



Rabi-Fest Univ. of Maryland, College Park, MD, Oct. 20-21, 2022

*The “old” and
the “new”
muon $g-2$ puzzles*

Antonio Masiero

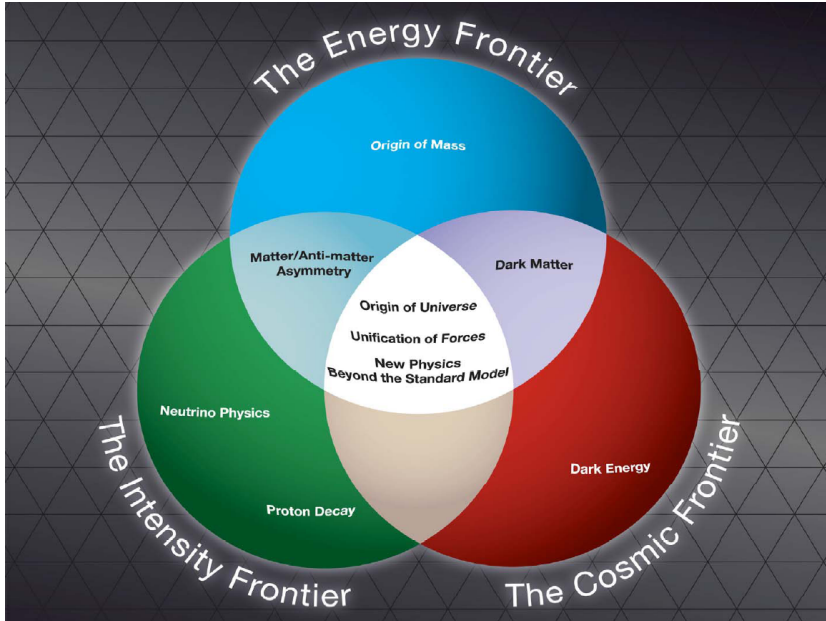
Univ. of Padova and INFN, Padova

Based on the work on the muon $g-2$ problem
done in these last years in collaboration with
**Luca Di Luzio, Bill Marciano, Paride Paradisi
and Massimo Passera**

EVEN MORE INTERESTING ...

my first lesson from Rabi, MPI, 1981

On the “old” muon g-2 puzzle



During the long sequel of restless attempts of finding experimental evidences or at least hints of **NEW PHYSICS** beyond the SM along the **traditional High-Energy (HE) and High-Intensity (HI) paths**, several 3 or even 4 σ signals at variance w.r.t. the SM expectations **have shown up**, but they have also (rather sooner than later) **invariably faded away**.

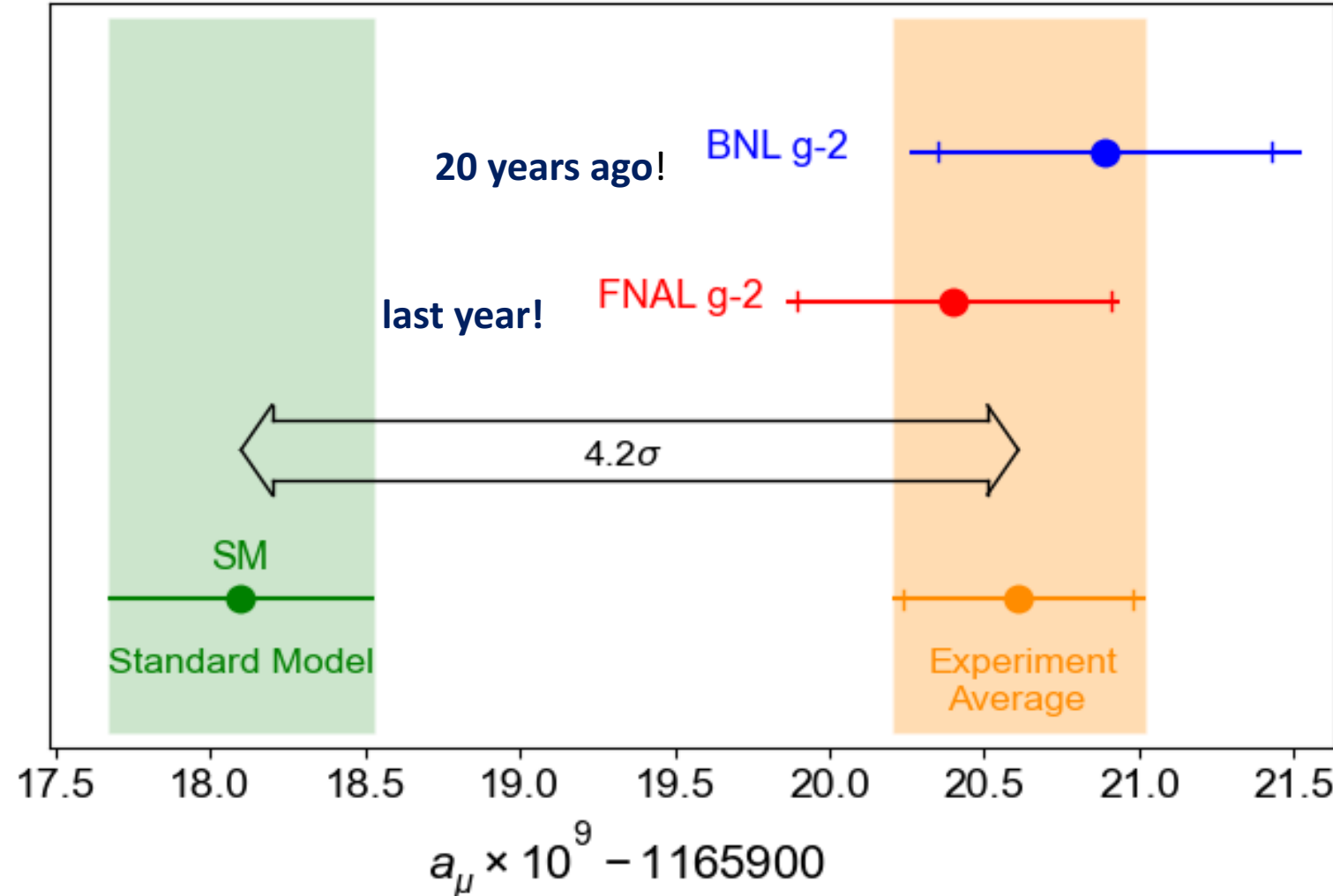
A remarkable exception is represented by

the anomalous magnetic moment of the muon

which has been for **several years now** and **still** represents a **major observational evidence along the HI frontier of the possible presence of NEW PHYSICS**

The other more recent hint of NEW PHYSICS along these two roads is again in the HI frontier, namely the possible **violation of lepton flavour universality in some B-meson semileptonic decays**.

The **EXP.** situation

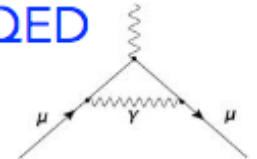
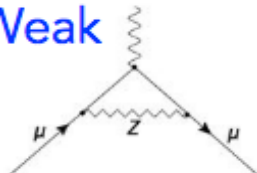
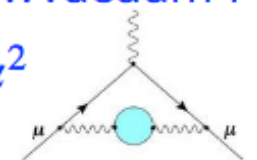
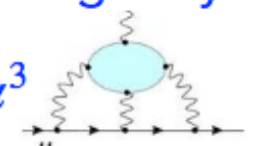


- $a_\mu^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821
- $a_\mu^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1
- $a_\mu^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

FNAL aims at 16×10^{-11}

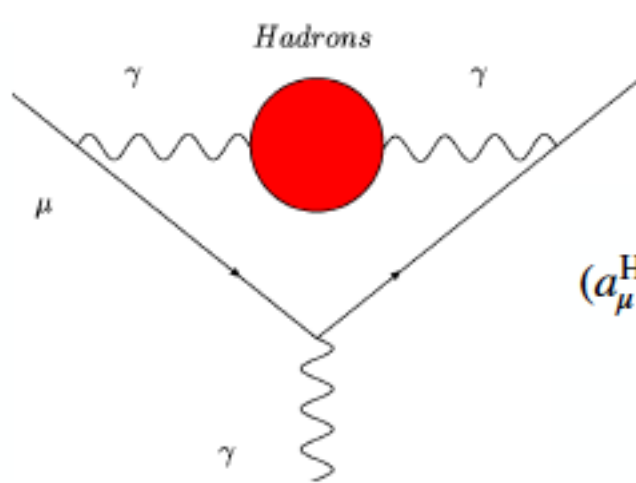
The 4 classes of SM contributions: **uncertainty largely dominated** by the **hadronic contributions** in **Vacuum Polarization (HVP)** and **Light-by-Light (HLbL)**

$$a_{\mu}(\text{SM}) = a_{\mu}(\text{QED}) + a_{\mu}(\text{Weak}) + a_{\mu}(\text{Hadronic})$$

<p>QED</p>  <p>+ ...</p>	$116\,584\,718.9(1) \times 10^{-11}$	0.001 ppm
<p>Weak</p>  <p>+ ...</p>	$153.6(1.0) \times 10^{-11}$	0.01 ppm
<p>Hadronic...</p>		
<p>...Vacuum Polarization (HVP)</p> <p>α^2</p>  <p>+ ...</p>	$6845(40) \times 10^{-11}$ [0.6%]	0.37 ppm
<p>...Light-by-Light (HLbL)</p> <p>α^3</p>  <p>+ ...</p>	$92(18) \times 10^{-11}$ [20%]	0.15 ppm

Numbers from Theory Initiative Whitepaper

C. Lehner, April 8, 2021 - CERN EP Seminar



$$\text{Im} \left[\text{wavy line} \text{---} \text{red circle} \text{---} \text{wavy line} \right] \sim \left| \text{wavy line} \text{---} \text{red lines} \right|^2 \sim \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$$

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} K(s) \text{Im} \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

dispersion relations

optical theorem

kernel function

$$K(s) \approx m_\mu^2/3s \quad \text{for} \quad \sqrt{s} \gg m_\mu$$

$$a_\mu^{\text{HLO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\mu^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{had})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$a_\mu^{\text{HLO}} = 6895 (33) \times 10^{-11}$$

F. Jegerlehner, arXiv:1711.06089

$$= 6939 (40) \times 10^{-11}$$

Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

$$= 6928 (24) \times 10^{-11}$$

Keshavarzi, Nomura, Teubner, arXiv:1911.00367

$$= 6931 (40) \times 10^{-11} (0.6\%)$$

WP20 value

$$a_{\mu}^{\text{EXP}} = 116592061 (41) \times 10^{-11}$$

BNL+FNAL

$$a_{\mu}^{\text{SM}} = 116591810 (43) \times 10^{-11}$$

WP20 $a_{\mu, e^+e^-}^{\text{HLO}} = 6931(40) \times 10^{-11}$

$$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = 251 (59) \times 10^{-11}$$

4.2 σ

The OLD $(g-2)_{\mu}$ puzzle

Can Δa_{μ} be due to a missing contribution in σ_{had} ?

The diagram shows a photon line with a red circle representing a hadronic vacuum polarization insertion. To the right, a vertical line with a branching structure represents the hadronic cross-section contribution, with a square symbol and a superscript 2 indicating the squared magnitude of the amplitude.

$$\text{Im} \sim \left| \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}) \right|^2$$

Shifts $\Delta\sigma(s)$ to fix Δa_{μ} are possible,
but conflict with the EW fit if they occur above ~ 1 GeV

Crivellin, Hoferichter, Manzari, Montuli;
de Rafael; Malaescu, Schott;
Colangelo, Hoferichter, Stoffer

Shifts below ~ 1 GeV conflict with the quoted exp. precision of $\sigma(s)$

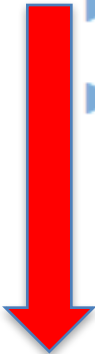
Keshavarzi, Marciano, Passera, Sirlin, PRD 2020 (updated 2021)

NEW PHYSICS for the muon $g-2$: at which scale?

$$\Delta a_\mu \equiv a_\mu^{\text{NP}} \approx (a_\mu^{\text{SM}})_{\text{weak}} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$


- ▶ A weakly interacting NP at $\Lambda \approx v$ can naturally explain $\Delta a_\mu \approx 2 \times 10^{-9}$
- ▶ $\Lambda \approx v$ favoured by the *hierarchy problem* and by a WIMP DM candidate.

On the other hand, HE experiments (LEP, Tevatron, LHC) have NOT provided any clue for the presence of new (charged) particles at the ELW. scale

- 
- ▶ NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.
 - ▶ NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.

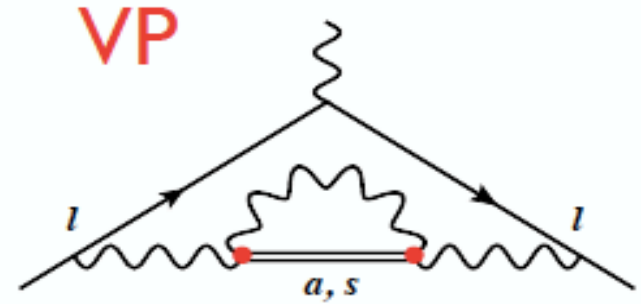
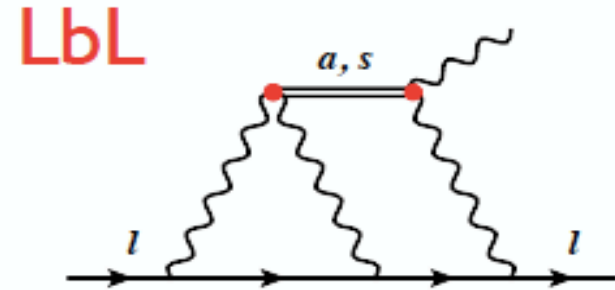
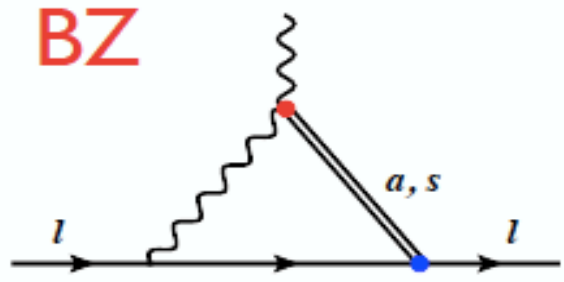
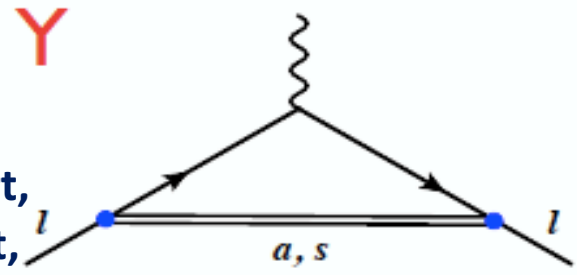
P. Paradisi, La Thuile 2021

The case of AXION-LIKE PARTICLES (ALPs)

ALPs contributions to the muon g-2?



Marciano, AM, Paradisi,
Passera '16; Bauer, Neubert,
Thamm '17; Bauer, Neubert,
Renner, Schnubel, Thamm '19;
Cornella, Paradisi, Sumensari '19

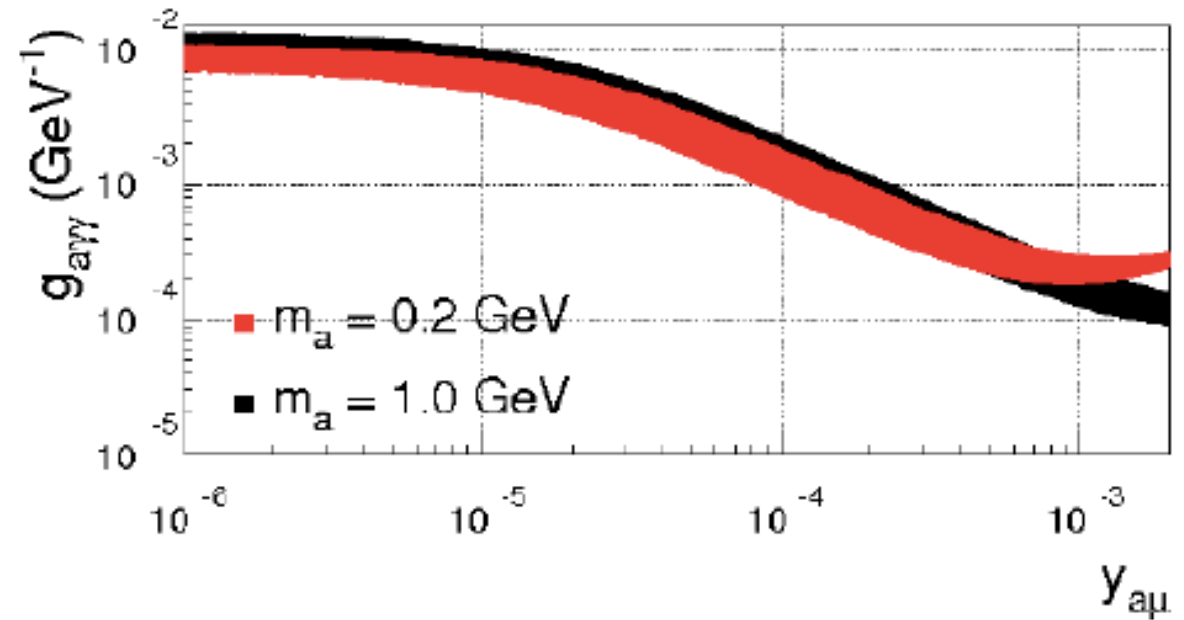
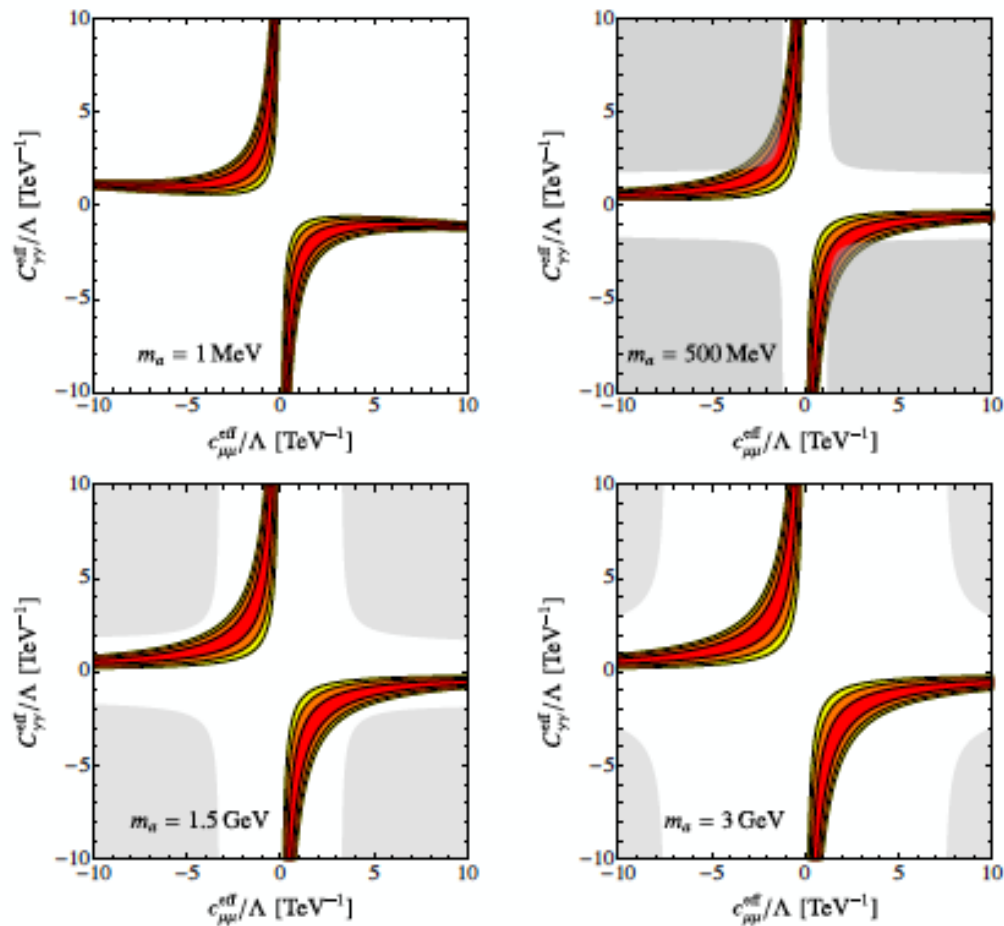


- Both scalar and pseudoscalar ALPs can solve Δa_μ for masses $\sim [100\text{MeV}-1\text{GeV}]$ and couplings allowed by current experimental constraints.
- They can be tested at present low-energy e^+e^- experiments, via dedicated $e^+e^- \rightarrow e^+e^- + \text{ALP}$ & $e^+e^- \rightarrow \gamma + \text{ALP}$ searches.

$$\mathcal{L} = e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{c_{\mu\mu}}{2} \frac{\partial^\nu a}{\Lambda} \bar{\mu} \gamma_\nu \gamma_5 \mu$$

$$\mathcal{L} = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{a\psi} a \bar{\psi} \gamma_5 \psi$$

$$g_{a\gamma\gamma} \equiv \frac{2\sqrt{2}\alpha}{\Lambda} c_{a\gamma\gamma}$$

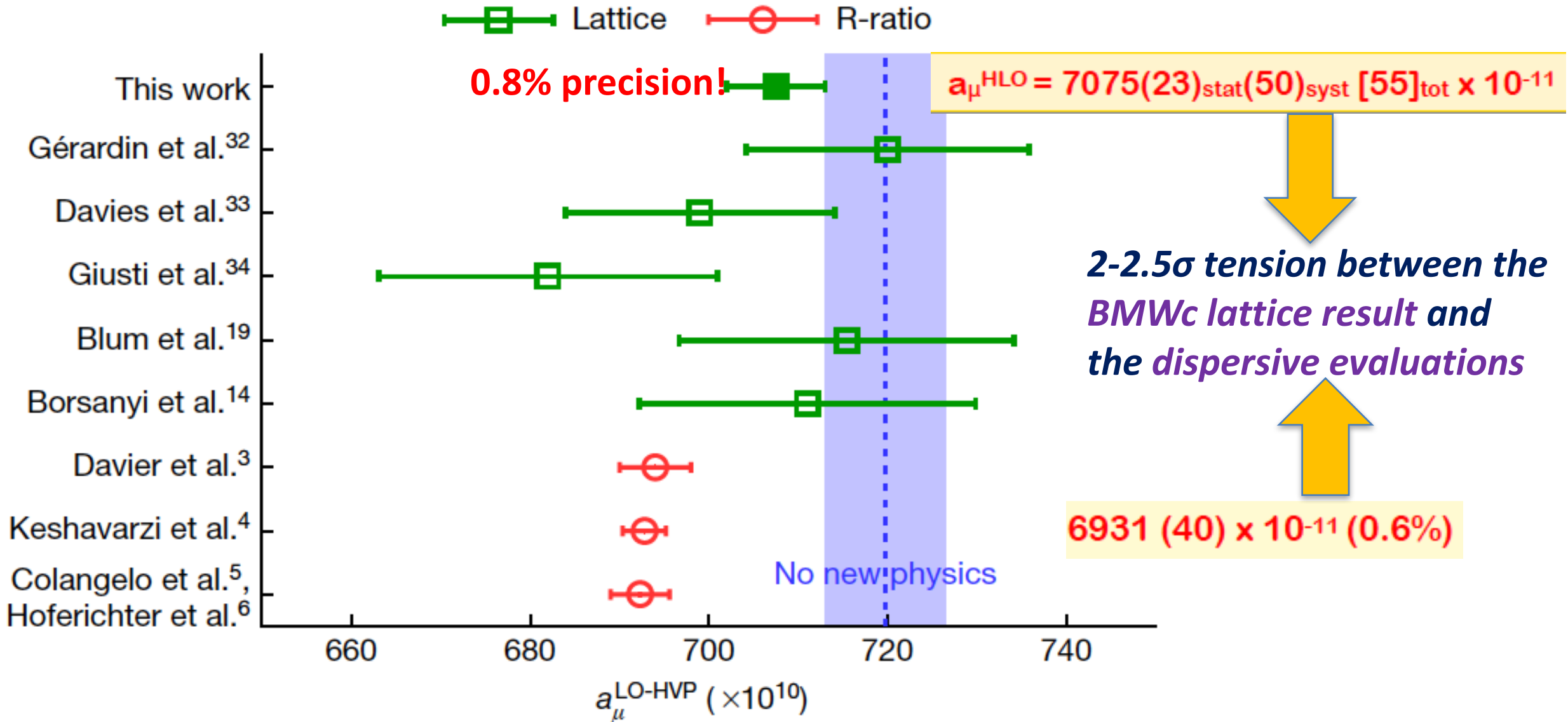


Pseudoscalar 1σ solution bands
to the $g-2$ muon anomaly taking
 $\Lambda = 1 \text{ TeV}$

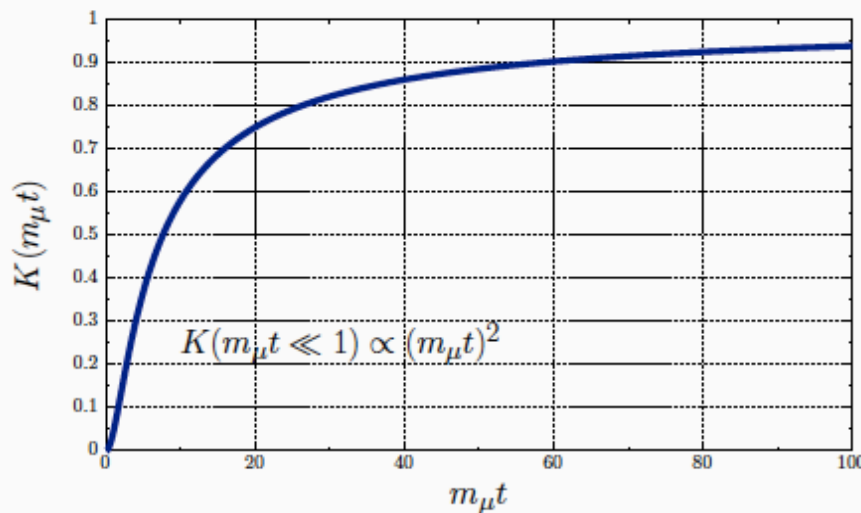
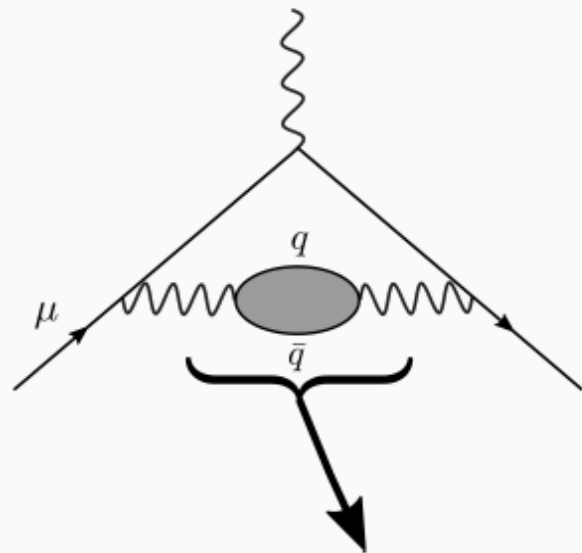
Figure: Δa_μ regions favoured at 68% (red), 95% (orange) and 99% (yellow) CL. Gray regions are excluded by the BaBar search $e^+e^- \rightarrow \mu^+\mu^- + \mu^+\mu^-$ [Bauer, Neubert, Thamm, '17]

Marciano, A.M.,
Paradisi, Passera '16

BMWc20: S. Borsanyi et al. 2002.12347, published on Nature, April 7, 2021
 first published lattice result with **sub-percent precision!**



LO-HVP from Lattice QCD



$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQ \cdot x} \langle J_\mu(x) J_\nu(0) \rangle = (\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi(Q^2)$$

$$a_\mu^{\text{LO-HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2}\right) \cdot (\Pi(Q^2) - \Pi(0)).$$

Time-Momentum representation (Bernecker & Meyer, 2011)

$$a_\mu^{\text{LO-HVP}} = 2\alpha_{em}^2 \int_0^\infty dt t^2 K(m_\mu t) V(t), \quad V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \langle J_i(\vec{x}, t) J_i(0) \rangle_i$$

Colangelo, El-Khadra, Hoferichter, Keshavarzi, Lehner, Stoffer, Teubner, arXiv:2205.12963v2 (2022)

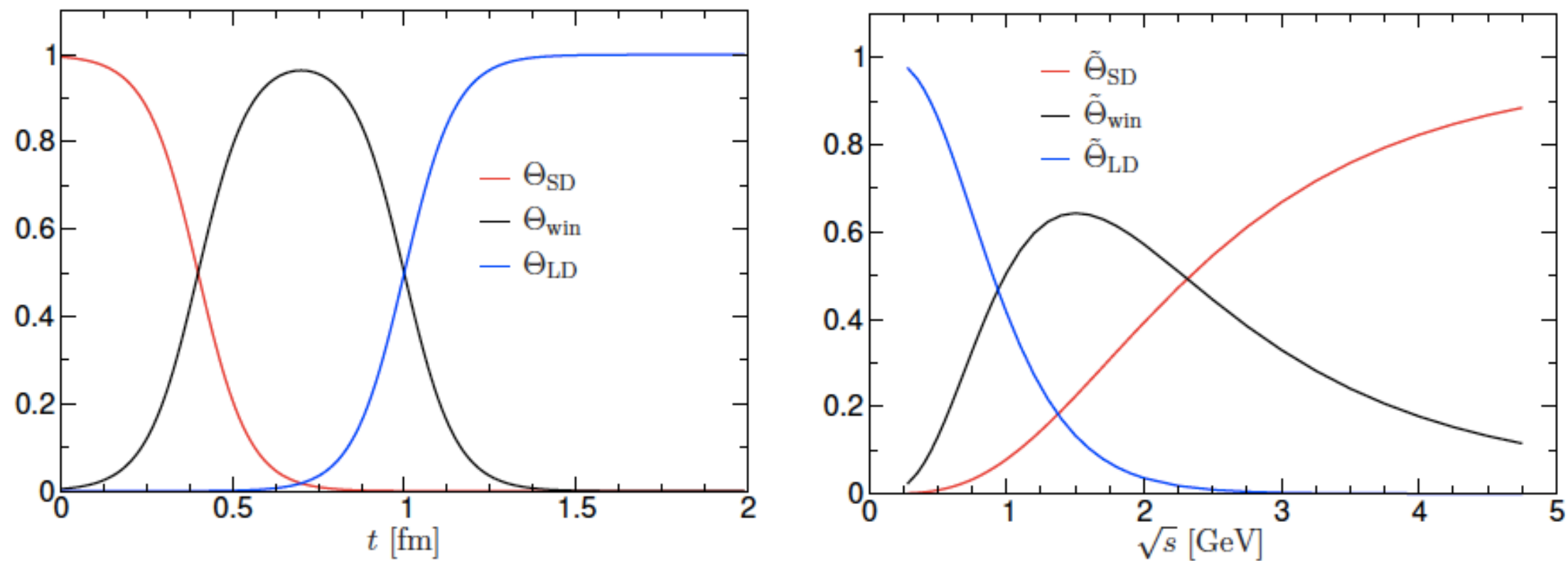
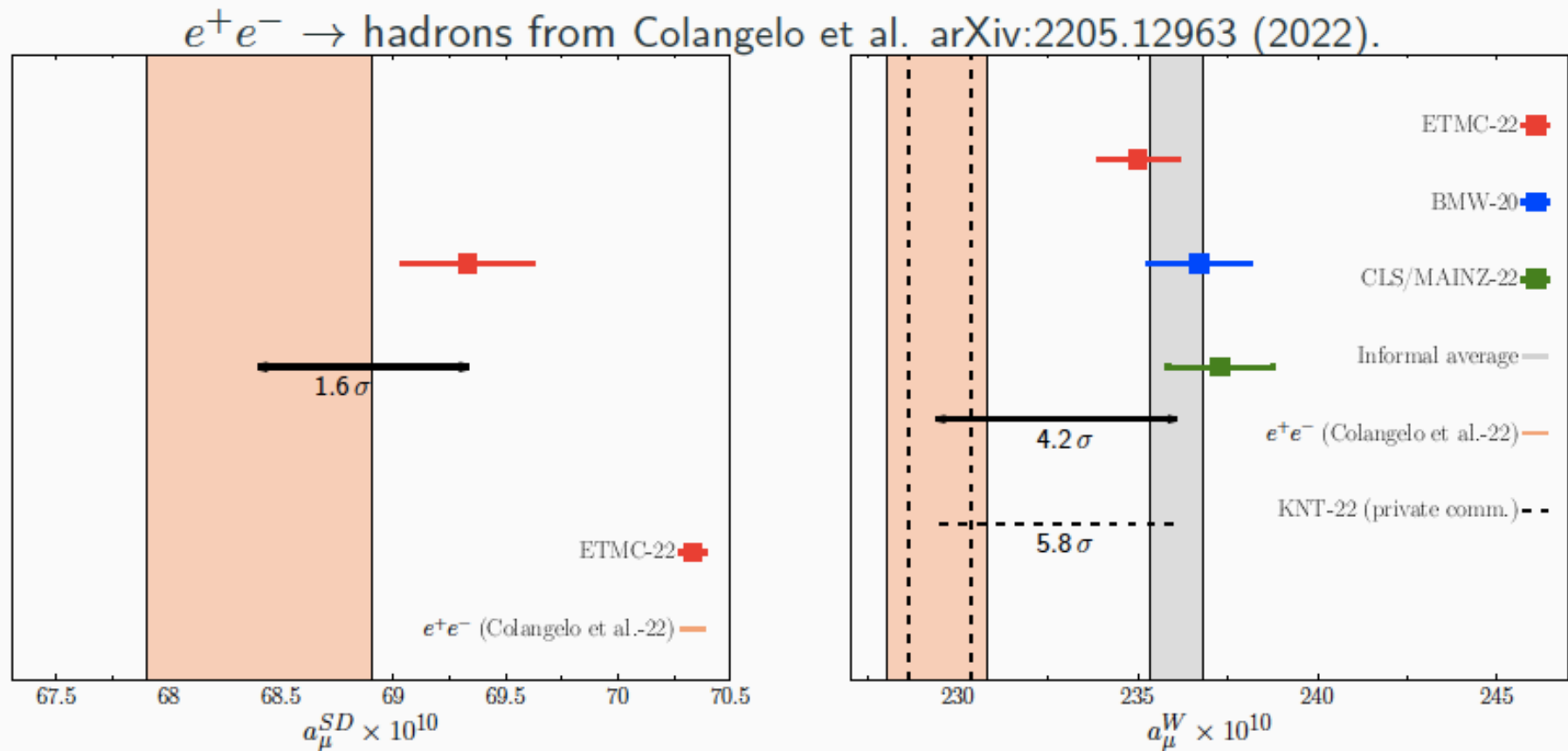


Figure 1: Short-distance, intermediate, and long-distance weight functions in Euclidean time (left), and their correspondence in center-of-mass energy (right).

Comparison with $e^+e^- \rightarrow \text{hadrons}$ results

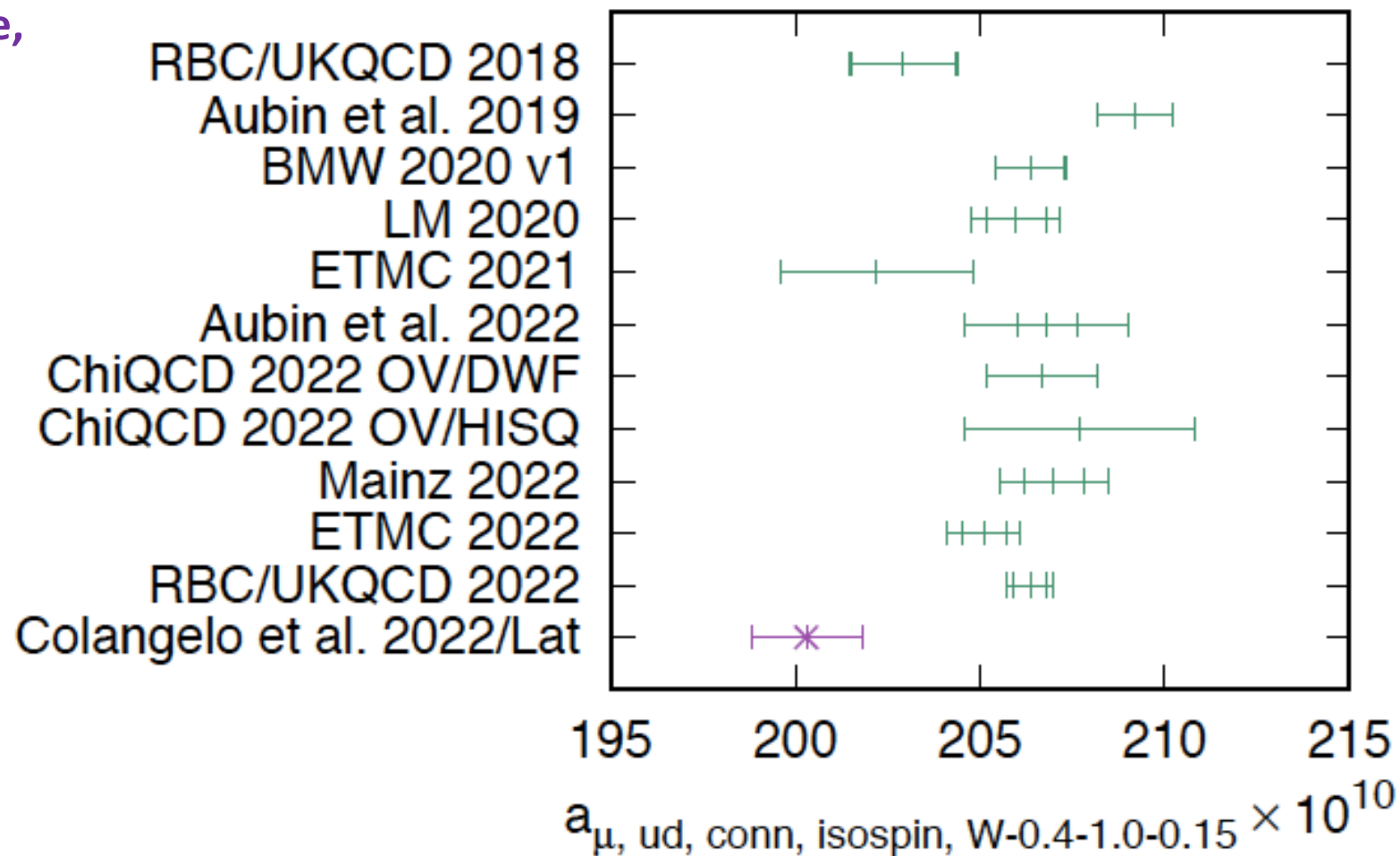
G. Gagliardi, Edinburgh
2022, on behalf of the
ETM Collaboration



- Tension in a_μ^W rises to 4.2σ if we combine ETMC '22, BMW '20 and CLS/Mainz '22 (informal average \rightarrow next WP).
- Deviation of $e^+e^- \rightarrow \text{hadrons}$ data w.r.t. the SM in the low and (possibly) intermediate energy regions, but not in the high energy region.

The RBC/UKQCD22 result in context

C. Lehner, Workshop of the
Muon g-2 Theory Initiative,
Edinburgh, Sept. 2022



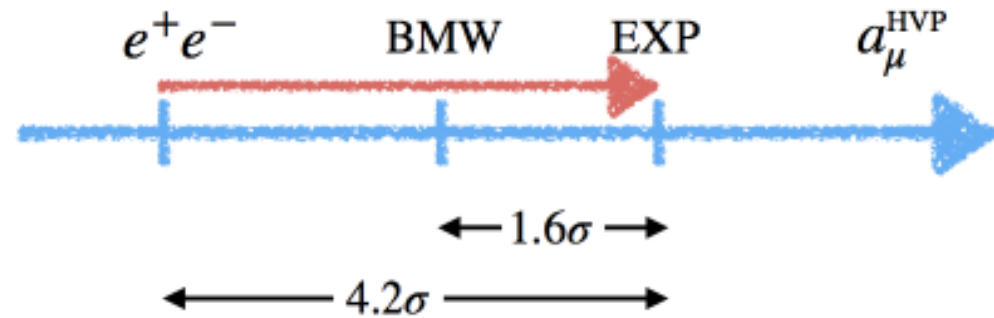
- ▶ 3.9 σ tension of RBC/UKQCD22 with Colangelo et al. 22/Lattice

The NEW g-2 puzzle

$$(a_{\mu}^{\text{HVP}})_{\text{EXP}} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM, rest}}$$

$$(a_{\mu}^{\text{HVP}})_{e^+e^-}^{\text{WP20}} = 6931(40) \times 10^{-11}$$

$$(a_{\mu}^{\text{HVP}})_{\text{BMW}} = 7075(55) \times 10^{-11}$$



If the new lattice results * – i.e., **BMWc** & (only for the (SD) + W windows, but not for the relevant LD window) **Mainz 2022+ETMC 2022 + RBC/UKQCD 2022** are correct (and will be confirmed also for the LD window!), then:

i) The “old” g-2 discrepancy would be basically gone, but

ii) A new significant discrepancy between the e^+e^- data- driven and lattice QCD evaluations of a_{μ}^{HVP} becomes quite significant ($> 4\sigma$)

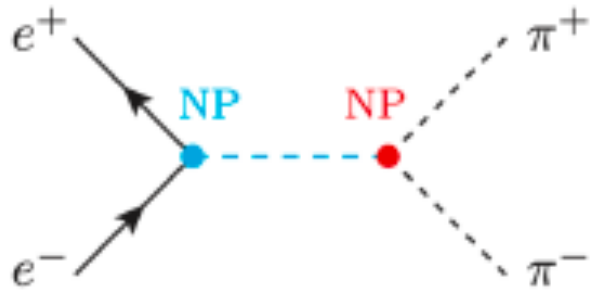
* The lattice FNAL/QCDMILC collaboration is going to unblind its data soon

New Physics to solve the new muon $g-2$ puzzle ?

NP in $\sigma_{\text{had}}(e^+e^- \rightarrow \text{hadrons})$ such that

1. $(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} \approx (a_\mu^{\text{HVP}})_{\text{EXP}}$
2. the approximate agreement between BMW and EXP is not spoiled
3. w/o a direct contribution a_μ^{NP} (i.e. NP not in muons)

L. Di Luzio, A.M., P. Paradisi, M. Passera, PLB 2022 (arXiv 2112.08312)



NP coupled both to **hadrons** and **electrons**

$$\text{Im} \left[\text{wavy line} \bullet \text{wavy line} \right] \sim \left| \text{wavy line} \rightarrow \text{hadrons} \right|^2 \sim \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$$

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} K(s) \text{Im} \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) \sigma_{\text{had}}(s) \quad \sigma_{\text{had}} = \sigma_{\text{had}}^{\text{SM}} + \Delta\sigma_{\text{had}}^{\text{NP}}$$

SUBTRACTION since NP does **NOT** contribute to the HVP at the LO, but it **DOES** contribute to the cross-section at the LO

$$\sigma_{\text{had}} - \Delta\sigma_{\text{had}}^{\text{NP}}$$

a **POSITIVE** SHIFT on

$(a_\mu^{\text{HVP}})_{e^+e^-}$ requires $\Delta\sigma_{\text{had}}^{\text{NP}} < 0$ (negative interference)

The unique scenario to obtain such a **SIZEABLE NEGATIVE interference**

- **SIZEABLE** → **TREE-LEVEL** contribution to modify σ_{had} at $\sqrt{s} < 1 \text{ GeV}$ (hence, **sub-GeV mediator** coupling to the hadronic and electron currents at tree-level)
- **NEGATIVE INTERF.** → NP particle couples via a **VECTOR** current to the u, d quarks (given the dominance of the $\pi^+\pi^-$ channel)

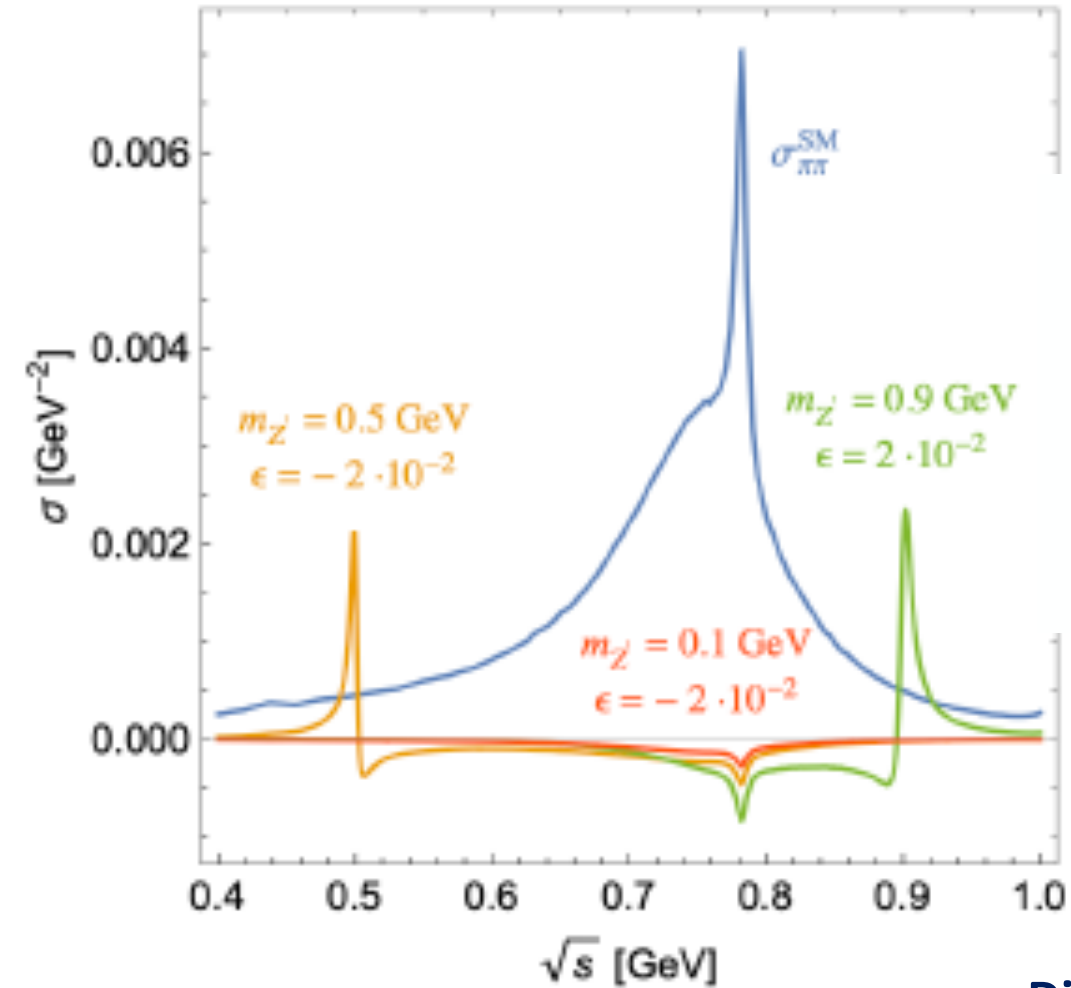
$$\mathcal{L}_{Z'} \supset (g_V^e \bar{e}\gamma^\mu e + g_V^q \bar{q}\gamma^\mu q) Z'_\mu \quad q = u, d \quad m_{Z'} \lesssim 1 \text{ GeV}$$

→ a light spin-1 mediator with vector couplings to first generation SM fermions

$$\frac{\sigma_{\pi\pi}^{\text{SM+NP}}}{\sigma_{\pi\pi}^{\text{SM}}} = \left| 1 + \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + im_{Z'}\Gamma_{Z'}} \right|^2$$

Examples of benchmark values for $m_{Z'}$ and Z' couplings to electrons and up- and down-quarks suitable to solve the $g-2$ discrepancy

$$\gamma = 10^{-2}$$



$$\Delta a_\mu = \frac{1}{4\pi^3} \int_{s_{\text{exp}}}^{\infty} ds K(s) (-\Delta\sigma_{\text{had}}^{\text{NP}}(s))$$

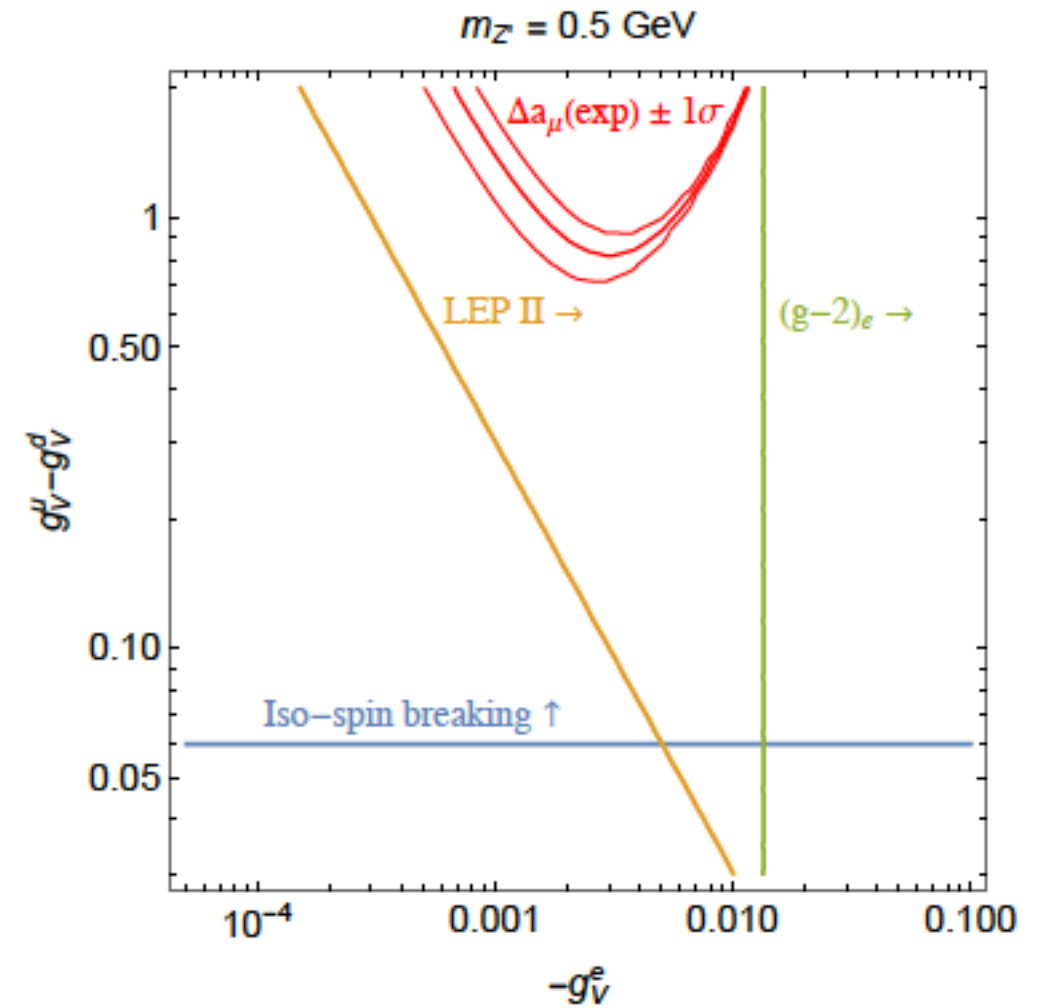
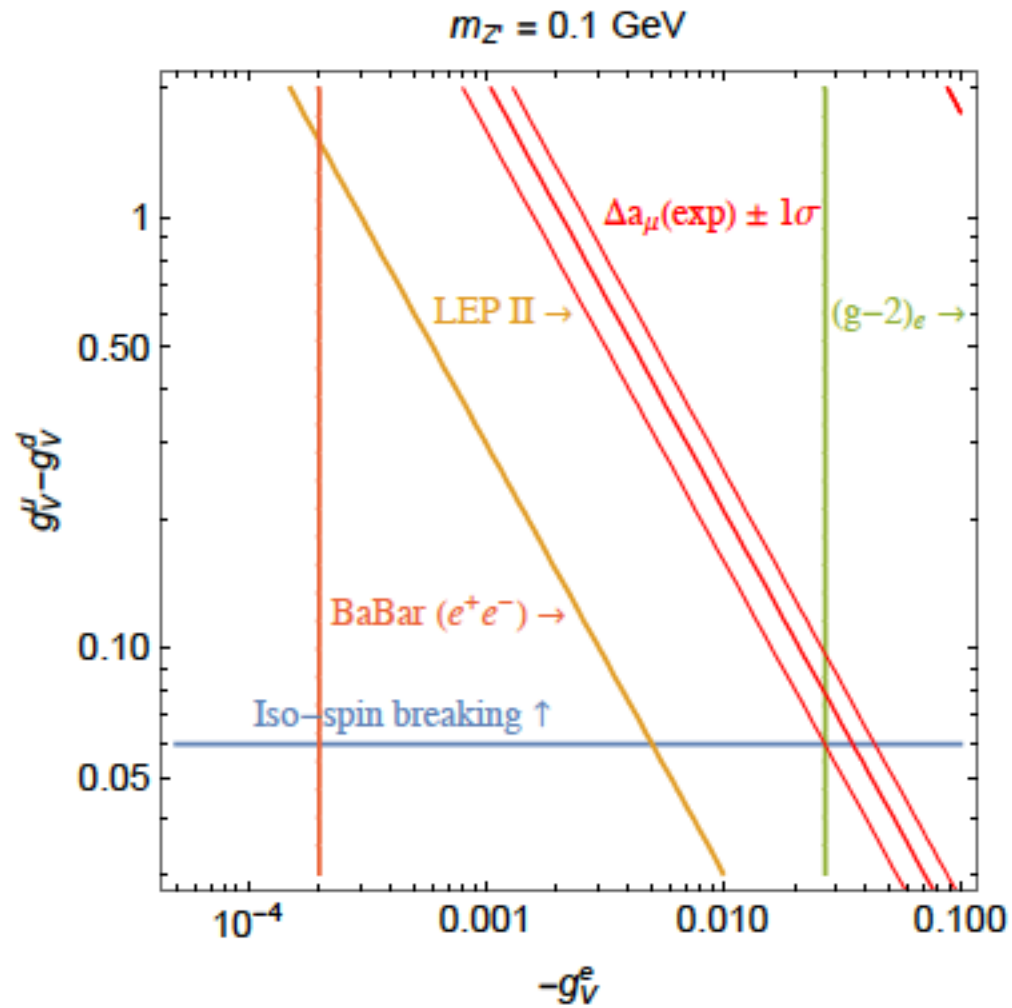
$\sqrt{s_{\text{exp}}} \approx 0.3 \text{ GeV}$
for $\pi^+\pi^-$ channel

$$\Delta\sigma_{\text{had}}^{\text{NP}}(s) \approx \sigma_{\pi\pi}^{\text{SM}}(s) \times \frac{2\epsilon s(s - m_{Z'}^2) + \epsilon^2 s^2}{(s - m_{Z'}^2)^2 + m_{Z'}^4 \gamma^2}$$

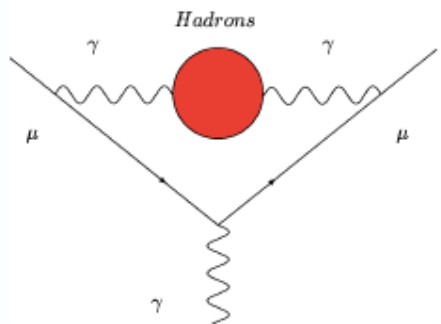
$$\epsilon \equiv g_V^e (g_V^u - g_V^d) / e^2$$

$$\gamma \equiv \Gamma_{Z'} / m_{Z'}$$

At least **TWO independent bounds prevent** to get a sizeable contribution to Δa_μ modifying σ_{had} via Z' exchange to **solve** the “**new**” μ $g-2$ puzzle



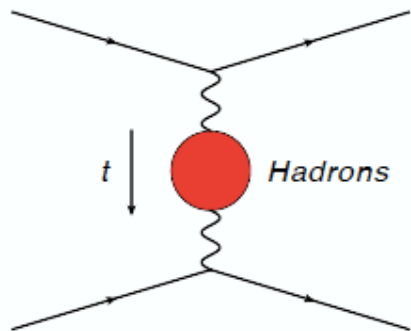
- At present, the leading hadronic contribution a_μ^{HLO} is computed via the **timelike** formula:



$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma_{\text{had}}^0(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)}$$

- Alternatively, exchanging the x and s integrations in a_μ^{HLO}



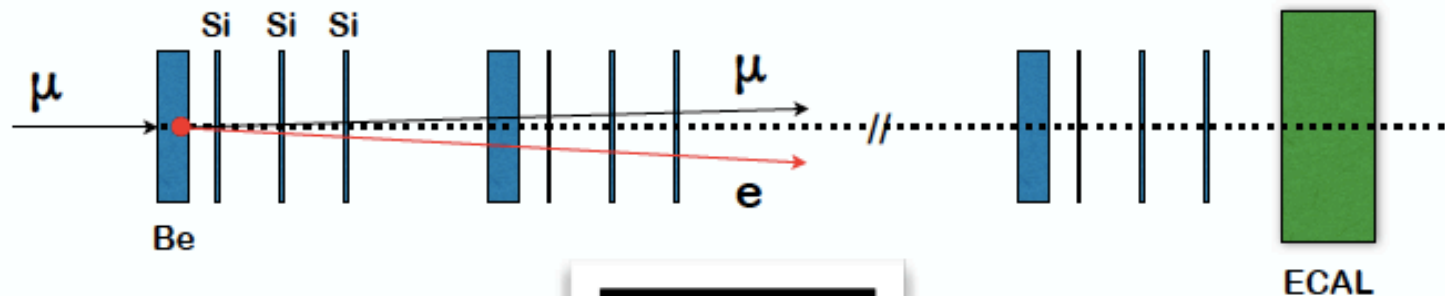
$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the running of α in the **spacelike region**: a_μ^{HLO} can be extracted from scattering data!

- $\Delta\alpha_{\text{had}}(t)$ can be measured via the **elastic scattering $\mu e \rightarrow \mu e$** .
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.

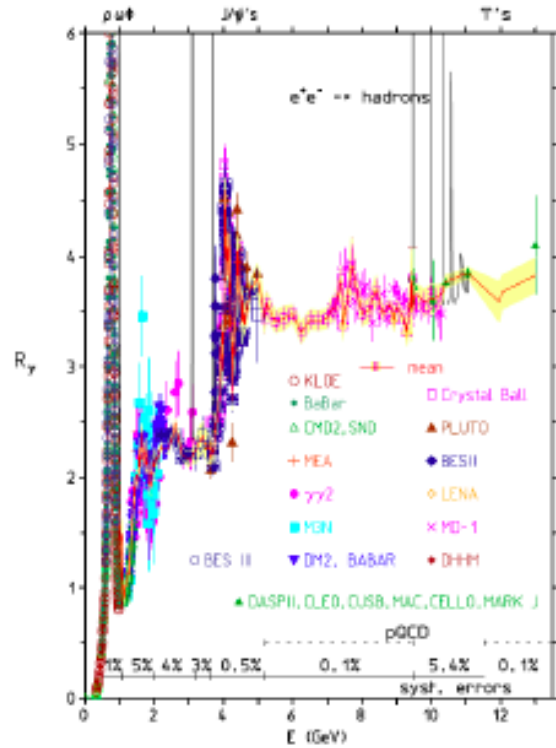


Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,
Nicosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

[Courtesy by M. Passera]

- Letter of Intent submitted to CERN SPSC in 2019: **Test run approved for 2021**

Timelike

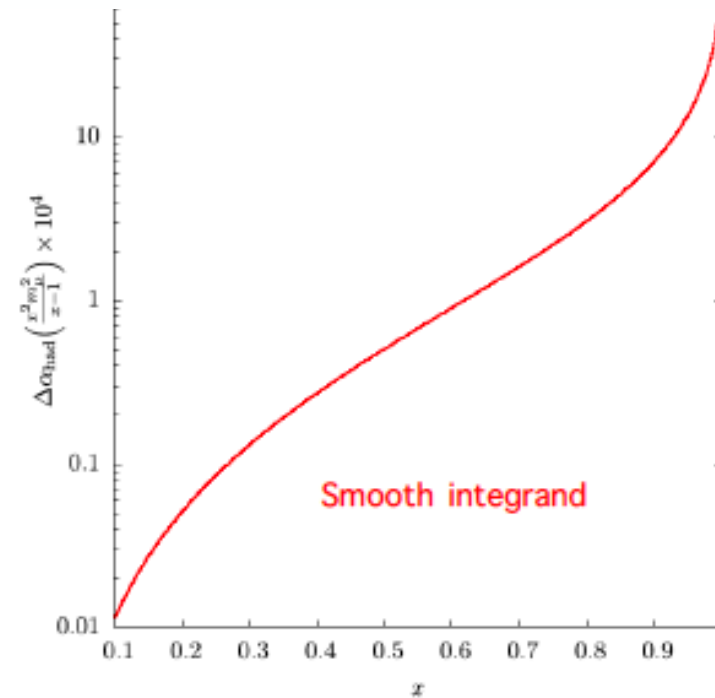


F. Jegerlehner, arXiv:1511.04473



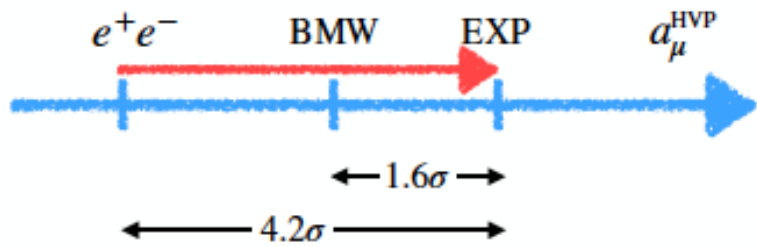
Spacelike

$\Delta\alpha_{\text{had}}(t)$ can be measured via the elastic scattering $\mu e \rightarrow \mu e$.



Carloni Calame, Passera, Trentadue, Venanzoni, PLB 2015

- ✓ Inclusive measurement
- ✓ Smooth integrand
- ✓ Direct interplay with lattice QCD



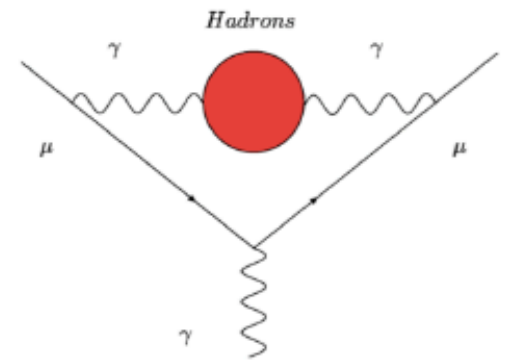
some conclusive thoughts:

- attempt to solve the "new" muon g-2 puzzle introducing NP which modifies $\sigma(e^+e^- \rightarrow \text{hadrons})$, but without affecting a_μ^{HVP} :
 - a) NP \rightarrow light (<1 GeV) vector Z' coupling only to electrons and hadrons;
 - b) the **experimental constraints** on the size of such couplings **prevent** the Z' exchange to provide the needed enhancement of the hadronic σ to suitably address the new g-2 puzzle
- **Two** directions to be vigorously pursued:
 - i) perform **new** independent **lattice QCD** computations of the HVP contribution to a_μ to assess the validity of the **BMWc result** ;
 - ii) identifies **new** experimental ways to probe a_μ^{HVP} (the **MUonE** exp. can (hopefully reasonably) soon provide an **independent determination** of the leading hadronic contributions to a_μ alternative to both the dispersive and lattice methods)

BACK-UP SLIDES

- dominated by $e^+e^- \rightarrow \pi^+\pi^-$ channel (70% of the full hadronic)

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} K(s) \text{Im} \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$



- what is $\sigma_{\text{had}}(s)$?
 - Includes Final State Radiation (FSR)
 - Initial State Radiation (ISR) and FSR/ISR interference are subtracted
 - Vacuum polarization also subtracted (by rescaling exp. cross-section by $|\alpha/\alpha(s)|^2$)

➔ part of higher-order HVP

[WP20, 2006.04822]

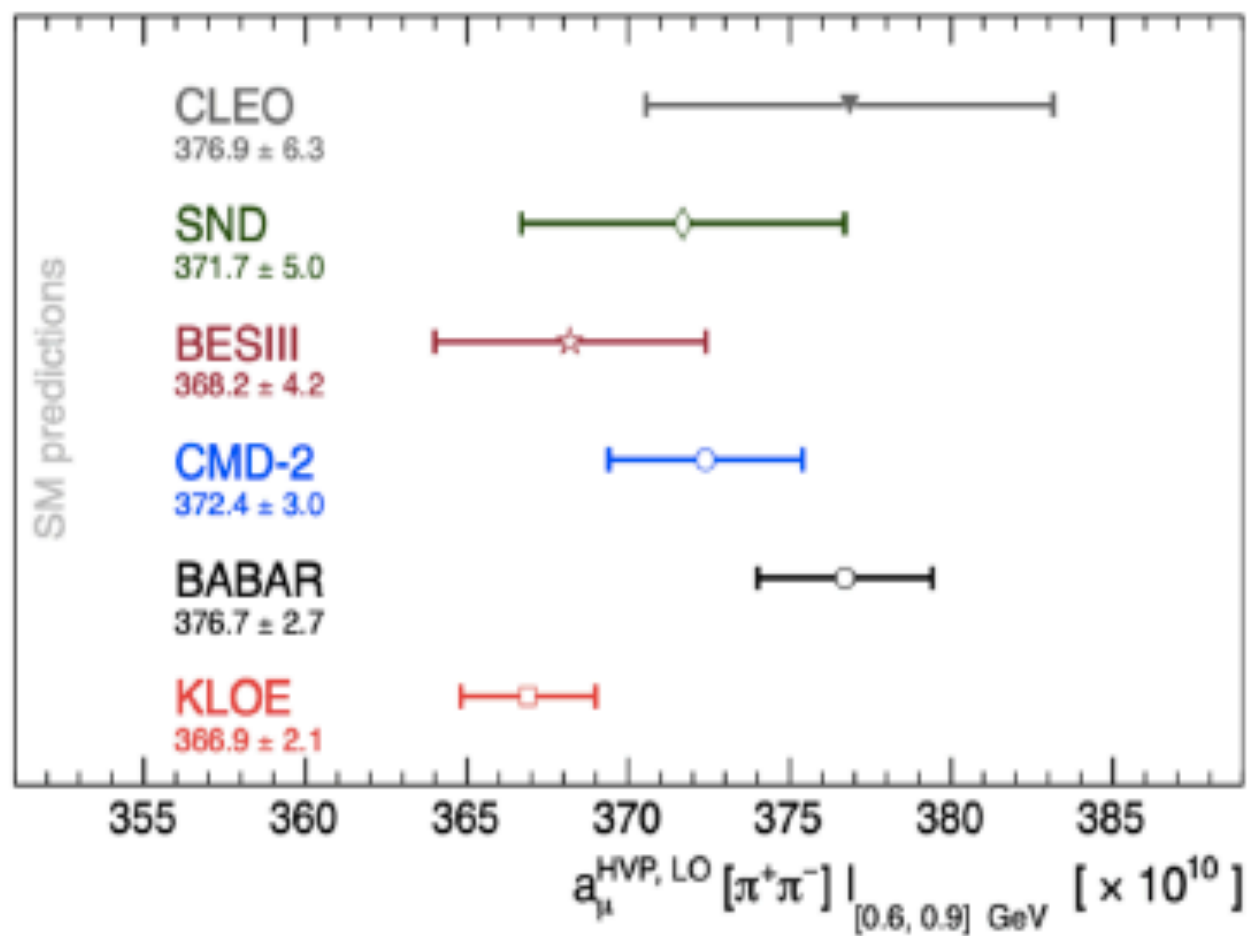


Figure 15: Comparison of results for $a_\mu^{\text{HVP, LO}}[\pi\pi]$, evaluated between 0.6 GeV and 0.9 GeV for the various experiments.

NP in Bhabha scattering?

- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona, Nardi 21/2.09/39]

$$\mathcal{L}_{e^+e^-}^{\text{SM}} = \frac{N_{\text{Bha}}}{\sigma_{\text{eff}}^{\text{SM}}} \quad \longrightarrow \quad \mathcal{L}_{e^+e^-} = \mathcal{L}_{e^+e^-}^{\text{SM}} \frac{\sigma_{\text{eff}}^{\text{SM}}}{\sigma_{\text{eff}}}$$

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}^{\text{SM}} (1 + \delta_R)$$

$$\sigma_{\text{had}} \propto N_{\text{had}} / \mathcal{L}_{e^+e^-} \quad \longrightarrow \quad \sigma_{\text{had}} \rightarrow \sigma_{\text{had}} (1 + \delta_R)$$

$$a_{\mu}^{\text{LO,HVP}} \rightarrow a_{\mu}^{\text{LO,HVP}} (1 + \delta_R)$$

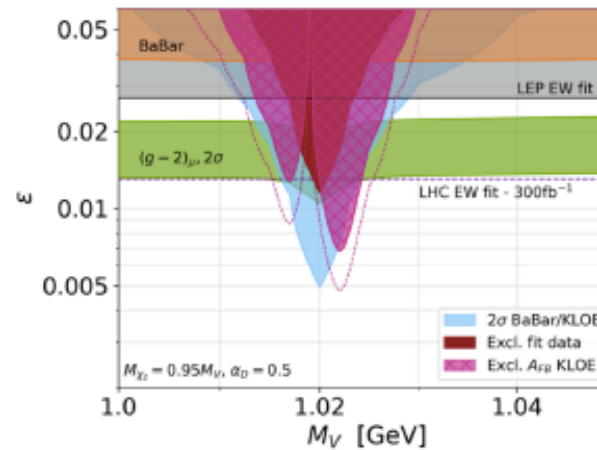
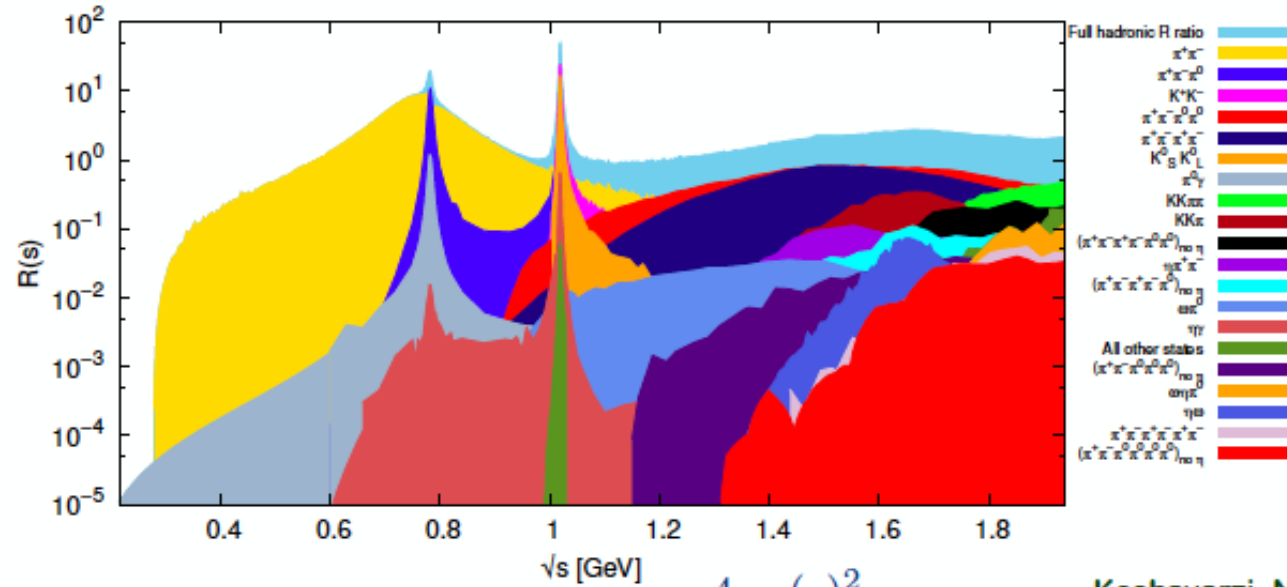
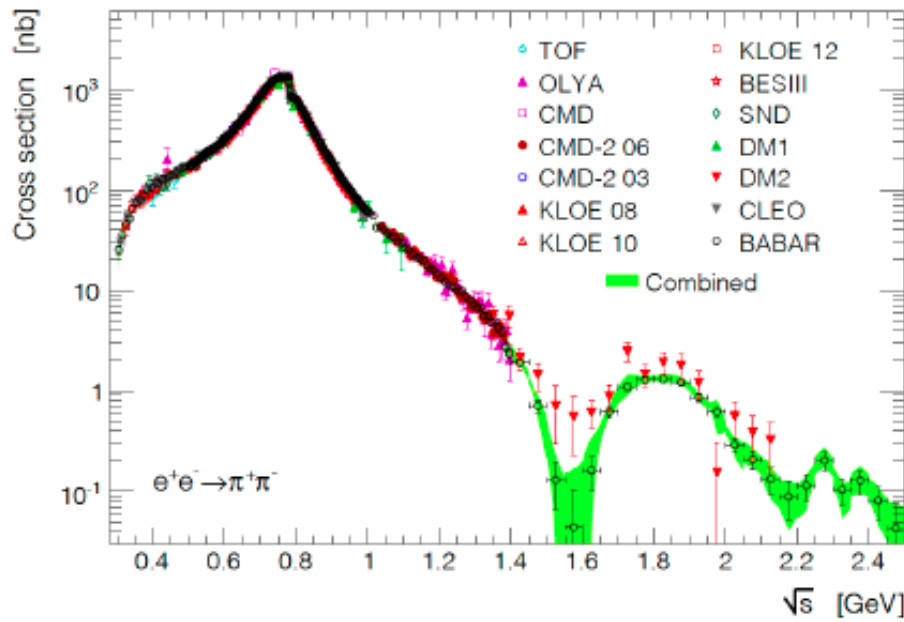


Figure 3. Parameter range compatible at 2σ with the experimental measurement of Δa_{μ} (green region) resulting from a redetermination of the KLOE luminosity, for $\alpha_D = 0.5$, $m_{X_2} = 0.95m_V$ and $m_{X_1} = 25$ MeV. In the blue region the KLOE and BaBar results for σ_{had} are brought into agreement at 2σ . The red region corresponds to a shift of the KLOE measurement in tension with BaBar (and with the other experiments) at more than 2σ .

$e^+e^- \rightarrow \pi^+\pi^-$ dominance of the low-energy hadronic cross-section



Keshavarzi, Nomura Teubner
PRD 2018



Davier, Hoecker, Malaescu, Zhang
EPJC 2020

$\Lambda \approx \nu$: SUSY and the muon ($g - 2$)

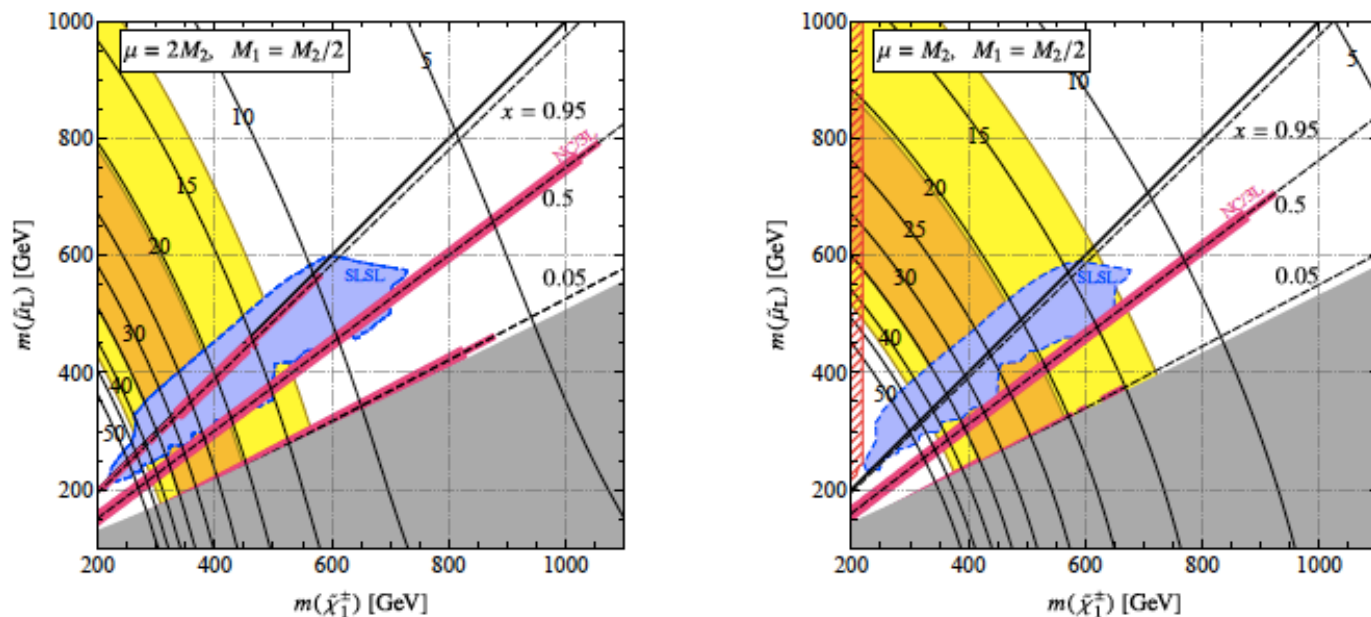


Figure: LHC Run 2 bounds on SUSY scenario for the muon $g - 2$ anomaly for $\tan \beta = 40$. Orange (yellow) regions satisfy the muon $g - 2$ anomaly at the 1σ (2σ) level [Endo et al., '20].

$$(a_{\mu}^{\text{SM}})_{\text{weak}} \approx \frac{g^2 m_{\mu}^2}{32\pi^2 M_W^2} \approx 2 \times 10^{-9}$$

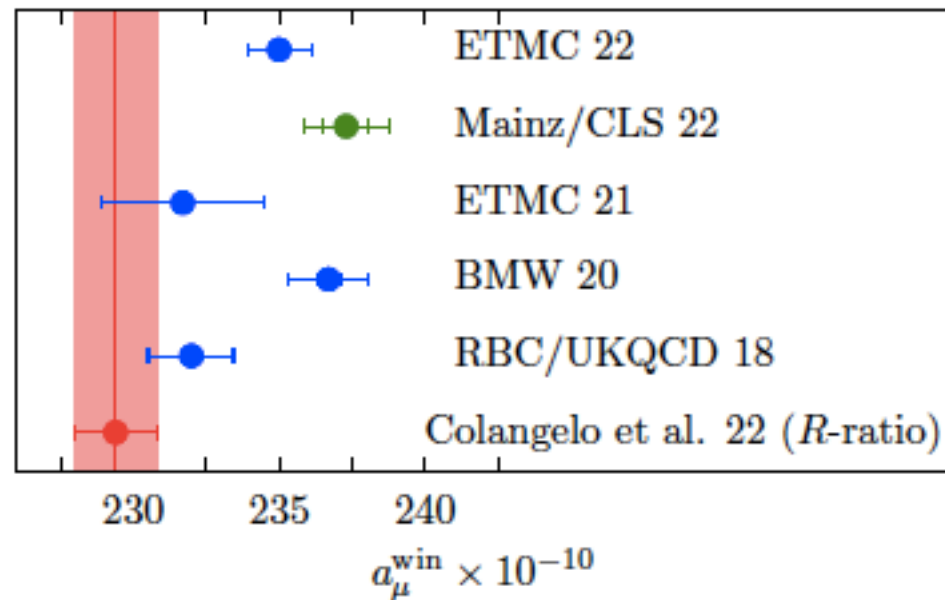
$$a_{\mu}^{\text{SUSY}} \approx \frac{g^2 m_{\mu}^2 \tan \beta}{32\pi^2 \tilde{m}^2} \approx 2 \times 10^{-9}$$

$\tilde{m} = 500\text{GeV} \ \& \ \tan \beta = 40$

COMPARISON WITH RESULTS FOR a_μ^{win}

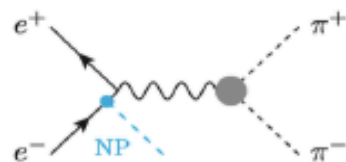
- Isospin-breaking correction $+(0.70 \pm 0.47) \times 10^{-10}$ included:

$$a_\mu^{\text{win}} = (237.30 \pm 0.79_{\text{stat}} \pm 1.13_{\text{syst}} \pm 0.05_{\text{Q}} \pm 0.47_{\text{IB}}) \times 10^{-10}$$



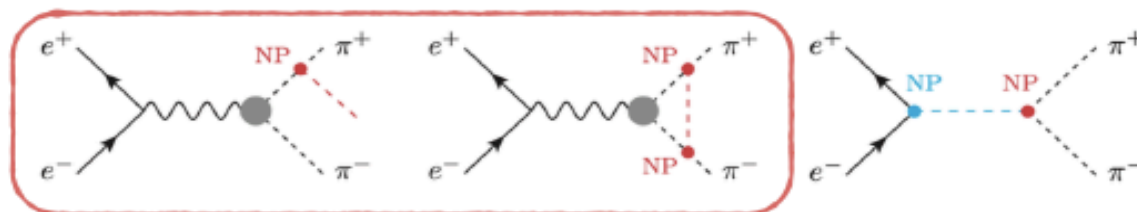
- 3.9σ tension with data-driven estimate in [2205.12963, Colangelo et al.].
- Genuine difference between lattice and data-driven results?

- Light new physics inducing a sub-GeV modification of σ_{had} is the only possibility



1. NP coupled only to **electrons** \rightarrow severe bounds

[See however Darmé, Grilli di Cortona, Nardi 2112.09139 NP in Bhabha scattering? \rightarrow backup slides]



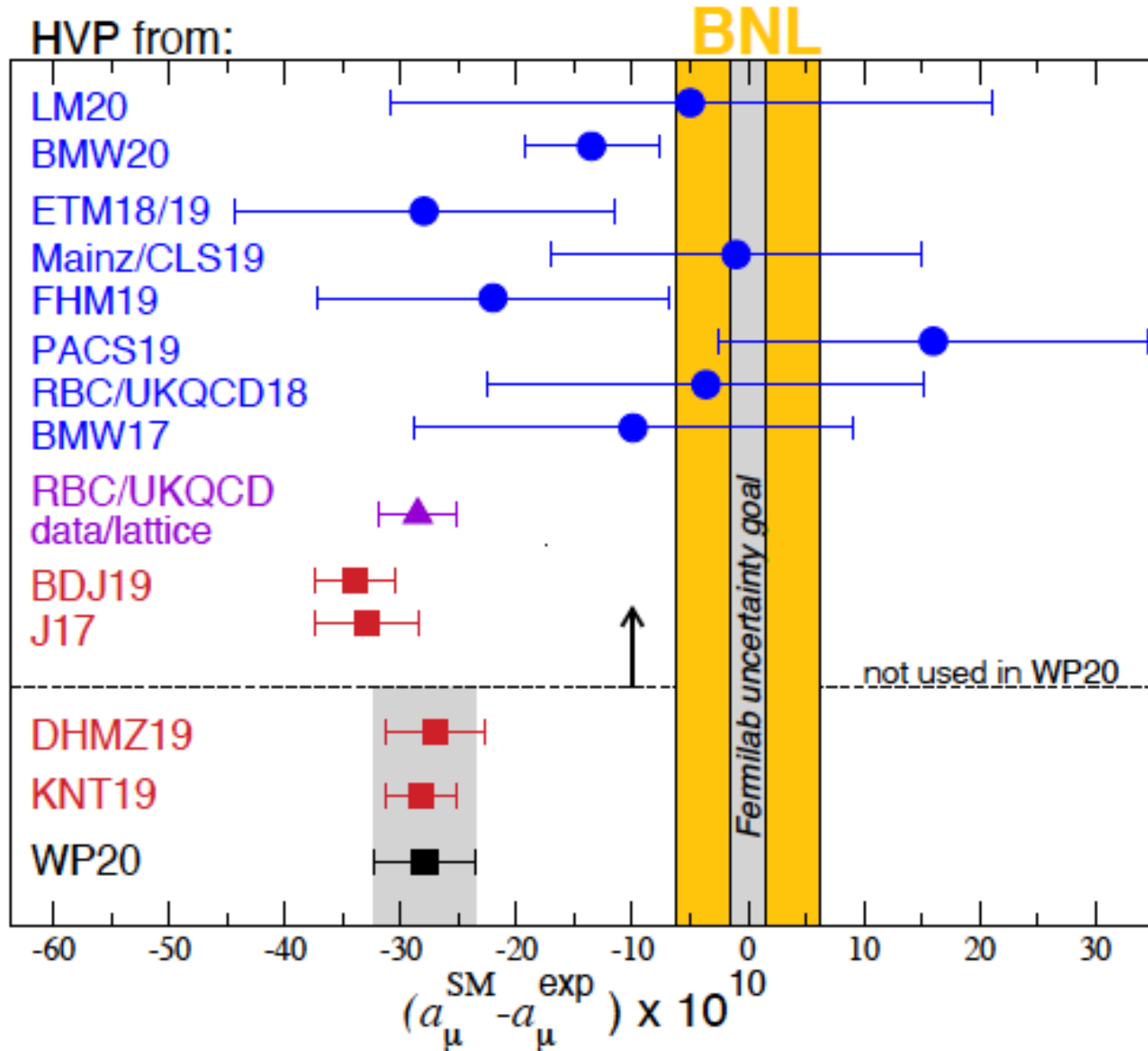
2. NP coupled only to **hadrons**

FSR effects due to NP should be included into $\sigma_{\text{had}}(s)$, not easy to be accounted for... (depend on exp. cuts and mass of NP)

\rightarrow however, we know that in the QED case

$$(a_{\mu}^{\text{HVP}})^{\text{FSR}}_{e^+e^-} \approx 50 \times 10^{-11} \quad \longleftrightarrow \quad |(a_{\mu}^{\text{HVP}})_{\text{BMW}} - (a_{\mu}^{\text{HVP}})^{\text{WP20}}_{e^+e^-}| \approx 150 \times 10^{-11}$$

HADRONIC VACUUM POLARIZATION CONTRIBUTION



Ab-initio lattice calculations

Dispersive relations,
 $e^+e^- \rightarrow$ hadrons exps.

However, **severe constraints on the Z' couplings** to electrons and to hadrons

- for $m_{Z'} \lesssim 0.3 \text{ GeV}$ ($Z' \rightarrow e^+e^-$ is the main decay mode)

$$e^+e^- \rightarrow \gamma Z' \text{ @ BaBar} \quad \longrightarrow \quad g_V^e \lesssim 2 \cdot 10^{-4}$$

- for $m_{Z'} \gtrsim \text{MeV}$

$$\text{electron } g-2 \quad \longrightarrow \quad |g_V^e| \lesssim 10^{-2} (m_{Z'}/0.5 \text{ GeV})$$

$e^+e^- \rightarrow q\bar{q}$ has been measured with per-cent accuracy at LEP-II

$$\frac{\sigma_{qq}^{\text{SM+NP}}}{\sigma_{qq}^{\text{SM}}} \approx 1 + 2 \frac{g_V^e g_V^q}{e^2 Q_q} \quad \longrightarrow \quad |g_V^e g_V^q| \lesssim 4.6 \cdot 10^{-4} |Q_q| \quad (\epsilon \lesssim 3.3 \cdot 10^{-3})$$

Iso-spin breaking observables

$$\longrightarrow \quad |g_V^u - g_V^d| \lesssim 0.06$$

charged vs. neutral pion mass² difference $\Delta m^2 = m_{\pi^+}^2 - m_{\pi^0}^2$ (rescaling the lattice QCD calculation of Frezzotti, Gagliardi, Lubicz, Martinelli, Sanfilippo and Simula 2112.01066)