# Beam lifetimes due to radiative Bhabha scattering

**Krzysztof PIOTRZKOWSKI**

**AGH University of Science & Technology**

$E_e$  $E_\gamma$  $E'_e$  $Q^2$  $E_p$  $E'_p$
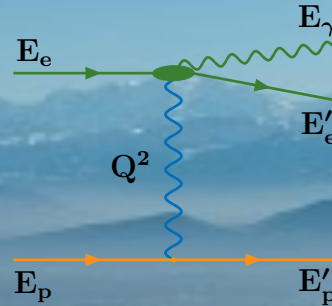
FCC Week, London
June 6, 2023

# $e^+e^-$ bremsstrahlung *aka* radiative Bhabha scattering at high energies



**electron-positron** bremsstrahlung $e^- + e^+ \rightarrow e^- + \gamma + e^+$ has following signatures:
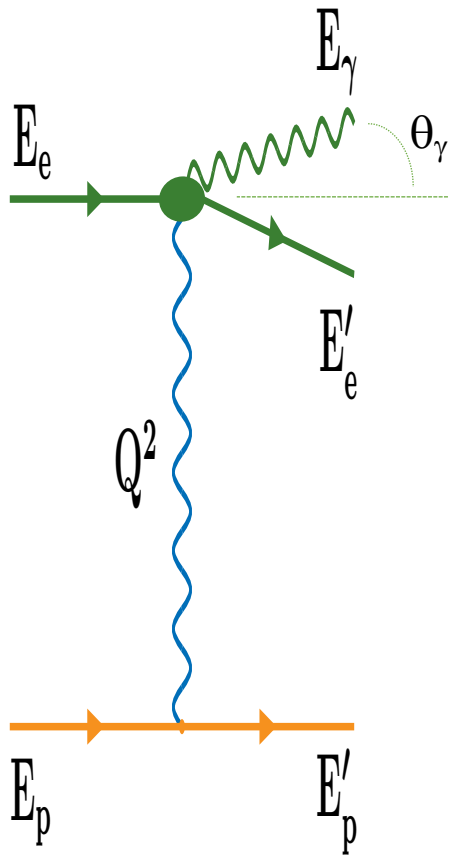
$E'_e + E_\gamma = E_e$ to a very (very) high accuracy, and it is a truly "zero-angle process"

$\Rightarrow$ typ. polar angles for photons/scattered electrons, $\theta_\gamma \approx \theta_e \approx m_e / E_e$

**It is** kinematically allowed that $\theta_\gamma = \theta_{e'} = \theta_p = 0$ − hence there is no transverse momentum transfer, which results in (for variables in LAB):

$$|q_{min}| = m_e^3 \, E_\gamma / (4 \, E_p \, E_e \, E'_e), \text{ where}$$

$$Q^2 = -q^2 \approx -q^2_{min} + q_T^2$$

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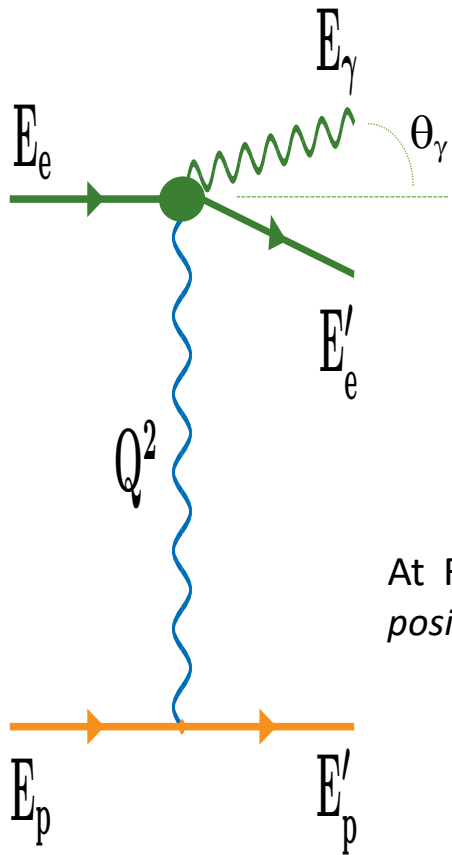
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At FCC-ee, for $E_e = E_p = 182.5$ GeV and $E_\gamma = 2$ GeV, minimal momentum transfer, *in positron rest-frame*, $\Delta p_z = |q_{min}|/c \simeq 10^{-8}$ eV/c (Corresponding energy transfer $= (\Delta p)^2/2m_e \approx 10^{-22}$ eV)

It corresponds to longitudinal *coherence* length $l_c = \hbar/\Delta p_z \approx$ **20 m !**

**Higher** beam energies/lower photon energy $\Rightarrow$ **more** extreme it becomes.

# Bremsstrahlung and *Beam-Size Effects*

$$d^3\sigma/dE_\gamma \, d\theta_e d\theta_\gamma \propto Q^{-4}$$

hence cross-section integrated over angles, that is bremsstrahlung spectrum, is dominated by large distance contributions

$p_T = 0 \to$ infinite impact parameter

$p_{T,typ} \approx |q_{min}|/c \to$ original **Beam-Size Effect** – *effective* bremsstrahlung *suppression* at colliders, at low $E_\gamma$ , due to finite beam-sizes

Discovered at VEPP-4 [*Phys. Lett.* **B113** (1982) 423], measured also at HERA I [*Z. Phys.* **C67** (1995) 577] , will be deeply studied at the EIC [Phys. Rev. **D103** (2021), no. 5 L051901]

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It is directly related to the ("text-book") **definition** of cross-section:

## Event rate = Luminosity × σ

where colliding particles are represented by **plane** waves. But this *assumption* is **invalid** if lateral beam sizes are **comparable** to relevant impact parameter of a process.

**Wave-packet formalism** must be used.
*Int. J. Mod. Phys.* **A7** (1992) 4707

# Beam-size effects at the FCC-ee, FCC-eh and LHeC

https://arxiv.org/abs/2305.12033

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ABSTRACT: Bremsstrahlung spectra will be strongly distorted due to small lateral beam sizes at future colliders. That in turn will have large consequences for the electron and positron beam lifetimes as well as for the luminosity measurements in case of electron-hadron colliders. We discuss in detail such consequences for the Future Circular Collider and Large Hadron electron Collider cases.
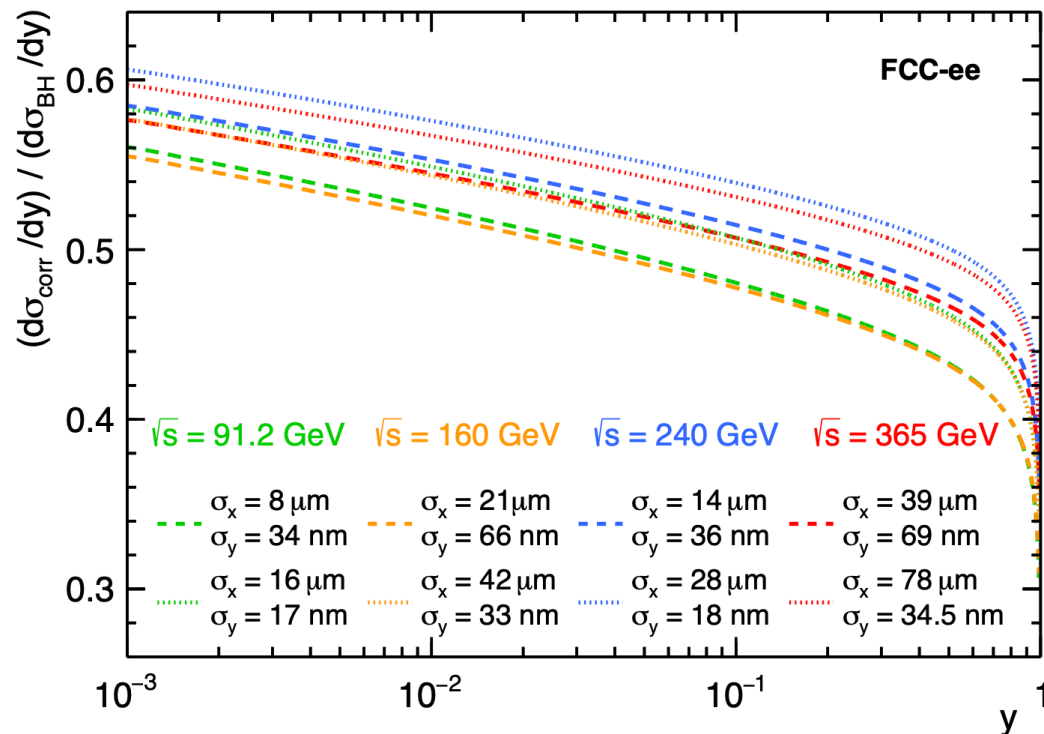
# Bremsstrahlung spectra @ FCC-ee
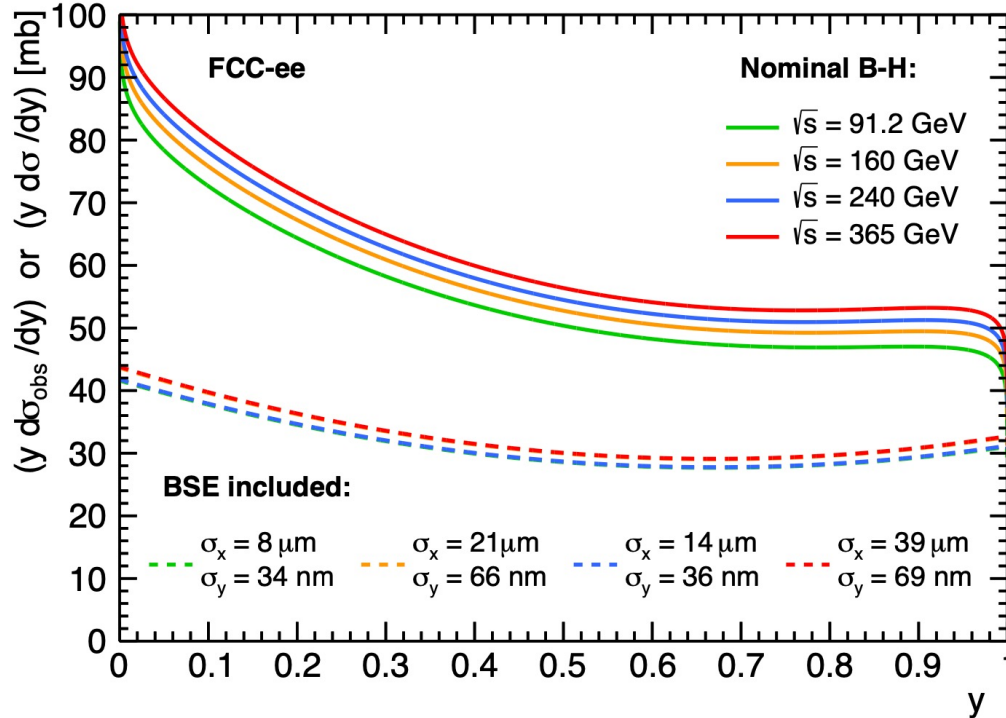


Strong distortions of whole bremsstrahlung spectra, where $y = E_\gamma/E_e$

arXiv:2305.12033

**Figure 2**. Bremsstrahlung spectra at the $\sqrt{s} = 91.2$ GeV, 160 GeV, 240 GeV and 365 GeV FCC-ee – solid lines $y\,\mathrm{d}\sigma/\mathrm{d}y$ are for the Bethe-Heitler nominal case and dashed ones $y\,\mathrm{d}\sigma_{\mathrm{obs}}/\mathrm{d}y$ when the BSE is included. Note that the spectra with the BSE included overlap due to similar $\sigma_y$: those at $\sqrt{s} = 160$ GeV and 365 GeV, as well as those at $\sqrt{s} = 91.2$ GeV and 240 GeV.

# Bremsstrahlung spectra @ FCC-ee



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**Figure 3**. Relative size of bremsstrahlung suppression at the $\sqrt{s} = 91.2$, 160, 240 and 365 GeV FCC-ee. Dashed lines represent the BSE for the nominal beam sizes and the dotted ones correspond to the BSE case with $\sigma_x \times 2$, $\sigma_y/2$.

# Beam lifetime @ FCC-*ee*

| $E_{\text{beam}}$ [GeV] | 45.6 | 80 | 120 | 182.5 |
|---|---|---|---|---|
| $N_b$ [$10^{13}$] | 239 | 25 | 5.3 | 0.93 |
| $L_{\text{tot}}$ [$10^{35}$ cm$^{-2}$s$^{-1}$] | 74 | 8.3 | 2.8 | 0.48 |
| $\sigma_a$ [mb] for $a = 0.01(0.02)$ | 166 (137) | 174 (144) | 167 (138) | 175 (145) |
| $\sigma_{\text{BH}}$ [mb] for $a = 0.01(0.02)$ | 319 (260) | 333 (271) | 343 (280) | 353 (288) |
| $\tau_b$ [min] for $a = 0.01(0.02)$ | 32 (39) | 29 (35) | 19 (23) | 18 (22) |

**Table 2**. FCC-ee beam lifetimes due to bremsstrahlung.

Assuming energy **aperture $a$ of 1–2 %** – electron and positron lifetimes (due to $e^+e^-$ bremsstrahlung) **improve** by about **factor of 2**.

Only decreasing of vertical beam size can (slightly) increase beam lifetimes.

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Reminder:

$$\tau = N_{\text{b}} / L \sigma_{\text{a}}$$

where $N_b$ is electron population and $L$ is luminosity and $\sigma_{\text{a}}$ is cross-section for events outside energy aperture.
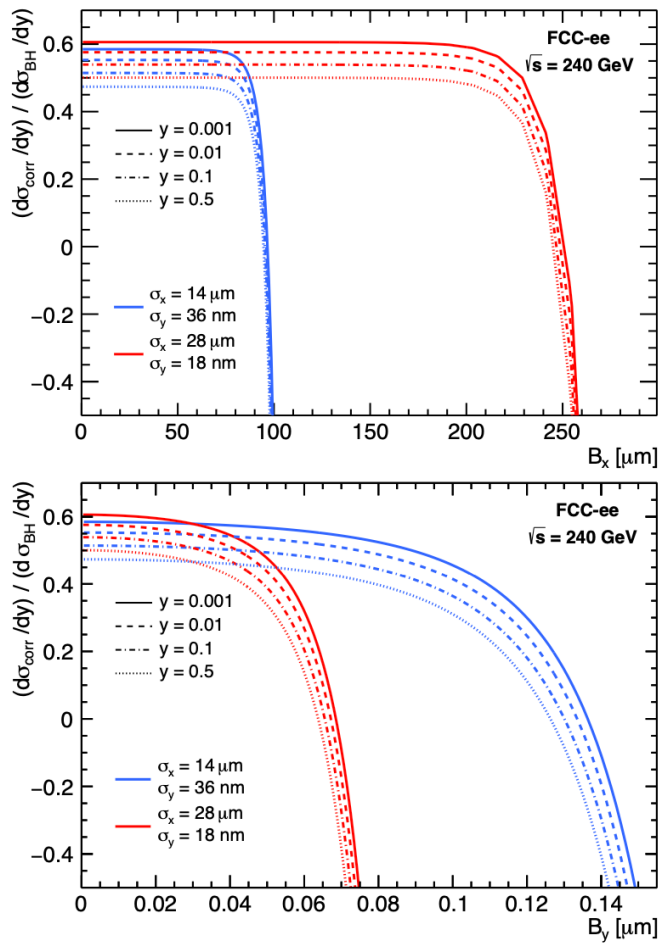
**Figure 4.** Variations of the bremsstrahlung cross-sections at the $\sqrt{s} = 240$ GeV FCC-ee for the horizontal (upper plots) and vertical (bottom plots) beam displacements or offsets. The blue lines correspond to the nominal beam sizes and the red ones to the case with a 4 times bigger beam aspect ratio. Results at different values of $y = E_\gamma/E_e$ are shown with different line styles.

# Beam offsets @ FCC-*ee*

Horizontal beam offsets do not change bremsstrahlung suppression unless offsets much exceed horizontal beam-size ⇒ no effect due to horizontal beam crossing angle

Vertical beam offsets change very slightly bremsstrahlung suppression unless the offsets much exceed vertical beam-size.

For large offsets **new effect** predicted ⇒ bremsstrahlung **amplification** [Phys. Rev. **D103** (2021), no. 5 L051901]

Reminder: $\sigma_{obs} = \sigma_{BH} - \sigma_{corr}$

**arXiv:2305.12033**

# Bremsstrahlung ↔ Coherent Bremsstrahlung ↔ Beamstrahlung

Coherence effects turn on when **coherence length $l_c$ > bunch length $\sigma_z$** but properties of *coherent radiation* are different for electron average deflection angles $\theta_d$ (in magnetic field of positron bunch) much larger or smaller than radiation angle $\theta_r \approx m_e/E_e$ – as measured by their ratio:

$$\eta = r_e\, N_p\, /\, \sigma_x \approx \langle \theta_d / \theta_r \rangle$$ where $r_e$ is classical electron radius, $N_p$ is number of positrons and $\sigma_x$ is bunch horizontal size

If $\eta \gtrsim 10$ then corresponding radiation is called *beamstrahlung* and if $\eta \lesssim 1$ that is CBS case – nota bene: *synchrotron radiation* is special case of beamstrahlung in uniform (external) field.

# Bremsstrahlung ⟷ Coherent Bremsstrahlung ⟷ Beamstrahlung

↳ Synchrotron radiation aka *magnetobremsstrahlung*

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At FCC-ee $\eta \simeq$ **53/19/23/11** for √s = 91/160/240/365 GeV, respectively. That is not very far from CBS regime, at 365 GeV in particular. Therefore, for significantly **larger** horizontal beam sizes or/and **smaller** bunch populations CBS takes place, resulting in different spectra shapes characterized by different critical energies:

$$E_{cr} \, (\text{CBS}) \simeq 4\gamma_e^2/\sigma_z \quad \text{vs.} \quad E_{cr} \, (\text{BS}) \approx 1.5\,\eta\,\gamma_e^2/\sigma_z$$

where $\gamma_e = E_e \, / m_e \approx 1/\langle \theta_r \rangle$

Radiated energy scales with $\eta^2$ for both CBS and BS.

# Summary

- *Beam-Size Effects* (BSE) in high energy bremsstrahlung – an apparent suppression (or amplification) of event rates – are due to small **lateral** sizes of **both** colliding beams.

- These are **not** coherent effects, in particular, not particle density effects.

- BSE **improve** FCC-*ee* beam lifetime limits, due to $e^+e^-$ bremsstrahlung, by **factor of 2**.

- Beam lateral offsets smaller than corresponding beam-sizes are not changing significantly bremsstrahlung suppression at FCC-*ee*.

- Impact of large horizontal beam crossing angle at the FCC-*ee* is expected to be also not significant.

- At low bunch intensities and/or for large horizontal beam-sizes **Coherent Bremsstrahlung** will occur at FCC-*ee* in place of Beamstrahlung.

- In-depth studies of BSE, and of Coherent Bremsstrahlung, are planned at the EIC.

# Thank you!

$$\frac{d\sigma_{\text{corr}}}{dy} = 2 \frac{\alpha^3}{m_e^2} \frac{1}{y} \int_0^\infty F(y,z) \, G(\omega) \, \frac{dz}{(1+z)^2} \qquad (6.7^\star)$$

which is twice as big as the correction in Eq. (6.7) in Ref. [11] due to summing over photon helicities, and

where
$$\omega = \frac{m_e^2 y(1+z)}{4E_e(1-y)} \quad \text{and} \quad \rho_m = E_p/(m_p\omega)$$

$$F(y,z) = 2 - y - 4\frac{(1-y)z}{(1+z)^2} - y(1-y) \qquad (6.3)$$

$$G(\omega) = 2\int_0^\infty \frac{\rho_\perp}{\rho_m} K_1^2\left(\frac{\rho_\perp}{\rho_m}\right) \left[1 - \frac{e^{-v_+}}{\pi} \int_0^\pi e^{v_- \cos\varphi} \cosh\left(t_x \cos\left(\frac{\varphi}{2}\right)\right) d\varphi\right] \frac{d\rho_\perp}{\rho_m} \qquad (1)$$

where $t_x = \rho_\perp B/a_x^2$ and $v_\pm = \rho_\perp^2(1 \pm a_y^2/a_x^2)/(4a_y^2)$, where $a_x^2 = \sigma_{x1}^2 + \sigma_{x2}^2$ and $a_y^2 = \sigma_{y1}^2 + \sigma_{y2}^2$ and $\sigma_{x1}$, $\sigma_{y1}$ and $\sigma_{x2}$, $\sigma_{y2}$ are, respectively, the horizontal and vertical beam sizes at the interaction point (that is, at $z = 0$) for two colliding beams; $K_1$ is the modified Bessel function of the second (third) kind.

The plots in Fig. 1 (left) below are effectively obtained by making cross-sections of the two-dimensional distributions shown in Fig. 5 at fixed $B$ - shapes of the obtained curves are driven by the beam displacement effect at high

$y$, and by the beam-size effect at low $y$. In Fig. 1 (right) the corresponding plots are shown for the horizontal displacement $B$ at the EIC, according to Eq. (1) above. Finally, for completeness, in Figs. 2 and 3 we show modifications of the photon angular distributions due to the vertical beam displacements and lateral beam-sizes.

All plots have been produced using the ROOT analysis framework [20]. Numerical integrations were performed using GNU Scientific Library [21] interfaced to ROOT. The library reimplements the algorithms used in QUADPACK which are described in Ref. [22].

# Predicted coherent bremsstrahlung at HERA

At HERA I, for $E_\gamma = 10$ keV, $\hbar/\Delta p_z \approx$ **11 cm** at LAB $\Rightarrow$ beam electron interacts with **whole** proton bunch and bremsstrahlung event rate becomes proportional to **number of protons squared**! Hence extraordinary signal **amplification**.

## The equivalent photon approximation for coherent processes at colliders

R. Engel, A. Schiller & V. G. Serbo

*Zeitschrift für Physik C Particles and Fields* **71**, Article number: 651 (1996) | Cite this article

**78** Accesses | Metrics

### Abstract

We consider coherent electromagnetic processes for colliders with short bunches, in particular the coherent bremsstrahlung (CBS). CBS is the radiation of one bunch particles in the collective field of the oncoming bunch. It can be a potential tool for optimizing collisions and for measuring beam parameters. A new simple and transparent method to calculate CBS is presented based on the equivalent photon approximation for this collective field. The results

*Critical energy* = $4\gamma_e^2$/p-bunch-length

$E_\gamma \dfrac{d\sigma_{\text{eff}}}{dE_\gamma}$ (barn)

$E_\gamma$ (GeV)
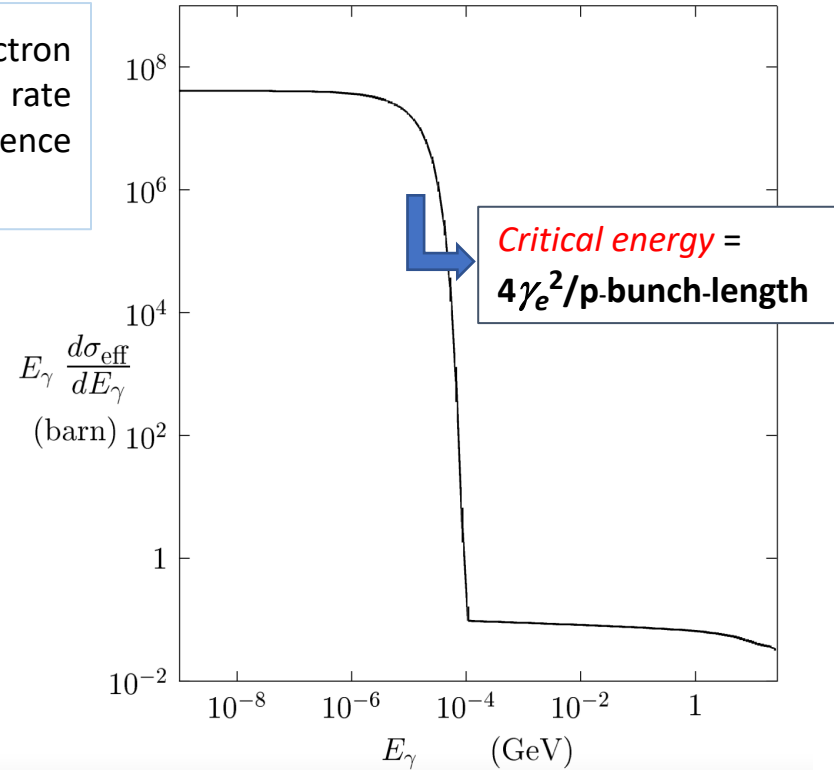
It has **same** origin as *beamstrahlung,* yet has never been confirmed experimentally...