Rabi Mohapatra and $n - \bar{n}$ Oscillations

Robert Shrock

C. N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy,
Stony Brook University

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Prologue

Rabi Mohapatra has made many outstanding contributions to particle physics over his long and very productive career. He is remarkable for his deep insights and innovative ideas. He has been an inspiration to many generations of physicists.

This conference highlights some of Rabi's contributions, including his co-development of the seesaw mechanism explaining light neutrino masses, the left-right-symmetric gauge theory and spontaneous parity violation, grand unified theories, baryon number violation, neutron-antineutron oscillations, baryogenesis, majorons, supersymmetric models, dark matter, and other ideas on physics beyond the Standard Model.

Truly, the breadth, depth, and importance of Rabi's research are remarkable.

Here I will focus on one of the many subjects on which Rabi has been a pioneer, namely neutron-antineutron $(n - \bar{n})$ oscillations. Rabi wrote a seminal paper in 1980 with Robert Marshak on this and has contributed many key ideas on this subject since then.

Further, Rabi has been a leading advocate for new experiments searching for $n - \bar{n}$ oscillations and has worked tirelessly toward this goal. There are, indeed, prospects for a new experimental search, as will be discussed by Yuri Kamyshkov.

My own interest in $n-\bar{n}$ oscillations started in 1982 with research that I did with one of my early Ph.D. thesis students, Sumathi Rao. I have had the pleasure of working with Rabi and Yuri on a number of white papers on this subject over the years (also Girmohanta, Mohapatra, and RS, PRD 103, 015021 (2021) [arXiv:2011.01237]).

General motivations for expecting violation of baryon number, B: producing the observed baryon asymmetry in the universe requires interactions that violate B (as well as CP violation and deviation from thermal equilibrium) (Sakharov, 1967).

Suggestion of $n-\bar{n}$ transitions as a mechanism involved in generating baryon asymmetry in the universe (Kuzmin, 1970).

Standard Model (SM) conserves B perturbatively. SU(2)$_L$ instantons produce nonperturbative violation of B and lepton number, L, while conserving $B-L$ ('t Hooft, 1976). These SU(2)$_L$ instantons have a negligibly small effect at temperatures $T \ll v_{EW}$, but are important for $T \gtrsim v_{EW}$ (Kuzmin, Rubakov, Shaposhnikov, 1985).

Since (anti)quarks and (anti)leptons are placed in same representations in grand unified theories (GUTs), the violation of B and L is natural in these theories.

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Local $B-L$ Symmetry of Electroweak Interactions, Majorana Neutrinos, and Neutron Oscillations

R. N. Mohapatra
Department of Physics, City College of the City University of New York, New York, New York 10031

and

R. E. Marshak
Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
(Received 11 January 1980)

Interpretation of the U(1) generator of the left-right-symmetric electroweak model in terms of $B-L$ enables us to study the spontaneous breaking of local $B-L$ symmetry. The same Higgs mechanism at the "partial unification" level of $SU(2)_L \otimes SU(2)_R \otimes SU(4')$ that produces $\Delta_L = 2$ processes (e.g., Majorana neutrinos) also yields $\Delta B = 2$ processes (e.g., "neutron oscillations"). The observation of "neutrinoless" double β decay and $\Delta B = 2$ nucleon transitions without proton decay would favor this model and an intermediate mass scale.

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Figure 1:

Rabi's 1980 paper with Marshak

In the very important paper R. N. Mohapatra and R. E. Mohapatra, "Local $B - L$ Symmetry or Electroweak Interactions, Majorana Neutrinos, and Neutron Oscillations", Phys. Rev. Lett. 44, 1316 (1980), Rabi and Marshak made several key advances on baryon number violation (BNV) and presented an explicit model in which:

- Baryon number violation is of $|\Delta B| = 2$ type, via $n - \bar{n}$ oscillations, rather than via $\Delta B = -1$ nucleon decays.
- The physics responsible for these $n - \bar{n}$ oscillations is characterized by a mass scale $M_{n\bar{n}}$ much smaller than a GUT scale.
- The scale $M_{n\bar{n}}$ characterizes both $|\Delta B| = 2$ BNV via $n - \bar{n}$ oscillations and the spontaneous breaking of left-right gauge symmetry and thus the spontaneous breaking of parity, in particular, the breaking of $\mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \to \mathrm{U}(1)_Y$.
- In this model, B and L are gauged as U(1)$_{B-L}$, in contrast to the SM, where B and L are global symmetries at the perturbative level.
- Owing to a Higgs vacuum expectation value (VEV) breaking a gauged U(1)$_{B-L}$ symmetry, transforming as $|\Delta(B-L)| = 2$, there is a natural connection between the $n - \bar{n}$ oscillations with $|\Delta B| = 2$, $\Delta L = 0$ and Majorana neutrino masses, which transform as $|\Delta L| = 2$ and $\Delta B = 0$.
- This also involves a deep connection between the seesaw mechanism for light neutrino masses $m_\nu \sim m_D^2/m_R$ and the scale m_R of BNV responsible for $n - \bar{n}$ oscillations.
- The explicit example in this 1980 paper by Mohapatra and Marshak used a partial unification gauge group $G_{422} = \mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, which is

spontaneously broken to the left-right-symmetric (LRS) group

$$G_{LRS} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$$

The $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{B-L}$ part of G_{LRS} forms a (maximal) subgroup of SU(4) (c.f. J. C. Pati and A. Salam, "Lepton Number as the Fourth Color", Phys. Rev. D 10, 275 (1974)).

- One of the appeals of the LRS model is the elegant relation for the electric charge:

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

in contrast to the relation $Q_{em} = T_{3L} + (Y/2)$ in the SM.

- Since $\mathrm{SU}(4) \approx SO(6)$ and $SU(2) \otimes SU(2) \approx SO(4)$, it follows that $G_{422} \approx \mathrm{SO}(6) \otimes \mathrm{SO}(4)$, which forms a (maximal) subgroup of the GUT group SO(10).

Feynman diagram in Fig. 2 of Mohapatra-Marshak PRL 44, 1316 (1980) paper:

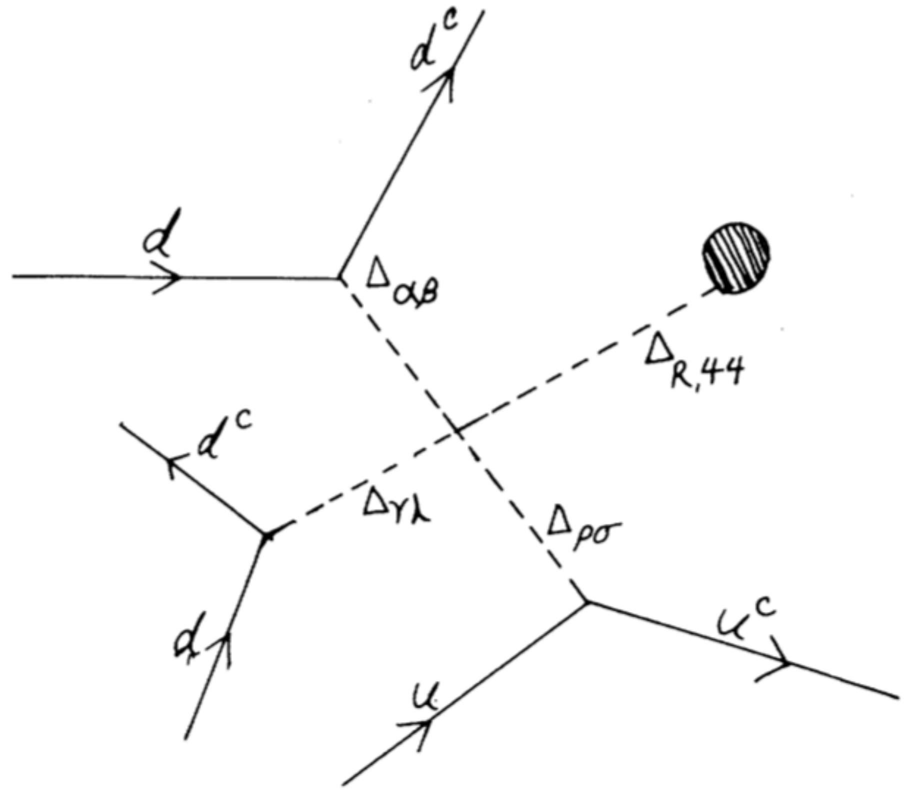
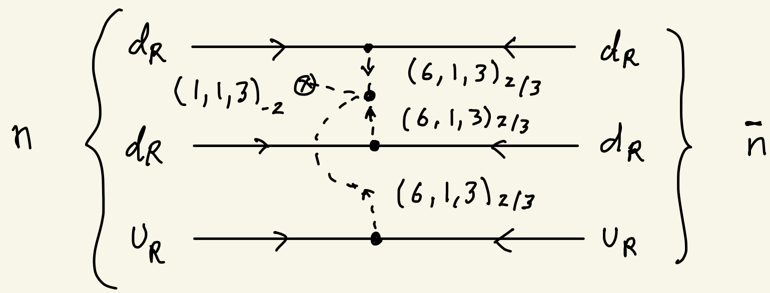


FIG. 1. The tree graph that induces the six-fermion $\Delta B = 2$ vertex that leads to $n \longleftrightarrow \bar{n}$ oscillation.

Figure 2:

$SU(3)_c: \; 6 \times 6 = \bar{6}_s + 15_a + 15'_s$



notation : $\left(\dim(R_{SU(3)_c}),\, \dim(R_{SU(2)_L}),\, \dim(R_{SU(2)_R})\right)_{B-L}$

$q_R^\alpha : (3,1,2)_{1/3}$, $\Delta_R^{\alpha\beta} : (6,1,3)_{2/3}$

$\Delta_R^{44} : (1,1,3)_{-2}$; $\langle \Delta_R^{44} \rangle = v_R$

Δ_R : 10-dim. symmetric tensor rep. of $SU(4) \supset SU(3)_c \times U(1)_{B-L}$

in $G_{422} = SU(4) \times SU(2)_L \times SU(2)_R$

$10 = 1_{-2} + 3_{-2/3} + 6_{2/3}$

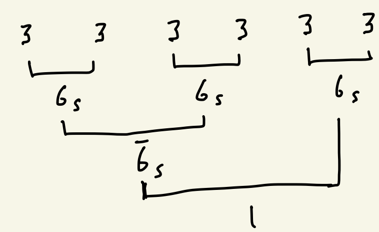
$SU(4) \supset SU(3)_c \times U(1)_{B-L}$

Contribution to $\mathcal{H}_{eff}^{(n\bar{n})}$:

$$\underbrace{\frac{v_R Y_\Delta^3}{(M_\Delta^2)^3}}_{} (T_s)_{\alpha\beta\gamma\delta\rho\sigma} \left[u_R^{T\alpha} C u_R^\beta\right]\left[d_R^{T\gamma} C d_R^\delta\right]\left[d_R^{T\rho} C d_R^\sigma\right]$$

coeff : $\dfrac{1}{M_{n-\bar{n}}^5}$

color contractions :



Some other papers on $n - \bar{n}$ oscillations from the early 1980s include those of S. Glashow, L. N. Chang and N.P. Chang, T. K. Kuo and S. Love, R. Cowsik and S. Nussinov, S. Rao and R.S., etc.

Some early papers on LRS theory:

- R. N. Mohapatra and J. C. Pati, "Left-Right Gauge Symmetry and an 'Isoconjugate' Model of CP Violation", Phys. Rev. D 11, 566 (1975)
- R. N. Mohapatra and J. C. Pati, " 'Natural' Left-Right Symmetry", Phys. Rev. D 11, 2558 (1975)
- G. Senjanović and R. N. Mohapatra, "Exact Left-Right Symmetry and Spontaneous Violation of Parity", Phys. Rev. D 12, 1502 (1975)
- R. E. Marshak and R. N.Mohapatra, "Quark - Lepton Symmetry and B-L as the U(1) Generator of the Electroweak Symmetry Group,", Phys. Lett. B 91, 222 (1980)
- R. N. Mohapatra and G. Senjanović, "Neutrino Mass and Spontaneous Parity Nonconservation", Phys. Rev. Lett. 44, 912 (1980) (seesaw mechanism)
- R. N. Mohapatra and G. Senjanović, "Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation", Phys. Rev. D $\mathbf{23}$, 165 (1981).

Neutrino masses and lepton mixing are confirmed physics beyond the SM, and the most natural mechanism to explain light neutrino masses is the seesaw mechanism, which involves a combination of Dirac mass terms $\bar{\nu}_{iL} M_{ij}^{(D)} \nu_{j,R} + h.c.$ and Majorana mass terms $\nu_{i,R}^T C M_{ij}^{(R)} \nu_{j,R} + h.c.$; the Majorana terms break L, as $\Delta L = 2$ operators.

One realization of the seesaw is in a GUT, where elts. of $M^{(R)}$ are $\sim$ GUT mass scale. But the seesaw relation $M^{(\nu)} = M^{(D)} [M^{(R)}]^{-1} [M^{(D)}]^T$ is invariant under simultaneous rescaling $M^{(D)} \to \epsilon M^{(D)}$ and $M^{(R)} \to \epsilon^2 M^{(R)}$, and hence so are the neutrino mass eigenvalues of $M^{(\nu)}$. For $\epsilon \ll 1$ this allows a low-scale seesaw mechanism with elements of $M^{(R)} \ll M_{GUT}$.

The occurrence of $\Delta L = 2$ operators, possibly in a low-scale seesaw, in neutrino mass models gives further motivation to explore the possibility that there might also be $\Delta B = 2$ operators at scales well below a GUT scale. This is particularly natural in models with a gauged $U(1)_{B-L}$, containing Higgs with $|B - L| = 2$, whose VEV thus lead to both $|\Delta L| = 2$ and $|\Delta B| = 2$ processes.

So there are good motivations for new experimental searches for $n - \bar{n}$ oscillations and associated $\Delta B = -2$ dinucleon decays as well as proton and bound neutron decay, as manifestations of baryon number violation (BNV).

The last exp. searching for $n-\bar{n}$ oscillations with free neutrons was done at the Institut Laue-Langevin (ILL) in Grenoble, France (1985-1994). Currently, there is a plan for $n-\bar{n}$ search exp. at European Spallation Source, ESS, in Lund, Sweden. Searches for $\Delta B=-2$ dinucleon decays at Super-K, in future at Hyper-K and DUNE.

Rabi has also played a key role in discussions of $n-\bar{n}$ oscillations at many workshops and in white papers and reviews, starting in the early 1980s and extending to the present. Some of these after 2000 (not a complete list) include

S. Raby,..., R. N. Mohapatra, et al., "DUSEL (Deep Underground Science and Engineering Laboratory) White Paper" arXiv:0810.4551 (and associated workshops at LBL in Sept. 2007 and Ohio State Univ. in Apr. 2008).

R. N. Mohapatra, "Neutron-Antineutron Oscillation: Theory and Phenomenology", J. Phys. G 36, 104006 (2009).

A. Kronfeld,..., R. N. Mohapatra, et al., "Physics Opportunities with Project X" for Snowmass 2013 (Intensity Frontier), arXiv:1306.5009.

K. Babu,..., R. N. Mohapatra, et al., Neutron-Antineutron Oscillations: a Snowmass 2013 White Paper, arXiv:1310.8593; K. Babu et al., "Baryon Number Violation" arXiv:1311.5268 (and associated Intensity Frontier Snowmass Workshop at Argonne National Lab. in Apr. 2013).

D. G. Phillips,... R. N. Mohapatra et al. “Neutron-Antineutron Oscillations: Theoretical Status and Experimental Prospects”, Phys. Repts., 612, 1 (2016) [arXiv:1410.1100] (and associated workshops at North Carolina State Univ. in May 2014 and CERN in June, 2014).

A. Addazi,..., R. N. Mohapatra, et al. “New High-Sensitivity Searches for Neutrons Converting into Antineutrons and/or Sterile Neutrons at the European Spallation Source”, Snowmass 2021 white paper, J. Phys. G 48, 070501 (2021) [arXiv:2006.04907] and an associated Snowmass meetings and ACFI workshop.

K. S. Babu,... R. N. Mohapatra, et al., “$|\Delta B| = 2$: State of the Field, and Looking Forward - A Brief Status Report of Theoretical and Experimental Physics Opportunities”, arXiv:2010.02299, for Snowmass 2021.

P. S. Bhupal Dev,... R. N. Mohapatra, et al., “Searches for Baryon Number Violation in Neutrino Experiments: A White Paper”, arXiv:2203.08771, Snowmass 2021.

General Formalism for $n - \bar{n}$ Oscillations

$n - \bar{n}$ Oscillations in Field-Free Vacuum:

CPT: $\langle n|H_{eff}|n\rangle = \langle \bar{n}|H_{eff}|\bar{n}\rangle = m_n - i\lambda_n/2$, where H_{eff} denotes relevant effective Hamiltonian and $\lambda_n^{-1} = \tau_n = 0.88 \times 10^3$ sec. H_{eff} may also mediate $n \leftrightarrow \bar{n}$ transitions: $\langle \bar{n}|H_{eff}|n\rangle \equiv \delta m$. Consider the matrix in $(n, \bar{n})$ basis:

$$\mathcal{M} = \begin{pmatrix} m_n - i\lambda_n/2 & \delta m \\ \delta m & m_n - i\lambda_n/2 \end{pmatrix}$$

Diagonalizing $\mathcal{M}$ yields mass eigenstates

$$|n_\pm\rangle = \frac{1}{\sqrt{2}}(|n\rangle \pm |\bar{n}\rangle)$$

with mass eigenvalues $m_\pm = (m_n \pm \delta m) - i\lambda_n/2$.

So if start with pure $|n\rangle$ state at $t = 0$, then there is a finite probability P for it to be an $|\bar{n}\rangle$ at $t \neq 0$. Denote $\tau_{n\bar{n}} = 1/|\delta m|$. Then

$$P(n(t) = \bar{n}) = |\langle \bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-\lambda_n t}$$

In general, in the $(n, \bar{n})$ basis, write

$$\mathcal{M} = \begin{pmatrix} M_{11} & \delta m \\ \delta m & M_{22} \end{pmatrix}$$

Diagonalization yields mass eigenstates $|n_1\rangle$ and $|n_2\rangle$:

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}$$

where

$$\tan(2\theta) = \frac{2\delta m}{\Delta M}$$

and $\Delta M = M_{11} - M_{22}$. The energy eigenvalues are

$$E_{1,2} = \frac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(\Delta M)^2 + 4(\delta m)^2} \right]$$

Let $\Delta E = E_1 - E_2 = \sqrt{(\Delta M)^2 + 4(\delta m)^2}$; transition probability:

$$P(n(t) \to \bar{n}) = |\langle \bar{n} | n(t) \rangle|^2 = \sin^2(2\theta)\ \sin^2[(\Delta E)t/2]\, e^{-\lambda_n t}$$

$$= \left[\frac{(\delta m)^2}{(\Delta M/2)^2 + (\delta m)^2} \right] \sin^2 \left[\sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t \right] e^{-\lambda_n t}$$

In realistic free-neutron experiment, $|\Delta M| \gg |\delta m|$, due to ambient magnetic field, but the exp. achieves sensitivity to δm by arranging that $[(1/2)\Delta E]t \ll 1$, i.e.,

$$[(\Delta M/2)^2 + (\delta m)^2]^{1/2}\, t \ll 1\ ,$$

then by Taylor-expanding the sine squared, the quantity $(\Delta M/2)^2 + (\delta m)^2$ cancels, so in this case

$$P(n(t) \to \bar{n}) \simeq [(\delta m)t]^2\, e^{-\lambda_n t} = (t/\tau_{n\bar{n}})^2\, e^{-\lambda_n t}$$

$n - \bar{n}$ Oscillations in a Magnetic Field $\vec{B}$:

Even with magnetic shielding, the neutrons in a free-neutron exp. searching for $n - \bar{n}$ oscillations are subject to a nonzero external magnetic field $\vec{B}$ due to the earth. The n and $\bar{n}$ interact with $\vec{B}$ via magnetic moment $\vec{\mu} = \mu\,\vec{\sigma}$, $\mu_n = -\mu_{\bar{n}} = \kappa\mu_N$, where $\kappa = -1.91$, $\mu_N = e/(2m_N) = 3.15 \times 10^{-14}$ MeV/Tesla, so

$$\mathcal{M} = \begin{pmatrix} m_n - \vec{\mu}_n \cdot \vec{B} - i\lambda_n/2 & \delta m \\ \delta m & m_n + \vec{\mu}_n \cdot \vec{B} - i\lambda_n/2 \end{pmatrix}$$

So $\Delta M = M_{11} - M_{22} = -2\vec{\mu}_n \cdot \vec{B}$ and diagonalization yields mass eigenstates $|n_1\rangle$, $|n_2\rangle$, with energy eigenvalues

$$E_{1,2} = m_n \pm \sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} - i\lambda_n/2$$

ILL experiment reduced $|\vec{B}| = B$ to $\sim 10^{-4}$ G $= 10^{-8}$ T, so

$$|\mu_n|B = (6.03 \times 10^{-22}\ \mathrm{MeV}) \left(\frac{B}{10^{-8}\ \mathrm{T}}\right)$$

Now $|\delta m| \lesssim 10^{-30}$ from Super-K, so $|\delta m| \ll |\mu_n|B$, and

$$\Delta E = 2\sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} \simeq 2|\vec{\mu}_n \cdot \vec{B}|$$

Experimentally, arrange that n's propagate a time t such that $[(1/2)\Delta E]t \ll 1$, i.e,

$$|\vec{\mu}_n \cdot \vec{B}|t = 0.92 \left(\frac{B}{10^{-8}\ \mathrm{T}}\right)\left(\frac{t}{1\ \mathrm{sec}}\right) \ll 1 \quad \text{and } t \ll \tau_n$$

Then the exp. is sensitive to δm

$$P(n(t) \to \bar{n}) \simeq [(\delta m)\, t]^2 = (t/\tau_{n\bar{n}})^2$$

Denoting the total number of neutrons measured as N_n, the resultant total number of $\bar{n}$'s produced in an exp. is

$$N_{\bar{n}} = P(n(t) \to \bar{n})\ N_n$$

Here, $N_n = \phi T_{run}$ where $\phi =$ is the neutron flux and $T_{run} =$ the exp. running time.

The sensitivity of exp. depends in part on the product

$$N_n\left(\frac{t}{\tau_{n\bar{n}}}\right)^2 = \phi T_{run}\left(\frac{t}{\tau_{n\bar{n}}}\right)^2$$

so, with adequate magnetic shielding, want to maximize t, subject to condition $|\vec{\mu}_n \cdot \vec{B}|t \ll 1$ (and neutrons not falling too far in gravity to reach detector).

Most sensitive reactor $n - \bar{n}$ exp. done with ILL High Flux Reactor (HFR) at Grenoble (Baldo-Ceolin, Fidecaro,.., 1985-1994; M. Baldo-Ceolin et al., Z. Phys. C63, 409 (1994))) with neutrons cooled to liquid D_2 temp., kinetic energy $E \simeq 2 \times 10^{-3}$ eV, typical velocity $v \simeq 700$ m/s, $L \simeq 76$ m, $t \simeq 0.11$ sec., $\phi \sim 1.25 \times 10^{11}$ n/s, so $\phi t^2 = 1.5 \times 10^9 \text{ n} \cdot \text{s}$; set limit

$$\tau_{n\bar{n}} \geq 0.86 \times 10^8 \text{ sec} \quad (90 \ \% \ CL)$$

i.e., $|\delta m| = \hbar/\tau_{n\bar{n}} = (6.58 \times 10^{-22} \text{ MeV} \cdot \text{s})/\tau_{n\bar{n}} \leq 0.77 \times 10^{-29}$ MeV.

In general, $|\delta m| = (0.658 \times 10^{-29} \text{ MeV})(10^8 \text{ s}/\tau_{n\bar{n}})$.

Note that $\tau_{n\bar{n}} \gg \tau_n$, i.e., the lower bound on $\tau_{n\bar{n}}$ is much larger than the free neutron lifetime, $\tau_n = 0.88 \times 10^3$ sec, so most free neutrons decay before they might undergo a transition to $\bar{n}$.

$n - \bar{n}$ Oscillations in Matter:

For $n - \bar{n}$ oscillations involving a neutron bound in a nucleus, consider

$$\mathcal{M} = \begin{pmatrix} m_{n,eff.} & \delta m \\ \delta m & m_{\bar{n},eff.} \end{pmatrix}$$

with

$$m_{n,eff} = m_n + V_n \ , \quad m_{\bar{n},eff.} = m_n + V_{\bar{n}}$$

where the nuclear potential V_n is real, $V_n = V_{nR}$, but $V_{\bar{n}}$ has an imaginary part representing the $\bar{n}N$ annihilation: $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$ with $V_{nR},\ V_{\bar{n}R},\ V_{\bar{n}I} \sim O(100)$ MeV (Dover, Gal, Richard; Friedman; recently work by Barrow, Golubeva, Ladd, Paryev, Richard for ^{12}C (ESS) and ^{40}Ar (DUNE)).

Mixing is thus strongly suppressed; $\tan(2\theta)$ is determined by

$$\frac{2\delta m}{|m_{n,eff.} - m_{\bar{n},eff.}|} = \frac{2\delta m}{\sqrt{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}} \ll 1$$

Using the reactor exp. bound on $|\delta m|$, this gives $|\theta| \lesssim 10^{-31}$. This suppression in mixing is compensated by the large number of nucleons in a nucleon decay detector, $\sim 10^{33}$ n's in Super-K.

Eigenvalues:

$$m_{1,2} = \frac{1}{2}\left[m_{n,eff.} + m_{\bar{n},eff.} \pm \sqrt{(m_{n,eff.} - m_{\bar{n},eff.})^2 + 4(\delta m)^2} \right]$$

Expanding m_1 for the mostly n mass eigenstate $|n_1\rangle \simeq |n\rangle$,

$$m_1 \simeq m_n + V_n - i \frac{(\delta m)^2 \, V_{\bar{n}I}}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

Imaginary part leads to matter instability, mainly via $\bar{n}n, \ \bar{n}p \to \pi$'s, with rate

$$\Gamma_{m.i.} = \frac{1}{\tau_{m.i.}} = \frac{2(\delta m)^2 |V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

So $\tau_{m.i.} \propto (\delta m)^{-2} = \tau_{n\bar{n}}^2$.

Writing $\tau_{m.i.} = R\,\tau_{n\bar{n}}^2$, one has $R \sim O(100)$ MeV, dependent on nucleus.

With $\hbar = 6.6 \times 10^{-22}$ MeV-sec, equiv. $R \sim 10^{23}$ sec^{-1}.

Searches for matter instability due to $n - \bar{n}$ oscillations with large nucleon decay detectors are complementary to searches with free neutrons at reactors or spallation sources. Searches for matter instability due to $n - \bar{n}$ oscillations were performed most recently by Soudan, Super-K, and SNO experiments.

Generic signature is a multipion final state resulting from the annihilation of the $\bar{n}$ with a neighboring neutron or proton.

A lower bound on $\tau_{m.i.}$ yields a lower bound on $\tau_{n\bar{n}}$ via $\tau_{n\bar{n}} = (\tau_{m.i.}/R)^{1/2}$. Current best bound is from Super-Kamiokande $\tau_{m.i.} > 3.6 \times 10^{32}$ yrs, giving

$\tau_{n\bar{n}} > 4.7 \times 10^{8}$ sec (90 % confidence level)

[Abe et al., Phys. Rev. D 103, 012008 (2021)] and thus

$$|\delta m| = \frac{1}{\tau_{n\bar{n}}} < 1.4 \times 10^{-30} \text{ MeV}$$

The future $n - \bar{n}$ search experiment at ESS plans to achieve improved sensitivity (Yuri Kamyshkov's talk).

The $n-\bar{n}$ transition operators $\mathcal{O}_i^{(n\bar{n})}$ are six-quark operators of the form $uudddd$, with mass dimension $6 \times (3/2) = 9$ (in $d = 4$ spacetime dimensions), so the coefficient of this operator is $\sim 1/M_{n\bar{n}}^5$, where $M_{n\bar{n}}$ is an effective scale characterizing the BNV physics responsible for the $n-\bar{n}$ transition. Then δm is determined by $M_{n\bar{n}}$ and the matrix elements $\langle \bar{n}|\mathcal{O}_i^{(n\bar{n})}|n\rangle \sim \Lambda_{QCD}^6$, where $\Lambda_{QCD} \simeq 0.25$ GeV.

So $\delta m \sim \Lambda_{QCD}^6/M_{n\bar{n}}^5$, i.e.,

$$M_{n\bar{n}} \sim \left(\frac{\Lambda_{QCD}}{|\delta m|}\right)^{1/5} \Lambda_{QCD}$$

Hence, an lower bound on $\tau_{n\bar{n}}$, i.e., upper bound on $|\delta m|$ yields a lower bound on $M_{n\bar{n}}$. Using the current lower limit on $\tau_{n\bar{n}}$ from Super-K, we have

$$M_{n\bar{n}} \gtrsim 700 \text{ TeV} \left(\frac{\tau_{n\bar{n}}}{4.7 \times 10^8 \text{ sec}}\right)^{1/5}$$

So $n-\bar{n}$ experiments probe baryon-number violating physics occurring at an mass scale of $\sim 10^2 - 10^3$ TeV.

Another interesting possibility is that there could be mixing between neutrons in our world and "mirror neutrons" (n') in a mirror world. Rabi Mohapatra and others (Z. Berezhiani, S. Nussinov, Y. Kamyshkov, L. Broussard,...) have studied neutron - mirror neutron oscillations and mirror dark matter in many papers. Recent exp. at SNS, ORNL: L. J. Broussard, J. L. Barrow,...,Y. Kamyshkov, et al., PRL 128, 212503 (2022) and future searches at ORNL and ESS (Kamyshkov talk). Thus, the future ESS exp. will also search for $n - n'$ and $n - \bar{n}'$ as well as $n - \bar{n}$ transitions (Addazi et al., 2006.04907). Some of Rabi's papers on this subject include

- Z. G. Berezhiani, A. D. Dolgov, and R. N. Mohapatra, "Asymmetric Inflationary Reheating and the Nature of the Mirror Universe", Phys. Lett. B 375, 26 (1996) [hep-ph/9511221].
- R. N. Mohapatra, S. Nussinov, and V. L. Teplitz, "Mirror Matter as Self-Interacting Dark Matter", Phys. Rev. D 66 063002 (2002) [hep-ph/0111381].
- R. N. Mohapatra, S. Nasri, S. Nussinov, "Some Implications of Neutron Mirror Neutron Oscillation", Phys. Lett. B 627 124 (2005) [hep-ph/0508109].
- R. N. Mohapatra and S. Nussinov, "Constraints on Mirror Models of Dark Matter from Observable Neutron-Mirror Neutron Oscillation", Phys. Lett. B 776, 22 (2018) [arXiv:1709.01637].
- I. Goldman, R. N. Mohapatra, and S. Nussinov, "Bounds on Neutron-Mirror Neutron

Mixing from Pulsar Timing", Phys. Rev. D 100, 12 (2019) [arXiv:1901.07077].

- I. Goldman, R. N. Mohapatra, S. Nussinov, and Y. Zhang, "Constraints on Neutron-Mirror-Neutron Oscillation from Neutron Star Cooling", arXiv:2203.08473.
- I. Goldman, R. N. Mohapatra, S. Nussinov, and Y. Zhang, "NeutronMirror-Neutron Oscillation and Neutron Star Cooling", Phys. Rev. Lett. 129, 061103 (2022) [arXiv:2208.03771].

Rabi has also worked on many other interesting related topics, often in collaboration with speakers at this conference. These topics include

- Further studies of left-right-symmetric gauge theories
- Further connections with neutrino masses and mixing
- Baryogenesis at various scales, including post-sphaleron baryogenesis
- Supersymmetric models
- Dark matter
- Other BSM topics

We next discuss a model in which proton decay can easily be suppressed well below experimental limits while $n - \bar{n}$ oscillations can occur at level comparable to existing limits (Shmuel Nussinov and RS, PRL 88, 171601 (2002) and recent work with Sudhakantha Girmohanta).

Thus, in this model, $n - \bar{n}$ oscillations are the main manifestation of baryon number violation, just as in the 1980 PRL by Mohapatra and Marshak.

Although we focus on $n - \bar{n}$ here and do not discuss neutrino masses and mixing, in

S. Girmohanta, R. N. Mohapatra, and RS, "Neutrino Masses and Mixing in Models with Large Extra Dimensions and Localized Fermions", Phys. Rev. D 103, 015021 (2021) [arXiv:2011.01237]

we have investigated the question of whether this model can produce a (low-scale) seesaw mechanism that can explain light neutrino masses and observed lepton mixing. We show that the answer is yes.

Refs. for next section:

S. Nussinov and RS, "$n - \bar{n}$ Oscillations in Models with Large Extra Dimensions", Phys. Rev. Lett. 88, 171601 (2002) [hep-ph/0112337].

S. Girmohanta and RS, "Baryon-Number-Violating Nucleon and Dinucleon Decays in a Model with Large Extra Dimensions", Phys. Rev. D 101, 015017 (2020) [arXiv:1911.05102].

S. Girmohanta and RS, "Improved Upper Limits on Baryon-Number Violating Dinucleon Decays to Dileptons", Phys. Lett. B 803, 135296 (2020) [arXiv:1910.08356].

S. Girmohanta and RS, "Baryon-Number-Violating Processes in a Left-Right Symmetric Model with Large Extra Dimensions", Phys. Rev. D 101, 095012 (2020) [arXiv:2003.14185].

S. Nussinov and RS, "Using $\bar{p}p$ and e^+e^- Annihilation Data to Refine Bounds on the Baryon-Number-Violating Dinucleon Decays $nn \to e^+e^-$ and $nn \to \mu^+\mu^-$", Phys. Rev. D 102, 035003 (2020) [arXiv:2005.12493].

S. Girmohanta, R. N. Mohapatra, and RS, "Neutrino Masses and Mixing in Models with Large Extra Dimensions and Localized Fermions", Phys. Rev. D 103, 015021 (2021) [arXiv:2011.01237].

$n-\bar{n}$ Oscillations in an Extra-Dimensional Model

Extra spatial dimensions have been of interest since Kaluza and Klein and received renewed attention with the development of string theory.

Consider a model with a $d=4+n$ dimensional spacetime, with n extra spatial dimensions. Denote usual spacetime coords. as x_ν, $\nu=0,1,2,3$ and consider n extra compact coordinates, y_λ with $0 \le y_\lambda \le L$, i.e., size of extra dimension(s) is L.

Each SM fermion f has the form

$$\Psi_f(x,y) = \psi_f(x)\chi_f(y)$$

with strong localization at a point y_f in the extra dimensions, with a Gaussian profile of half-width $\sigma \equiv 1/\mu \ll L$:

$$\chi_f(y) = Ae^{-\mu^2\|y-y_f\|^2} = Ae^{-\|\eta-\eta_f\|^2}$$

where $\|y_f\| = (\sum_{\lambda=1}^n y_{f,\lambda}^2)^{1/2}$, A is a normalization constant, and we define a convenient dimensionless variable $\eta_f = \mu y_f$.

Such models are of interest partly because they can provide a mechanism for obtaining a generational hierarchy in fermion masses and quark mixing.

We use a low-energy effective field theory (EFT) approach with an ultraviolet cutoff M_*, where $M_* > \mu$ for self-consistency. Only the lowest mode in the Kaluza-Klein (KK) mode decompositions of each Ψ field will be needed here; effects of higher modes are considered in our papers.

Starting from the Lagrangian in the d-dimensional spacetime, one obtains the resultant low-energy EFT in 4D by integrating over the extra n dimension(s). For canonical normalization of the 4D fermion kinetic term,

$$\boldsymbol{A = \left(\frac{2}{\pi}\right)^{n/4} \mu^{n/2}}$$

The localization is achieved by coupling to auxiliary "localizer" scalar fields with kink form for $n = 1$, and similarly for higher n (Arkani-Hamed + Schmaltz; Mirabelli+Schmaltz, 2000). Higgs fields are taken flat in extra dims.

Define $\Lambda_L \equiv 1/L$; take $\Lambda \sim 10^2$ TeV, $\sigma \equiv 1/\mu \sim L/30$; this gives adequate separation of fermions while fitting in interval $[0, L]$, consistent with precision electroweak data, collider bounds, flavor-changing neutral current constraints. Corresponding compactification length: $L = 2 \times 10^{-19}$ cm.

With $\Lambda_L = 10^2$ TeV, this yields $\mu \sim 3 \times 10^3$ TeV.

This type of extra-dimensional (ED) model is often called a split-fermion model because different chiral components of quark and lepton fields have localized wave function centers at different positions in the extra dimensions.

N.B.: This split fermion ED model is quite different from ED models with low quantum gravity scales (Arkani-Hamed, Dimopoulos, Dvali; Dienes, Dudas, Gherghetta, 1998), as is clear from the fact that, e.g., for $n = 2$ and quantum gravity scale of 30 TeV, the ADD-DDG models have a compactification size $\sim 3 \times 10^{-4}$ cm., much larger than the scale $L \simeq 2 \times 10^{-19}$ cm in the ED model that we use.

The split-fermion model used here is also different from warped (Randall-Sundrum) models.

Given the localization of fermion wavefunctions on scale $\sigma \ll L$, in the integration over the extra dimensions, can extend $\int_0^L \longrightarrow \int_{-\infty}^{\infty}$ to good approximation.

Integrals over extra dimensions have the general form (with $\int d^n\eta = \int_{-\infty}^{\infty} d^n\eta$)

$$\int d^n\eta \, \exp\Big[- \sum_{i=1}^{m} a_i \|\eta - \eta_{f_i}\|^2 \Big] = \Big[\frac{\pi}{\sum_{i=1}^m a_i} \Big]^{n/2} \exp\left[\frac{-\sum_{j,k=1;\ j<k}^m a_j a_k \|\eta_{f_j} - \eta_{f_k}\|^2}{\sum_{s=1}^m a_s} \right] .$$

For example, for $m = 3$,

$$\int d^n\eta \, \exp\Big[- \Big(a_1\|\eta - \eta_{f_1}\|^2 + a_2\|\eta - \eta_{f_2}\|^2 + a_3\|\eta - \eta_{f_3}\|^2 \Big) \Big] =$$

$$= \Big[\frac{\pi}{a_1 + a_2 + a_3} \Big]^{n/2} \exp\left[\frac{-\Big(a_1a_2\|\eta_{f_1} - \eta_{f_2}\|^2 + a_2a_3\|\eta_{f_2} - \eta_{f_3}\|^2 + a_3a_1\|\eta_{f_3} - \eta_{f_1}\|^2 \Big)}{a_1 + a_2 + a_3} \right] .$$

If only one fermion involved in integrand, then no exponential suppression:

$$\int d^n\eta \, \exp\Big[- a_1\|\eta - \eta_{f_1}\|^2 \Big] = \Big[\frac{\pi}{a_1} \Big]^{n/2}$$

A Yukawa interaction in the d-dimensional space with coefficients of order unity and moderate separation of localized fermion wavefunction centers yields a strong hierarchy in the low-energy 4D Yukawa interaction,

$$\int d^n y\, \bar{\chi}(y_{f_L})\chi(y_{f_R}) \sim \int d^n \eta\, e^{-\|\eta-\eta_{f_L}\|^2} e^{-\|\eta-\eta_{f_R}\|^2} \sim e^{-(1/2)\|\eta_{f_L}-\eta_{f_R}\|^2}$$

Resultant fermion masses m_f:

$$m_f \simeq h^{(f)} \frac{v}{\sqrt{2}} \exp\Big[-\frac{1}{2}\|\eta_{f_L} - \eta_{f_R}\|^2 \Big] \, ,$$

where $v/\sqrt{2}$ is SM Higgs VEV. With $h^{(f)} \simeq 1$, produce fermion generational hierarchy via different separation distances $\|\eta_{f_L} - \eta_{f_R}\|$ for different generations.

Leading nucleon decay operators are of the form $qqq\ell$. Hence, one can suppress nucleon decay well below experimental limits by arranging that the wavefunction centers of the u and d quarks are separated far from those of the leptons.

Key point: this does not suppress $n - \bar{n}$ oscillations because the $n - \bar{n}$ transition operators do not involve leptons.

For example, one nucleon decay operator is (with $\ell = e,\ \mu$)

$$\mathcal{O}_1^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha\ T} C d_R^{\beta}][u_R^{\gamma\ T} C \ell_R]$$

where $\alpha,\ \beta,\ \gamma$ are SU(3)$_c$ color indices.

The product of y-dependent fermion wavefunctions in this operator is

$$A^4 \exp\left[- \left\{ 2\|\eta - \eta_{u_R}\|^2 + \|\eta - \eta_{d_R}\|^2 + \|\eta - \eta_{\ell_R}\|^2 \right\} \right]$$

The integral over y yields

$$I_1^{(Nd)} = b_4 \ \exp\left[- \frac{1}{4} \left\{ 2\|\eta_{u_R} - \eta_{d_R}\|^2 + 2\|\eta_{u_R} - \eta_{\ell_R}\|^2 + \|\eta_{d_R} - \eta_{\ell_R}\|^2 \right\} \right]$$

where $b_4 = (\mu/\sqrt{\pi})^n$.

One can guarantee that this is sufficiently small by taking the distances between wavefunction centers $\|\eta_{u_R} - \eta_{\ell_R}\|$ and/or $\|\eta_{d_R} - \eta_{\ell_R}\|^2$ sufficiently large.

Similarly for other nucleon decay operators.

At the quark level $n \to \bar{n}$ is $(udd) \to (u^c d^c d^c)$. This is mediated by 6-quark operators $\mathcal{O}_r^{(n\bar{n})} \sim uddudd$.

In $d = 4$ dims., effective Lagrangian for the $n - \bar{n}$ transition is

$$\mathcal{L}_{eff}^{(n\bar{n})}(x) = \sum_r c_r^{(n\bar{n})}\, \mathcal{O}_r^{(n\bar{n})}(x) + h.c.\ .$$

Correspondingly, in $d = 4 + n$ dimensions,

$$\mathcal{L}_{eff,4+n}^{(n\bar{n})}(x,y) = \sum_r \kappa_r^{(n\bar{n})} O_r^{(n\bar{n})}(x,y) + h.c.\ .$$

where the $\mathcal{O}_r^{(n\bar{n})}(x)$ and $O_r^{(n\bar{n})}(x,y)$ are 6-quark operators in $d = 4$ and $d = 4 + n$ dims.

In d-dimensional spacetime the dimension of a fermion field ψ in mass units is $\mathrm{dim}(\psi) = (d-1)/2$, so $\mathrm{dim}(\mathcal{O}_r^{(n\bar{n})}) = 6d_\psi = 3(d-1)$ and

$$\mathrm{dim}(\kappa_r) = d - \mathrm{dim}(\mathcal{O}_r^{(n\bar{n})}) = 3 - 2d = 3 - 2(4+n) = -(5+2n)$$

So the coefficients κ_r have the form

$$\kappa_r^{(n\bar{n})} = \frac{\bar{\kappa}_r^{(n\bar{n})}}{M_{n\bar{n}}^{5+2n}}$$

where $\bar{\kappa}_r^{(n\bar{n})}$ are dimensionless and $M_{n\bar{n}}$ is the effective mass characterizing the physics responsible for the $n - \bar{n}$ oscillation. We can set $\bar{\kappa}_r^{(n\bar{n})} = 1$ for the dominant $O_r^{(n\bar{n})}$ in defining $M_{n\bar{n}}$.

Integration of fermion wavefunctions in the $O_r^{(n\bar{n})}(x,y)$ over y yield the coeffs. $c_r^{(n\bar{n})}$ in terms of $\kappa_r^{(n\bar{n})}$

Operators $\mathcal{O}_r^{(n\bar{n})}$ must be color singlets and, for $M_{n\bar{n}}$ larger than the electroweak symmetry breaking scale, also $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$-singlets. Relevant operators in SM EFT:

$$\mathcal{O}_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_2^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}\epsilon_{km}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

where $Q_L = \binom{u}{d}_L$, $i, j..$ are SU(2)$_L$ indices, and color tensors are

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$$

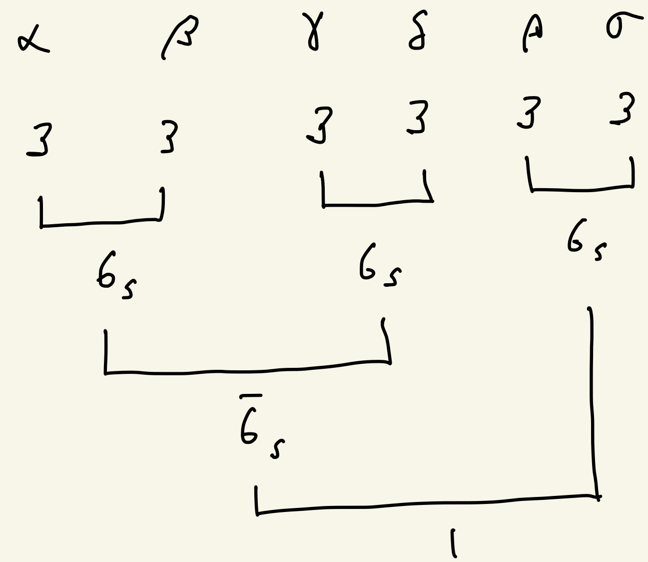
$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$ is symmetric in the indices $(\alpha\beta)$, $(\gamma\delta)$, $(\rho\sigma)$.

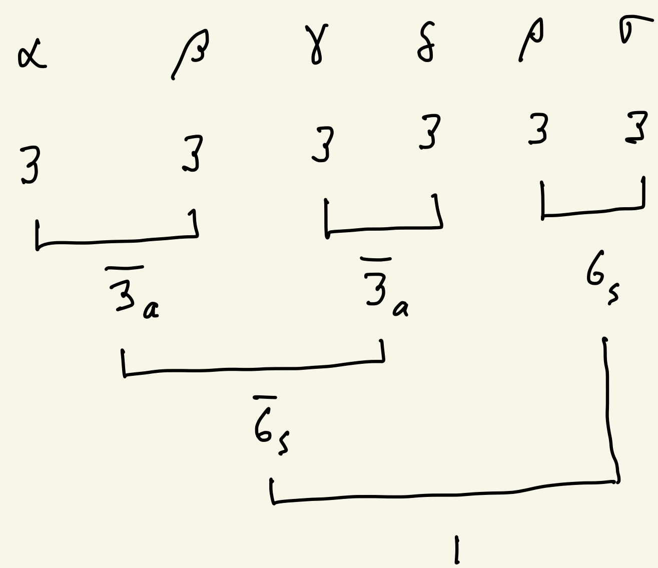
$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$ is antisymmetric in $(\alpha\beta)$ and $(\gamma\delta)$ and symmetric in $(\rho\sigma)$.

$SU(3)_c$ color contractions

$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$



$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$



The integrals of these operators over y comprise three classes: operators $O_1^{(n\bar{n})}$ and $O_2^{(n\bar{n})}$ yield the integral

$$I_{r12}^{(n\bar{n})} = b_6 \ \exp\left[-\frac{4}{3}\|\eta_{u_R} - \eta_{d_R}\|^2 \right] ,$$

$O_3^{(n\bar{n})}$ yields the integral

$$I_{r3}^{(n\bar{n})} = b_6 \ \exp\left[-\frac{1}{6}\Big\{ 2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 6\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2 \Big\} \right] .$$

$O_4^{(n\bar{n})}$ yields the integral

$$I_{r4}^{(n\bar{n})} = b_6 \ \exp\left[-\frac{4}{3}\|\eta_{Q_L} - \eta_{d_R}\|^2 \right] .$$

where $b_6 = (2 \cdot 3^{-1/2} \pi^{-1} \mu^2)^n$.

The coeffs. $c_r^{(n\bar{n})} = \bar{\kappa}_r^{(n\bar{n})}/(M_{n\bar{n}})^5$ times these $I_r^{(n\bar{n})}$ integrals.

Consider, e.g., case $n = 2$: one can fit data on quark masses, mixing with

$$\|\eta_{Q_L} - \eta_{u_R}\| = 4.75, \quad \|\eta_{Q_L} - \eta_{d_R}\| \simeq 4.60$$

$$\|\eta_{u_R} - \eta_{d_R}\| \simeq 7$$

We find that the $|c_r^{(n\bar{n})}|$ for $r = 1, 2, 3$ are $\ll |c_4^{(n\bar{n})}|$, and hence we focus on $c_4^{(n\bar{n})}$:

To leading order (neglecting small CKM mixings), $\|\eta_{Q_L} - \eta_{d_R}\|$ is determined by m_d via relation (with Higgs vev $v = 246$ GeV)

$$m_d = h_d \frac{v}{\sqrt{2}}$$

with

$$h_d = h_{d,0} \exp[-(1/2)\|\eta_{Q_L} - \eta_{d_R}\|^2]$$

where $h_{d,0}$ is the Yukawa coupling in $(4 + n)$-dims. so that

$$\exp\left[-(1/2)\|\eta_{Q_L} - \eta_{d_R}\|^2\right] = \frac{2^{1/2} m_d}{h_{d,0} v}$$

With $h_{d,0} \sim 1$

$$\delta m \simeq c_4^{(n\bar{n})}\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle \simeq \left(\frac{4\mu^4}{3\pi^2 M_{n\bar{n}}^9}\right)\left(\frac{2^{1/2}m_d}{v}\right)^{8/3}\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle$$

Requiring that $\tau_{n\bar{n}} = 1/|\delta m|$ agree with the lower limit from Super-K, $\tau_{n\bar{n}} > 4.7 \times 10^8$ sec. yields the lower bound on the mass scale of $n - \bar{n}$ oscillations:

$$M_{n\bar{n}} > (51\,\mathrm{TeV})\left(\frac{\tau_{n\bar{n}}}{4.7\times10^8\ \mathrm{sec}}\right)^{1/9}\left(\frac{\mu}{3\times10^3\ \mathrm{TeV}}\right)^{4/9}\left(\frac{|\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle|}{\Lambda_{QCD}^6}\right)^{1/9}.$$

where $\Lambda_{QCD} = 0.25$ GeV. This bound is not very sensitive to the precise size of $\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle$ because of the 1/9 power in the exponent.

$\mathcal{O}_4^{(n\bar{n})} = -Q_3$ in notation of lattice QCD calculation (Syritsyn et al. 2019), with LQCD matrix element $|\langle\bar{n}|Q_3|n\rangle| \simeq 5\times10^{-4}$ GeV$^6 = 2\Lambda_{QCD}^6$; substituting this yields factor $2^{1/9} = 1.08$ so lower bound is (1.08)51 TeV = 55 TeV.

Hence, for relevant values of $M_{n\bar{n}}$ in this model, $n - \bar{n}$ oscillations could occur at levels that are close to the current limit.

This model also illustrates how baryon number violation can occur via $n - \bar{n}$ oscillations with strongly suppressed proton decay.

With SM fermion wavefunction centers chosen to suppress BNV nucleon decays adequately in this model, an interesting question is what are the predictions for other $\Delta B = -1$ nucleon and $\Delta B = -2$ dinucleon decays, including

- (i) the $\Delta L = -3$ nucleon decays $p \to \ell^+ \bar{\nu} \bar{\nu}'$ and $n \to \bar{\nu} \bar{\nu}' \bar{\nu}''$
- (ii) the $\Delta L = 1$ nucleon decays $p \to \ell^+ \nu \nu'$ and $n \to \bar{\nu} \nu' \nu''$
- (iii) the $\Delta L = -2$ dinucleon decays $pp \to (e^+ e^+, \mu^+ \mu^+,\ e^+ \mu^+,\ e^+ \tau^+,\ \mathrm{or}\ \mu^+ \tau^+)$, $np \to \ell^+ \bar{\nu}$, and $nn \to \bar{\nu} \bar{\nu}'$, where $\ell^+ = e^+,\ \mu^+$, or τ^+;
- (iv) the $\Delta L = 2$ dineutron decays $nn \to \nu \nu'$.

The decays of type (i) and (ii) are mediated by 6-fermion operators, while the decays of type (iii) and (iv) are mediated by 8-fermion operators. In S. Girmohanta and RS, PRD 101, 015017 (2020) we show that the predictions of the model are in accord with experimental constraints.

$n - \bar{n}$ Oscillations in an Extra-Dimensional Model with G_{LRS} Gauge Group

We have also studied $n - \bar{n}$ oscillations in an extra-dimensional model with the gauge group $G_{LRS} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$

in Girmohanta + RS, PRD 101, 095012 (2020) [arXiv:2003.14185].

This model provides a useful contrast to the previous study because in the SM the $n - \bar{n}$ oscillations do not break the SM gauge symmetry, while in the LRS model, they occur via the breaking of the $\mathrm{U}(1)_{B-L}$ gauge symmetry.

Recall field content of LRS model (Mohapatra, Pati, Senjanović, 1975...) for fermions (first gen.):

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L : \ (3,2,1)_{1/3,L} \ , \quad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R : \ (3,1,2)_{1/3,R}$$

$$L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L : \ (1,2,1)_{-1,L} \ , \quad L_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R : \ (1,1,2)_{-1,R} \ ,$$

Higgs sector:

$$\mathbf{\Phi} : (\mathbf{1}, \mathbf{2}, \mathbf{2})_0 : \quad \mathbf{\Phi} = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} .$$

$$\mathbf{\Delta}_L : (\mathbf{1}, \mathbf{3}, \mathbf{1})_2, \quad \mathbf{\Delta}_R : (\mathbf{1}, \mathbf{1}, \mathbf{3})_2$$

$$\mathbf{\Delta}_{L,R} = \begin{pmatrix} \Delta_{L,R}^+/\sqrt{2} & \Delta_{L,R}^{++} \\ \Delta_{L,R}^0 & -\Delta_{L,R}^+/\sqrt{2} \end{pmatrix} ,$$

Minimization of Higgs potential (recent study: P. S. Bhupal Dev, R. N. Mohapatra, W. Rodejohann, and X.-J. Xu, JHEP 02 (2019) 154) yields VEVs

$$\langle \mathbf{\Phi} \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 e^{i\theta_\Phi} \end{pmatrix} ,$$

$$\langle \mathbf{\Delta}_L \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_\Delta} & 0 \end{pmatrix}$$

$$\langle \mathbf{\Delta}_R \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} .$$

At highest scale, v_R breaks $\mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \longrightarrow \mathrm{U}(1)_Y$ with $|\Delta(B-L)| = 2$. This naturally yields $n - \bar{n}$ oscillations and connects them with the Majorana neutrino mass generation. So in this model,

$$M_{n\bar{n}} = v_R$$

At electroweak level, $\kappa,\ \kappa'$ break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_{em}$. Take $v_L \ll \kappa,\ \kappa'$ to preserve $\rho = 1$ where $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$.

As in the SM EFT, nucleon decay can be suppressed well below experimental limits by separating the wavefunction centers of the quarks from those of the leptons.

Since the adjoint rep. of SU(2) is the rank-2 symmetric tensor, can write Δ_L as $(\Delta_L)^{ij}$ and Δ_R as $(\Delta_R)^{i'j'}$, where i, j are SU(2)$_L$ indices and i', j' are SU(2)$_R$ indices.

$O_r^{(n\bar{n})}$ operators:

$$O_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma} (\epsilon_{i'k'}\epsilon_{j'm'} + \epsilon_{j'k'}\epsilon_{i'm'})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$
$$\times\ [Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_2^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{i'j'}\epsilon_{k'm'}\,(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$
$$\times\ [Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{k'm'}\,(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{km}\,(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_5^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}(\epsilon_{ik}\epsilon_{jm} + \epsilon_{jk}\epsilon_{im})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$
$$\times\ [Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

After symmetry breaking of U(1)$_{B-L}$, replace Δ_R by VEV, v_R.

In the same way as before, we obtain the low-energy 4D EFT by integrating the operator products over the n extra dimensions.

Because $O_1^{(n\bar{n})}$ and $O_2^{(n\bar{n})}$ involve only one kind of fermion field (namely, Q_R), we find that for these two operators the integral over y does not yield any exponential (Gaussian) suppression factor. Coeffs. $\bar{\kappa}_r^{(n\bar{n})}$ can naturally be $\sim$ O(1) in the model for these operators.

This is in contrast to the SM EFT, where the integrals of all $n - \bar{n}$ operators involved exponential suppression factors.

Because of this, the constraint that this model should agree with the experimental lower limit on $\tau_{n\bar{n}}$ imposes a more stringent lower bound on the scale $M_{n\bar{n}}$ in this model than in the SM EFT analysis:

$$M_{n\bar{n}} \gtrsim \max\Bigg[(1 \times 10^3 \text{ TeV}) \Big(\frac{\tau_{n\bar{n}}}{4.7 \times 10^8 \text{ sec}} \Big)^{1/9} \times \Big(\frac{\mu}{3 \times 10^3 \text{ TeV}} \Big)^{4/9} \Bigg(\frac{|\bar{\kappa}_r^{(n\bar{n})} \langle \bar{n} | \mathcal{O}_r^{(n\bar{n})} | n \rangle |}{\Lambda_{QCD}^6} \Bigg)^{1/9} \Bigg] , \quad r = 1, 2$$

Closing Remarks

Here we have discussed one of the many areas of particle physics where Rabi Mohapatra has made contributions of pioneering, profound, and far-reaching importance, namely $n - \bar{n}$ oscillations.

This festschrift is a wonderul recognition of Rabi's research contributions throughout the many years of his remarkable career.

We look forward to results from future experimental searches for $n - \bar{n}$ oscillations in free neutron propagation experiments and via searches for matter instability in deep underground nucleon decay detectors.

We wish Rabi Mohapatra the best for good health and many more years of very productive research and mentoring.

Thank you