

Rabi's scenarios for neutrino mass and a “big picture”

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Scenario for neutrino mass

L-R Seesaw
GUT (or kind of the g - l unification)
SUSY (in some form)

➡ Neutrinos are Majorana

Rabi's
criteria

Natural
Plausible
Testable

Understanding neutrino mass and mixing requires larger framework

➡ a “big picture”

Content

L-R seesaw and its incarnations

Embedding into big picture

Status, challenges and perspectives

R.N. Mohapatra , AYS,

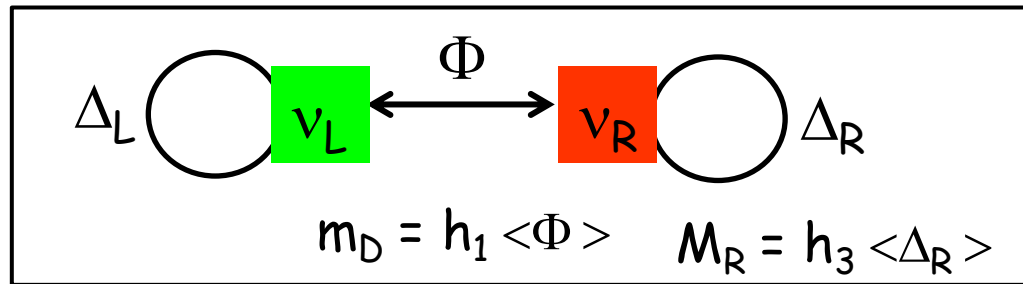
Seesaw and incarnations

From L-R to see-saw

*R. N. Mohapatra and G. Senjanovic,
PRL 44, 14 (1980) 912*

Explanation of smallness of neutrino mass in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Key: Higgs triplets Δ_L, Δ_R instead of doublets



Φ is bi-doublet

$\langle \Delta_R \rangle$ $M_R = h_3 \langle \Delta_R \rangle$
 $m_{W_R} = g \langle \Delta_R \rangle$

$M_R \gg m_D$

$\langle \Delta_L \rangle = 0$

$$m_\nu = - \frac{(h_1 \langle \Phi \rangle)^2}{M_R} = \frac{g (h_1 \langle \Phi \rangle)^2}{h_3 m_{W_R}}$$

Seesaw scale is determined by the scale of L- R symmetry violation.

Smallness of neutrino mass is related to the suppression of the RH weak interactions

L-R seesaw

Minimal?

If $\langle \Delta_L \rangle \neq 0 \rightarrow$ seesaw type II

In general: seesaw type I + II more parameters/ freedom

Where are Higgs doublets? Why Nature starts with higher representation - should be a good reason?

Possible explanation: no elementary scalars

Higgses - composite states made of fermion doublets F_L, F_R

$$\Delta_L = (F_L F_L), \Delta_R = (F_R F_R), \Phi = (F_L F_R)$$

But in the Nambu Jona-Lasinio approach -
no parity violation

*E. K. Akhmedov, et al
PRD 53 (1996) 2752,
PLB 368 (1996) 270*

New fermions instead Higgs triplets:

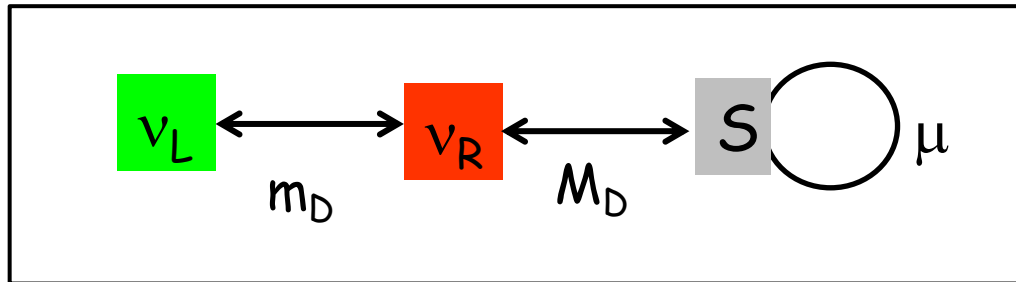
Scheme: L, R doublets (+ bi-doublet) + new fermions

Extended seesaws

R. N. Mohapatra, PRL 56, 6, (1986) 561

R. N. Mohapatra and J W F Valle PRD 34, 5, (1986) 1642

New fermions - singlets of L-R symmetry



$$M_R \gg m_D$$

No representations/interactions which give Majorana mass of ν_R

In M paper S = gauge fermion: mixture of SUSY partner of W_{3R} and Z_{B-L} . μ can be generated radiatively in 2 loops?

In M - V paper: S is singlet of $E_6 \times E_6$

One needs to start with superstrings, Calabi-Yao compactification, Ten dimensional $E_8 \times E_8$ Yang-Mills theory, symmetry breaking ... to arrive at neutrino mass matrix

Double seesaw and inverse seesaw

Mass matrix in the basis (ν, ν^c, S) :

$$\begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & \mu \end{pmatrix}$$

for 3 generations three S
or 3 combinations of many S

$\mu \gg M_D$ - double seesaw

RH neutrinos get mass via see-saw $M_R = M_D^T \mu^{-1} M_D$

$$\mu \sim M_{\text{Pl}}, M_D \sim M_{\text{GUT}}$$

$\mu \ll M_D$ - inverse seesaw

ν^c, S form pseudoDirac neutrino with mass M_D

In both cases for light neutrinos $m_\nu = m_D^T M_D^{-1T} \mu M_D^{-1} m_D$

If $m_D = A M_D$ ($A = \text{const}$) $\rightarrow m_\nu = A^2 \mu$

M. Lindner, M. Schmidt, A.S.

Structure of m_ν is determined by μ , it does not depend on the Dirac mass matrix structure (Dirac screening)

LHC-filic L-R models

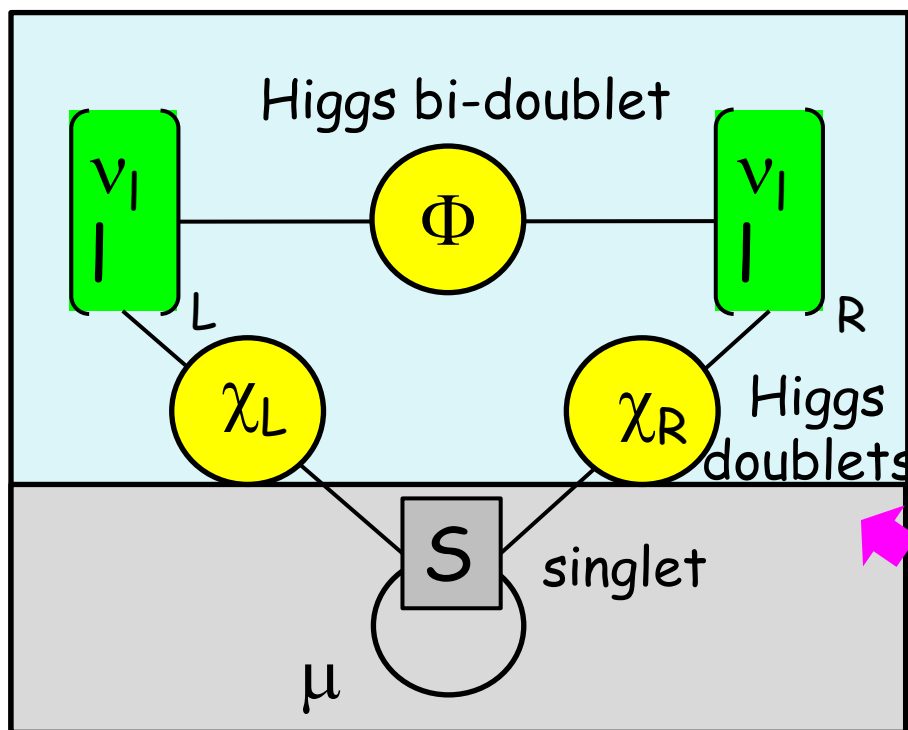
*P.S.Bhupal Dev,
R.N. Mohapatra,
0910.3924 [hep-ph]*

with GUT embeddings

*V.Brdar, A. Yu.S
JHEP 02 (2019) 045*

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$$

with q-l similarity $m_q \sim m_l \sim m_{\nu}^D$



| Fields | L_L | L_R | χ_L | χ_R | S |
|--------|-------|-------|----------|----------|-----|
| $B-L$ | -1 | -1 | 1 | 1 | 0 |

inverse seesaw

$$M_D \sim M_R \sim \text{PeV}$$

$$\mu \sim 10 \text{ keV}$$

flavor symmetry in μ

breaks
L-R symmetry

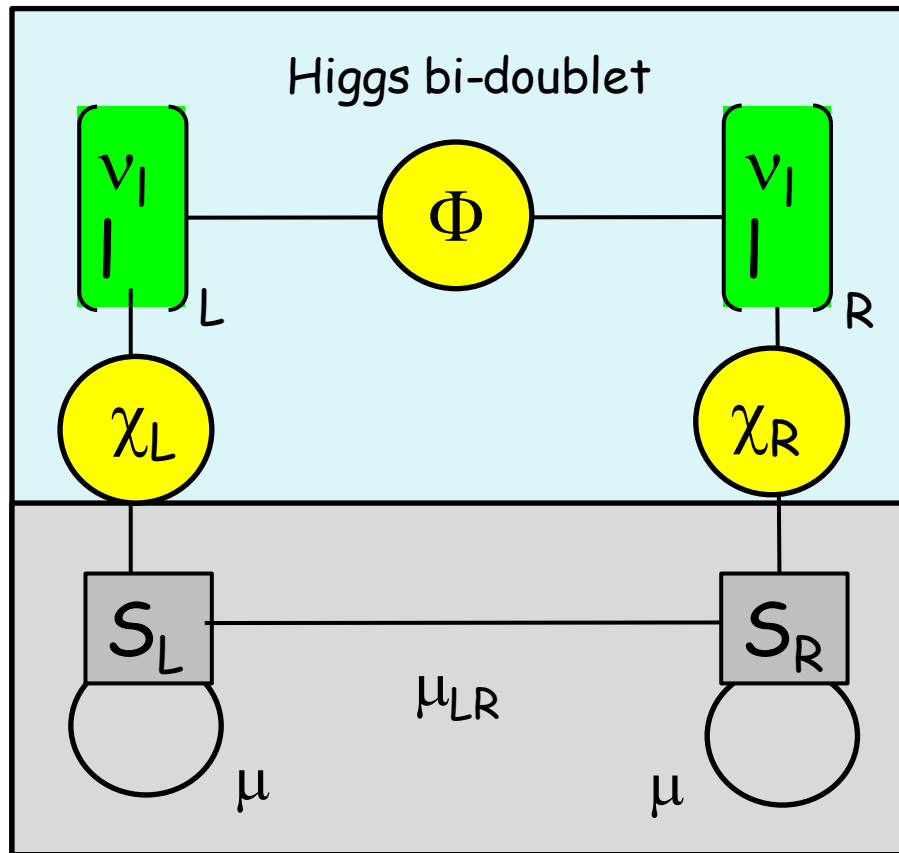
with Majorana mass terms

Dark
sector

Model with L and R singlets

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$$

V.Brdar, A. Yu.S
JHEP 02 (2019) 045
1809.09115 [hep-ph]



invariant under global $U(1)$
broken by μ -terms

keV scale sterile neutrino
- DM

Local or Global? Majoron

In L-R $U(1)_{L+R} = U(1)_{B-L}$ local symmetry

R.N. Mohapatra and R.E. Marshak
PRL 44, 20 (1980) 1316

Y. Chikashige R.N. Mohapatra and R. D. Peccei
PLB, 4 (1981) 265, PRL 45, 24 (1980) 1926

Lepton number is spontaneously broken global symmetry

→ Goldstone boson = Majoron

Rich phenomenology, long range forces, neutrino decay, $\beta\beta_{0\nu}$ - decay, Supernova neutrinos, DM ...

Neutrino decay into light or massless scalar

A.Y.S. Sov.J.Nucl. Phys.
34(6) (1981), 859

How heavy are neutrinos?

A. Yu. Smirnov

Institute of Nuclear Research, USSR Academy of Sciences

[Submitted 19 March 1981]

Yad. Fiz. 34, 1547-1553 (December 1981)

Conditions are analysed under which masses of the ν_μ and ν_τ neutrinos may exceed the cosmological bound $\sum m(\nu_i) \lesssim 50$ eV and attain the upper laboratory limits. The most preferable variant is as follows: a light scalar particle χ^0 exists [$m(\chi^0) < m(\nu_\mu)$], has only off-diagonal interactions with the neutrinos, and is responsible for sufficiently rapid decays $\nu_\mu \rightarrow \nu_e \chi^0, \nu_\tau \rightarrow \nu_\mu \chi^0, \dots$. The special status of the neutrinos among other fermions is related here to the fact that ν is the only particle that may have a Majorana-type mass.

Let us examine the following potential of the fields φ and Φ :

$$-\eta^2 \varphi^2 + i \kappa \varphi^4 - \mu^2 \Phi^2 + i \lambda \Phi^4 + \delta \Phi^2 (\varphi^+ + \varphi). \quad (4)$$

In the case $\delta = 0$ the total Lagrangian of the model will be invariant with respect to the continuous transformations $\varphi \rightarrow e^{2i\alpha} \varphi; (\nu, l) \rightarrow e^{-i\alpha} (\nu, l)$, so that upon spontaneous symmetry breaking the field ζ_1 is massless (i.e.,

Seesaw and lepton mixing

Mixing pattern can be the key for understanding neutrino mass

Two facts: Large lepton mixing with certain pattern: \sim TBM

Weak mass hierarchy, if any

Can be related

Seesaw enhancement of lepton mixing

Smallness of neutrino mass and large mixing have the same origin - seesaw

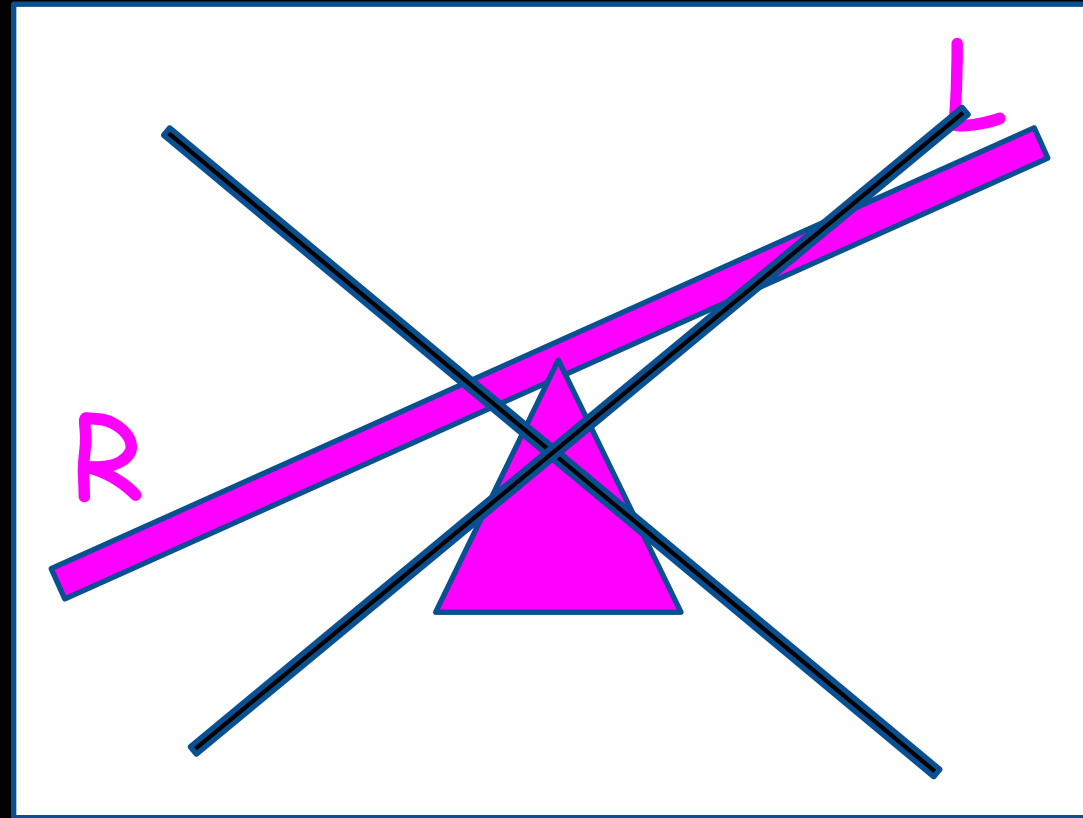
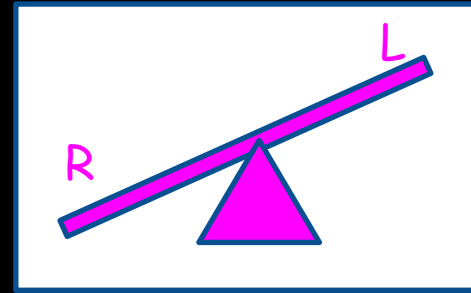
*A Y S, Phys.Rev.D
48 (1993) 3264*

*E. Akhmedov,
M. Frigerio,
AYS*

Mixing \rightarrow structure of neutrino mass matrix, properties, symmetries of m_ν $L_\mu - L_\tau$

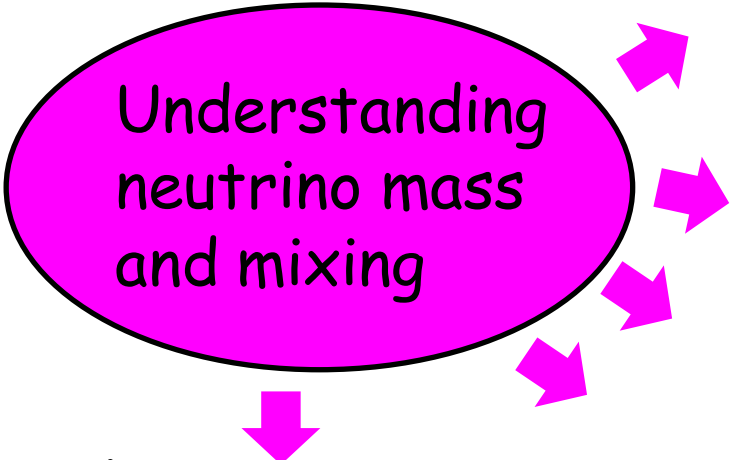
Mixing, mass hierarchy (flavor physics), demand further extensions, big picture

A Big picture



A big picture

Understanding
neutrino mass
and mixing



Quark-lepton
correspondence,
symmetry,
Unification, GUT

Flavor physics

SUSY

Connections:

Dark Matter

Baryon asymmetry
of the Universe

Inflation

Anomalies

$(g - 2)_\mu$

B- anomalies

Hubble tension

Quarks and Leptons. QLC

Similarity of mass hierarchy of quarks and charged leptons
Famous equality $m(b) = m(\tau)$ at GUT scale

Puzzle: strongly different patterns of quarks and lepton mixings

But relations:

$$\theta_{12}^l \sim \pi/4 - \theta_{12}^q$$

$$\theta_{23}^l \sim \pi/4 - \theta_{23}^q$$

data

$$33.4^\circ \sim 45^\circ - 13^\circ$$

Lepton mixing \sim maximal minus quark mixing

Quark - lepton complementarity

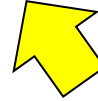
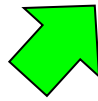
H. Minakata , AYS

$$1 \gg \theta_{13}^l \gg \theta_{13}^q$$

The relations are approximate and at low energy scale.
Renormalization, corrections ...

Framework

$$U_{\text{PMNS}} = U_{\text{lept}} + U_X$$



Common sector for quarks and leptons:

$$U_{\text{lept}} \sim V_{\text{CKM}}$$

From the "dark sector" coupled to neutrinos. Responsible for large neutrino mixing and smallness of neutrino mass

Implies

Q - L unification, GUT

CKM physics: hierarchy of masses and mixings, relations between masses and mixing

may have special symmetries which lead to BM or TBM mixing

$$U_X = U_{\text{BM}}, U_{\text{TBM}}$$

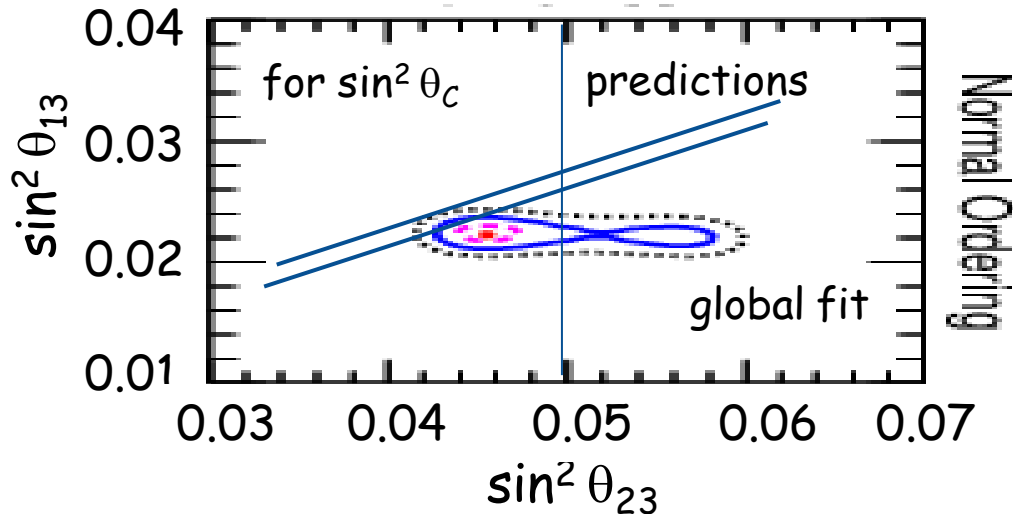
→ easy to realize flavor symmetry

Prediction

for the 1-3 leptonic mixing:

$$\sin^2\theta_{13} \sim \frac{1}{2} \sin^2\theta_c$$

$$\sin^2\theta_{13} = \sin^2\theta_{23} \sin^2\theta_c (1 + O(\lambda^2))$$



θ_c - Cabibbo angle

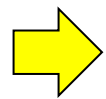
Difference can be due to deviation of θ_{12}^l from θ_c related to difference of q and l - masses

Renormalization effects from GUT to low energies

Predictions for δ_{CP}^l

B. Dasgupta, A.Y.S.,
N.P. B884 (2014) 357
1404.0272 [hep-ph]

If $U_X = U_{BM}, U_{TBM}$, then $U_{lept} \sim V_{CKM}$ can be the only source of CP violation. Leads to the relation



$$\sin \delta_{CP}^l = - \sin \delta_{CP}^q \cos \theta_{23}^l \frac{\sin \theta_{13}^q}{\sin \theta_{13}^l}$$

0.93

0.75

λ^2

λ^3

λ

$\lambda = \sin \theta_C$

$$\sin \delta_{CP}^l \sim \lambda^2 \sim 0.046 \text{ or } \delta_{CP}^l = 2.6^\circ$$

CP phase: $\sim -\delta_{CP}^l$ or $\pi + \delta_{CP}^l$

Leptonic CP is small because the leptonic 1-3 mixing is large

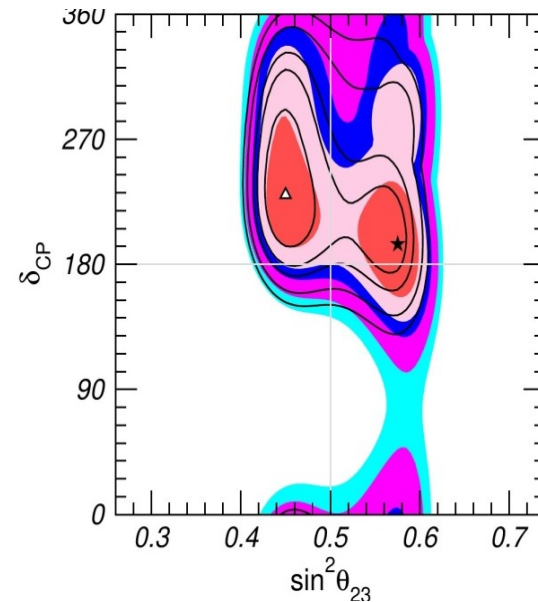
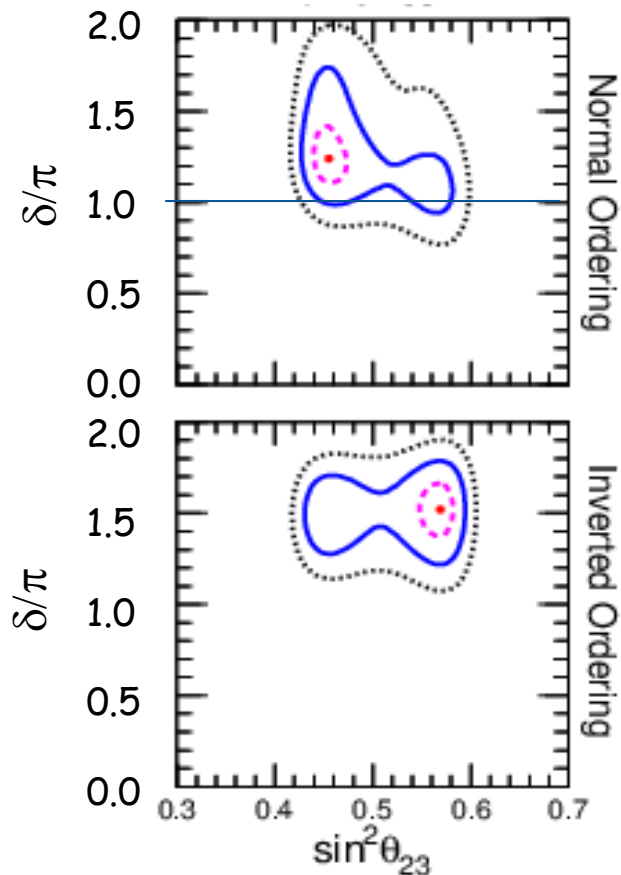
Experiment: CP-phase is close to π ?

NOvA-T2K 2σ tension NOvA: $\delta_{CP} = 0.82\pi$, T2K $\delta_{CP} = 1.5\pi$

Global fits

F Capozzi, et al
2107.00532 [hep-ph]

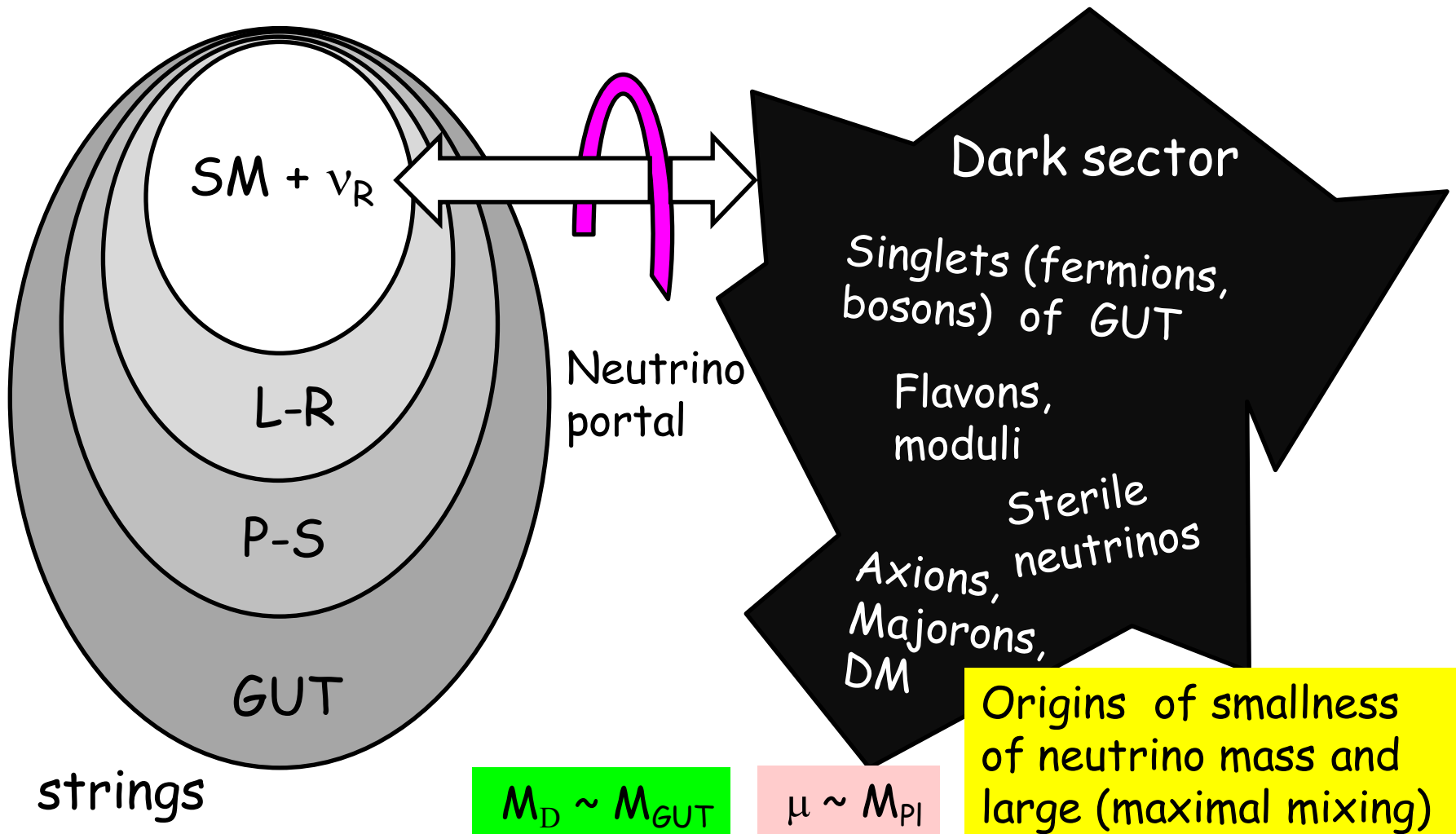
NuFIT 5.1 (2021),
www.nu-fit.org



even closer to π

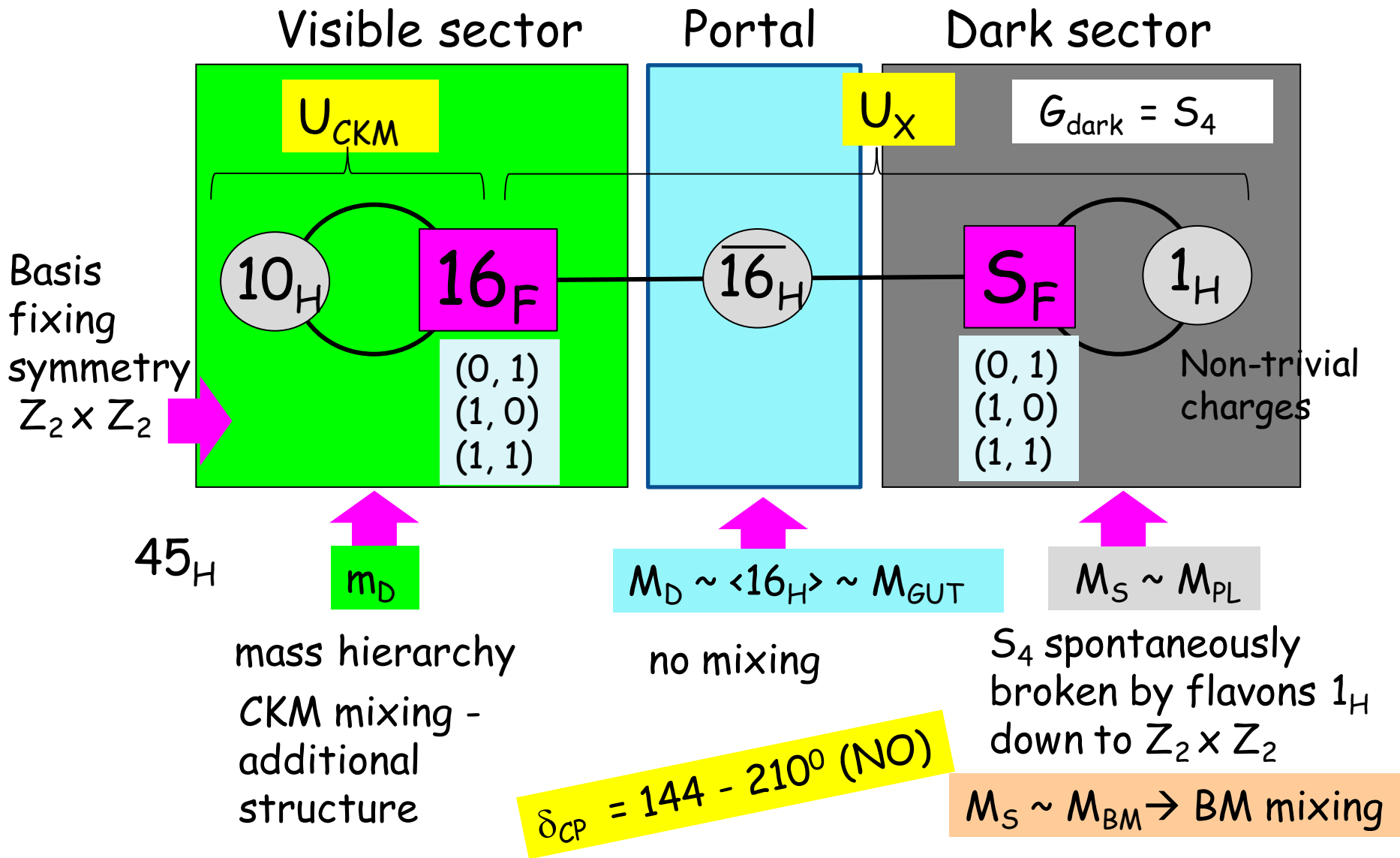
A scenario

From double seesaw to
neutrino portal and Dark sector



Realization in SO(10) GUT

Xun-Jie Xu, A.Y.S.
1803.07933 [hep-ph]



Status, challenges perspectives

Neutrino data and neutrino mass

Scenarios are in agreement with most of the data

Cosmology: $\sum_i m_{\nu_i} < (0.09 - 0.12) \text{ eV}$ Quasi degenerate spectrum is excluded, $m_2 / m_3 < 0.5$ (NO), IO spectrum - disfavored

*KamLAND-Zen, S. Abe et al.
2203.02139 [hep-ex]*

$m_{ee}^0 < (0.05 - 0.156) \text{ eV}$ (90% C.L.)
touching IO band

NuFIT 5.1 (2021), www.nu-fit.org

$\sin^2\theta_{12}$ closer to 0.3, $\theta_{12} = 33^\circ$ consistent with the QLC value 32°
Normal mass ordering is preferable by $(2 - 3\sigma)$

Deviation from maximal $|\Delta \sin^2\theta_{23}| = 0.05$, $|\Delta\theta_{23}| = 3^\circ$
Maximal mixing is disfavored at 2σ . First octant is preferred?

No dependence of neutrino parameters on neutrino energy, time, environment has been found

Sterile “damage”. eV seesaw

$$\begin{pmatrix} m_{ee} & \dots & m_{es} \\ \dots & \dots & \dots \\ m_{es} & \dots & m_{ss} \end{pmatrix}$$

$$m_{ss} = 1 - 3 \text{ eV}$$

$$\sin^2 \theta_{es} \sim m_{es}/m_{ss} \sim 0.02 - 0.1$$

BEST, Gallium,
LSND, MiniBooNE
Neutrino 4

Correction to the mass matrix of active neutrinos

$$\delta m = \sin^2 \theta_s m_s \sim (0.02 - 0.3) \text{ eV}$$

- at the level of largest elements ($\sim 0.03 \text{ eV}$) of the 3ν mass matrix

ν_s effect is not a small perturbation of 3ν picture and its symmetries

It requires cancellations, fine tuning (cancellation), and should be included in theory/symmetry constructions from the beginning

Rabi: smallest damage, if ν_s are mirror neutrinos

On the other hand it allows to explain difference of the quark and lepton mixings, large lepton mixing

Perspectives

High scale L-R seesaw

Indirect
confirmations

No new physics
at low scales

Proton decay

Low scale L-R seesaw

Inverse
seesaw

Discovery of
 N_R, W_R, Δ^{++}

PseudoDirac
heavy leptons

Confirmation of
anomalies and their
interpretations

Non-unitarity,
non-universality

ν MSM

Alternative mechanisms

Radiative
Sneutrino
condensate

Discovery of
new particles
involved in ν
mass generation

Completely different

Another nature
of neutrino mass

Effective mass due to refraction: ν scattering on background
Modification of dispersion relations \rightarrow oscillations

Scenarios based on NSI with heavy mediators are excluded,
New possibilities - light mediators and light scatterers

Tests:

Dark Matter identification

Discovery very light scalars, dark photons

Neutrino condensation $\langle \nu\nu \rangle$

may emerge e.g. if gravity contains a non-zero topological
vacuum susceptibility

Search for time, space dependence of oscillation parameters

Vacuum and properties of oscillations

G.Dvali, L Funcke,
1602.03191 [hep-ph]

Neutrino vacuum condensate due to gravity. Order parameter

$$\langle \Phi_{\alpha\beta} \rangle = \langle v_{\alpha}^T C v_{\beta} \rangle \sim \Lambda_G = \text{meV} - 0.1 \text{ eV}$$

Cosmological phase transition at $T \sim \Lambda_G$

Neutrinos get masses, in flavor basis $m_{\alpha\beta} \sim \langle \Phi_{\alpha\beta} \rangle$
(charged leptons mass generated by usual Higgs field)

$$m \sim U(\theta)^T \langle \Phi \rangle U(\theta)$$

$\langle \Phi \rangle = \text{diag}(\Phi_{11}, \Phi_{22}, \Phi_{33})$, $U(\theta)$ - mixing matrix

$T < \Lambda_G$ Relic neutrinos form bound states $\phi = (v_{\alpha}^T v_{\beta})$
decay and annihilate into ϕ (neutrinoless Universe)

Symmetry of system, $SU(3) \times U(1)$, spontaneously broken by neutrino condensate - ϕ are goldstone bosons

ϕ get small masses due explicit symmetry breaking by WI via loops

Mixing and topological defects

G.Dvali , L Funcke,
T Vachaspati
2112.02107 [hep-ph]

Symmetry breaking: $SU(3) \rightarrow Z_2 \times Z_2 \rightarrow I$



string-wall
network

Length scale of strings \sim inter-string separation

$$\xi = 10^{14} \text{ m } (\lambda/a_G) \left(\frac{\Lambda_G}{1 \text{ meV}} \right)^{7/2}$$

(self-coupling of string field Φ /scale factor of phase transition)

Travelling around string winds VEV $\langle \Phi \rangle$ by the $SU(3)$ transformation:

$$\langle \Phi(\theta_S) \rangle = \omega(\theta_W)^T \langle \Phi \rangle \omega(\theta_W)$$

$\omega(\theta_W)$ path - $O(3)$ transformation with angles $\theta_W = (\theta_W^{12}, \theta_W^{13}, \theta_W^{23})$.

After the path ω mixing changes as $U = U(\theta) \omega(\theta_W)$

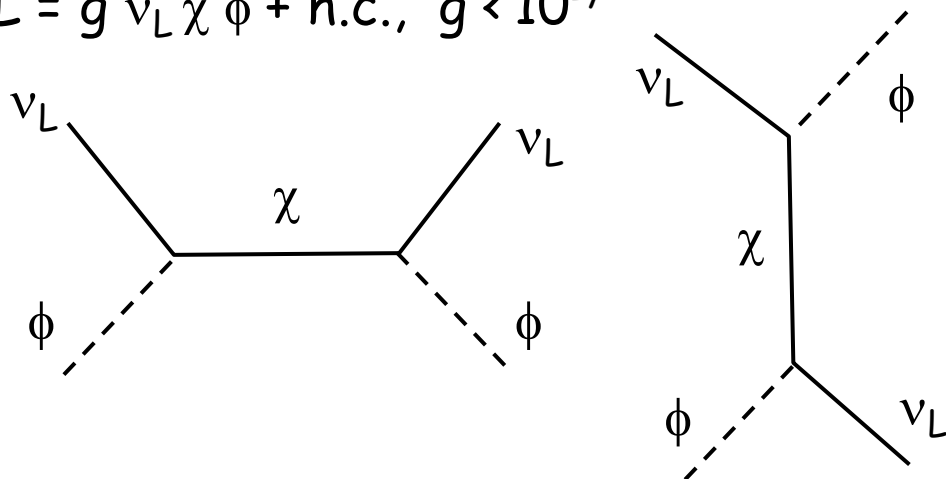
over length ξ : $\theta_W = O(1)$

Solar system moves through frozen string-DW background with $v = 230 \text{ km/sec}$. For 6 years (operation of Daya Bay) $d = vt = 4 \times 10^{13} \text{ m}$
- comparable with expected ξ

VEV or refraction on scalar DM?

Elastic forward scattering of ν on background scalars ϕ with fermionic χ mediator:

$$L = g \nu_L \bar{\chi} \phi + \text{h.c.}, \quad g < 10^{-7}$$



Resonance: $s = m_\chi^2$

For ϕ at rest the resonance ν energy:

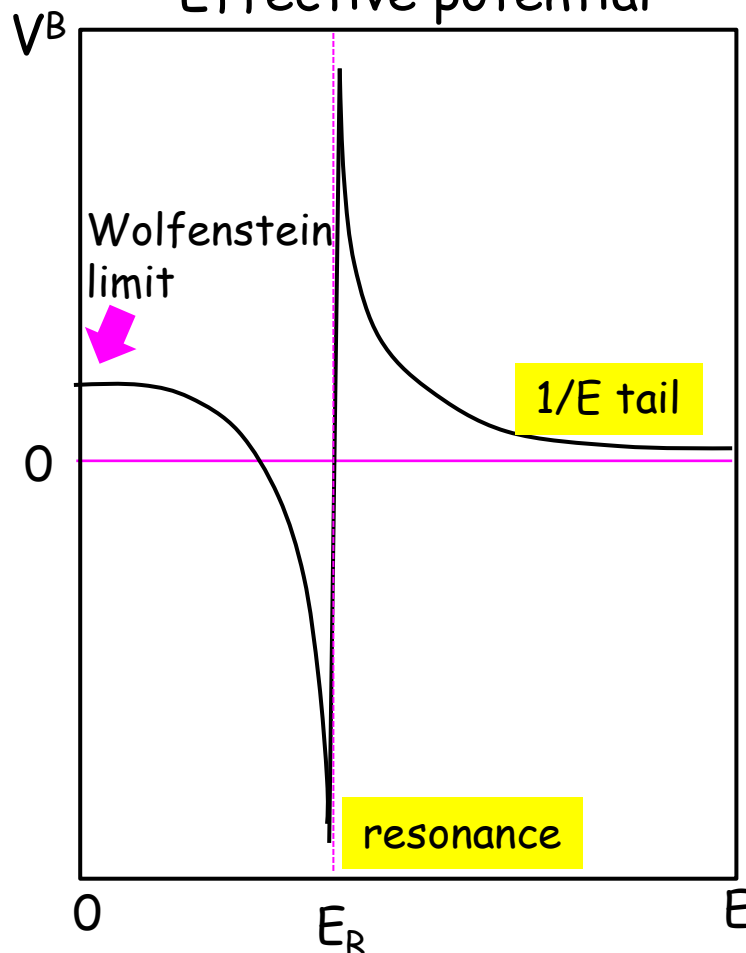
$$E_R = \frac{m_\chi^2}{2m_\phi}$$

For small m_ϕ resonance at low, observable energies

A.Y.S. , V. Valera, 2106.13829 [hep-ph]

*S. F Ge and H Murayama, 1904.02518 [hep-ph]
Ki-Yong Choi, Eung Jin Chun, Jongkuk Kim, 1909.10478 [hep-ph]
2012.09474 [hep-ph]*

Effective potential



Effective Δm^2

$$\Delta m_{\text{eff}}^2 \sim \frac{y^2 n_\phi}{4 m_\chi} \begin{cases} 1, & E \gg E_R \\ \varepsilon \frac{E}{E_R}, & E \ll E_R \end{cases}$$

$\Delta m_{\text{eff}}^2 = \text{constant}$ - checked down to 0.1 MeV

→ take $E_R \ll 0.1 \text{ MeV}$

For $E_R = 0.01 \text{ MeV}$:

KATRIN, $E \sim 1 \text{ eV}$: $m_{\text{eff}} < 2 \cdot 10^{-4} \text{ eV}$ - undetectable

A.Y.S. ...

COSMOLOGY

$m_{\text{eff}}^2 \sim n_\phi \sim (1+z)^3 \rightarrow$ increased in the past while $\text{VEV} = \text{const}$

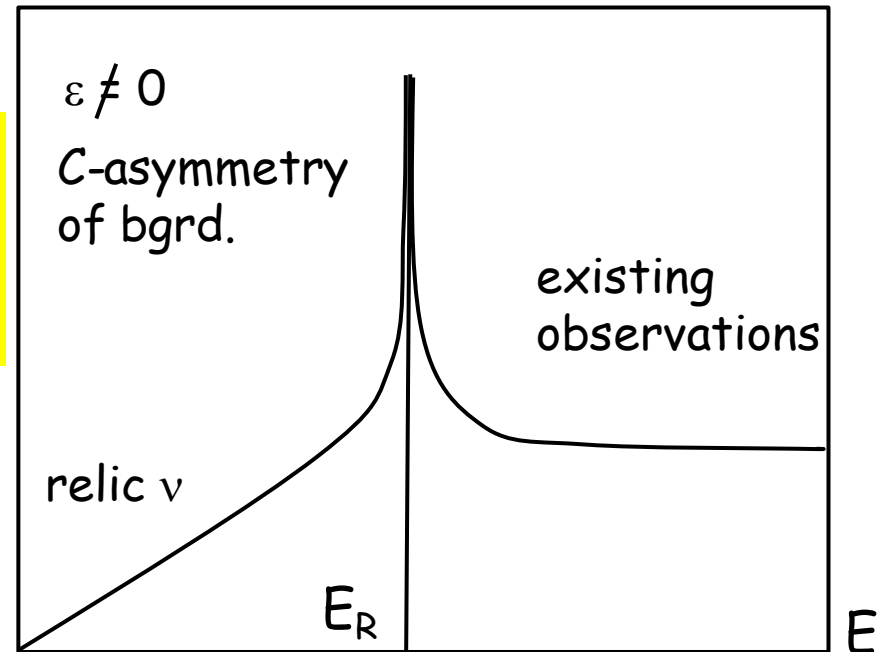
Relic ν , $E = 10^{-4} \text{ eV}$: $m_{\text{eff}}(0) < 5 \cdot 10^{-6} \text{ eV}$; $m_{\text{eff}}(z = 1000) \sim 0.15 \text{ eV}$,

$\Delta m^2 = 0$ in vacuum?

no problem

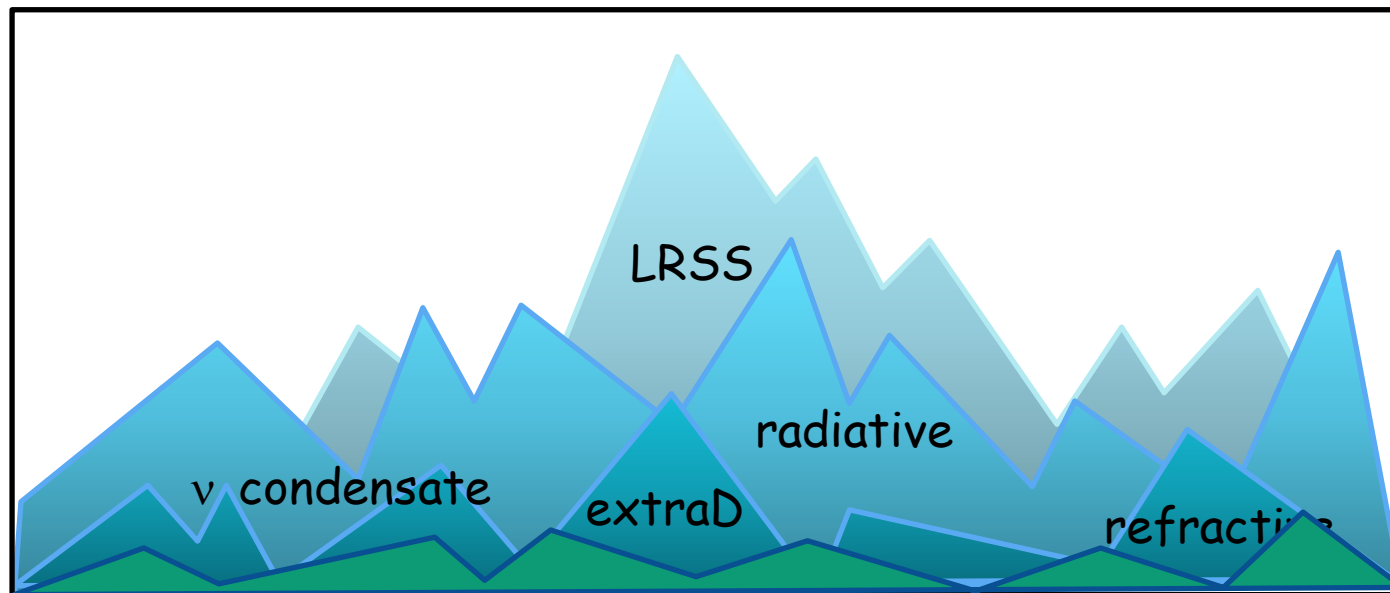
$$\Delta m_{\text{eff}}^2 \sim 2EV^B$$

$|\Delta m_{\text{eff}}^2|$



Summary

Rabi's scenarios of neutrino mass alive and flourishing



Backup

L and B-L

*R.N. Mohapatra and R.E. Marshak
PRL 44, 20 (1980) 1316*

In L-R $U(1)$ can be interpreted as

$$U(1)_{L+R} = U(1)_{B-L}$$

Quark-lepton correspondence

$$SU(2)_L \times SU(2)_R \times SU(4)$$

$$SU(3)_c \times U(1)_{B-L} \rightarrow \text{unification of color and B-L}$$

The same symmetry breaking

$\Delta L = 2$ effects

Majorana neutrino mass

$\beta\beta_{0\nu}$ - decay

$\Delta B = 2$ effects

$n - \bar{n}$ oscillations

Rates close to experimental bounds for $\langle \Delta_R \rangle \sim 10^4 \text{ GeV}$

Features

Neutrino masses from the Double seesaw with

$$M_S \sim M_{PL}, M_D \sim \langle 16_H \rangle \sim M_{GUT} \quad m_D, M_D = \text{diag}$$

$$\delta_{CP} = 144 - 210^\circ \text{ (NO)}$$

Similar non-SUSY structure with $10_H, 16_H, 45_H$
(responsible for gauge symmetry breaking)

*A. Preda, G. Senjanovic,
M. Zantedeschi,
2201.02785[hep-ph]*

High dimensional operators, correspond to the integrated out
Dark sector but with $\Lambda \sim 10 M_{GUT}, M_{GUT} \sim 4 \cdot 10^{15} \text{ GeV}$

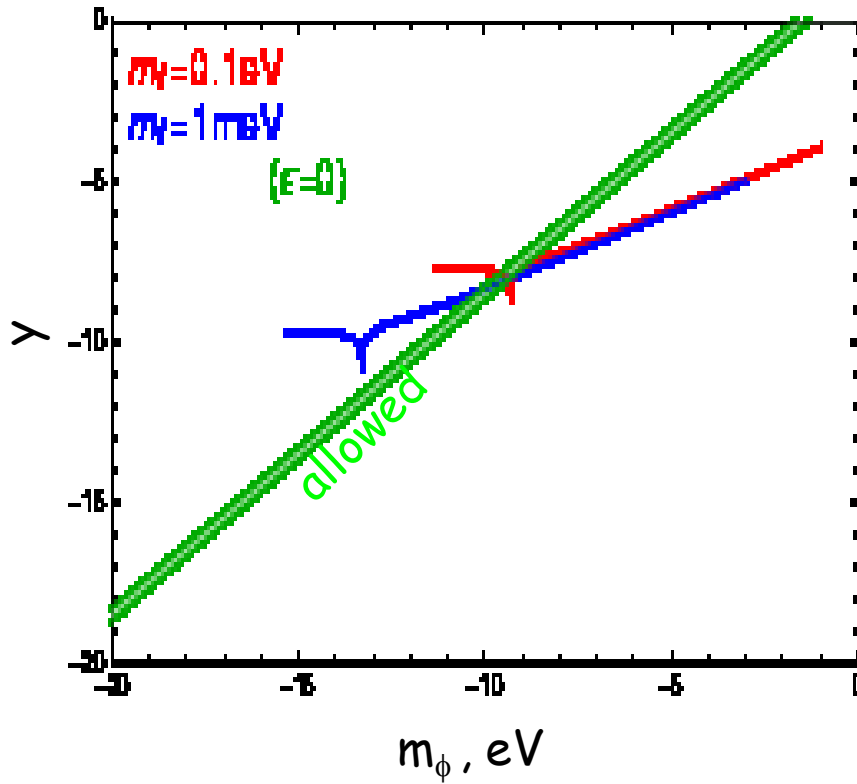
$$M_D \sim \langle 16_H \rangle \sim M_I \sim 5 \cdot 10^{14} \text{ GeV}$$

Unification is achieved by strong mass splitting in 45_H with weak tripled and color octet, as well as in 10_H with scalar quark doublet having masses below 10 TeV

accessible to LHC?

Bounds on parameters

*Ki-Young Choi, Eung Jin Chun,
Jongkuk Kim, 2012.09474 [hep-ph]*



Green band: $\Delta m_{\text{eff}}^2 = \Delta m_{\text{atm}}^2$

Upper bounds on γ from scattering of neutrinos from SN1987A on DM ϕ with zero C -asymmetry and two different masses of mediator f

Similar bound from $\text{Ly}\alpha$ (relic neutrinos).

Allowed values:

$$\begin{aligned}
 m_f &< 10^{-3} \text{ eV} \\
 m_\phi &< 10^{-10} \text{ eV} \\
 \gamma &< 10^{-9}
 \end{aligned}$$

the corresponding resonance energy $E_R = 0.01 \text{ MeV}$

Cosmological bound is satisfied

Summary

Rabi's scenarios of neutrino mass alive and flourishing

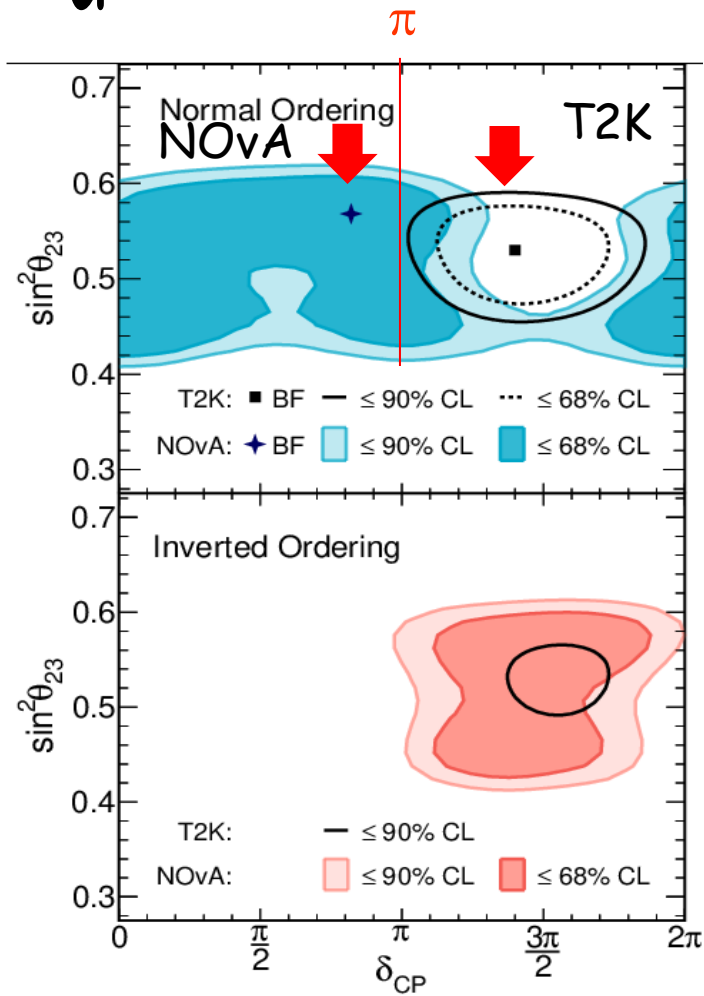
Practically nothing is excluded yet, although crucial tests can be around the corner

Any new observation can affect the picture

Gallium anomaly, BEST, LSND/MB: light sterile neutrinos representatives of light dark sector?

Neutrino interactions with light dark sector, resonance refraction at low energies, medium induced neutrino mass

δ_{CP} : NOvA - T2K tension?



2108.08219 [hep-ex]

NOvA: $\delta_{CP} = 0.82\pi$, disfavors $\delta_{CP} = 1.5\pi$ by 2σ

NOvA-T2K difference can be related to different baselines and matter effects

Reconcile with NSI

Sterile neutrinos do not help?

S. Chatterje, A. Palazzo, 2008.04161 [hep-ph],
2005.103338 [hep-ph]

Statistical fluctuations/systematics?

Global fit: $\delta_{CP} \rightarrow \pi$

No tension in the case of inverted ordering

Effective neutrino mass and cosmology

★ Due to dependence on energy and number density of scatterers m_{eff} can be different in different space-time points

★ Red shift dependence

$$m_{\text{eff}}(z) \sim \sqrt{n(z)} \quad n(z) = n_0 (1+z)^3$$

- effective mass increased in the past in contrast to standard mass generated by coupling with VEV (does not depend on z).

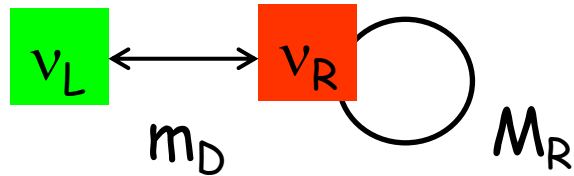
★
$$m_{\text{eff}}(z) \sim [\xi (1+z)^3]^{1/2} m_{\text{eff}}(\text{loc})$$

$1/\xi \sim 10^5$ - local (near the Earth) over-density of the background.

★ For $m_{\text{eff}}(\text{loc}) = 5 \cdot 10^{-6}$ eV: $m_{\text{eff}}(1000) \sim 5 \cdot 10^{-4}$ eV, and
 $m_{\text{eff}}(10^5) \sim 0.5$ eV

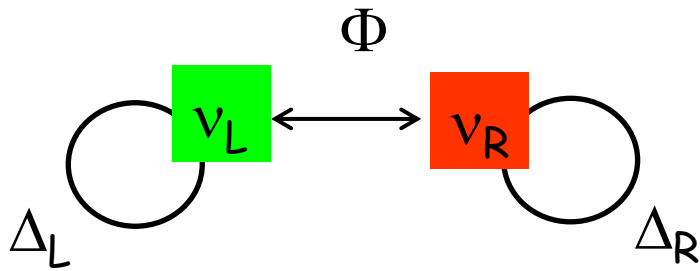
Bound on sum of neutrino masses from structure formation is satisfied
 m_{eff} dependence on E for not very small E_R resolve the problem

See-saw and L-R

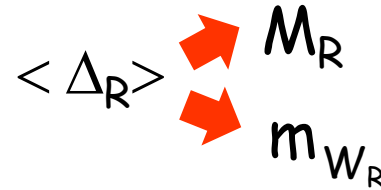


LSND

Naturally embedded into $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$



Historically:
getting neutrino
mass in LR



Seesaw scale is determined by the scale of L- R symmetry violation

$\langle \Delta_L \rangle \neq 0 \rightarrow$ type II

Type I + II

LHC-filic scenarios

$$\Delta m_{41}^2 = 1 - 2 \text{ eV}^2$$

Small neutrino mass with small masses of RH neutrinos

*J Kersten, AYS, Phys.Rev.D 76
(2007) 073005, 0705.3221
[hep-ph]*

Inverse seesaw

Low scale seesaw, a la ν MSM

Bounds on the eV sterile neutrinos

BEST, reactors,
LSND/MB

$$\begin{pmatrix} m_{ee}^0 & \dots & m_{es}^0 \\ \dots & \dots & \dots \\ m_{es}^0 & \dots & m_{ss}^0 \end{pmatrix}$$

m_{ee}^0 should be large enough to cancel large induced mass $(m_{ee}^0)^2 / m_{ss}^0 \sim \sin^2 \theta_{41} m_4$

After decoupling of sterile neutrino:

$$m_{ee} = m_{ee}^0 - \frac{1}{4} \sin^2 2\theta_{41} \sqrt{\Delta m_{41}^2}$$

oscillations of active neutrinos, Cosmology

$\beta\beta_{0\nu}$ - decay

sterile neutrino

$$\Delta m_{41}^2 < \frac{16(m_{ee}^0 - m_{ee})^2}{\sin^4 2\theta_{41}}$$

$$m_{ee}^0 < 0.156 \text{ eV (90\% C.L.)}$$

$$\sin^2 2\theta_{41} = 0.4$$

*KamLAND-Zen, S. Abe et al.
2203.02139 [hep-ex]*

For NH: $m_{ee} \sim m_2 \sin^2 \theta_{21} = 2.5 \cdot 10^{-3} \text{ eV}$

$$\Delta m_{41}^2 < 2.4 \text{ eV}^2$$

disfavour
BEST,

For quasi-degenerate: $m_{ee} < 0.03 \text{ eV}$

$$\Delta m_{41}^2 < 1.6 \text{ eV}^2$$

Neutrino-4