Introduction to „Transverse Beam Dynamics“

Bernhard Holzer

What we will do instead …

… introduce some "funny" keywords that you always wanted to understand and never really asked for.

trajectory / closed orbit / tune / resonances / chromaticity & dispersion
Higgs / structure of matter / beam emittance / adiabatic shrinking
beam size / beta function, focusing matrix / lattice cell
mini-beta insertion / "L-star" and "beta-star" / dynamic aperture

… and ask some "interesting" questions … LOL (MDR for the French)

like … why do the particles not follow gravity and just drop down to the bottom of the vacuum chamber (… or do they do so ?)

Accelerator without transverse beam dynamics
Tandem van de Graaf

Electrostatic Linear Acceleration System
Injection —> Acceleration —> target (i.e. dump)
- No RF,
- no quadrupoles

Example for such a „steam engine“:
12 MV-Tandem van de Graaff
Accelerator at MPI Heidelberg

Gretchen Frage (J.W. Goethe, Faust)

Fallen die Dinger eigentlich runter ?

Do they actually drop ?

Gretchen Frage (J.W. Goethe, Faust)

Fallen die Dinger eigentlich runter ?

Antwort: JA !!

Gretchen Frage (J.W. Goethe, Faust)

Do they actually drop ?

Yes, they do !!

$$l_{VdG} = 30\,m$$

$$v \approx 10\%\, c \approx 3 * 10^{7}\, m/s$$

$$\Delta t = 1\,\mu s$$

Free Fall in Vacuum:

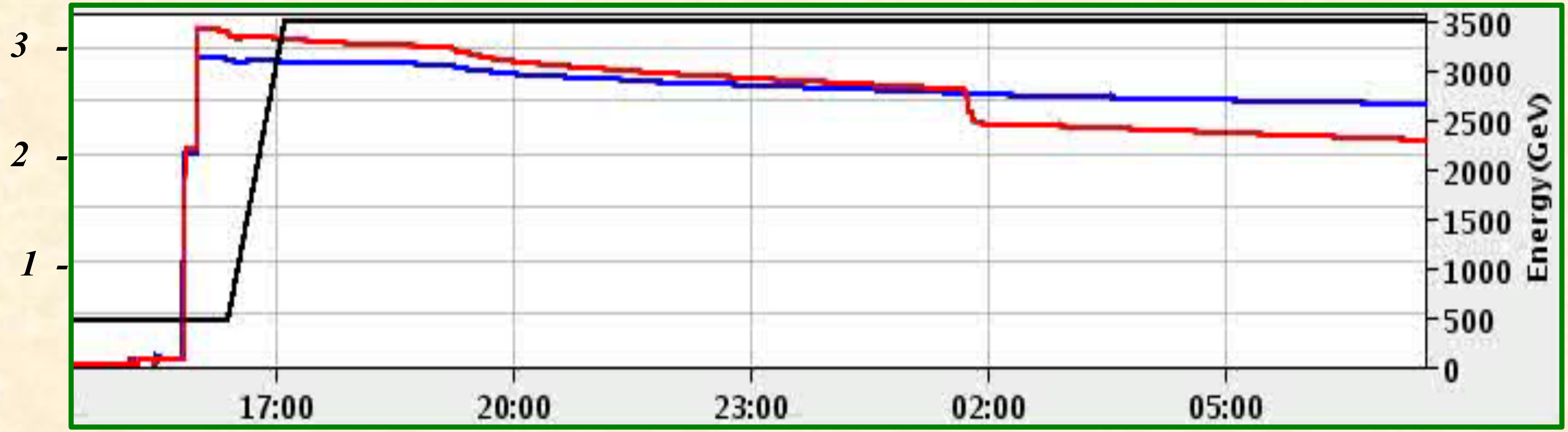
$$\begin{aligned} s &= \frac{1}{2} g t^2 \\ &= \frac{1}{2} 10 \frac{m}{s^2} * (1\,\mu s)^2 \\ &= 5 * 10^{-12}\, m = 5\, pm \end{aligned}$$

Transverse Beam Dynamics I

Linear Beam Optics / Single Particle Trajectories / Magnets and Focusing Fields / Tune & Orbit

Luminosity Run of a typical storage ring:

intensity (10^{11})



→ ***guide the particles** on a well defined orbit („design orbit")*

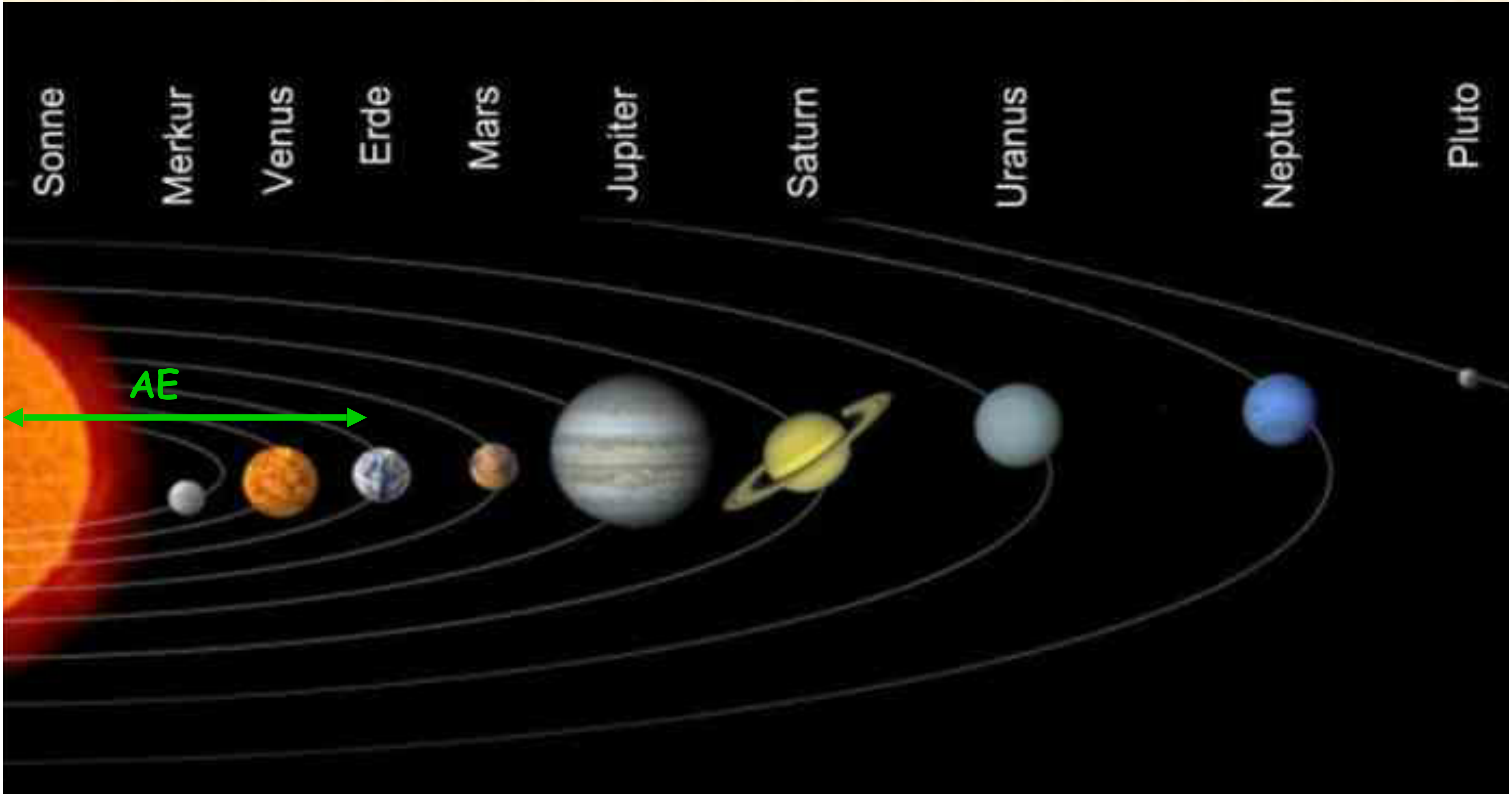
→ ***focus the particles** to keep each single particle trajectory within the vacuum chamber of the storage ring, i.e. close to the design orbit.*

Largest storage ring: The Solar System

astronomical unit: average distance earth-sun

$1AE \approx 150 * 10^6$ km

Distance Pluto-Sun ≈ 40 AE



1.) Introduction and Basic Ideas

„ … in the end and after all it should be a kind of circular machine"
→ need transverse deflecting force

Lorentz force $\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3 * 10^8 \, m/s$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

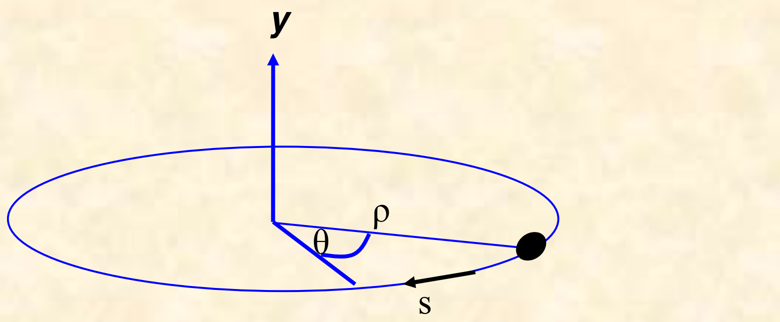
$$F = q * \underbrace{300 \frac{MV}{m}}_{E}$$

equivalent electrical field: E

Technical limit for electrical fields:

$$E \leq 1 \frac{MV}{m}$$

1.) Introduction and Basic Ideas



circular coordinate system

Dipole Magnets define an ideal circular orbit

condition for circular orbit:

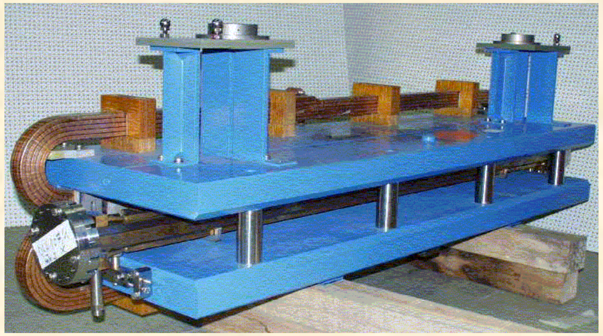
Lorentz force = centrifugal force

$$F_L = e\,v\,B \longleftrightarrow F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\,\cancel{v}\,B$$

$$\boxed{\frac{p}{e} = B\,\rho}$$

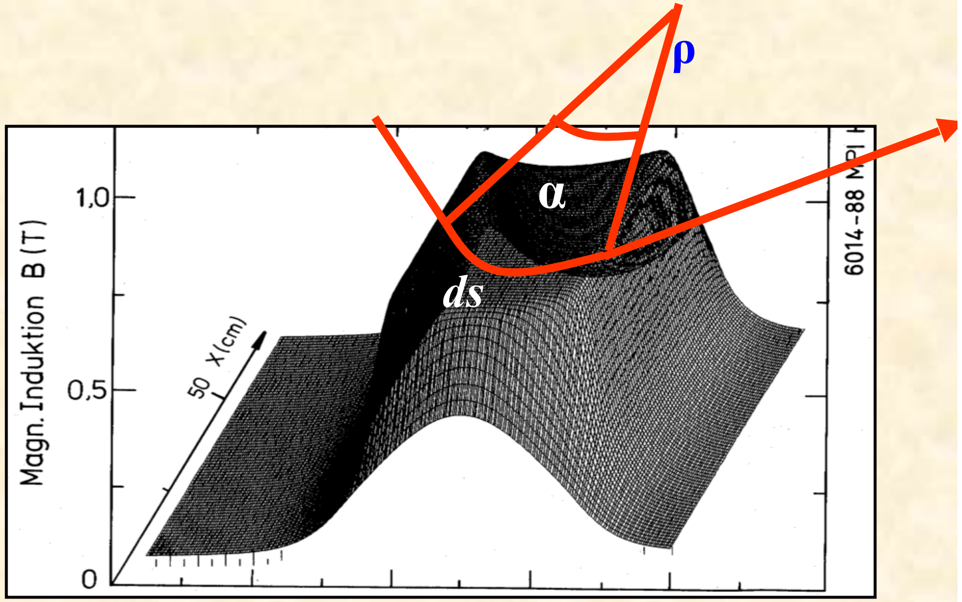
"beam rigidity"



The field of the dipole magnets defines the particle momentum

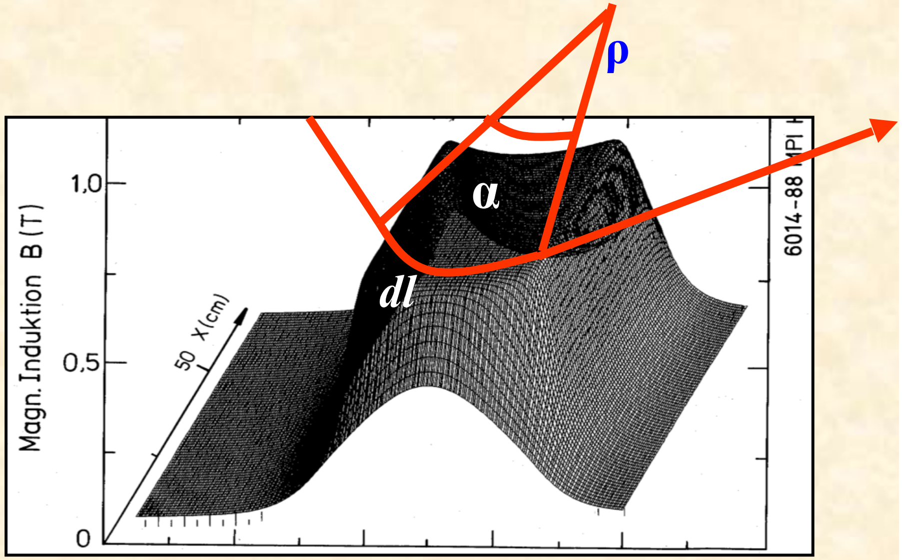
nomalise field to the momentum:

$$\frac{B}{p/e} = \frac{B}{B*\rho} = \frac{1}{\rho}$$



field map of a storage ring dipole magnet

Bending Angle



„integrated field strength"

$$\alpha = \frac{B * dl}{B * \rho}$$

bending angle of one dipole

The angle of all dipoles, swept out in one revolution must be 2π, so

$$\Sigma\ \alpha = \frac{\oint B dl}{B\rho} = 2\pi \quad \rightarrow \quad \oint B dl = 2\pi \cdot \frac{p}{q}$$

… for a full circle

Example LHC:

$$2\pi\rho = 17.6\ km$$
$$\approx 66\%$$

We cover typically 2/3 of the ring with dipole magnets.

The integrated dipole strength (along "s") defines the momentum of the particle beam.

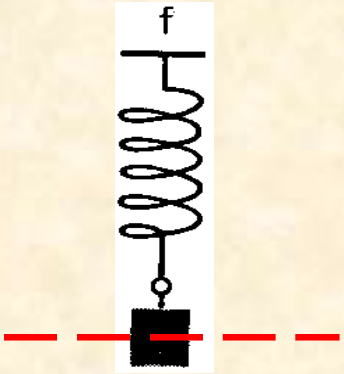
$$\alpha = \frac{\int B dl}{B * \rho} = 2\pi \qquad \rightarrow \qquad \int B dl = 2\pi * \frac{p}{q}$$

3.) Focusing Properties – Quadrupoles

... keeping the flocs together:
In addition to the pure bending of the beam
we have to keep 10^{11} particles close together

focusing force

And here we borrow the idea from classical mechanics:
The pendulum



there is a restoring force, proportional to the elongation x:

$$F = m * a = -\, const * x$$

$$F = m * \frac{d^2x}{dt^2} = -\, const * x$$

general solution:
free harmonic oscillation

$$x(t) = A * \cos(\omega t + \varphi)$$

*...**this is how grandma's Kuckuck's clock is working!!!***

Quadrupole Magnets:

… have a linear increasing magnetic field $B_y = g * x$

… with the constant of proportionality "g" called "gradient" $g = \frac{dB_y}{dx} = const$

and so they lead to a linear restoring Lorentz force

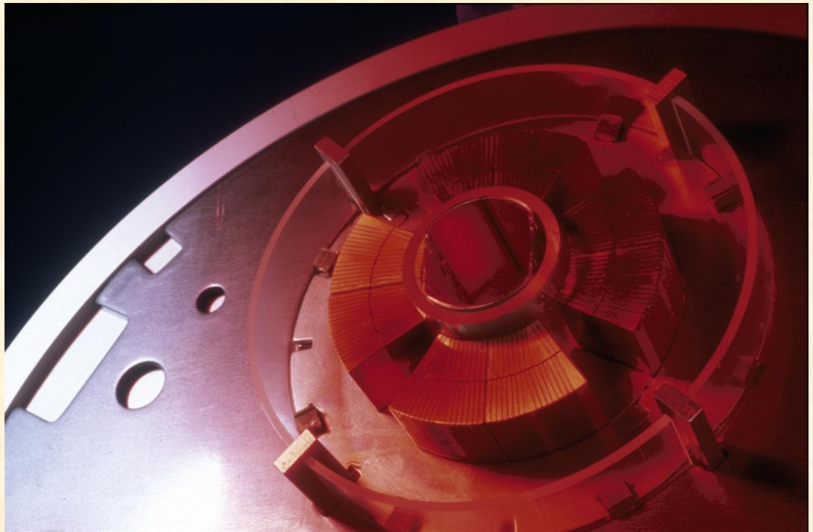
$$F = e * v * B = -\underbrace{(e * v * g)}_{= -\, const} * x$$

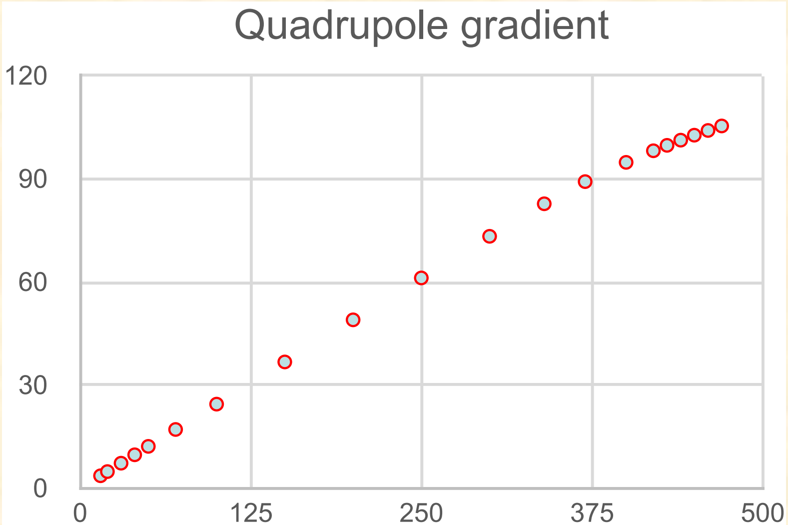
$$= -\, const * x$$

gradient is nomalised to the momentum:

$$k = \frac{g}{B * \rho} = \frac{g}{p/e}$$

LHC main quadrupoles: g = 25 … 220 T/m





A linear increasing restoring force leads always (!) to a harmonic oscillation.
==> quadrupoles do that for us.

$$B_y = g * x \qquad\qquad k = \frac{g}{B*\rho} = \frac{g}{p/e}$$

Focusing forces and particle trajectories:

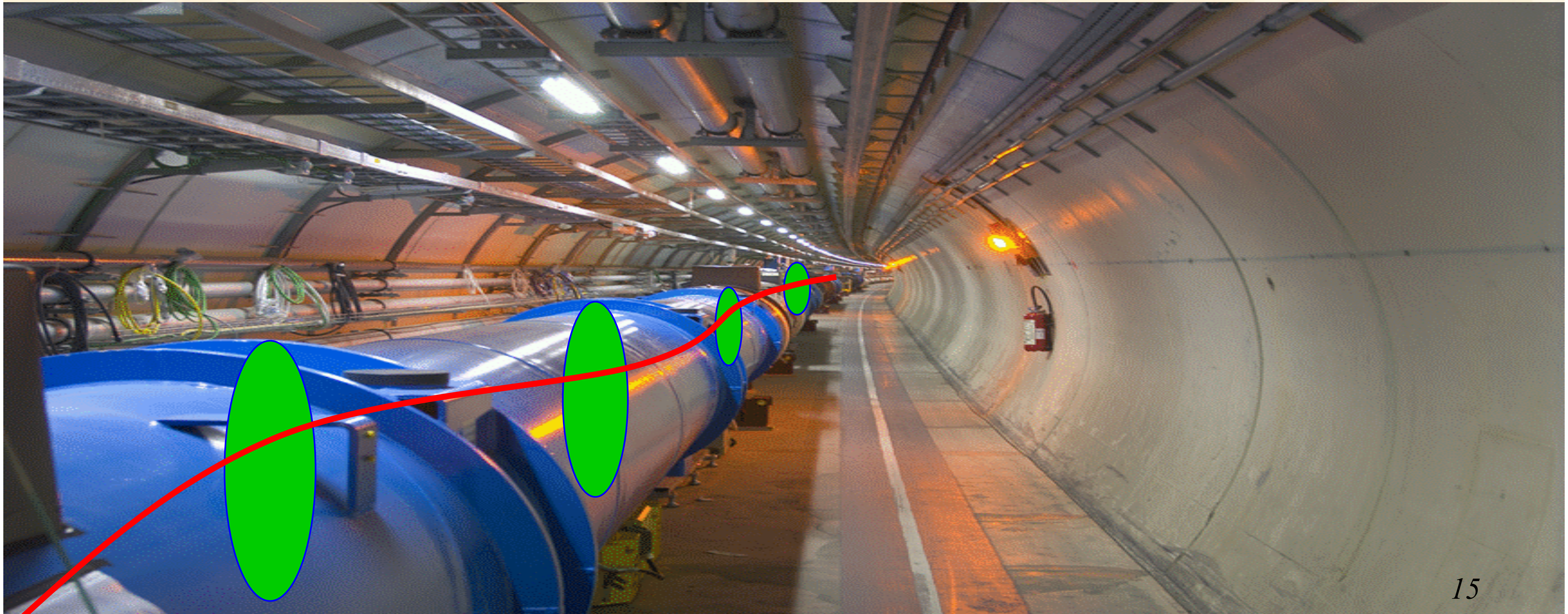
normalise magnet fields to momentum
(remember: $B*\rho = p/q$ *)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

$$k := \frac{g}{p/q}$$



A Storage Ring

… or "synchrotron" (… which is basicaly the same)

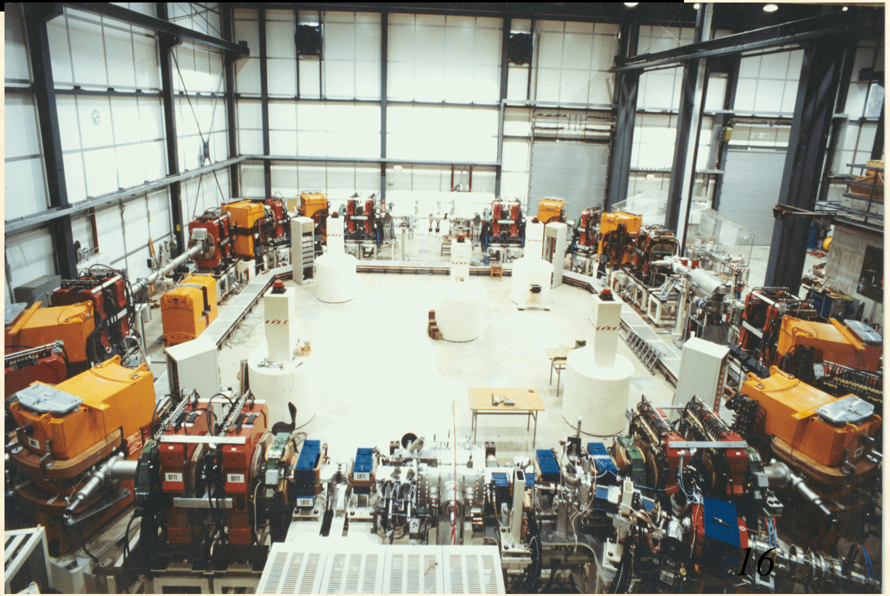
... is a device that creates a pattern of magnetic fields.

This magnetic fields …,
—> keep the particles floating in the air ‼

—> guide a beam of charged particles on a circular orbit

—> and focuses these foggy particle clouds together to form a "bunch".

Typical storage ring:
Dipoles for bending,
Quadrupoles for focusing



Dipole Magnets ...

... bend the particle trajectories onto a „polygon“ (... well a kind of ring),
... define the *geometry* of the machine
... define the maximum momentum (… or energy) of the particle beam
... have a small contribution to the focusing of the beam
... have constant field → not critical for mis-alignment but for roll angles

Quadrupole Magnets ...

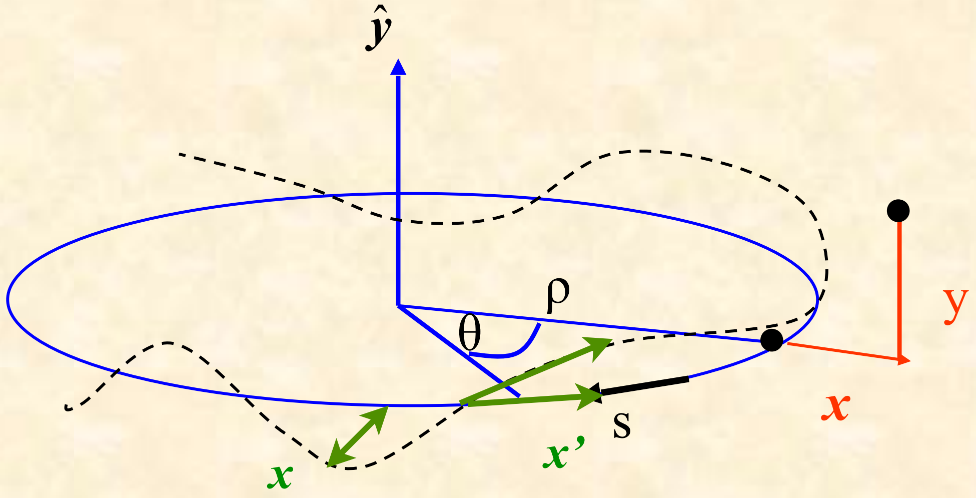
... focus every single particle trajectory towards the centre of the vacuum chamber
... define the beam size
... „produce“ the tune
... increase the luminosity
... Are most (!) critical for mis-alignment

Trajectory ...

... under the influence of the focusing fields the beam centre follows a certain path along the machine: the closed orbit.
The individual particles oscillate transversely around this closed orbit, while moving around the “ring”.

4.) A Bit of Theory
The large Storage Rings and „Synchrotrons“

The Equation of Motion:



* *Equation for the* ***horizontal motion:***

$$\boxed{x'' + x(\frac{1}{\rho^2} + k) = 0}$$

***x* = particle amplitude**
***x'* = angle of particle trajectory (wrt ideal path line)**

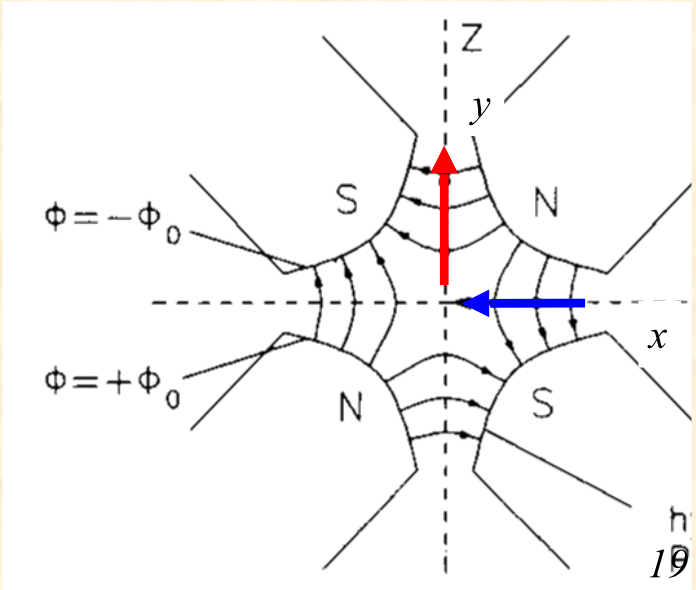
* *Equation for the* ***vertical motion:***

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$$k \leftrightarrow -k$$ ***quadrupole field changes sign***

→ ***Upssssss***

$$\boxed{y'' - k * y = 0}$$



Remarks:

* $$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$

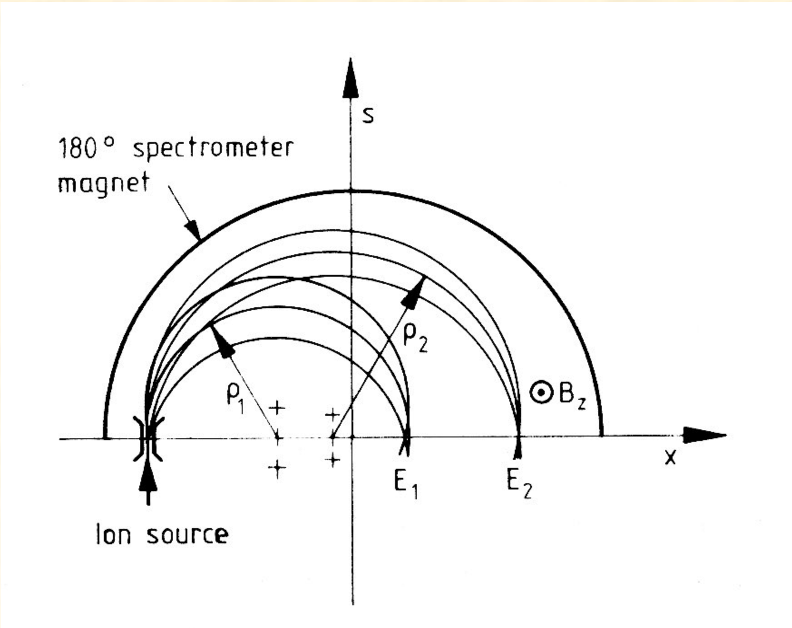
… there seems to be a focusing even without a quadrupole gradient

„weak focusing of dipole magnets“

$$k = 0 \quad \Rightarrow \quad x'' = -\frac{1}{\rho^2}\,x$$

even without quadrupoles there is a retriving force (i.e. focusing) in the bending plane of the dipole magnets

… in large machines it is weak. (!)



Mass spectrometer: particles are separated according to their energy and focused due to the $1/\rho^2$ effect of the dipole

* *Hard Edge Model:*

$$x'' + x(\frac{1}{\rho^2} + k) = 0$$

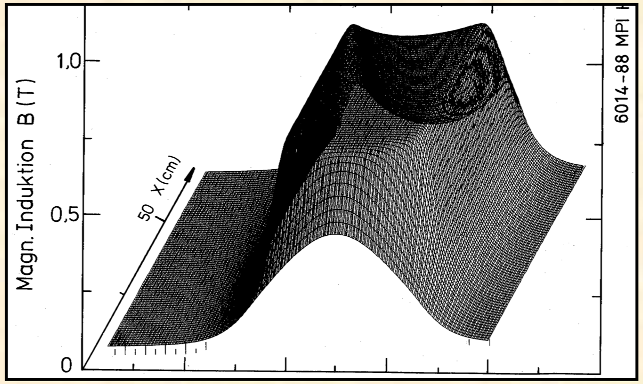
… this equation is not correct !!!

$$x''(s) + x(s)(\frac{1}{\rho^2(s)} + k(s)) = 0$$

bending and focusing fields … are functions of the independent variable „s"

The fields change, when we enter (or exit) the magnet

!



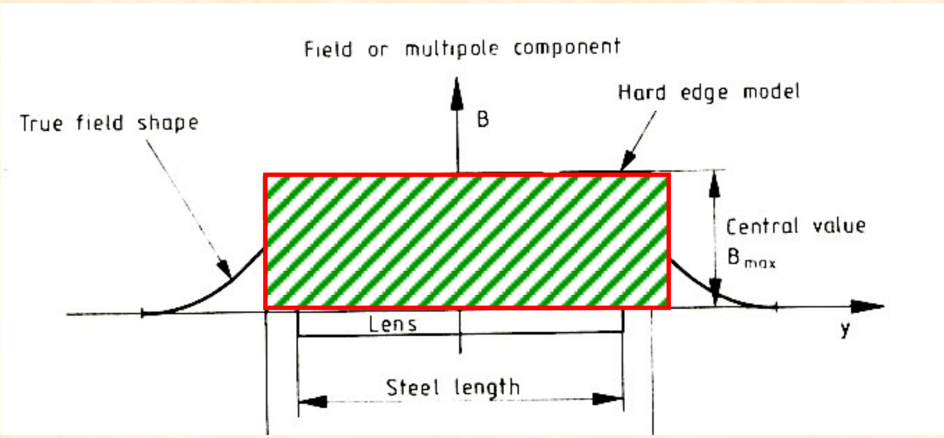
Inside a magnet we assume constant magnetic field

$$\frac{1}{\rho} = const$$

$$k = const$$

… and define a "effective magnetic length"

$$B\, l_{eff} = \int_0^{l_{mag}} B\, ds$$



==> SM 18

$$\frac{\Delta B}{B} \approx 10^{-4}$$ *is usually required !!*

5.) Solution of Trajectory Equations

Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

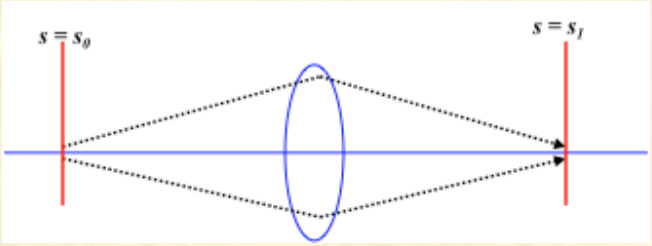
$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$

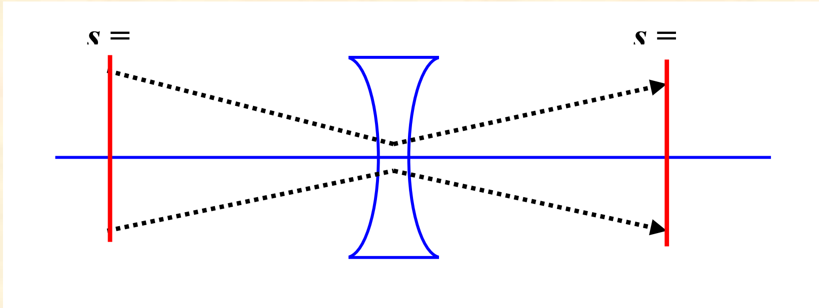


For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$\boldsymbol{M_{foc}} = \begin{pmatrix} \cos\left(\sqrt{|K|}\,l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\,l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}\,l\right) & \cos\left(\sqrt{|K|}\,l\right) \end{pmatrix}$$

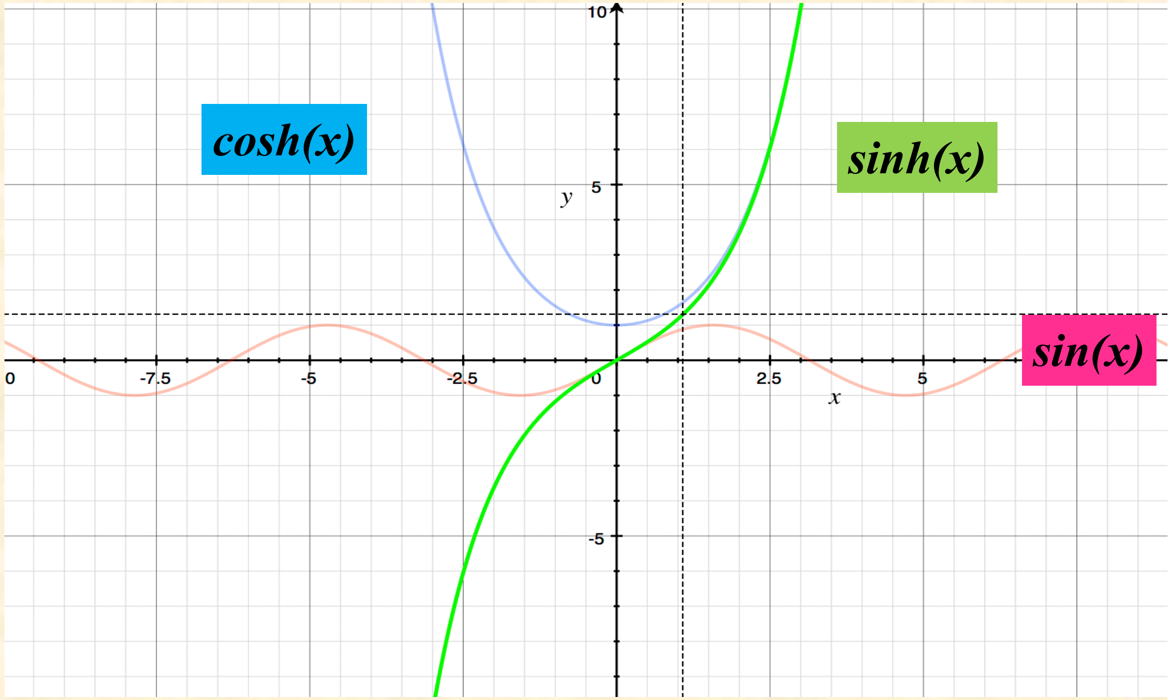
hor. defocusing quadrupole: $\boldsymbol{x'' - K\ x} = 0$



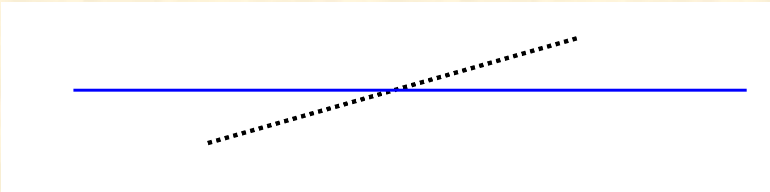
Remember from school:

Ansatz: $x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$



drift space: $\boldsymbol{K = 0}$



$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

Magnet Lattice …

… is the arrangement of the magnets in the tunnel.
Usually they are grouped together to make the alignment easier and technically more feasible.

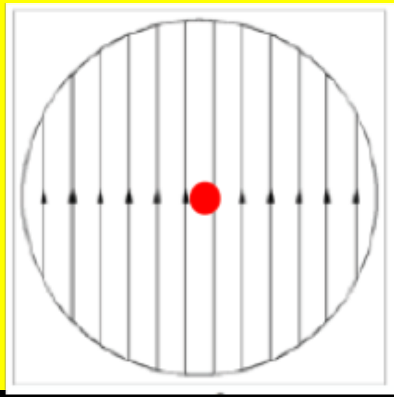
to make it very clear ...

... as long as the particle trajectories are perfectly centred in the magnetic fields they are floating without any oscillation …
… while they are travelling at the speed of light around the machine.

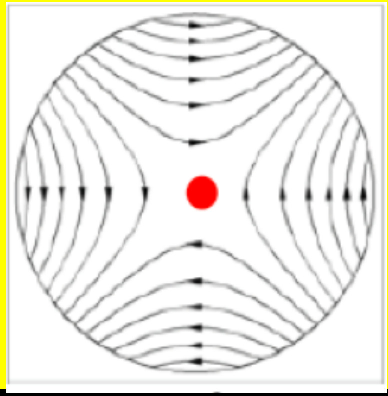
The problems come as soon as we are not perfect:

as soon as the beam is offset in the quadrupoles or the beam has a finite transverse dimension, the Lorentz force is acting and life gets difficult.

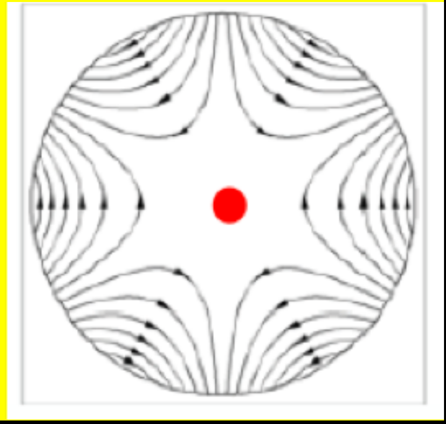
dipole



4-pole



6-pole



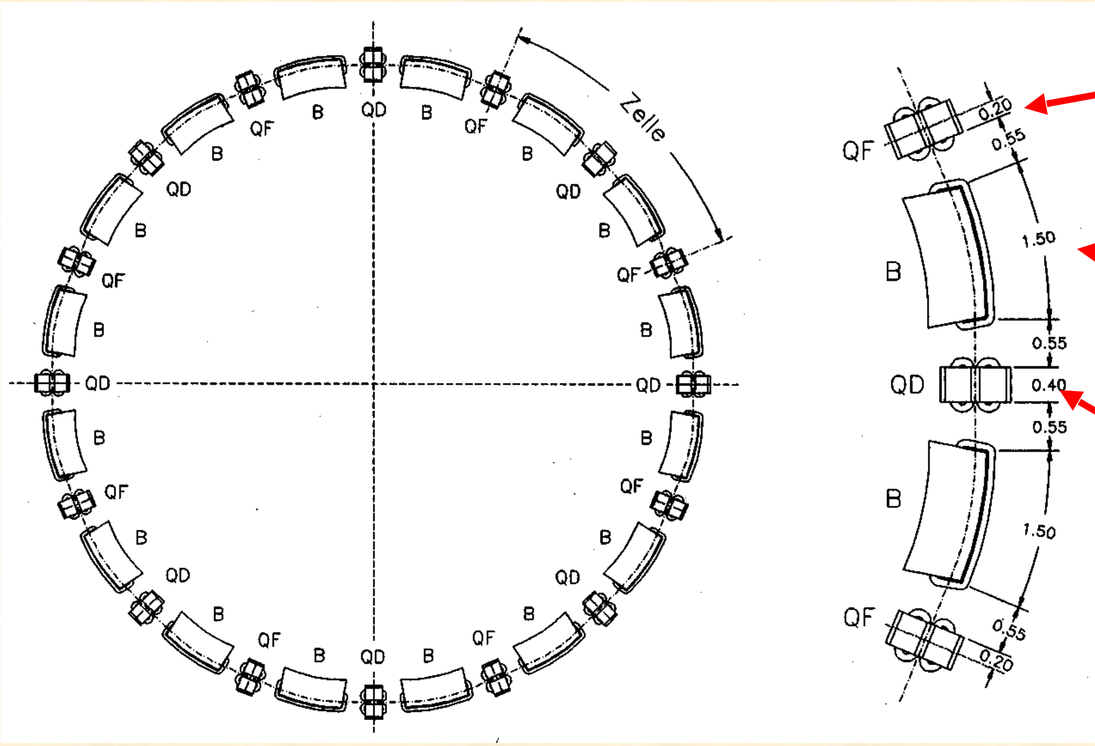
… in the end you do not need physics to understand that.
Just go for a walk in spring.

„veni vidi vici ...“ .... or in english .... „we got it !“

* we can calculate the trajectory of a single particle, inside a storage ring magnet (lattice element)
* for arbitrary initial conditions x_0, x'_0
* we can combine these trajectory parts (also mathematically) and so get the complete transverse trajectory around the storage ring

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D^*}.....$$

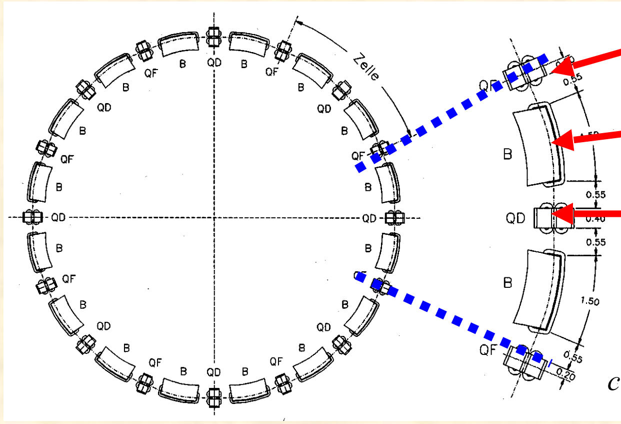
Beispiel: Speichering für Fußgänger (Wille)



combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D^*}\ldots\ldots$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M_{s1->s2} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



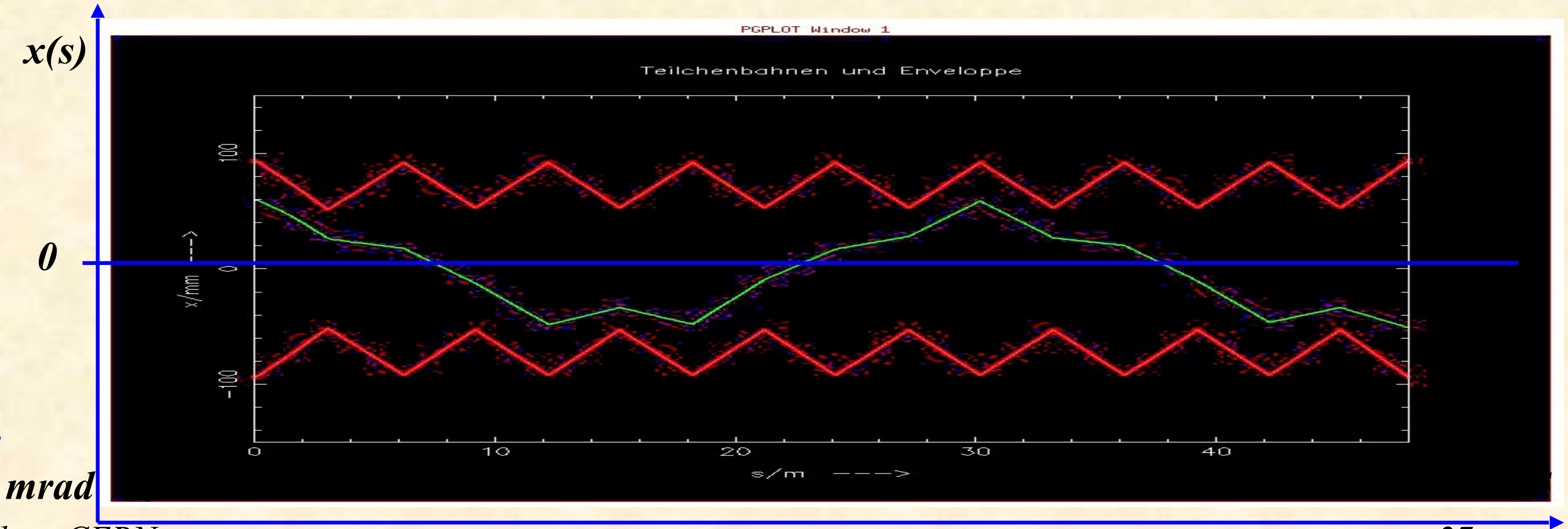
focusing lens

dipole magnet

defocusing lens

court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator ! ! !



typical values in a strong foc. machine:

x ≈ mm, x´ ≤ mrad

Ok … ok ... it's a bit complicated and cosh and sinh and all that is a pain.
BUT ... compare ...

Weak Focusing / Strong Focusing

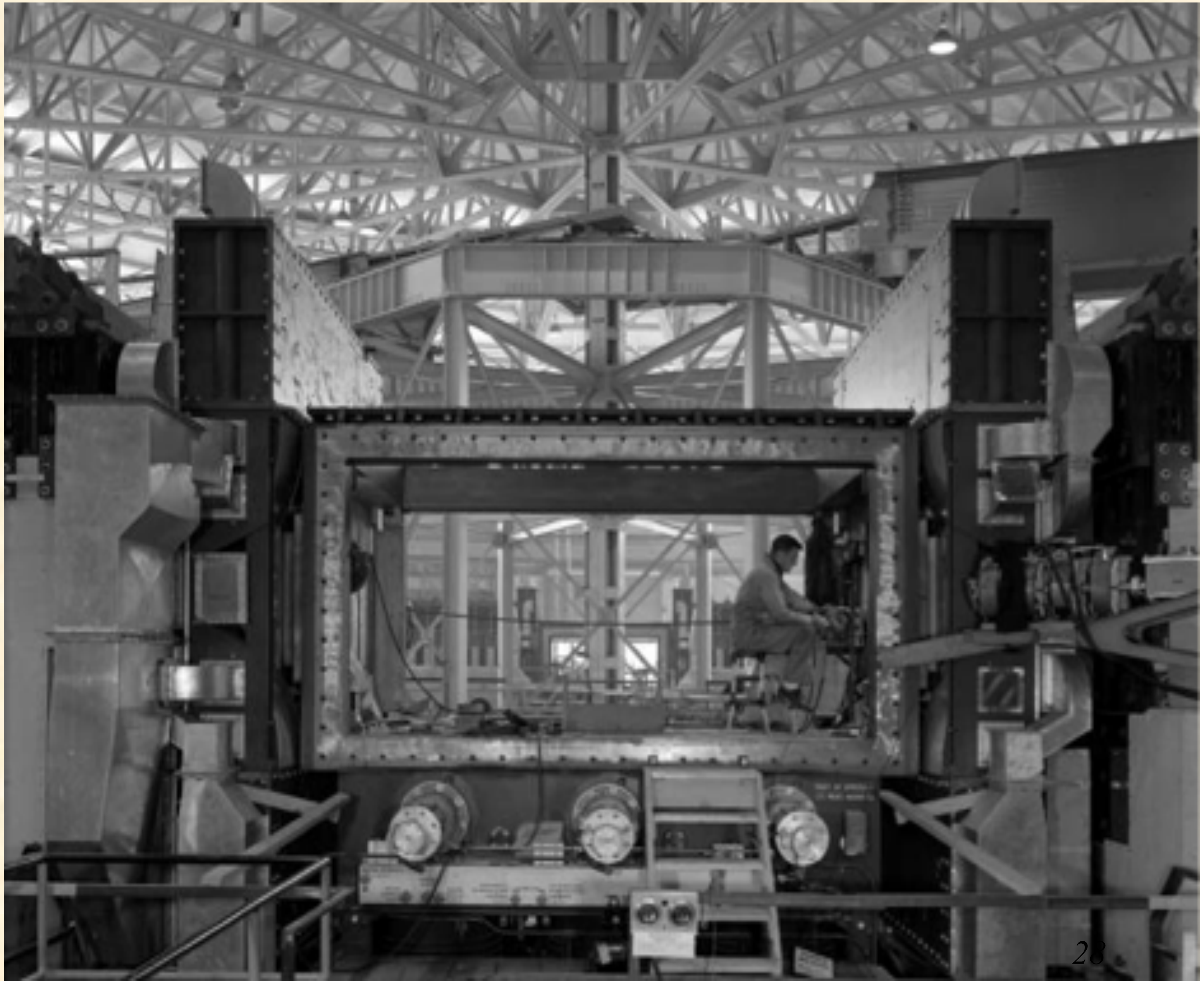
weak focusing term = 1/ρ²

$$x'' + x\left(\frac{1}{\rho^2} + \cancel{k}\right) = 0$$

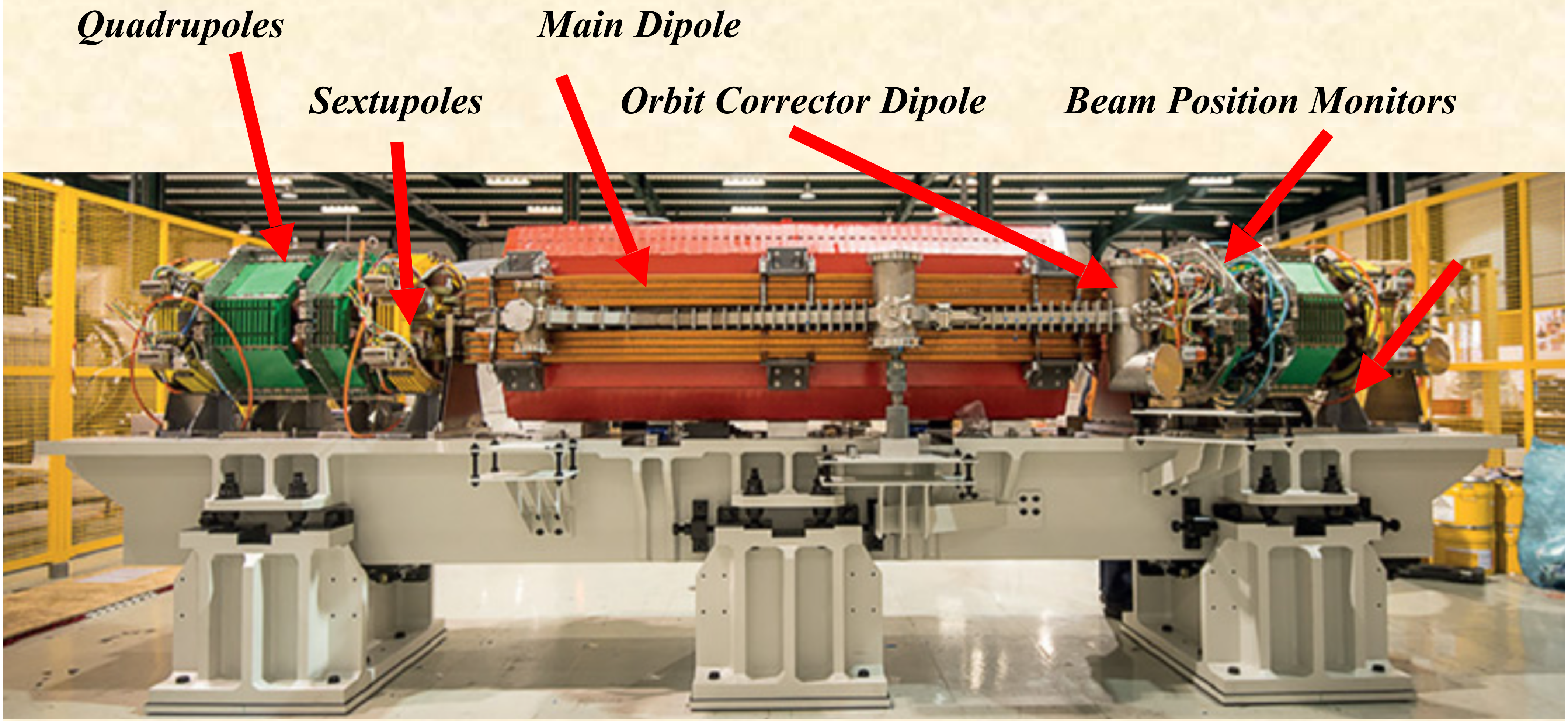
Problem: the higher the energy,
the larger the machine (ρ)
and the weaker the focusing 1/ρ²

The last weak focusing
high energy machine …
BEVATRON

→ *large apertures needed*
→ *very expensive magnets*



Lattice Elements and Beam Instrumentation in a Storage Ring



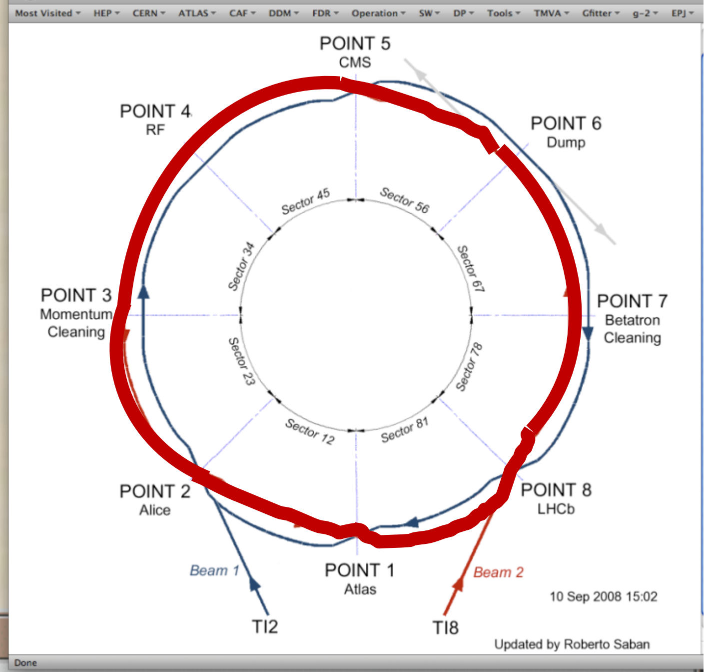
SESAME light source

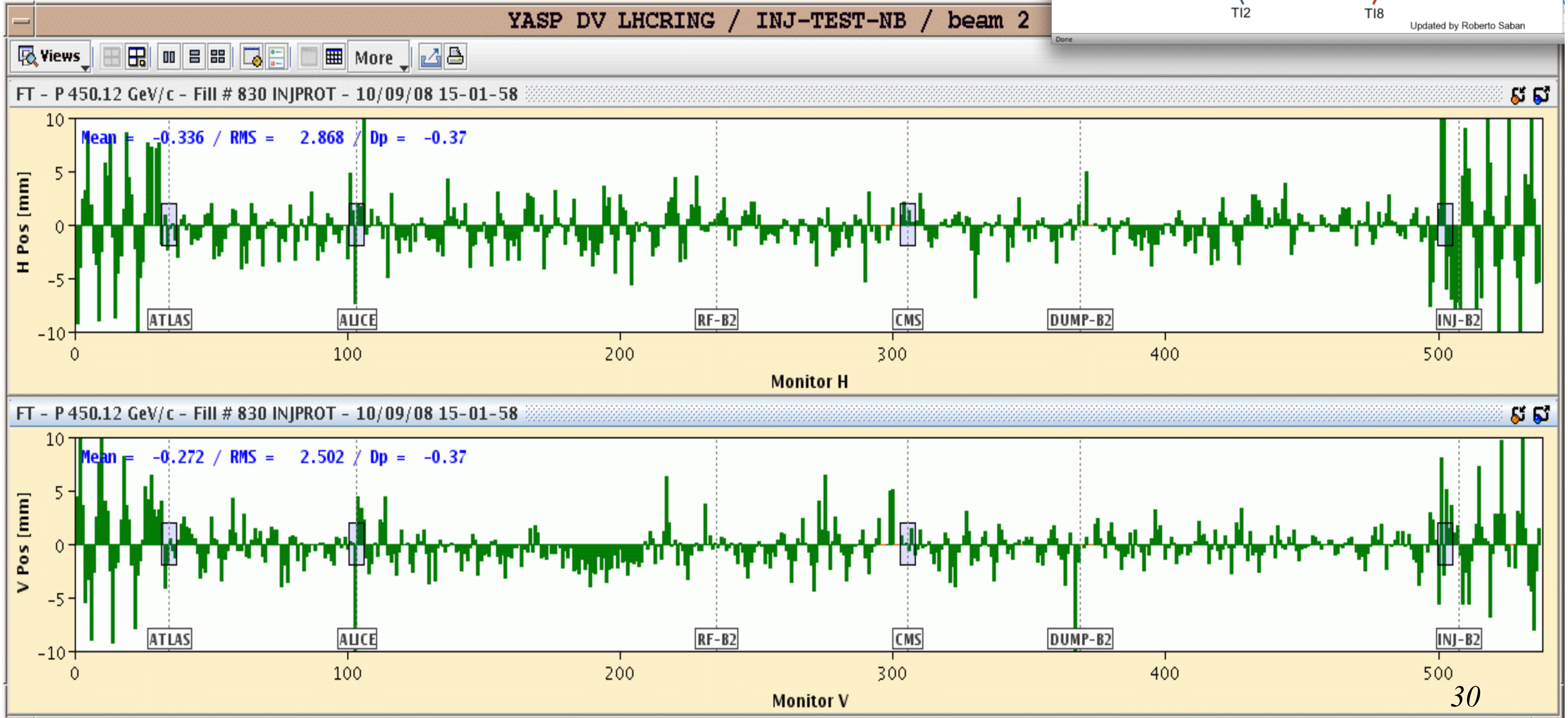
Knowing strength and length of each magnet we can calculate the matrices and determine the trajetory of every single particle !!!

LHC Operation: Beam Commissioning

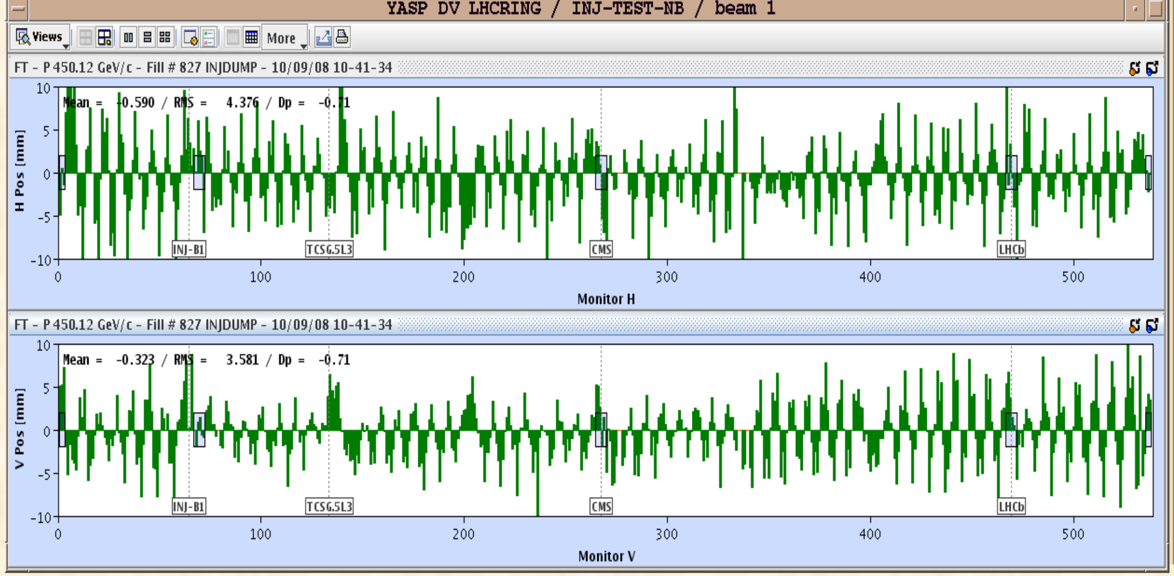
First turn steering "by sector:"

- One beam at the time
- Beam through 1 sector (1/8 ring),
correct trajectory, open collimator and move on.





6.) Orbit & Tune:



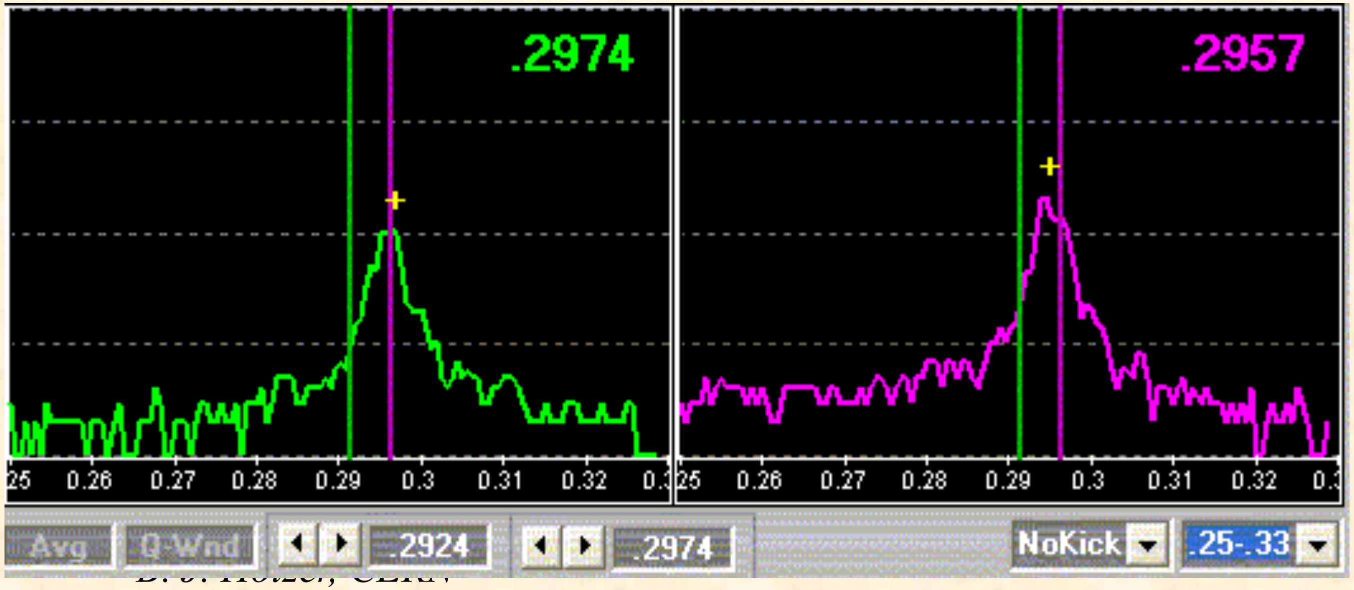
Tune: number of oscillations per turn

64.31
59.32

Relevant for beam stability:
non integer part

LHC revolution frequency: 11.3 kHz

$0.31 * 11.3 = 3.5 kHz$



... and the tunes in x and y are different.

i.e. we can apply different focusing forces in the two planes

i.e. we can create different beam sizes in the two planes

Closed Orbit ...

... There is one (!) trajectory that closes upon itself. It is given by the foc. fields and it is what we „see“ when we observe the BPM readings of the stored beam.

... The single particle will perform transverse oscillations and so the Single Particle Trajectories will oscillate (= betatron oscillations) around this closed orbit.

The Tune ...

... is the number of these transverse oscillations per turn and corresponds to the „Eigenfrequency“ or sound of the particle oscilations.

There is a tune for the horizontal, the vertical and the longitudinal oscillation.

And we could even hear it ... if there were no vacuum.

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