Introduction to „Transverse Beam Dynamics“

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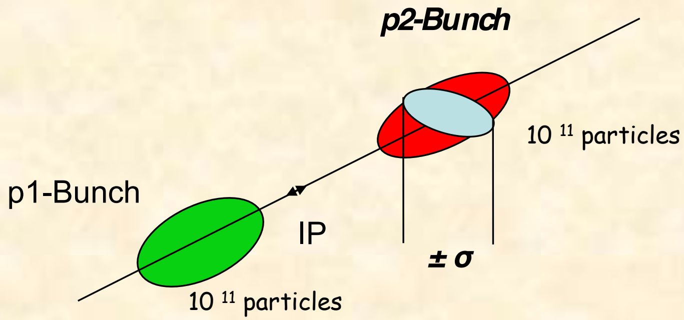
Transverse Beam Dynamics III

I) Linear Beam Optics
- *Single Particle Trajectories*
- *Magnets and Focusing Fields*
- *Tune & Orbit*

II) The State of the Art in High Energy Machines:
- *The Beam as Particle Ensemble*
- *Emittance and Beta-Function*
- *Colliding Beams & Luminosity*

III) Errors in Field and Gradient:
- *Liouville during Acceleration*
- *The $\Delta p/p \neq 0$ problem*
- *Dispersion*
- *Chromaticity*

Luminosity



$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$\sqrt{\varepsilon\beta}$$

Example: Luminosity at LHC

$$\beta^*_{x,y} = 0.55\, m$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\, m$$

$$\sigma_{x,y} = 16\ \mu m$$

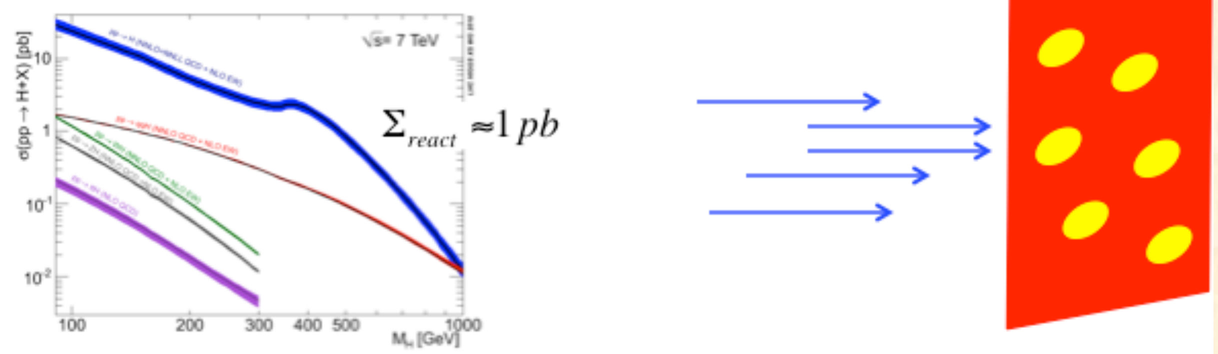
$$I_p = 584\ mA$$

$$f_0 = 11.245\ kHz$$

$$n_b = 2808$$

$$L = 1.0 * 10^{34}\ {}^{1}\!/\!{}_{cm^2 s}$$

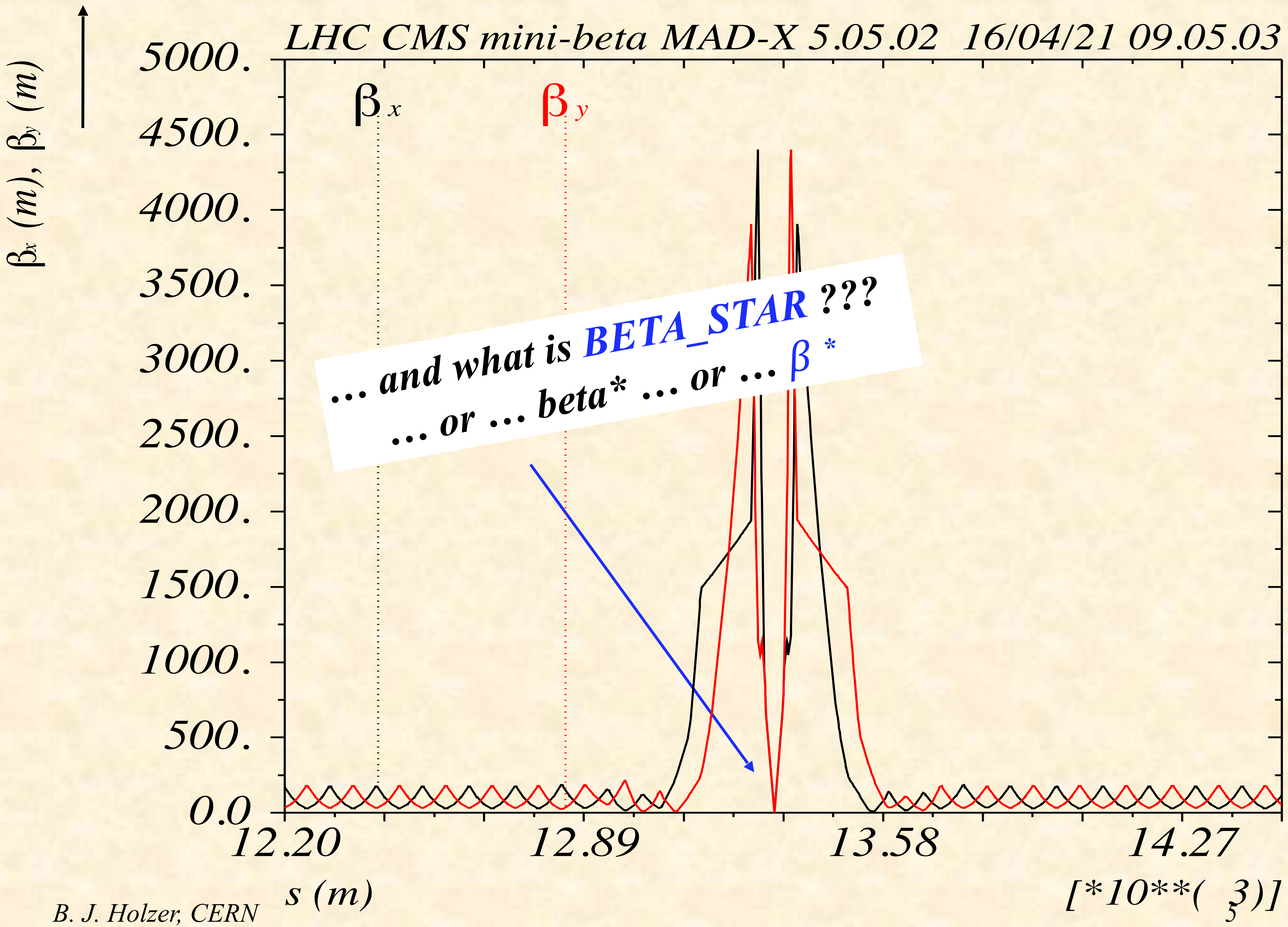
Overall cross section of the Higgs:



Make β* as small as possible to achieve μm beam size at the IP !!!

What is a μm ?

12.) Insertions

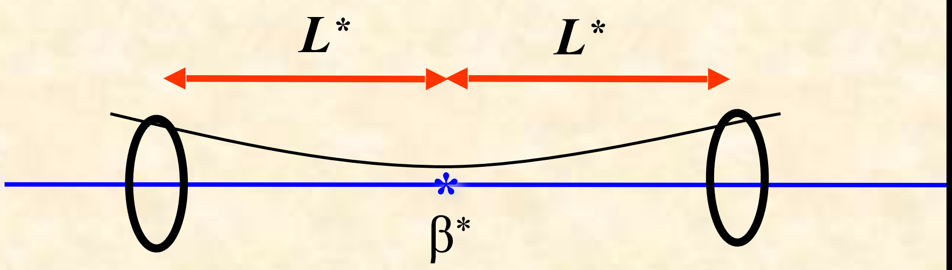


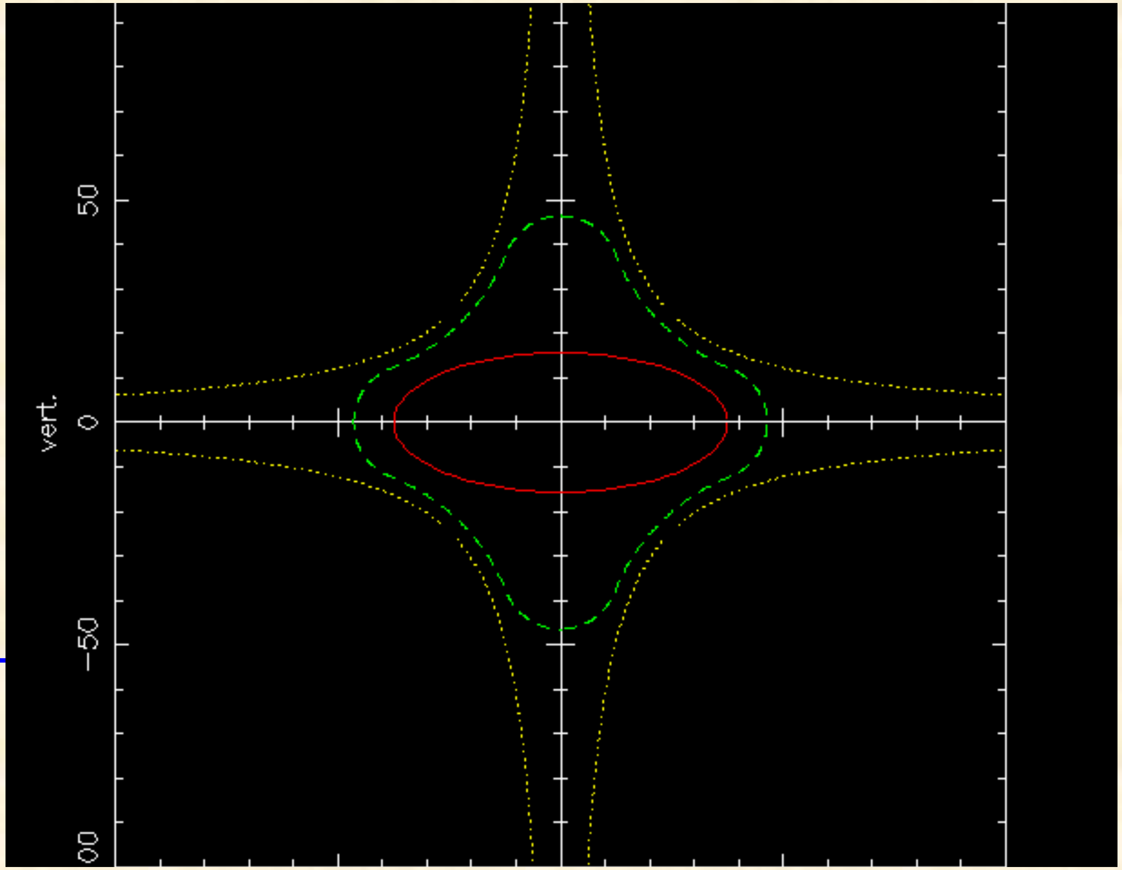
β-Function in a Drift

In a drift, without focusing, the β-function is increasing quadratically.
At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.

-> keep L* as small as possible

$$\beta(L) = \beta^* + \frac{L^2}{\beta^*}$$

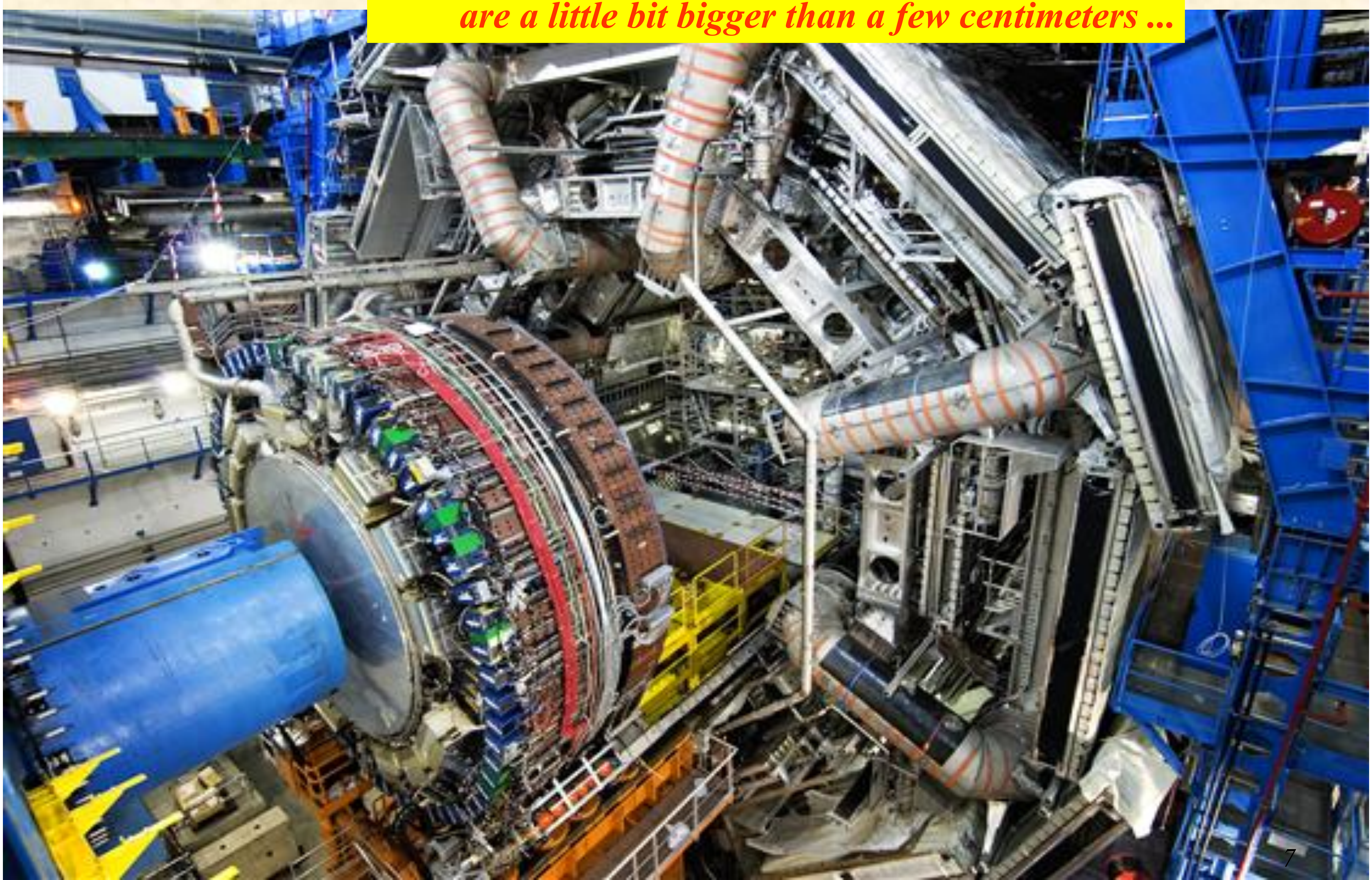




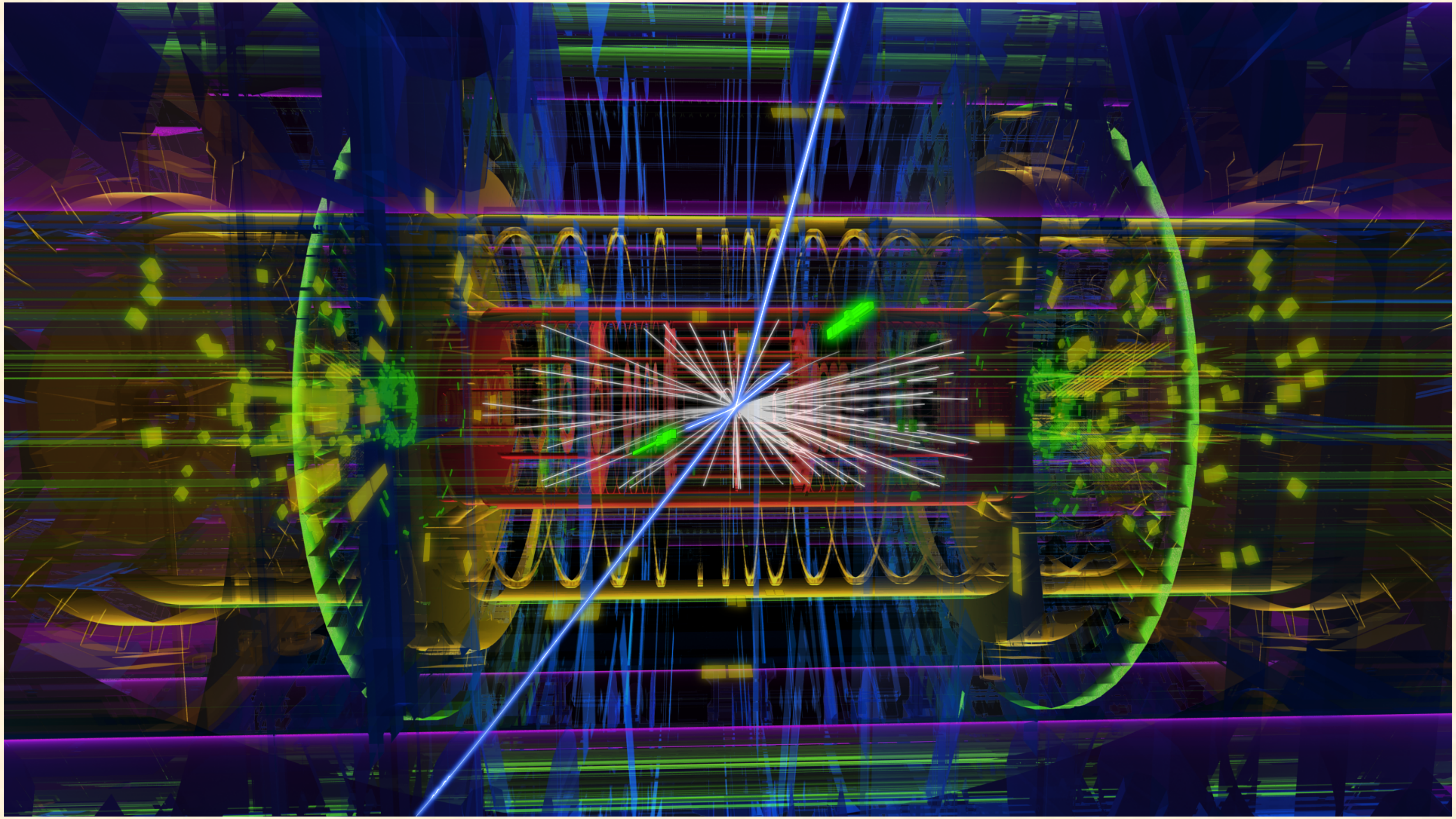
7 sigma beam size inside a mini beta quadrupole

... clearly there is an

... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

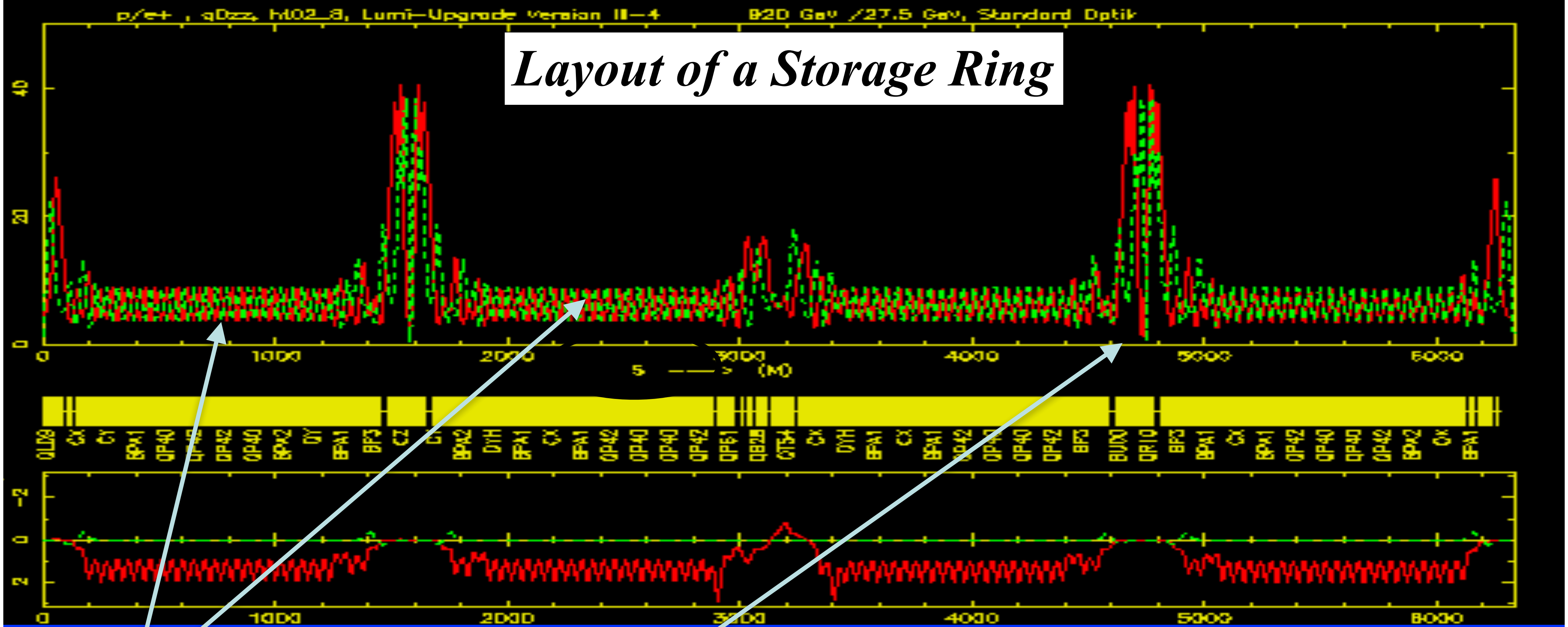


yes ... yes ... there is NO talk without it ...
The Higgs



ATLAS event display: Higgs => two electrons & two muons

Layout of a Storage Ring



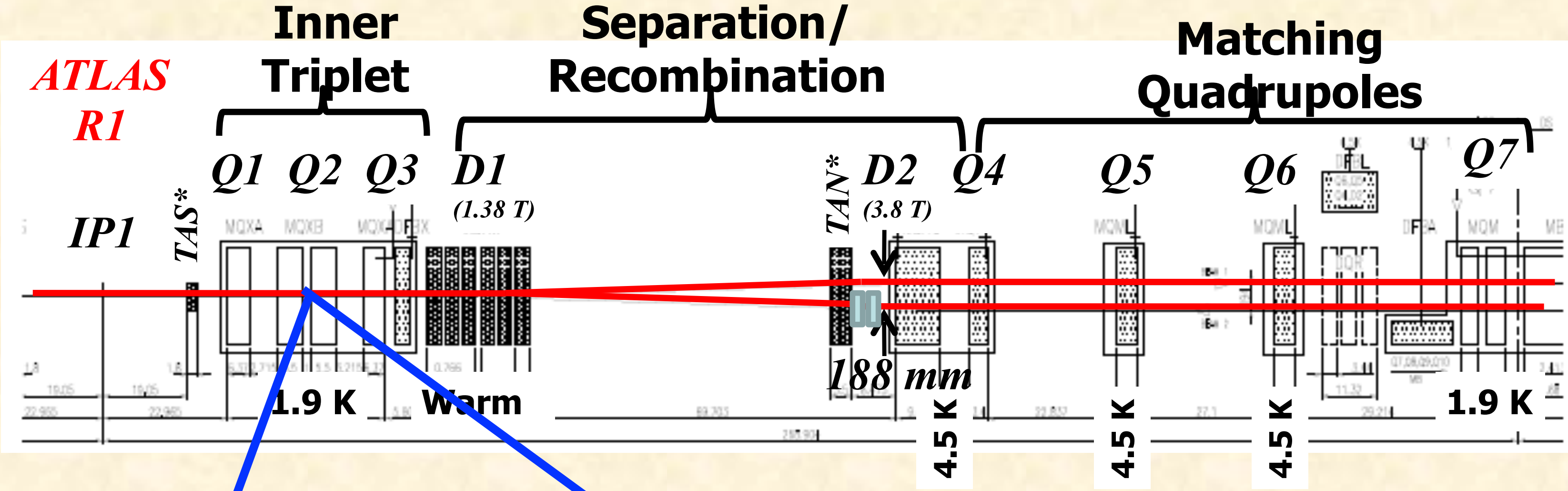
Arc: regular (periodic) magnet structure:

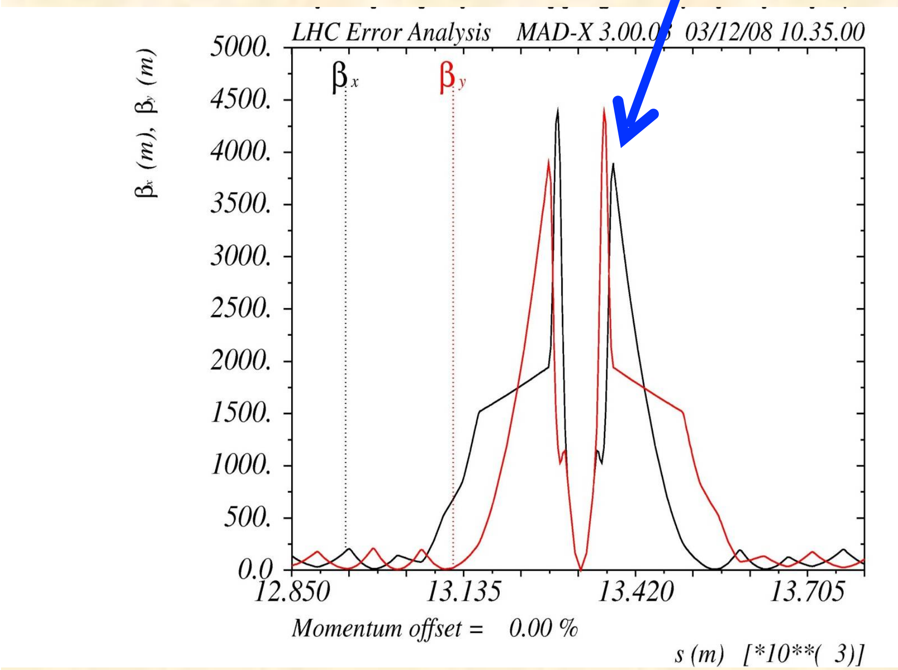
bending magnets B define the energy of the ring
main focusing & tune control, chromaticity correction,
multipoles for higher order corrections

Straight sections: *drift spaces for injection, dispersion suppressors, low beta insertions, RF cavities, etc....*

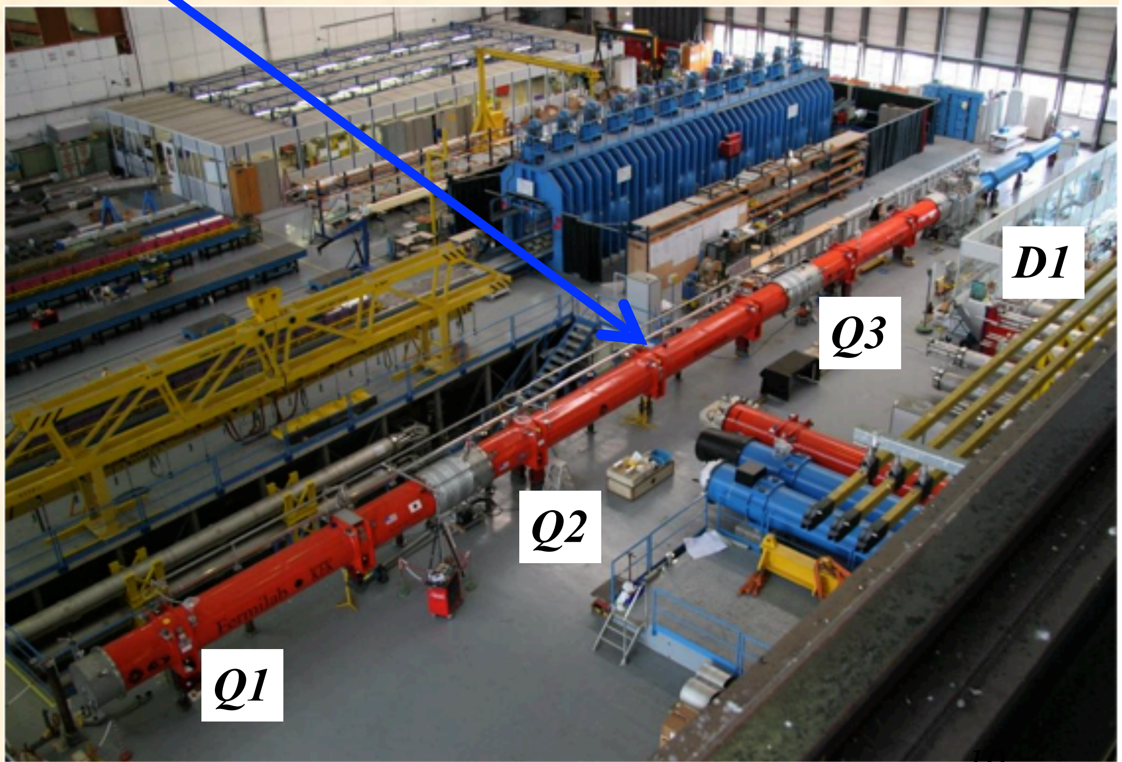
… and the high energy experiments if they cannot be avoided

The LHC Insertions

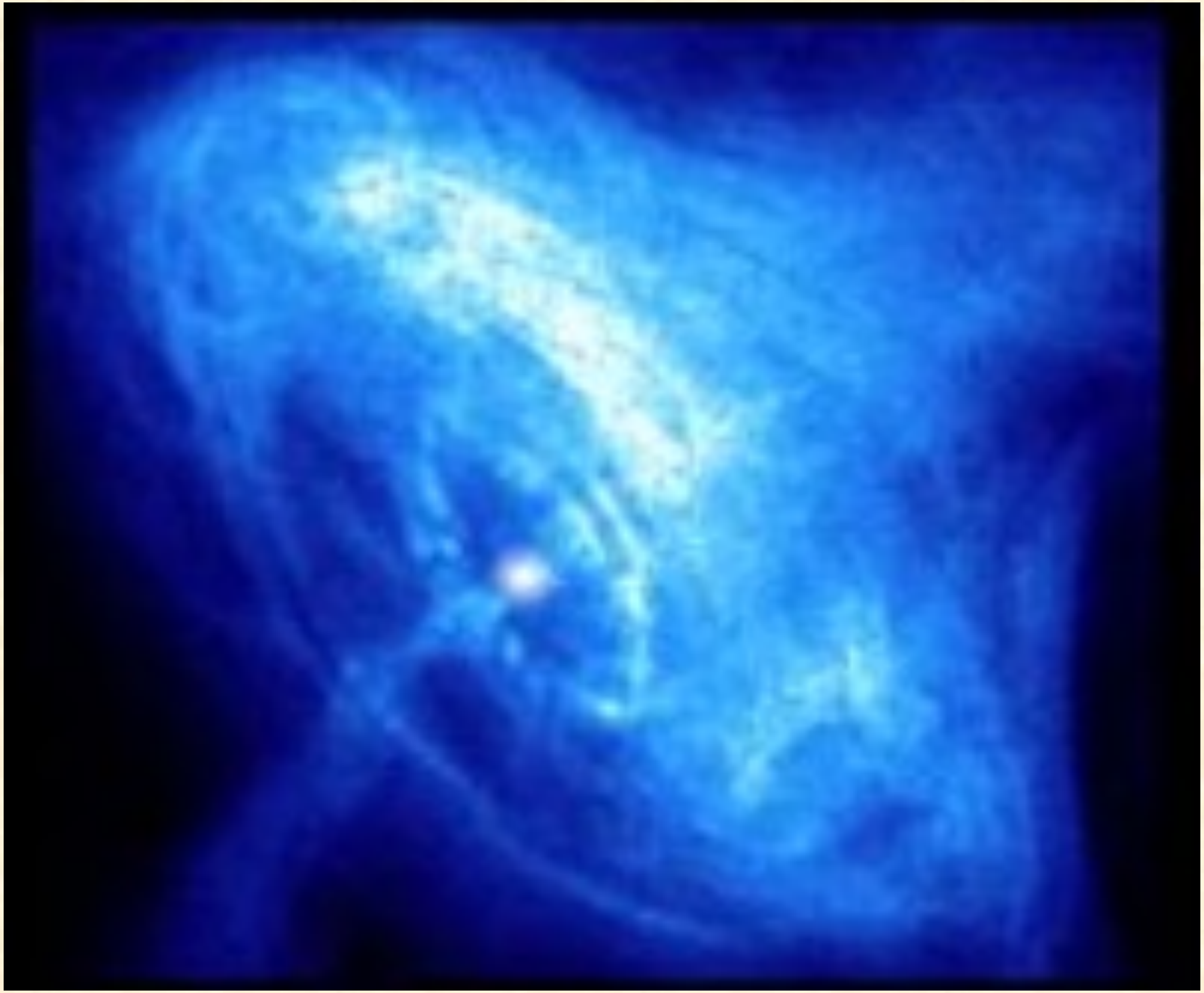




mini β optics



… finally … let´s talk about acceleration

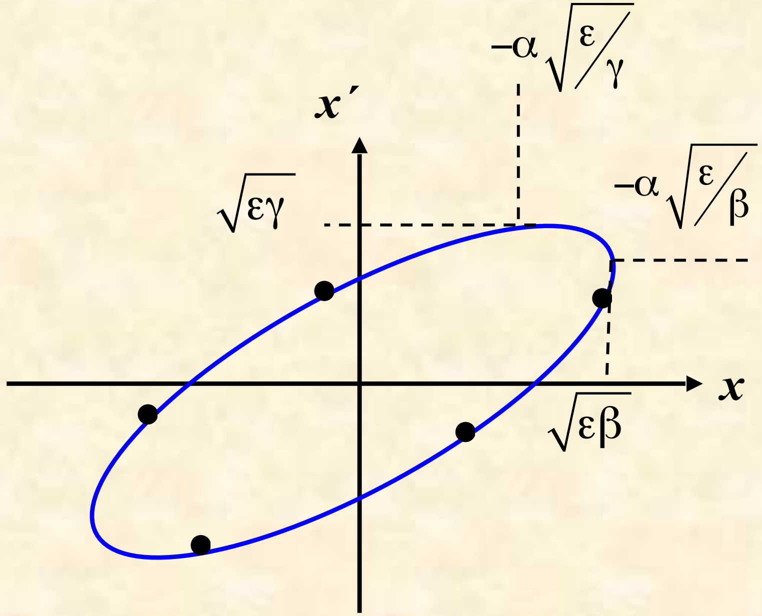


crab nebula,

burst of charged particles E = 10 ^{20} eV

14.) Liouville during Acceleration

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$



***Beam Emittance** corresponds to the area covered in the*
x, x´ Phase Space Ellipse

***Liouville:** Area in phase space is constant.*

But so sorry ... $\varepsilon \neq$ const !

Classical Mechanics:

***phase space** = diagram of the two canonical variables*

position &	***momentum***
x	p_x

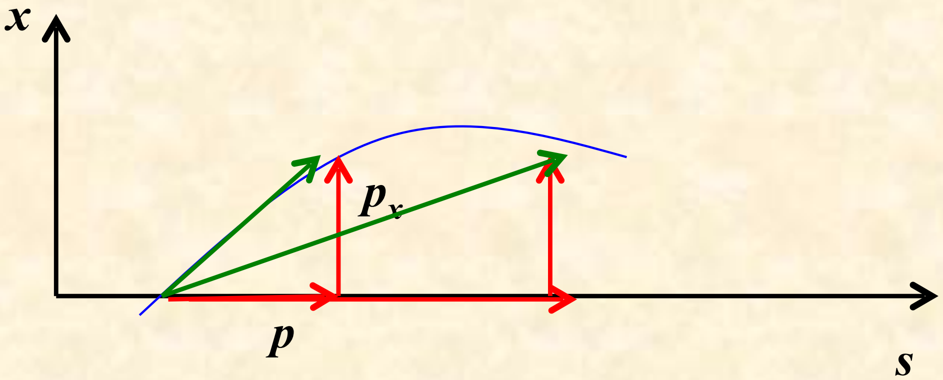
According to Hamiltonian mechanics:
phase space diagram relates the variables q and p

Liouvilles Theorem: $\int p\,dq = const$

… referring to the hor. plane $\int p_x dx = const$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\,\frac{dt}{ds} = \frac{\beta_x}{\beta} = \frac{p_x}{p}$$



$$\underbrace{\int x' dx}_{\varepsilon} = \frac{\int p_x dx}{p} \propto \frac{const}{m_0 c\ \gamma\beta}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\beta_x = \frac{v_x}{c}$$

$$\Rightarrow \quad \varepsilon = \int x' dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration $\varepsilon \sim 1/\gamma$

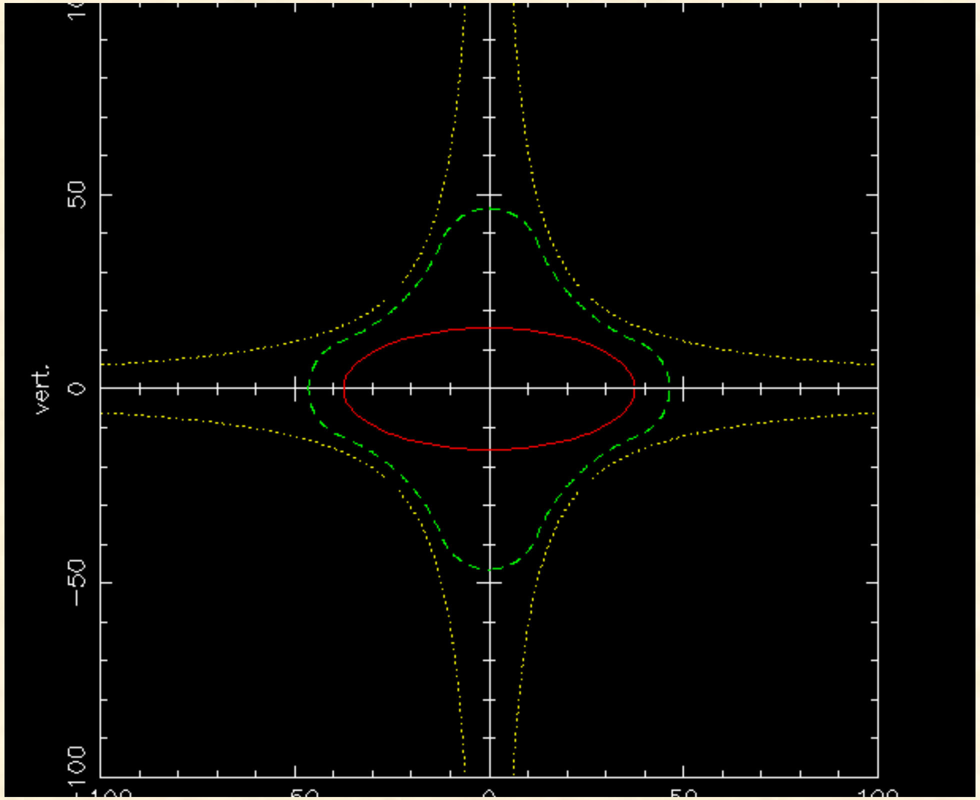
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$

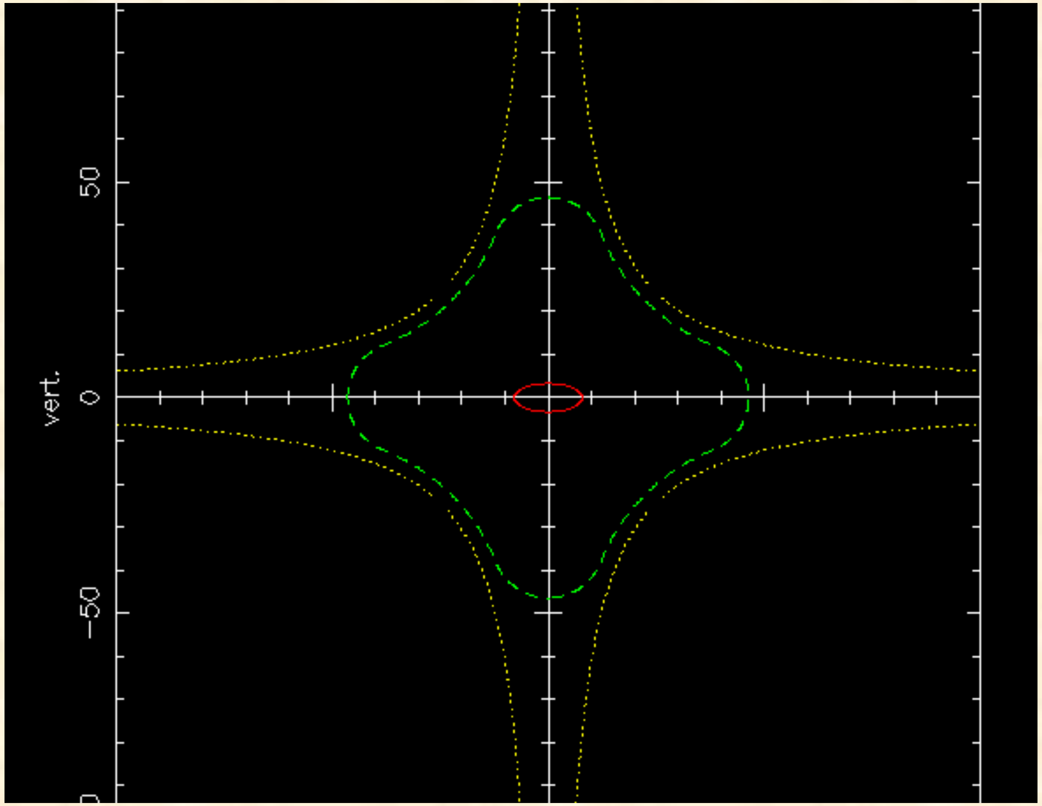
flat top energy: 920 GeV $\gamma = 980$

emittance $\varepsilon\ (40GeV) = 1.2 * 10^{-7}$

$\varepsilon\ (920GeV) = 5.1 * 10^{-9}$



7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

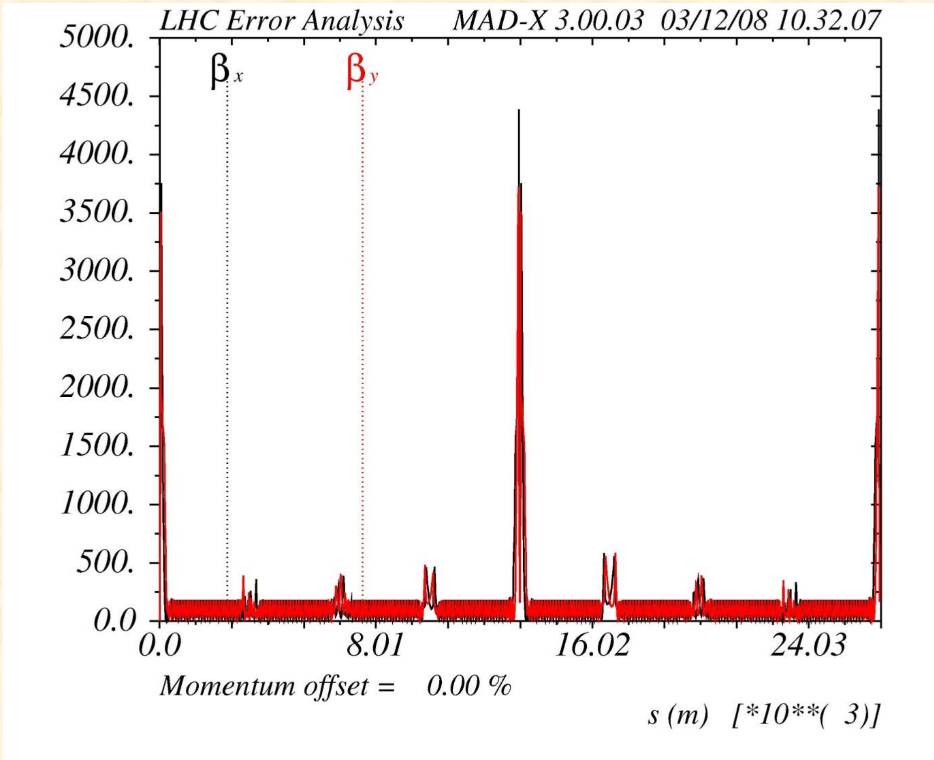
Nota bene:

1.) A proton machine … or an electron linac … needs the highest aperture at injection energy !!!
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

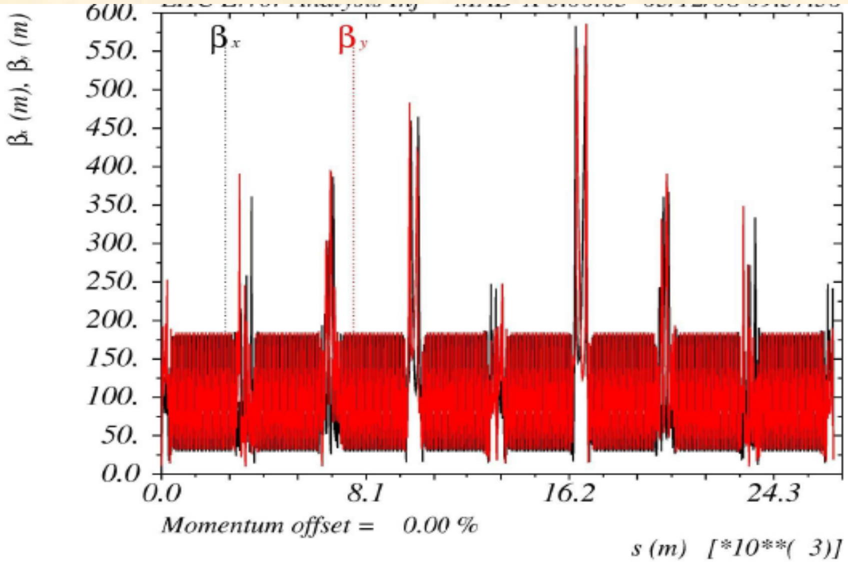
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) At lowest energy the machine will have the major aperture problems,
→ here we have to minimise $\hat{\beta}$

3.) we need different beam optics adopted to the energy:
A Mini Beta concept will only be adequate at flat top.



LHC mini beta optics at 7000 GeV



LHC injection optics at 450 GeV

Liouville during Acceleration

Protons

… shrink during acceleration

ATTENTION !!!

Electron beams *in a storage ring are determined by light emission and behave completely different.*

… they grow.

The „ not so ideal world “

15.) The „ Δp / p ≠ 0“ Problem

ideal accelerator: *all particles will see the same accelerating voltage.*

→ $\Delta p / p = 0$

„nearly ideal“ accelerator: *Cockroft Walton or van de Graaf*

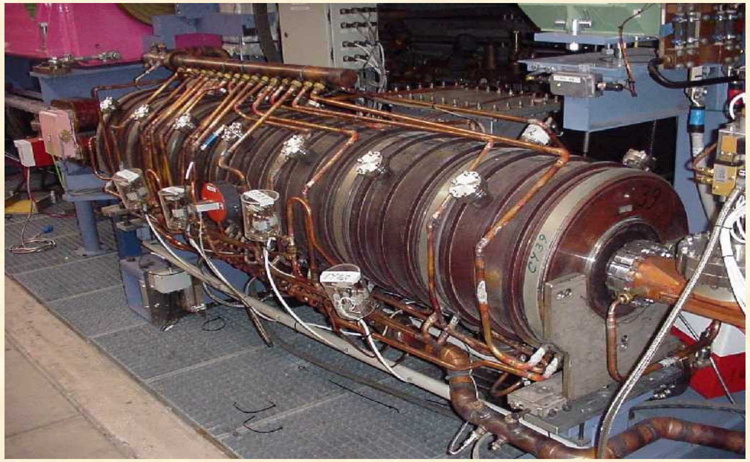
$\Delta p / p \approx 10^{-5}$

Vivitron, Straßbourg, inner structure of the acc. section

MP Tandem van de Graaf Accelerator at MPI for Nucl. Phys. Heidelberg

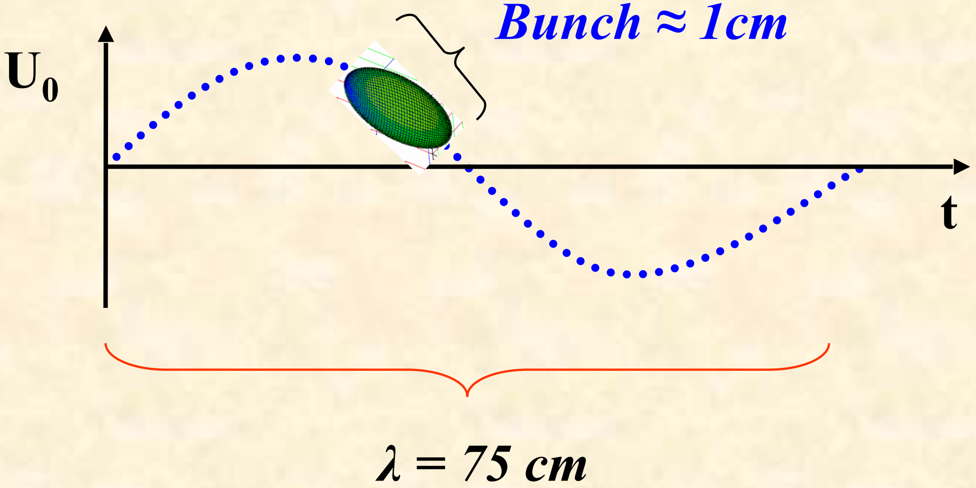
RF Acceleration-Problem:
panta rhei !!!

(Heraklit: 540-480 v. Chr.)



storage ring rf cavity

just a stupid (and a little bit wrong) example)



$$\left.\begin{array}{l} \nu = 400\,MHz \\ c = \lambda\,\nu \end{array}\right\} \lambda = 75\ cm$$

$$\sin(90^o) = 1$$

$$\sin(84^o) = 0.994$$

$$\frac{\Delta U}{U} = 6.0\ \ 10^{-3}$$

typical momentum spread of an electron bunch:

$$\frac{\Delta p}{p} \approx 1.0\ \ 10^{-3}$$

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???

Sure there are !!!

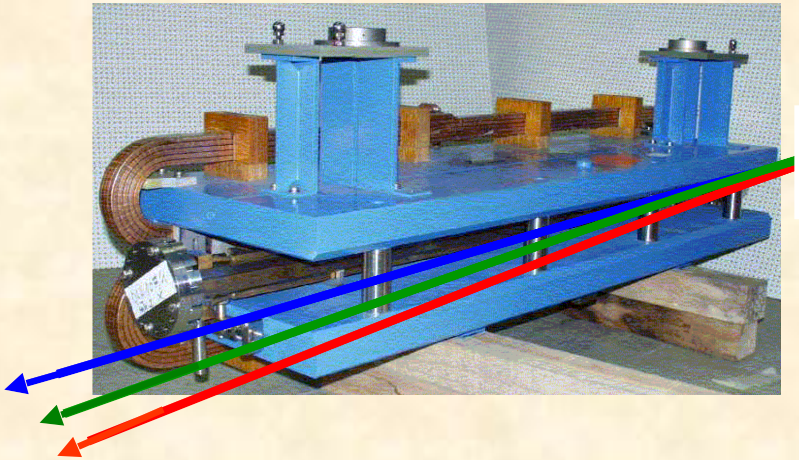
font colors due to pedagogical reasons

16.) Dispersion and Chromaticity: Magnet Errors for $\Delta p/p \neq 0$

***Influence of external fields on the beam:** prop. to magn. field & prop. zu 1/p*

dipole magnet

$$\alpha = \frac{\int B\,dl}{p/e}$$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens

$$k = \frac{g}{p/e}$$

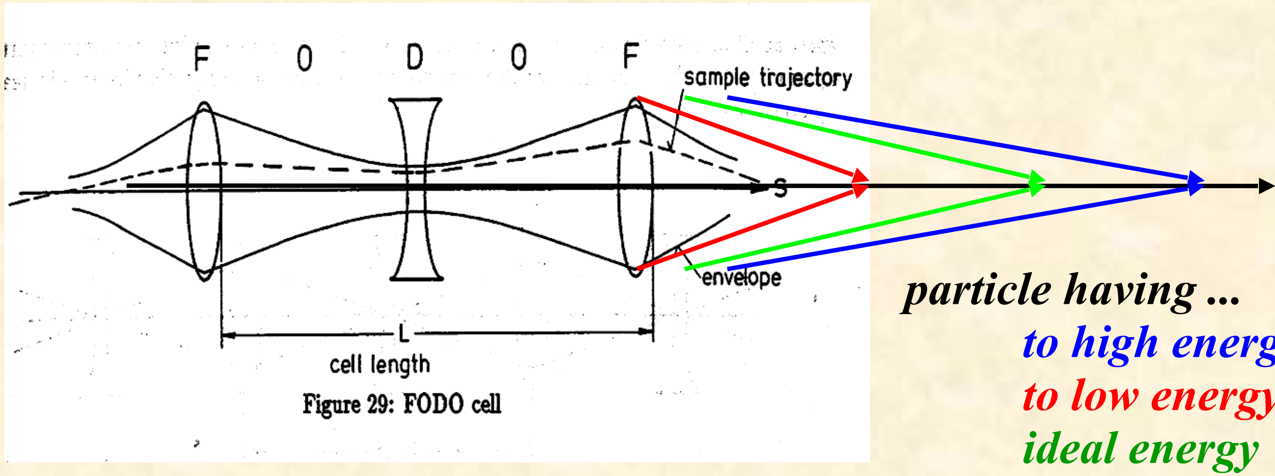


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

Dispersion

***the** typical Formula 1 effect:*

Those who are faster (have higher momentum) …

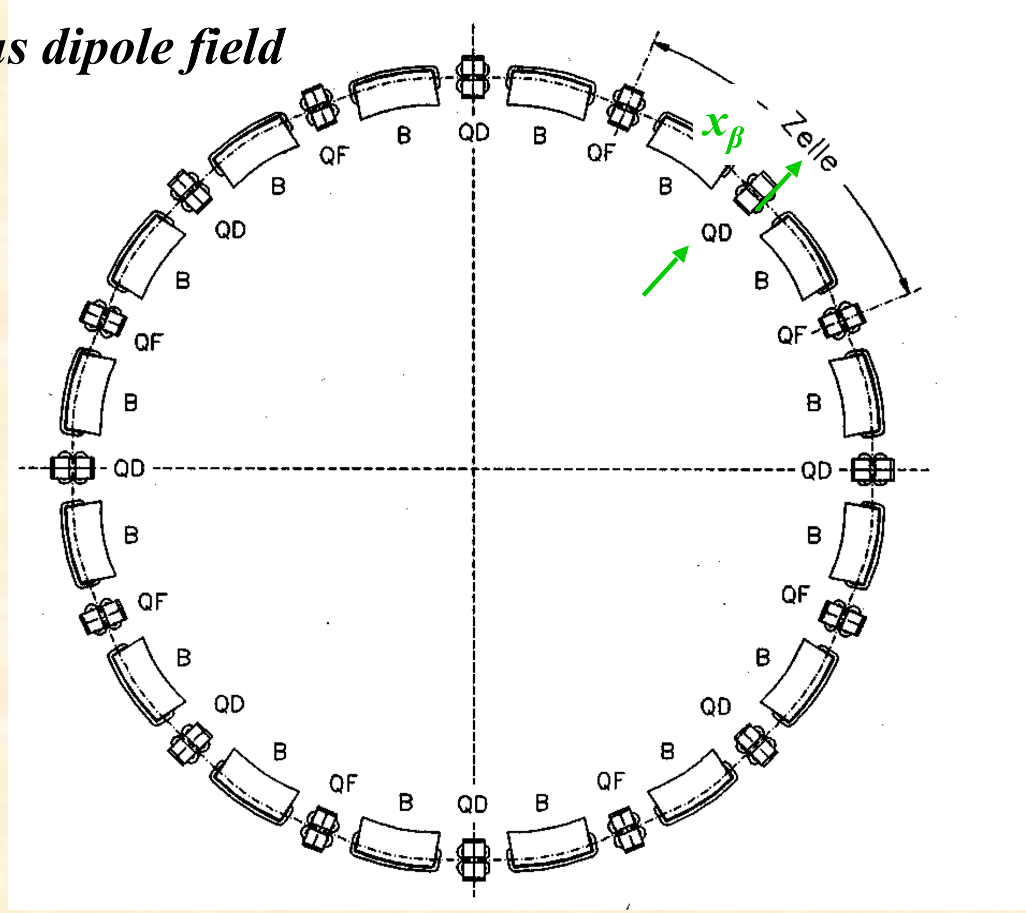
… are running on a larger circle.

BUT

they are focused nevertheless.

Dispersion

Example: homogeneous dipole field



it for Δp/p > 0

$$D(s)\cdot\frac{\Delta p}{p}$$

Matrix formalism:

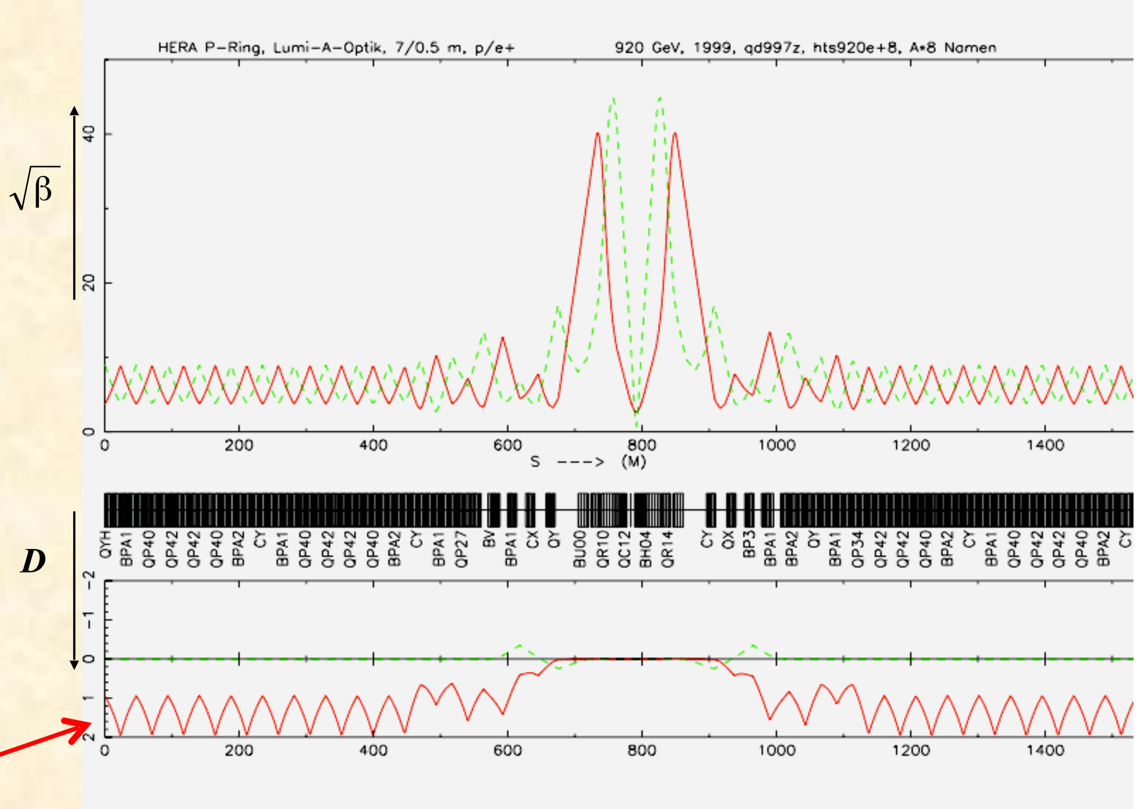
$$x(s) = x_\beta(s) + D(s)\cdot\frac{\Delta p}{p}$$

$$x(s) = C(s)\cdot x_0 + S(s)\cdot x_0' + D(s)\cdot\frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}\begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p}\begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \ldots 2\,mm$$
$$D(s) \approx 1 \ldots 2\,m$$
$$\Delta p/p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation contribution due to Dispersion ≈ beam size

→ *Dispersion must vanish at the collision point*

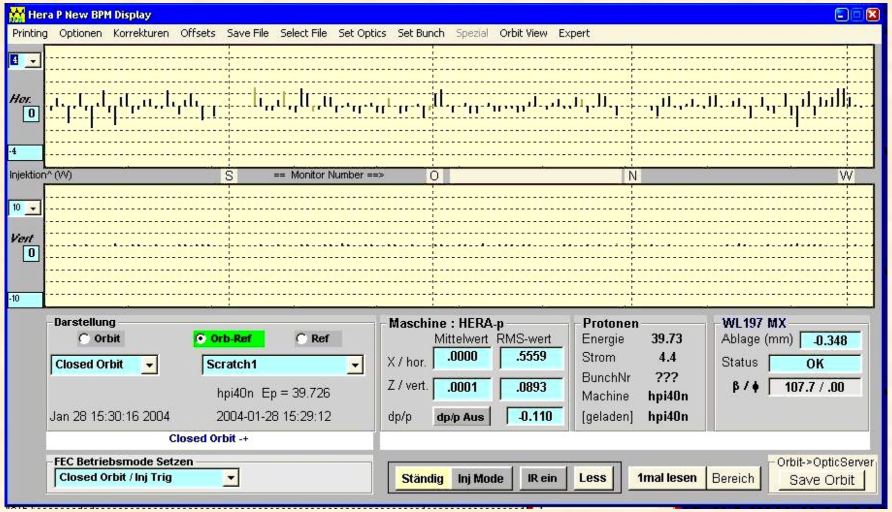
!

Calculate D, D´: ... takes a couple of sunny Sunday evenings !

$$D(s) = S(s) \int_{s_0}^{s_1} \frac{1}{\rho} C(\tilde{s})\,d\tilde{s} - C(s) \int_{s_0}^{s_1} \frac{1}{\rho} S(\tilde{s})\,d\tilde{s}$$

(proof see CAS proc.)

Dispersion is visible

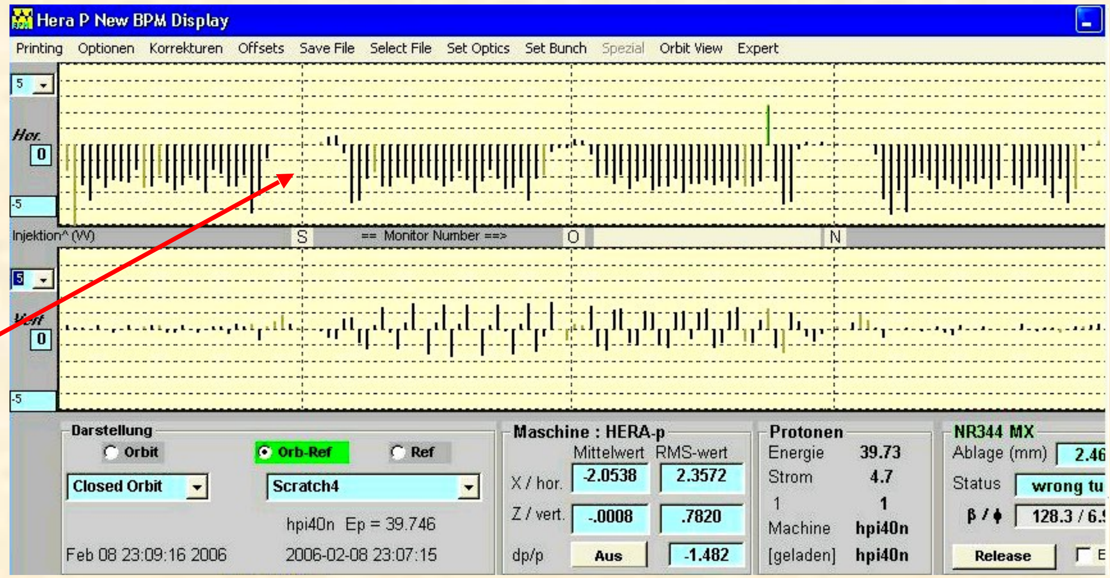


HERA Standard Orbit

HERA Dispersion Orbit

dedicated energy change of the stored beam

→ closed orbit is moved to a dispersions trajectory



$$x_D = D(s) * \frac{\Delta p}{p}$$

Attention: at the Interaction Points we require D=D´= 0

17.) Chromaticity: A Quadrupole Error for $\Delta p/p \neq 0$

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

Remember the normalisation of the external fields:

focusing lens $$k = \frac{g}{p/e}$$

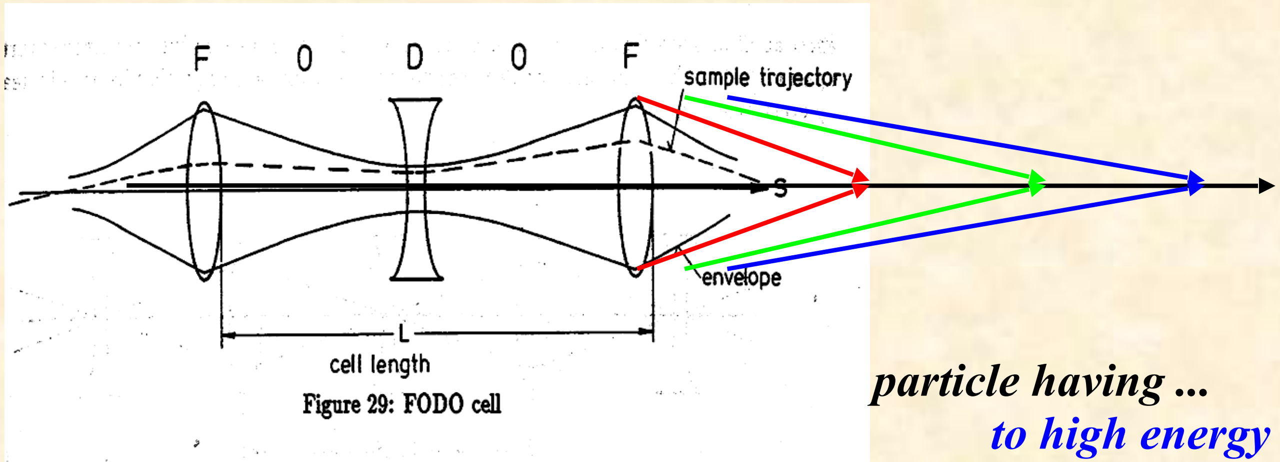


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

a particle that has a higher momentum feels a weaker quadrupole gradient and has a lower tune.

$$k \rightarrow k - \Delta k \qquad Q \rightarrow Q - \Delta Q$$

definition of chromaticity: $$\Delta Q = Q' \frac{\Delta p}{p}$$

… what is wrong about Chromaticity:

Every individual particle has an individual momentum and thus an individual tune.

Q' is a number indicating the size of the tune spot in the working diagram,
Q' is always created if the beam is focussed
→ *it is determined by the focusing strength k of all quadrupoles*

$$\Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0} k_0 \beta(s) ds \qquad Q' = -\frac{1}{4\pi}\oint k(s)\beta(s) ds$$

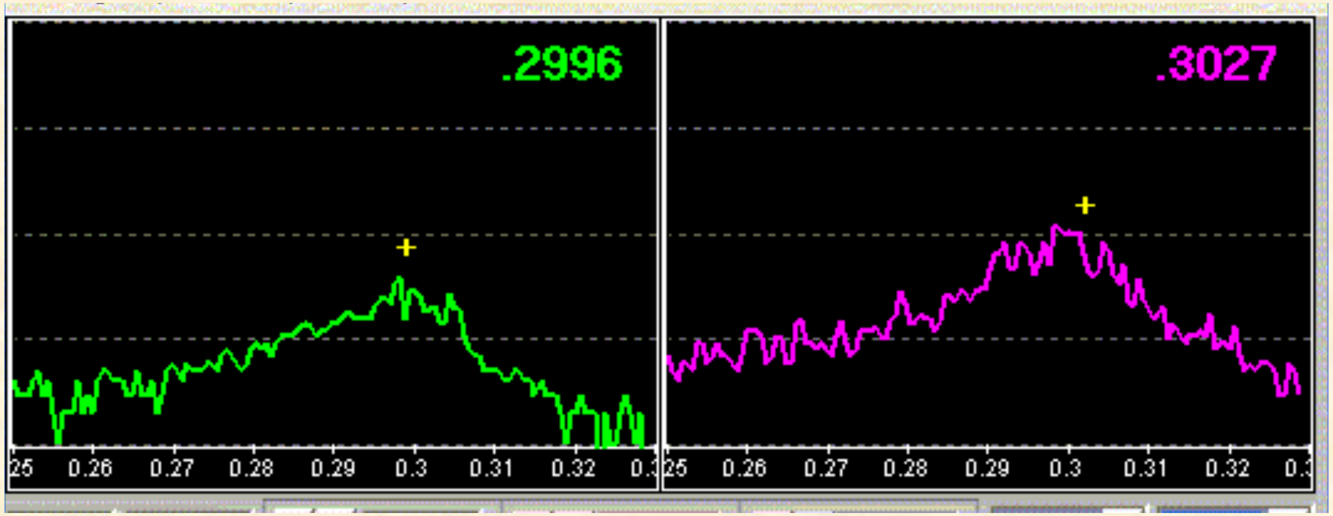
k = quadrupole strength
β = betafunction indicates the beam size … and even more the sensitivity of the beam to external fields

Example: LHC

$$\left.\begin{array}{l} Q' = 250 \\ \Delta p/p = \pm 0.2 \cdot 10^{-3} \\ \Delta Q = 0.256 \ldots 0.36 \end{array}\right\}$$

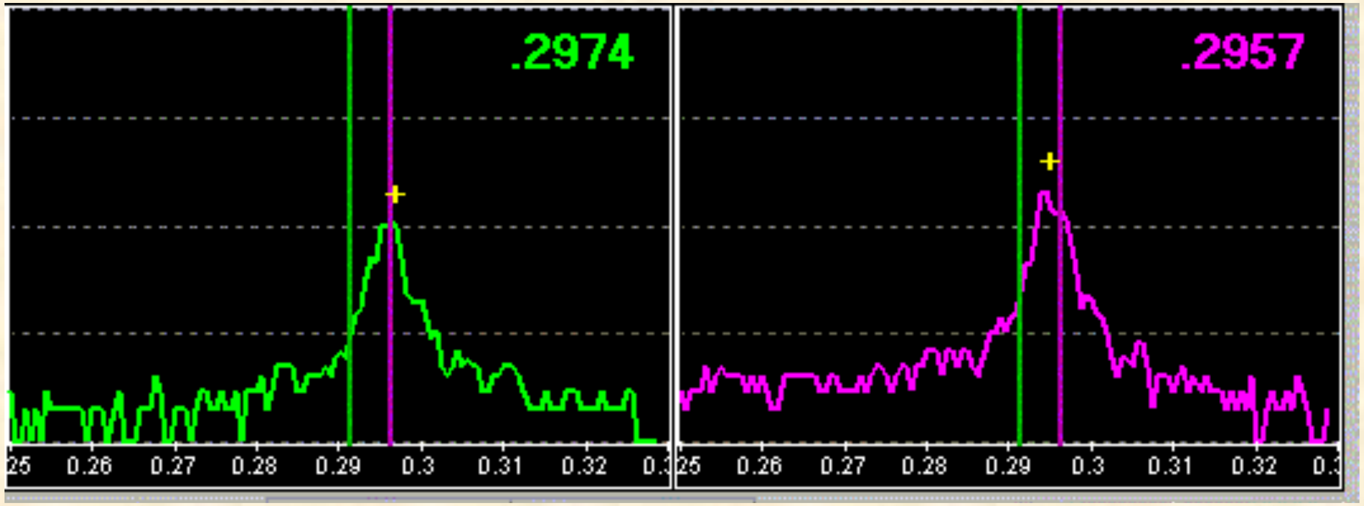
→Some particles get very close to resonances and are lost

in other words: the tune is not a point it is a pancake



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)

Ideal situation: cromaticity well corrected, (Q' ≈ 1)

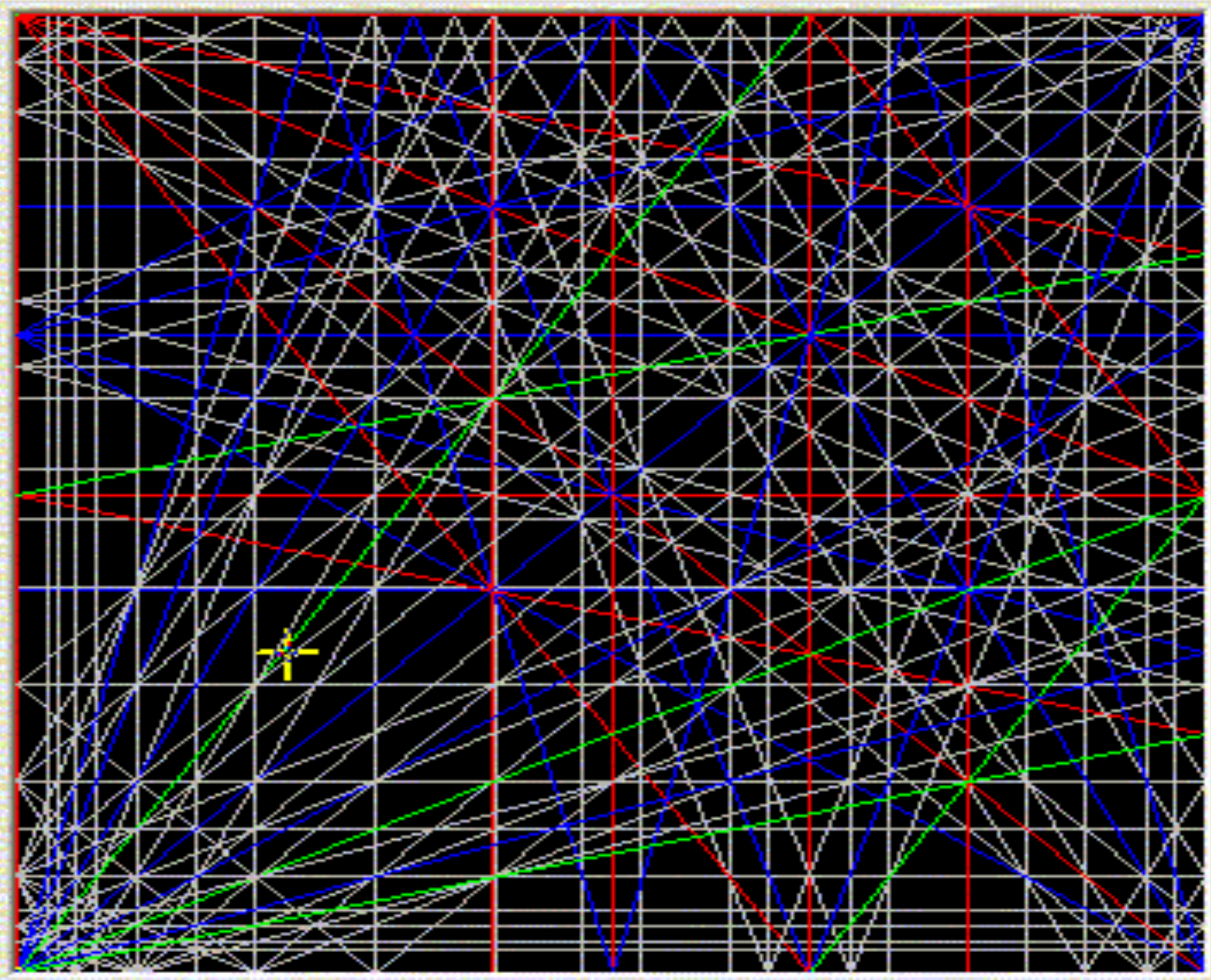


Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

Tune diagram up to 3rd order

… and up to 7th order



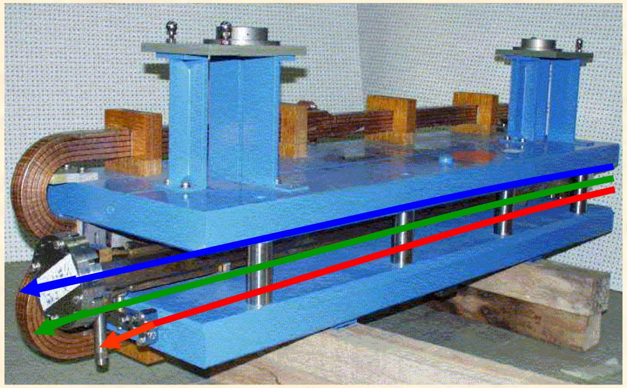
Homework for the operateurs:
find a nice place for the tune
where against all probability
the beam will survive

Correction of Chromaticity:

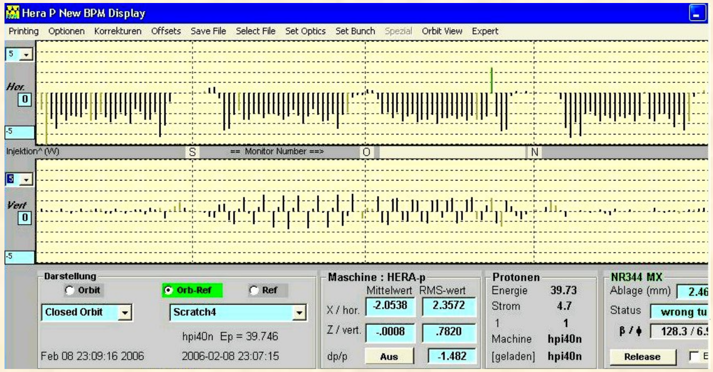
Need: additional quadrupole strength for each momentum deviation Δp/p

1.) sort the particles acording to their momentum

$$x_D(s) = D(s)\frac{\Delta p}{p}$$



… using the dispersion function



2.) apply a magnetic field that rises quadratically with x (sextupole field)

$$\left.\begin{aligned} B_x &= \tilde{g}xy \\ B_y &= \frac{1}{2}\tilde{g}(x^2 - y^2) \end{aligned}\right\} \qquad \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \tilde{g}x$$

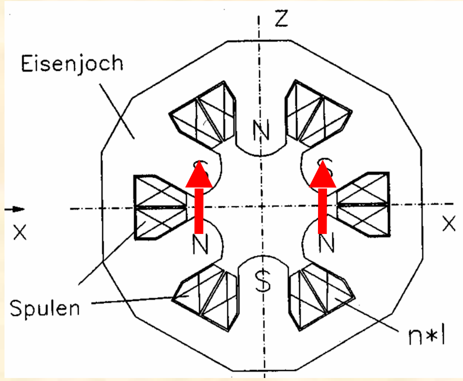
—> amplitude dependent gradient

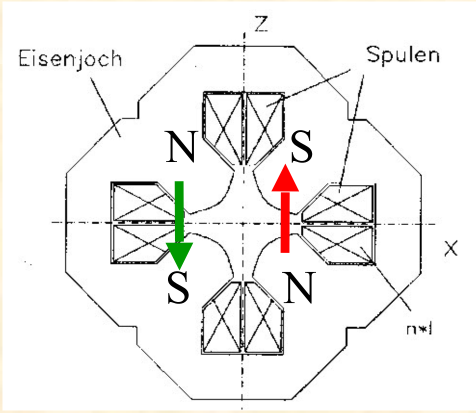
Correction of Q':

k_1 normalised quadrupole strength

k_2 normalised sextupole strength

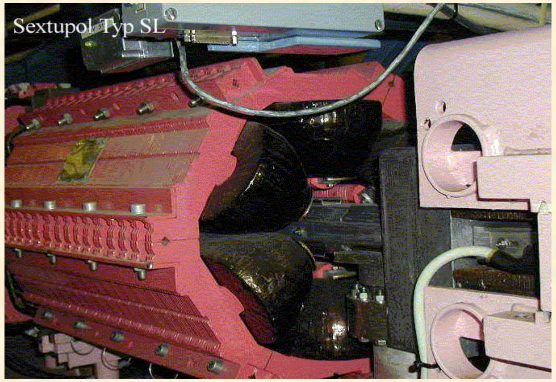
Sextupole Magnets:





$$k_1(sext) = \frac{\tilde{g}x}{p/e} = k_2 * x$$

$$= k_2 * D\frac{\Delta p}{p}$$



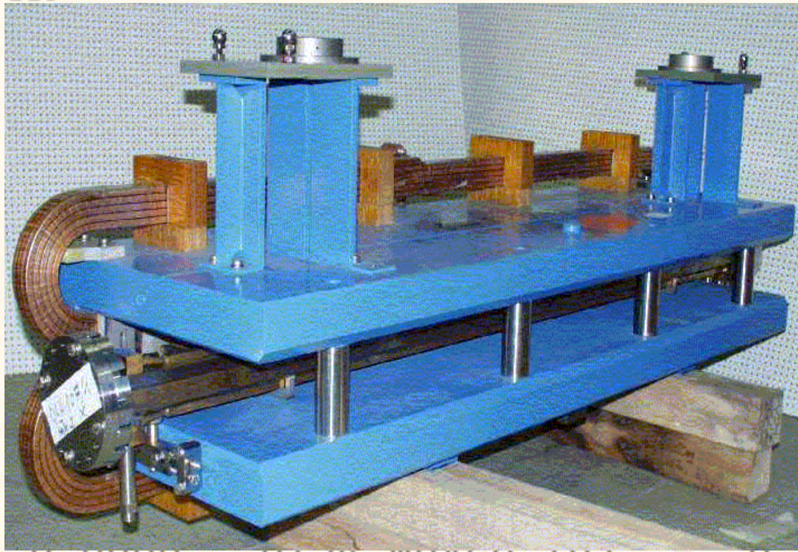
Combined effect of „natural chromaticity“ and Sextupole Magnets:

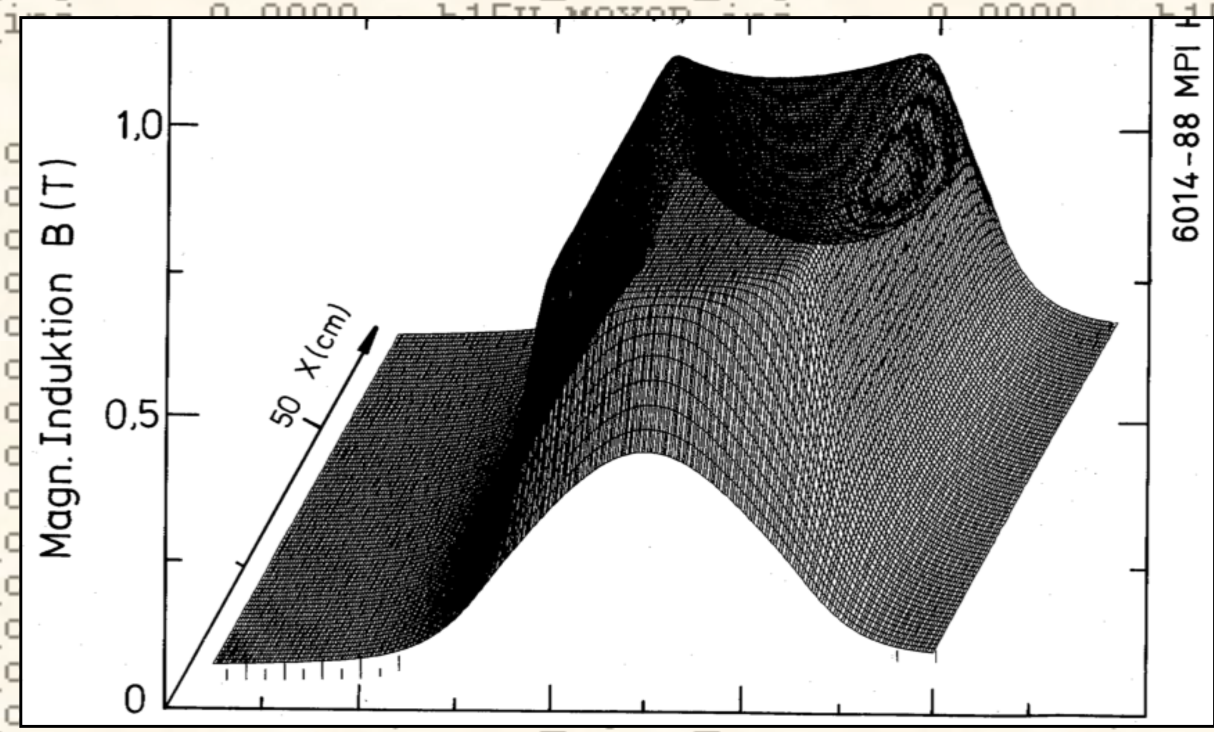
$$Q' = -\frac{1}{4\pi}\{\int k_1(s)\beta(s)\ ds + \int k_2(s)D(s)\beta(s)\ ds\}$$

You only should not forget to correct Q‘ in both planes ...
and take into account the contribution from quadrupoles of both polarities.

A word of caution: keep non-linear terms in your storage ring low.

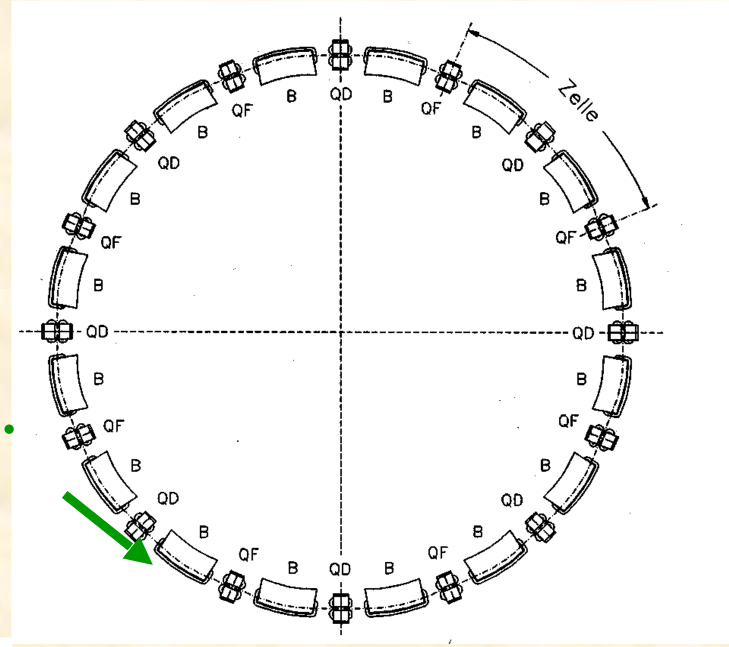
$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} \left(b_n + ia_n\right)\left(\frac{x+iy}{r_0}\right)^{n-1}$$





Clearly there is another problem ...

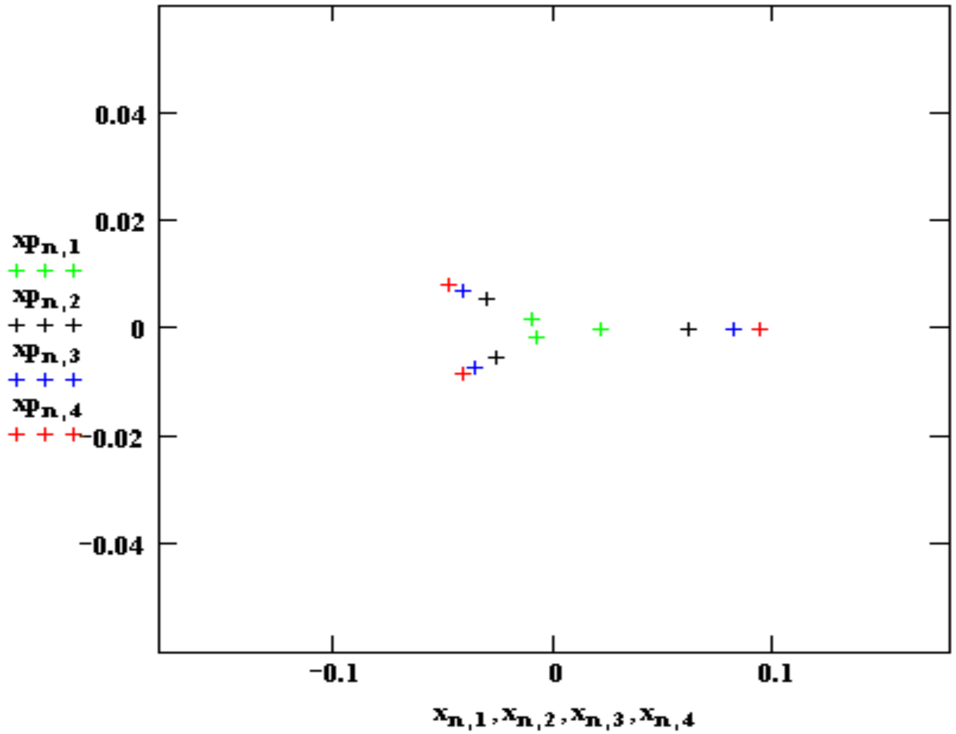
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s“ in the ring – the single partilce amplitude x and the angle x´ ... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



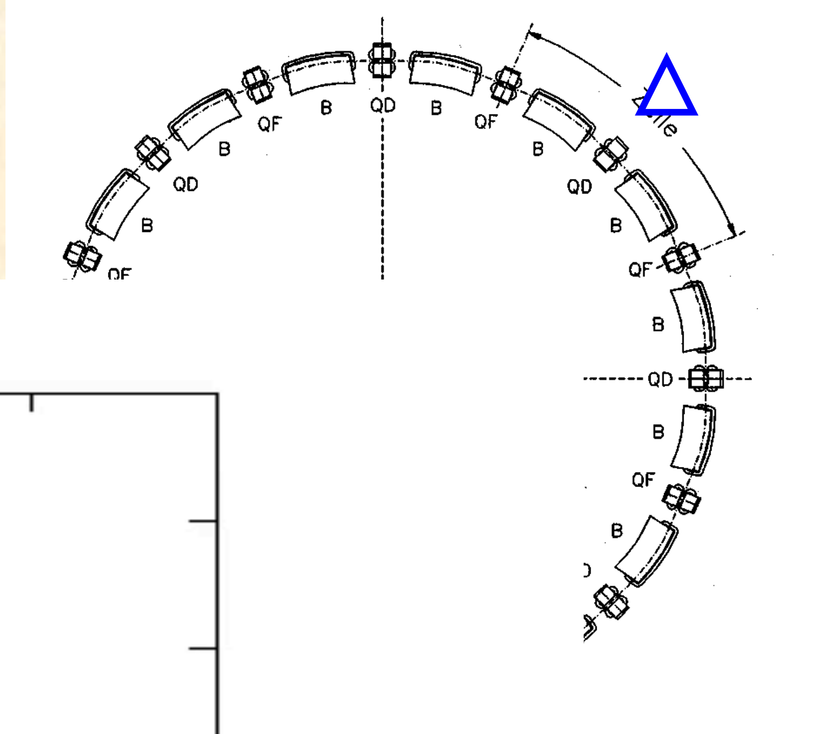
A beam of 4 particles

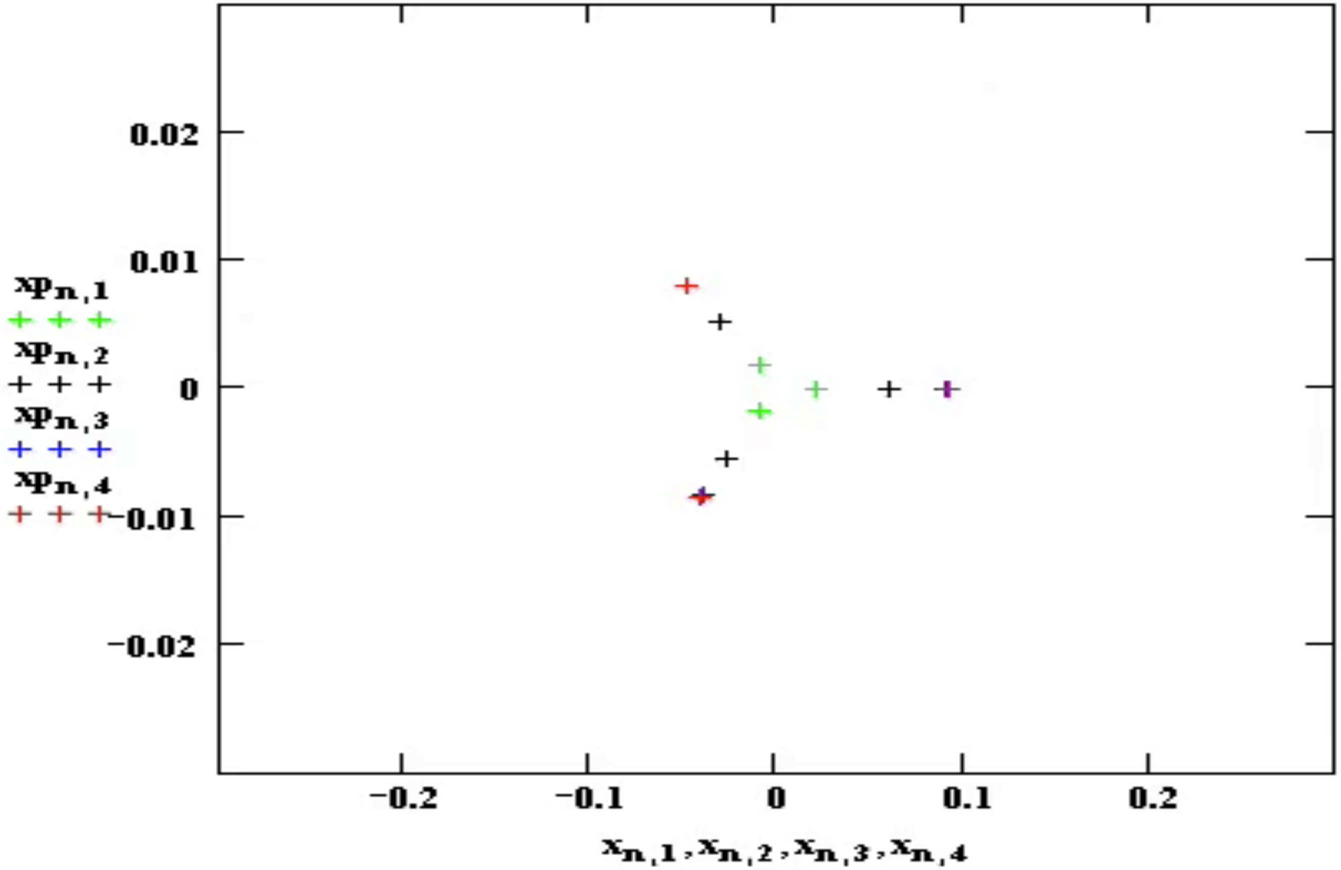
– each having a slightly different emittance:

Installation of a weak (!!!) sextupole magnet

The good news: sextupole fields in accelerators cannot be treated analytically anymore.

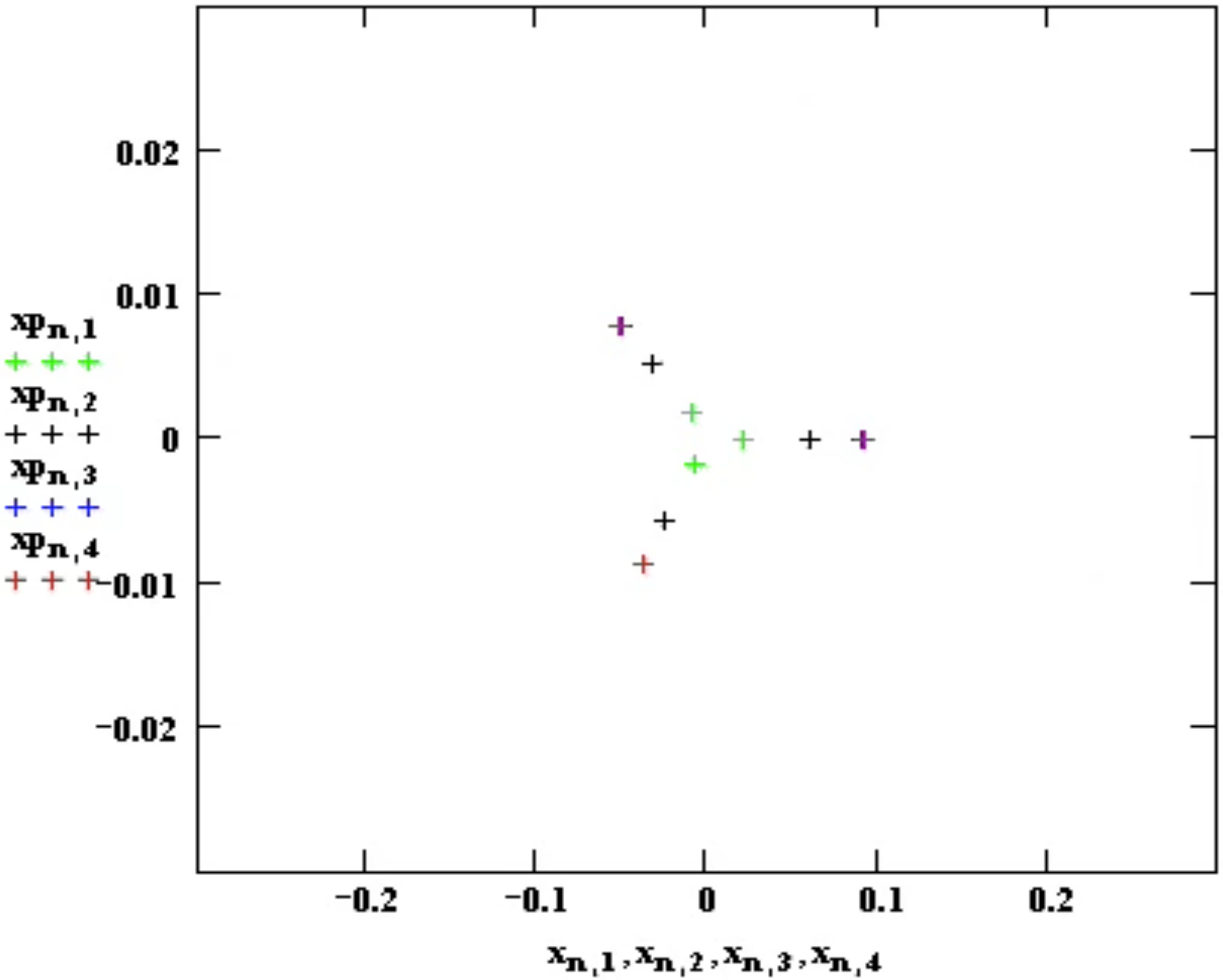
→ no equatiuons; instead: Computer simulation „ particle tracking "

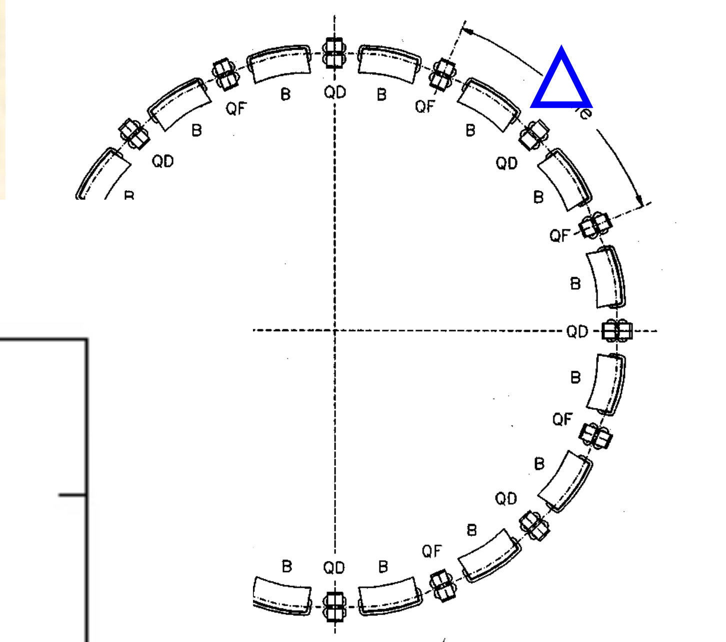




Effect of a strong (!!!) Sextupole …

→ ***Catastrophy !***





„dynamic aperture“

The Mini-Beta scheme ...

... focusses strongly the beams to achieve smallest possible beam sizes at the IP. The obtained small beta function at the IP is called β*.
Don't forget the cat.

Beam dimension during acceleration: A proton beam shrinks during acceleration in both ytransverse dimensions. We call it unfortunately „adiabatic shrinking".
Nota bene: An electron beam in a ring is *growing* with energy !!

Dispersion ...

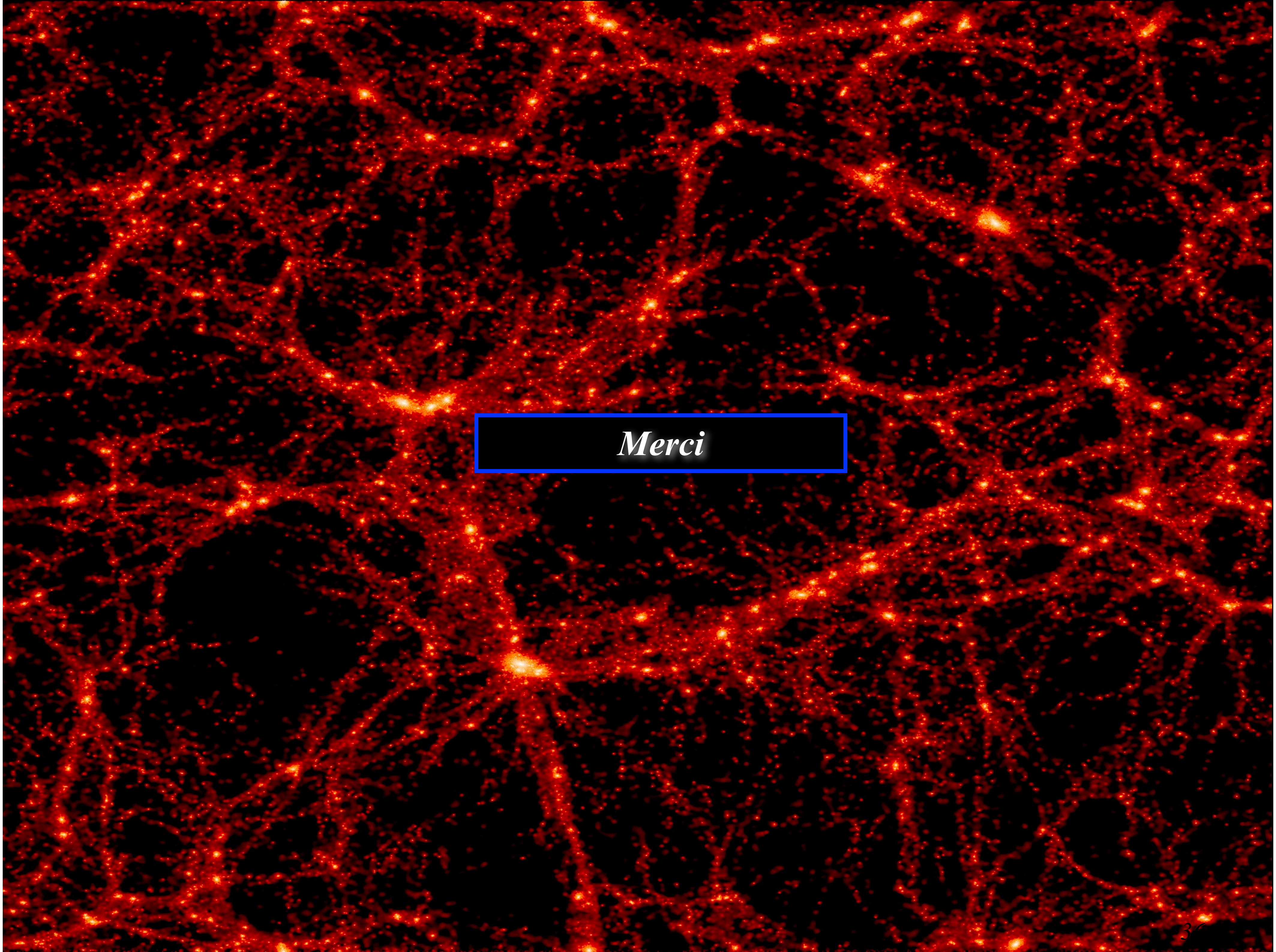
... is the particle orbit for a *given* momentum difference.

Chromaticity ...

... is a focusing problem. Different momenta lead to different tunes
→ attention ... resonances !!

Sextupoles ...

have non-linear fields and are used to compensate chromaticity. However we have to be careful: Strong non-linear fields can lead to particle losses (dynamic aperture)


Merci