Rabi-Fest 2022

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Does SUSY Like Rabi?

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Dreams Fulfilled

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	–1	d down	0.006	–1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	–1	s strange	0.1	–1/3
ν_τ tau neutrino	<0.02	0	t top	175	2/3
τ tau	1.7771	–1	b bottom	4.3	–1/3

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1			Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge	Name	Mass GeV/c^2	Electric charge
γ photon	0	0	g gluon	0	0
W^-	80.4	–1			
W^+	80.4	+1			
Z^0	91.187	0			

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

FERMIONS

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spin = 1/2, 3/2, 5/2, ...

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ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0
H^0	125	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
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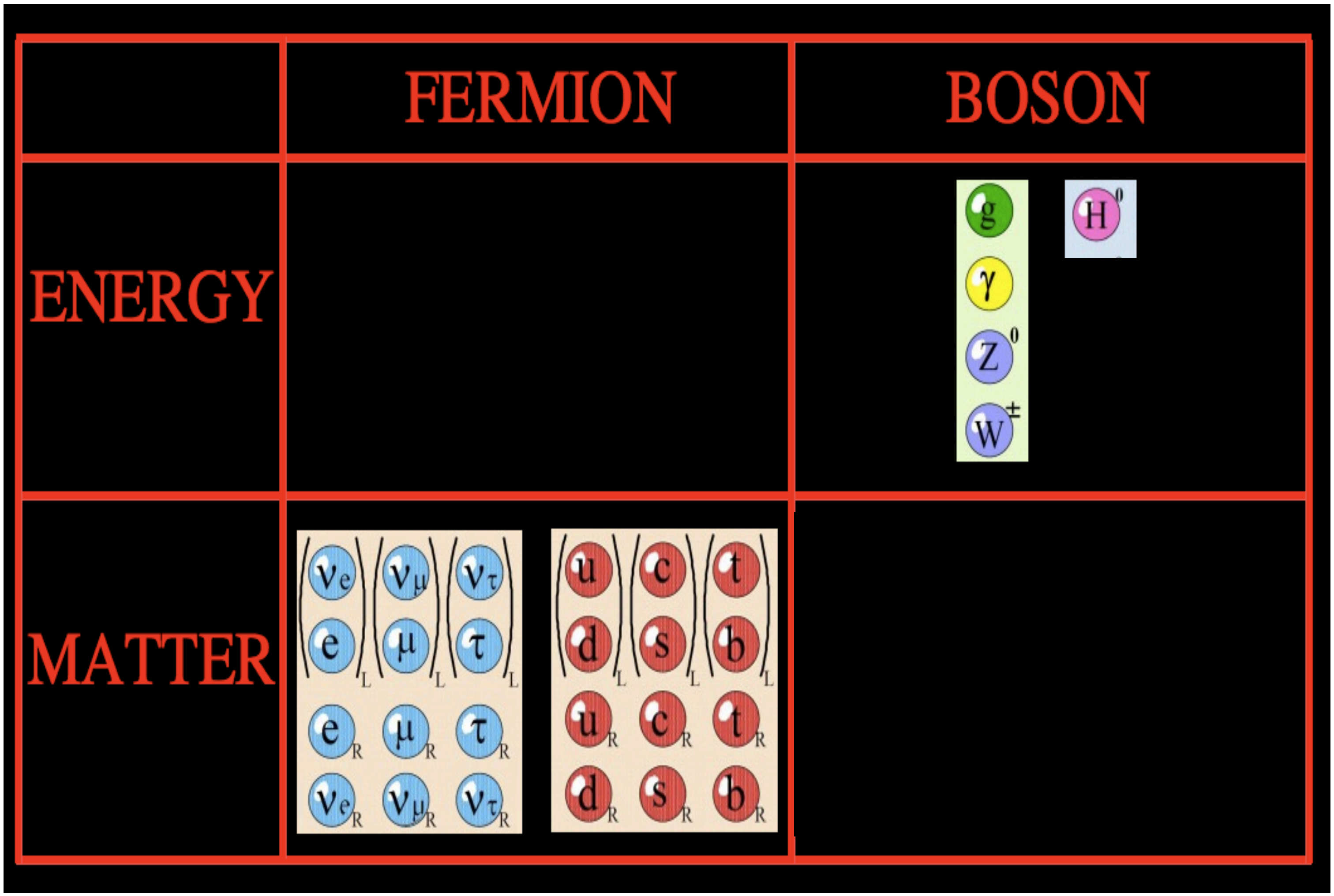
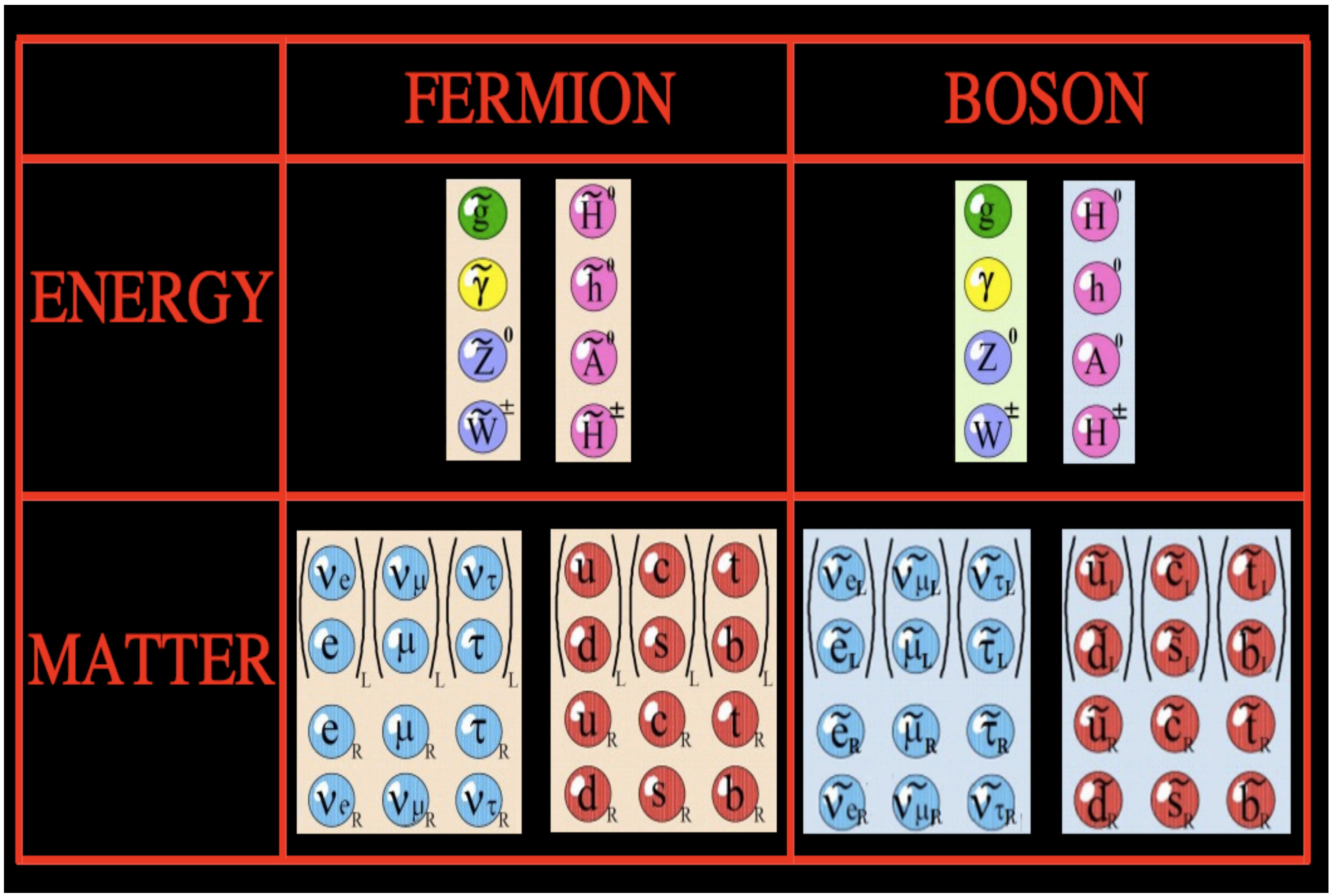
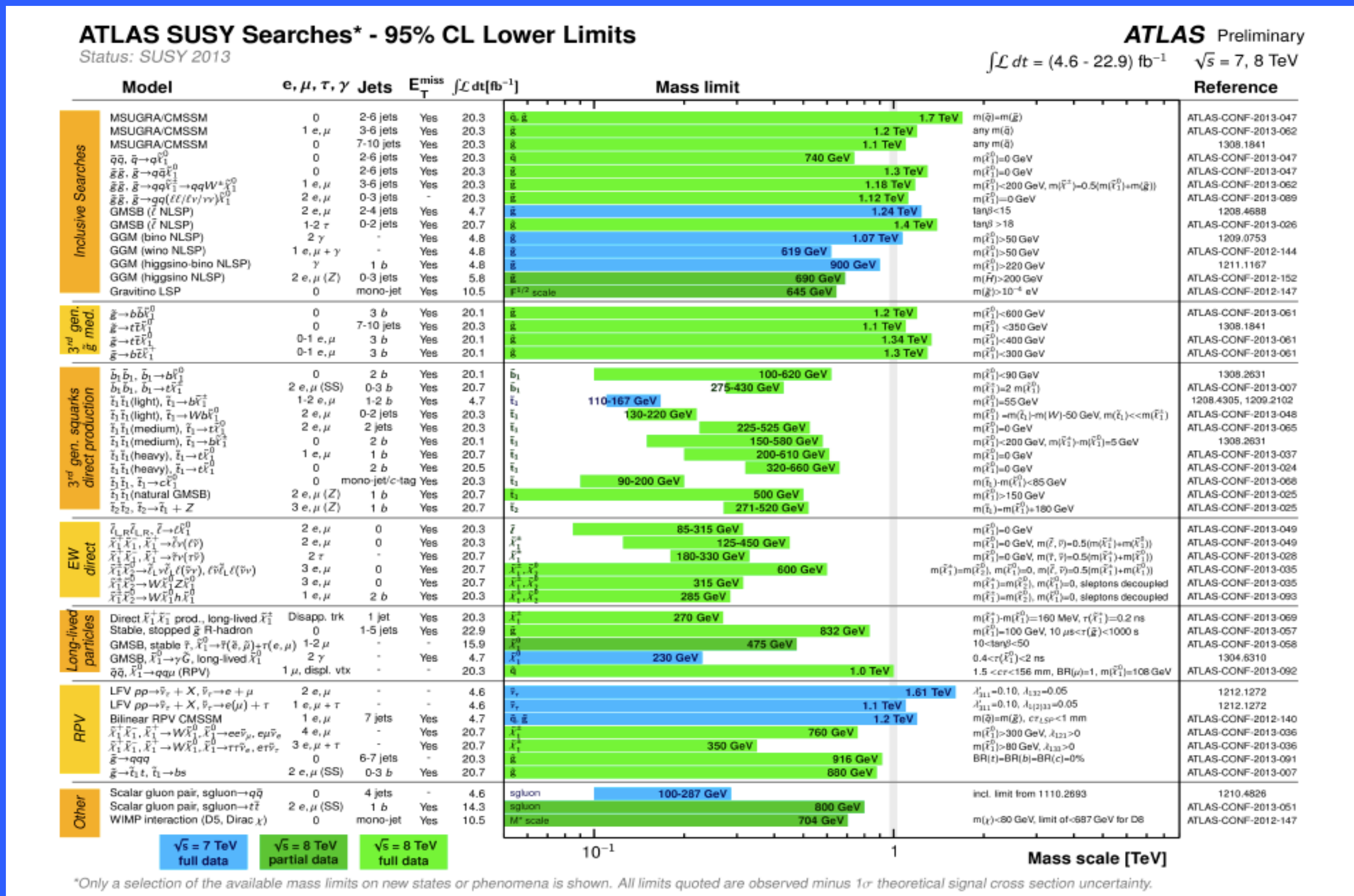
Dreams Unfulfilled

When all the particles of today's Standard Model are classified according to their spins (bosons or fermions) and matter/energy properties, the image is highly asymmetrical.

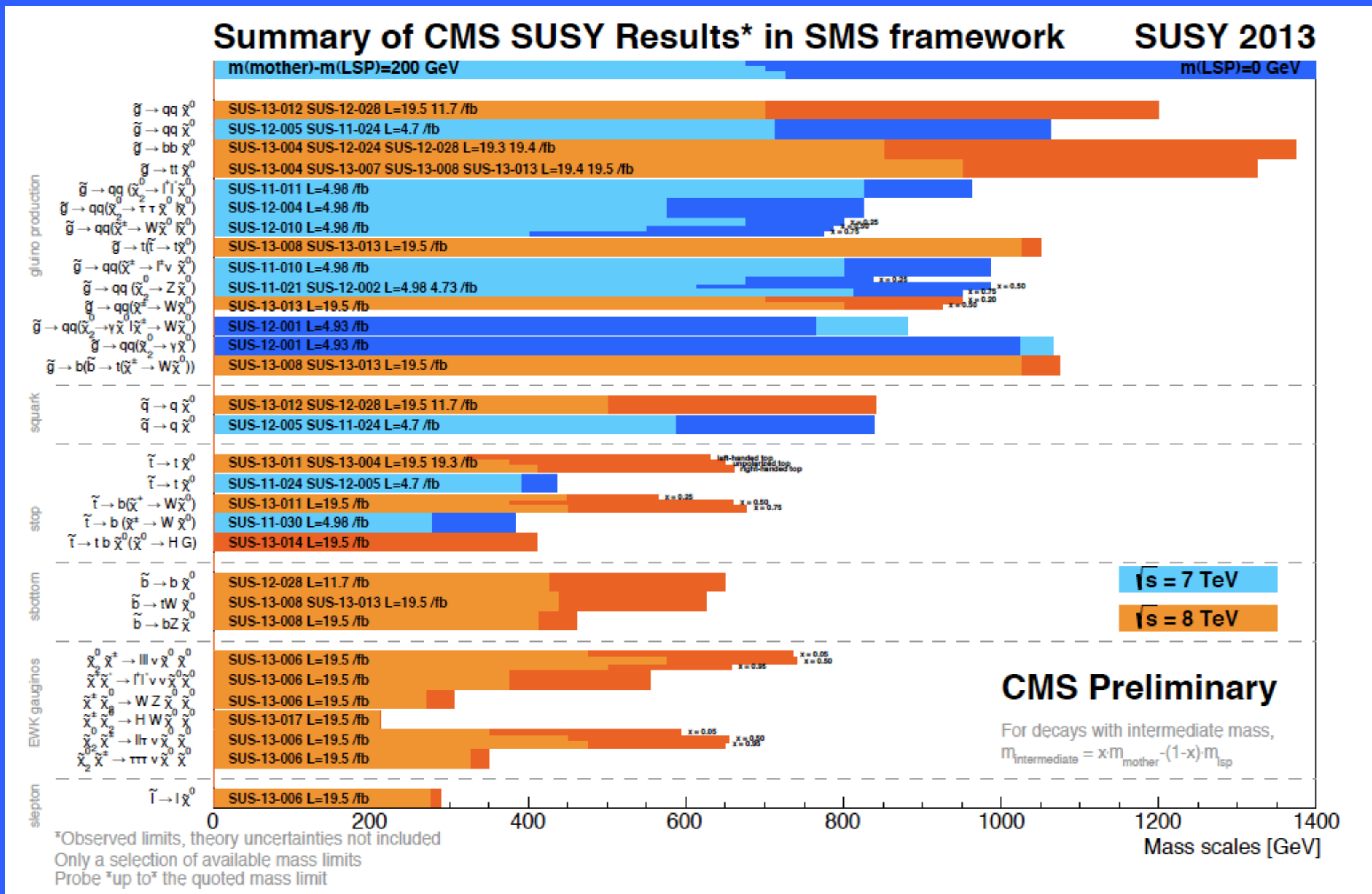


Should ‘sparticles’ or ‘superpartners’ be later observed in laboratories, once more there would he a high symmetrical table to describe physical reality.





ATLAS SUSY Searches* - 95% CL Lower Limits
Status: SUSY 2013
ATLAS Preliminary
∫L dt = (4.6 - 22.9) fb⁻¹
√s = 7, 8 TeV
Model
e, μ, τ, γ
Jets
E_T^miss
∫L dt[fb⁻¹]
Mass limit
Reference
Inclusive Searches
MSUGRA/CMSSM
1.7 TeV
1.2 TeV
1.1 TeV
740 GeV
1.3 TeV
1.18 TeV
1.12 TeV
GMSB (ℓ̃ NLSP)
1.24 TeV
1.4 TeV
GGM (bino NLSP)
1.07 TeV
GGM (wino NLSP)
619 GeV
GGM (higgsino-bino NLSP)
900 GeV
GGM (higgsino NLSP)
690 GeV
Gravitino LSP
645 GeV
3rd gen. g̃ med.
1.2 TeV
1.1 TeV
1.34 TeV
1.3 TeV
3rd gen. squarks direct production
100-620 GeV
275-430 GeV
110-167 GeV
130-220 GeV
225-525 GeV
150-580 GeV
200-610 GeV
320-660 GeV
90-200 GeV
500 GeV
271-520 GeV
EW direct
85-315 GeV
125-450 GeV
180-330 GeV
600 GeV
315 GeV
285 GeV
Long-lived particles
Stable, stopped g̃ R-hadron
270 GeV
832 GeV
475 GeV
230 GeV
1.0 TeV
RPV
Bilinear RPV CMSSM
1.61 TeV
1.1 TeV
1.2 TeV
760 GeV
350 GeV
916 GeV
880 GeV
Other
Scalar gluon pair, sgluon→qq̄
Scalar gluon pair, sgluon→tt̄
WIMP interaction (D5, Dirac χ)
100-287 GeV
800 GeV
704 GeV
√s = 7 TeV full data
√s = 8 TeV partial data
√s = 8 TeV full data
10⁻¹
1
Mass scale [TeV]
*Only a selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.


Summary of CMS SUSY Results* in SMS framework
SUSY 2013
m(mother)-m(LSP)=200 GeV
m(LSP)=0 GeV
gluino production
SUS-13-012 SUS-12-028 L=19.5 11.7 /fb
SUS-12-005 SUS-11-024 L=4.7 /fb
SUS-13-004 SUS-12-024 SUS-12-028 L=19.3 19.4 /fb
SUS-13-004 SUS-13-007 SUS-13-008 SUS-13-013 L=19.4 19.5 /fb
SUS-11-011 L=4.98 /fb
SUS-12-004 L=4.98 /fb
SUS-12-010 L=4.98 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
SUS-11-010 L=4.98 /fb
SUS-11-021 SUS-12-002 L=4.98 4.73 /fb
SUS-13-013 L=19.5 /fb
SUS-12-001 L=4.93 /fb
SUS-12-001 L=4.93 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
squark
SUS-13-012 SUS-12-028 L=19.5 11.7 /fb
SUS-12-005 SUS-11-024 L=4.7 /fb
stop
SUS-13-011 SUS-13-004 L=19.5 19.3 /fb
left-handed top
unpolarized top
right-handed top
SUS-11-024 SUS-12-005 L=4.7 /fb
SUS-13-011 L=19.5 /fb
SUS-11-030 L=4.98 /fb
SUS-13-014 L=19.5 /fb
sbottom
SUS-12-028 L=11.7 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
SUS-13-008 L=19.5 /fb
EWK gauginos
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-017 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
slepton
SUS-13-006 L=19.5 /fb
√s = 7 TeV
√s = 8 TeV
CMS Preliminary
For decays with intermediate mass,
m_intermediate = x·m_mother -(1-x)·m_lsp
0 200 400 600 800 1000 1200 1400
Mass scales [GeV]
*Observed limits, theory uncertainties not included
Only a selection of available mass limits
Probe *up to* the quoted mass limit

Constrained Minimal Supersymmetric

Phenomenological Minimal Supersymmetric

Symbol	Description	number of parameters
$\tan\beta$	the ratio of the vacuum expectation values of the two Higgs doublets	1
M_A	the mass of the pseudoscalar Higgs boson	1
μ	the higgsino mass parameter	1
M_1	the bino mass parameter	1
M_2	the wino mass parameter	1
M_3	the gluino mass parameter	1
$m_{\tilde{q}}, m_{\tilde{u}_R}, m_{\tilde{d}_R}$	the first and second generation squark masses	3
$m_{\tilde{l}}, m_{\tilde{e}_R}$	the first and second generation slepton masses	2
$m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}$	the third generation squark masses	3
$m_{\tilde{L}}, m_{\tilde{\tau}_R}$	the third generation slepton masses	2
A_t, A_b, A_τ	the third generation trilinear couplings	3

Constrained Minimal Supersymmetric Standard Model (CMSSM)

G. L. Kane, C. F. Kolda, L. Roszkowski and J. D. Wells, Phys. Rev. D 49 (1994) 6173

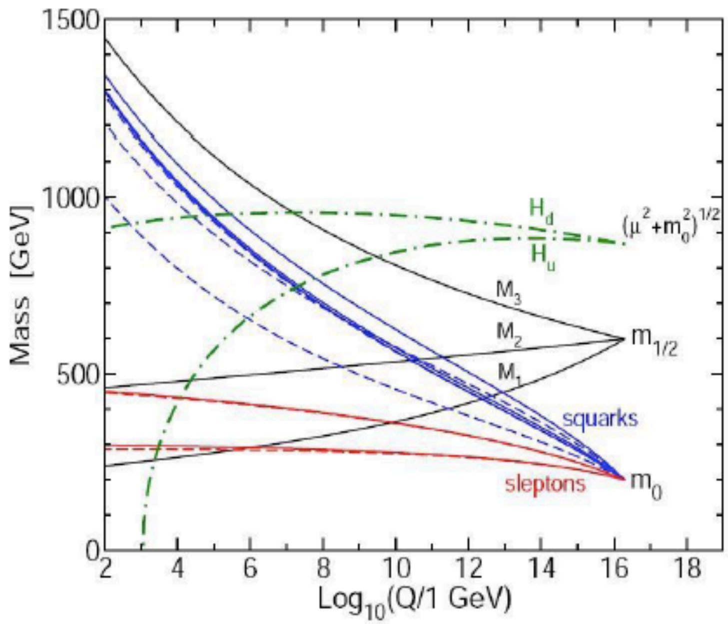


figure from hep-ph/9709356

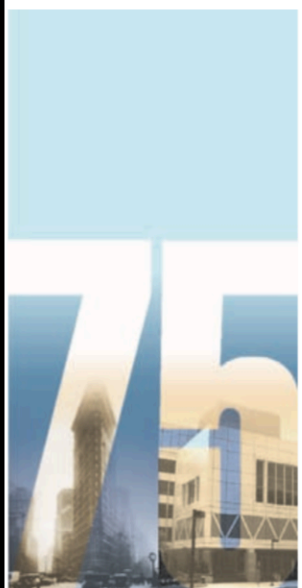
At $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}\,\mathrm{GeV}$:

- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars $m^2_{\tilde{q}_i} = m^2_{\tilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m_0^2$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB $\mu^2 = \frac{m^2_{H_b} - m^2_{H_t}\tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$
- five independent parameters: $\boxed{m_{1/2},\ m_0,\ A_0,\ \tan\beta,\ \mathrm{sgn}(\mu)}$
- well developed machinery to compute masses and couplings

Bayes FITS

In general supersymmetric SM too many free parameter

PHYSICS TODAY



Is string theory phenomenologically viable?

S. James Gates Jr

String theory is entering an era in which its theoretical constructs will be confronted by experimental data. Some cherished ideas just might fail to pass the test.

Jim Gates is the John S. Toll Professor of Physics and director of the Center for String and Particle Theory at the University of Maryland in College Park.

Physics Today 59, 6, 54 (2006); https://doi.org/10.1063/1.2218556

With the dates of discovery and the masses of the neutron and W boson as benchmarks, one can crudely estimate the rate at which humanity is progressing in its ability to detect massive particles… about 1.5 GeV/c^2 per year.

"Thus, if Nature is kind enough to provide light superpartners, one might still expect about a century to pass before a superparticle is directly observed."

– *Physics Today, 59N6 (2006) 54.*

“Much more likely, evidence for supersymmetry will emerge by indirect means. Such evidence might be provided by precision measurements of the rates of change of coupling constants, anomalies in lifetimes or branching ratios in decays of known particles, and so forth. ”

– *Physics Today,*
59N6 (2006) 54.

Some Past Lessons

Lesson 1-A

ELSEVIER

Nuclear Physics B

Volume 238, Issue 2, 11 June 1984, Pages 349-366

Superspace formulation of new non-linear sigma models

S. James Gates Jr. [1,2]

Lesson 1-B

ELSEVIER

Nuclear Physics B

Volume 248, Issue 1, 17 December 1984, Pages 157-186

Twisted multiplets and new supersymmetric non-linear σ-models ☆, ☆☆, ★

S.J. Gates Jr., C.M. Hull, M. Roček

Lesson 1-A

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Lesson 1-B

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Twisted multiplets and new supersymmetric non-linear σ-models ☆, ☆☆, ★

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Lesson 2

ELSEVIER
Nuclear Physics B
Volume 254, 1985, Pages 187-200

Comments on nonminimal $N = 1$ scalar multiplets

B.B. Deo *, S.J. Gates Jr. **

Lesson 1: Useful Inequivalence In Strings

$$(\mathrm{P}^{(\pm)})^{ab} \;=\; \tfrac{1}{2}\left[\, C^{ab} \pm (\gamma^5)^{ab} \,\right] \quad ,$$

$$(\mathrm{P}^{(\pm)})_a{}^b(\mathrm{P}^{(\pm)})_b{}^c \;=\; (\mathrm{P}^{(\pm)})_a{}^c \quad , \quad (\mathrm{P}^{(\pm)})_a{}^b(\mathrm{P}^{(\mp)})_b{}^c \;=\; 0 \quad ,$$

$$\begin{aligned}
&(\mathrm{P}^{(\pm)})_a{}^b(\gamma^5)_b{}^c \;=\; (\gamma^5)_a{}^b(\mathrm{P}^{(\pm)})_b{}^c \;=\; \pm\,(\mathrm{P}^{(\pm)})_a{}^c \quad , \quad (\mathrm{P}^{(\pm)})_{ab} \;=\; -\,(\mathrm{P}^{(\pm)})_{ba} \quad , \\
&(\gamma^\mu)_a{}^b(\mathrm{P}^{(\pm)})_b{}^c \;=\; (\mathrm{P}^{(\mp)})_a{}^b(\gamma^\mu)_b{}^c \quad , \quad (\mathrm{P}^{(\pm)}\gamma^\mu)_{ab} \;=\; (\mathrm{P}^{(\mp)}\gamma^\mu)_{ba} \quad , \\
&(\gamma^{\mu\nu})_a{}^b(\mathrm{P}^{(\pm)})_b{}^c \;=\; (\mathrm{P}^{(\pm)})_a{}^b(\gamma^{\mu\nu})_b{}^c \quad , \quad (\mathrm{P}^{(\pm)}\gamma^{\mu\nu})_{ab} \;=\; (\mathrm{P}^{(\pm)}\gamma^{\mu\nu})_{ba} \quad .
\end{aligned}$$

$$(\mathrm{P}^{(+)})^* \;=\; \mathrm{P}^{(-)} \quad , \quad (\mathrm{P}^{(+)}\gamma^\mu)^* \;=\; \mathrm{P}^{(-)}\gamma^\mu \quad , \quad (\mathrm{P}^{(+)}\gamma^{\mu\nu})^* \;=\; \mathrm{P}^{(-)}\gamma^{\mu\nu} \quad .$$

$$\mathrm{D}_a^{(\pm)} \;=\; (\mathrm{P}^{(\pm)})_a{}^b \mathrm{D}_b \quad ,$$

Lesson 1: Useful Inequivalence In Strings

The Chiral Supermultiplet in 4D, N = 1 & 2D, N = 2 Superspace

$$\mathrm{D}_a^{(-)}\Phi = 0 \quad , \quad \mathrm{D}_a^{(+)}\overline{\Phi} = 0 \quad .$$

$$\begin{aligned}
\mathrm{D}_a \boldsymbol{A} &= \boldsymbol{\psi}_a \quad , \quad \mathrm{D}_a \boldsymbol{B} = -\, i\,(\gamma^5)_a{}^b\, \boldsymbol{\psi}_b \quad , \\
\mathrm{D}_a \boldsymbol{\psi}_b &= i\,(\gamma^\mu)_{a\,b}\,\partial_\mu \boldsymbol{A} + (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu \boldsymbol{B} - i\,C_{a\,b}\,\boldsymbol{F} + (\gamma^5)_{a\,b}\boldsymbol{G} \quad , \\
\mathrm{D}_a \boldsymbol{F} &= (\gamma^\mu)_a{}^b\,\partial_\mu\,\boldsymbol{\psi}_b \quad , \quad \mathrm{D}_a \boldsymbol{G} = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\boldsymbol{\psi}_b \quad .
\end{aligned}$$

$$\boldsymbol{\Phi} = \boldsymbol{A} + i\,\boldsymbol{B} \qquad\qquad \boldsymbol{X} = \boldsymbol{F} + i\,\boldsymbol{G}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{CS}} &= -\,\tfrac{1}{32}\,\mathrm{D}^a\,\mathrm{D}_a^{(+)}\,\mathrm{D}^b\mathrm{D}_b^{(-)}\,\overline{\boldsymbol{\Phi}}\,\boldsymbol{\Phi} \\
&= -\,\tfrac{1}{2}\partial_\mu\boldsymbol{\Phi}\partial^\mu\overline{\boldsymbol{\Phi}} + \tfrac{1}{2}\boldsymbol{X}\overline{\boldsymbol{X}} + i\,\tfrac{1}{2}(\gamma^\mu)^{ab}\boldsymbol{\psi}_a\partial_\mu\boldsymbol{\psi}_b
\end{aligned}$$

Lesson 1: Useful Inequivalence In Strings

The ‘Vector’ Supermultiplet in 4D, N = 1 & 2D, N = 2 Superspace

$$\mathrm{D}_a^{(-)} \left((\mathrm{P}^{(+)} \boldsymbol{\lambda})_b \right) ~=~ 0 \quad ,$$

$$\begin{aligned}
\mathrm{D}_a^{(\pm)} \boldsymbol{A}_\mu ~&=~ (\mathrm{P}^{(\pm)} \gamma_\mu)_a{}^b \, \boldsymbol{\lambda}_b \quad , \\
\mathrm{D}_a^{(\pm)} \mathbf{d} ~&=~ \pm \, i \, (\mathrm{P}^{(\pm)} \gamma^\mu)_a{}^b \, \partial_\mu \boldsymbol{\lambda}_b \quad , \\
\mathrm{D}_a^{(\pm)} \boldsymbol{\lambda}_b ~&=~ - \, i \, \tfrac{1}{2} (\mathrm{P}^{(\pm)} \gamma^{\mu\nu})_{ab} \, \boldsymbol{F}_{\mu\nu} ~\pm~ (\mathrm{P}^{(\pm)})_{ab} \, \mathbf{d} \quad ,
\end{aligned}$$

$$\boldsymbol{F}_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{VS}} ~&=~ - \, \tfrac{1}{16} \, \mathrm{D}^a \, \mathrm{D}_a^{(+)} \, \boldsymbol{\lambda}^b (\mathrm{P}^{(+)} \boldsymbol{\lambda})_b ~+~ \mathrm{h.\,c.} \quad . \\
~&=~ - \, \tfrac{1}{4} \boldsymbol{F}_{\mu\nu} \boldsymbol{F}^{\mu\nu} ~+~ i \, \tfrac{1}{2} \, (\gamma^\mu)^{ab} \boldsymbol{\lambda}_a \partial_\mu \boldsymbol{\lambda}_b ~+~ \tfrac{1}{2} \mathbf{d}^2
\end{aligned}$$

Lesson 1: Useful Inequivalence In Strings

The 'Tensor' Supermultiplet in 4D, N = 1 & 2D, N = 2 Superspace

$$\mathrm{D}^a \mathrm{D}_a^{(+)} \boldsymbol{\varphi} = 0 \quad , \quad \mathrm{D}^b \mathrm{D}_b^{(-)} \boldsymbol{\varphi} = 0 \quad ,$$

$$\begin{aligned} \mathrm{D}_a^{(\pm)} \boldsymbol{\varphi} &= (\mathrm{P}^{(\pm)})_a{}^b \boldsymbol{\chi}_b \quad , \\ \mathrm{D}_a^{(\pm)} \boldsymbol{H}_\mu &= \mp i (\mathrm{P}^{(\pm)} \gamma_\mu{}^\rho)_a{}^b \partial_\rho \boldsymbol{\chi}_b \quad , \\ \mathrm{D}_a^{(\pm)} \boldsymbol{\chi}_b &= i (\mathrm{P}^{(\pm)} \gamma^\mu)_{ab} \left[\, \partial_\mu \boldsymbol{\varphi} \pm i \boldsymbol{H}_\mu \,\right] \quad . \end{aligned}$$

$$\boldsymbol{H}_{\rho\alpha\beta} = \partial_{[\rho} \boldsymbol{B}_{\alpha\beta]} \quad , \qquad \boldsymbol{H}_\mu = \tfrac{1}{3!} \epsilon_\mu{}^{\rho\alpha\beta} \boldsymbol{H}_{\rho\alpha\beta} \quad .$$

$$\begin{aligned} \mathcal{L}_{\mathrm{TS}} &= \tfrac{1}{32} \mathrm{D}^a \mathrm{D}_a^{(+)} \mathrm{D}^b \mathrm{D}_b^{(-)} \boldsymbol{\varphi}^2 + \mathrm{h.\,c.} \quad . \\ &= - \tfrac{1}{2} \partial_\mu \boldsymbol{\varphi} \partial^\mu \boldsymbol{\varphi} - \tfrac{1}{12} \boldsymbol{H}_{\mu\nu\rho} \boldsymbol{H}^{\mu\nu\rho} + i \tfrac{1}{2} (\gamma^\mu)^{bc} \boldsymbol{\chi}_b \partial_\mu \boldsymbol{\chi}_c \\ &= - \tfrac{1}{2} \partial_\mu \boldsymbol{\varphi} \partial^\mu \boldsymbol{\varphi} + \tfrac{1}{2} \boldsymbol{H}_\mu \boldsymbol{H}^\mu + i \tfrac{1}{2} (\gamma^\mu)^{bc} \boldsymbol{\chi}_b \partial_\mu \boldsymbol{\chi}_c \quad . \end{aligned}$$

Lesson 1: Useful Inequivalence In Strings

In the GHR paper:

When the dimensionally reduced version of the last two super-multiplets were studied it was concluded that the latter two theories are in fact equivalent to each other. In less than four dimensions these were given the name of the ‘twisted chiral supermultiplet.’ (‘C-Map’)

But when compared to the chiral supermultiplet an Interesting observation was made.

Lesson 1: Useful Inequivalence In Strings

Important Mathematical Lesson:

When sigma-models are described using ONLY one these supermultiplets in 2D, their geometry is Kahler.

When sigma-models are described using MORE than one of these supermultiplets in 2D, their geometry is NOT Kahler, but contains torsion.

This was the first hint of a new topic in mathematics now called ‘complex geometry.’

Lesson 1: Useful Inequivalence In Strings

Important Physics Lesson:

The physics of sigma-models described using ‘complex geometry’ is totally different from Kahler models. This has been observed many times in the context of string theories where different compactifications lead to the algebraic geometrical structures (homology) describing the zero-modes of the string.

Lesson 1: Useful Inequivalence In Strings

Important Physics Lesson:

This phenomenon where distinct supermultiplets with the same spectrum of states when simultaneously appearing in actions leads to distinct physical results has been given the name of "useful inequivalence" by T. Hubsch.

Lesson 2: Useful Inequivalence In 4D SUSY?

A Brief Aside On Majorana Spinors & Gamma Matrices

A natural from the view of Salam-Strathdee superfields is to introduce Majorana Spinors as a basis to describe spinors and thus we describe such fields by the introduction of

$$\psi^a(x) \ = \ \begin{bmatrix} \psi^1(x) \\ \psi^2(x) \\ \psi^3(x) \\ \psi^4(x) \end{bmatrix}$$

where the four anticommuting functions $\psi^a(x)$ (with $a = 1$, 2, 3, and 4) are real.

The four dimensional gamma matrices we use are defined by

$$(\gamma^0)_a{}^b = i(\sigma^3 \otimes \sigma^2)_a{}^b \quad , \qquad (\gamma^1)_a{}^b = (\mathbf{I}_2 \otimes \sigma^1)_a{}^b \quad ,$$
$$(\gamma^2)_a{}^b = (\sigma^2 \otimes \sigma^2)_a{}^b \quad , \qquad (\gamma^3)_a{}^b = (\mathbf{I}_2 \otimes \sigma^3)_a{}^b \quad .$$

which can all be seen to be purely real satisfying the conditions

$$\gamma^\mu \gamma^\nu \ + \ \gamma^\nu \gamma^\mu \ = \ 2\eta^{\mu\nu} \mathbf{I}_4 \ , \ \ \gamma^\mu \gamma_\mu \ = \ 4\,\mathbf{I}_4 \quad ,$$

Lesson 2: Useful Inequivalence In 4D SUSY?

The corresponding gamma-5 matrix is given by

$$(\gamma^5)_a{}^b = -(\sigma^1 \otimes \sigma^2)_a{}^b \quad .$$

which is purely imaginary. Hence onfy the products $\pm i\,(\gamma^5)$ can be multiplied by the Majorana spinor.

In order to raise and lower spinor indices, we define a spinor metric by

$$C_{ab} \equiv -i(\sigma^3 \otimes \sigma^2)_{ab} \;=\; \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \;\rightarrow\; C_{ab} = -C_{ba} \quad .$$

The inverse spinor metric is defined by the condition $C^{ab}C_{ac} = \delta_c{}^b$.

We construct a Dirac spinor $\Psi^a(x)$ by simply a doublet of Majorana $\psi^a_{(1)}(x)$ and $\psi^a_{(2)}(x)$ spinors to form a complex one.

$$\Psi^a(x) \;=\; \frac{1}{\sqrt{2}}\left[\; \psi^a_{(1)}(x) \;+\; i\,\psi^a_{(2)}(x)\right]$$

Lesson 2: Useful Inequivalence In 4D SUSY?

Introducing Dirac spinors in a system of 4D, N = 1 SUSY requires doubling.

$$\mathrm{D}_a A_{(I)} = \psi_{a(I)} \quad ,$$

$$\mathrm{D}_a B_{(I)} = i\,(\gamma^5)_a{}^b\,\psi_{b(I)} \quad ,$$

$$\mathrm{D}_a \psi_{b(I)} = i\,(\gamma^\mu)_{ab}\,\partial_\mu A_{(I)} - (\gamma^5\gamma^\mu)_{ab}\,\partial_\mu B_{(I)} - i C_{ab}\,F_{(I)} + (\gamma^5)_{ab}\,G_{(I)} \quad ,$$

$$\mathrm{D}_a F_{(I)} = (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_{b(I)} \quad ,$$

$$\mathrm{D}_a G_{(I)} = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_{b(I)} \quad .$$

Lesson 2: Useful Inequivalence In 4D SUSY?

The standard way to obtain the chiral components in SUSY phenomenological models from Dirac spinors

$$\Psi_{\alpha}(x) \;=\; \tfrac{1}{2}\,\Big[\;(\mathbf{I}_4)_{ab} \;+\; (\gamma^5)_{ab}\Big]\,\Psi^b(x)$$

$$\overline{\Psi}_{\dot{\alpha}}(x) \;=\; \tfrac{1}{2}\,\Big[\;(\mathbf{I}_4)_{ab} \;-\; (\gamma^5)_{ab}\Big]\,[\Psi^b(x)]^*$$

Lesson 2: Useful Inequivalence In 4D SUSY?

But could there be alternatives?

Lesson 2: Useful Inequivalence In 4D SUSY?

The Complex Linear Supermultiplet (CLS)

$$\overline{\mathrm{D}}^2 \Sigma = 0 \quad .$$

$$\mathrm{D}_\alpha B = \rho_\alpha \ , \ \overline{\mathrm{D}}_{\dot\alpha} B = \bar\zeta_{\dot\alpha} \ ,$$

$$\mathrm{D}_\alpha \rho_\beta = -C_{\alpha\beta} H \ , \ \overline{\mathrm{D}}_{\dot\alpha} \rho_\beta = U_{\beta\dot\alpha} \ ,$$

$$\mathrm{D}_\alpha \bar\zeta_{\dot\beta} = i\partial_{\alpha\dot\beta} B - U_{\alpha\dot\beta} \ , \ \overline{\mathrm{D}}_{\dot\alpha} \bar\zeta_{\dot\beta} = 0 \ ,$$

$$\mathrm{D}_\alpha H = 0 \ , \ \overline{\mathrm{D}}_{\dot\alpha} H = \tfrac{i}{2}\partial^\alpha{}_{\dot\alpha} \rho_\alpha - \bar\beta_{\dot\alpha} \ ,$$

$$\mathrm{D}_\alpha U_{\beta\dot\beta} = i\partial_{\alpha\dot\beta}\rho_\beta + \tfrac{i}{2} C_{\alpha\beta}\partial^\gamma{}_{\dot\beta}\rho_\gamma - C_{\alpha\beta}\bar\beta_{\dot\beta} \ , \ \overline{\mathrm{D}}_{\dot\alpha} U_{\beta\dot\beta} = iC_{\dot\alpha\dot\beta}\partial_\beta{}^{\dot\gamma}\bar\zeta_{\dot\gamma} \ ,$$

$$\mathrm{D}_\alpha \bar\beta_{\dot\beta} = -\tfrac{i}{2}\partial_{\alpha\dot\beta} H \ , \ \overline{\mathrm{D}}_{\dot\alpha} \bar\beta_{\dot\beta} = \tfrac{i}{2}\partial_{\alpha\dot\beta} U^\alpha{}_{\dot\alpha} + \partial^\alpha{}_{\dot\alpha}\partial_{\alpha\dot\beta} B + i\,\partial^\alpha{}_{\dot\alpha} U_{\alpha\dot\beta} \ .$$

Lesson 2: Useful Inequivalence In 4D SUSY?

The Complex Linear Supermultiplet (CLS)

$$
\begin{aligned}
B &= \Sigma| \quad , \\
\rho_\alpha = \mathrm{D}_\alpha \Sigma| \quad &, \quad \bar{\zeta}_{\dot{\alpha}} = \overline{\mathrm{D}}_{\dot{\alpha}} \Sigma| \quad , \\
H = \mathrm{D}^2 \Sigma| \quad , \quad U_{\alpha\dot{\alpha}} &= \overline{\mathrm{D}}_{\dot{\alpha}} \mathrm{D}_\alpha \Sigma| \quad , \quad \overline{U}_{\alpha\dot{\alpha}} = -\, \mathrm{D}_\alpha \overline{\mathrm{D}}_{\dot{\alpha}} \overline{\Sigma}| \quad , \\
\bar{\beta}_{\dot{\alpha}} &= \frac{1}{2} \mathrm{D}^\alpha \overline{\mathrm{D}}_{\dot{\alpha}} \mathrm{D}_\alpha \Sigma| \quad ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{CLS}} &= -\int d^2\theta d^2\bar{\theta}\, \overline{\Sigma}\Sigma = -\,\frac{1}{4} \mathrm{D}^\alpha \mathrm{D}_\alpha \overline{\mathrm{D}}^{\dot{\alpha}} \overline{\mathrm{D}}_{\dot{\alpha}} \{ \overline{\Sigma}\Sigma \}| \\
&= (\Box \overline{B}) B - H\overline{H} + \overline{U}^{\alpha\dot{\alpha}} U_{\alpha\dot{\alpha}} - i\,\zeta_\alpha \partial^{\alpha\dot{\alpha}} \bar{\zeta}_{\dot{\alpha}} + \beta^\alpha \rho_\alpha + \bar{\beta}^{\dot{\alpha}} \bar{\rho}_{\dot{\alpha}}
\end{aligned}
$$

Another Past Lesson

Lesson 3: S-T-P vs. V-A

$$\vec{S} = \{\, \Psi^a(x)\vec{T}\,\Psi_a(x)\}$$

$$\vec{V}_\mu = \{\, \Psi^a(x)\vec{T}(\gamma_\mu)_a{}^b\,\Psi_b(x)\}$$

$$\vec{t}_{\mu\nu} = \{\, \Psi^a(x)\vec{T}[(\gamma_\mu\ ,\ \gamma_\nu\,])_a{}^b\,\Psi_b(x)\}$$

$$\vec{A}_\mu = \{\, \Psi^a(x)\vec{T}(\gamma^5\gamma_\mu)_a{}^b\,\Psi_b(x)\}$$

$$\vec{P}_\mu = \{\, \Psi^a(x)\vec{T}(\gamma_5)_a{}^b\,\Psi_b(x)\}$$

$$\begin{aligned}\mathcal{L}(current\,\mathrm{x}\,current) \ &= \ c_s\,\vec{S}\cdot\vec{S} \ + \ c_v\,\vec{V}^\mu\cdot\vec{V}_\mu \ + \ c_t\,\vec{t}^{\mu\nu}\cdot\vec{t}_{\mu\nu} \\ &\quad + \ c_a\,\vec{A}^\mu\cdot\vec{A}_\mu \ + \ c_s\,\vec{S}\cdot\vec{S}\end{aligned}$$

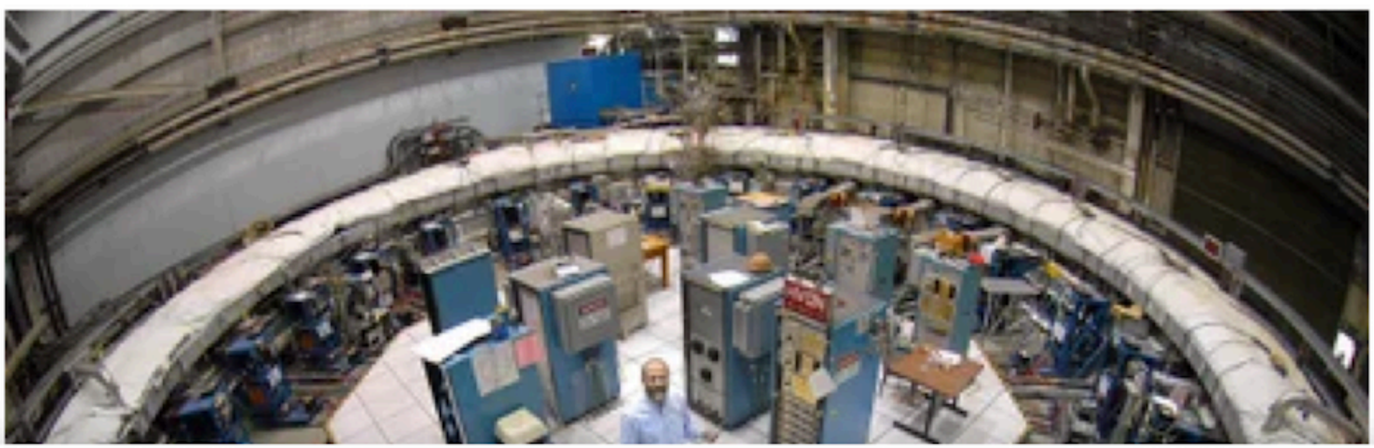
Lessons Possibility

PHYSICS

Long-Awaited Muon Measurement Boosts Evidence for New Physics

Initial data from the Muon g-2 experiment have excited particle physicists searching for undiscovered subatomic particles and forces

By Daniel Garisto on April 7, 2021

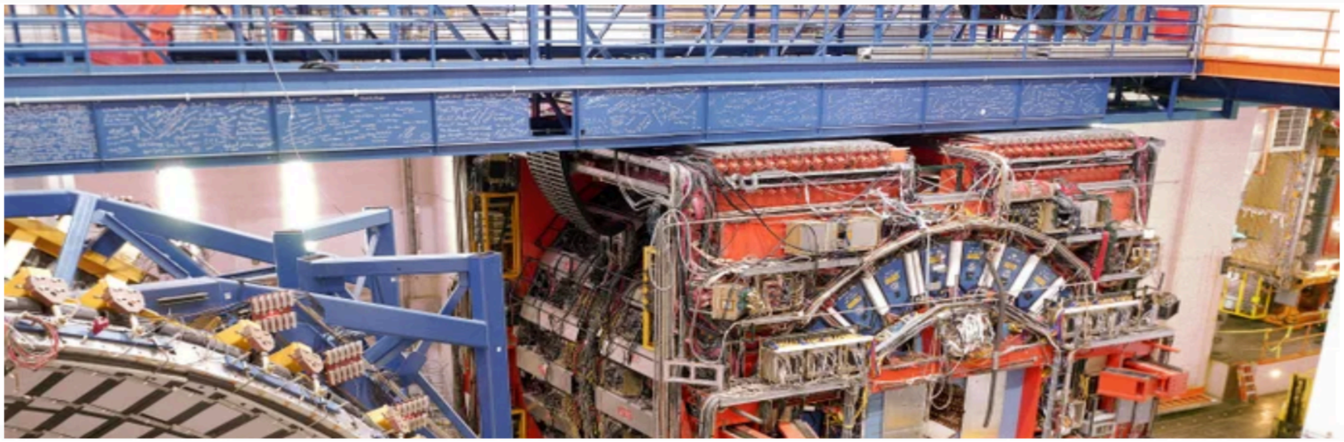


PARTICLE PHYSICS

Elementary Particle's Unexpected Heft Stuns Physicists

A new analysis by the CDF collaboration is a bolt from the blue, finding that the W boson is significantly heavier than suggested by previous measurements and theoretical prediction

By Daniel Garisto on April 7, 2022



Queries For Future Possibilities

On 1D, $\mathcal{N} = 4$ supersymmetric SYK-type models. Part II

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Abstract: This paper is an extension of our last 1D, $\mathcal{N} = 4$ supersymmetric SYK paper [1]. In this paper we introduced the complex linear supermultiplet (CLS), which is "usefully inequivalent" to the chiral supermultiplet. We construct three types of models based on the complex linear supermultiplet containing quartic interactions from modified CLS kinetic term, quartic interactions from 3-pt vertices integrated over the whole superspace, and $2(q-1)$-pt interactions generated via superpotentials respectively. A strong evidence for the inevitability of dynamical bosons for 1D, $\mathcal{N} = 4$ SYK is also presented.

Keywords: Extended Supersymmetry, Supersymmetric Effective Theories

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Queries For Future Possibilities

$$
\begin{aligned}
\mathcal{L}_{\rm CS+CLS+3PT-B} \;=\; & \int d^2\theta \, d^2\bar{\theta} \, \Big[\, \overline{\mathbf{\Phi}}^{\mathcal{A}} \, \mathbf{\Phi}_{\mathcal{A}} \;-\; \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}} \, \mathbf{\Sigma}_{\widehat{\mathcal{A}}} \, \Big] \;+ \\
& \Big\{ \int d^2\theta \, d^2\bar{\theta} \, \Big[\, \widehat{\kappa}_{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}} \, \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}} \, \mathbf{\Phi}^{\mathcal{B}} \, \mathbf{\Phi}^{\mathcal{C}} \, \Big] \;+\; {\rm h.\,c.} \Big\} \quad ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{\rm off-shell}_{\rm CS+CLS+3PT-B} = & \; (\Box \overline{A}^{\mathcal{A}}) A_{\mathcal{A}} \,+\, F^{\mathcal{A}} \overline{F}_{\mathcal{A}} \,+\, (\Box \overline{B}^{\widehat{\mathcal{A}}}) B_{\widehat{\mathcal{A}}} \,-\, H^{\widehat{\mathcal{A}}} \overline{H}_{\widehat{\mathcal{A}}} \,+\, \overline{U}^{\alpha\dot{\alpha}}_{\widehat{\mathcal{A}}} U^{\widehat{\mathcal{A}}}_{\alpha\dot{\alpha}} \\
& - \, B^{\widehat{\mathcal{A}}} \overline{\mathcal{Q}}'_{\widehat{\mathcal{A}}\mathcal{B}}(\overline{A}) \overline{F}^{\mathcal{B}} \,-\, \overline{B}^{\widehat{\mathcal{A}}} \mathcal{Q}'_{\widehat{\mathcal{A}}\mathcal{B}}(A) F^{\mathcal{B}} \,-\, \overline{\mathcal{Q}}^{\widehat{\mathcal{A}}}(\overline{A}) \mathcal{Q}_{\widehat{\mathcal{A}}}(A) \\
& - \, i \, \psi^{\mathcal{A}}_{\alpha} \partial^{\alpha\dot{\alpha}} \overline{\psi}_{\mathcal{A}\dot{\alpha}} \,-\, i \, \zeta^{\widehat{\mathcal{A}}}_{\alpha} \partial^{\alpha\dot{\alpha}} \overline{\zeta}_{\widehat{\mathcal{A}}\dot{\alpha}} \,-\, \mathcal{Q}' \psi^{\alpha} \zeta_{\alpha} \,-\, \overline{\mathcal{Q}}'^{\widehat{\mathcal{A}}\mathcal{B}} \overline{\psi}^{\dot{\alpha}}_{\mathcal{B}} \overline{\zeta}_{\widehat{\mathcal{A}}\dot{\alpha}} \\
& - \, \frac{1}{2} \, \mathcal{Q}''^{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}(A) \psi^{\alpha}_{\mathcal{B}} \psi_{\mathcal{C}\alpha} \overline{B}_{\widehat{\mathcal{A}}} \,-\, \frac{1}{2} \, \overline{\mathcal{Q}}''^{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}(\overline{A}) \, \overline{\psi}^{\dot{\alpha}}_{\mathcal{B}} \overline{\psi}_{\mathcal{C}\dot{\alpha}} B_{\widehat{\mathcal{A}}} \,+\, \beta^{\widehat{\mathcal{A}}\alpha} \rho_{\widehat{\mathcal{A}}\alpha} \,+\, \overline{\beta}^{\widehat{\mathcal{A}}\dot{\alpha}} \overline{\rho}_{\widehat{\mathcal{A}}\dot{\alpha}} \\
& + \, \Big\{ \widehat{\kappa}_{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}} \Big[- (\Box \overline{B}^{\widehat{\mathcal{A}}}) A^{\mathcal{B}} A^{\mathcal{C}} \,+\, \overline{\mathcal{Q}}'^{\widehat{\mathcal{A}}}{}_{\mathcal{D}}(\overline{A}) \, A^{\mathcal{B}} A^{\mathcal{C}} \overline{F}^{\mathcal{D}} \,-\, 2i \, \overline{U}^{\widehat{\mathcal{A}}}_{\alpha\dot{\alpha}} (\partial^{\alpha\dot{\alpha}} A^{\mathcal{B}}) A^{\mathcal{C}} \\
& + \, \tfrac{1}{2} \, \overline{\mathcal{Q}}''^{\widehat{\mathcal{A}}}{}_{\mathcal{D}\mathcal{E}}(\overline{A}) \, A^{\mathcal{B}} A^{\mathcal{C}} \overline{\psi}^{\mathcal{D}\dot{\alpha}} \overline{\psi}^{\mathcal{E}}_{\dot{\alpha}} \,-\, i \, \overline{\rho}^{\widehat{\mathcal{A}}}_{\dot{\alpha}} (\partial^{\alpha\dot{\alpha}} \psi^{\mathcal{B}}_{\alpha}) A^{\mathcal{C}} \,-\, i \, \overline{\rho}^{\widehat{\mathcal{A}}}_{\dot{\alpha}} \psi^{\mathcal{B}}_{\alpha} (\partial^{\alpha\dot{\alpha}} A^{\mathcal{C}}) \,-\, 2 \, \beta^{\widehat{\mathcal{A}}\alpha} \psi^{\mathcal{B}}_{\alpha} A^{\mathcal{C}} \\
& + \, 2 \, \overline{H}^{\widehat{\mathcal{A}}} F^{\mathcal{B}} A^{\mathcal{C}} \,+\, \overline{H}^{\widehat{\mathcal{A}}} \psi^{\mathcal{B}\alpha} \psi^{\mathcal{C}}_{\alpha} \Big] \,+\, {\rm h.\,c.} \Big\}
\end{aligned}
$$

Queries For Future Possibilities

$$\mathcal{L}_{\rm CS+CLS+3PT-B} = \int d^2\theta\, d^2\overline{\theta}\, \Big[\, \overline{\mathbf{\Phi}}^{\mathcal{A}}\, \mathbf{\Phi}_{\mathcal{A}} - \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}}\, \mathbf{\Sigma}_{\widehat{\mathcal{A}}} \,\Big] + \Big\{ \int d^2\theta\, d^2\overline{\theta}\, \Big[\, \widehat{\kappa}_{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}\, \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}}\, \mathbf{\Phi}^{\mathcal{B}}\, \mathbf{\Phi}^{\mathcal{C}} \,\Big] + {\rm h.c.} \Big\} \quad ,$$

$$\begin{aligned} \mathcal{L}^{\rm on-shell}_{\rm CS+CLS+3PT-B} &= \frac{\widehat{\kappa}^{\widehat{\mathcal{A}}}{}_{\mathcal{B}\mathcal{C}}\, \widehat{\kappa}^{*\widehat{\mathcal{G}}}{}_{\mathcal{D}\mathcal{E}}}{\delta^{\widehat{\mathcal{A}}\widehat{\mathcal{G}}} + \mathcal{Y}^{\widehat{\mathcal{A}}\widehat{\mathcal{G}}}}\, \psi^{\mathcal{B}\alpha}\, \psi^{\mathcal{C}}_{\alpha}\, \overline{\psi}^{\mathcal{D}\dot{\alpha}}\, \overline{\psi}^{\mathcal{E}}_{\dot{\alpha}} + \cdots \\ &= \widehat{\kappa}^{\widehat{\mathcal{A}}}{}_{\mathcal{B}\mathcal{C}}\, \widehat{\kappa}^{*}_{\widehat{\mathcal{A}}\mathcal{D}\mathcal{E}}\, \psi^{\mathcal{B}\alpha}\, \psi^{\mathcal{C}}_{\alpha}\, \overline{\psi}^{\mathcal{D}\dot{\alpha}}\, \overline{\psi}^{\mathcal{E}}_{\dot{\alpha}} + \cdots \end{aligned}$$

$$\mathcal{Y}^{\widehat{\mathcal{A}}\widehat{\mathcal{G}}} = 4\, \widehat{\kappa}^{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}\, \widehat{\kappa}^{*\widehat{\mathcal{G}}}{}_{\mathcal{B}\mathcal{D}}\, A_{\mathcal{C}}\, \overline{A}^{\mathcal{D}} \quad .$$

Queries For Future Possibilities

$$\mathcal{L}_{\rm CS+CLS+3PT-B} = \int d^2\theta\, d^2\overline{\theta} \left[\overline{\mathbf{\Phi}}^{\mathcal{A}} \mathbf{\Phi}_{\mathcal{A}} - \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}} \mathbf{\Sigma}_{\widehat{\mathcal{A}}} \right] + \left\{ \int d^2\theta\, d^2\overline{\theta} \left[\widehat{\kappa}_{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}} \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}} \mathbf{\Phi}^{\mathcal{B}} \mathbf{\Phi}^{\mathcal{C}} \right] + \text{h.c.} \right\} \quad ,$$

$$\begin{aligned}
\mathcal{L}_{\rm 3PT-A} &= \int d^2\theta\, d^2\overline{\theta}\, \kappa_{\mathcal{A}\mathcal{B}\mathcal{C}} \overline{\mathbf{\Phi}}^{\mathcal{A}} \mathbf{\Phi}^{\mathcal{B}} \mathbf{\Phi}^{\mathcal{C}} + \text{h.c.} \\
&= \frac{1}{4} \kappa_{\mathcal{A}\mathcal{B}\mathcal{C}}\, \mathrm{D}^{\alpha} \mathrm{D}_{\alpha} \overline{\mathrm{D}}^{\dot{\beta}} \overline{\mathrm{D}}_{\dot{\beta}} \left[\overline{\mathbf{\Phi}}^{\mathcal{A}} \mathbf{\Phi}^{\mathcal{B}} \mathbf{\Phi}^{\mathcal{C}} \right] \Big| + \text{h.c.}
\end{aligned}$$

Queries For Future Possibilities

$$
\begin{aligned}
\mathcal{L}^{\rm off-shell}_{\rm CS+CLS++3PT-A+3PT-B} = & (\Box\overline{A}^{\mathcal{A}})A_{\mathcal{A}} + F^{\mathcal{A}}\overline{F}_{\mathcal{A}} + (\Box\overline{B}^{\widehat{\mathcal{A}}})B_{\widehat{\mathcal{A}}} - H^{\widehat{\mathcal{A}}}\overline{H}_{\widehat{\mathcal{A}}} + \overline{U}^{\alpha\dot{\alpha}}_{\widehat{\mathcal{A}}} U^{\widehat{\mathcal{A}}}_{\alpha\dot{\alpha}} \\
& - B^{\widehat{\mathcal{A}}}\overline{Q}'_{\widehat{\mathcal{A}}\mathcal{B}}(\overline{A})\overline{F}^{\mathcal{B}} - \overline{B}^{\widehat{\mathcal{A}}} Q'_{\widehat{\mathcal{A}}\mathcal{B}}(A)F^{\mathcal{B}} - \overline{Q}^{\widehat{\mathcal{A}}}(\overline{A})Q_{\widehat{\mathcal{A}}}(A) \\
& - i\,\psi^{\mathcal{A}}_{\alpha}\,\partial^{\alpha\dot{\alpha}}\overline{\psi}_{\mathcal{A}\dot{\alpha}} - i\,\zeta^{\widehat{\mathcal{A}}}_{\alpha}\,\partial^{\alpha\dot{\alpha}}\overline{\zeta}_{\widehat{\mathcal{A}}\dot{\alpha}} - Q'\psi^{\alpha}\zeta_{\alpha} - \overline{Q}'^{\widehat{\mathcal{A}}\mathcal{B}}\overline{\psi}^{\dot{\alpha}}_{\mathcal{B}}\overline{\zeta}_{\widehat{\mathcal{A}}\dot{\alpha}} \\
& - \frac{1}{2}\,Q''^{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}(A)\psi^{\alpha}_{\mathcal{B}}\psi_{\mathcal{C}\alpha}\overline{B}_{\widehat{\mathcal{A}}} - \frac{1}{2}\,\overline{Q}''^{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}(\overline{A})\,\overline{\psi}^{\dot{\alpha}}_{\mathcal{B}}\overline{\psi}_{\mathcal{C}\dot{\alpha}}B_{\widehat{\mathcal{A}}} + \beta^{\widehat{\mathcal{A}}\alpha}\rho_{\widehat{\mathcal{A}}\alpha} + \overline{\beta}^{\widehat{\mathcal{A}}\dot{\alpha}}\overline{\rho}_{\widehat{\mathcal{A}}\dot{\alpha}} \\
& + \Big\{ \kappa_{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}} \Big[(\Box\overline{A}^{\widehat{\mathcal{A}}})A^{\mathcal{B}}A^{\mathcal{C}} + 2\,(i\partial^{\alpha\dot{\alpha}}\overline{\psi}^{\mathcal{A}}_{\dot{\alpha}})\psi^{\mathcal{B}}_{\alpha}A^{\mathcal{C}} \\
& + 2\,\overline{F}^{\mathcal{A}}F^{\mathcal{B}}A^{\mathcal{C}} + \overline{F}^{\mathcal{A}}\psi^{\mathcal{B}\alpha}\psi^{\mathcal{C}}_{\alpha} \Big] + {\rm h.c.} \Big\} \\
& + \Big\{ \widehat{\kappa}_{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}} \Big[-(\Box\overline{B}^{\widehat{\mathcal{A}}})A^{\mathcal{B}}A^{\mathcal{C}} + \overline{Q}'^{\widehat{\mathcal{A}}}{}_{\mathcal{D}}(\overline{A})\,A^{\mathcal{B}}A^{\mathcal{C}}\overline{F}^{\mathcal{D}} - 2i\,\overline{U}^{\widehat{\mathcal{A}}}_{\alpha\dot{\alpha}}(\partial^{\alpha\dot{\alpha}}A^{\mathcal{B}})A^{\mathcal{C}} \\
& + \tfrac{1}{2}\,\overline{Q}''^{\widehat{\mathcal{A}}}{}_{\mathcal{D}\mathcal{E}}(\overline{A})\,A^{\mathcal{B}}A^{\mathcal{C}}\overline{\psi}^{\mathcal{D}\dot{\alpha}}\overline{\psi}^{\mathcal{E}}_{\dot{\alpha}} - i\,\overline{\rho}^{\widehat{\mathcal{A}}}_{\dot{\alpha}}(\partial^{\alpha\dot{\alpha}}\psi^{\mathcal{B}}_{\alpha})A^{\mathcal{C}} - i\,\overline{\rho}^{\widehat{\mathcal{A}}}_{\dot{\alpha}}\psi^{\mathcal{B}}_{\alpha}(\partial^{\alpha\dot{\alpha}}A^{\mathcal{C}}) \\
& - 2\,\beta^{\widehat{\mathcal{A}}\alpha}\psi^{\mathcal{B}}_{\alpha}A^{\mathcal{C}} + 2\,\overline{H}^{\widehat{\mathcal{A}}}F^{\mathcal{B}}A^{\mathcal{C}} + \overline{H}^{\widehat{\mathcal{A}}}\psi^{\mathcal{B}\alpha}\psi^{\mathcal{C}}_{\alpha} \Big] + {\rm h.c.} \Big\} ~~,
\end{aligned}
$$

$$
\mathcal{L}^{\rm on-shell}_{\rm CS+CLS+3PT-A+3PT-B} = -\,\kappa^{\mathcal{A}}{}_{\mathcal{B}\mathcal{C}}\,\kappa^{*}_{\mathcal{A}\mathcal{D}\mathcal{E}}\,\psi^{\mathcal{B}\alpha}\,\psi^{\mathcal{C}}_{\alpha}\,\overline{\psi}^{\mathcal{D}\dot{\alpha}}\,\overline{\psi}^{\mathcal{E}}_{\dot{\alpha}} + \widehat{\kappa}^{\widehat{\mathcal{A}}}{}_{\mathcal{B}\mathcal{C}}\,\widehat{\kappa}^{*}_{\widehat{\mathcal{A}}\mathcal{D}\mathcal{E}}\,\psi^{\mathcal{B}\alpha}\,\psi^{\mathcal{C}}_{\alpha}\,\overline{\psi}^{\mathcal{D}\dot{\alpha}}\,\overline{\psi}^{\mathcal{E}}_{\dot{\alpha}} + \cdots
$$

Queries For Future Possibilities

$$\mathcal{L}_{\rm CS+CLS+nCLS-A} = \int d^2\theta\, d^2\bar{\theta} \left\{ \overline{\mathbf{\Phi}}^{\mathcal{A}} \mathbf{\Phi}_{\mathcal{A}} - \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}} \mathbf{\Sigma}_{\widehat{\mathcal{A}}} + \left[\mathbf{\Phi}^{\mathcal{A}} \mathcal{P}_{\mathcal{A}}(\mathbf{\Sigma}) + {\rm h.\,c.} \right] \right\} \quad .$$

$$\mathcal{L}_{\rm nCLS-A} = \tfrac{1}{4}\, {\rm D}^{\alpha} {\rm D}_{\alpha}\, \overline{\rm D}^{\dot{\alpha}} \overline{\rm D}_{\dot{\alpha}} \left[\mathbf{\Phi}^{\mathcal{A}} \mathcal{P}_{\mathcal{A}}(\mathbf{\Sigma}) \right] + {\rm h.\,c.} \quad .$$

$$\mathcal{P}_{\mathcal{A}}(\mathbf{\Sigma}) = \sum_{i=2}^{P} \kappa^{(i)}_{\mathcal{A}\widehat{\mathcal{B}}_1 \cdots \widehat{\mathcal{B}}_i} \prod_{k=1}^{i} \mathbf{\Sigma}^{\widehat{\mathcal{B}}_k} \quad ,$$

Queries For Future Possibilities

$$
\begin{aligned}
\mathcal{L}^{\rm off-shell}_{\rm CS+CLS+nCLS-A} =&\ (\Box \overline{A}^{\mathcal{A}}) A_{\mathcal{A}} + F^{\mathcal{A}} \overline{F}_{\mathcal{A}} + (\Box \overline{B}^{\widehat{\mathcal{A}}}) B_{\widehat{\mathcal{A}}} - H^{\widehat{\mathcal{A}}} \overline{H}_{\widehat{\mathcal{A}}} + \overline{U}^{\alpha\dot{\alpha}}_{\widehat{\mathcal{A}}} U^{\widehat{\mathcal{A}}}_{\alpha\dot{\alpha}} \\
&- B^{\widehat{\mathcal{A}}} \overline{\mathcal{Q}}'_{\widehat{\mathcal{A}}\mathcal{B}}(\overline{A})\overline{F}^{\mathcal{B}} - \overline{B}^{\widehat{\mathcal{A}}} \mathcal{Q}'_{\widehat{\mathcal{A}}\mathcal{B}}(A) F^{\mathcal{B}} - \overline{\mathcal{Q}}^{\widehat{\mathcal{A}}}(\overline{A})\mathcal{Q}_{\widehat{\mathcal{A}}}(A) \\
&- i\,\psi^{\mathcal{A}}_{\alpha} \partial^{\alpha\dot{\alpha}}\overline{\psi}_{\mathcal{A}\dot{\alpha}} - i\,\zeta^{\widehat{\mathcal{A}}}_{\alpha} \partial^{\alpha\dot{\alpha}}\overline{\zeta}_{\widehat{\mathcal{A}}\dot{\alpha}} - \mathcal{Q}'\psi^{\alpha}\zeta_{\alpha} - \overline{\mathcal{Q}}'^{\widehat{\mathcal{A}}\mathcal{B}}\overline{\psi}^{\dot{\alpha}}_{\mathcal{B}}\overline{\zeta}_{\widehat{\mathcal{A}}\dot{\alpha}} \\
&- \frac{1}{2}\, \mathcal{Q}''^{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}(A)\psi^{\alpha}_{\mathcal{B}}\psi_{\mathcal{C}\alpha}\overline{B}_{\widehat{\mathcal{A}}} - \frac{1}{2}\, \overline{\mathcal{Q}}''^{\widehat{\mathcal{A}}\mathcal{B}\mathcal{C}}(\overline{A})\, \overline{\psi}^{\dot{\alpha}}_{\mathcal{B}}\overline{\psi}_{\mathcal{C}\dot{\alpha}} B_{\widehat{\mathcal{A}}} + \beta^{\widehat{\mathcal{A}}\alpha}\rho_{\widehat{\mathcal{A}}\alpha} + \overline{\beta}^{\widehat{\mathcal{A}}\dot{\alpha}}\overline{\rho}_{\widehat{\mathcal{A}}\dot{\alpha}} \\
&+ \Big\{ F^{\mathcal{A}}\Big[\tfrac{1}{2}\, \mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})\overline{\zeta}^{\widehat{\mathcal{B}}_1\dot{\alpha}}\overline{\zeta}^{\widehat{\mathcal{B}}_2}_{\dot{\alpha}} + \mathcal{P}'_{\mathcal{A}\widehat{\mathcal{B}}_1}(\boldsymbol{\Sigma})\mathcal{Q}^{\widehat{\mathcal{B}}_1}(\boldsymbol{\Phi})\Big] \\
&+ \psi^{\mathcal{A}\alpha}\Big[\tfrac{1}{2}\, \mathcal{P}'''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2\widehat{\mathcal{B}}_3}(\boldsymbol{\Sigma})\rho^{\widehat{\mathcal{B}}_1}_{\alpha}\overline{\zeta}^{\widehat{\mathcal{B}}_2\dot{\alpha}}\overline{\zeta}^{\widehat{\mathcal{B}}_3}_{\dot{\alpha}} - \mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})\overline{\zeta}^{\widehat{\mathcal{B}}_1\dot{\alpha}}\Big(i\partial_{\alpha\dot{\alpha}}B^{\widehat{\mathcal{B}}_2} - U^{\widehat{\mathcal{B}}_2}_{\alpha\dot{\alpha}}\Big) \\
&\qquad + \mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})\rho^{\widehat{\mathcal{B}}_1}_{\alpha}\mathcal{Q}^{\widehat{\mathcal{B}}_2}(\boldsymbol{\Phi}) + \mathcal{P}'_{\mathcal{A}\widehat{\mathcal{B}}}(\boldsymbol{\Sigma})\mathcal{Q}'^{\widehat{\mathcal{B}}}{}_{\mathcal{C}}\psi^{\mathcal{C}}_{\alpha}\Big] \\
&+ A^{\mathcal{A}}\Big[\tfrac{1}{4}\, \mathcal{P}''''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2\widehat{\mathcal{B}}_3\widehat{\mathcal{B}}_4}(\boldsymbol{\Sigma})\rho^{\widehat{\mathcal{B}}_1\alpha}\rho^{\widehat{\mathcal{B}}_2}_{\alpha}\overline{\zeta}^{\widehat{\mathcal{B}}_3\dot{\alpha}}\overline{\zeta}^{\widehat{\mathcal{B}}_4}_{\dot{\alpha}} + \tfrac{1}{2}\, \mathcal{P}'''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2\widehat{\mathcal{B}}_3}(\boldsymbol{\Sigma})H^{\widehat{\mathcal{B}}_1}\overline{\zeta}^{\widehat{\mathcal{B}}_2\dot{\alpha}}\overline{\zeta}^{\widehat{\mathcal{B}}_3}_{\dot{\alpha}} \\
&\qquad - \mathcal{P}'''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2\widehat{\mathcal{B}}_3}(\boldsymbol{\Sigma})\rho^{\widehat{\mathcal{B}}_1}_{\alpha}\Big(i\partial^{\alpha\dot{\alpha}}B^{\widehat{\mathcal{B}}_2} - U^{\widehat{\mathcal{B}}_2\alpha\dot{\alpha}}\Big)\overline{\zeta}^{\widehat{\mathcal{B}}_3}_{\dot{\alpha}} \\
&\qquad - \tfrac{1}{2}\, \mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})\Big(i\partial^{\alpha\dot{\alpha}}B^{\widehat{\mathcal{B}}_1} - U^{\widehat{\mathcal{B}}_1\alpha\dot{\alpha}}\Big)\Big(i\partial_{\alpha\dot{\alpha}}B^{\widehat{\mathcal{B}}_2} - U^{\widehat{\mathcal{B}}_2}_{\alpha\dot{\alpha}}\Big) \\
&\qquad - \mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})\overline{\zeta}^{\widehat{\mathcal{B}}_1\dot{\alpha}}\overline{\beta}^{\widehat{\mathcal{B}}_2}_{\dot{\alpha}} + \tfrac{1}{2}\, \mathcal{P}'''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2\widehat{\mathcal{B}}_3}(\boldsymbol{\Sigma})\rho^{\widehat{\mathcal{B}}_1\alpha}\rho^{\widehat{\mathcal{B}}_2}_{\alpha}\mathcal{Q}^{\widehat{\mathcal{B}}_3}(\boldsymbol{\Sigma}) \\
&\qquad + \mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})H^{\widehat{\mathcal{B}}_1}\mathcal{Q}^{\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma}) - \mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})\rho^{\widehat{\mathcal{B}}_1}_{\alpha}\mathcal{Q}'^{\widehat{\mathcal{B}}_2}{}_{\mathcal{C}}(\boldsymbol{\Sigma})\psi^{\mathcal{C}\alpha} \\
&\qquad + \mathcal{P}'_{\mathcal{A}\widehat{\mathcal{B}}_1}(\boldsymbol{\Sigma})\Big(\tfrac{1}{2}\, \mathcal{Q}''^{\widehat{\mathcal{B}}_1}{}_{\mathcal{C}\mathcal{D}}\psi^{\mathcal{C}\alpha}\psi^{\mathcal{D}}_{\alpha} + \mathcal{Q}'^{\widehat{\mathcal{B}}_1}{}_{\mathcal{C}}F^{\mathcal{C}}\Big)\Big] \qquad \cdots \\
&- \frac{i}{2}\,\partial_{\alpha\dot{\alpha}}\Big(A^{\mathcal{A}}\mathcal{P}''_{\mathcal{A}\widehat{\mathcal{B}}_1\widehat{\mathcal{B}}_2}(\boldsymbol{\Sigma})\overline{\zeta}^{\widehat{\mathcal{B}}_1\dot{\alpha}}\Big)\rho^{\widehat{\mathcal{B}}_2\alpha} + \text{h.c.}\Big\} \quad .
\end{aligned}
$$

Queries For Future Possibilities

$$\mathcal{L}_{\mathrm{CS+CLS+nCLS-A}} = \int d^2\theta \, d^2\bar{\theta} \left\{ \overline{\mathbf{\Phi}}^{\mathcal{A}} \mathbf{\Phi}_{\mathcal{A}} - \overline{\mathbf{\Sigma}}^{\widehat{\mathcal{A}}} \mathbf{\Sigma}_{\widehat{\mathcal{A}}} + \left[\mathbf{\Phi}^{\mathcal{A}} \mathcal{P}_{\mathcal{A}}(\mathbf{\Sigma}) + \mathrm{h.\,c.} \right] \right\} \quad .$$

$$\mathcal{L}_{\mathrm{nCLS-A}} = \tfrac{1}{4} \, \mathrm{D}^{\alpha} \mathrm{D}_{\alpha} \, \overline{\mathrm{D}}^{\dot{\alpha}} \overline{\mathrm{D}}_{\dot{\alpha}} \left[\mathbf{\Phi}^{\mathcal{A}} \mathcal{P}_{\mathcal{A}}(\mathbf{\Sigma}) \right] + \mathrm{h.\,c.} \quad .$$

$$\mathcal{P}_{\mathcal{A}}(\mathbf{\Sigma}) = \sum_{i=2}^{P} \kappa^{(i)}_{\mathcal{A}\widehat{\mathcal{B}}_1 \cdots \widehat{\mathcal{B}}_i} \prod_{k=1}^{i} \mathbf{\Sigma}^{\widehat{\mathcal{B}}_k} \quad ,$$

$$\mathcal{L}^{\mathrm{on-shell}}_{\mathrm{CS+CLS+nCLS-A}} = - \, \kappa^{(2)*\mathcal{A}}{}_{\widehat{\mathcal{B}}_1 \widehat{\mathcal{B}}_2} \, \kappa^{(2)}_{\mathcal{A}\widehat{\mathcal{C}}_1 \widehat{\mathcal{C}}_2} \, \zeta^{\widehat{\mathcal{B}}_1 \alpha} \zeta_{\alpha}^{\widehat{\mathcal{B}}_2} \, \bar{\zeta}^{\widehat{\mathcal{C}}_1 \dot{\alpha}} \bar{\zeta}_{\dot{\alpha}}^{\widehat{\mathcal{C}}_2} + 4 \, \kappa^{(2)}_{\mathcal{A}\widehat{\mathcal{B}}\widehat{\mathcal{C}}} \, \kappa^{(2)*\widehat{\mathcal{C}}}_{\mathcal{D}\widehat{\mathcal{E}}} \, \psi^{\mathcal{A}\alpha} \bar{\zeta}^{\widehat{\mathcal{B}}\dot{\alpha}} \bar{\psi}_{\dot{\alpha}}^{\mathcal{D}} \zeta_{\alpha}^{\widehat{\mathcal{E}}} + \cdots$$

Queries For Future Possibilities

Important Physics Question:

We have presented an argument that the two different spin 0-1/2 supermultiplets (CS vs. CLS), while equivalent as free field theories, are usefully inequivalent when higher current x current operators are included.

The conventional way of represented Dirac fermions in the presence of SUSY (via CS pairs) seems to rule out products of V-A fermion currents in effective actions.

Queries For Future Possibilities

Important Physics Question:

We have presented an argument that the two different spin 0-1/2 supermultiplets (CS vs. CLS), while equivalent As free field theories, are usefully inequivalent when higher current x current operators are included.

The representation of Dirac fermions in the presence of SUSY (via CS-CLS pairs) seems to allow products of V-A fermion currents in effective actions.

Queries For Future Possibilities

Important Physics Question:

In the presence of right-left symmetry, in all other ways, does the CS-CLS description give a usefully inequivalent theory realizing the breaking of this symmetry in an important way?

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