Bayesian PDF fits to ZEUS high-x data

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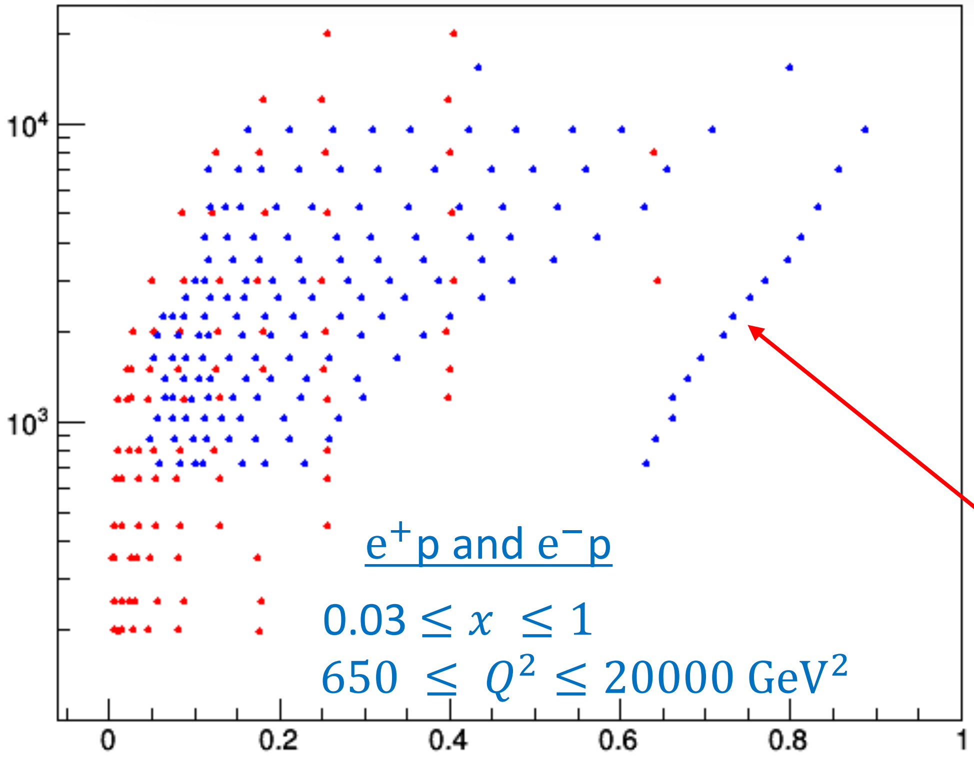
arXiv:2209.06571 [hep-ph]

Measurement of neutral current $e^{\pm}p$ cross sections at high Bjorken x with the ZEUS detector

ZEUS collab., Phys.Rev. D **89**, 072007, hep-ex:1312.4438 (2014)



- Unique e^+p and e^-p data set extending to $x = 1$
- Data not used in any global pdf fit
- No higher twists which plague analysis of other high-x data
- Event numbers and differential cross-sections are given in 153 x-Q^2 bins for each data set
- At high x only integrated cross-sections are given

Use the full data set in a Bayesian pdf fit of bin-counts that can handle the Poisson statistics of low event numbers at very large x

Bayesian forward model approach

- Parameterise pdfs at some Q_0^2 and evolve at NNLO
- Compute at NNLO F_2, F_L and xF_3 and Born neutral current $\mathrm{e}^{\pm}\mathrm{p}$ cross-sections
- Integrate x-sections over bins in x-Q^2 and compute event numbers
- Apply radiative and detector effects to get predictions for the observed events n
- Compute Poisson likelihood $P(n|\theta)$ for the set of fit parameters θ
- Define prior probabilities $P(\theta)$ for the fit parameters
- Get posterior from Bayes theorem

$$P(\theta|n, \mathrm{model}) \propto P(n|\theta)P(\theta)$$

Posterior is also conditional on the choice of parameterisation

- **Bayesian approach is very attractive because …**
  - No Gaussian assumptions
  - Constraints are easily implemented (posterior cannot extend beyond prior range)
  - Badly constrained parameters do not spoil the fit
  - Uncertainties in badly constrained parameters (as encoded in the prior) are automatically propagated to the posterior of other parameters
  - Can easily judge information content of the data by comparing posterior to prior
  - Marginalisation of the posterior gives easy access to single-parameter distributions and correlations
- **But also …**
  - Priors should be chosen with care to not introduce bias in the posterior
  - Need lots of CPU to map-out the posterior in multi-dimensional parameter space

For this analysis we use the Bayesian Analysis Toolkit and QCDNUM

Bayesian Analysis Toolkit

O. Schulz *et al.*, SN computer Science **2**, 210 (2021)

- High-performance toolkit for Bayesian inference
  - Tools for definition of likelihoods, priors and posteriors
  - Provides MCMC sampling techniques to explore the posterior
  - Location and interval estimation, marginalisation, visualisation, *etc.*
  - And much more …
- Written in Julia (with Julia interface to QCDNUM)

BAT.jl	https://bat.github.io
Analysis	https://github.com/cescalara/PartonDensity.jl

QCDNUM

QCDNUM and SPLINT

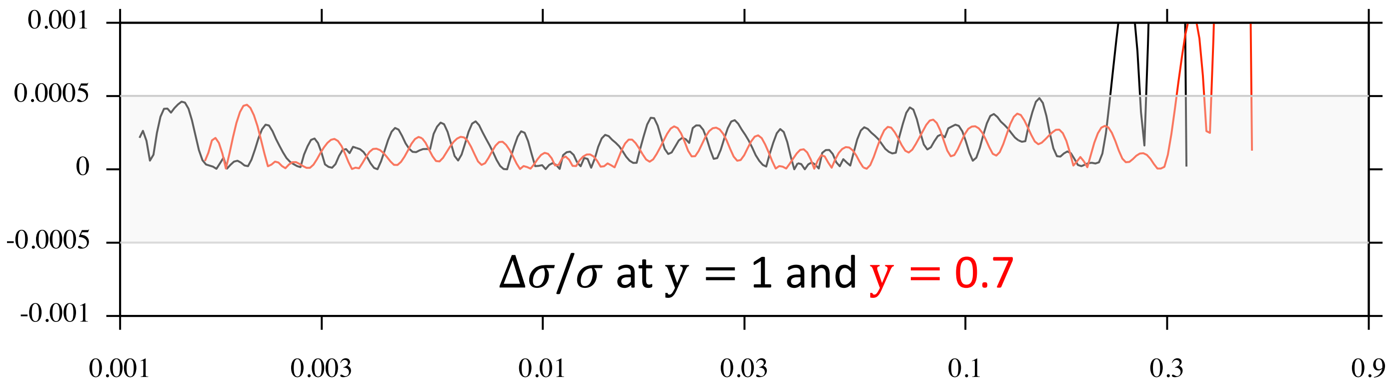
www.nikhef.nl/~h24/qcdnum

- Use QCDNUM for NNLO evolution and structure functions
- SPLINT add-on provides cubic spline interpolation and integration
  - Spline interpolation much faster than computing stfs and xsecs from scratch
  - Needs some tuning of spline-grid to balance speed vs accuracy
  - SPLINT provides fast integration over bins taking kinematic limit into account
  - SPLINT integration is factor 300 faster than 2-dim Gauss integration

Timing (MacBook Pro 2018)

	n_x	n_q	t [ms]
Evolution	100	50	3.6
6 Stf splines	22	7	2.9
Xsec spline	100	25	2.2
Integration	429 bins		0.8

$\Delta\sigma/\sigma \lesssim 5\times10^{-4}$ along the kinematic cut

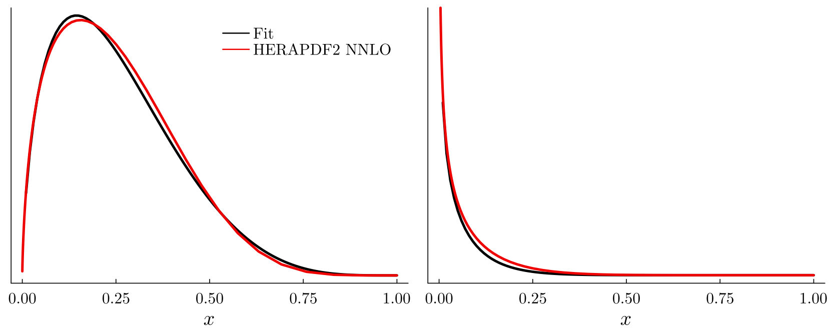


Parameterisation

- Parameterise pdf as beta distribution $xf(x) = A\,x^{\lambda}(1-x)^{K}$
- Integrable for $\lambda > -1$ $\quad \Delta \equiv \int_0^1 xf(x)\mathrm{d}x = A\,\frac{\Gamma(\lambda+1)\Gamma(K+1)}{\Gamma(\lambda+K+2)}$
- Replacement $\lambda \to \lambda - 1$ for valence pdf gives number density
- Integrable for $\lambda > 0$ giving $\quad A_i = N_i^{\mathrm{v}}\,\frac{\Gamma(\lambda_i+K_i+1)}{\Gamma(\lambda_i)\Gamma(K_i+1)} \quad \text{with} \begin{cases} N_d^{\mathrm{v}} = 1 \\ N_u^{\mathrm{v}} = 2 \end{cases}$
- Can fix valence λ for given N and Δ through $\quad \Delta_i = N_i^{\mathrm{v}}\,\frac{\lambda_i}{\lambda_i+K_i+1}$
- Easy to control low-x behaviour
  - Integrable and decreasing towards low x for $\lambda > 0$ (valence)
  - Integrable and increasing towards low x for $-1 < \lambda < 0$ (sea, gluon)

Parameterise 5 flavours at input scale $Q_0^2 = 100$ GeV^2

$$
\begin{aligned}
xd^{\mathrm{v}}(x) &= A_d\, x^{\lambda_d}(1-x)^{K_d} \\
xu^{\mathrm{v}}(x) &= A_u\, x^{\lambda_u}(1-x)^{K_u} \\
x\bar{q}(x) &= A_i\, x^{\lambda_{\bar{q}}}(1-x)^{K_{\bar{q}}} \quad i = \{\bar{d}, \bar{u}, \bar{s}, \bar{c}, \bar{b}\} \\
xg(x) &= A_g^{\mathrm{v}}\, x^{\lambda_g^{\mathrm{v}}}(1-x)^{K_g} + A_g^{\mathrm{s}}\, x^{\lambda_g^{\mathrm{s}}}(1-x)^{K_{\bar{q}}}
\end{aligned}
$$



Parameterise HERAPDF at $Q^2 = 100$ GeV^2 to check flexibility as shown here for u^{V} and $\bar{u}$

- Gluon with a valence and sea component
- Fix λ_d and λ_u via the quark counting rules
- All $x\bar{q}$ have the same shape but different normalisations
- Gluon sea component has same high-x power as the anti-quarks
- Do not fit normalisation constants but momentum fractions Δ which are more meaningful
- Momentum sum constraint $\Delta_u + \Delta_d + 2\sum_{\bar{q}} \Delta_{\bar{q}} + \Delta_g^{\mathrm{v}} + \Delta_g^{\mathrm{s}} = 1$

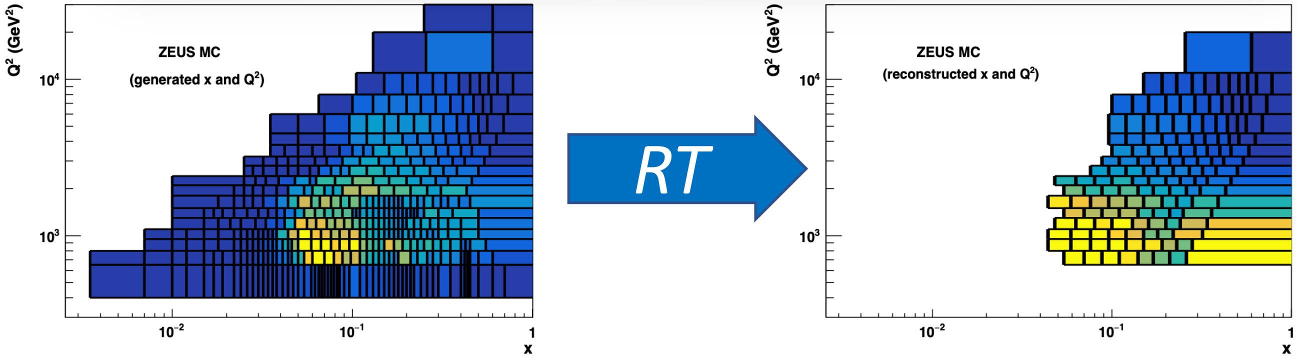
7 shape parameters + 9 momentum fractions with sum rule constraint contribute 15 degrees of freedom to the fit

Event predictions

- Evolve at NNLO with QCDNUM and compute neutral current $e^{\pm}p$ cross-sections
- Integrate over 429 bins and compute vector of event predictions $\vec{\nu}$
- Multiply by matrix RT to correct for radiative and detector effects: $\vec{u} = RT\vec{\nu}$
- This gives the observed event predictions $\vec{u}$ in 153 ZEUS bins for each data set
- For parameters θ the predictions $\vec{u}(\theta)$ give for the likelihood of observing the events $\vec{n}$:

$$\boxed{P(\boldsymbol{n}|\boldsymbol{\theta}) = \prod_{\text{bins}} \frac{u^n e^{-u}}{n!}}$$

ZEUS, Phys. Rev. D **101**, 11209, hep-ex:2003.08732 (2020)



- Added to RT is the weighted sum of 10 systematic matrices
- The 10 weights δ_i are left free parameters in the fit giving a total of 16+10=26 parameters

Priors

M. Betancourt, Proc. AIP Conf., **1443**, 157 (2012)

- Take 9-dimensional Dirichlet distribution for momentum prior
  - $\mathrm{dir}(\vec{\alpha})$ with 9 shape parameters α is multivariate extension of beta distribution
  - Lives on an 8-dimensional manifold in the space $\Delta_i \in [0,1]$ with $\sum \Delta_i = 1$

- Set Dirichlet shape parameters $\vec{\alpha}$ according to asymptotic expectations
  - $\Delta(\mathrm{gluon}) \approx \Delta(\mathrm{quarks}), \quad \Delta(u^{\mathrm{V}}) \approx 2\Delta(d^{\mathrm{V}}), \quad \Delta(s,c,b) \sim \mathrm{small}$

- Pdf shape priors set to truncated Normal or Uniform such that the pdfs are integrable and have the required low-x behaviour

- Priors of systematic δ parameters set to truncated Normal with zero mean and unit width

	Prior	Range
$\boldsymbol{\alpha}$	Dir(20, 10, 20, 20, 5, 2.5, 1.5, 1.5, 0.5)	$[0, 1]$
K_u	Normal(3.5, 0.5)	$[2, 5]$
K_d	Normal(3.5, 0.5)	$[2, 5]$
λ_g^{v}	Uniform	$[0, 1]$
λ_g^{s}	Uniform	$[-1, -0.1]$
K_g	Normal(4, 1.5)	$[2, 7]$
$\lambda_{\bar{q}}$	Uniform	$[-1, -0.1]$
$K_{\bar{q}}$	Normal(4, 1.5)	$[3, 10]$
$\boldsymbol{\delta}$	Normal(0, 1)	$[-5, 5]$

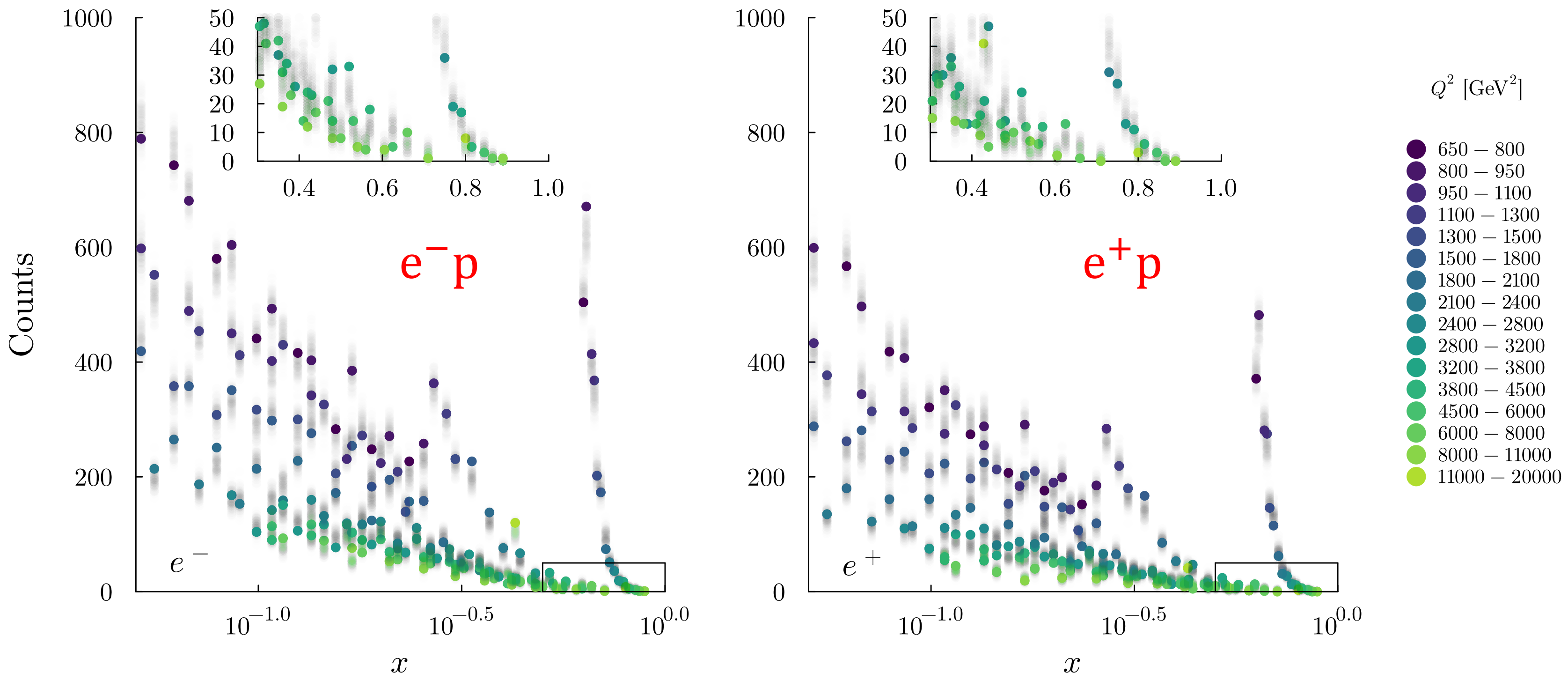
Results

- The real result of the analysis is the 26-dim posterior in parameter space
- Parameter values and errors are defined in two ways as
  - Position of the mode of the posterior in parameter space
  - Mode of the marginal parameter distribution with error corresponding to the smallest credible interval around the mode that contains 68% probability
- Here are the parameters that are reasonably well constrained by the data

	Global mode	Marginal mode		Global mode	Marginal mode
Δ_u	0.219	$0.219^{+0.008}_{-0.010}$	K_u	3.76	$3.78^{+0.14}_{-0.17}$
Δ_d	0.099	$0.088^{+0.028}_{-0.020}$	K_d	3.66	$3.69^{+0.33}_{-0.60}$
$\lambda_{\bar{q}}$	-0.55	$-0.52^{+0.06}_{-0.11}$	$K_{\bar{q}}$	6.01	$6.38^{+1.13}_{-1.40}$
K_g	4.92	$5.22^{+0.91}_{-1.57}$			
$2\Delta_{\bar{u}}$	0.126	$0.104^{+0.022}_{-0.027}$	$2\Delta_{\bar{d}}$	0.031	$0.024^{+0.020}_{-0.017}$
Δ_g^{v}	0.265	$0.239^{+0.043}_{-0.037}$	Δ_g^{s}	0.245	$0.241^{+0.047}_{-0.036}$

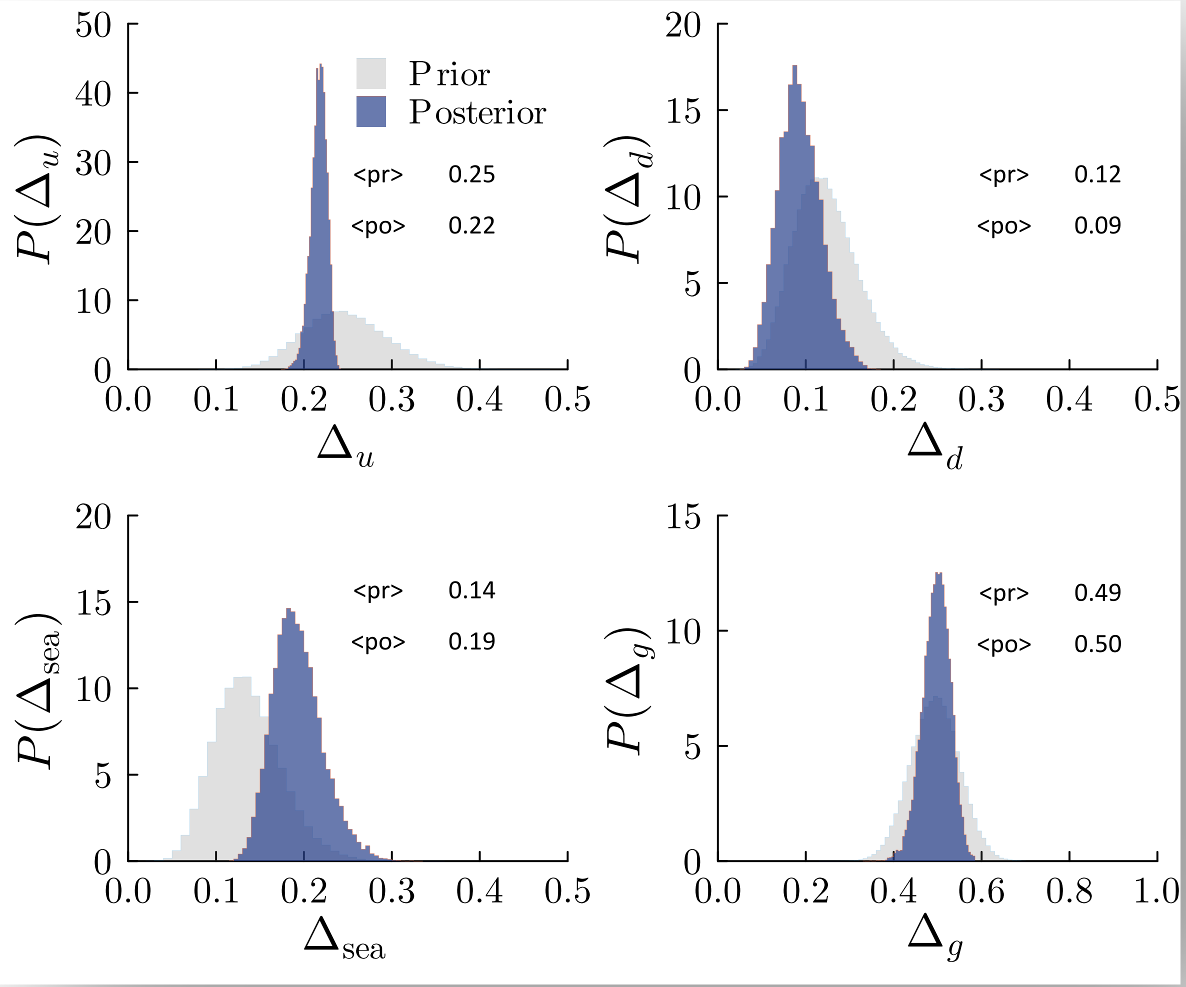
2×10^5 samples ~24h on MacBook Pro

Fitted event counts versus data



- Compute event predictions from posterior mode parameters (bands) and compare to observed event counts (dots), plotted at the bin centers
- Pierson chi-squared gives $\chi^2/\mathrm{pt} = 321/306$ with a p-value of 0.27

Our parameterisation yields an excellent description of the data



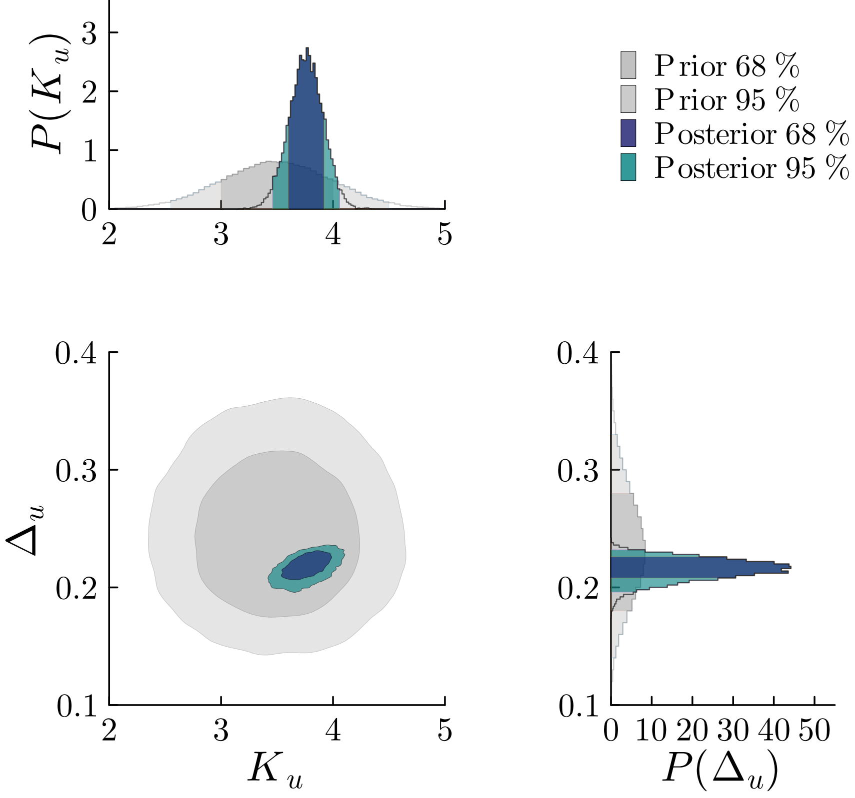
Momentum fraction priors and posteriors

- The data very much constrain the momentum carried by the up-valence
- Weaker constraints on the down-valence, sea and gluon
- From global mode we find

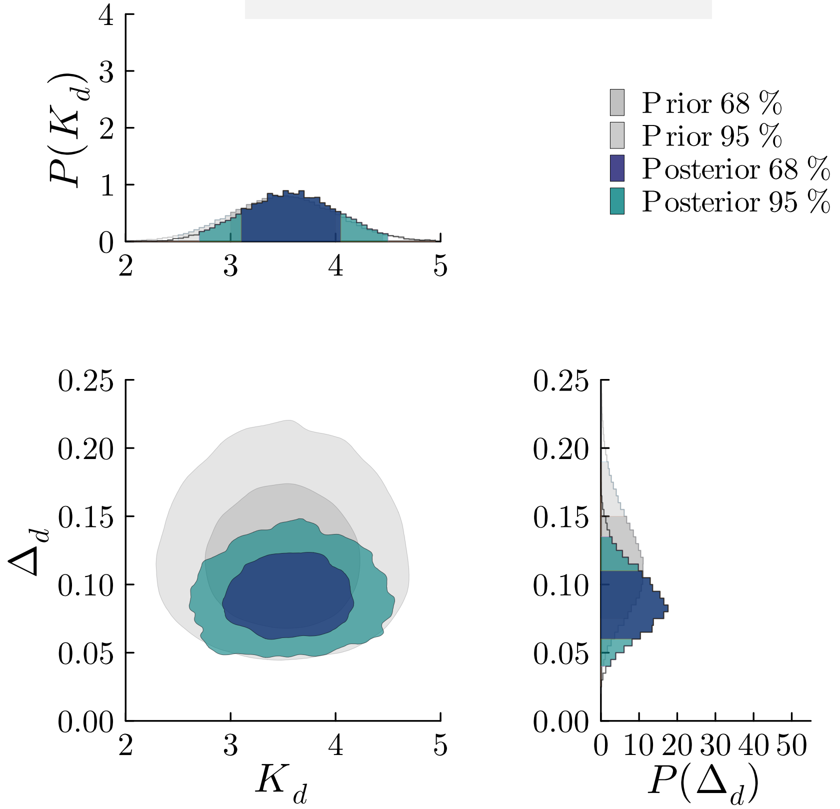
u_v	d_v	sea	gluon
0.22	0.10	0.17	0.51

Momentum Δ versus 1− x power K

up-valence

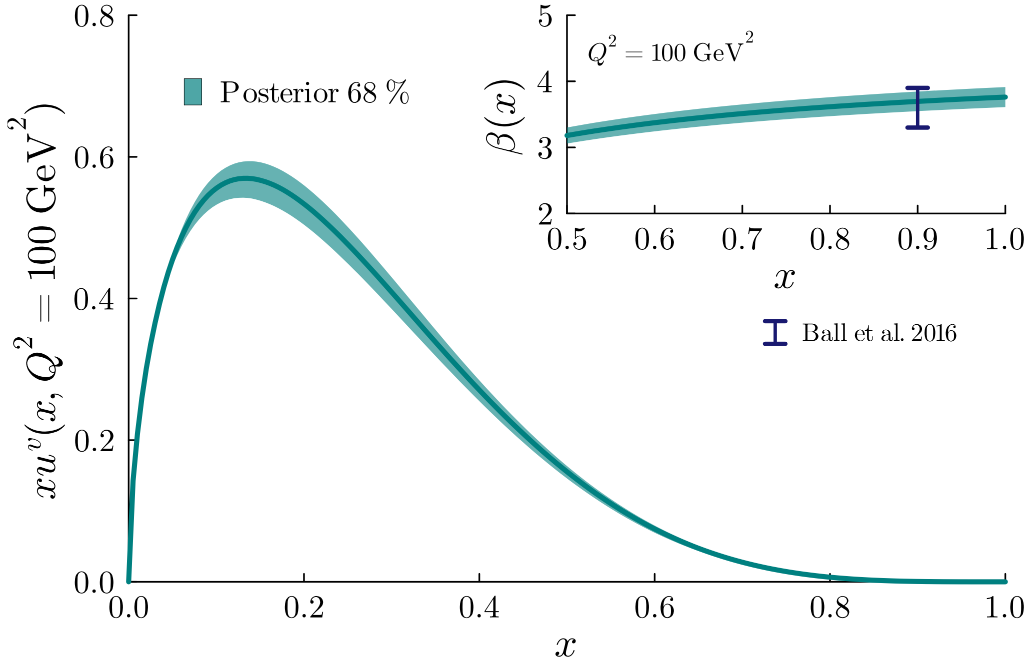


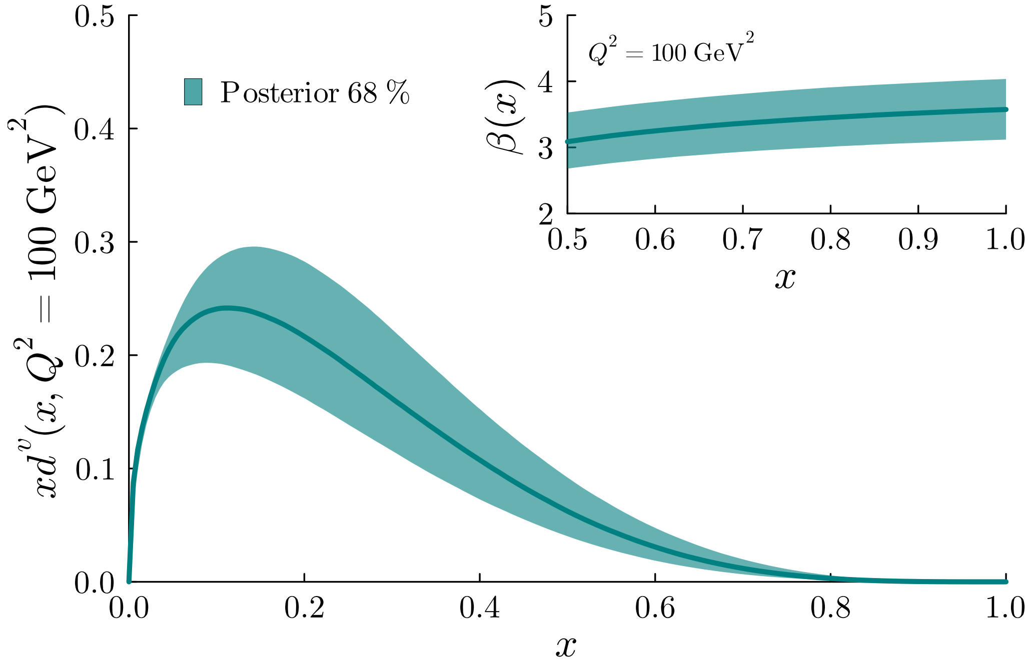
down-valence



Very strong constraint on the up-valence parameters

Up and down valence distributions



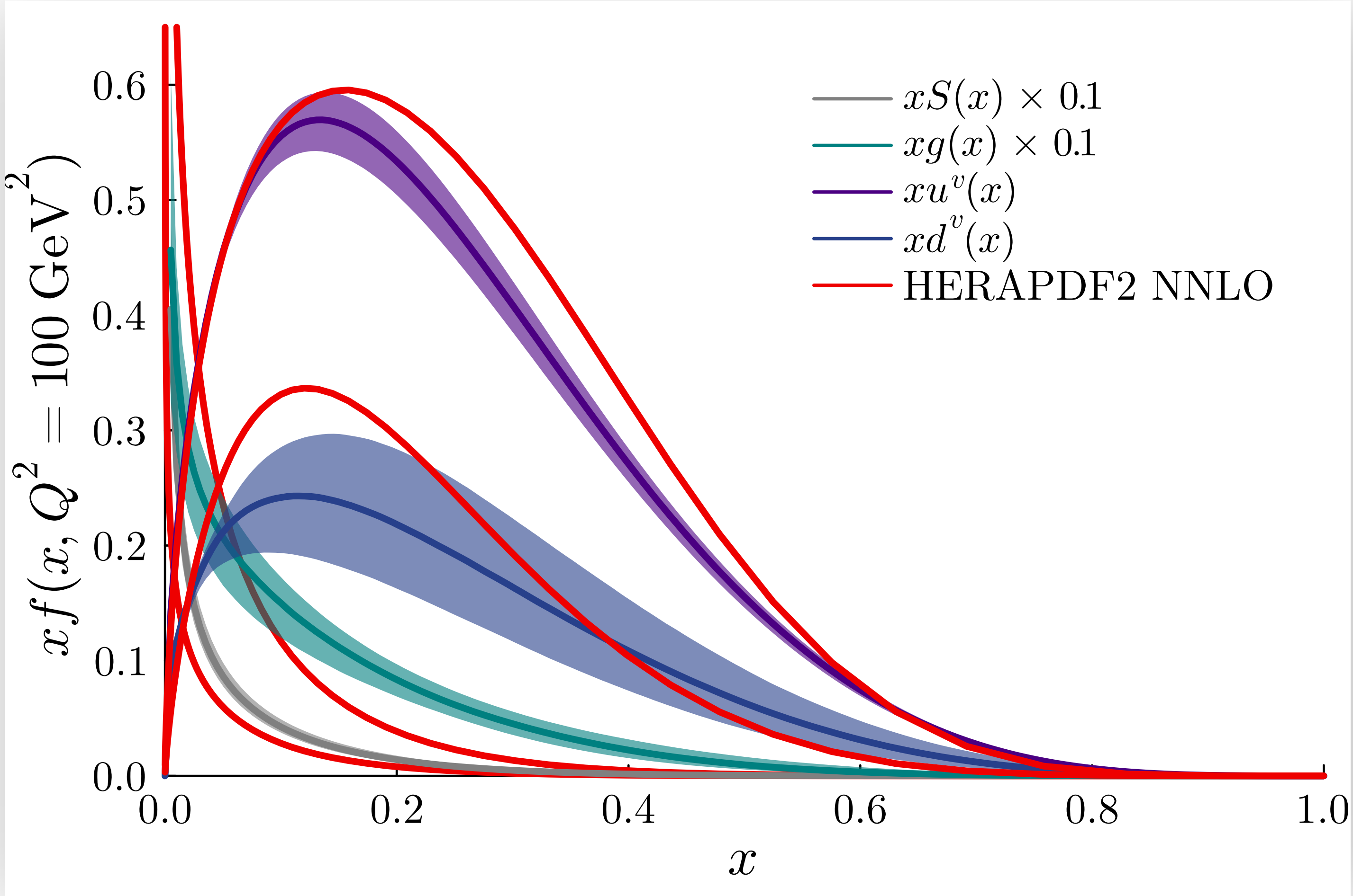


- The insets show the effective 1-x power $\beta(x) = \mathrm{d}\ln xf/\mathrm{d}\ln(1-x)$
- The β slope of u_v agrees well with a recent summary from Ball *et al.*

R. Ball, E. Nocera and J. Rojo, Eur. Phys. J. C **76** (2016) , 383 , hep-ph:1604.00024

Compare to HERAPDF

H. Abramowicz *et al.*, Eur. Phys. J. C **75**, (2015) 12, 580



- Analytic parameterisations strongly couple regions of small and large x
- HERAPDF parameterisation is similar to ours but fitted at much lower x (not using the ZEUS high-x data)
- This may explain at least part of the observed differences

Summary

- The Bayesian analysis of ZEUS high-x data shows that these data carry a lot of information on the up-valence distribution
- Given our parameterisation we obtain accurate results on the momentum Δ carried by the up-valence quark and its $(1-x)$ power K (marginal mode and 68% credible interval)

$$\Delta = 0.22^{+0.01}_{-0.01} \qquad K = 3.8^{+0.1}_{-0.2}$$

What is next ...

- Paper with detailed description of our analysis is in preparation
- Extend the analysis to investigate parameterisations with Bayesian model-selection techniques
- Exploit in the extended analysis the many opportunities for parallel computing

Bayesian pdf fitting is a viable, challenging, and highly interesting undertaking!