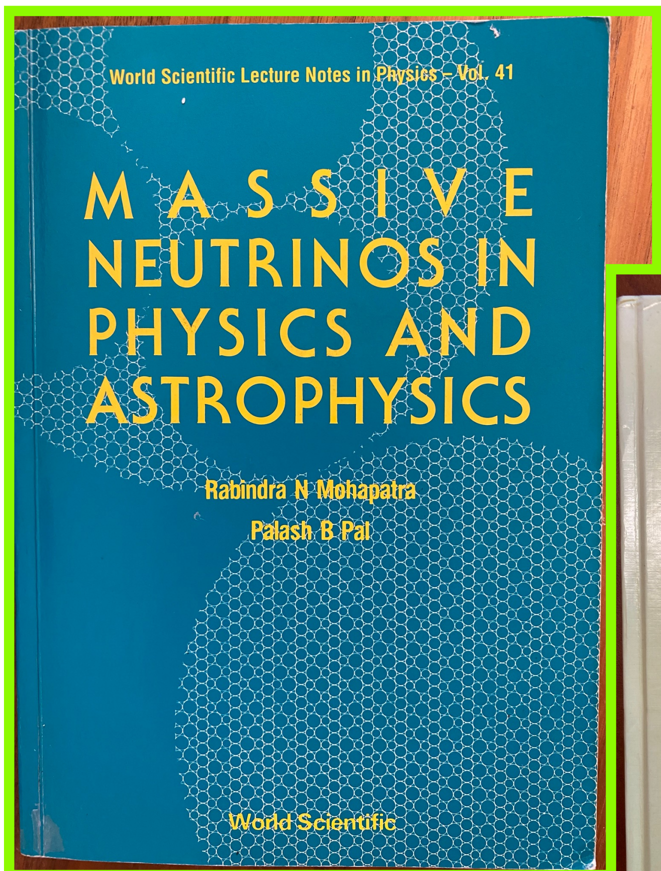
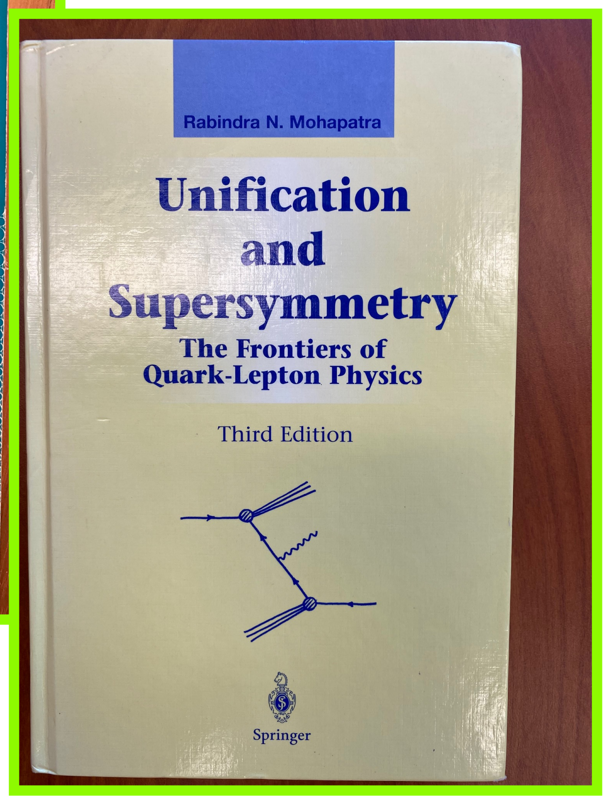
Rabi's Always Bright Insights

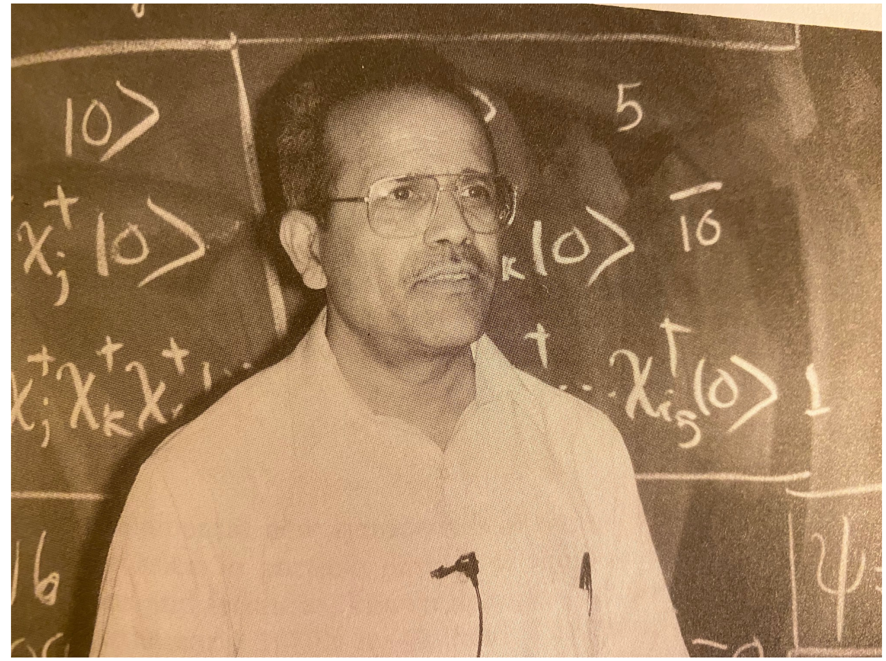
Mu-Chun Chen, University of California at Irvine

Rabi Fest, University of Maryland, College Park, Oct 20, 2022

My first learning about Neutrinos & SUSY







TASI 1997, Boulder Colorado

SUPERSYMMETRIC GRAND UNIFICATION:
Lectures at TASI97

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MD 20742, USA
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One of the key ingredients of physics beyond the standard model is widely believed to be a symmetry between the fermions and the bosons known as supersymmetry. The reason for this is that the milder divergence structure of field theories with this symmetry may explain why the electroweak scale (or the Higgs mass) is stable under radiative corrections. Two other reasons adding to this belief are : (i) a way to understand the origin of the electroweak symmetry breaking as a consequence of radiative corrections and (ii) the particle content of the minimal supersymmetric model that leads in a natural way to the unification of the three gauge couplings of the standard model at a high scale. This last observation suggests that at scales close to the Planck scale, all matter and all forces may unify into a single matter and a single force leading to a supersymmetric grand unified theory. It is the purpose of these lectures to provide a pedagogical discussion of the various kinds of supersymmetric unified theories beyond the minimal supersymmetric standard model (MSSM) including SUSY GUTs and present a brief overview of their implications. Questions such as proton decay, R-parity violation, doublet triplet splitting etc. are discussed. Exhaustive discussion of $SU(5)$ and $SO(10)$ models and less detailed ones for other GUT models such as those based on E_6, $SU(5) \times SU(5)$, flipped SU(5) and $SU(6)$ are presented.

Two Physics lessons from Rabi:

Symmetries ⇒ reduction in # of parameters

Novel Fundamental Origin of CP Violation

Models for Geometric CP Violation with Extra Dimensions

Darwin Chang[a,b1], Wai-Yee Keung[b2], and Rabindra N. Mohapatra[c3]

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ysics Department, University of Illinois at Chicago, IL 60607-7059
rtment of Physics, University of Maryland, College Park, MD, 20742

May, 2001

Abstract

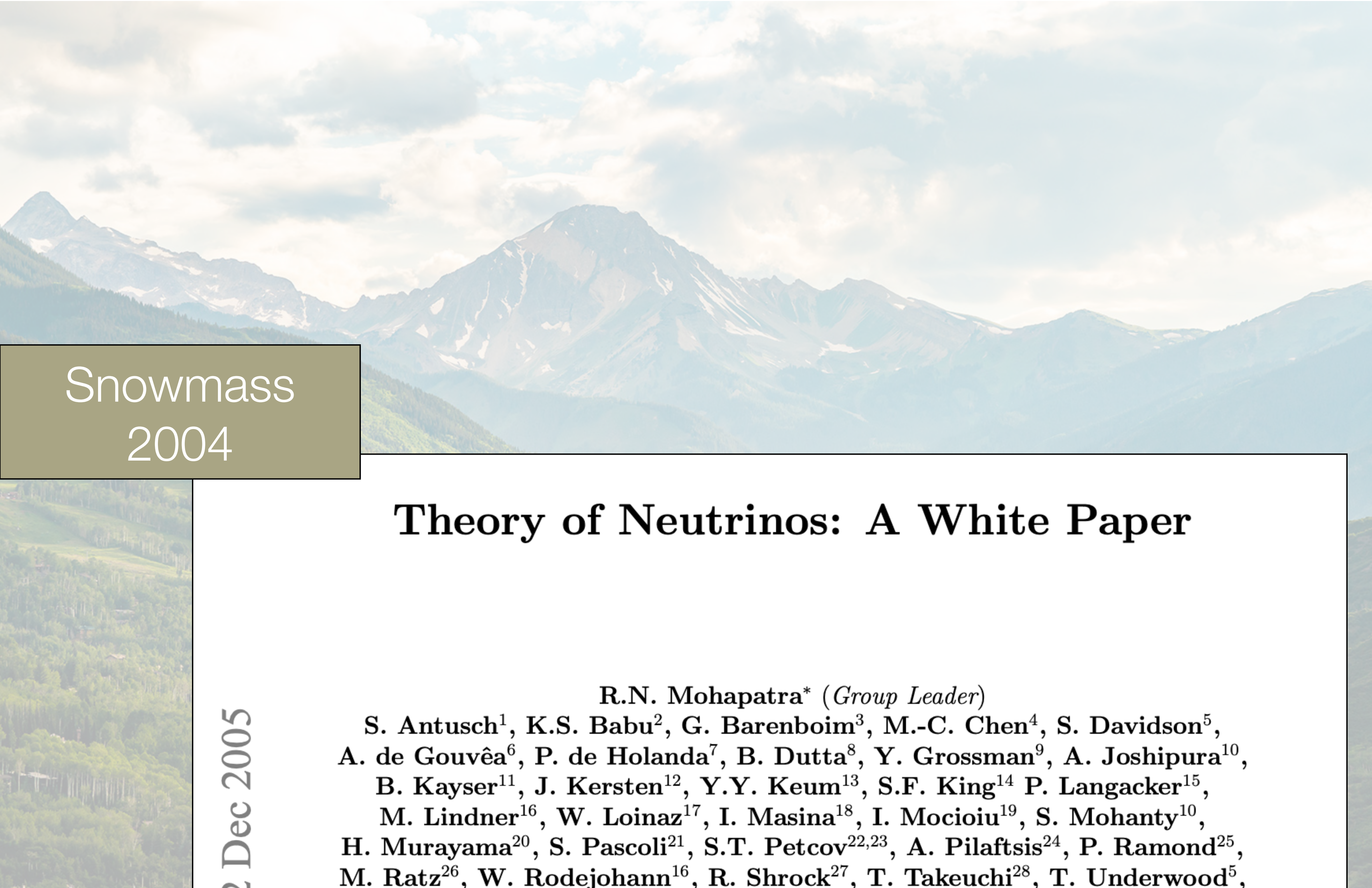
ent paper, two of us (D.C. and R.N.M.) proposed a new way to break CP
geometrically using orbifold projections. The mechanism can be realized
dimensional brane bulk picture. In this paper, we elaborate on this
nd provide additional examples of models of this type. We also note the
ological implications of some of these models.

Geometric CP Violation with Extra Dimensions

Darwin Chang[1,3*] and R. N. Mohapatra[2†],
[1] Physics Department, National Tsing-Hua University, Hsinchu, Taiwan, 30043, ROC
[2] Department of Physics, University of Maryland, College Park, MD, 20742
[3] Physics Department, University of Illinois at Chicago, Chicago, IL, 60607-7059
(March, 2001)

Abstract

We discuss how CP symmetry can be broken geometrically through orbifold construction in hidden extra dimensions in the context of D-brane models for particle unifications. We present a few toy models to illustrate the idea and suggest ways to incorporate this technique in the context of realistic models.



Snowmass 2004

Theory of Neutrinos: A White Paper

R.N. Mohapatra* (*Group Leader*)
S. Antusch[1], K.S. Babu[2], G. Barenboim[3], M.-C. Chen[4], S. Davidson[5], A. de Gouvêa[6], P. de Holanda[7], B. Dutta[8], Y. Grossman[9], A. Joshipura[10], B. Kayser[11], J. Kersten[12], Y.Y. Keum[13], S.F. King[14] P. Langacker[15], M. Lindner[16], W. Loinaz[17], I. Masina[18], I. Mocioiu[19], S. Mohanty[10], H. Murayama[20], S. Pascoli[21], S.T. Petcov[22,23], A. Pilaftsis[24], P. Ramond[25], M. Ratz[26], W. Rodejohann[16], R. Shrock[27], T. Takeuchi[28], T. Underwood[5], L. Wolfenstein[29]

2 Dec 2005

Where Do We Stand?

- Latest 3 neutrino global analysis:

Gonzalez-Garcia, Maltoni, Schwetz (NuFIT), 2111.03086

with SK atmospheric data

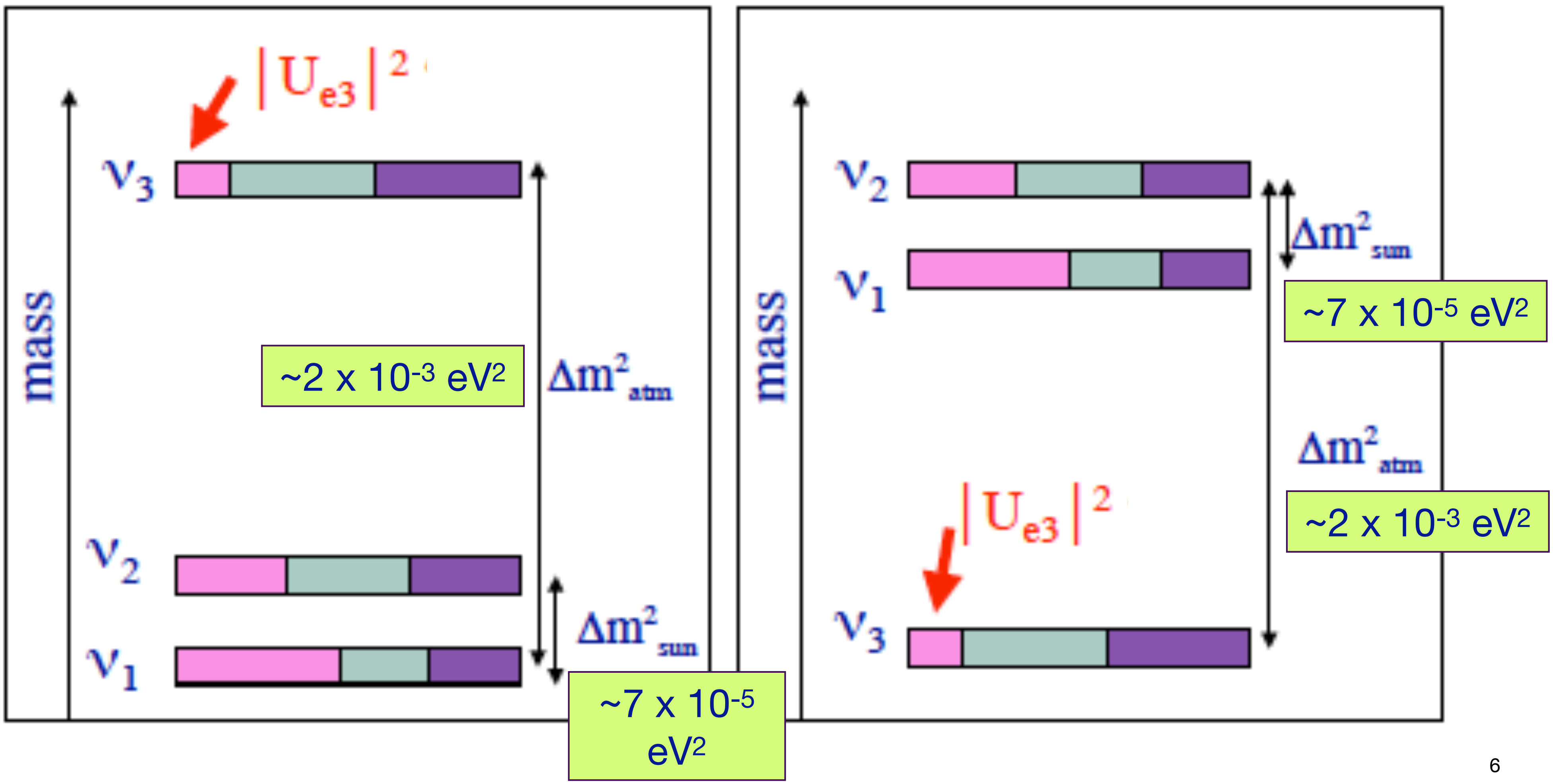
	Normal Ordering (Best Fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

- ➡ hints of $\theta_{23} \neq \pi/4$
- ➡ expectation of Dirac CP phase δ
- ➡ slight preference for normal mass ordering

Where Do We Stand?

Normal Ordering

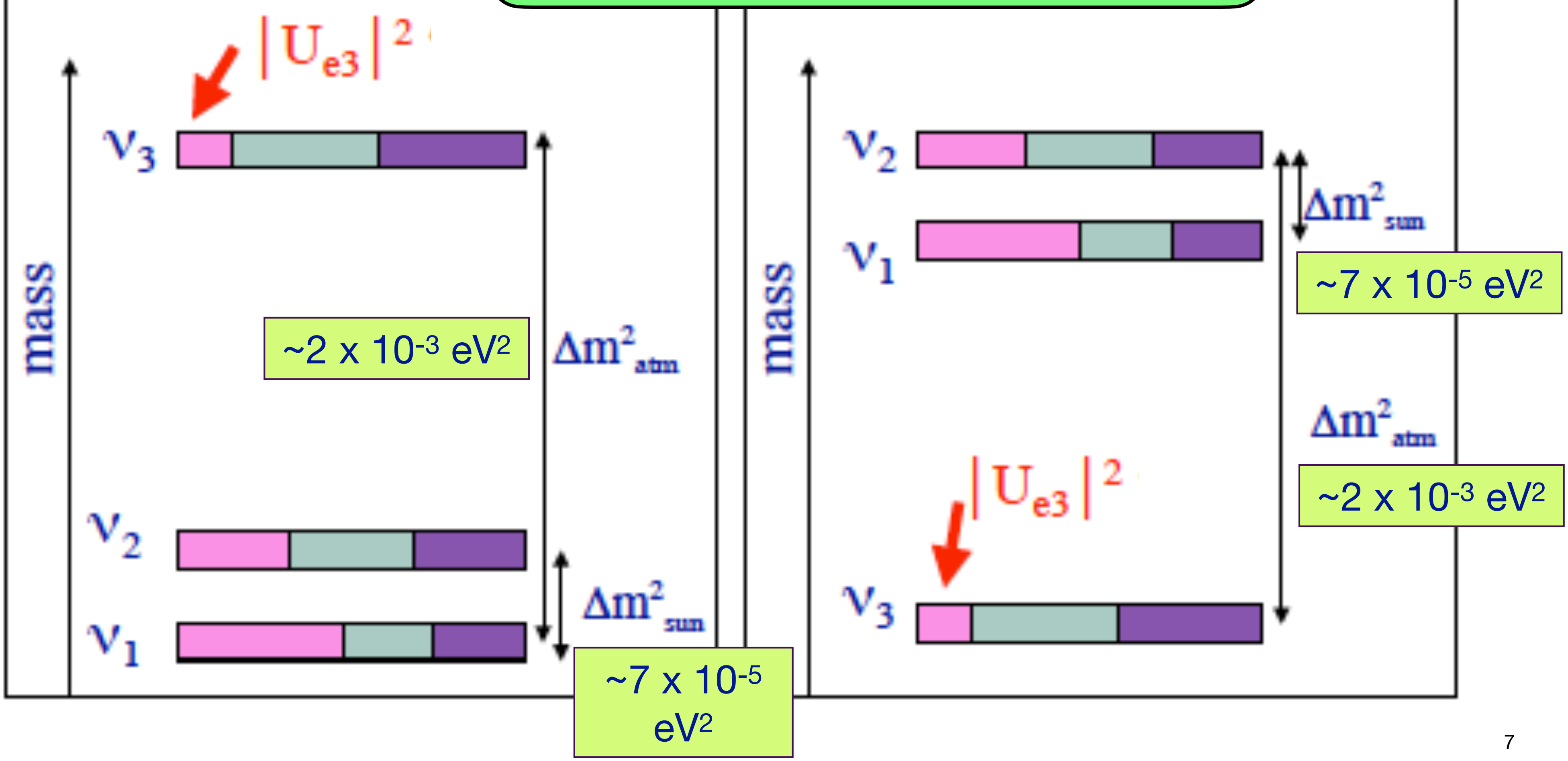
Inverted Ordering



Where Do We Stand?

Normal Ordering

$$\sin^2\theta_{23} \sim \frac{1}{2}, \quad \sin^2\theta_{12} \sim \frac{1}{3}, \quad \sin\ \ \theta_{13} \sim \frac{1}{6}$$



Open Questions - Neutrino Properties

- ☞ Majorana vs Dirac?
- ☞ CP violation in lepton sector?
- ☞ Absolute mass scale of neutrinos?
- ☞ Mass ordering: sign of (Δm_{13}^2)?
- ☞ Sterile neutrino(s)?
- ☞ Precision: $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?
- ☞ Additional Neutrino Interactions?

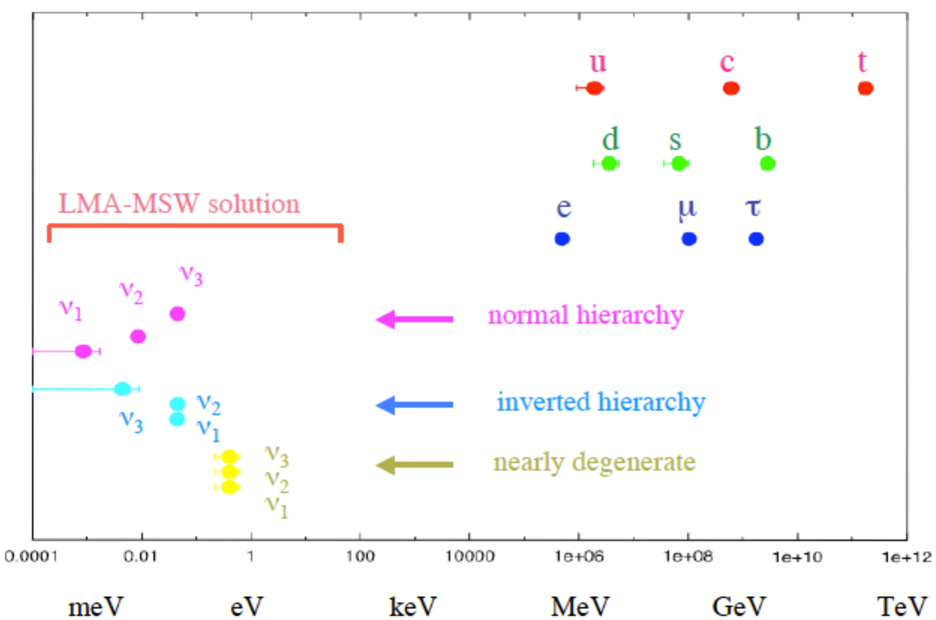
To understand some of these properties
⇒ BSM Physics

a suite of current and upcoming experiments to address these puzzles

Open Questions - Theoretical

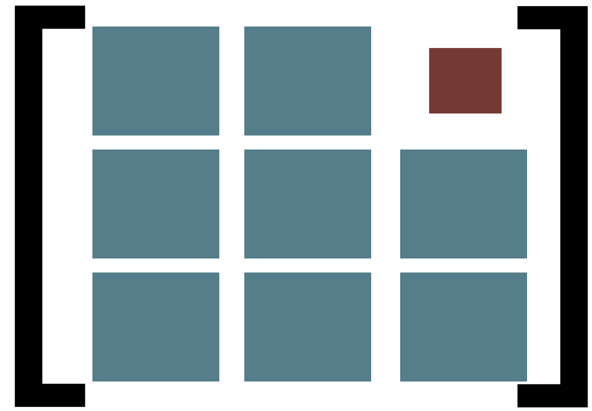
☞ Smallness of neutrino mass:

$$m_\nu \ll m_{e,\,u,\,d}$$

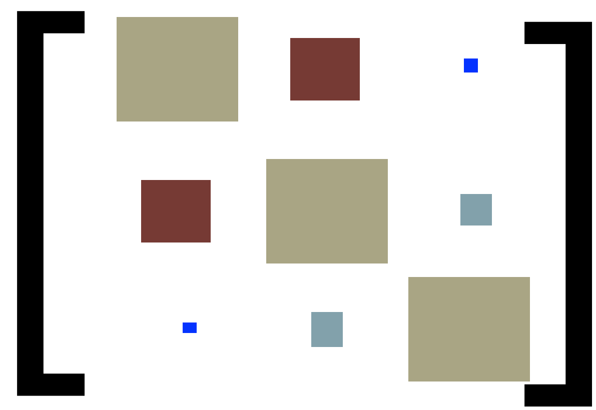


Fermion mass and hierarchy problem ➟ Many free parameters in the Yukawa sector of SM

☞ Flavor structure:



leptonic mixing



quark mixing

Why Should We Care?

- Understanding a wealth of data, fundamentally
- SM flavor sector: no understanding of significant fraction (22/28) of SM parameters; (c.f. SM gauge sector)
- Neutrinos as window into BSM physics
  - neutrino mass generation unknown (suppression mechanism, scale)
  - Uniqueness of neutrino masses ➜ connections w/ NP frameworks
- **Neutrinos affords opportunities for new explorations**
  - New Tools
  - May address other puzzles in particle physics
    - Window into early Universe
  - UV connection

Smallness of Neutrino Masses

VOLUME 44, NUMBER 14 PHYSICAL REVIEW LETTERS 7 APRIL 1980

Neutrino Mass and Spontaneous Parity Nonconservation

Rabindra N. Mohapatra
Department of Physics, City College of New York, New York, New York 10031

and

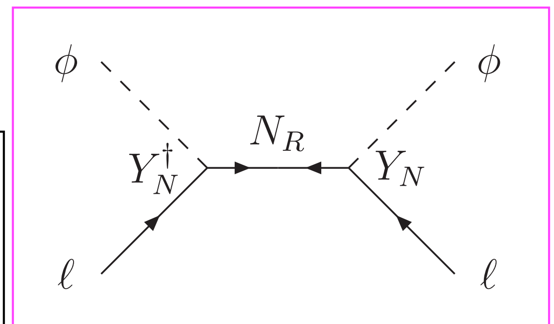
Goran Senjanović
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

(Received 10 December 1979)

In weak-interaction models with spontaneous parity nonconservation, based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$, we obtain the following formula for the neutrino mass: $m_{\nu_e} \simeq m_e^2/gm_{W_R}$, where W_R is the gauge boson which mediates right-handed weak interactions. This formula, valid for each lepton generation, relates the maximality of observed parity nonconservation at low energies to the smallness of neutrino masses.

PACS numbers: 11.30.Er, 11.30.Ly, 12.30.Ez, 14.60.Gh.

It is attractive to suppose that observed parity nonconservation in weak interactions is only a low-energy phenomenon, which ought to disappear at high energies. This idea has been implemented in unified gauge theories of electroweak interactions based on the gauge group $SU(2)_L \otimes SU(2)_R$



Type I Seesaw

PHYSICAL REVIEW D VOLUME 23, NUMBER 1 1 JANUARY 1981

Neutrino masses and mixings in gauge models with spontaneous parity violation

Rabindra N. Mohapatra*
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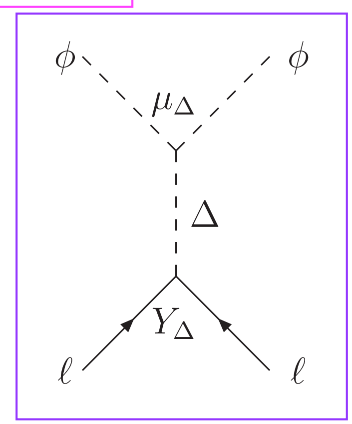
Goran Senjanović
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and Department of Physics, University of Maryland, College Park, Maryland 20742

(Received 8 August 1980)

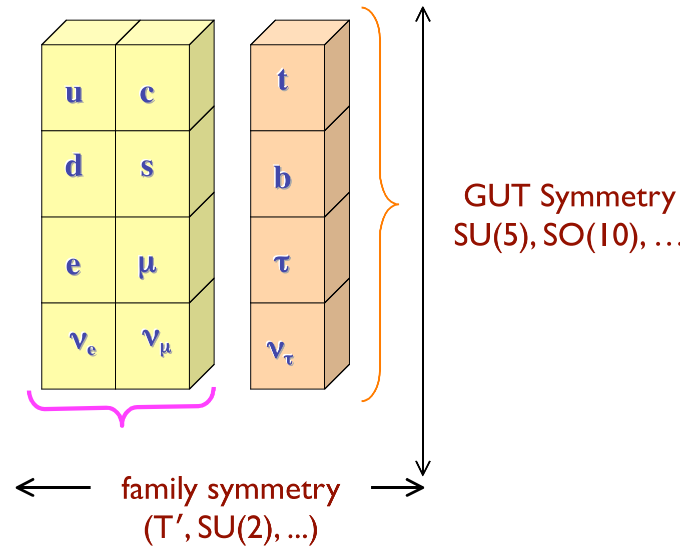
Unified electroweak gauge theories based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, in which the breakdown of parity invariance is spontaneous, lead most naturally to a massive neutrino. Assuming the neutrino to be a Majorana particle, we show that smallness of its mass can be understood as a result of the observed maximality of parity violation in low-energy weak interactions. This result is shown to be independent of the number of generations and unaffected by renormalization effects. Phenomenological consequences of this model at low energies are studied. Observation of neutrinoless double-β decay will provide a crucial test of this class of models. Implications for rare decays such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$, etc. are also noted. It is pointed out that in the realm of neutral-current phenomena, departure from the predictions of the standard model for polarized-electron–hadron scattering, forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$, and neutrino interactions has a universal character and may be therefore used as a test of the model.

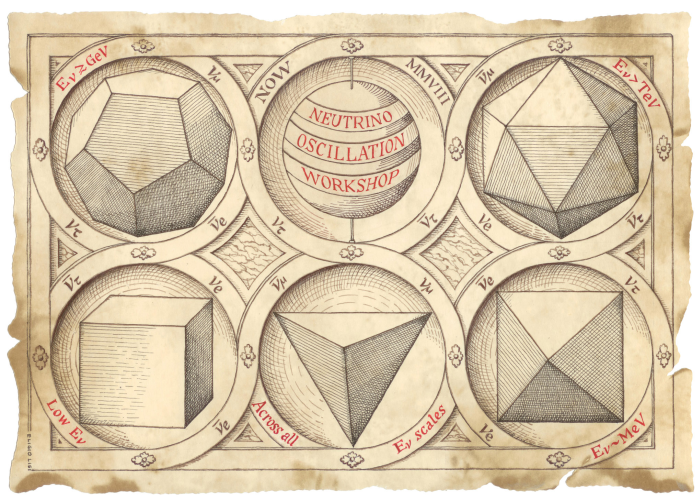
Type II Seesaw



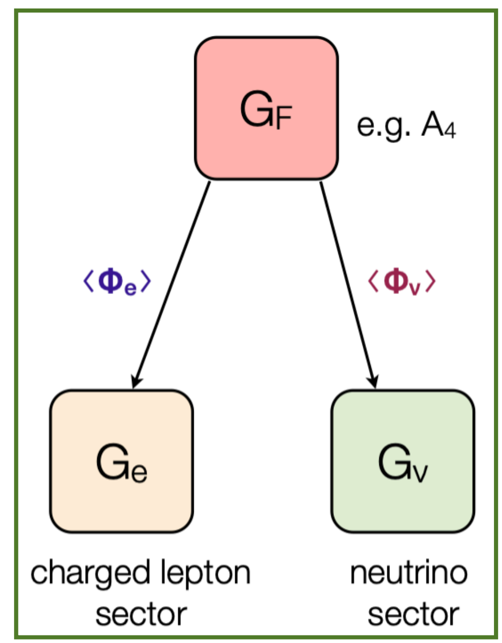
Non-Abelian Discrete Flavor Symmetries

- Large neutrino mixing motivates discrete flavor symmetries
    - A_4 (tetrahedron)
    - T´ (double tetrahedron)
    - S_3 (equilateral triangle)
    - S_4 (octahedron, cube)
    - A_5 (icosahedron, dodecahedron)
    - Δ_{27}
    - Q_6
    - .....





[Eligio Lisi for NOW2008]



Tri-bimaximal Neutrino Mixing

- **Latest Global Fit (3σ)**

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou (2020)

$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$ [$\theta^{lep}_{23} \sim 49.2°$]

$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$ [$\theta^{lep}_{12} \sim 33.4°$]

$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$ [$\theta^{lep}_{13} \sim 8.57°$]

- **Tri-bimaximal Mixing Pattern**

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\text{atm, TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\text{TBM}} = 1/3$$

$$\sin\theta_{13,\text{TBM}} = 0.$$

Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

- Imposing A4 flavor symmetry on the Lagrangian
- A4 spontaneously broken by flavon fields

2 free parameters

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Neutrino Mixing Angles from Group Theory

CP Violation in Nature

- CP violation: required to explain matter-antimatter asymmetry
- So far observed only in flavor sector
  - SM: CKM matrix for the quark sector
    - experimentally established δ_{CKM} as major source of CP violation
    - not sufficient for observed cosmological matter-antimatter asymmetry
- Search for new source of CP violation:
  - CP violation in neutrino sector
  - if found $\Rightarrow$ phase in PMNS matrix
- Discrete family symmetries:
  - suggested by large neutrino mixing angles
  - neutrino mixing angles from group theoretical CG coefficients
  - may come from orbifold compactification

Discrete (family) symmetries $\Leftrightarrow$ Physical CP violation

Outer Automorphisms ⇔ CP

- Outer automorphisms of the Lorentz group: P, T, C
- C and P violation tied to parity, but CP violation less understood
- Left-Right parity in left-right symmetric/Pati-Salam models
- Gauge Origin of Left-Right Parity:
    - Additional Z_2 in Pati-Salam models

Pati, Salam (1974)

$$[\mathrm{SU}(4) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}] \rtimes \mathbb{Z}_2$$

    - Additional Z_2 in left-right symmetric models

Mohapatra, Senjanovic (1980)

$$[\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{B-L}] \rtimes \mathbb{Z}_2$$

    - Z_2^{LR} can be preserved in SO(10) GUTs by giving a VEV to 54-plet

Kibble, Lazarides, Shafi (1982); Chang, Mohapatra, Parida (1984)

    - Automatic preservation through orbifold compactification

Biermann, Mütter, Parr, Ratz, Vaudrevange (2019)

Outer Automorphisms ⇔ CP

- Not all models can have left-right parity
  - Gauge symmetry
  - Particle content

- For a long time, it was assumed that CP (which is also an outer automorphism) can be imposed on models with arbitrary (gauge) symmetry and particle content
  - True for continuous symmetries
  - Fail for some discrete symmetries
- CP is an outer automorphism; But not all outer automorphisms are CP
  - e.g. Left-right parity for Strong CP problem

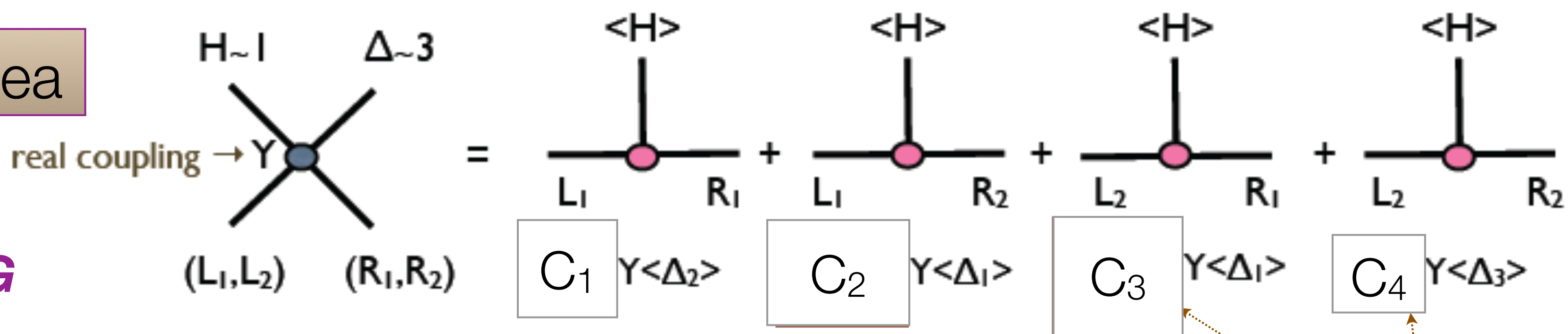
A Novel Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

- Complex CG coefficients in certain discrete groups ⇒ explicit CP violation
  - Real Yukawa couplings, real scalar VEVs
  - CPV in quark and lepton sectors purely from complex CG coefficients
  - No additional parameters needed ⇒ extremely predictive model!

Basic idea

Discrete symmetry ***G***



- Scalar potential: if Z_3 symmetric ⇒ $\langle\Delta_1\rangle = \langle\Delta_2\rangle = \langle\Delta_3\rangle \equiv \langle\Delta\rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$$(L_1 \quad L_2)$$

$$M = \begin{pmatrix} C_1 & C_3 \\ C_2 & C_4 \end{pmatrix} Y \langle\Delta\rangle \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

$C_{1,2,3,4}$: complex CG coefficients of ***G***

Generalized CP Transformation

- setting w/ discrete symmetry G

G and CP transformations do not commute

- generalized CP transformation

Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)

- invariant contraction/coupling in A_4 or T'

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi\,\left(x_1\,y_1 + \omega^2\,x_2\,y_2 + \omega\,x_3\,y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

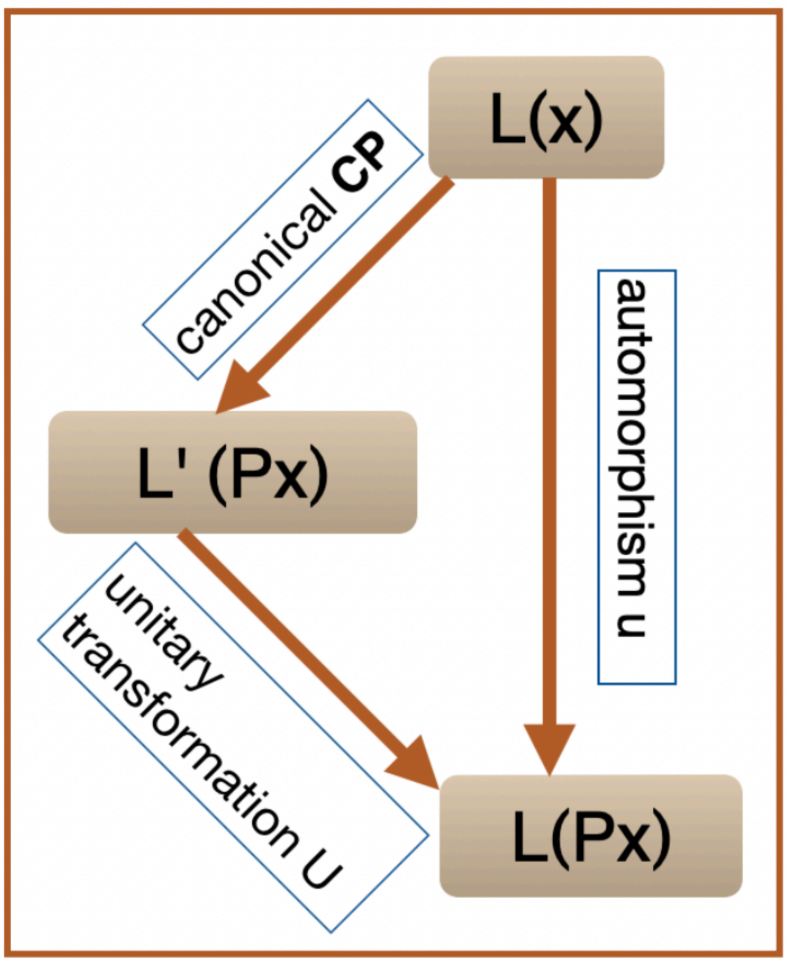
- canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

➥ need generalized CP transformation $\widetilde{CP}$: $\phi \xmapsto{\widetilde{CP}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{CP}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{CP}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

Physical CP vs. Generalized CP Transformations

complex CGs ➪ G and physical CP transformations do not commute



Generalized CP transformation:

$$\Phi(x) \xrightarrow{\widetilde{\mathcal{CP}}} U_{\mathrm{CP}}\, \Phi^*(\mathcal{P}\ x)$$

contains all reps in model

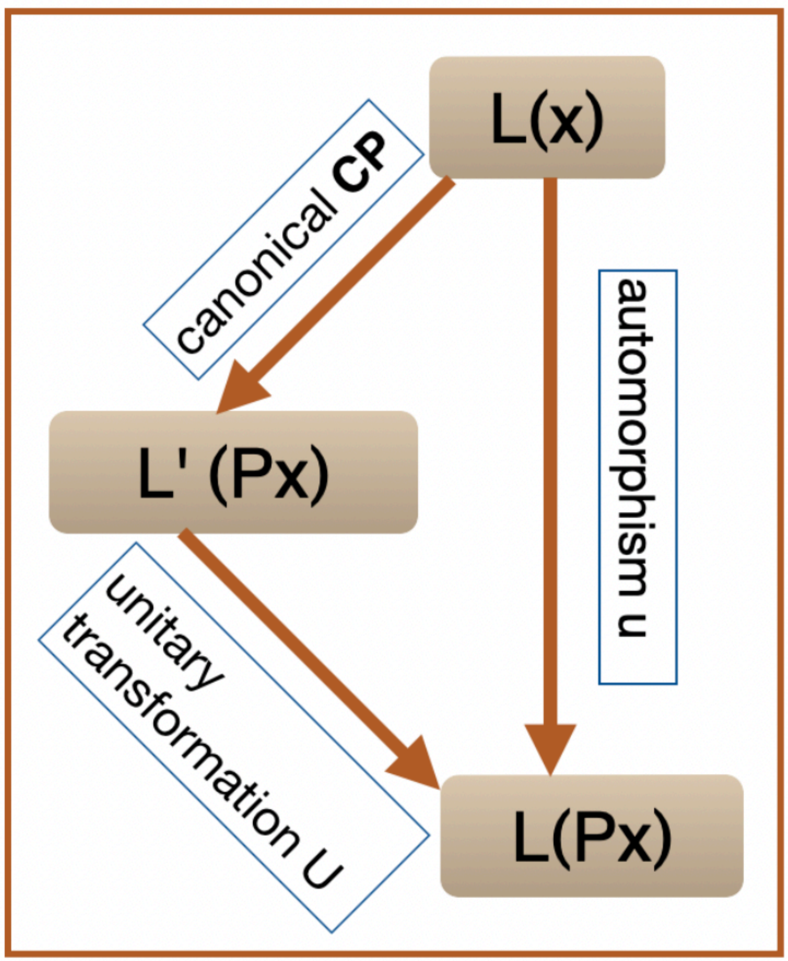
Necessary Consistency condition:

Holthausen, Lindner, Schmidt (2013)

$$\rho\big(u(g)\big) \;=\; U_{\mathrm{CP}}\ \rho(g)^*\, U_{\mathrm{CP}}^\dagger \quad \forall\, g \in G$$

Physical CP vs. Generalized CP Transformations

complex CGs ➪ G and physical CP transformations do not commute



Generalized CP transformation:

$$\Phi(x) \overset{\widetilde{CP}}{\longmapsto} U_{\mathrm{CP}}\, \Phi^*(\,\mathcal{P}\ x)$$

Necessary Consistency condition:

Holthausen, Lindner, Schmidt (2013)

$$\rho\big(u(g)\big) \;=\; U_{\mathrm{CP}}\ \rho(g)^*\, U_{\mathrm{CP}}{}^{\dagger} \quad \forall\, g \in G$$

However, GCP may not correspond to physical CP transformation
➪ for GCP = physical CP: more stringent consistency condition

Physical CP vs. Generalized CP Transformations

- generalized CP transformation $\Phi(x) \xmapsto{\widetilde{CP}} U_{\mathrm{CP}}\, \Phi^*(\mathcal{P}\ x)$
- Necessary consistency condition

$$\rho\big(u(g)\big) \;=\; U_{\mathrm{CP}}\ \rho(g)^*\, {U_{\mathrm{CP}}}^\dagger \quad \forall\, g \in G$$

Holthausen, Lindner, Schmidt (2013)

- **Necessary and sufficient consistency condition**

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i$$

⟷ **physical *CP***

Physical CP vs. Generalized CP Transformations

- generalized CP transformation $\Phi(x) \xmapsto{\widetilde{CP}} U_{\mathrm{CP}}\, \Phi^*(\,\mathcal{P}\; x)$
- Necessary consistency condition

$$\rho\big(u(g)\big) \;=\; U_{\mathrm{CP}}\;\rho(g)^*\, U_{\mathrm{CP}}{}^{\dagger} \quad \forall\, g \in G$$

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- **Necessary and sufficient consistency condition**

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⟷ physical *CP*

u has to be a class-inverting, involuntary automorphism of G

➪ non-existence of such automorphism in certain groups

➪ explicit physical CP violation

How (Not) to Generalize CP

proper CP transformations

- ☞ map field operators to *their own* Hermitean conjugates
- ☞ violation of physical CP is prerequisite for a non–trivial

$$\varepsilon_{i\to f} = \frac{\left|\Gamma\left(i\to f\right)\right|^2 - \left|\Gamma\left(\bar{\imath}\to \bar{f}\right)\right|^2}{\left|\Gamma\left(i\to f\right)\right|^2 + \left|\Gamma\left(\bar{\imath}\to \bar{f}\right)\right|^2}$$

➥ connection to observed ~~CP~~, baryogenesis & . . .

CP–like transformations

- ☞ map some field operators to some other operators
- ☞ such transformations have sometimes been called "generalized CP transformations" in the literature
- ☞ however, imposing CP–like transformations does **not** imply physical CP conservation

➥ **NO** connection to observed ~~CP~~, baryogenesis & . . .

The Bickerstaff-Damhus automorphism (BDA)

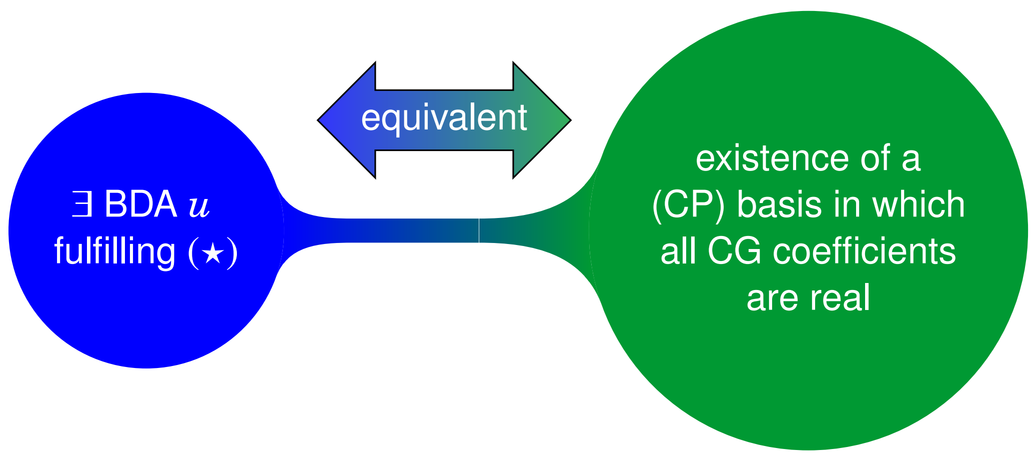
- Bickerstaff-Damhus automorphism (BDA) **u**

Bickerstaff, Damhus (1985)

$$\rho_{\boldsymbol{r}_i}(u(g)) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i \qquad (\,\star\,)$$

unitary & symmetric

- BDA vs. Clebsch-Gordan (CG) coefficients



Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius-Schur indicator

Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

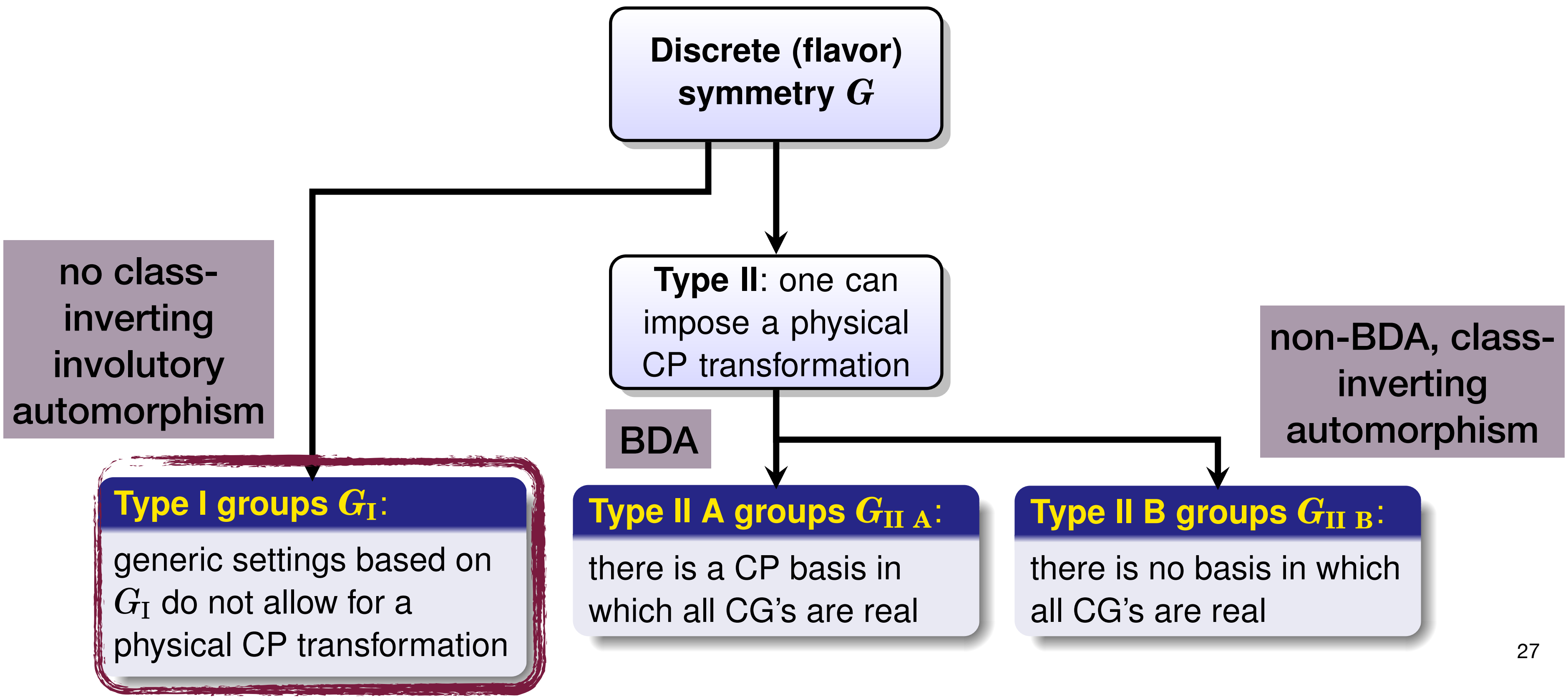
$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

A Novel Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

- **For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ Physical CP violation**

CP Violation from Group Theory!



Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

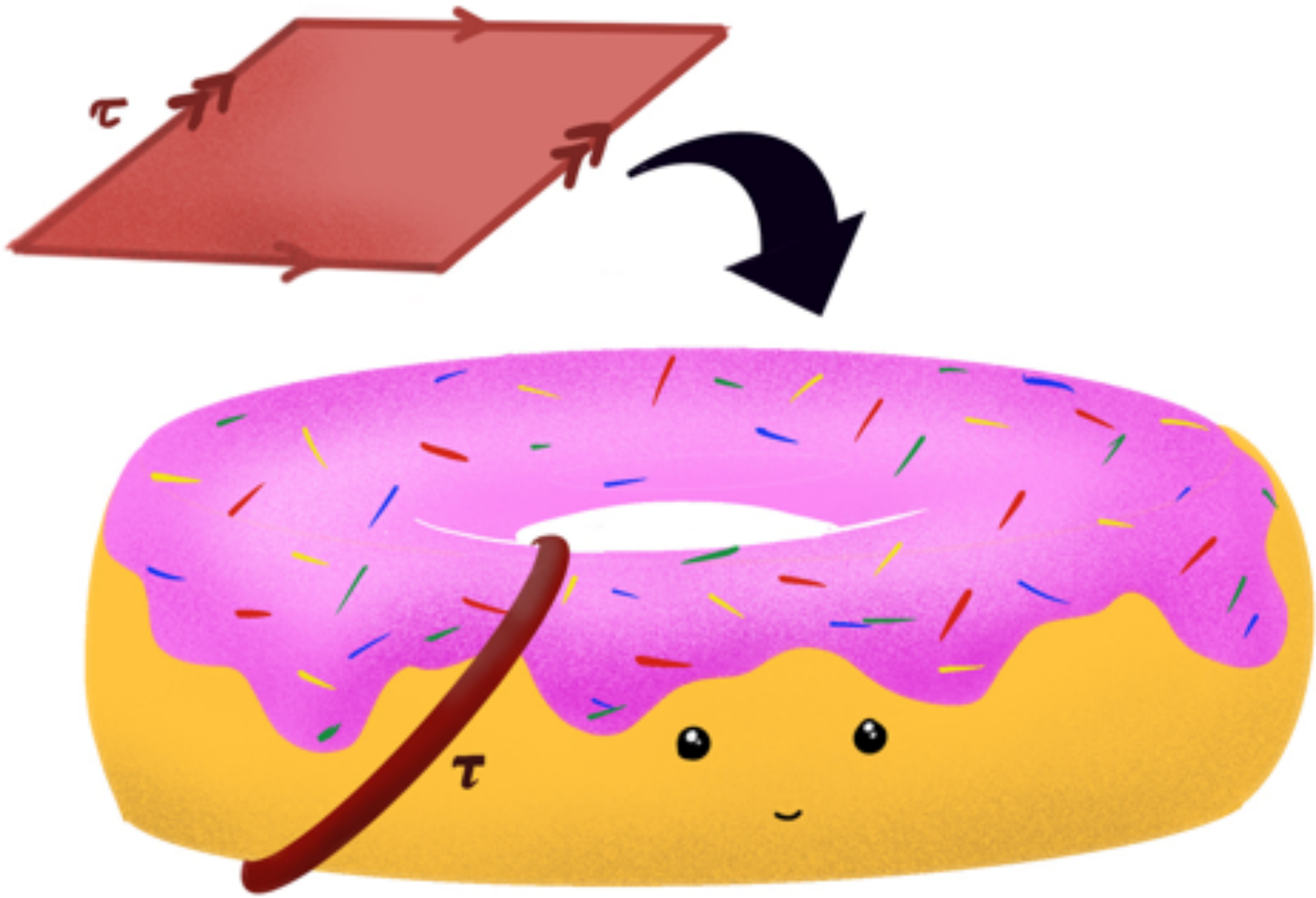
- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T′	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

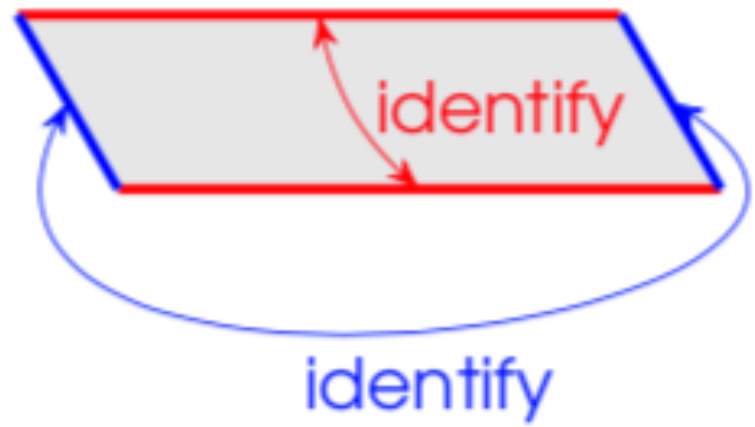
Modular Flavor Symmetries

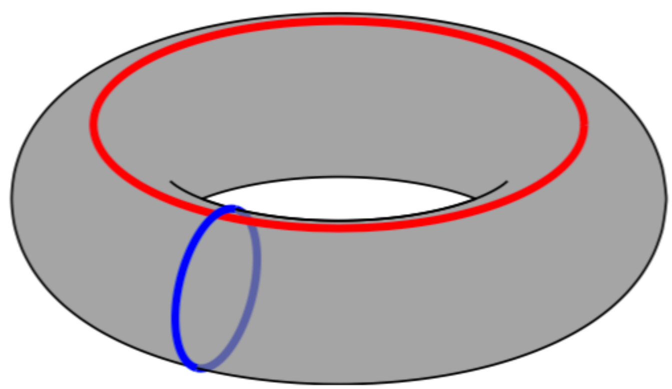


Artwork by Shreya Shukla

Donuts = TORI

constructed
from
parallelogram

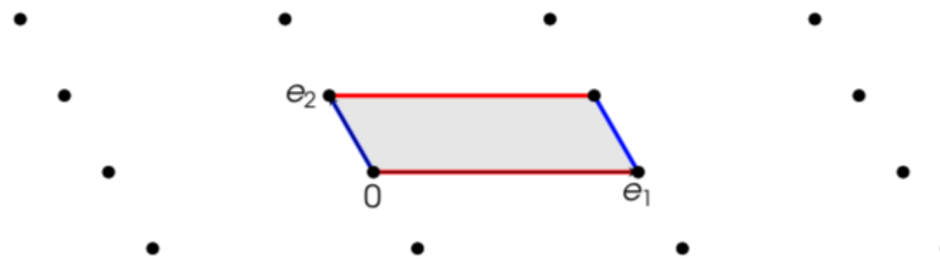




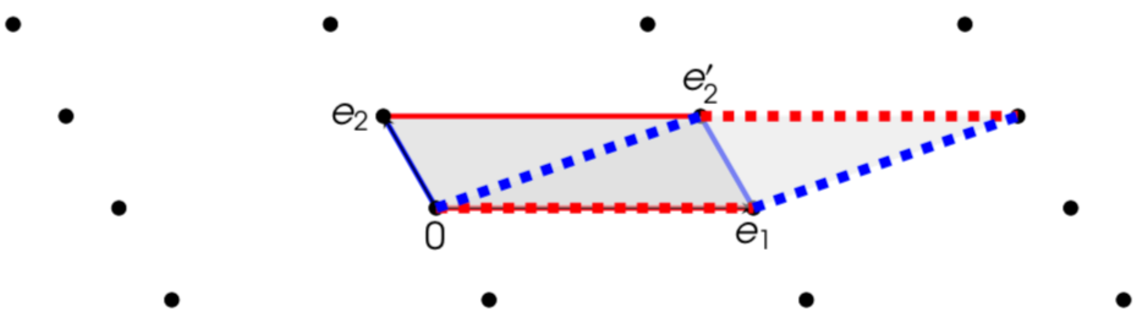
two cycles

Modular Symmetries

edges ⇒ lattice basis vectors



points in plane identified if differ by a lattice translation



Equivalent TORI related by Modular Symmetries

Modular Symmetries

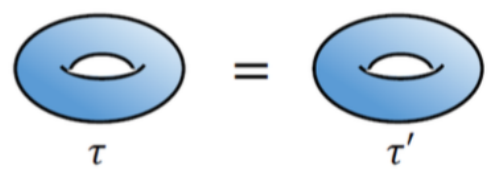
- Two basic transformations:

$$T \; : \; e_2 \mapsto e_2' = e_2 + e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} =: T$$

$$S \; : \; e_1 \mapsto e_1' = e_2 \quad \text{and} \quad e_2 \mapsto e_2' = -e_1 \quad \curvearrowright \gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =: S$$

- In complex coordinates: modulus $\tau = e_2/e_1$

$$\tau \overset{S}{\longmapsto} \frac{-1}{\tau} \quad \text{and} \quad \tau \overset{T}{\longmapsto} \tau + 1$$



- S and T generate $\mathrm{SL}(2, \mathbb{Z})$ and satisfy

$$\boxed{S^2 = (S\,T)^3 = \mathbb{1}}$$

Modular Symmetries

- Finite Modular Group (quotient group): $\Gamma_N := \Gamma/\Gamma(N)$ 'here principal congruence group $\Gamma(N)$ is

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2 ; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

Γ

- Generators of the quotient group Γ_N satisfy

$$S^2 = 1, \quad (ST)^3 = 1, \quad T^N = 1$$

- Some examples

$$\Gamma_2 \simeq S_3, \quad \Gamma_3 \simeq A_4, \quad \Gamma_4 \simeq S_4, \quad \Gamma_5 \simeq A_5$$

Modular Symmetries

Feruglio (2017)

- Imposing modular symmetry Γ on the Lagrangian:

$$\mathscr{L} \supset \sum Y_{i_1,\, i_2,\, \ldots\ldots\, i_n} \Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n}$$

$$\tau \xrightarrow{\gamma} \gamma\tau := \frac{a\tau + b}{c\tau + d} \,,$$

$$\Phi_j \xrightarrow{\gamma} (c\tau + d)^{k_j} \rho_{r_j}(\gamma) \Phi_j \,, \quad \text{where } \gamma := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

k_i : integers

representation matrix of Γ_N

- Yukawa Couplings = Modular Forms at level "N" w/ weight "k"

$$f_i(\gamma\tau) = (c\tau + d)^{-k} [\rho_N(\gamma)]_{ij} f_j(\tau)$$

$k = k_{i_1} + k_{i_2} + \ldots + k_{i_n}$

representation matrix of Γ_N

A Toy Modular A_4 Model

Feruglio (2017)

- Weinberg Operator $$\mathscr{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\ Y\ (H_u \cdot L)]_1$$
- Traditional A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Flavon VEVs (A_4 triplet, 6 real parameters)

$$Y \rightarrow \langle \phi \rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix}$$

- Modular A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Modular Forms (A4 triplet, 2 real parameters)

$$Y \rightarrow \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

Modular Forms

Feruglio (2017)

- Level (N) = 3, Weight (k) = 2, in terms of Dedekind eta-function

$$
\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right] \\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega^2\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \\
Y_3(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)} + \omega\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)} + \omega^2\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \quad .
\end{aligned}
$$

$$
\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad\qquad q \equiv e^{i2\pi\tau}
$$

A Toy Modular A_4 Model

Feruglio (2017)

- Input Parameters:

$$\tau = 0.0111 + 0.9946i$$

$$v_u^2/\Lambda$$

- Predictions:

$$\frac{\Delta m^2_{sol}}{|\Delta m^2_{atm}|} = 0.0292$$

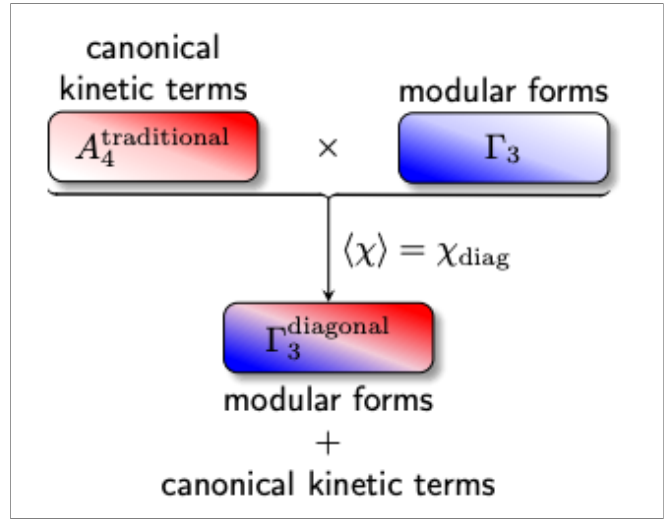
$$\sin^2\theta_{12} = 0.295 \qquad \sin^2\theta_{13} = 0.0447 \qquad \sin^2\theta_{23} = 0.651$$

$$\frac{\delta_{CP}}{\pi} = 1.55 \qquad \frac{\alpha_{21}}{\pi} = 0.22 \qquad \frac{\alpha_{31}}{\pi} = 1.80 \quad .$$

$$m_1 = 4.998 \times 10^{-2}\ eV \qquad m_2 = 5.071 \times 10^{-2}\ eV \qquad m_3 = 7.338 \times 10^{-4}\ eV$$

Modular Symmetry: Bottom-Up Meet Top-Down

- Bottom-Up:
  - reducing the number of parameters: in extreme case, entire neutrino mass matrix controlled by τ Feruglio (2017)
  - Traditional flavor symmetries: corrections to kinetic terms generally sizable Leurer, Nir, Seiberg ('93); Dudas, Pokorski, Savoy ('95); M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
  - Setup with modular symmetries: corrections to kinetic terms can be under control MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)



- Top-Down:
  - Modular flavor symmetries from strings e.g. Baur, Nilles, Trautner, Vaudrevange
  - Modular Symmetries from magnetized tori e.g. Almumin, MCC, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

Rabi as a role model on Mentoring

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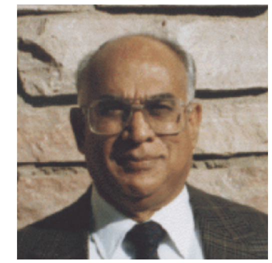

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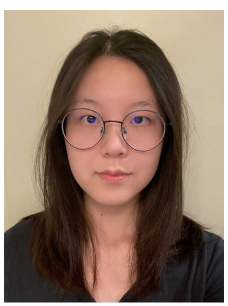
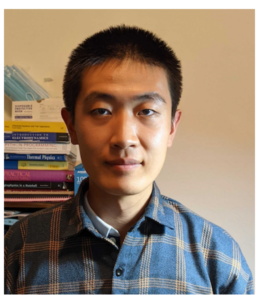
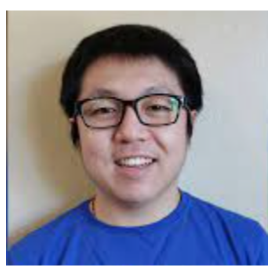
Saúl Ramos-Sánchez (UNAM, Mexico)



Murong Cheng (UIUC Grad; former UCI UG)



Martin Yulun Li (Virginia Tech Grad; former UCI UG)



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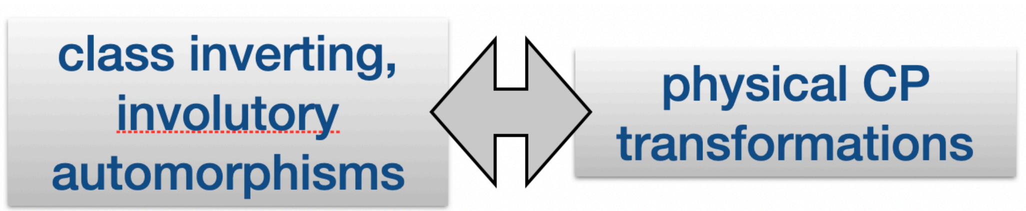
Maximilian Fallbacher (former TUM Grad)

Andreas Trautner (MPI Heidelberg PD; former TUM Grad)

Michael Ratz (UCI)

Outlook

- Fundamental origin of fermion mass & mixing patterns still unknown
- Uniqueness of Neutrino masses offers exciting opportunities to explore BSM Physics
- New Tools/insights:
  - Non-Abelian Discrete Flavor Symmetries
    - Deep connection between outer automorphisms and CP



  - Modular Flavor Symmetries
    - Enhanced predictivity of flavor models
    - Possible connection to string theories
- **Having diverse perspectives/approaches drives intellectual excellence**

Thank you, Rabi, for guiding us with your always bright insights. Congratulations!

Back Up Slides

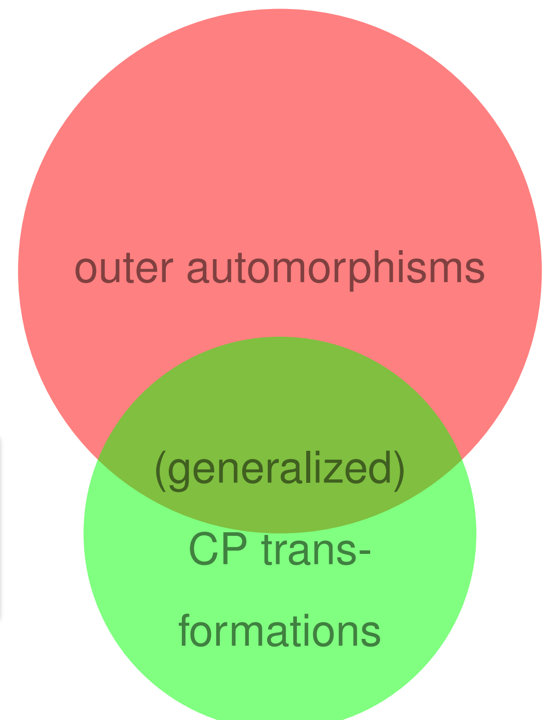
Summary

- **NOT all outer automorphisms correspond to physical CP transformations**

- **Condition on automorphism for *physical* CP transformation**

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \qquad \forall\, g \in G \text{ and } \forall\, i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

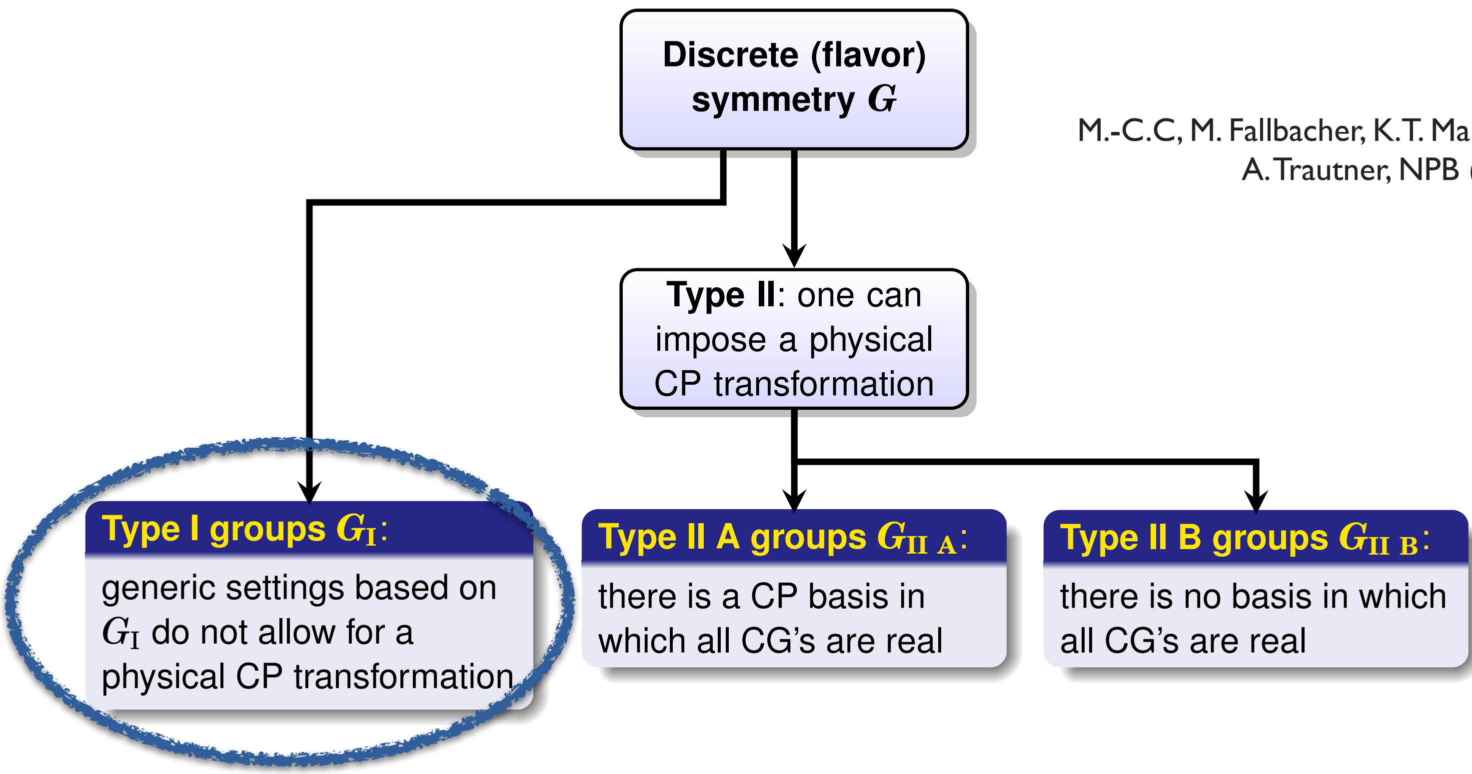


class inverting, involutory automorphisms ⇔ **physical CP transformations**

Summary

- For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)
- Non-existence of such automorphism ⇔ physical CP violation

CP Violation from Group Theory!



M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

Example for a type I group:

$$\boldsymbol{\Delta(27)}$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\text{toy}} = F^{ij}\, S\, \overline{\Psi}_i \Sigma_j + G^{ij}\, X\, \overline{\Psi}_i \Sigma_j + H^{ij}_\Psi\, Y\, \overline{\Psi}_i \Psi_j + H^{ij}_\Sigma\, Y\, \overline{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f\, \mathbb{1}_3$$

$$G = g \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega := e^{2\pi\, i/3}$

"flavor" structures determined by (complex) CG coefficients

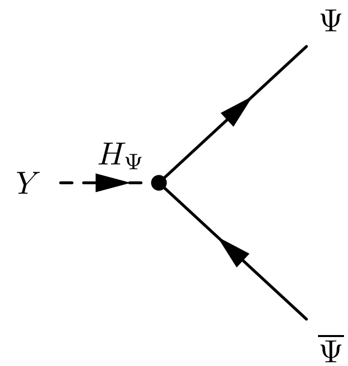
arbitrary coupling constants: f, g, h_Ψ, h_Σ

Toy Model based on Δ(27)

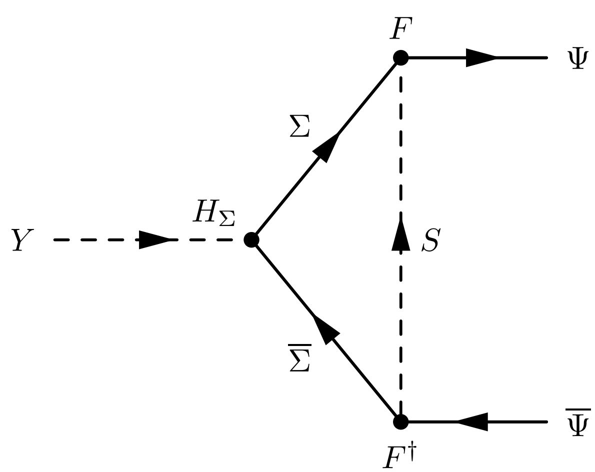
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

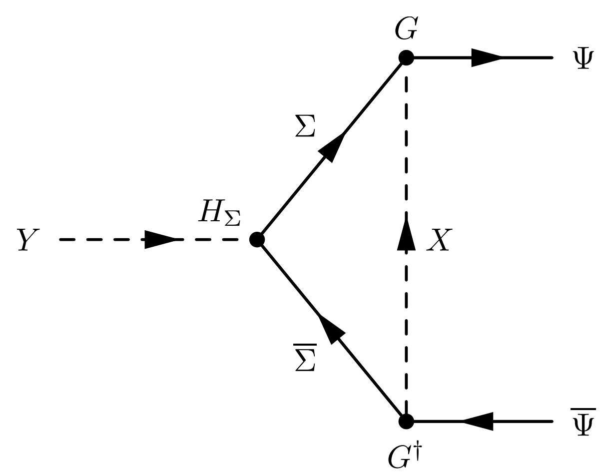
- Particle decay $Y \longrightarrow \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |f|^2 \; \mathrm{Im}\left[I_S\right] \; \mathrm{Im}\left[h_\Psi h_\Sigma^*\right] + |g|^2 \; \mathrm{Im}\left[I_X\right] \; \mathrm{Im}\left[\omega h_\Psi h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by outer automorphism of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = \frac{\Gamma(Y\to\overline{\Psi}\Psi) - \Gamma(Y^*\to\overline{\Psi}\Psi)}{\Gamma(Y\to\overline{\Psi}\Psi) + \Gamma(Y^*\to\overline{\Psi}\Psi)}$$

$$\propto \ \mathrm{Im}\,[I_S]\ \mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\, H_\Psi\, F\, H_\Sigma^\dagger\right)\right] + \mathrm{Im}\,[I_X]\ \mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\, H_\Psi\, G\, H_\Sigma^\dagger\right)\right]$$

$$= \ |f|^2\ \mathrm{Im}\,[I_S]\ \mathrm{Im}\,[h_\Psi\, h_\Sigma^*] + |g|^2\ \mathrm{Im}\,[I_X]\ \mathrm{Im}\,[\omega\, h_\Psi\, h_\Sigma^*]\ .$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

CP Conservation vs Symmetry Enhancement

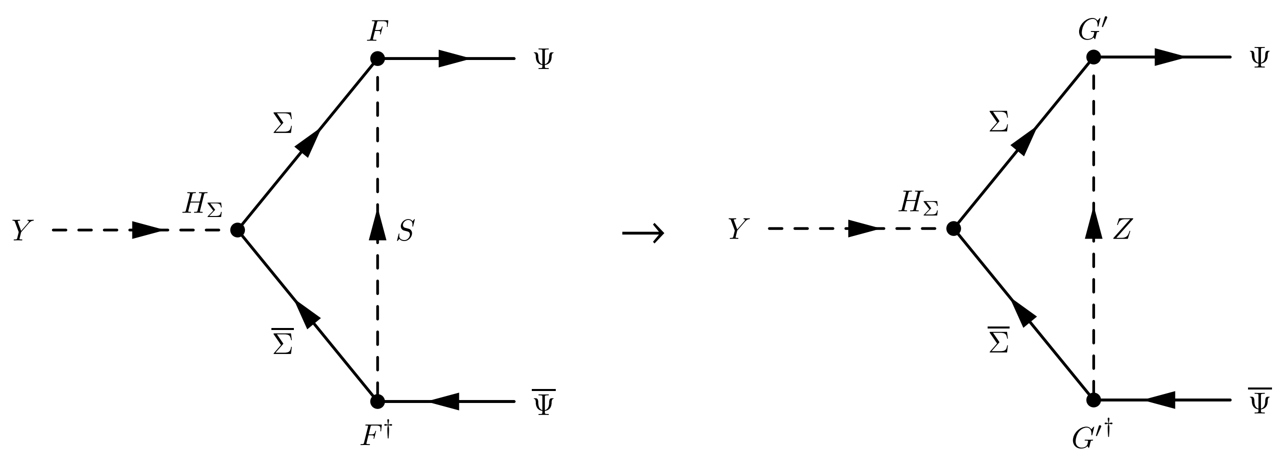
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

☞ replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8$ ↷ interaction

$$\mathscr{L}_{\mathrm{toy}}^{Z} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij} \, Z \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

$$G' = g' \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}$$

and leads to new interference diagram



CP Conservation vs Symmetry Enhancement

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8$ $\curvearrowright$ interaction

$$\mathscr{L}_{\text{toy}}^{Z} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij} \; Z \, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

- different contribution to decay asymmetry: $\varepsilon^{S}_{Y \to \overline{\Psi}\Psi} \;\to\; \varepsilon^{Z}_{Y \to \overline{\Psi}\Psi}$

- total CP asymmetry of the Y decay vanishes if $\begin{cases} \text{(i)} & M_Z = M_X \\ \text{(ii)} & |g| = |g'| \\ \text{(iii)} & \varphi = 0 \end{cases}$

- relations (i)—(iii) can be due to an outer automorphism

$$X \overset{u_3}{\longleftrightarrow} Z\,, \quad Y \overset{u_3}{\longrightarrow} Y\,, \quad \Psi \overset{u_3}{\longrightarrow} U_{u_3}\,\Sigma^C \quad \& \quad \Sigma \overset{u_3}{\longrightarrow} U_{u_3}\,\Psi^C$$

requires $q_\Sigma = -q_\Psi$

$$U_{u_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

... BUT this enlarges $\Delta(27) \to \text{SG}(54,5) \simeq \Delta(27) \rtimes \mathbb{Z}_2^{u_3}$

SG(54, 5): group name from GAP library

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

$$\Delta(27) \subset \mathrm{SG}(54,5): \begin{cases} (X, Z) & : \text{doublet} \\ (\Psi, \Sigma^C) & : \text{hexaplet} \\ \phi & : \text{non–trivial 1–dim. representation} \end{cases}$$

☞ non–trivial $\langle\phi\rangle$ breaks $\mathrm{SG}(54,5) \to \Delta(27)$

☞ allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\text{toy}} \supset M^2 \left(|X|^2 + |Z|^2\right) + \left[\frac{\mu}{\sqrt{2}} \langle\phi\rangle \left(|X|^2 - |Z|^2\right) + \text{h.c.}\right]$

CG coefficient of SG(54, 5)

➥ CP asymmetry with calculable phases

$$\varepsilon_{Y\to\overline{\Psi}\Psi} \propto |g|^2 |h_\Psi|^2 \,\mathrm{Im}\left[\,\omega\,\right] \left(\mathrm{Im}\left[I_X\right] - \mathrm{Im}\left[I_Z\right]\right)$$

phase predicted by group theory

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

Some Outer Automorphisms of Δ(27)

- sample outer automorphisms of Δ(27)

$$u_1 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_2 \;,\; \mathbf{1}_4 \leftrightarrow \mathbf{1}_5 \;,\; \mathbf{1}_7 \leftrightarrow \mathbf{1}_8 \;,\; \mathbf{3} \rightarrow U_{u_1}\,\mathbf{3}^*$$
$$u_2 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_4 \;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_8 \;,\; \mathbf{1}_3 \leftrightarrow \mathbf{1}_6 \;,\; \mathbf{3} \rightarrow U_{u_2}\,\mathbf{3}^*$$
$$u_3 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_8 \;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_4 \;,\; \mathbf{1}_5 \leftrightarrow \mathbf{1}_7 \;,\; \mathbf{3} \rightarrow U_{u_3}\,\mathbf{3}^*$$
$$u_4 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_7 \;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_5 \;,\; \mathbf{1}_3 \leftrightarrow \mathbf{1}_6 \;,\; \mathbf{3} \rightarrow U_{u_4}\,\mathbf{3}^*$$
$$u_5 \;:\; \mathbf{1}_i \leftrightarrow \mathbf{1}_i{}^* \;,\; \mathbf{3} \rightarrow U_{u_5}\,\mathbf{3}$$

- twisted Frobenius-Schur indicators

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_8$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{FS}_{u_1}(\boldsymbol{R})$	1	1	1	0	0	0	0	0	0	1	1
$\mathrm{FS}_{u_2}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_3}(\boldsymbol{R})$	1	0	0	0	0	1	0	1	0	1	1
$\mathrm{FS}_{u_4}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_5}(\boldsymbol{R})$	1	1	1	1	1	1	1	1	1	0	0

- none of the u_i maps all representations to their conjugates
- however, it is possible to impose CP in (non-generic) models, where only a subset of representations are present, e.g. $\{\boldsymbol{r}_i\} \subset \{\mathbf{1}_0, \mathbf{1}_5, \mathbf{1}_7, \mathbf{3}, \overline{\mathbf{3}}\}$
- CP conservation possible in non-generic models
  - e.g. some well-known multiple Higgs model Branco, Gerard, and Grimus (1984)

CP-like Symmetries

- ☞ outer automorphism u_5

$$X \to X^* \,, \quad Z \to Z^* \,, \quad Y \to Y^* \,, \quad \Psi \to U_{u_5}\,\Sigma \quad \& \quad \Sigma \to U_{u_5}\,\Psi$$

$$U_{u_5} = \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & 1 & 0 \\ \omega & 0 & 0 \end{pmatrix}$$

- ☞ does **not** lead to a vanishing decay asymmetry

➥ in general, imposing an outer automorphism as a symmetry does not lead to physical CP conservation!

➥ CP–like symmetry