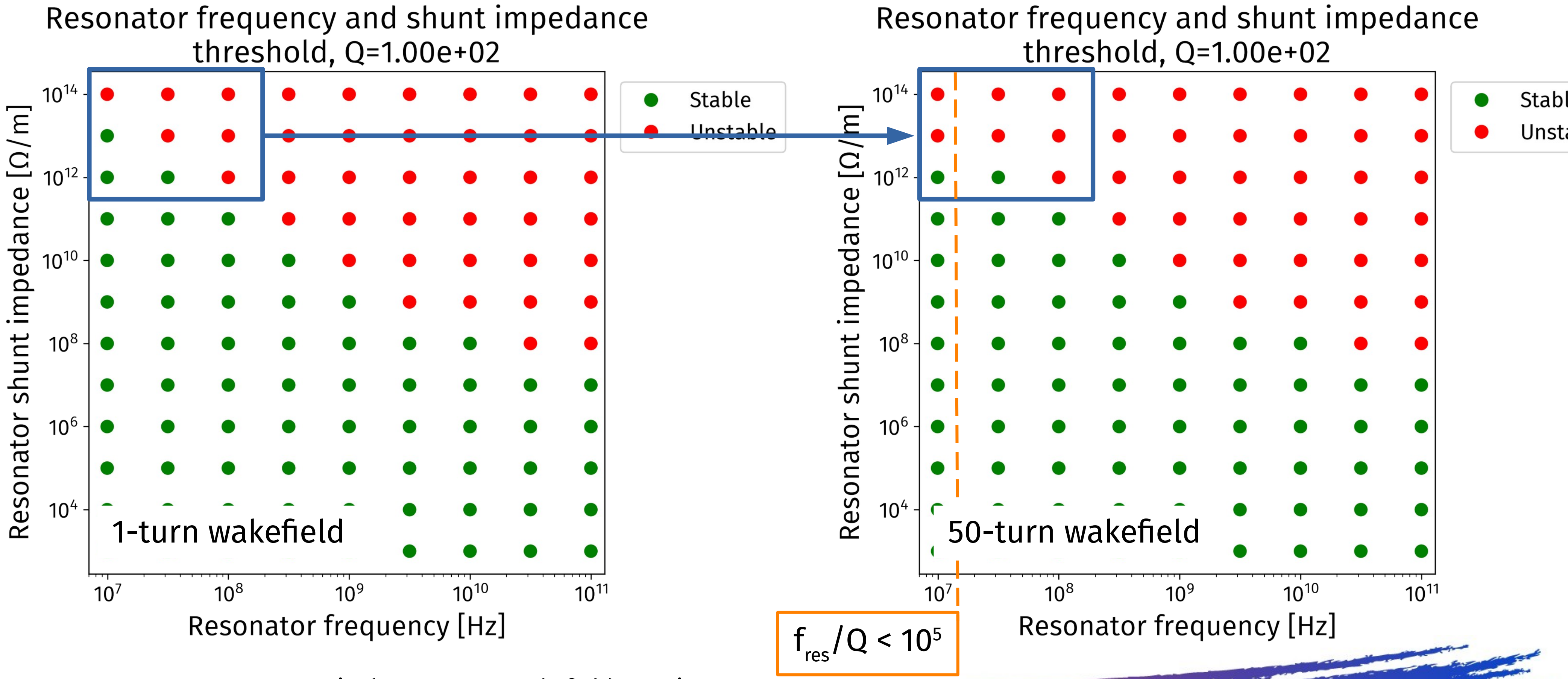
Q = 100, 2-turn wakefield



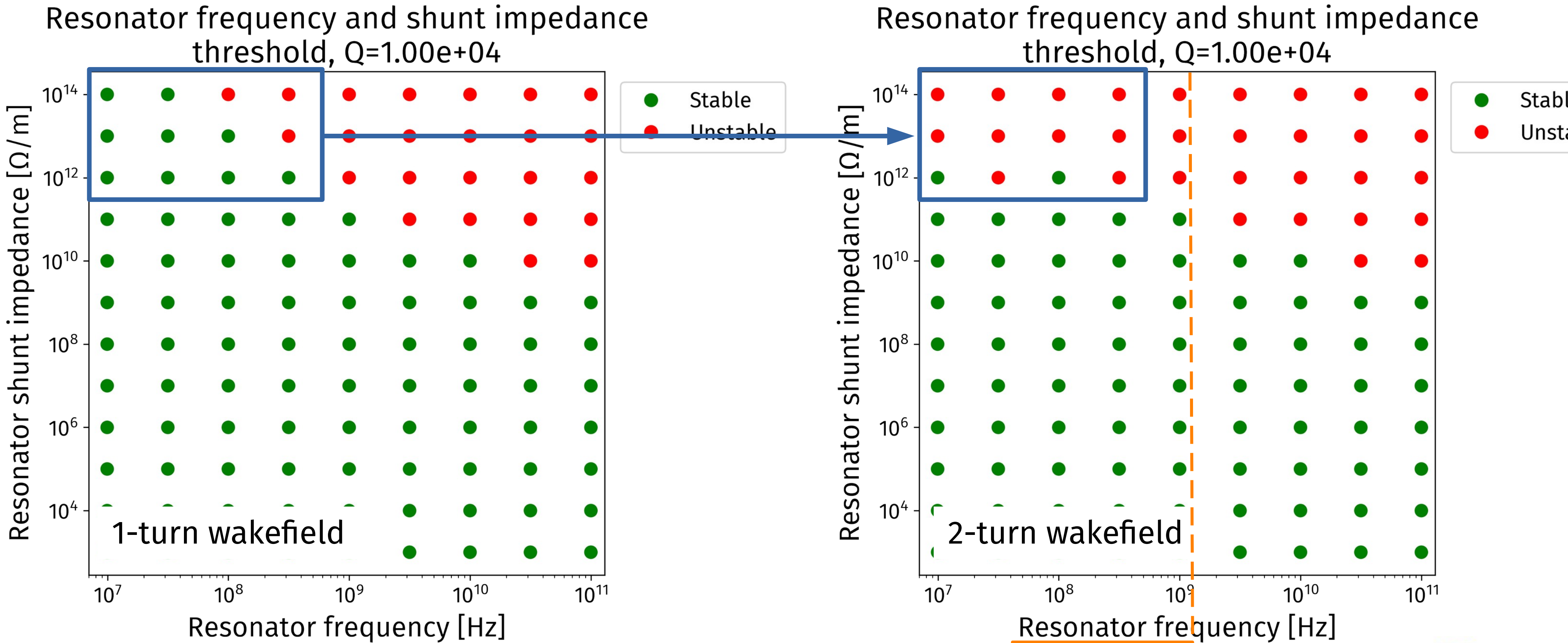
Q = 100, 10-turn wakefield



Q = 100, 50-turn wakefield

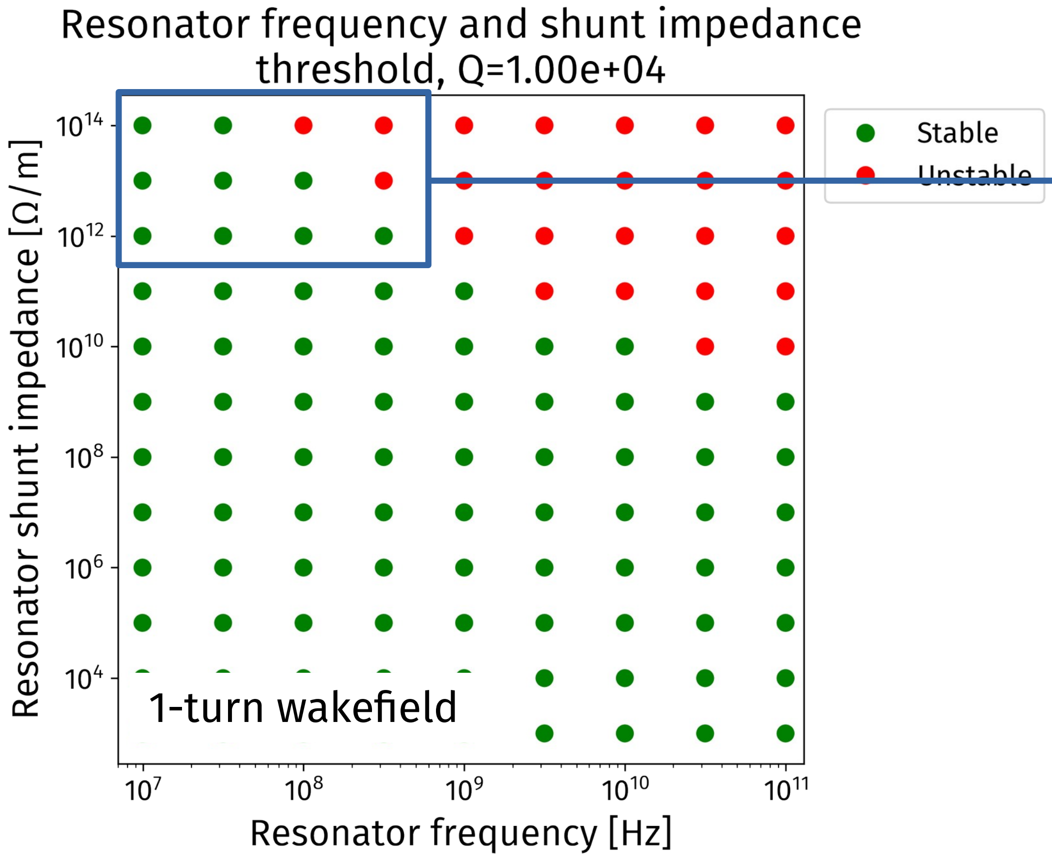


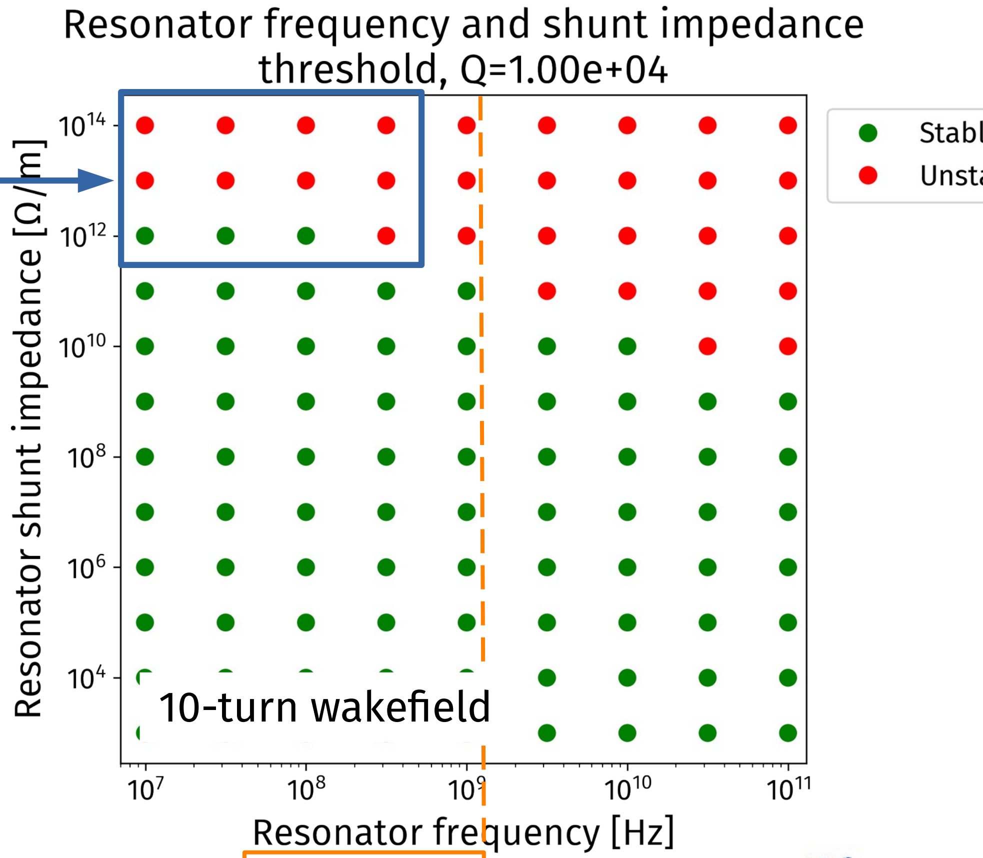
Q = 10000, 2-turn wakefield



$f_{res}/Q < 10^5$

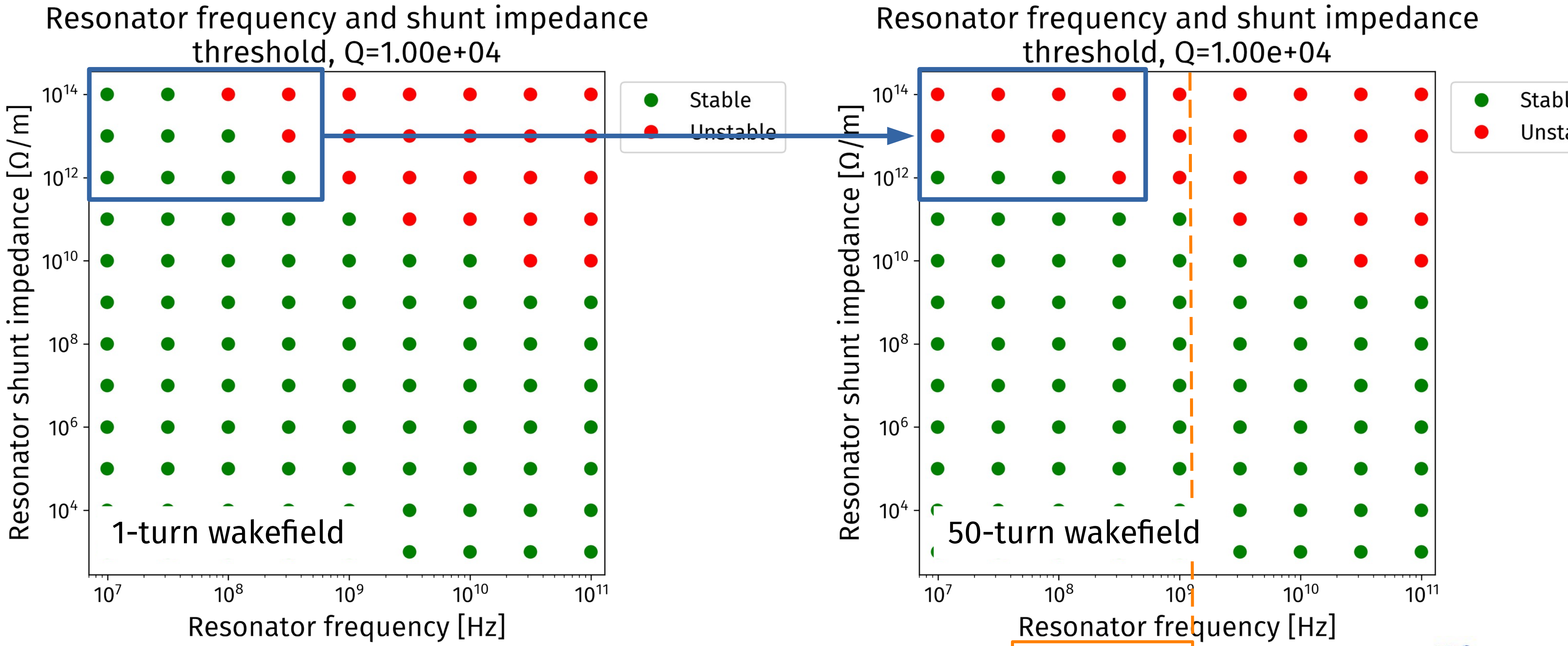
Q = 10000, 10-turn wakefield





$f_{res}/Q < 10^5$

Q = 10000, 50-turn wakefield



$f_{res}/Q < 10^5$