QCD & Event Generators for FCC-ee

P. Monni (CERN)

FCC Week 2023 - 6 June 2023
Role of precision QCD at FCC-ee

Monte Carlo generators

jet physics (algorithms, flavour, S/B, …)

Precision calcns (pert. & non-pert.)

Reaching the foreseen precision poses outstanding challenges on theory calculations. Evolution in many areas is required to meet the goals.

* See also Janus Gluza’s talk for EW
Role of precision QCD at FCC-ee

This talk addresses mainly QCD aspects*. EW corrections will be discussed in detail in the EW sessions.

* See Janus Gluza’s talk for EW calculations
Outline of the talk: please visit indico pages for more info

Precision calculations for future e+e− colliders: targets and tools

7–17 Jun 2022
CERN
Europe/Zurich timezone

https://indico.cern.ch/event/1140580/

The main goal is to identify clear theoretical and computational targets for high-precision predictions of relevance to the programme of future e+e− colliders.

Parton Showers for future e+e- colliders

24–28 Apr 2023
CERN
Europe/Zurich timezone

https://indico.cern.ch/event/1233329/

The unprecedented experimental performance expected by the next generation of lepton colliders poses an outstanding challenge for theoretical computations that must be pushed far beyond the current state of the art to guarantee an optimal exploitation of the data. Among the theoretical aspects of this programme, Monte Carlo event generators play a special role due to their versatility in bridging theoretical predictions and experimental measurements. The precision reached by current event generation algorithms is dramatically insufficient for this task, thus demanding a dedicated effort to improve their formal accuracy and achieve a higher precision in event simulations.
Perturbative calculations
Physics at the Z pole

- Theory crucial in 3 ways: measurement/calibration (e.g. QED ISR); interpretation of results (EWPO); parametric uncertainties (i.e. couplings, masses)

- QCD uncertainties concern all three categories

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**Numbers are given here for FCC-ee (best prospects)**

<table>
<thead>
<tr>
<th>Observables</th>
<th>Present value</th>
<th>FCC-ee stat.</th>
<th>FCC-ee current syst.</th>
<th>FCC-ee ultimate syst.</th>
<th>Theory input (not exhaustive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ (keV)</td>
<td>911.87500 ± 2100</td>
<td>4</td>
<td>100</td>
<td>10 ?</td>
<td>Lineshape QED unfolding</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Relation to measured quantities</td>
</tr>
<tr>
<td>$\Gamma_Z$ (keV)</td>
<td>249.5500 ± 2300 [*]</td>
<td>4</td>
<td>25</td>
<td>5 ?</td>
<td>Lineshape QED unfolding</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Relation to measured quantities</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}$ (pb)</td>
<td>414.80 ± 32.5 [*]</td>
<td>0.04</td>
<td>4</td>
<td>0.8</td>
<td>Bhabha cross section to 0.01%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$e^+e^- \rightarrow \gamma\gamma$ cross section to 0.002%</td>
</tr>
<tr>
<td>$N_\nu (\times10^3)$ from $\sigma_{\text{had}}$</td>
<td>2996.3 ± 7.4</td>
<td>0.007</td>
<td>1</td>
<td>0.2</td>
<td>Lineshape QED unfolding</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(\Gamma_{\nu\nu},\Gamma_{ee})_{\text{SM}}$</td>
</tr>
<tr>
<td>$R_\ell (\times10^3)$</td>
<td>2076.6 ± 24.7</td>
<td>0.04</td>
<td>1</td>
<td>0.2 ?</td>
<td>Lepton angular distribution</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(QED ISR/FSR/IFI, EW corrections)</td>
</tr>
<tr>
<td>$\alpha_s(m_Z) (\times10^4)$ from $R_\ell$</td>
<td>1196 ± 30</td>
<td>0.1</td>
<td>1.5</td>
<td>0.4 ?</td>
<td>Higher order QCD corrections for $\Gamma_{\text{had}}$</td>
</tr>
</tbody>
</table>
| $R_0 (\times10^6)$             | 216290 ± 660 | 0.3          | ?                    | < 60 ?                | QCD (gluon radiation, gluon splitting, fragmentation, decays, ...)

[P. Janot’s talk @ CERN FC workshop 2022]
Main computational challenges from EW aspects:

- EWPO $Z \rightarrow q\bar{q} + X$ @ 3 loops EW and beyond
- Beam calibration [$e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \gamma\gamma$ @ NNLO EW - still beyond reach]

But high potential for precision QCD studies at the Z pole and above:

- Strong coupling constant
- Jet dynamics and substructure: spin correlations, fragmentation & track functions, multi-jet observables (global/non-global)
- Non-perturbative effects & modelling
- Heavy quarks (Q) studies (e.g. asymmetries, fragmentation) & jet tagging (e.g. $q/Q$ vs. $g$ jets)
- $\tau$ decays ($\alpha_s$)
- Calibration/tuning of ML & MC models (instrumental for higher-energy runs)
Precision physics in $Z/\gamma^* \rightarrow$ jets

- Significant room for improvement for QCD calculations, e.g.

- Heavy quarks: $R_b$, $A_{FB}$ requires QQg and qg($\rightarrow$ QQ) @ 2 loops with $m_b$ dependence (NLO known)

- Fragmentation functions

- Multi-jet final states
  - 3 jets @ $N^3$LO QCD
  - 4 & 5 jets at NNLO QCD

Some of this is within the reach of technology developed at LHC (e.g. $Z/\gamma^*+2$ jets @ 2 loops, subtraction methods)

[e. g. five-point amplitudes in Abreu et al. '18-'23; Badger et al '19-'22; Chawdhry et al. '20-'21]

- Promising new directions for loop calculations: e.g. numerical approaches for total rates at $N^{(2/3)}$LO (e.g. Feynman parameters, local unitarity, AMFlow, diffExp), though further progress needed for distributions
All-order logarithmic corrections (resummations) desirable for phenomenology. A lot of new techniques refined in recent years for jet observables (SCET(s), numerical methods, generating functionals, ...)

Precision physics in $Z/\gamma^* \rightarrow$ jets

- Room for improvement in high-multiplicity ($\geq 3$ jets) observables, possibly requires algorithmic methods
Non-perturbative QCD corrections

- Better understanding of hadronisation in jet observables appears to be essential (event shapes, jet rates, jet substructure): serious limitation of TH accuracy. Possible avenues (possibly in combination):
  - Techniques to calculate leading corrections as $1/Q$ expansion, recently first important steps for 3-jet configurations (largely based on large-$n_F$ approximation)
  - New observables with reduced NP sensitivity, e.g. through jet grooming. Preliminary studies on strong coupling extractions
  - Tuning of MC generators across $\sqrt{s}$ values (Q/q/g samples). High perturbative accuracy demanded, a lot of recent progress

\[ \frac{d\sigma}{d\phi}(\phi) \approx \frac{d\sigma^{\text{pert}}}{d\phi} \left( \phi - \zeta(\phi) \Delta_{\text{NP}} \frac{\Lambda}{Q} \right) \]

SD thrust @ NLL+NLO

[Luisoni, PM, Salam '20]
[Caola, Ferrario Ravasio, Limatola, Melnikov, Nason '21+ '22]
[Nason, Zanderighi '23]
Experimental precision approaching 0.1% in many cases at ZH threshold

Example: total cross section will be measured with precision in the range 0.2%-0.5%. Necessary ingredients:

- $e^+e^- \rightarrow ZH$ (now available), $H \nu \nu (e^+e^-)$ @ 2 loops EW (hard at the moment)
- Mixed QCD$\otimes$EW @ 2 loops under control
- Wealth of data in hadronic decays of the Higgs boson (demanding also excellent jet tagging performance*)

<table>
<thead>
<tr>
<th>Decay</th>
<th>current unc. $\delta \Gamma$ [%]</th>
<th>future unc. $\delta \Gamma$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \rightarrow b\bar{b}$</td>
<td>$&lt; 0.4$ 1.4 0.4 -</td>
<td>$0.2$ 0.6 $&lt; 0.1$ -</td>
</tr>
<tr>
<td>$H \rightarrow \tau^+\tau^-$</td>
<td>$&lt; 0.3$ - - -</td>
<td>$&lt; 0.1$ - - -</td>
</tr>
<tr>
<td>$H \rightarrow c\bar{c}$</td>
<td>$&lt; 0.4$ 4.0 0.4 -</td>
<td>$0.2$ 1.0 $&lt; 0.1$ -</td>
</tr>
<tr>
<td>$H \rightarrow \mu^+\mu^-$</td>
<td>$&lt; 0.3$ - - -</td>
<td>$&lt; 0.1$ - - -</td>
</tr>
<tr>
<td>$H \rightarrow W^+W^-$</td>
<td>0.5 - - 2.6</td>
<td>0.3 - - 0.1</td>
</tr>
<tr>
<td>$H \rightarrow gg$</td>
<td>3.2 &lt; 0.2 3.7 -</td>
<td>1.0 - 0.5 -</td>
</tr>
<tr>
<td>$H \rightarrow ZZ$</td>
<td>0.5 - - 3.0</td>
<td>0.3 - - 0.1</td>
</tr>
<tr>
<td>$H \rightarrow \gamma\gamma$</td>
<td>$&lt; 1.0$ &lt; 0.2 - -</td>
<td>$&lt; 1.0$ - - -</td>
</tr>
<tr>
<td>$H \rightarrow Z\gamma$</td>
<td>5.0 - - 2.1</td>
<td>1.0 - - 0.1</td>
</tr>
</tbody>
</table>

Projected reduction of intrinsic TH uncertainties in line with what can be achieved with future calculations (total rates); improvement needed in parametric unc.

* See e.g. L. Gouskos’ talk

[Chen, Guan, He, Liu, Ma ’22; Freitas, Song ’21–’22]

[Credit: J. de Blas]
Hadronic Higgs decays

- Accuracy significantly lower for differential distributions (e.g. potential sensitivity to light-quarks Yukawa)
- NNLO (+resummations) achievable in the coming years (already available in $H \to bb$ and partly $H \to gg$); sufficient for several-% precision (3loops needed for few-% level)

* All ingredients for HO in $H \to gg$ known (also with full mass dependence)

[Czakon et al. ’20; Bonciani et al. ’22 Melnikov, Penin ’16; Liu, Penin ’17–’19; Anastasiou, Penin ’20; Chen, Jakubcik, Marcoli, Stagnitto ’23]

[Ju, Zu, Yang, Zhou ’23]
• Accuracy significantly lower for differential distributions (e.g. potential sensitivity to light-quarks Yukawa)

• However, hadronisation remains the main bottleneck
  
  • e.g. thrust in Higgs decays (MC variation in plot)

• Increase in energy insufficient for suppression ($Q \sim m_H$)

• Runs at lower energies are essential for a robust tuning of NP models in MCs

• Also crucial for training of ML algorithms for jet tagging, instrumental in extraction of Higgs couplings
TH cross section currently known accurately at NLO (EW) + NNLO (unstable particles EFT) sufficient for $\delta m_W \sim 5$-6 MeV

Can be further improved using NLL ISR

Effect of tight selection cuts in the EFT to be understood

Can be further improved using NLL ISR

Effect of tight selection cuts in the EFT to be understood
W mass extraction from hadronic and semi-leptonic decays

- Very good experimental resolution with momentum conservation fit (4C or 5C), competitive with threshold scan

- Theory modelling harder, with systematics yet to be precisely assessed
  - Control over QED ISR (NLL available)
  - EFT resonant aspects near threshold
  - Backgrounds: 2f & 4f final states
  - Colour reconnection in hadronic channels

\[
B_h^2 = 45.4\%
\]

\[
6B_\ell B_h = 43.9\%
\]

[15]

G. Wilson’s talk @ CERN FC workshop 2022
Top physics

- Huge potential from threshold scan: up to per-mille accuracy on cross section & asymmetries
- Access to top mass and width, as well as strong coupling and top Yukawa coupling
- e.g. projected exp. target for top mass $\delta m_t \sim 20$ MeV

Great challenge for theory to match this precision; intrinsic (e.g. higher order) & parametric (e.g. strong coupling from Z pole) uncertainties
Top physics: theory for threshold scan

- PNRQCD predictions known to N^3LO (also including EW+non-resonant effects @ NNLO)

\[ R \sim v \sum_k \left( \frac{\alpha_s}{v} \right)^k \cdot \left\{ \begin{array}{c} 1 \text{ (LO)} \; ; \; \alpha_s, v \text{ (NLO)}; \; \alpha_s^2, \alpha_s v, v^2 \text{ (NNLO)}; \; \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 \text{ (N^3LO)}; \ldots \end{array} \right\} \]

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser '15]

- Uncertainty in top mass (potential subtracted) \( \delta m_t \sim 40 \text{ MeV} \). Towards exp. target (20 MeV):
  - Some improvements already from matching of N^3LO+NNLL (NNLL from Hoang et al.)
  - Needs NLL ISR (possibly including soft modes)
  - Ultimately might require N^4LO in PNRQCD needed (currently out of reach)
Top physics: above threshold & continuum (mainly ILC/CLIC)

- **Continuum:** target is 0.1% on cross section. $N^3$LO QCD recently calculated but NNLO EW is necessary

- **Top mass from radiative return from ISR photon:** required matching of continuum and threshold calcns
  - TH unc. doesn’t seem to be dominant source of unc.
  - Possible access to running of (MSR) mass

<table>
<thead>
<tr>
<th>cms energy</th>
<th>CLIC, $\sqrt{s} = 380$ GeV</th>
<th>ILC, $\sqrt{s} = 500$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>luminosity [fb$^{-1}$]</td>
<td>500</td>
<td>1000</td>
</tr>
<tr>
<td>statistical</td>
<td>140 MeV</td>
<td>90 MeV</td>
</tr>
<tr>
<td>theory</td>
<td>46 MeV</td>
<td>55 MeV</td>
</tr>
<tr>
<td>lum. spectrum</td>
<td>20 MeV</td>
<td>20 MeV</td>
</tr>
<tr>
<td>photon response</td>
<td>16 MeV</td>
<td>85 MeV</td>
</tr>
<tr>
<td>total</td>
<td>150 MeV</td>
<td>110 MeV</td>
</tr>
</tbody>
</table>

[Chen, Guan, He, Liu, Ma '22]

$N^3$LO uncertainties @ 500 GeV: 0.15%
Central component in FCCee precision phenomenology (Z, WW, tt, ZH, …)

Recently important progress in formulating collinear factorisation (as opposed to YFS) beyond LO/LL. NLL sizeable (% level) and process/observable dependent. E.g. corrections to total rates \( \tau_{\text{min}} = \frac{M^2}{s} \)

- NNLL hard but within reach of modern perturbative techniques

- Ongoing discussions as to whether a simultaneous resummation of soft and collinear corrections is necessary

- \( \sqrt{Q^2} = 500 \text{ GeV} \)

\[ \ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}, \quad L = \log \frac{Q^2}{m^2} \]

[Example from S. Frixione 2022]

\[ L = 24.59 \quad \Rightarrow \quad \frac{\alpha}{\pi} L = 0.068 \]

\[ 0 \leq m_{ul} \leq m_Z, \quad \ell = 1.46 \quad \Rightarrow \quad \frac{\alpha}{\pi} \ell = 0.0036 \]

\[ m_Z - 1 \text{ GeV} \leq m_{ul} \leq m_Z, \quad \ell = 4.51 \quad \Rightarrow \quad \frac{\alpha}{\pi} \ell = 0.01 \]
Parton showers & event generators
Event generators impact FCC physics programme in toto

- Perturbative calculations often available for (semi-)inclusive observables. Event generators vital for, e.g.
  - Exclusive hadronic observable (e.g. jets)
  - Beam & detector calibration
  - Training of Machine Learning tools for jet/flavour tagging
- Matching the accuracy goals of FCCee poses an outstanding challenge:
  - Perturbative accuracy of parton shower algorithms
  - Matching to higher order calculations for hard scattering
  - Treatment of heavy resonances
  - Non-perturbative QCD
  - QED corrections (jointly with QCD)
Perturbative accuracy of parton showers

### Possible NLL dipole-shower solutions for $e^+e^-$

<table>
<thead>
<tr>
<th>[M. van Beekveld 2023]</th>
<th>Ordering</th>
<th>Kinematic map</th>
<th>Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>PanScales showers</td>
<td>$0 &lt; \beta &lt; 1$</td>
<td>Dipole-local</td>
<td>+, −, ⊥</td>
</tr>
<tr>
<td>(Dipole and antenna)</td>
<td></td>
<td>Global</td>
<td></td>
</tr>
<tr>
<td>PanLocal</td>
<td>$0 \leq \beta &lt; 1$</td>
<td>+, −</td>
<td>−, ⊥</td>
</tr>
<tr>
<td>PanGlobal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alaric [2208.06057]</td>
<td>$\beta = 0$</td>
<td>+</td>
<td>−, ⊥</td>
</tr>
<tr>
<td>Deductor $k_t$ [2011.04777]</td>
<td>$\beta = 0$</td>
<td>+</td>
<td>−, ⊥</td>
</tr>
<tr>
<td>Deductor $\Lambda$</td>
<td>$\beta = 1$</td>
<td>+</td>
<td>−, ⊥</td>
</tr>
<tr>
<td>Manchester-Vienna [2003.06400]</td>
<td>$\beta = 0$</td>
<td>+</td>
<td>−, ⊥</td>
</tr>
</tbody>
</table>

Showers also differ on the implementation of the splitting functions and how the global imbalance is redistributed.

**All have different approaches to assess NLL accuracy**

- New technology to improve logarithmic accuracy on a more systematic basis: current status is NLL, with uncertainties at the ≳10% level
- Promising developments also re. subleading colour effects
- FCCee demands at least NNLL QCD accuracy, and arguably higher

![Diagram](image-url)
Logarithmic accuracy aware matching

Newly developed NLL showers constrain matching to N(N)LO

Well known matching schemes may be affected by breaking of logarithmic accuracy in specific observables

Further work needed to upgrade NNLO generators to NLL accuracy

\begin{align}
\Sigma (O < e^L) &= e^{-\bar{\alpha} L^2}, \quad \bar{\alpha} = \alpha_s \\
\Sigma (O < e^L) &= e^{-\bar{\alpha} L^2} \left[ 1 + 2 \left( e^{-\bar{\alpha} L^2} - 1 \right) \bar{\alpha} \Delta \right] \\
\bar{\alpha} \Delta &= \frac{2C_F \alpha_s}{\pi} \frac{4\pi^2 - 15}{24} \\
\Delta \text{ is the effective area of one shaded green region, which for PanLocal and } \gamma \to q\bar{q} \text{ is given by}
\end{align}

[Hamilton, Karlberg, Salam, Scyboz, Verheyen 2023]

e.g. Soft-drop jets $k_t$

\begin{align}
SD_z > 0.25, \beta_{SD} = 0 \ln k_t/Q, \sqrt{s} = 2 \text{ TeV}
\end{align}
### Treatment of heavy resonances

- **Correct handling of resonances at higher orders essential at FCA**
- **Technology for resonance-aware NLO+PS already available in main MC generators.**
  - Logarithmic accuracy of matching with virtuality-preserving mappings
  - \[ \text{MG5}_\text{aMC@NLO} \] [Frederix, Frixione, Papanastasiou, Prestel, Torrielli ‘16]
    - Phase space remapping & prescription for including the resonance in LHE
    - \( pp \rightarrow t\bar{t} \)
    - SMC: Herwig6, Pythia8
  - \[ \text{Whizard} \] [Chokoufé Nejad, Kilian, Lindert, Pozzorini, Reuter, Weiss ‘16]
    - Resonance-aware FKS with \( Z \rightarrow b\bar{b} \) and \( H \rightarrow b\bar{b} \) RH
    - Fixed order \( e^+e^- \rightarrow t\bar{t} \) & \( e^+e^- \rightarrow ttH \)
  - \[ \text{Sherpa} \] [Höche, Liebschner, Siegert ‘18]
    - Resonance-aware CS subtraction
    - Fixed order \( e^+e^- \rightarrow t\bar{t} \) & \( pp \rightarrow t\bar{t} \)
- Must be analysed/revisited in light of recent and future NLL developments
  - Logarithmic accuracy of matching with virtuality-preserving mappings
  - Higher order showers for reactions with massive quarks (e.g. \( tt, WW \rightarrow \text{jets} \))
  - Non-relativistic effects (NRQCD, unstable particle EFT), currently out of reach in MCs
Non-perturbative QCD

- Modelling of NP effects is a crucial goal for precision programme
  - Spectrum of old&new models of NP physics
  - Input from FCCee is highly beneficial:
    - Span of c.o.m. energies crucial for tuning, jointly w/ higher order MCs
    - High-purity samples of gluon/heavy-quark jets beneficial for fragmentation models (used e.g. in jet tagging)
    - Potential of cross-benefit between stages of FCCee (e.g. tunes in $Z \rightarrow$ jets useful for $ZH, CR \rightarrow WW \rightarrow$ jets, …)
  - Crucial to explore implications of recent analytic calcns (in large-$n_F$) for MC generators (e.g. mappings)

- e.g. CR inspired by amplitude-level evolution in PS
  - e.g. GANs as hadronisation model ($\pi$s only)

[Images of diagrams showing various processes and models]
First steps towards NLL QED (ISR) effects in parton showers

QED Parton Shower

see for instance review in 0912.0749

[G. Stagnitto 2023]

Introduction of a cutoff $x_+ = 1 - \epsilon$, with $\epsilon \ll 1$, to regularise splitting kernels:

$$P_+(z) = \theta(x_+ - z)P(z) - \delta(1 - z) \int_0^{x_+} dx\, P(x)$$

By introducing a Sukudov form factor: $\Pi(s_1, s_2) = \exp\left( -\frac{\alpha}{2\pi} \int_{s_2}^{s_1} \frac{ds'}{s'} \int_0^{s_1} dz\, P(z) \right)$

one can recast the evolution equation in an iterative integral form:

$$D(x, s) = \sum_{n=0}^{\infty} \prod_{i=1}^{n} \left\{ \int_{m_c^2}^{s_i} \frac{ds_i}{s_i} \Pi(s_{i-1}, s_i) \frac{\alpha}{2\pi} \int_{x(s_{i-1})}^{s_i} \frac{dz_i}{z_i} P(z_i) \right\} \Pi(s_n, m_c^2) D \left( \frac{x}{z_1 \cdots z_n}, m_c^2 \right)$$

which can be solved by means of a MC algorithm

- New extension of QED collinear factorisation to NLL provides the ingredients for a NLL (next-to-single-log) accurate evolution

- Currently inclusive treatment of radiation, more work needed for fully differential generator & interleaved QED\(\times\)QCD PS

Non-singlet NLL PDF

PRELIMINARY

\(\mu = 100\, \text{GeV}\)
• Astounding experimental programme at FCCee, drastic reduction of statistical (and systematic) uncertainties: theory precision likely to be among the main bottlenecks

• Many (if not all) areas of theory calculations need to be involved (fixed order QCD + EW, resummations in QCD & QED, effective field theories, non-perturbative QCD, event generators, …)

• Most challenges are technical in nature: hard calculations, currently beyond reach but likely to become achievable with the evolution of the field at the LHC in the coming decade(s), and substantial work

• Some deep conceptual issues, which need significant breakthroughs to improve their understanding: e.g. non-perturbative QCD (hadronisation, colour reconnection), currently a bottleneck in several studies

• Many new opportunities from high-quality experimental data, crucial to think of how to exploit it to improve on modelling aspects and theory uncertainties (e.g. heavy flavour & gluon fragmentation, hadronisation modelling, …)