The pion and nucleon within Minkowski-space dynamics

Emanuel Ydrefors

Institute of Modern Physics, China

Collaborators: T. Frederico, V. A. Karmanov, W. de Paula, G. Salmè

References: PRD 104 (2021) 114012, PRD 103 (2021) 014002, PRD 105 (2022) L071505, PLB 820 (2021) 136494

Annual Swedish Nuclear Physicist's Meeting 2022
(KTH)
Oct 25, 2022

Motivation: Importance of hadron structure for nuclear physics

- The proton used as a probe in nuclear physics, so important to know its structure.
- The pion is the most simple hadron, but still contain the full complexity of QCD.
- In calculations of e.g. cross sections for neutrino-nucleus scattering it is needed the electromagnetic and weak form factors of the nucleons.
- The nuclear force is a remnant of the strong force (described by QCD) between quarks and gluons, i.e. knowledge about the properties of the QCD at low energy is important to understand the nuclear force from first principles.

Light-front dynamics

- Dynamical system is characterized by ten fundamental quantities, i.e. energy, momentum, angular momentum and boosts.
- Convential form (instant form): dynamical variables refer to physical conditions at some instant time, e.g. $x^0 = 0$. But, other choices are possible. In the Light-front (LF) dynamics refer to conditions on a front $x^+ = t + z = 0$. So, commutation relations defined at equal LF time ($x^+ = 0$).
- LF variables: $x^{\pm} = t \pm z$ and similarly for the momenta.
- After integration over relative momentum k^- and putting $x^+ = 0$, the four-dimensional space reduced to a three-dimensional one ($k^+, \vec{k}_\perp$).

Light-front Dynamics (LFD) turns out to be convenient for description of, in particular, relativistic bound states:

- It allows a Fock space expansion of a state vector in terms of contribution with well-defined particle-number. For example, for a pion:

$$|p\rangle = |n=2\rangle + |n=3\rangle + ... \tag{1}$$

  where each term has an associated boost-invariant wave function Ψ_n with probability

$$P_n = \left\{ \prod_{i=1}^{n} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta\left(\sum_{i=1}^{n} \vec{k}_{i\perp}\right) |\Psi_n(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, ...)|^2 \tag{2}$$

  In Eq. (1), the leading contribution is referred to as valence component.

- Using the Fock space expansion one can derive a Schroedinger like equation of the form $(P_\perp = 0)$

$$H_{LC}|\Psi\rangle = M^2|\Psi\rangle, \quad H_{LC} = P^+P^-, \tag{3}$$

  with P^+ diagonal and P^- a functional. But, in practice Fock-expansion has to be truncated to finite order.

- The light-front wave function gives access to various observables in momentum space.
- For example:
  - Electromagnetic form factors
  - The parton distribution function, $f_1(x_1)$, i.e. probability distribution for a quark having a momentum fraction. Extracted from inclusive deep inelastic scattering, only scattered lepton detected.
  - Transverse momentum distribution. Dependence on both momentum fraction x and transverse one $\vec{k}_\perp$. Associated with semi-inclusive deeply inelastic scattering (SIDIS), also high-momentum hadron detected.
- Additionally, in the double parton scattering cross section enters the double parton distribution function (DPDF) [1]:

$$D(x_1,x_2,\vec{\eta}_\perp) = \sum_{n=3}^{\infty} D_n(x_1,x_2,\vec{q}_\perp) = \sum_{n=3}^{\infty} \int \frac{d^2k_{1\perp}}{(2\pi)^2}\frac{d^2k_{2\perp}}{(2\pi)^2} \left\{ \prod_{i\neq 1,2} \int \frac{d^2k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\}$$
$$\times \delta\left(1-\sum_{i=1}^{n} x_i\right) \delta\left(\sum_{i=1}^{n} \vec{k}_{i\perp}\right) \Psi_n^\dagger(x_1,\vec{k}_{1\perp}+\vec{\eta}_\perp,x_2,\vec{k}_{2\perp}-\vec{\eta}_\perp,...)\Psi_n(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},...)\,, \quad (4)$$

[1] B. Blok et al, PRD 83 (2011) 071501 (R).

Models

- Pion:
  - The so-called Bethe-Salpeter (BS) equation is used in Minkowski space together with a quark-gluon vertex (finite-range interaction).
  - An effective model but with parameters inspired by Lattice QCD.
  - The full BS amplitude used, i.e. no truncation in Fock space.
- Proton:
  - More simple, at this stage;
  - Fock basis truncated to valence order and spin degree-of-freedom not included.
  - The quark-quark transition amplitude has a pole representing the s-wave diquark introduced through the zero-range interaction between two of the quarks. In that sense it is an effective low-energy model.

Pion Bethe-Salpeter equation

- The BS amplitude, describing π^+, obeys the equation [1]

$$\Phi(k,p) = S(k+p/2)\int \frac{d^4k'}{(2\pi)^4} S^{\mu\nu}(q)\Gamma_\mu(q)\phi(k',p)\widehat{\Gamma}_\nu(q)S(k-p/2); \\ \widehat{\Gamma}_\nu(q) = C\Gamma_\nu(q)C^{-1}, \tag{5}$$

where we currently use bare progagators for the quarks and gluons, i.e.

$$S(k) = i\frac{\not{k}+m}{k^2-m^2+i\epsilon}, \quad S^{\mu\nu}(q) = -i\frac{g^{\mu\nu}}{q^2-\mu^2+i\epsilon}, \tag{6}$$

and the quark-gluon vertex is described by

$$\Gamma^\mu(q) = ig\frac{\mu^2-\Lambda^2}{q^2-\Lambda^2+i\epsilon}\gamma^\mu, \tag{7}$$

i.e. dressed by a simple form factor characterized by the scale parameter Λ.

[1] W. de Paula et al, Eur. Phys. C (2017) 77

- The BS amplitude is decomposed as

$$\Phi(k,p) = \sum_{i=1}^{4} S_i(k,p)\phi_i(k,p) \tag{8}$$

where S_i is a Dirac structure.

- Each scalar function ϕ_i written in terms of Nakanishi integral representation:

$$\phi_i(k,p) = \int_{-1}^{1} dz' \int_0^{\infty} d\gamma' \frac{g_i(\gamma', z')}{[k^2 + z' p \cdot k - \gamma' - \kappa^2 + i\epsilon]^3} \tag{9}$$

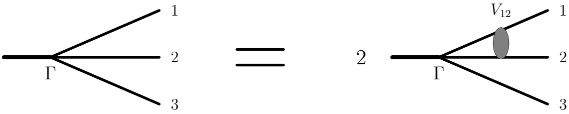
- The following system of coupled integral equations can be obtained:

$$\int_0^{\infty} d\gamma' \frac{g_i(\gamma', z')}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2)\kappa^2]^2} = iMg^2 \sum_j \int_0^{\infty} d\gamma' \int_{-1}^{1} dz' \mathcal{L}_{ij}(\gamma, z; \gamma' z') g_j(\gamma, z'), \tag{10}$$

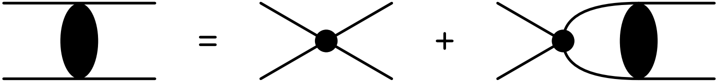
which are solved for the coupling constant g^2 and the Nakanishi weight functions g_i.

- Once the g_i's are known physical observables can be calculated as integrals over these functions.

Three-body model for the proton



- Three spinless particles of mass m. Spectator + pair of interacting particles. Factor of two due to symmetry of wave function with respect to exchange of the particles.



- In the present work a zero-range interaction with four-leg-vertex $i\lambda$ used. Then, for the two-body amplitude (see figure)

$$i\mathcal{F}(M_{12}^2)) = i\lambda + (i\lambda)^2\mathcal{B} + (i\lambda)^3\mathcal{B}^2 + ... = \frac{1}{(i\lambda)^{-1} - \mathcal{B}(M_{12}^2)} \tag{11}$$

with the bubble diagram

$$\mathcal{B}(M_{12}^2) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{(k^2 - m^2 + i\epsilon)} \frac{i}{[(k-P)^2 - m^2 + i\epsilon]}, \tag{12}$$

where $M_{12}^2 = P^2$. The bubble diagram regularized by assuming a pole in the scattering amplitude.

Valence LF integral equation for the proton

- The valence three-body LF equation given by [1, 2]:

$$\Gamma(x,k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3}\int_0^{1-x}\frac{dx'}{x'(1-x-x')}\int_0^\infty d^2k'_\perp\left[\frac{1}{M_0^2-M_N^2}-\frac{1}{M_0^2+\mu^2}\right]\Gamma(x',k'_\perp) \tag{13}$$

where μ is a cut-off, $k_\perp$ transverse momentum and x momentum fraction of spectator. Furthemore, the squared free three-body mass

$$M_0^2 = (k'^2_\perp + m^2)/x' + (k^2_\perp + m^2)/x + ((k'_\perp + k_\perp)^2 + m^2)/(1-x-x') \tag{14}$$

- The three-body valence LF wave function is given by

$$\Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}) = \frac{\Gamma(x_1,\vec{k}_{1\perp})+\Gamma(x_2,\vec{k}_{2\perp})+\Gamma(x_3,\vec{k}_{3\perp})}{\sqrt{x_1x_2x_3}(M_N^2-M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))}, \tag{15}$$

where due to momentum conservation: $x_3 = 1 - x_2 - x_3$ and $\vec{k}_{3\perp} = -\vec{k}_{1\perp} - \vec{k}_{2\perp}$.

[1] J. Carbonell and V.A. Karmanov, PRC 67 (2003) 037001

[2] T. Frederico, PLB 282 (1992) 409

Results for the pion: Static properties

Set	m (MeV)	B/m	μ/m	Λ/m	P_{val}	$P_{\uparrow\downarrow}$	$P_{\uparrow\uparrow}$	f_π (MeV)
I	187	1.25	0.15	2	0.64	0.55	0.09	77
II	255	1.45	1.5	1	0.65	0.55	0.10	112
III	255	1.45	2	1	0.66	0.56	0.11	117
IV	215	1.35	2	1	0.67	0.57	0.11	98
V	187	1.25	2	1	0.67	0.56	0.11	84
VI	255	1.45	2.5	1	0.68	0.56	0.11	122
VII	255	1.45	2.5	1.1	0.69	0.56	0.12	127
VIII	255	1.45	2.5	1.2	0.70	0.57	0.13	130
IX	255	1.45	1	2	0.70	0.57	0.14	134
X	215	1.35	1	2	0.71	0.57	0.14	112
XI	187	1.25	1	2	0.71	0.58	0.14	96

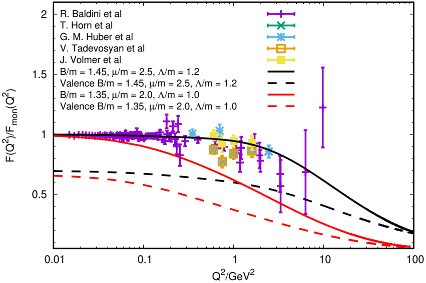
- Parameters: B (binding energy), μ (gluon mass), Λ.
- The set VIII gives an f_π in good agreement with the experimental value.
- The valence probability is 64-71%, i.e. rather large contributions beyond the valence component.

Pion electromagnetic form factors

- In impulse approximation, with bare photon vertex $i\gamma^\mu$,

$$(p+p')^\mu F(Q^2) = -i\frac{N_c}{4M^2+Q^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[(-\not{k}-m)\bar{\Phi}_2(k_2;p')(\not{p}+\not{p}')\Phi_1(k_1;p)], \quad (16)$$

where $Q^2 = -(p-p')^2$.



- We have a good agreement with experimental data for all Q^2.
- At large values of Q^2 the valence contribution dominates.

Emperical access to the PDF

- The PDF gives the probability distribution amplitude for a quark to have a certain momentum fraction x within the pion/proton.
- Information about the PDF can be obtained from e.g. deep inelastic scattering

$$e + p \longrightarrow e + X, \tag{17}$$

  where X is undetected.
- The differential cross section is of the form

$$\frac{d\sigma}{dxdQ^2} \sim \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_i(z, Q^2) f_{i/p}(x/z, Q^2), \tag{18}$$

  with the sum running over active quark flavors and gluon and $f_{i/p}$ the corresponding PDF. The coefficient functions C_i is obtained from perturbative QCD.

PDF evolution

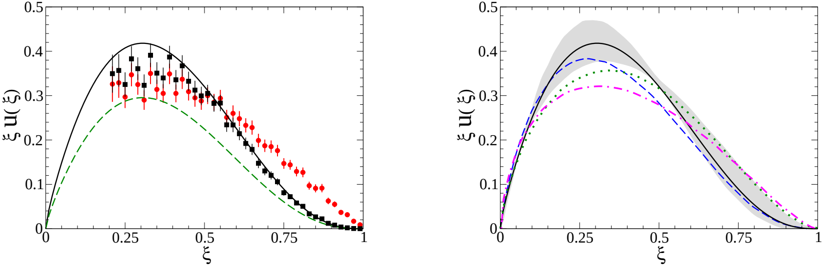
- For the comparison with other frameworks and/or experimental data the PDF should be evolved from the model scale to a higher scale.
- This is done by using the DGLAP equation

$$\frac{dq}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 P(x/y,\alpha_s(Q)^2)q(y,Q^2) \tag{19}$$

- We will use the effective coupling (EPJC 80 (2020) 1064):

$$\alpha_s(k^2) = \frac{\gamma_m \pi}{\log[\mathcal{K}^2(k^2)/\Lambda^2_{QCD}]}, \quad \mathcal{K}^2(y) = (a_0^2 + a_1 y + y^2)/(b_0 + y) \tag{20}$$

- The initial scale is given by the hadron scale $Q_0 = 0.330 \pm 0.03$ GeV.



- Left panel; Results of this work compared with two sets of experimental data. Our results in good agreement with the so-called res-summed data (black squares), predicting $u(\xi) \sim (1-x)^2$ as $x \to 1$.
- Right panel; Shaded Area: Lattice QCD (PRD 104 05404), solid line: (this work), dotted line: BLFQ (PRD 101 034024), dashed line: DSE (EPJA 58 10), dashed-dotted line: DSE (PRL 124 042002).

- The valence contribution to the Dirac form factor is obtained from the matrix element of γ^+. In the frame $q^+ = 0$ and $q^2 = -Q^2 = -q_\perp^2$ it is given by

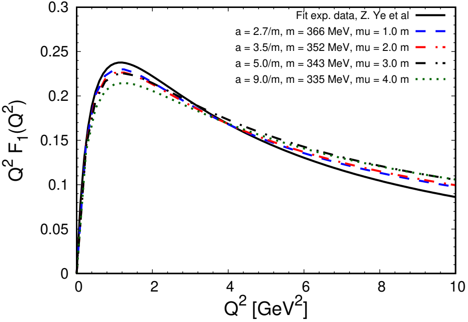
$$F_1(Q^2) = \left\{ \prod_{i=1}^{3} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \delta\left(1 - \sum_{i=1}^{3} x_i\right) \delta\left(\sum_{i=1}^{3} \vec{k}^{\mathrm{f}}_{i\perp}\right) \Psi_3^\dagger(x_1, \vec{k}^{\mathrm{f}}_{1\perp}, ...) \Psi_3(x_1, \vec{k}^{\mathrm{i}}_{1\perp}, ...), \tag{21}$$

where $Q^2 = \vec{q}_\perp \cdot \vec{q}_\perp$ and the magnitudes of the momenta read

$$\left|\vec{k}^{\mathrm{f(i)}}_{i\perp}\right|^2 = \left|\vec{k}_{i\perp} \pm \frac{\vec{q}_\perp}{2} x_i\right|^2 = \vec{k}^2_{i\perp} + \frac{Q^2}{4} x_i^2 \pm \vec{k}_{i\perp} \cdot \vec{q}_\perp x_i \quad (i = 1, 2), \tag{22}$$

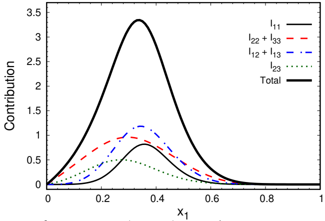
and

$$\left|\vec{k}^{\mathrm{f(i)}}_{3\perp}\right|^2 = \left|\pm \frac{\vec{q}_\perp}{2}(x_3 - 1) - \vec{k}_{1\perp} - \vec{k}_{2\perp}\right|^2 = (1 - x_3)^2 \frac{Q^2}{4} \pm (1 - x_3)\vec{q}_\perp \cdot (\vec{k}_{1\perp} + \vec{k}_{2\perp}) + (\vec{k}_{1\perp} + \vec{k}_{2\perp})^2. \tag{23}$$



- In figure $Q^2F(Q^2)$ for different values of a and μ compared with fit to exp. data by Z. Ye et al [1].
- Best agreement obtained for $a \approx 1.46$ fm and $\mu = m = 366$ MeV, and this parameters will be used in the following.
- Fair agreement with exp. data for $Q^2 < 5\,\text{GeV}^2$ but for larger values of Q^2 they deviate, presumably due to lack of a finite-range interaction.

Z. Ye et al, PLB 777 (2018) 8.



- The single parton distribution function (PDF), is the integrand of the form factor at $Q^2 = 0$, i.e.

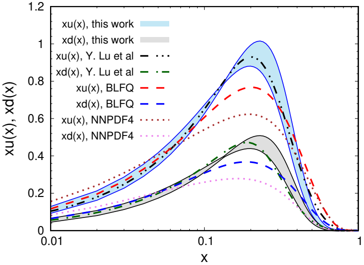
$$f_1(x_1) = \frac{1}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} |\Psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp})|^2 = I_{11} + I_{22} + I_{33} + I_{12} + I_{13} + I_{23}. \tag{24}$$

with the Faddeev contributions

$$\begin{aligned} I_{ii} &= \frac{1}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma^2(x_i, \vec{k}_{i\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}))^2} \\ I_{ij} &= \frac{2}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma(x_i, \vec{k}_{i\perp})\Gamma(x_j, \vec{k}_{j\perp})}{x_1x_2x_3(M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}))^2}; \quad i \neq j. \end{aligned} \tag{25}$$

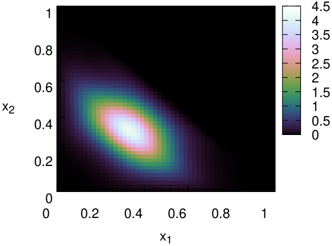
- The PDF at model scale is peaked around $x = 1/3$ and quite narrow. None of the Faddeev contributions are negligible.

Proton PDF at $Q = 3.097$ GeV



- Colored areas: Computed u and d-quark xpdfs at $Q = 3.097$ GeV with the areas corresponding to the uncertainty in initial scale $Q_0 = 0.330 \pm 0.03$ GeV.
- Dash-dotted lines: Results from quark-diquark by Y. Lu et al [1]. Reasonable agreement. Disagreement at large x probably due to the use of contact interaction in our model.
- Dashed-lines: Basis Light-front Quantization (BLFQ) [1] but evolved using same framework as in this work. Only good agreement for small x.
- Dotted lines: Results from the NNPDF 4.0 global fit. None of the models agree well with these results.
- A few remarks:
  - Model of this work and the one by Y. Lu et al, are both quark-diquark models, but the latter one has also axial-vector diquark and a more realistic quark-quark interaction.
  - The BLFQ which is a Hamiltonian approach include (at least effectively) confinement, which is lacking in the two other models.

[1] arXiv:2203.00753 [hep-th], [2] PRD 104, 094036 (2021), [3] arXiv:2109.02653 [hep-ph]



- The valence double parton distribution function (DPDF) is given by

$$D_3(x_1,x_2;\vec{\eta}_\perp) = \frac{1}{(2\pi)^6}\int d^2k_{1\perp}d^2k_{2\perp} \times \Psi_3^\dagger(x_1,\vec{k}_{1\perp}+\vec{\eta}_\perp;x_2,\vec{k}_{2\perp}-\vec{\eta}_\perp;x_3,\vec{k}_{3\perp})\Psi_3(x_1,\vec{k}_{1\perp};x_2,\vec{k}_{2\perp};x_3,\vec{k}_{3\perp}). \tag{26}$$

- Fourier transform of $D_3(x_1,x_2,\vec{\eta}_\perp)$ in $\vec{\eta}_\perp$ gives the probability of finding the quarks 1 and 2 with momentum fractions x_1 and x_2 at a relative distance $\vec{y}_\perp$ within the proton.
- In the figure is shown results for $\eta_\perp = 0$, showing a distribution centered around $x_1 = x_2 = 1/3$.

Conclusions

- The pion and proton have been studied in Minkowski space.
- For the pion we have a good agreement with experimental data for both the EM form factor and the PDF.
- Currently, we are attempting to compute tranverse momentum distributions.
- Our present model for the nucleon is much more simple.
- Spin degree freedom and a more realistic interaction should be included in the future. Also, we will attempt to go beyond the valence approximation.