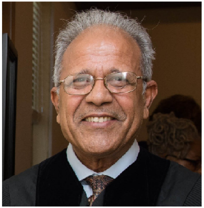
Rabi and the Parity Solution to the Strong CP Problem

K.S. Babu

Oklahoma State University



Rabi-Fest

University of Maryland

October 20-21, 2022

Rabi's Major Contributions in Research

- Proposal and development of of left-right symmetric gauge theories
- Seesaw mechanism for neutrino masses (type-I, type-II, Inverse)
- Origin of $B - L$ symmetry and neutron-antineutron oscillation
- Spontaneous lepton number breaking and the Majoron idea
- Neutrino mixing pattern in unified theories based on $SO(10)$
- Asymmetric inflationary models
- **Parity as a solution to the strong CP problem**
- Many others, including neutrino mass models, neutrino magnetic moment, flavor models, proton decay, spontaneous R-parity breaking, supersymmetric models,,,,

Rabi's Outstanding Mentorship

- Rabi has been an outstanding mentor to a large group of students, postdocs and junior researchers
- He continues to look after their well-being even after they move on from his group
- Personally I have been a beneficiary of Rabi's kindness and caring mentorship
- Apparently I have published over 50 research papers in collaboration with Rabi
- I wish to congratulate Rabi for his accomplishments on this front, and also express my deep appreciation

The Strong CP Problem

- QCD interactions appear to conserve CP symmetry. However,

$$\overline{\theta} = \theta_{QCD} + \mathrm{ArgDet}(M_Q)$$

  is a physical parameter of the theory

- $\overline{\theta}$ contributes to neutron EDM

- $d_n \sim 10^{-16}\ \overline{\theta}$ e-cm $\Rightarrow \overline{\theta} < 10^{-10}$

- The smallness of a dimensionless parameter is the strong CP problem

- Setting $\overline{\theta}$ to zero is unnatural, since weak interactions require $\mathcal{O}(1)$ CP violation in that sector

- Note that $\overline{\theta}$ is P- and T-odd

- Naturally, Rabi sought a solution to the "strong P problem" with spontaneously broken Parity

Mohapatra, Senjanovic (1978)

Rabi's Solution to the Strong P Problem

- Imagine Parity is spontaneously broken. $\Rightarrow$

$$\theta_{QCD} = 0 \text{ by Parity.}$$

- If the quark mass matrix is hermitian, also by Parity, then $\overline{\theta} = 0$ at tree-level.
- Quantum corrections could induce small nonzero $\overline{\theta}$.
- In left-right symmetric models, Parity symmetry is exact, with

$$q_L \leftrightarrow q_R, \qquad \Phi \leftrightarrow \Phi^\dagger$$

- Consequently, the Yukawa coupling $\left(Y_q \, \overline{q}_L \, \Phi \, q_R \right)$ is hermitian:

$$Y_q = Y_q^\dagger$$

- However, the quark mass matrix is

$$M_q = Y_q \langle \Phi \rangle$$

- It is a challenge to make the VEVs of Φ real.
- Rabi and Goran used discrete symmetries to achieve this goal.

Parity Solution to the Strong P Problem

- The Higgs potential of the standard left-right symmetric model has a single complex coupling:

$$V \supset \left\{ \alpha_2 e^{i\delta_2} \left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\Delta_L \Delta_L^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\mathrm{Tr}(\Delta_R \Delta_R^\dagger) \right] + h.c. \right\}$$

  Here Δ_R is an $SU(2)_R$ triplet or doublet, with Δ_L being its Parity partner.

- For nonzero phase δ_2, the VEVs of Φ would develop a relative phase of order one, spoiling the Parity solution to strong CP problem.
  See talk by Ravi Kuchimanchi tomorrow

- Supersymmetric Higgs sector would not admit such couplings, and would lead to real VEVs of Φ

  Kuchimanchi (1996)
  Mohapatra, Rasin (1996)
  Mohapatra, Rasin, Senjanovic (1997)
  Babu, Dutta, Mohapatra (2002)

SUSY-Assistance to the Strong P Problem

- Several SUSY models have been constructed within left-right symmetry that solves the strong P problem

- If the theory has two hermitian flavor matrices Y_u and Y_d, and if all flavor singlets are real, the lowest order contribution to $\overline{\theta}$ would arise from:

$$c_1 \mathrm{ImTr}(Y_u^2 Y_d^4 Y_u^4 Y_d^2) + c_2 \mathrm{ImTr}(Y_d^2 Y_u^4 Y_d^4 Y_u^2)$$

- In explicit models the coefficients $c_{1,2}$ are of order

$$c_{1,2} \sim \left(\frac{\ln(M_{W_R}/M_{W_L})}{16\pi^2} \right)^4$$

- This leads to and induced $\overline{\theta}$ of order

$$\overline{\theta} \sim 3 \times 10^{-27} (\tan\beta)^6 (c_1 - c_2)$$

Babu, Dutta, Mohapatra (2002)

- Argument similar to Eliis, Gaillard (1979) for SM contribution to $\overline{\theta}$

Solution with P Symmetry Alone

- Parity alone can solve the strong CP problem
- Key point is to go easy with the Higgs sector
- If only an $SU(2)_L$ doublet Higgs χ_L and an $SU(2)_R$ doublet Higgs χ_R are used for symmetry breaking, gauge rotations would guarantee that their VEVs are real
- Fermion mass generation is achieved via mixing of the usual fermions with vector-like fermions via χ_L and χ_R
- This class of left-right symmetric models belong to "universal seesaw" class Davidson, Wali (1987)
- Parity is softly broken by the mass terms of χ_L and χ_R, which leads to consistent phenomenology
- This setup can solve the strong P problem via parity symmetry alone. Babu, Mohapatra (1990)

Left-Right Symmetry with Universal Seesaw

- Gauge symmetry is extended to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$
- These models are motivated on several grounds:
  - Provide understanding of Parity violation
  - Better understanding of smallness of Yukawa couplings
  - Requires right-handed neutrinos to exist
  - Provide a solution to the strong CP problem via Parity
  - Naturally light *Dirac neutrinos* may be realized
  - Possible relevance to experimental anomalies

Davidson, Wali (1987) – universal seesaw
Babu, He (1989) – Dirac neutrino
Babu, Mohapatra (1990) – solution to strong CP problem via parity
Babu, Dutta, Mohapatra (2018) – R_{D^*} solution
Craig, Garcia Garcia, Koszegi, McCune (2020) – flavor constraints
Babu, He, Su, Thapa (2022) – neutrino oscillations with Dirac neutrinos
Babu, Dcruz (2022) – Cabibbo anomaly, W mass anomaly

Left-Right Symmetric Model

- Fermion transformation: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$:

$$Q_L\ (3,2,1,1/3) = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_R\ (3,1,2,1/3) = \begin{pmatrix} u_R \\ d_R \end{pmatrix},$$

$$\Psi_L\ (1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \Psi_R\ (1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}.$$

- Vector-like fermions are introduced to realize seesaw for charged fermion masses:

$$P(3,1,1,4/3), \quad N(3,1,1,-2/3), \quad E(1,1,1,-2)\ .$$

- Higgs sector is very simple:

$$\chi_L\ (1,2,1,1) = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R\ (1,1,2,1) = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$$

- $\langle \chi_R^0 \rangle = \kappa_R$ breaks $SU(2)_R \times U(1)_X$ down to $U(1)_Y$, and $\langle \chi_L^0 \rangle = \kappa_L$ breaks the electroweak symmetry with $\kappa_R \gg \kappa_L$

Seesaw for Charged Fermion Masses

- Yukaw interactions:

$$\mathcal{L} = y_u\ (\bar{Q}_L\tilde{\chi}_L + \bar{Q}_R\tilde{\chi}_R)P + y_d\ (\bar{Q}_L\chi_L + \bar{Q}_R\chi_R)N \\ + y_\ell\ (\bar{\Psi}_L\chi_L + \bar{\Psi}_R\chi_R)E + h.c.$$

- Vector-like fermion masses:

$$\mathcal{L}_{\rm mass} = M_{p^0}\ \bar{P}P + M_{N^0}\ \bar{N}N + M_{E^0}\ \bar{E}E$$

- Seesaw for charged fermion masses:

$$M_F = \begin{pmatrix} 0 & y\kappa_L \\ y^\dagger\kappa_R & M \end{pmatrix} \Rightarrow m_f = \frac{y^2\kappa_L\kappa_R}{M}$$

- Under Parity, fields transform as:

$$Q_L \leftrightarrow Q_R, \quad \Psi_L \leftrightarrow \Psi_R, \quad F_L \leftrightarrow F_R, \quad \chi_L \leftrightarrow \chi_R$$

  Consquently $y_{u,d,\ell} = y^\dagger_{u,d,\ell}$, and $M_{F^0} = M^\dagger_{F^0}$

- $\theta_{QCD} = 0$ due to Parity; $\mathrm{ArgDet}(M_U M_D) = 0$; induced $\overline{\theta} = 0$ at one-loop; small and finite $\overline{\theta}$ arises at two-loop

Vanishing $\overline{\theta}$ at one-loop

- Correction to the quark mass matrix:

$$\mathcal{M}_U = \mathcal{M}_U^0 (1 + C)$$

- $\overline{\theta}$ is given by

$$\overline{\theta} = \mathrm{ArgDet}(1 + C) = \mathrm{ImTr}(1 + C) = \mathrm{ImTr}\, C_1$$

where a loop-expansion is used:

$$C = C_1 + C_2 + \ldots$$

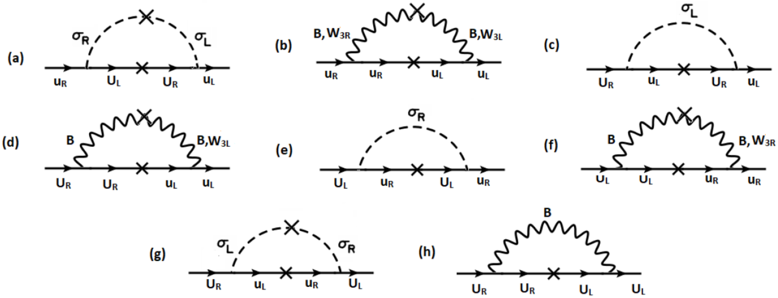
- The corrected mass matrix has a form:

$$\delta\mathcal{M}_U = \begin{bmatrix} \delta M_{LL}^U & \delta M_{LH}^U \\ \delta M_{HL}^U & \delta M_{HH}^U \end{bmatrix}$$

- From here $\overline{\theta}$ can be computed to be:

$$\overline{\theta} = \mathrm{ImTr}\left[-\frac{1}{\kappa_L \kappa_R} \delta M_{LL}^U (Y_U^\dagger)^{-1} M_U Y_U^{-1} + \frac{1}{\kappa_L} \delta M_{LH}^U Y_U^{-1} + \frac{1}{\kappa_R} \delta M_{HL}^U (Y_U^\dagger)^{-1} \right] .$$

Feynman Diagrams for induced $\overline{\theta}$



- Each diagram separately gives zero contribution to $\overline{\theta}$
- Induced value of $\overline{\theta}$ at two-loop is of order 10^{-11}

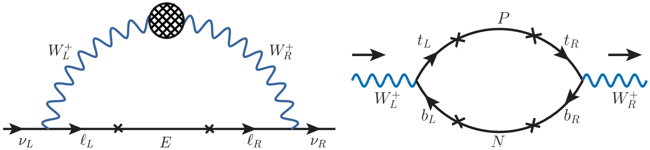
Matter Content from $SU(5)_L \times SU(5)_R$

$$\psi_{L,R} = \begin{bmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{bmatrix}_{L,R} \qquad \chi_{L,R} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3^c & -U_2^c & -u_1 & -d_1 \\ -U_3^c & 0 & U_1^c & -u_2 & -d_2 \\ U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -E^c \\ d_1 & d_2 & d_3 & E^c & 0 \end{bmatrix}_{L,R} ,$$

- All left-handed SM fermions are in $\{(10,1)+(\bar{5},1)\}$, while all right-handed SM fermions are in $\{(1,10)+(1,\bar{5})\}$
- There is ν_R in the theory, but no seesaw for neutrino sector
- Small *Dirac neutrino masses* arise as two-loop radiative corrections
- We have evaluated the flavor structure of the two-loop diagrams and shown consistency with neutrino data

Naturally Light Dirac Neutrinos

- Higgs sector is very simple: $\chi_L(1,2,1,1/2) + \chi_R(1,1,2,1/2)$
- $W_L^+ - W_R^+$ mixing is absent at tree-level in the model
- $W_L^+ - W_R^+$ mixing induced at loop level, which in turn generates Dirac neutrino mass at two loop Babu, He (1989)



- Flavor structure of two loop diagram needs to be studied to check consistency
- Oscillation date fits well within the model regardless of Parity breaking scale Babu, He, Su, Thapa (2022)

Loop Integrals

$$M_{\nu^D} = \frac{-g^4}{2} y_t^2 y_b^2 y_\ell^2 \kappa_L^3 \kappa_R^3 \frac{r\ M_P M_N M_{E_\ell}}{M_{W_L}^2 M_{W_R}^2}\ I_{E_\ell}$$

$$I_{E_\ell} = \int\int \frac{d^4k d^4p}{(2\pi)^8} \frac{3M_{W_L}^2 M_{W_R}^2 + (p^2 - M_{W_L}^2)(p^2 - M_{W_R}^2)}{k^2(p+k)^2(k^2 - M_N^2)((p+k)^2 - M_P^2)p^2(p^2 - M_{E_\ell}^2)(p^2 - M_{W_L}^2)(p^2 - M_{W_R}^2)}$$

$$\begin{aligned}
G_1 = \frac{3}{(r_3-1)(r_4-1)(r_4-r_3)} \Bigg[& -\frac{\pi^2}{6}(r_1+r_2)(r_3-1)(r_3-r_4)(r_4-1) \\
& + r_3 r_4 (r_4 - r_3)\left(r_1 F\left[\frac{1}{r_1}, \frac{r_2}{r_1}\right] + r_2 F\left[\frac{1}{r_2}, \frac{r_1}{r_2}\right] + F\left[r_1, r_2\right]\right) \\
& - (r_4-1)r_4 \left(r_1 F\left[\frac{r_3}{r_1}, \frac{r_2}{r_1}\right] + r_2 F\left[\frac{r_3}{r_2}, \frac{r_1}{r_2}\right] + r_3 F\left[\frac{r_1}{r_3}, \frac{r_2}{r_3}\right]\right) \\
& + (r_3-1)r_3 \left(r_1 F\left[\frac{r_4}{r_1}, \frac{r_2}{r_1}\right] + r_2 F\left[\frac{r_4}{r_2}, \frac{r_1}{r_2}\right] + r_4 F\left[\frac{r_1}{r_4}, \frac{r_2}{r_4}\right]\right) \\
& + (r_3-r_4)(r_3-1)(r_4-1)\left(r_2 Li_2\left[1 - \frac{r_1}{r_2}\right] + r_1 Li_2\left[1 - \frac{r_2}{r_1}\right]\right) \\
& + r_3 r_4 (r_3 - r_4)\left(Li_2[1-r_1] + Li_2[1-r_2] + r_1 Li_2\left[\frac{r_1 - 1}{r_1}\right] + r_2 Li_2\left[\frac{r_2-1}{r_2}\right]\right) \\
& + r_4(r_4-1)\left(r_3 Li_2\left[1 - \frac{r_1}{r_3}\right] + r_3 Li_2\left[1 - \frac{r_2}{r_3}\right] + r_1 Li_2[1 - \frac{r_3}{r_1}] + r_2 Li_2[1 - \frac{r_3}{r_2}]\right) \\
& - r_3(r_3-1)\left(r_4 Li_2\left[1 - \frac{r_1}{r_4}\right] + r_4 Li_2\left[1 - \frac{r_2}{r_4}\right] + r_1 Li_2[1 - \frac{r_4}{r_1}] + r_2 Li_2[1 - \frac{r_4}{r_2}]\right) \Bigg] .
\end{aligned} \tag{-1}$$

Neutrino Fit in Two-loop Dirac Mass Model

Oscillation parameters	3σ range NuFit5.1	Model prediction			
		BP I (NH)	BP II (NH)	BP III (IH)	BP IV (IH)
$\Delta m^2_{21}(10^{-5}\ \mathrm{eV}^2)$	6.82 - 8.04	7.42	7.32	7.35	7.30
$\Delta m^2_{23}(10^{-3}\ \mathrm{eV}^2)$(IH)	2.410 - 2.574	-	-	2.48	2.52
$\Delta m^2_{31}(10^{-3}\ \mathrm{eV}^2)$(NH)	2.43 - 2.593	2.49	2.46	-	-
$\sin^2\theta_{12}$	0.269 - 0.343	0.324	0.315	0.303	0.321
$\sin^2\theta_{23}$ (IH)	0.410 - 0.613	-	-	0.542	0.475
$\sin^2\theta_{23}$ (NH)	0.408 - 0.603	0.491	0.452	-	-
$\sin^2\theta_{13}$ (IH)	0.02055 - 0.02457	-	-	0.0230	0.0234
$\sin^2\theta_{13}$(NH)	0.02060 - 0.02435	0.0234	0.0223	-	-
δ_{CP} (IH)	192 - 361	-	-	271°	296°
δ_{CP} (NH)	105 - 405	199°	200°	-	-
m_{light} (10^{-3}) eV		0.66	0.17	0.078	4.95
M_{E_1}/M_{W_R}		917	321.3	639	3595
M_{E_2}/M_{W_R}		0.650	19.3	1.54	5.03
M_{E_3}/M_{W_R}		0.019	1.26	0.054	2.94

- Ten parameters to fit oscillation data
- Both normal ordering and inverted ordering allowed
- Dirac CP phase is unconstrained
- Left-right symmetry breaking scale is not constrained

Tests with $N_{\rm eff}$ in Cosmology

- Dirac neutrino models of this type will modify $N_{\rm eff}$ by about 0.14

$$\Delta N_{\rm eff} \simeq 0.027 \left(\frac{106.75}{g_\star\left(T_{\rm dec}\right)}\right)^{4/3} g_{\rm eff}$$

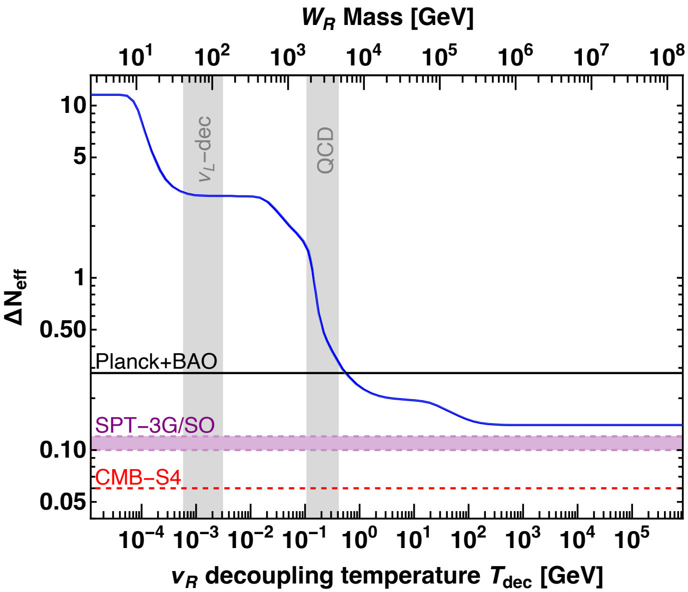
$$g_{\rm eff} = (7/8) \times (2) \times (3) = 21/4$$

- Can be tested in CMB measurements: $N_{\rm eff} = 2.99 \pm 0.17$ (Planck+BAO)

$$G_F^2 \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 T_{\rm dec}^5 \approx \sqrt{g^*(T_{\rm dec})}\,\frac{T_{\rm dec}^2}{M_{\rm Pl}}$$

$$T_{\rm dec} \simeq 400 \text{ MeV} \left(\frac{g_*\left(T_{\rm dec}\right)}{70}\right)^{1/6} \left(\frac{M_{W_R}}{5 \text{ TeV}}\right)^{4/3}$$

- Present data sets a lower limit of 7 TeV on W_R mass



W_R Mass [GeV]
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
ΔN_{eff}
10
5
1
0.50
0.10
0.05
ν_L-dec
QCD
Planck+BAO
SPT−3G/SO
CMB−S4
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
10^4
10^5
ν_R decoupling temperature T_{dec} [GeV]

Anomalies and the P Symmetric Model

- Currently there are several experimental anomalies. The P symmetric model may be relevant to some of these
- Anomalies include:
  - Muon $g-2$
  - R_K, R_{K^*} in B meson decay
  - R_D, R_{D^*} in B deays
  - W-boson mass shift
  - Cabibbo anomaly
- Not all anomalies find resolution here
- Notably, muon $g-2$ is hard to explain, without further ingredients
- Cabibbo anomaly and W mass shift fit in nicely with testable predictions

Babu, Dcruz (2022)

Explaining the Cabibbo Anomaly

- The first row of the CKM matrix appears to show a 3 sigma deviation from unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$$

- The sum of the first column also deviates slightly from unity:

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9970(18)$$

  Suggestive of mixing of up or down-quark with a vector-like quark

- Occurs naturally in the quark seesaw model. However, if the up-quark mixes with a heavy U-quark via

$$M_{\rm up} = \begin{bmatrix} 0 & y_u \kappa_L \\ y_u^* \kappa_R & M_U \end{bmatrix},$$

  $u_L - U_L$ mixing is too small, suppressed by u-quark mass.

- This is a consequence of Parity symmetry

Explaining the Cabibbo Anomaly (cont.)

- A way out: Mix up-quark with two of the U-quarks:

$$M_{\rm up} = \begin{bmatrix} 0 & y_u\kappa_L & 0 \\ y_u^*\kappa_L & M_1 & M_2 \\ 0 & M_2 & 0 \end{bmatrix}$$

- In this case large value of $y_u\kappa_L \sim 200$ GeV is allowed, without generating large u-quark mass. Note: $\mathrm{Det}(M_{\rm up}) = 0$
- Assume CKM angles arise primarily from down sector. Then the full 5×3 CKM matrix spanning (u, c, t, U_1, U_2) and (d, s, b) is:

$$V_{CKM} = \begin{bmatrix} c_L V_{ud} & c_L V_{us} & c_L V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ -s_L s_L' V_{ud} & -s_L s_L' V_{us} & -s_L s_L' V_{ub} \\ -s_L c_L' V_{ud} & -s_L c_L' V_{us} & -s_L c_L' V_{ub} \end{bmatrix}$$

- $s_L = 0.0387$ explains the apparent unitarity violation

Consistency with other constraints

- In order to get $s_L = 0.038$, one of the U-quark mass should be below 5 TeV.
- Owing to the $u_L - U_L$ mixing, Z coupling to u_L is modified to

$$\left(\frac{g}{c_W}\right)\left(\frac{1}{2} - \frac{2}{3}s_W^2 - \frac{s_L^2}{2}\right)$$

- This shifts the Z hadronic width by about 1 MeV, which is consistent. The total Z width has an uncertainty of 2.3 MeV.
- There are no FCNC induced by Z boson at tree-level. The box diagram contribution to $K - \bar{K}$ mixing gets new contributions from VLQ, which is a factor of few below experimental value.
- Di-Higgs production via t-channel exchange of U quark is a possible way to test this model at LHC.

Explaining the W boson mass shift

- CDF collaboration recently reported a new measurement of W boson mass that is about 7 sigma away from SM prediction:

$$M_W^{\mathrm{CDF}} = (80,433.5 \pm 9.4)\ \mathrm{MeV}, \quad M_W^{\mathrm{SM}} = (80,357 \pm 6)\ \mathrm{MeV}$$

- Vector-like quark that mixes with SM quark can modify T, S, U parameters. This occurs in the quark seesaw model
- Needed mixing between SM quark and VLQ is or order 0.15. $t - T$ mixing alone won't suffice, as it is constrained by top mass.
- t-quark mixing with two VLQs with the mixing angle of order 0.15 can consistently explain the W mass anomaly
- Source of custodial $SU(2)$ violation is the $t_L - U_L$ mixing
- Mixing of light quarks with VLQs cannot explain the anomaly, since these mixings are constrained by Z hadronic width

W boson mass shift

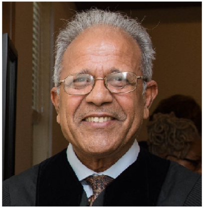
- $(t,\ U_2,\ U_3)$ mass matrix:

$$M_up = \begin{pmatrix} 0 & 0 & y_t\kappa_L \\ .\ 0 & 0 & M_1 \\ y_t\kappa_R & M_1 & M_2 \end{pmatrix}$$

- $m_t \to 0$ approximation is realized
- In the simplified verions with $M_2 = 0$, the oblique T-parameter is:

$$T = \frac{N_c M_T^2 s_L^4}{16\pi s_W^2 m_W^2}$$

- $t_L - U_L$ mixing angle s_L is contrained from $|V_{td}|$ measurement to be $|s_L| < 0.17$
- $T = 0.16$ is obtained for $M_T = 2.1$ TeV. $T = \{0.15, 0.26\}$ needed to explain W mass shift implies $M_T = \{2.1, 2.6\}$ TeV



Congratulations Rabi on all your amazing achievements!

And Thank You for all the wisdom you shared!

Wishing you all the best for the next chapter!