

# Conformal Extensions of the Standard Model

**Manfred Lindner**



Rabi-Fest, University of Maryland, Oct. 20-22, 2022 (virtual)

## Rabi and I:

- admired his work as a late-comer
- we met at different occasions
- joint interests  $\leftrightarrow$  Rabi's wide interests!



Alexander von Humboldt  
Stiftung/Foundation

### Humboldt Research Award

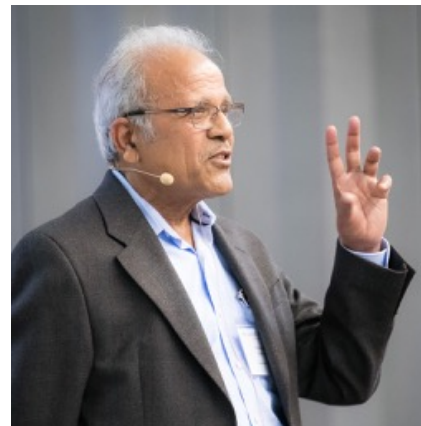
- internationally recognized researchers
- all countries (excluding Germany)
- **research stay of up to 12 months in Germany**

- 11/2004 proposal → awarded in 2005
- first visit at TUM in Munich: 2005
- 2006, 2007 TUM ...
- problem: ... I had meanwhile moved to MPI in Heidelberg

Later @Heidelberg:

→ 2009

→ 2017 at MPIK →



ML →

- 1983-present  
SENIOR, Maryland U.
- 1974-1982  
SENIOR, City Coll., N.Y.
- 1971-1974  
POSTDOC, Maryland U.
- 1969-1971  
POSTDOC, SUNY, Stony Brook
- 1966-1969  
PHD, Rochester U.
- 1964-1966  
UNDERGRADUATE, Delhi U.
- 1960-1964  
UNDERGRADUATE, Utkal U.

Rabi's wishes:

Say "yes" to celebrating,  
to big plans,  
to everything that matters to you.  
There's no better time  
than now!

→ planned to attend in-person and celebrate with all the Rabi-Fest...

...but unfortunately grounded in the last moment by covid

**Problem:** What to talk about that is not covered by the all the former collaborators?

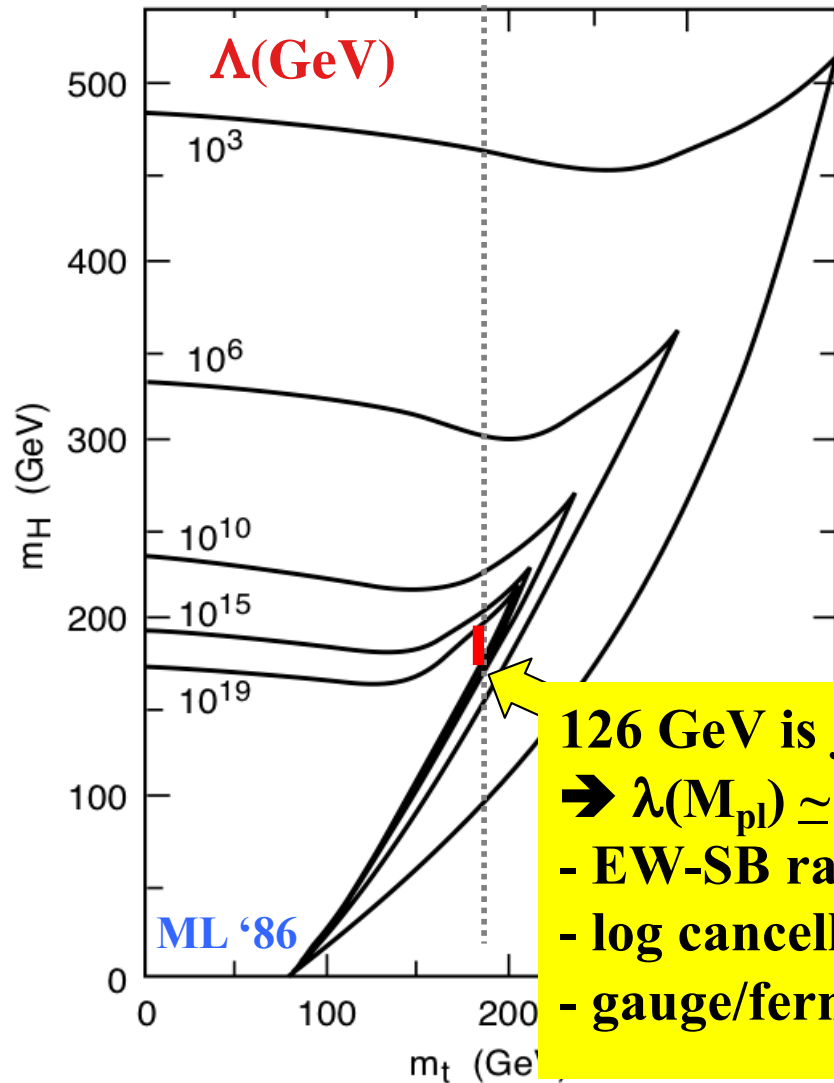
**Solution:** Something Rabi did \*NOT\* work on!

experimental: ... XENON, CONUS, ... → theory: conformal symmetry, but...

# Back to the basics: A remarkable coincidence

→ SM is a renormalizable QFT like QED w/o hierarchy problem

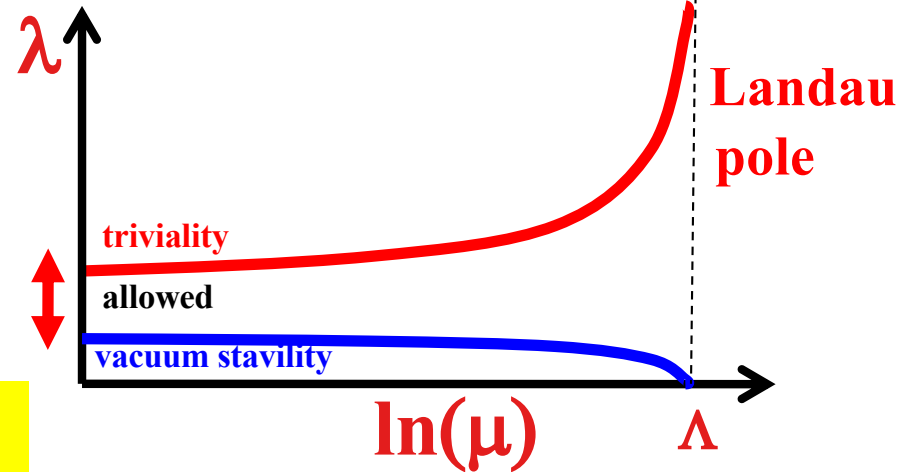
→ Cutoff “ $\Lambda$ ” has no meaning → **triviality, vacuum stability**



**126 GeV is just here!**  
 →  $\lambda(M_{pl}) \simeq 0$   
 - EW-SB radiative  
 - log cancellations  
 - gauge/fermion/scalar

$$126 \text{ GeV} < m_H < 174 \text{ GeV}$$

SM does not exist w/o embedding  
 - U(1) coupling, Higgs self-coupling



→ RGE arguments seem to work  
 → but we need some embedding

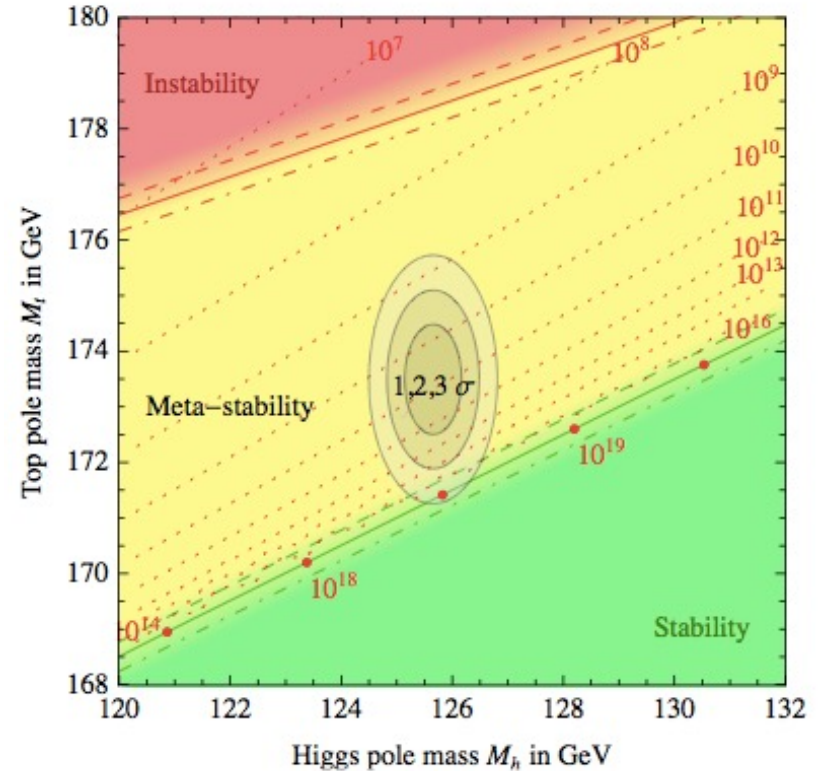
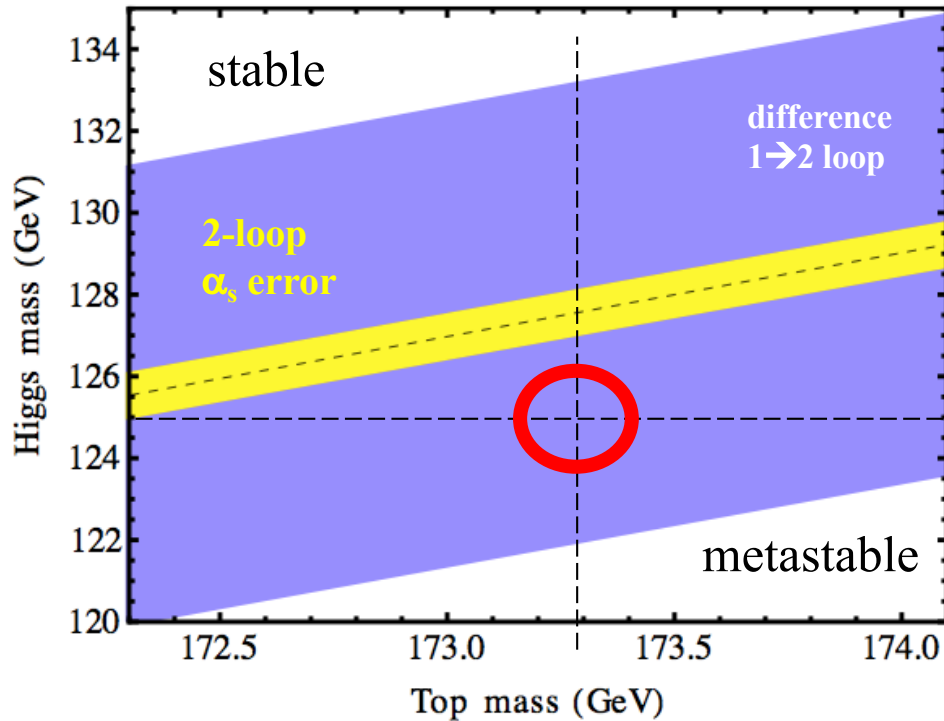
# Is the Higgs Potential at $M_{\text{Planck}}$ flat?

Holthausen, ML, Lim

12 Dec 2011

Elias-Miro, Espinosa, Giudice, Isidori, Riotto, Strumia

13 Dec 2011



**Experimental values indicate metastability, but,**

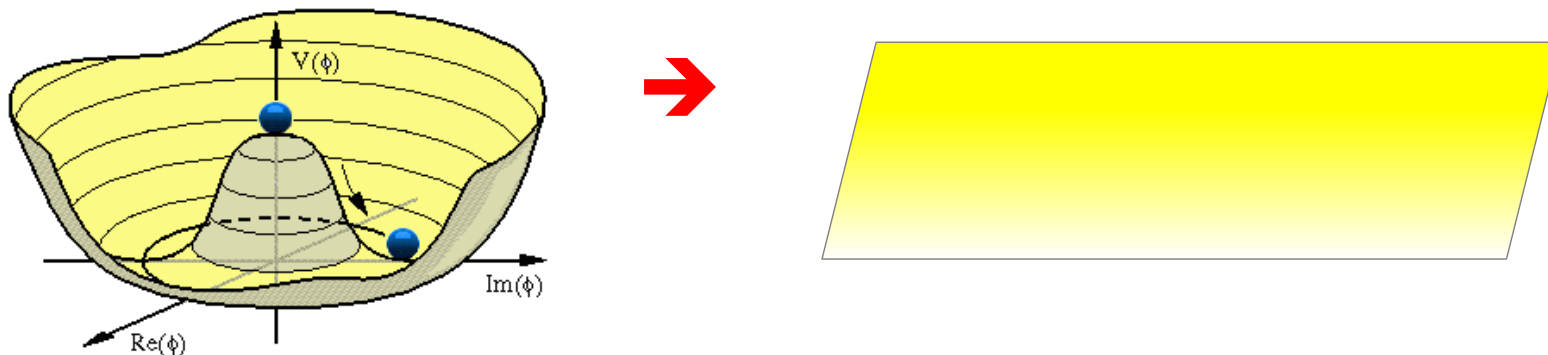
- $\rightarrow$  we need to include DM, neutrino masses, ...? are all errors (EX+TH) fully included?
- $\rightarrow$  be cautious about claiming that metastability is established

**$\rightarrow$  Important observation:**

- remarkable relation between weak scale,  $m_t$ , couplings and  $M_{\text{Planck}}$   $\leftrightarrow$  precision
- interplay between gauge, Higgs and top loops: log divergences – not quadratic div.

# Is there a Message?

- $\lambda(M_{\text{Planck}}) \simeq 0$ ?  $\rightarrow$  remarkable log cancellations  $\leftrightarrow$  CA  $\sim$   $\beta$ -fcts.  
 $M_{\text{planck}}$ ,  $M_{\text{weak}}$ , gauge, Higgs & Yukawa couplings are unrelated
- remember:  $\mu$  is the only single scale of the SM  $\rightarrow$  special role
  - $\rightarrow$  if in addition  $\mu^2 = 0 \rightarrow V(M_{\text{Planck}}) \simeq 0$
  - $\rightarrow$  flat Mexican hat (<1%) at the Planck scale!



$\rightarrow$  conformal (or shift) symmetry as solution to the HP?

$\rightarrow$  combined conformal & EW symmetry breaking

- conceptual issues

- minimal realizations  $\leftrightarrow$  SM seems to know about high scales  $\rightarrow$  bottom-up

$\leftrightarrow$  many new d.o.f. (fields, big reps.)  $\sim$  UV-instabilities

# Hierarchy Problems

- 1) why are scales vastly different
- 2) why do scales remain vastly different under quantum corrections

## SM + embedding at $\Lambda$

$$\delta M_H^2 = \frac{\Lambda^2}{32\pi^2 V^2} (6M_W^2 + 3M_Z^2 + 3M_H^2 - 12M_t^2) \sim \Lambda^2 \gg M_H^2$$

**SM + Dirac neutrino masses:** no problem – just like SM

## SM + Majorana neutrinos:

- more than one scale: VEV and the Majorana mass(es)  $M$

→ generates a HP problem for large  $M$  even if  $y_\nu$  is tiny

$$\delta m_H^2 \simeq \frac{y_\nu^2}{16\pi^2} M^2 \quad y_\nu^2 = M m_\nu / v^2$$

→  $M \lesssim 10^7 - 10^8 \text{ GeV}$  ↔ see-saw, leptogenesis, ...



- there should be some new symmetry (SUSY) at  $\Lambda = O(\text{TeV})$   
to solve the hierarchy problem
- new particles @  $O(\text{TeV})$

**BUT: So far nothing seen!?**

- SUSY at higher scales
- other options? → **conformal symmetry**

### Nevertheless: Very interesting lessons

- SM works perfectly
- triumph (precision) of concepts (QFT, SM symmetries)
- ☺ Higgs discovered  $\leftrightarrow$  SM particle masses
- ☺ quantum structure of SM
- ☺ neutrino masses, - DM, - DE ... → very exciting  
→ **requires BSM**



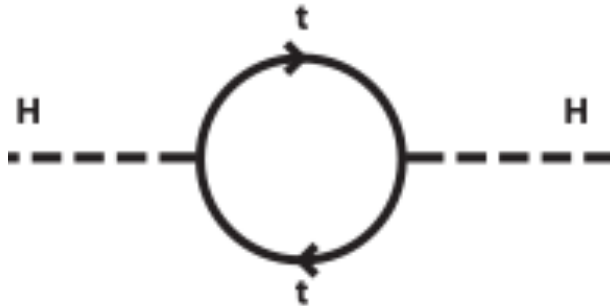
# The Problem: EXPLICIT Scales

- Renormalizable QFT with two scalars  $\varphi$ ,  $\Phi$  with masses  $m$ ,  $M$  and a hierarchy  $m \ll M$
  - These scalars must interact since  $\varphi^+\varphi$  and  $\Phi^+\Phi$  are singlets  
→  $\lambda_{\text{mix}}(\varphi^+\varphi)(\Phi^+\Phi)$  (= portal) in addition to  $\varphi^4$  and  $\Phi^4$
  - Quantum corrections  $\sim M^2$  drives  $m$  to the (heavy) scale  $M$   
→ vastly different explicit scalar scales are generically unstable
- 
- Since SM Higgs exists → problem: embedding with a 2<sup>nd</sup> scalar
    - gauge extensions: LR, PS, GUTs → must be broken...
    - even for SUSY GUTS → doublet-triplet splitting...
    - also for fashionable Higgs-portal scenarios...

## Ways out:

- no Higgs ...
- symmetry: SUSY, ... → conformal symmetry = no explicit scales!

# Theories without any explicit scale!



Observed scales  $\leftrightarrow$

Non-linear realizations of CS:

- naïve power counting invalid
- no  $\Lambda^2$  divergence

**Obstacle:** Conformal Anomaly = breaking of CS by loops

→ requirement for particle content which cancels CA in UV

→ anomaly  $\sim$  trace of energy momentum tensor

$\leftrightarrow$   $\beta$ -functions  $\leftrightarrow$   $\log(\Lambda)$  → UV fixed points

→ a path to avoid hierarchy problems

→ dimensional transmutation of conformal theories  
by log running of couplings like in chiral QCD

# Conformal Symmetry and SM Extensions

## Main idea:

- **Do not introduce any fundamental (explicit) scales**  
→ **theories with conformal or shift symmetry**
- **Dynamical breaking of CS → Coleman Weinberg  $V_{\text{eff}}$**   
→ **all scale(s) by dimensional transmutation**  
→ non-linear realization of CS:
  - naïve power counting ( $\sim \Lambda^2$ ) misleading
  - similar to gauge symmetry and vector boson masses
- **An UV complete theory should have UV fixedpoints...**  
... the SM parameters may point in that direction...

# Bottom-up realizations

# Why the minimalistic SM does not work

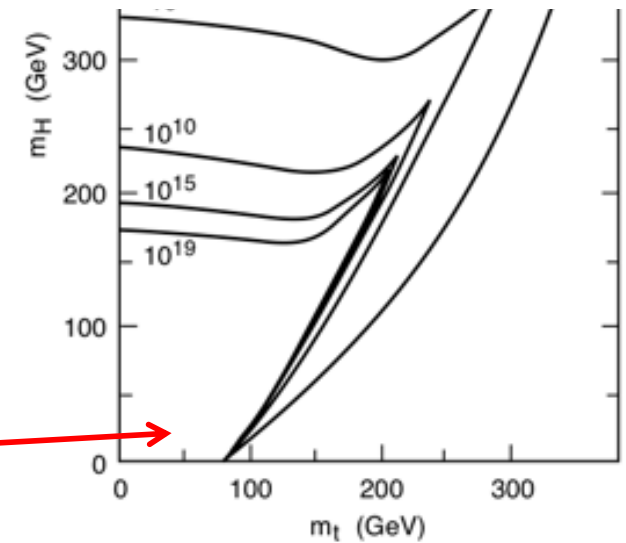
**Minimalistic version:  $\rightarrow$  “SM-”**

**SM + with  $\mu=0$   $\leftrightarrow$  CS**

**Coleman Weinberg: effective potential**

$\rightarrow$  CS breaking (**dimensional transmutation**)

$\rightarrow$  induces for  $m_t < 79$  GeV  
a Higgs mass  $m_H = 8.9$  GeV

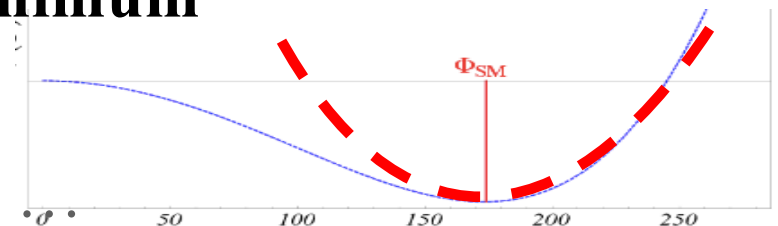


- **Success: no-scale SM  $\rightarrow$  broken SM but: Higgs and top do not fit**

- **DSB for weak coupling  $\leftrightarrow$  CS= phase boundary**  
 **$\rightarrow$  scale set by log-running couplings  $\leftrightarrow$  gap eqn: hierarchical!**

- **Reason for  $m_H \ll v$ :  $V_{\text{eff}}$  flat around minimum**  
 **$\leftrightarrow m_H \sim$  loop factor  $\sim 1/16\pi^2$**

AND: We need neutrino masses, dark matter,



# Realizing the Idea via Higgs Portals

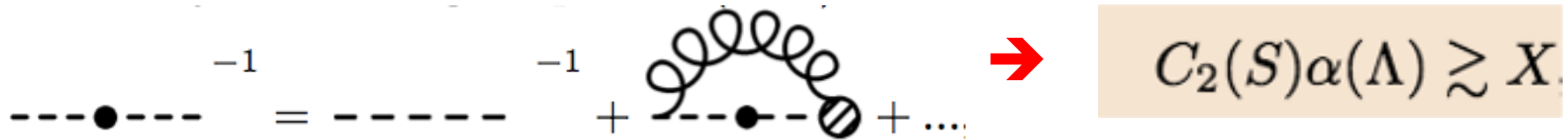
- SM scalar  $\Phi$  plus some new scalar  $\varphi$  (or more scalars)
- CS  $\rightarrow$  no scalar mass terms
- the scalar portal  $\lambda_{\text{mix}}(\varphi^+\varphi)(\Phi^+\Phi)$  must exist

$\rightarrow$  a condensate of  $\langle\varphi^+\varphi\rangle$  produces  $\lambda_{\text{mix}}\langle\varphi^+\varphi\rangle(\Phi^+\Phi) = \mu^2(\Phi^+\Phi)$   
 $\rightarrow$  effective mass term for  $\Phi$

- no CA...  $\rightarrow$  breaking only  $\ln(\Lambda)$   
 $\rightarrow$  implies a TeV-ish condensate for  $\varphi$  to obtain  $\langle\Phi\rangle = 246$  GeV
- Many model building possibilities / phenomenological aspects:
  - $\varphi$  could be an effective field of some hidden sector DSB
  - further particles could exist in hidden sector; e.g. confining...
  - extra hidden U(1) potentially problematic  $\leftrightarrow$  U(1) mixing
  - avoid Yukawas which couple visible and hidden sector $\rightarrow$ phenomenology safe due to Higgs portal  $\rightarrow$ suppressed TeV-ish BSM physics!

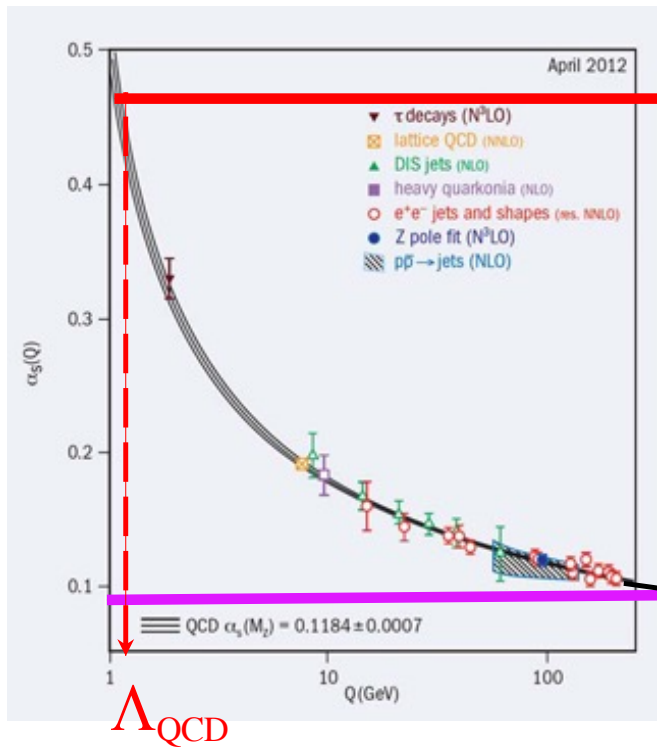
# Rather minimalistic: SM + QCD Scalar S

J. Kubo, K.S. Lim, ML New scalar representation S  $\rightarrow$  QCD gap equation:



$C_2(\Lambda)$  increases with larger representations

$\leftrightarrow$  condensation for smaller values of running  $\alpha$



$q=3$   $\mathcal{L} = \mathcal{L}_{\text{SM}, m^2 \rightarrow 0} + (D_{\mu, ij} S_j)^\dagger (D_{ik}^\mu S_k) + \lambda_{HS} H^\dagger H S^\dagger S - \lambda_{1_i} [\bar{\mathbf{S}} \times \mathbf{S} \times \bar{\mathbf{S}} \times \mathbf{S}]_{1_i}$

$\lambda_{HS} \langle S^\dagger S \rangle H^\dagger H \rightarrow \lambda_{HS} \Lambda^2 H^\dagger H$

$m_h^2 = 2\lambda_{HS} \Lambda^2 \quad \frac{\lambda_h}{\lambda_{HS}} = \frac{\Lambda^2}{v^2}$



# SM $\otimes$ hidden $SU(3)_H$ Gauge Sector

Holthausen, Kubo, Lim, ML

- hidden  $SU(3)_H$ :

$$\mathcal{L}_H = -\frac{1}{2} \text{Tr} F^2 + \text{Tr} \bar{\psi} (i\gamma^\mu D_\mu - yS) \psi$$

gauge fields ;  $\psi = 3_H$  with  $SU(3)_F$  ; **S = real singlet scalar**

- SM coupled by S via a Higgs portal:

$$V_{SM+S} = \lambda_H (H^\dagger H)^2 + \frac{1}{4} \lambda_S S^4 - \frac{1}{2} \lambda_{HS} S^2 (H^\dagger H)$$

- no scalar mass terms
- use similarity to QCD, use NJL approximation, ...
- $\chi$ -ral symmetry breaking in hidden sector:  $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \rightarrow$  generation of TeV scale
- $\rightarrow$  transferred into the SM sector through the singlet S**
- $\rightarrow$  dark pions are PGBs: naturally stable  $\rightarrow$  DM**

# Realizing the Idea: Many more Models

SM + extra singlet or doublet:  $\Phi, \varphi$

Nicolai, Meissner Farzinnia, He, Ren, Foot, Kobakhidze, Volkas, Hill, ...

Minimal B-L extension of SM:  $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

Iso, Okada, Orikasa

**Minimal LR-model:**  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  Holthausen, ML, Schmidt

SM  $\otimes$   $SU(N)_H$  with new N-plet in a hidden sector

Ko, Carone, Ramos, Holthausen, Kubo, Lim, ML, Hambye, Strumia, ...

SM + QCD colored scalar which condenses at TeV scale Kubo, Lim, ML

SM  $\otimes$  [ $SU(2)_X \otimes U(1)_X$ ]

Altmannshofer, Bardeen, Bauer, Carena, Lykken

... more ...

**Since the SM-only version does not work  $\rightarrow$  observable effects:**

- Higgs coupling to other scalars (singlet, hidden sector, ...)
- dark matter candidates  $\leftrightarrow$  hidden sectors & Higgs portals
- consequences for neutrino masses

# Conformal Symmetry & Neutrino Masses

ML, S. Schmidt and J. Smirnov

- **No explicit scale  $\rightarrow$  no explicit (Dirac or Majorana) mass term  $\rightarrow$  only Yukawa couplings  $\otimes$  generic scales**
- **Enlarge the Standard Model field spectrum like in 0706.1829 - R. Foot, A. Kobakhidze, K.L. McDonald, R. Volkas**
- **Consider direct product groups: SM  $\otimes$  HS**
- **Two scales: CS breaking scale at O(TeV) + induced EW scale**

**Important consequence for fermion mass terms:**

- $\rightarrow$  spectrum of Yukawa couplings  $\otimes$  TeV or  $\otimes$  EW scale
- $\rightarrow$  interesting consequences  $\leftrightarrow$  Majorana mass terms are no longer expected at the generic L-breaking scale  $\rightarrow$  anywhere

# Examples

$$\mathcal{M} = \begin{pmatrix} 0 & y_D \langle H \rangle \\ y_D^T \langle H \rangle & y_M \langle \phi \rangle \end{pmatrix}$$

→ generically expect a TeV seesaw

BUT:  $y_M$  can be tiny

→ wide range of sterile masses → including pseudo-Dirac case

→ suppressed  $0\nu\beta\beta$

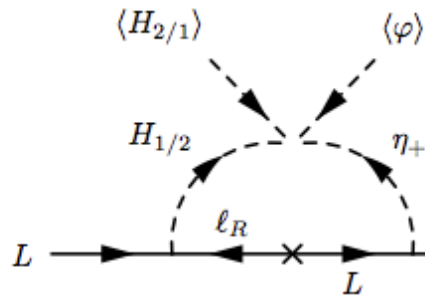
## Yukawa seesaw:

SM +  $\nu_R$  + singlet

$$\langle \phi \rangle \approx \text{TeV}$$

$$\langle H \rangle \approx 1/4 \text{ TeV}$$

## Radiative masses



$$\mathcal{M} = m_L \quad \text{or}$$

$$\mathcal{M} = \begin{pmatrix} \mu_1 & y_D \langle H \rangle \\ y_D^T \langle H \rangle & \mu_2 \end{pmatrix}$$

→ pseudo-Dirac case

## The punch line:

all usual neutrino mass terms can be generated

→ suitable scalars required

→ no explicit masses:

**all via Yukawa couplings**

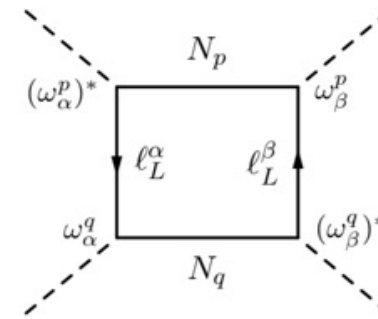
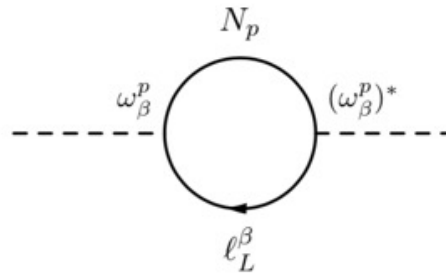
→ different numerical expectations  $\leftrightarrow$  could easily explain keV masses

# The Neutrino Option

Interesting possibility: Connection between EWSB and neutrinos  $\leftrightarrow$  v-hierarchy problem

Neutrino option: Brivio

$\rightarrow$   $V_{\text{eff}}$  from neutrino loops



Conformal Realization of the Neutrino Option: Brdar, Emonds, Helmboldt, ML

$\rightarrow$  conformal symmetry +  $V_{\text{eff}}$  from neutrino loops (not from Higgs portal)

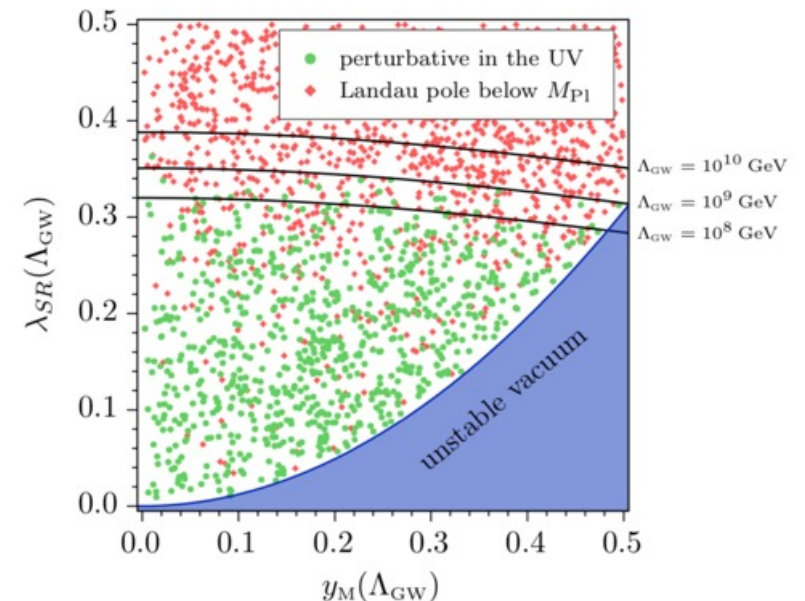
SM particle content

3x NR

2x scalar SM singlets: S, R

$$\mathcal{L} \supseteq \frac{1}{2} \partial_\mu S \partial^\mu S + \frac{1}{2} \partial_\mu R \partial^\mu R + i \bar{N}_R \not{\partial} N_R - V(H, S, R) - \left( \frac{1}{2} y_M S \bar{N}_R N_R^c + y_\nu \bar{L} \tilde{H} N_R + \text{h.c.} \right)$$

$\rightarrow$  consistent UV-complete realization of the idea



# Conformal Symmetry & Dark Matter

## Different natural and viable options:

- 1) eV, **keV = DM**, TeV, ... sterile neutrino mass easily possible  $\leftrightarrow$  not so easy in standard see-saw's
  - 2) New particles which are fundamental or composite DM candidates:
    - hidden sector pseudo-Goldstone-bosons
    - stable color neutral bound states from new QCD representations
- some look like WIMPs
- others are extremely weakly coupled (via Higgs portal)
- or even coupled to QCD (threshold suppressed...)

**Including the Planck Scale**



# The Planck Scale from CS Breaking

## Conformal Gravity (CG):

- more symmetry → CG claimed to be power counting renormalizable
- CG may have a ghost... → see later

**Idea: Generate  $M_{\text{Planck}}$  from conformal gravity  $\otimes$  SU(N)**

→ gauge assisted condensate via SU(N) field

→  $M_{\text{Planck}}$  becomes an effective scale

**Kubo, ML, Schmitz, Yamada**      similar ideas: **Donoghue, Menezes, ...**

$$S_C = \int d^4x \sqrt{-g} \left[ -\hat{\beta} S^\dagger S R + \hat{\gamma} R^2 - \frac{1}{2} \text{Tr} F^2 + \right. \\ \left. + g^{\mu\nu} (D_\mu S)^\dagger D_\nu S - \hat{\lambda} (S^\dagger S)^2 + a R_{\mu\nu} R^{\mu\nu} + b R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right]$$

R = Ricci curvature scalar,  $R_{\mu\nu}$  = Ricci tensor,  $R_{\mu\nu\alpha\beta}$  = Riemann tensor

F = field-strength tensor of the SU( $N_c$ ) gauge theory, **S = complex scalar in fund. rep. →  $N_c$**

→ most general diffeomorphism invariance, gauge invariance, and global scale invariance

## Condensation in $SU(N_c)$ gauge sector

→ **dimensional transmutation:**  $\langle S^+ S \rangle \rightarrow$  effective Planck mass

$$M_{\text{planck}} = \sqrt{2\beta f_0} = \frac{N_c \beta}{16\pi^2} (2\lambda f_0) \left( 1 + 2 \ln \frac{2\lambda f_0}{\Lambda^2} \right) \quad \text{with } f_0 = \langle S^+ S \rangle$$

→ Effectively normal gravity with a dynamically generated  $M_{\text{Planck}}$

## What about the ghost problem of CG?

...new ideas: [J. Kubo](#) and [J. Kuntz](#)

# Dilaton-Scalaron Inflation

Effective Jordan-frame Lagrangian:

$$\frac{\mathcal{L}_{\text{eff}}^J}{\sqrt{-g_J}} = -\frac{1}{2} B(\chi) M_{\text{Pl}}^2 R_J + G(\chi) R_J^2 + \frac{1}{2} g_J^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \quad \rightarrow \text{auxiliary field } \Psi \rightarrow$$

$$\frac{\mathcal{L}_{\text{eff}}^J}{\sqrt{-g_J}} = -\left[ \frac{1}{2} B(\chi) M_{\text{Pl}}^2 - 2G(\chi) \psi \right] R_J + \frac{1}{2} g_J^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) - G(\chi) \psi^2$$

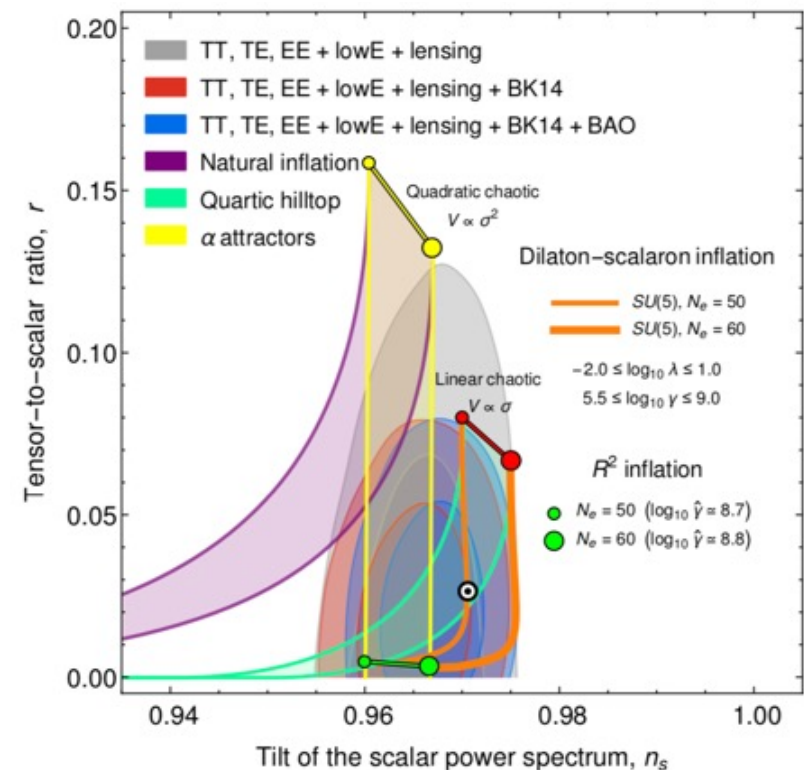
Weyl rescaling:  $g_{\mu\nu} = \Omega^2 g_{\mu\nu}^J$       $\Omega^2 = e^{\Phi(\phi)}$ ,      $\Phi(\phi) = \frac{\sqrt{2} \phi}{\sqrt{3} M_{\text{Pl}}}$

Einstein-frame scalar potential:

$$V(\chi, \phi) = e^{-2\Phi(\phi)} \left[ U(\chi) + \frac{M_{\text{Pl}}^4}{16G(\chi)} \left( B(\chi) - e^{\Phi(\phi)} \right)^2 \right]$$

→ Slow role inflation

→ fits data very well!

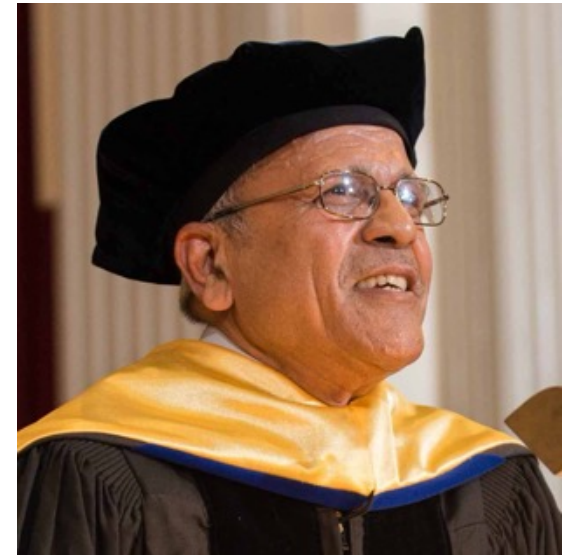


# Summary

- **SM works (so far) perfectly**
  - the expected TeV physics did so far not show up
  - be a bit more patient: hints ... new physics may be around the corner...
  - or maybe it is time to re-consider some ingredients...?
- **SM embeddings into QFTs with conformal symmetry**
  - combined conformal & electro-weak symmetry breaking
  - implications for BSM phenomenology
  - implications for Higgs couplings, neutrino physics, dark matter, ...
  - ➔ **testable consequences: @LHC, dark matter, neutrinos**
- **Planck scale generation by gauge induced breaking of conformal GR**
  - very nice phenomenology: inflation...
  - consistent quantum gravity: renormalizability?, ghost?
    - ↔ normal GR from a theory with more symmetry
  - stabilizing large scale hierarchies...
  - trans-Planck: just be a different phase - no new concept required

## Congratulations Rabi

- on your impressive achievements
- in a remarkable wide range of topics
- and
- in guiding / mentoring many young colleauges



Say “yes” to celebrating,  
to big plans,  
to everything that matters to you.

There’s no better time  
than now!