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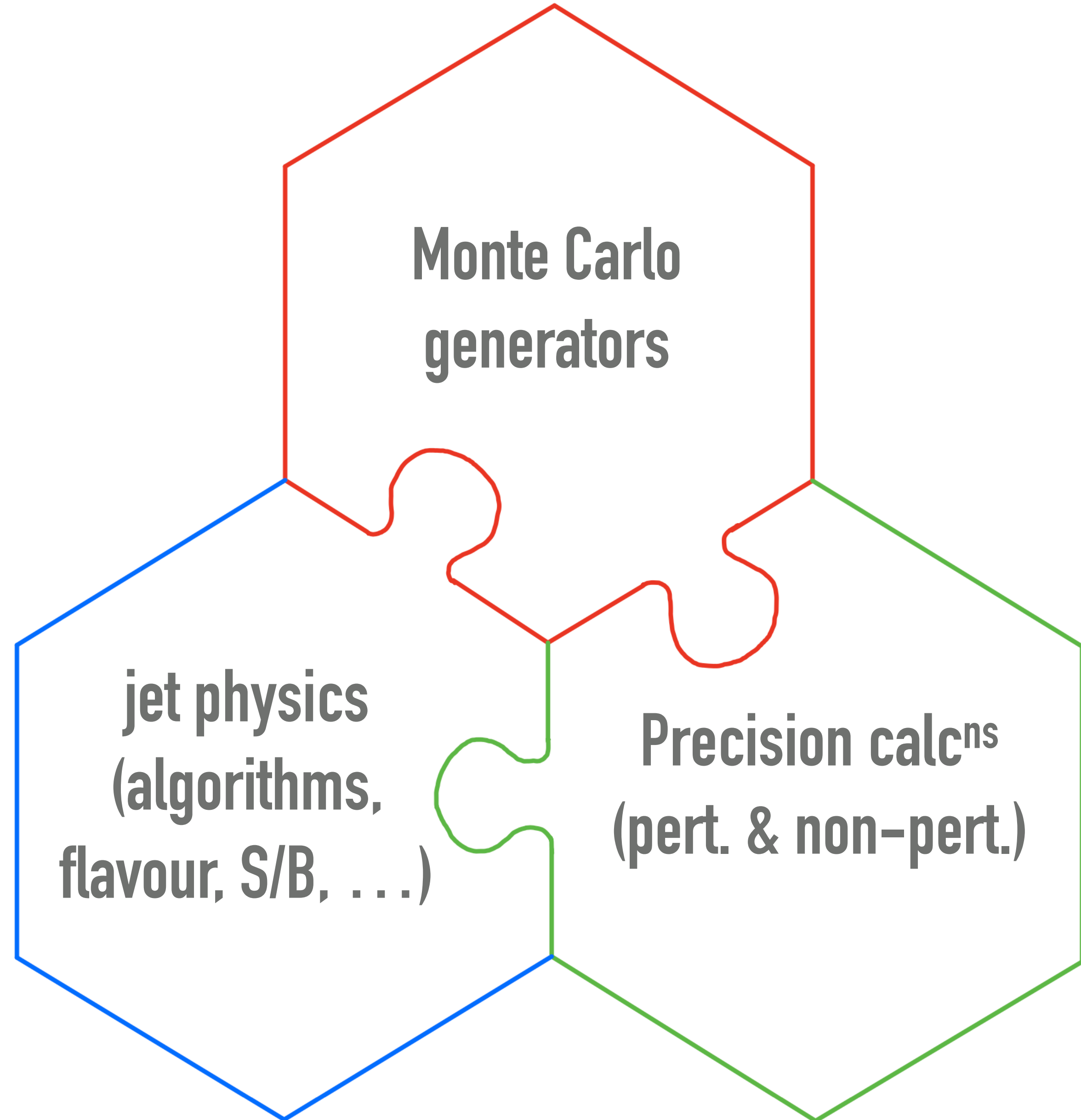
# QCD & Event Generators for FCC-ee

P. Monni (CERN)

FCC Week 2023 - 6 June 2023

# Role of precision QCD at FCC-ee

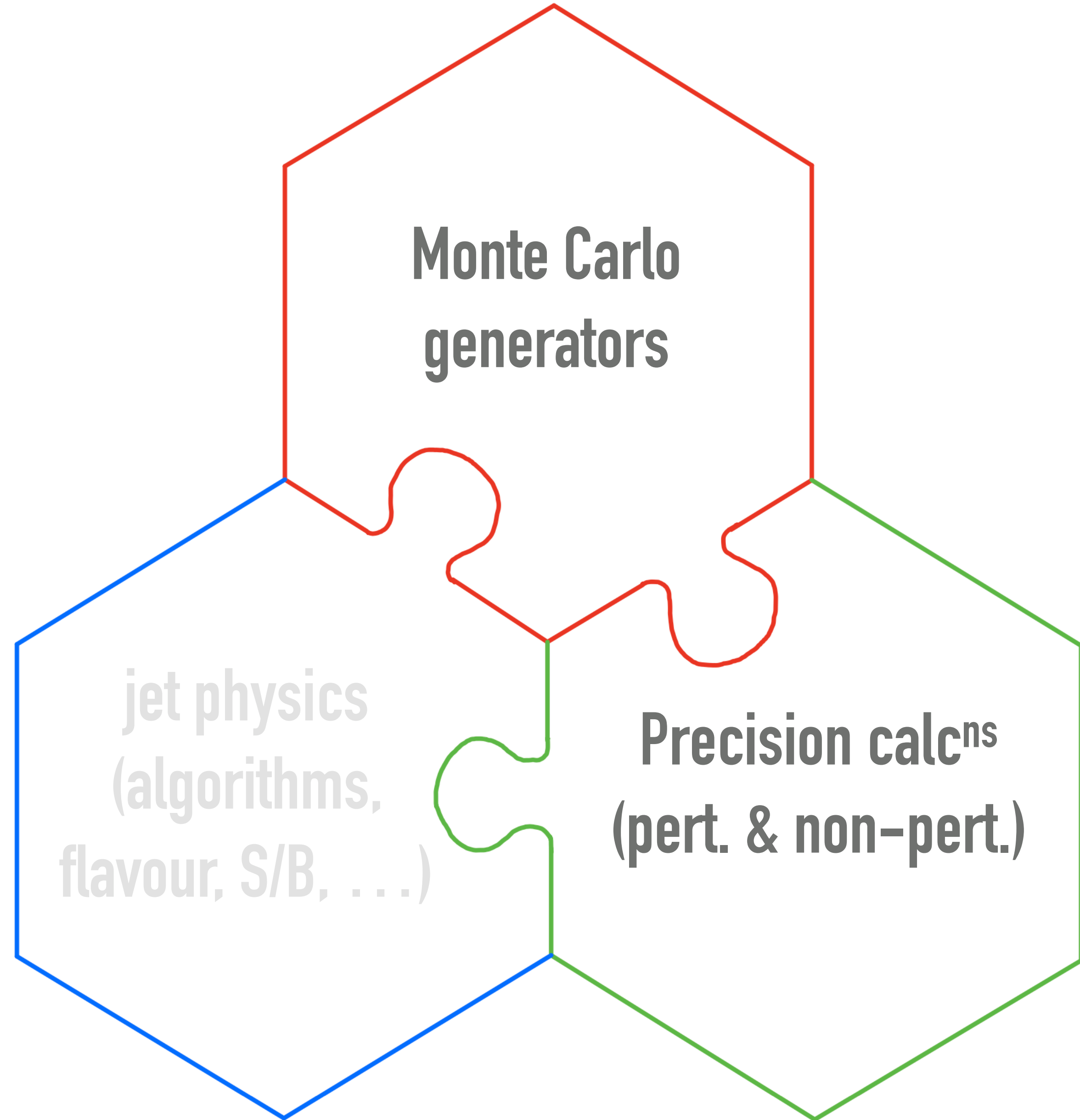
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**Reaching the foreseen precision poses outstanding challenges on theory calculations. Evolution in many areas is required to meet the goals**

# Role of precision QCD at FCC-ee

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**This talk addresses mainly QCD aspects\*,  
EW corrections will be discussed in  
detail in the EW sessions**

\* See Janus Gluza's talk for EW calculations

# Outline of the talk: please visit indico pages for more info

## Precision calculations for future $e^+e^-$ colliders: targets and tools

7–17 Jun 2022  
CERN

Europe/Zurich timezone

<https://indico.cern.ch/event/1140580/>

Enter your search term



Overview

Programme Committee

Timetable

Application Form

Participant List

The main goal is to identify clear theoretical and computational targets for high-precision predictions of relevance to the programme of future  $e^+e^-$  colliders.

## Parton Showers for future $e^+e^-$ colliders

24–28 Apr 2023  
CERN

Europe/Zurich timezone

<https://indico.cern.ch/event/1233329/>

Enter your search term



Overview

Timetable

Registration

Participant List

Videoconference

The unprecedented experimental performance expected by the next generation of lepton colliders poses an outstanding challenge for theoretical computations that must be pushed far beyond the current state of the art to guarantee an optimal exploitation of the data. Among the theoretical aspects of this programme, Monte Carlo event generators play a special role due to their versatility in bridging theoretical predictions and experimental measurements. The precision reached by current event generation algorithms is dramatically insufficient for this task, thus demanding a dedicated effort to improve their formal accuracy and achieve a higher precision in event simulations.



# Perturbative calculations

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# Physics at the Z pole

❑ Numbers are given here for FCC-ee (best prospects)

[P. Janot's talk @ CERN FC workshop 2022]

Observables	Present value	FCC-ee stat.	FCC-ee current syst.	FCC-ee ultimate syst.	Theory input (not exhaustive)
$m_Z$ (keV)	$91187500 \pm 2100$	4	100	10 ?	Lineshape QED unfolding Relation to measured quantities
$\Gamma_Z$ (keV)	$2495500 \pm 2300$ [*]	4	25	5 ?	Lineshape QED unfolding Relation to measured quantities
$\sigma^0_{\text{had}}$ (pb)	$41480.2 \pm 32.5$ [*]	0.04	4	0.8	Bhabha cross section to 0.01% $e^+e^- \rightarrow \gamma\gamma$ cross section to 0.002%
$N_v (\times 10^3)$ from $\sigma_{\text{had}}$	$2996.3 \pm 7.4$	0.007	1	0.2	Lineshape QED unfolding $(\Gamma_{\nu\nu}/\Gamma_{\ell\ell})_{\text{SM}}$
$R_\ell (\times 10^3)$	$20766.6 \pm 24.7$	0.04	1	0.2 ?	Lepton angular distribution (QED ISR/FSR/IFI, EW corrections)
$\alpha_s(m_Z) (\times 10^4)$ from $R_\ell$	$1196 \pm 30$	0.1	1.5	0.4 ?	Higher order QCD corrections for $\Gamma_{\text{had}}$
$R_b (\times 10^6)$	$216290 \pm 660$	0.3	?	< 60 ?	QCD (gluon radiation, gluon splitting, fragmentation, decays, ...)

- Theory crucial in 3 ways: measurement/calibration (e.g. QED ISR); interpretation of results (EWPO); parametric uncertainties (i.e. couplings, masses)
- QCD uncertainties concern all three categories

# Precision physics in $Z/\gamma^* \rightarrow \text{jets}$

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- Main computational challenges from EW aspects:
  - EWPO  $Z \rightarrow qq+X$  @ 3 loops EW and beyond
  - Beam calibration [ $e^+e^- \rightarrow e^+e^-$ ,  $\mu^+\mu^-$ ,  $\gamma\gamma$  @ NNLO EW - still beyond reach]
- But **high potential for precision QCD** studies at the Z pole and above:
  - Strong coupling constant
  - Jet dynamics and substructure: spin correlations, fragmentation & track functions, multi-jet observables (global/non-global)
  - Non-perturbative effects & modelling
  - Heavy quarks (Q) studies (e.g. asymmetries, fragmentation) & jet tagging (e.g.  $q/Q$  vs.  $g$  jets)
  - $\tau$  decays ( $\alpha_s$ )
  - Calibration/tuning of ML & MC models (instrumental for higher-energy runs)

# Precision physics in $Z/\gamma^* \rightarrow \text{jets}$

- Significant room for improvement for QCD calculations, e.g.
- Heavy quarks:  $R_b$ ,  $A_{\text{FB}}$  requires  $QQg$  and  $qqg(\rightarrow QQ)$  @ 2 loops with  $m_b$  dependence (NLO known)

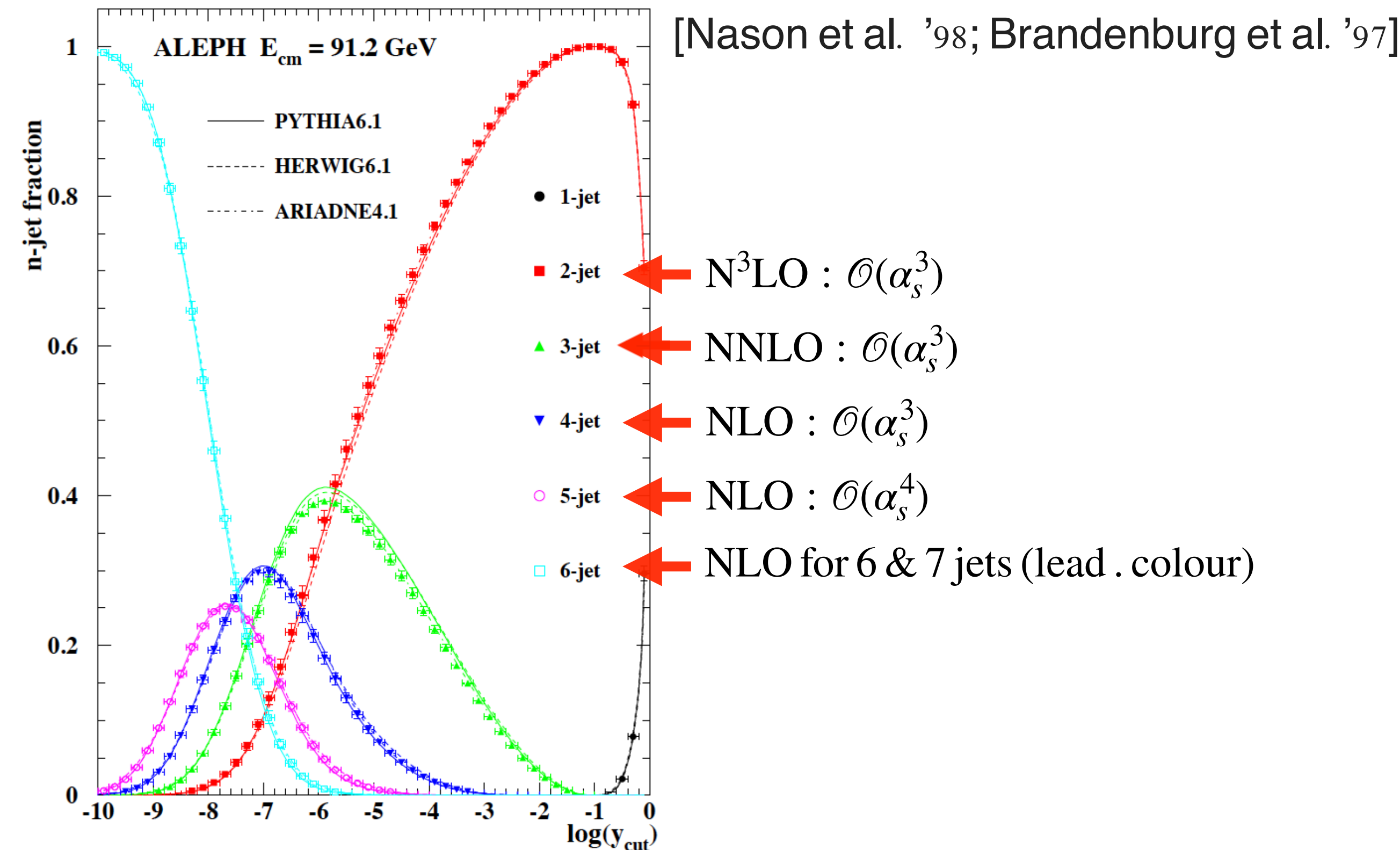
- Fragmentation functions

- Multi-jet final states

- 3 jets @  $N^3\text{LO}$  QCD
- 4 & 5 jets at NNLO QCD

Some of this is within the reach of  
technology developed at LHC  
(e.g.  $Z/\gamma^* + 2$  jets @ 2 loops, subtraction methods)

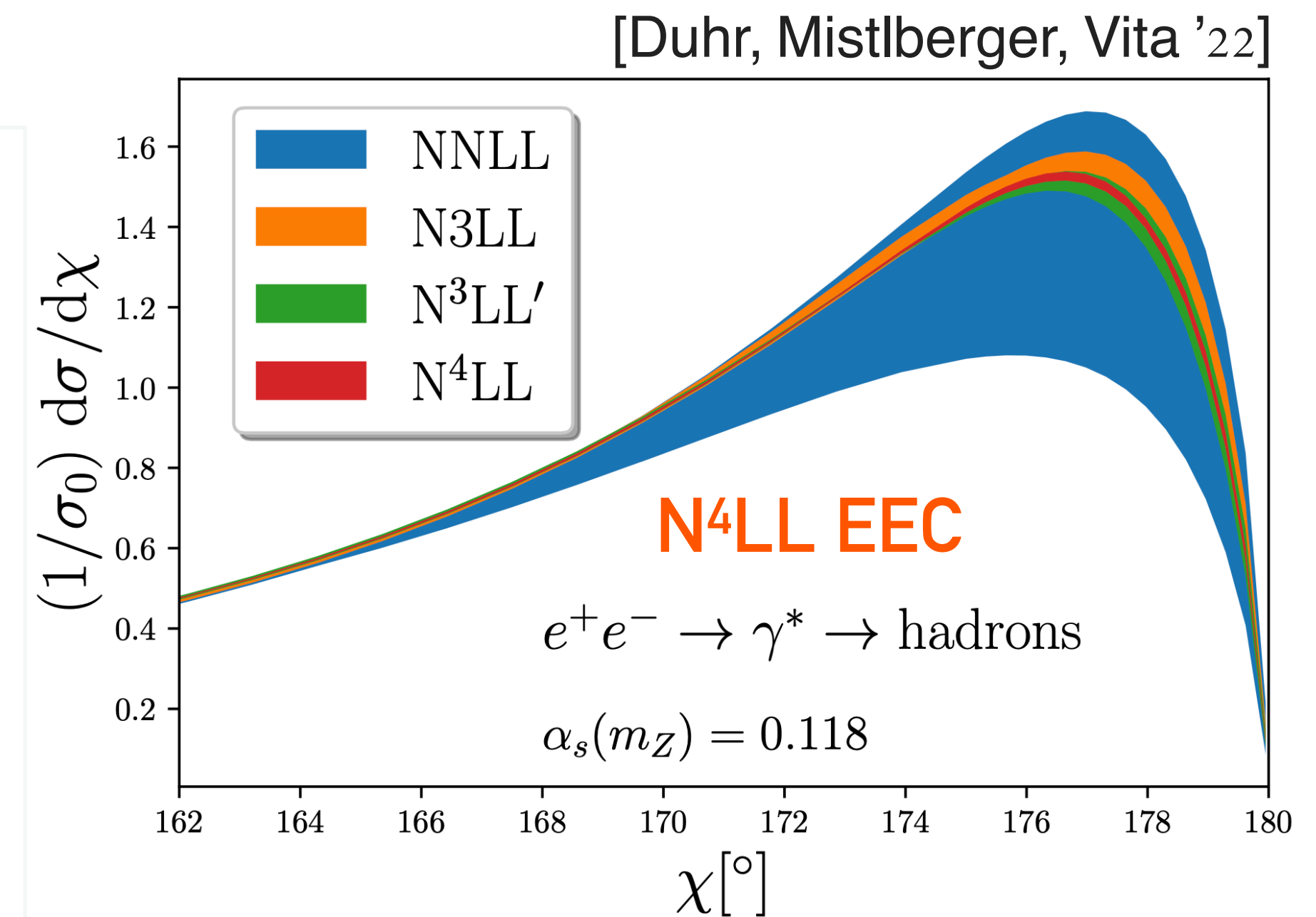
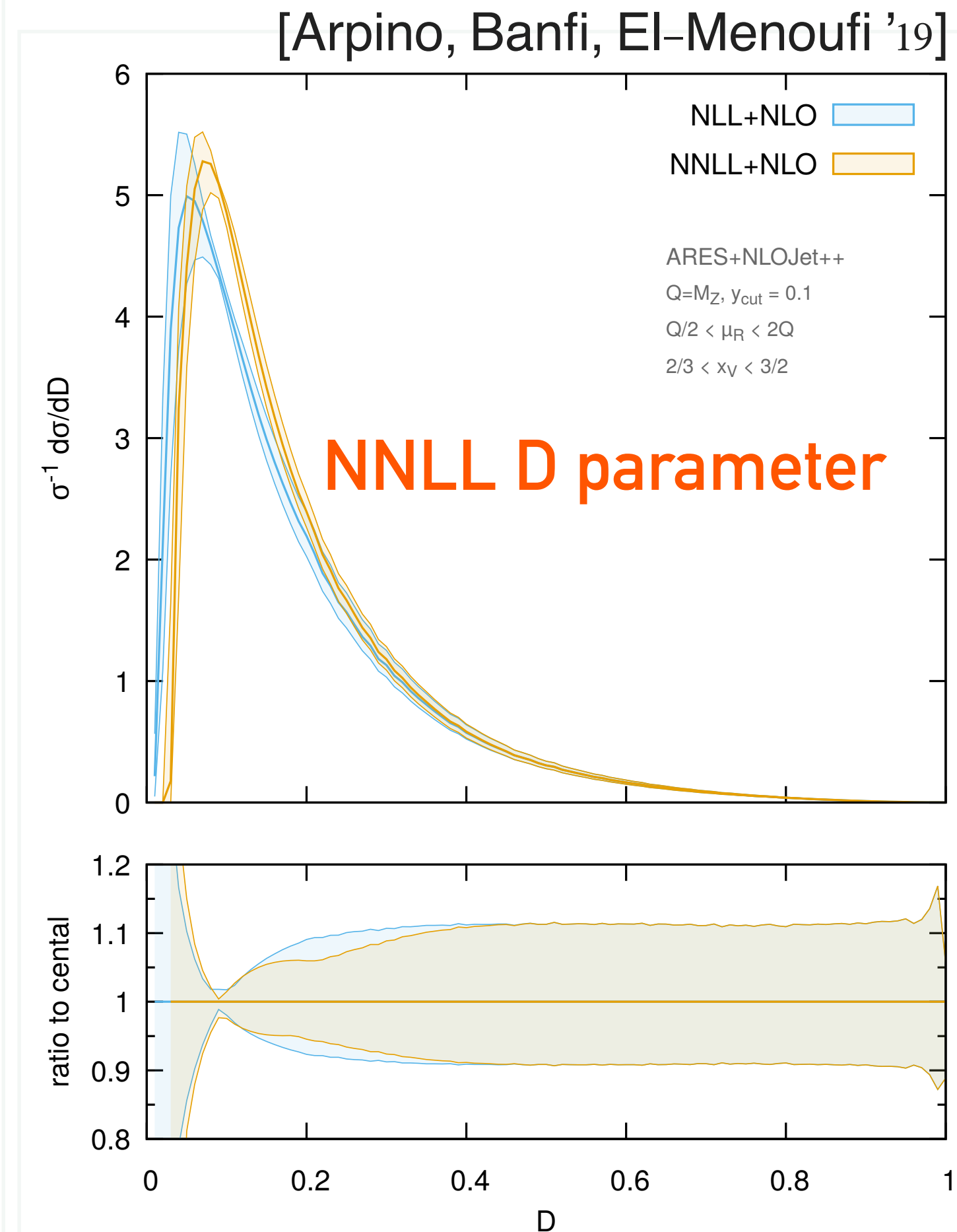
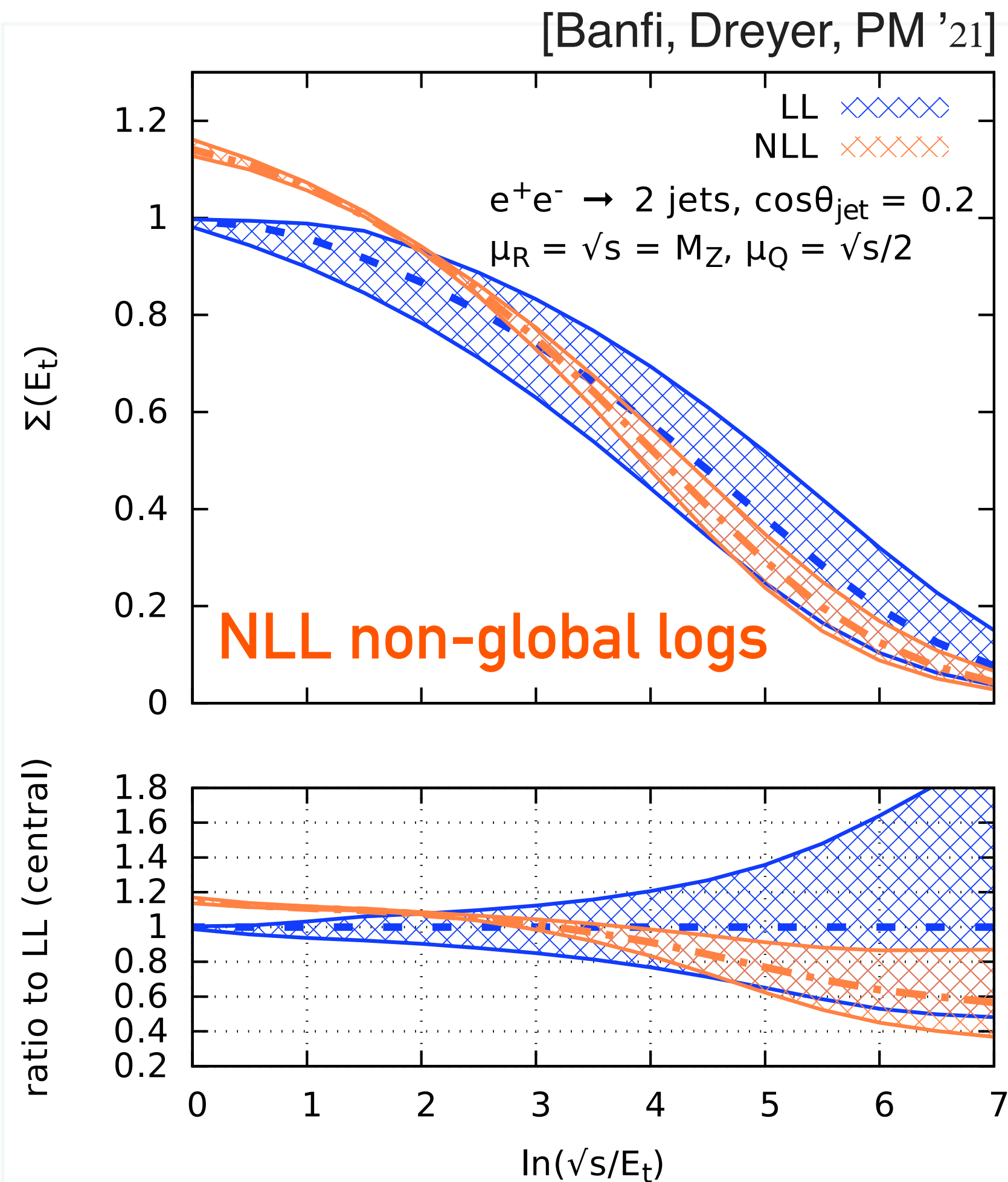
[e. g. five-point amplitudes in Abreu et al. '18–'23; Badger et al '19–'22; Chawdhry et al. '20–'21]



- Promising new directions for loop calculations: e.g. numerical approaches for total rates at  $N^{(2/3)}\text{LO}$  (e.g. Feynman parameters, local unitarity, AMFlow, diffExp), though further progress needed for distributions

# Precision physics in $Z/\gamma^* \rightarrow \text{jets}$

- All-order logarithmic corrections (resummations) desirable for phenomenology. A lot of new techniques refined in recent years for jet observables (SCET(s), numerical methods, generating functionals, ...)

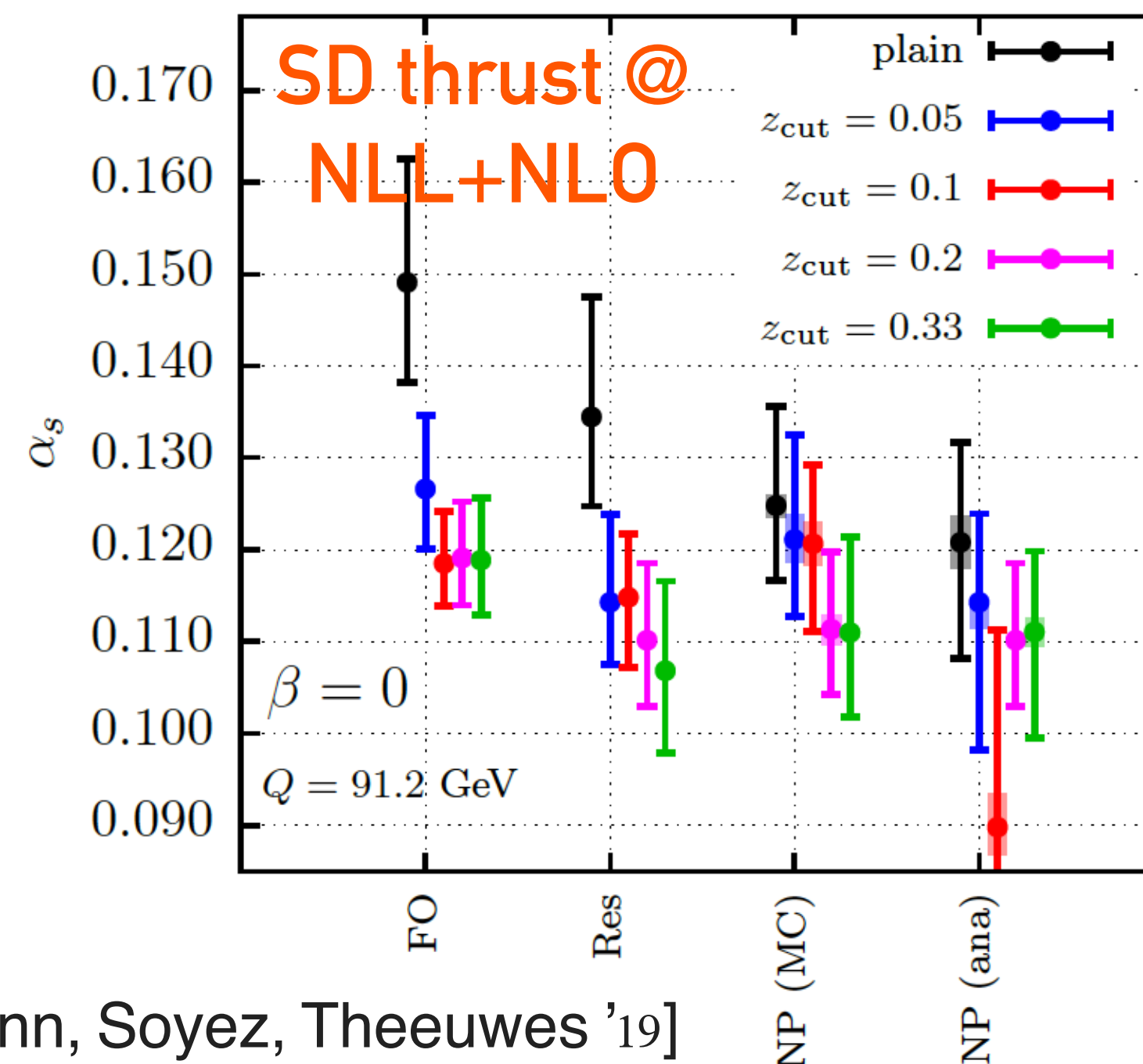


Room for improvement in high-multiplicity ( $\geq 3$  jets) observables, possibly requires algorithmic methods

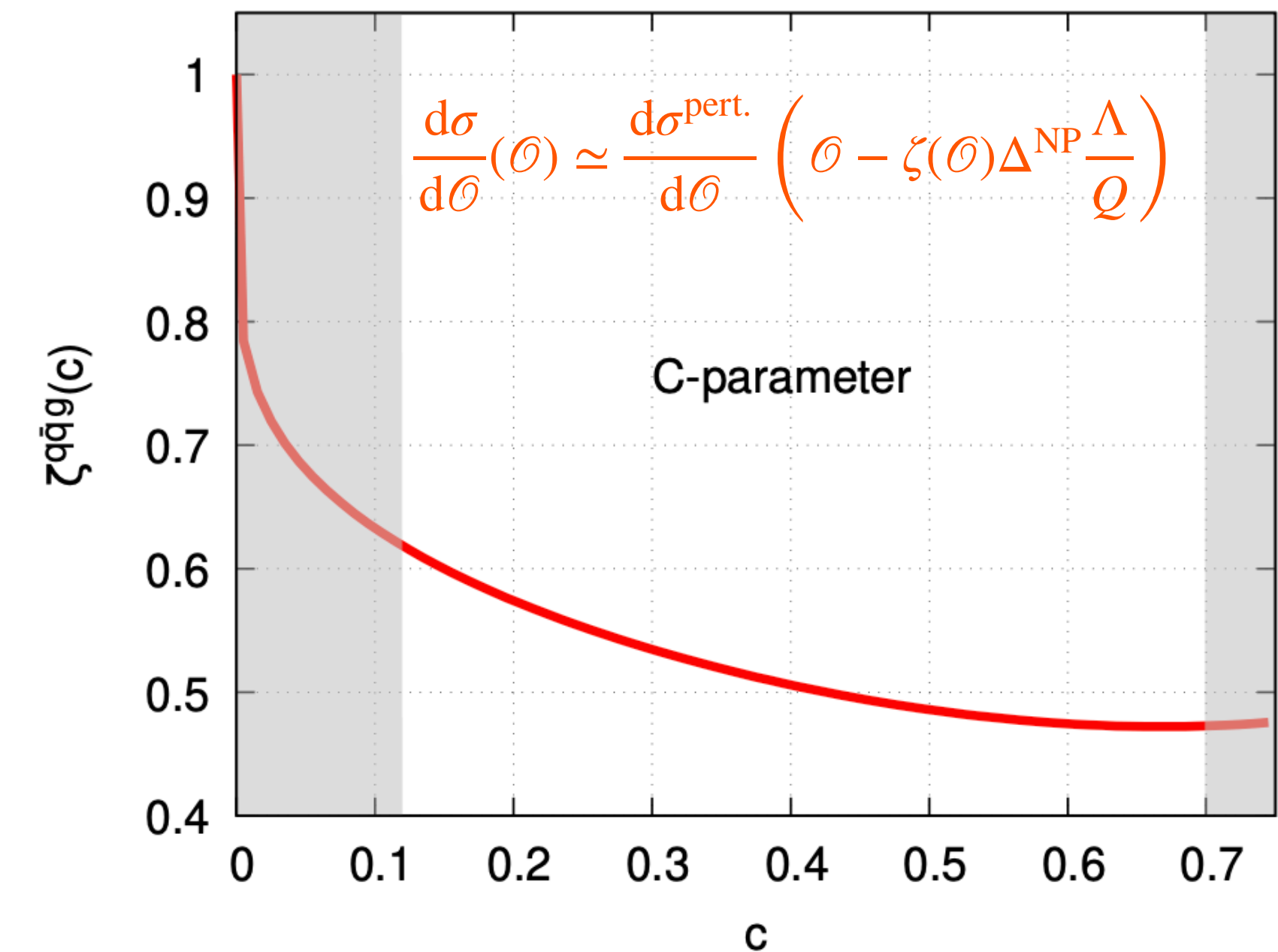


# Non-perturbative QCD corrections

- **Better understanding of hadronisation in jet observables appears to be essential** (event shapes, jet rates, jet substructure); **serious limitation of TH accuracy**. Possible avenues (possibly in combination):
- Techniques to calculate leading corrections as  $1/Q$  expansion, recently first important steps for 3-jet configurations (largely based on large- $n_F$  approximation)
- New observables with reduced NP sensitivity, e.g. through jet grooming. Preliminary studies on strong coupling extractions
- Tuning of MC generators across  $\sqrt{s}$  values (Q/q/g samples). High perturbative accuracy demanded, a lot of recent progress



[Marzani, Reichelt, Schumann, Soyez, Theeuwes '19]

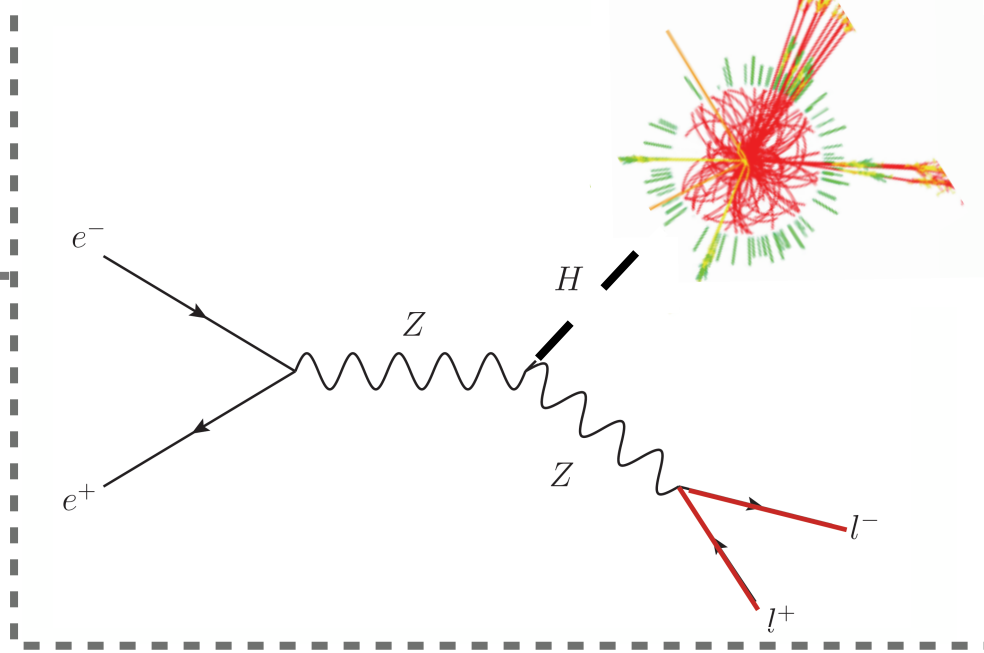


[Luisoni, PM, Salam '20]

[Caola, Ferrario Ravasio, Limatola, Melnikov, Nason '21+'22]

[Nason, Zanderighi '23]

# ZH threshold



- Experimental precision approaching 0.1% in many cases at ZH threshold
- Example: **total cross section will be measured with precision in the range 0.2%-0.5%.** Necessary ingredients:
  - $e^+e^- \rightarrow Z H$  (now available),  $H \nu \nu$  ( $e^+e^-$ ) @ 2 loops EW (hard at the moment)  
[Chen, Guan, He, Liu, Ma '22; Freitas, Song '21-'22]
  - Mixed QCD $\otimes$ EW @ 2 loops under control  
[Gong et al. '17]
- Wealth of data in hadronic decays of the Higgs boson (demanding also excellent jet tagging performance\*)

\* See e.g. L. Gouskos' talk

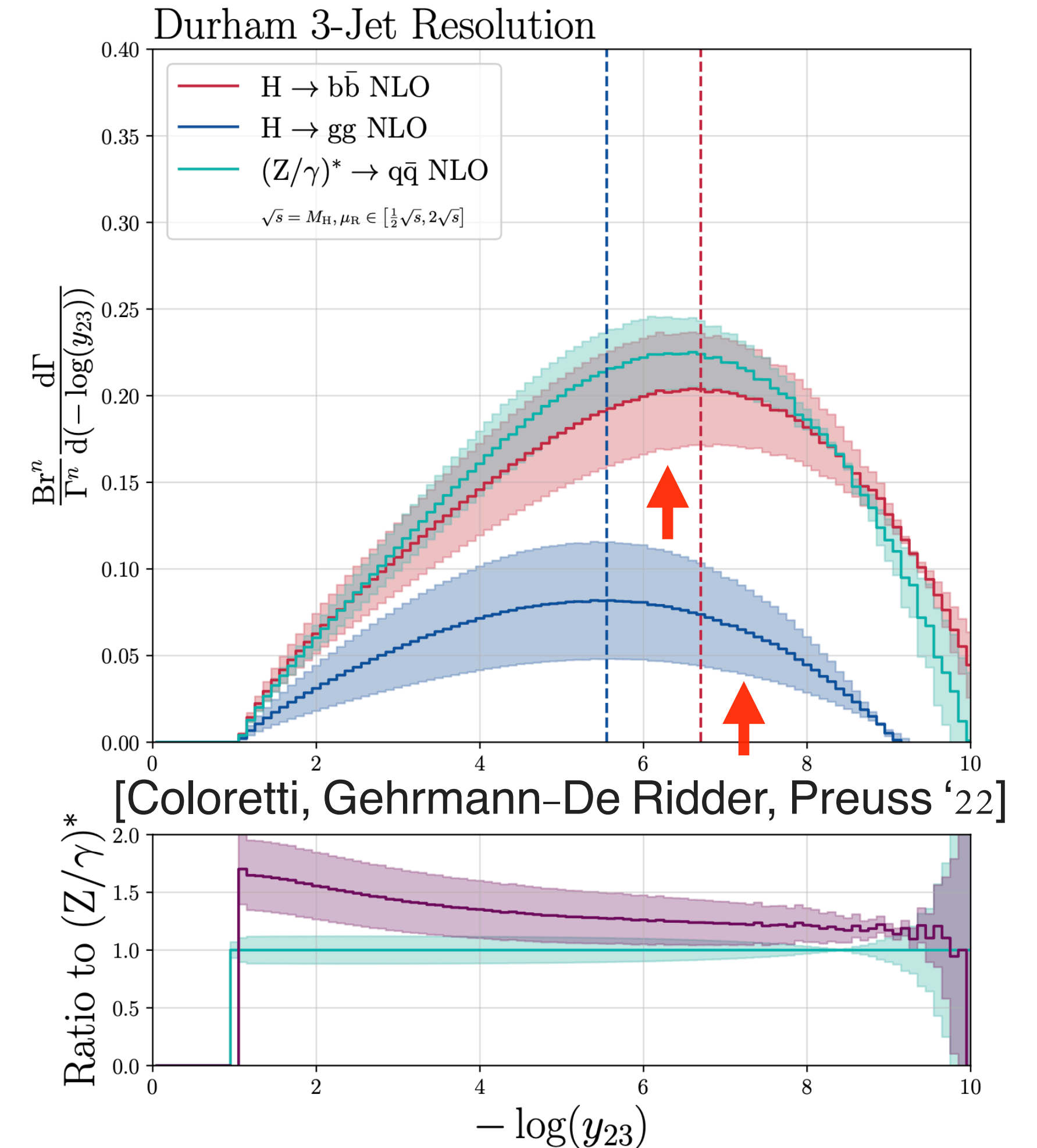
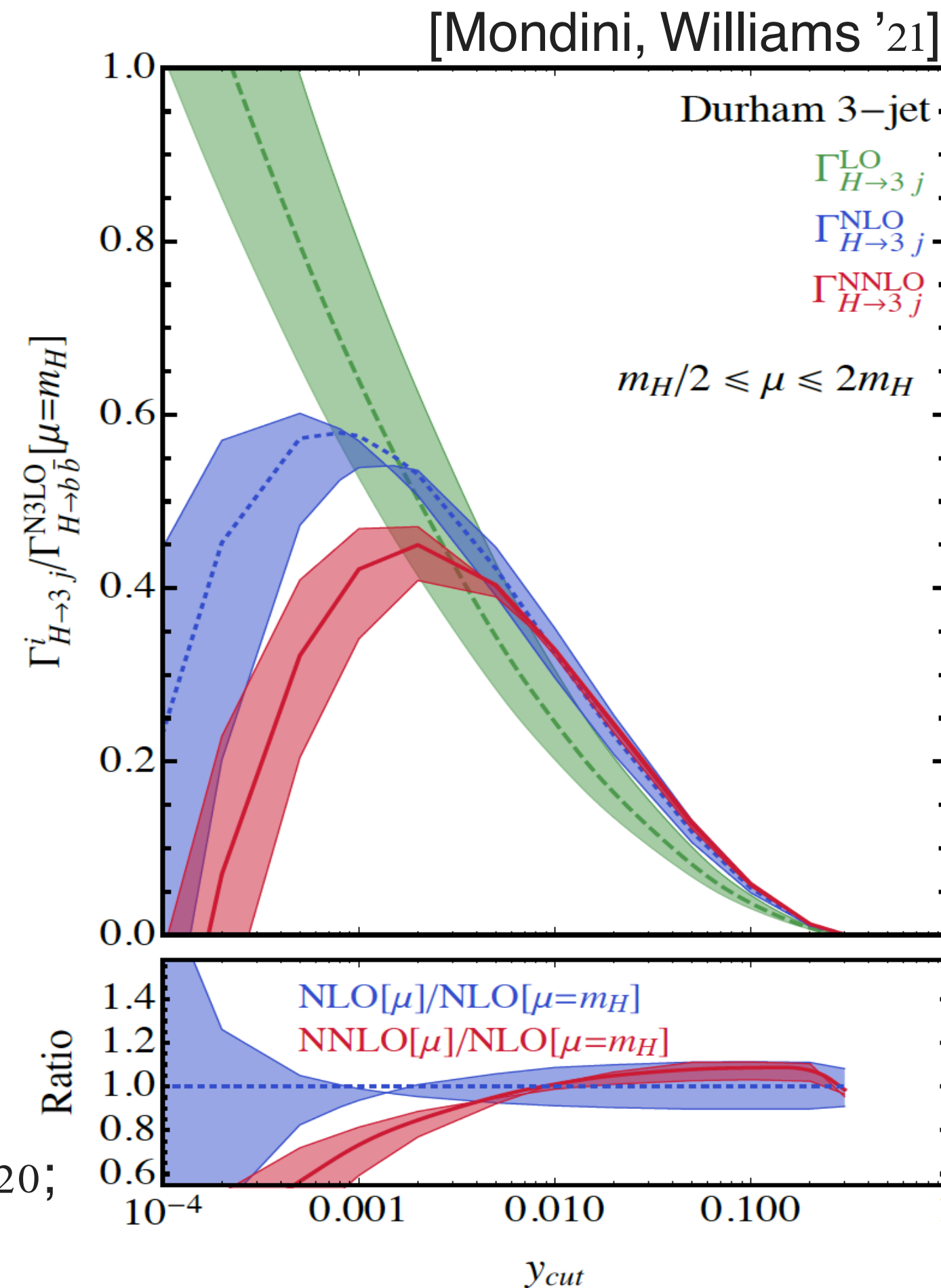
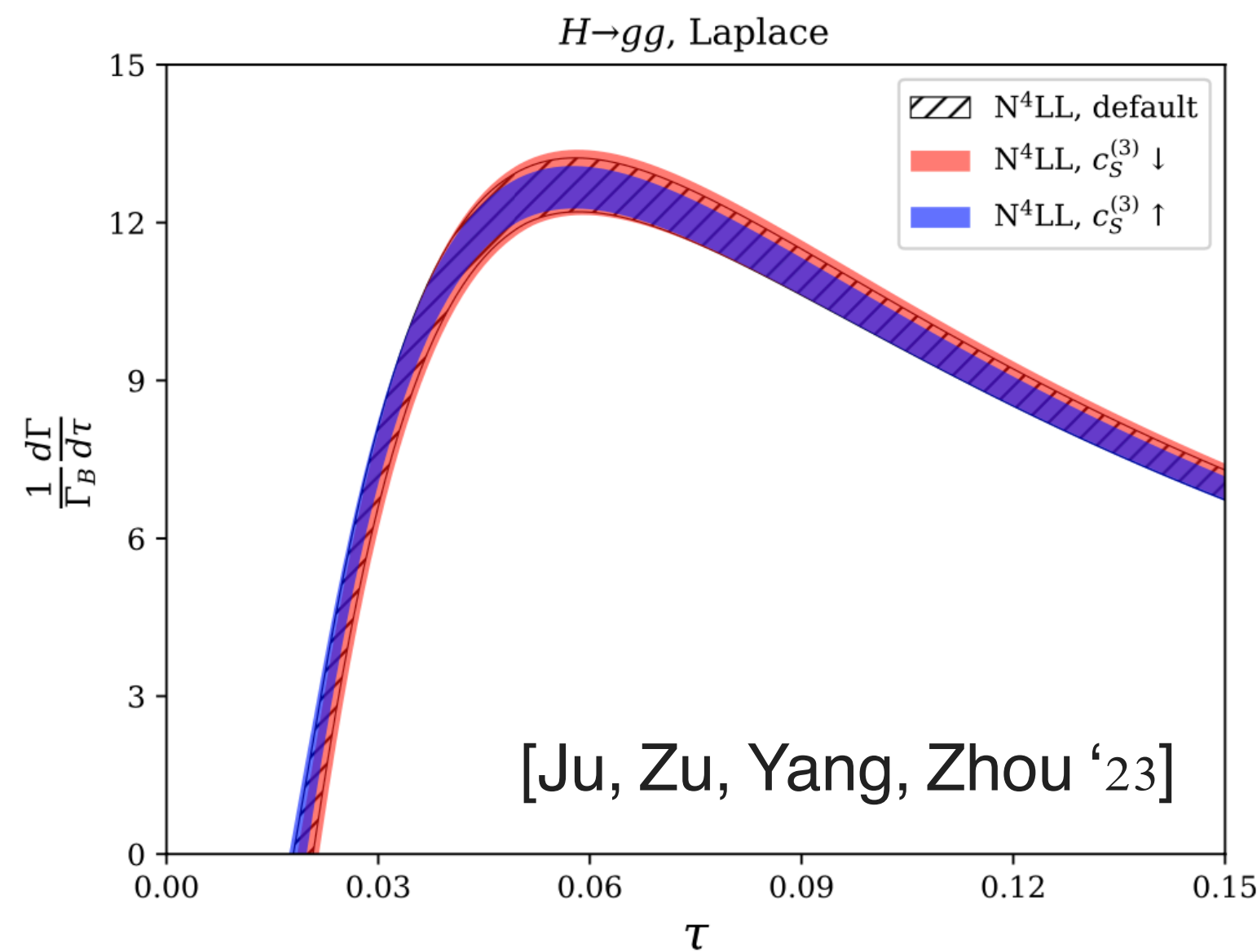
Decay	current unc. $\delta\Gamma$ [%]				future unc. $\delta\Gamma$ [%]			
	Th <sub>Intr</sub>	Th <sub>Par</sub> <sup><math>m_q</math></sup>	Th <sub>Par</sub> <sup><math>\alpha_s</math></sup>	Th <sub>Par</sub> <sup><math>m_H</math></sup>	Th <sub>Intr</sub>	Th <sub>Par</sub> <sup><math>m_q</math></sup>	Th <sub>Par</sub> <sup><math>\alpha_s</math></sup>	Th <sub>Par</sub> <sup><math>m_H</math></sup>
$H \rightarrow b\bar{b}$	< 0.4	1.4	0.4	—	0.2	0.6	< 0.1	—
$H \rightarrow \tau^+\tau^-$	< 0.3	—	—	—	< 0.1	—	—	—
$H \rightarrow c\bar{c}$	< 0.4	4.0	0.4	—	0.2	1.0	< 0.1	—
$H \rightarrow \mu^+\mu^-$	< 0.3	—	—	—	< 0.1	—	—	—
$H \rightarrow W^+W^-$	0.5	—	—	2.6	0.3	—	—	0.1
$H \rightarrow gg$	3.2	< 0.2	3.7	—	1.0	—	0.5	—
$H \rightarrow ZZ$	0.5	—	—	3.0	0.3	—	—	0.1
$H \rightarrow \gamma\gamma$	< 1.0	< 0.2	—	—	< 1.0	—	—	—
$H \rightarrow Z\gamma$	5.0	—	—	2.1	1.0	—	—	0.1

**Projected reduction of intrinsic TH uncertainties in line with what can be achieved with future calculations (total rates); improvement needed in parametric unc.**

[Credit: J. de Blas]

# Hadronic Higgs decays

- Accuracy significantly lower for differential distributions (e.g. potential sensitivity to light-quarks Yukawa)
- NNLO (+resummations) achievable in the coming years (already available in  $H \rightarrow b\bar{b}$  and partly  $H \rightarrow g\bar{g}^*$ ); sufficient for several-% precision (3loops needed for few-% level)



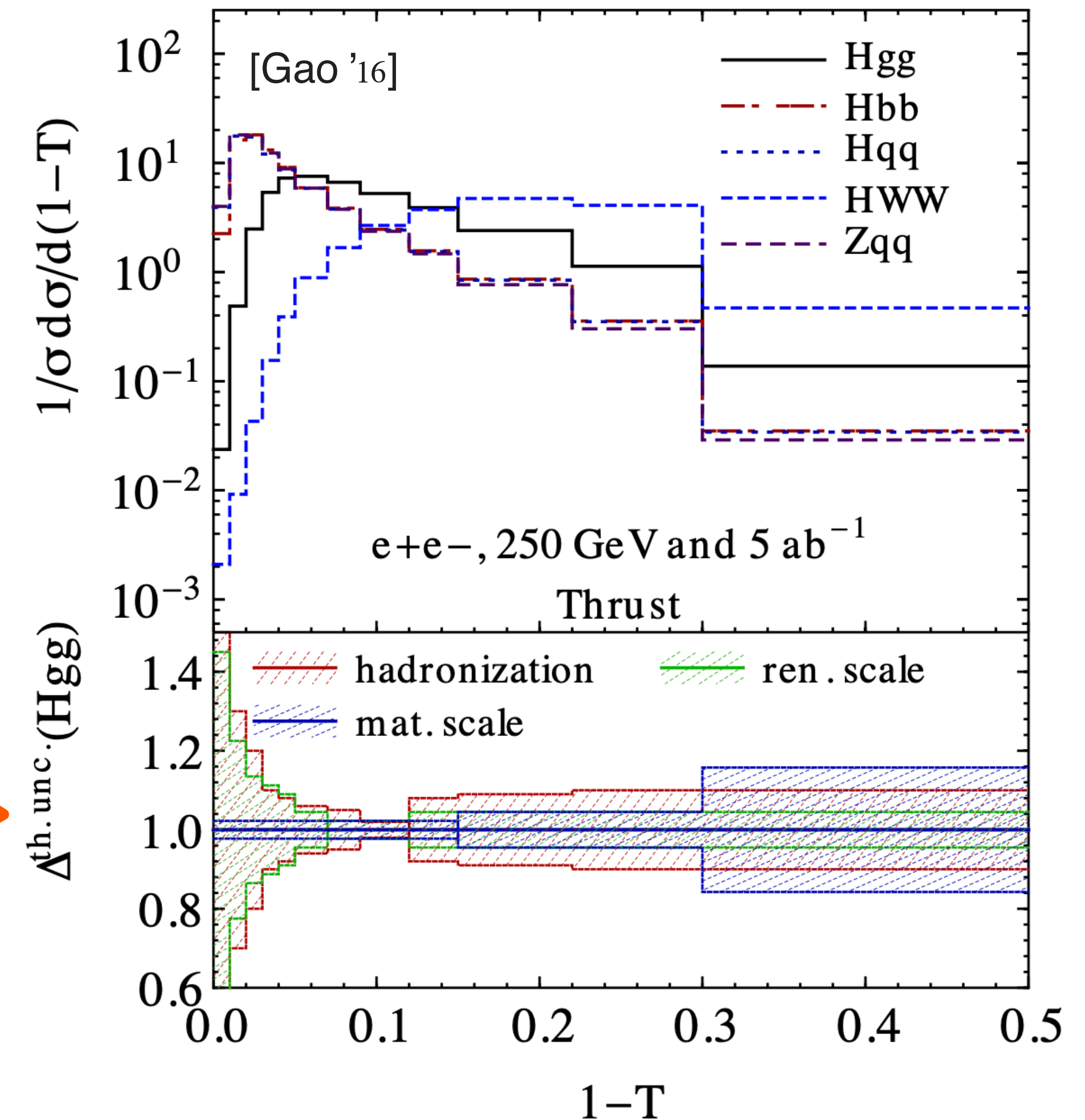
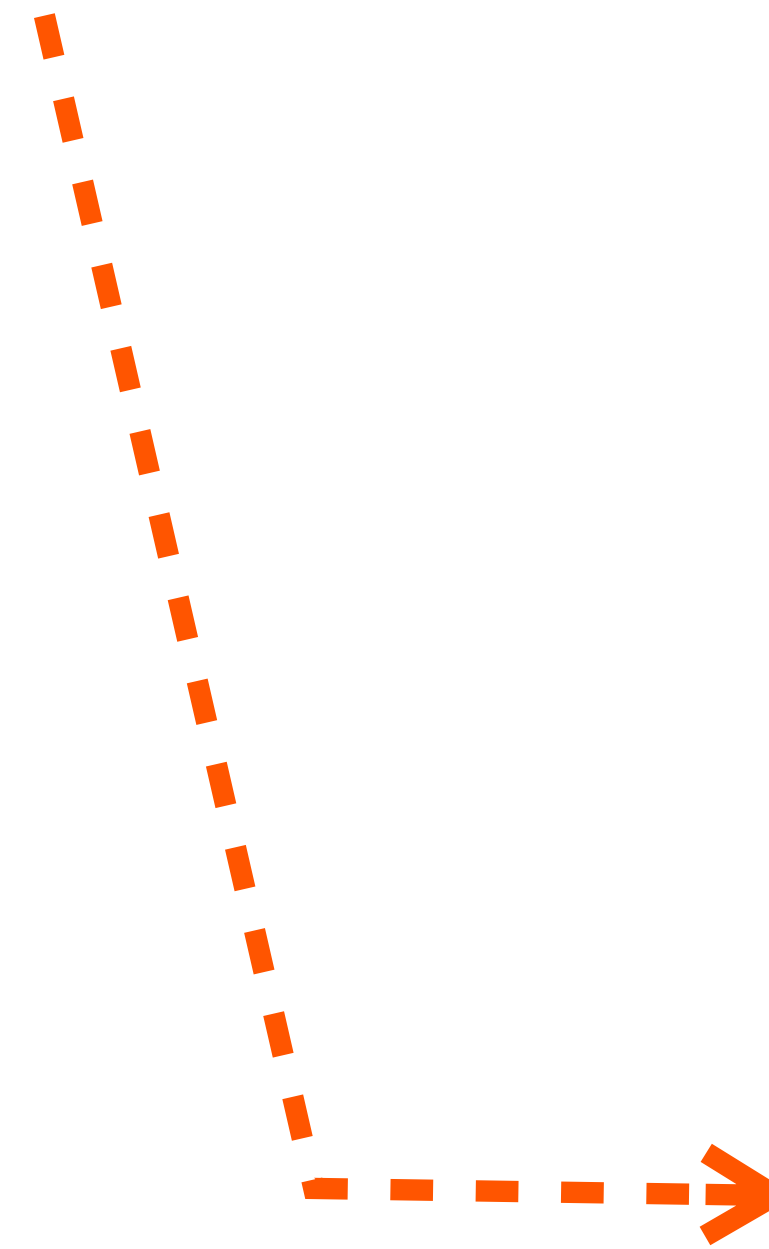
\* All ingredients for HO in  $H \rightarrow gg$  known  
(also with full mass dependence)

[Czakon et al. '20; Bonciani et al. '22 Melnikov,  
Penin '16; Liu, Penin '17–'19; Anastasiou, Penin '20;  
Chen, Jakubcik, Marcoli, Stagnitto '23]



# Hadronic Higgs decays

- Accuracy significantly lower for differential distributions (e.g. potential sensitivity to light-quarks Yukawa)
- However, hadronisation remains the main bottleneck
  - e.g. thrust in Higgs decays (MC variation in plot)
- Increase in energy insufficient for suppression ( $Q \sim m_H$ )
- Runs at lower energies are essential for a robust tuning of NP models in MCs
- Also crucial for training of ML algorithms for jet tagging, instrumental in extraction of Higgs couplings



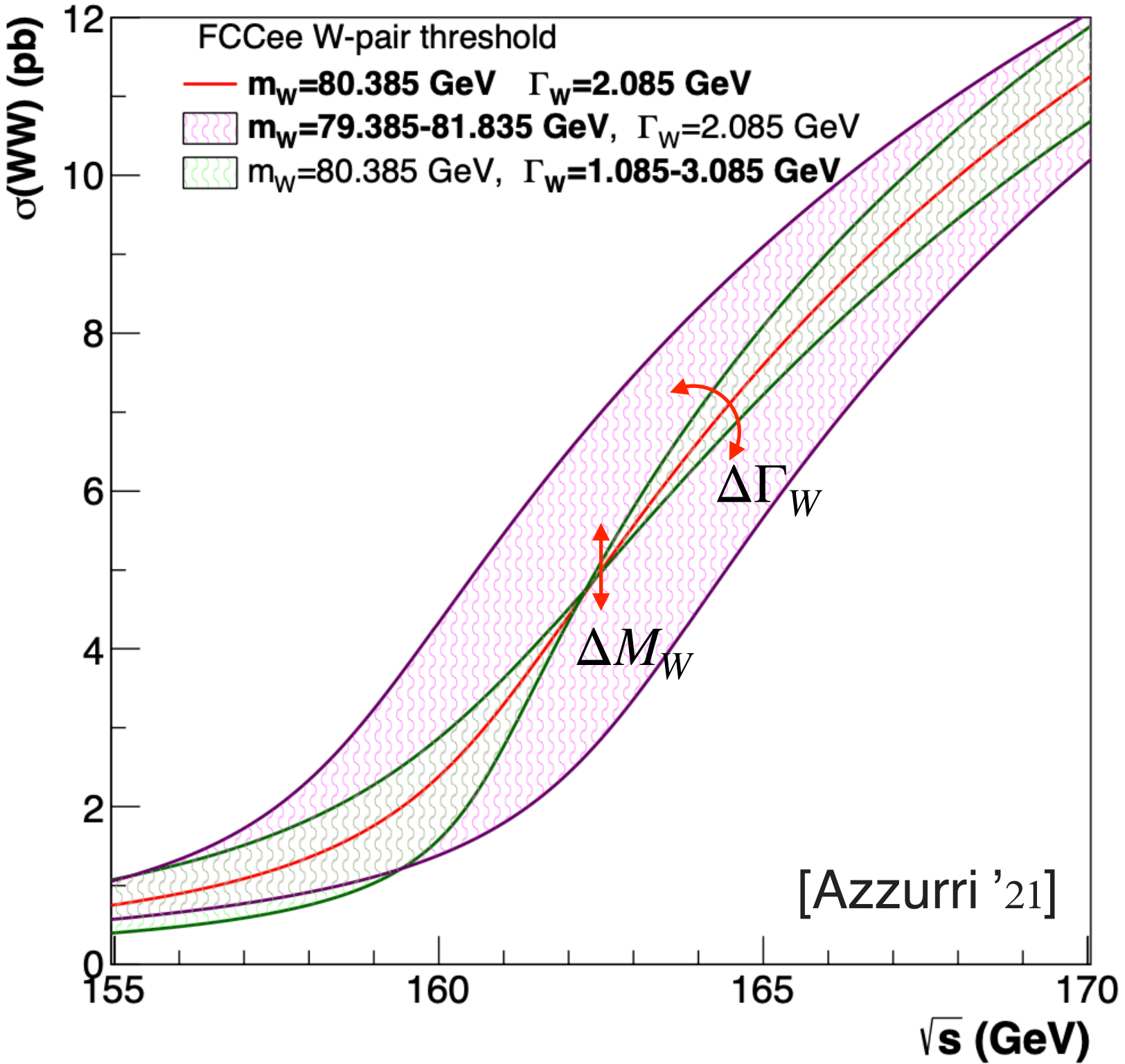
# WW threshold scan and W mass and width

- TH cross section currently known accurately at NLO (EW) + NNLO (unstable particles EFT) sufficient for  $\delta m_W \sim 5\text{-}6\text{ MeV}$

[Denner, Dittmaier, Roth, Wieders '05; Actis, Beneke, Falgari, Schwinn '08]

	$\sigma(e^-e^+ \rightarrow \mu^- \bar{\nu}_\mu u \bar{d} X)(\text{fb})$				
$\sqrt{s}$ [GeV]	Born	Born (ISR)	NLO	$\hat{\sigma}^{(3/2)}$	$\sigma_{\text{ISR}}^{(3/2)}$
158	61.67(2)	45.64(2) [−26.0%]	49.19(2) [−20.2%]	−0.001 [−0.0‰]	0.000 [+0.0‰]
161	154.19(6)	108.60(4) [−29.6%]	117.81(5) [−23.6%]	0.147 [+1.0‰]	0.087 [+0.6‰]
164	303.0(1)	219.7(1) [−27.5%]	234.9(1) [−22.5%]	0.811 [+2.7‰]	0.544 [+1.8‰]
167	408.8(2)	310.2(1) [−24.1%]	328.2(1) [−19.7%]	1.287 [+3.1‰]	0.936 [+2.3‰]
170	481.7(2)	378.4(2) [−21.4%]	398.0(2) [−17.4%]	1.577 [+3.3‰]	1.207 [+2.5‰]

- Can be further improved using NLL ISR
- Effect of tight selection cuts in the EFT to be understood



Reaching the stat. uncertainty of 0.3–0.5 MeV is very demanding

$$\Delta m_W(T) = \left( \frac{d\sigma_{WW}}{dm_W} \right)^{-1} \Delta\sigma_{WW}(T)$$

$$\Delta\sigma_{WW}(T) < 0.8 \text{ fb}$$



# W mass extraction from hadronic and semi-leptonic decays

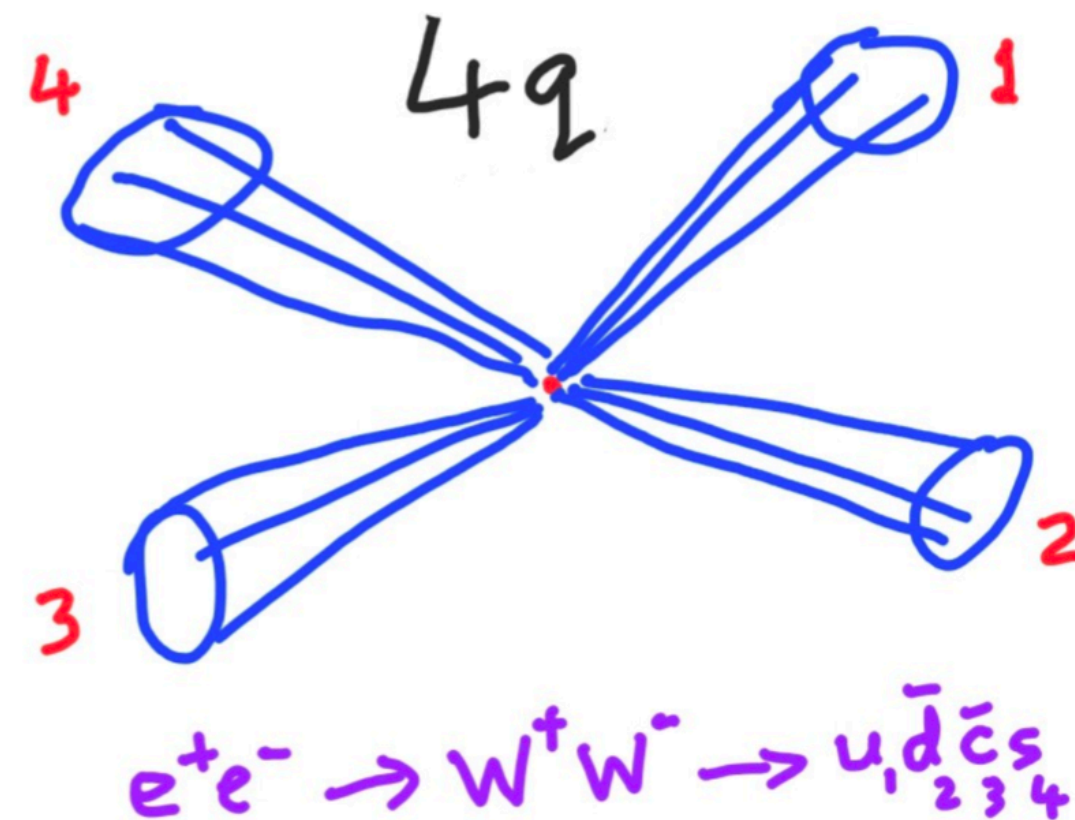
- Very good experimental resolution with momentum conservation fit (4C or 5C), competitive with threshold scan

- Theory modelling harder, with systematics **yet to be precisely assessed**

- ▶ Control over QED ISR (NLL available)
- ▶ EFT resonant aspects near threshold
- ▶ Backgrounds: 2f & 4f final states
- ▶ Colour reconnection in hadronic channels

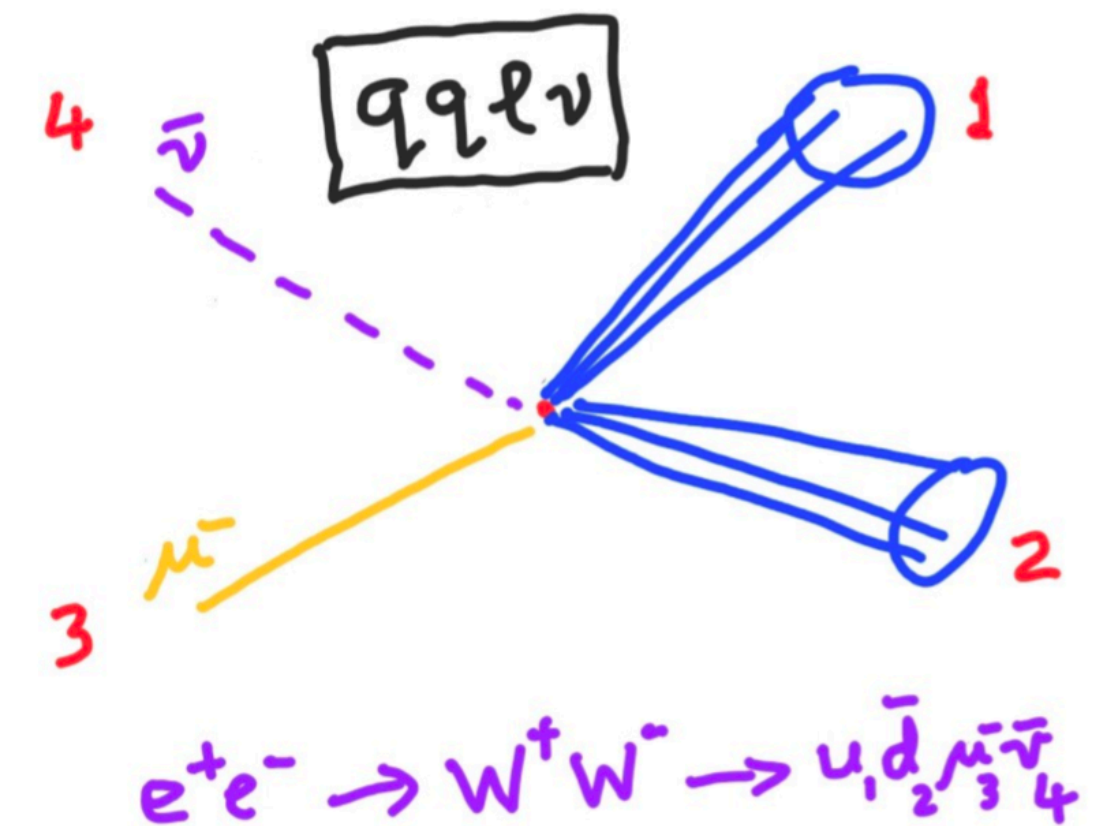
[G. Wilson's talk @ CERN FC workshop 2022]

fully hadronic  $q\bar{q}q\bar{q}$



$$B_h^2 = 45.4\%$$

semi-leptonic  $q\bar{q}\ell\nu_\ell$

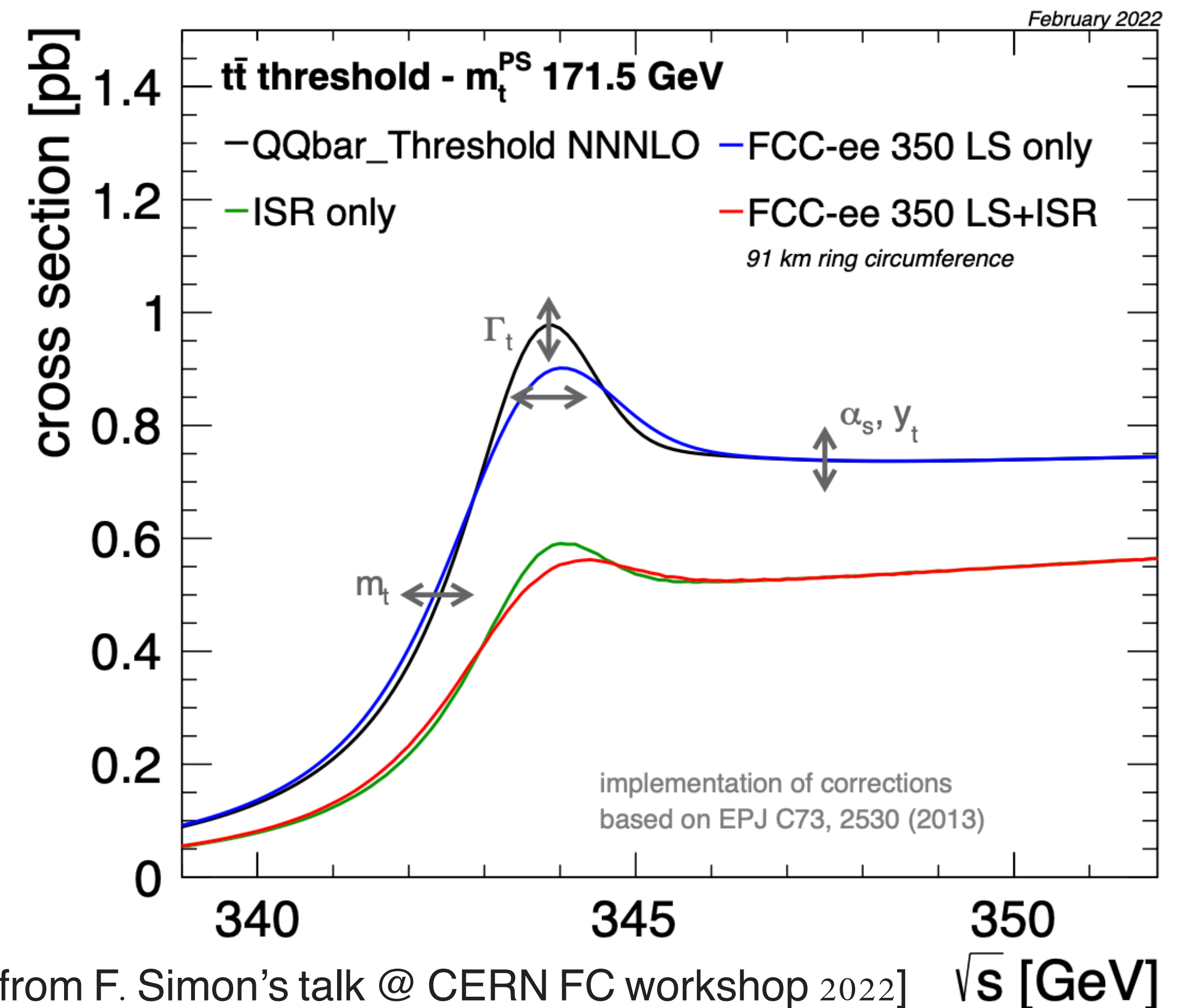


$$6B_\ell B_h = 43.9\%$$

# Top physics

- Huge potential from threshold scan: up to per-mille accuracy on cross section & asymmetries
- Access to top mass and width, as well as strong coupling and top Yukawa coupling
- e.g. projected exp. target for top mass  $\delta m_t \sim 20 \text{ MeV}$

Great challenge for theory to match this precision;  
intrinsic (e.g. higher order) & parametric (e.g. strong coupling from Z pole) uncertainties



# Top physics: theory for threshold scan

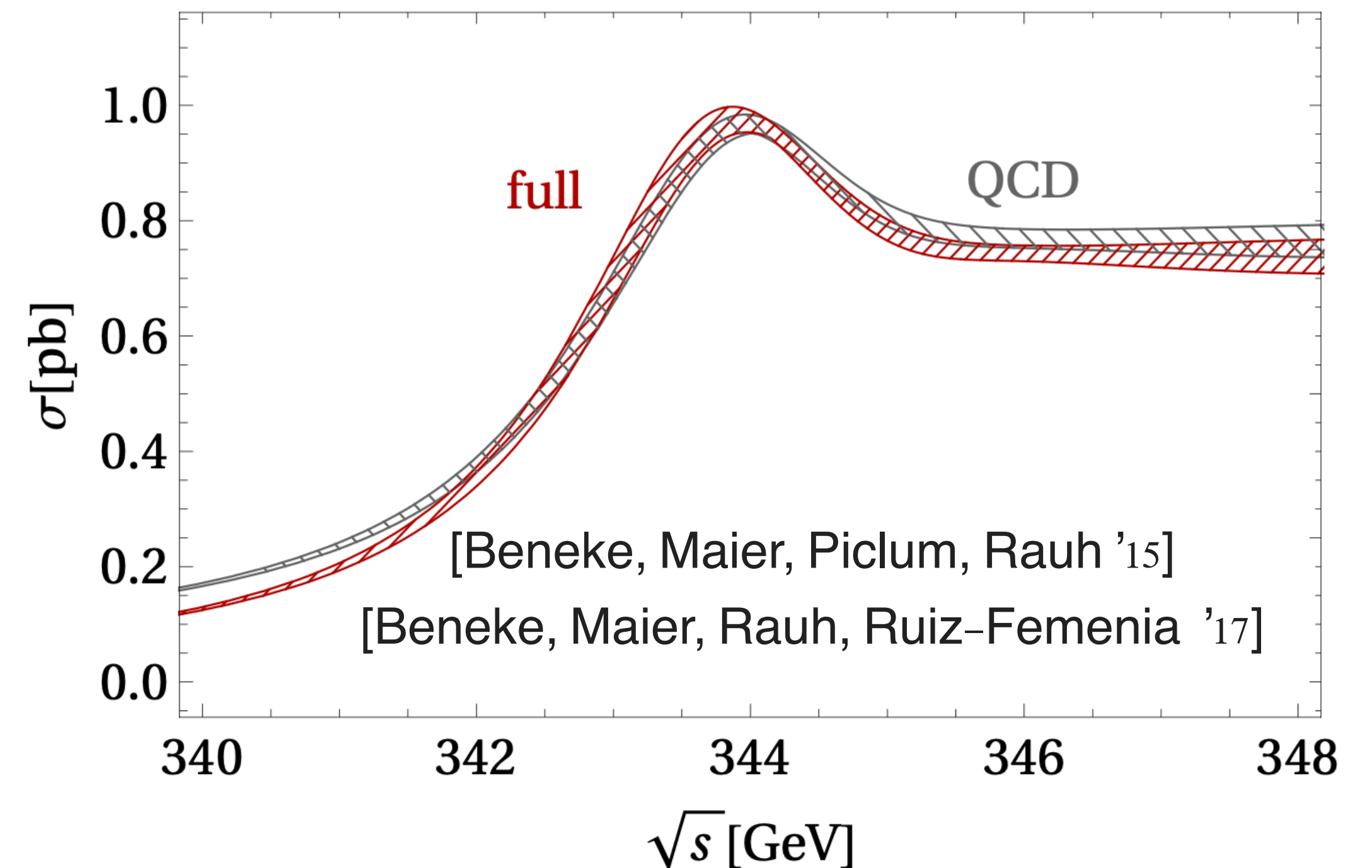
- PNRQCD predictions known to N<sup>3</sup>LO (also including EW+non-resonant effects @ NNLO)

$$R \sim v \sum_k \left( \frac{\alpha_s}{v} \right)^k \cdot \left\{ \underbrace{1}_{\text{(LO)}} ; \underbrace{\alpha_s, v}_{\text{(NLO)}}; \underbrace{\alpha_s^2, \alpha_s v, v^2}_{\text{(NNLO)}}; \underbrace{\alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3}_{\text{(N3LO)}}; \dots \right\}$$

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser '15]

- Uncertainty in top mass (potential subtracted)  $\delta m_t \sim 40 \text{ MeV}$ . Towards exp. target (20 MeV):

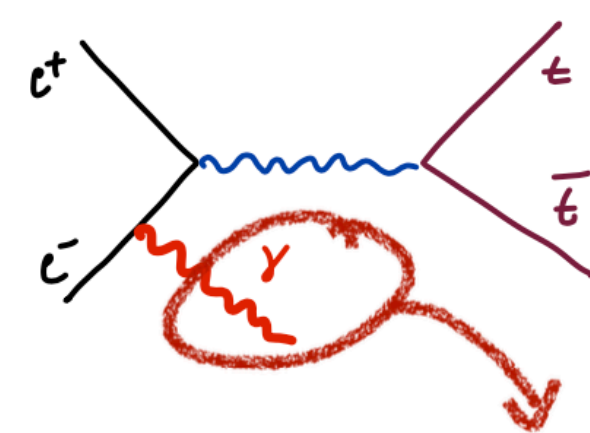
- Some improvements already from **matching of N<sup>3</sup>LO+NNLL** (NNLL from Hoang et al.)
- Needs **NLL ISR** (possibly including soft modes)
- Ultimately might require **N<sup>4</sup>LO in PNRQCD needed** (currently out of reach)





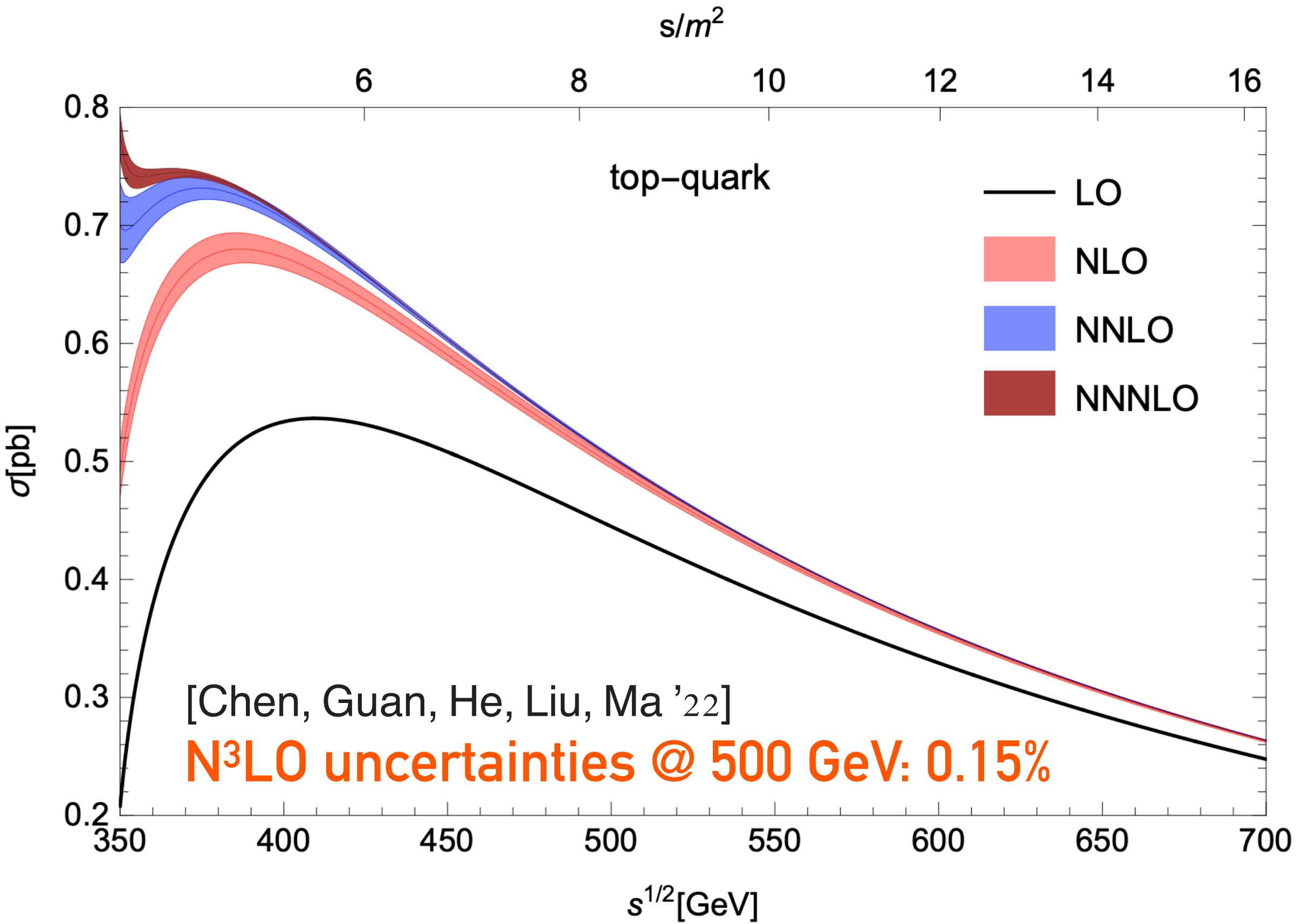
# Top physics: above threshold & continuum (mainly ILC/CLIC)

- Continuum: **target is 0.1% on cross section**. N<sup>3</sup>LO QCD recently calculated but NNLO EW is necessary
- Top mass from radiative return from ISR photon: required matching of continuum and threshold calc<sup>ns</sup>
  - TH unc. doesn't seem to be dominant source of unc.
  - Possible access to running of (MSR) mass



[Boronat, et al. '19]

cms energy	CLIC, $\sqrt{s} = 380$ GeV		ILC, $\sqrt{s} = 500$ GeV	
luminosity [fb <sup>-1</sup> ]	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV



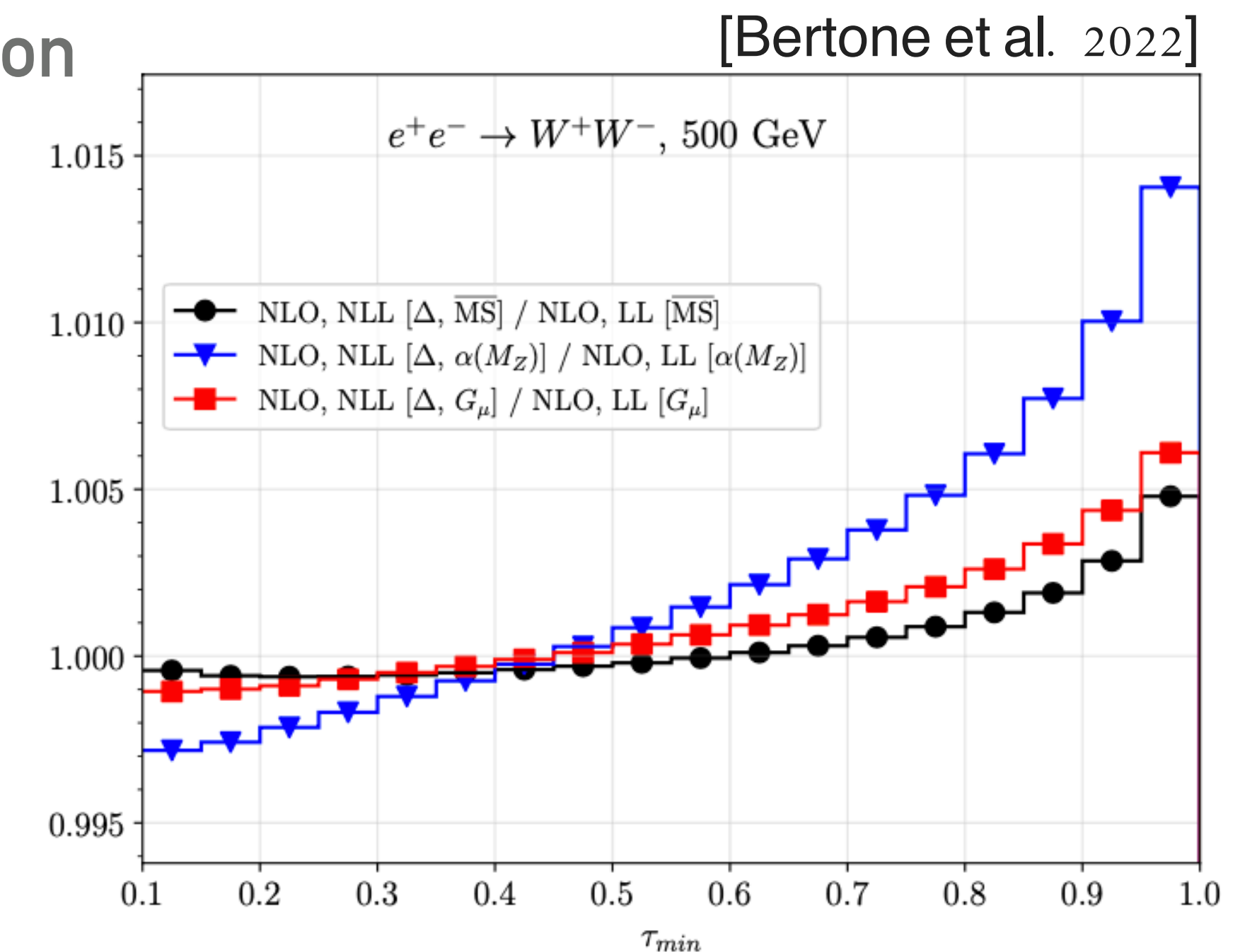
# QED collinear factorisation

- Central component in FCCee precision phenomenology (Z, WW, tt, ZH,...)
- Recently important progress in formulating collinear factorisation (as opposed to YFS) beyond LO/LL.  
**NLL sizeable (% level) and process/observable dependent.** E.g. corrections to total rates ( $\tau_{\min} = \frac{M^2}{s}$ )
  - ▶ NNLL hard but within reach of modern perturbative techniques  
e. g. [Bluemlein et al. '12-'21]
  - ▶ Ongoing discussions as to whether a simultaneous resummation of soft and collinear corrections is necessary

•  $\sqrt{Q^2} = 500 \text{ GeV}$        $\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}, \quad L = \log \frac{Q^2}{m^2}$

[Example from S. Frixione 2022]

$$\begin{aligned}
 L = 24.59 &\implies \frac{\alpha}{\pi} L = 0.068 \\
 0 \leq m_{ll} \leq m_Z, \quad \ell = 1.46 &\implies \frac{\alpha}{\pi} \ell = 0.0036 \\
 m_Z - 1 \text{ GeV} \leq m_{ll} \leq m_Z, \quad \ell = 4.51 &\implies \frac{\alpha}{\pi} \ell = 0.01
 \end{aligned}$$





# Parton showers & event generators

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# Event generators impact FCC physics programme in toto

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- Perturbative calculations often available for (semi-)inclusive observables. Event generators vital for, e.g.
  - Exclusive hadronic observable (e.g. jets)
  - Beam & detector calibration
  - Training of Machine Learning tools for jet/flavour tagging
- Matching the accuracy goals of FCCee poses an **outstanding challenge**:
  - Perturbative accuracy of parton shower algorithms
  - Matching to higher order calculations for hard scattering
  - Treatment of heavy resonances
  - Non-perturbative QCD
  - QED corrections (jointly with QCD)

# Perturbative accuracy of parton showers

## Possible NLL dipole-shower solutions for $e^+e^-$

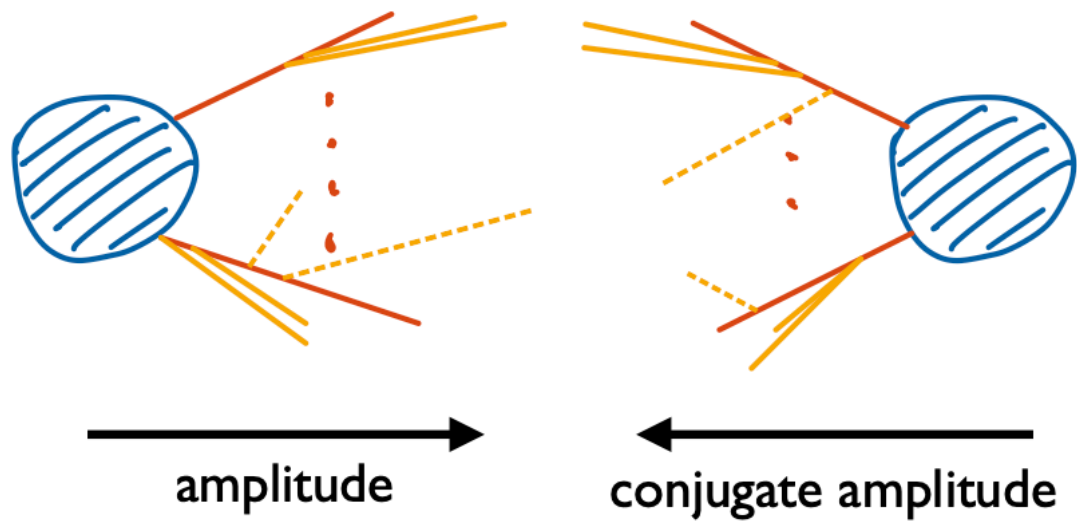
[M. van Beekveld 2023]	Ordering	Kinematic map		Tests
		Dipole-local	Global	
<b>PanScales showers</b> [2002.11114]	<b>PanLocal</b> (Dipole and antenna) $0 < \beta < 1$	$+, -, \perp$		Fixed- and all-order numerical tests for different observables for $e^+e^-$ and $pp$ (colour singlet)
	<b>PanGlobal</b> $0 \leq \beta < 1$	$+, -$	$\perp$	
<b>Alaric</b> [2208.06057]	$\beta = 0$	$+$	$-, \perp$	Numerical tests for global event shapes
<b>Deductor</b> [2011.04777]	<b>Deductor <math>k_t</math></b> $\beta = 0$	$+$ (Also formulation with $+, -, \perp$ )	$-, \perp$	Analytical and to some extent numerical for thrust
	<b>Deductor <math>\Lambda</math></b> $\beta = 1$	$+$	$-, \perp$	
<b>Manchester-Vienna</b> [2003.06400]	$\beta = 0$	$+$	$-, \perp$	Analytical for thrust and multiplicity

Showers also differ on the implementation of the splitting functions and how the global imbalance is redistributed

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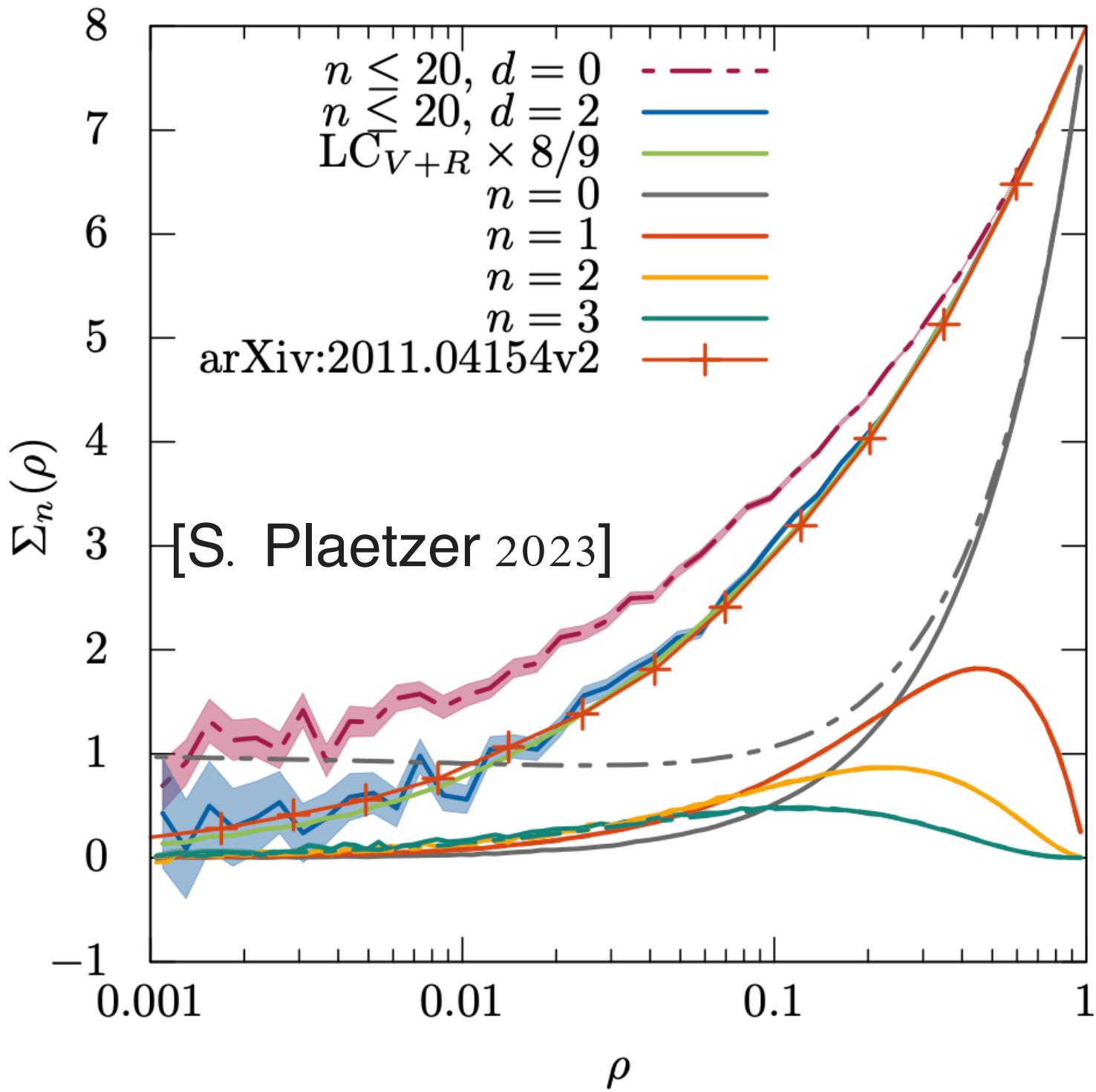
## All have different approaches to assess NLL accuracy

- New technology to improve logarithmic accuracy on a more systematic basis: current status is NLL, with uncertainties at the  $\gtrsim 10\%$  level
- Promising developments also re. subleading colour effects
- FCCee demands at least NNLL QCD accuracy, and arguably higher



## e.g. NGLs in rapidity slice

singlet  $\rightarrow gg$  spectrum

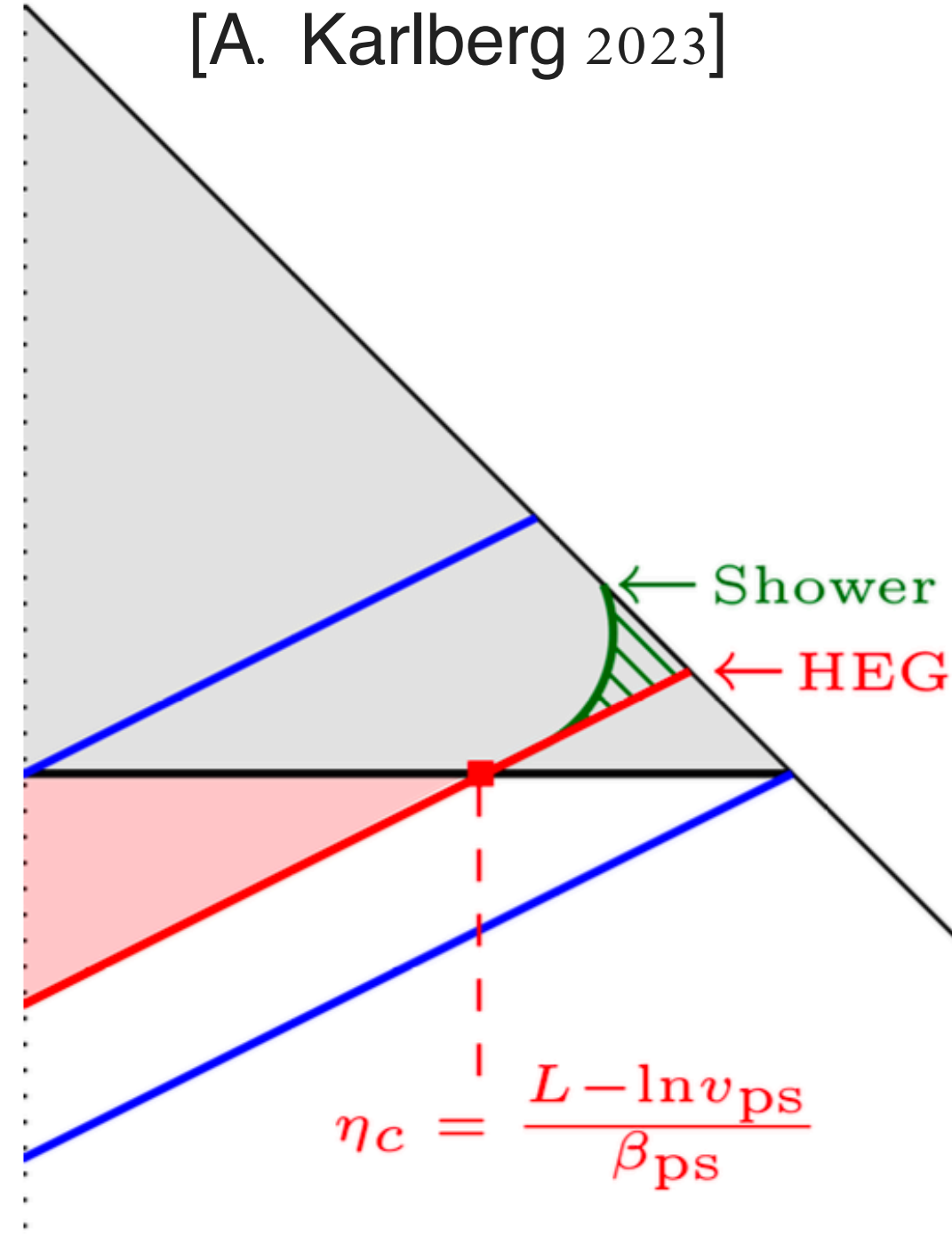




# Logarithmic accuracy aware matching

## POWHEG $_{\beta}$ and NNDL accuracy

[A. Karlberg 2023]



- At DL accuracy the answer we are after is given by

$$\Sigma(O < e^L) = e^{-\bar{\alpha}L^2}, \quad \bar{\alpha} = \alpha_s$$

- If the shower and HEG contours line up everywhere, we would get that answer. If they disagree in the hard-collinear region, we instead get (neglecting terms beyond NNDL)

$$\Sigma(O < e^L) = e^{-\bar{\alpha}L^2} \left[ 1 + 2 \left( e^{-\bar{\alpha}\beta L^2} - 1 \right) \bar{\alpha}\Delta \right] \quad (1)$$

- $\Delta$  is the effective area of one shaded green region, which for PanLocal and  $\gamma \rightarrow q\bar{q}$  is given by

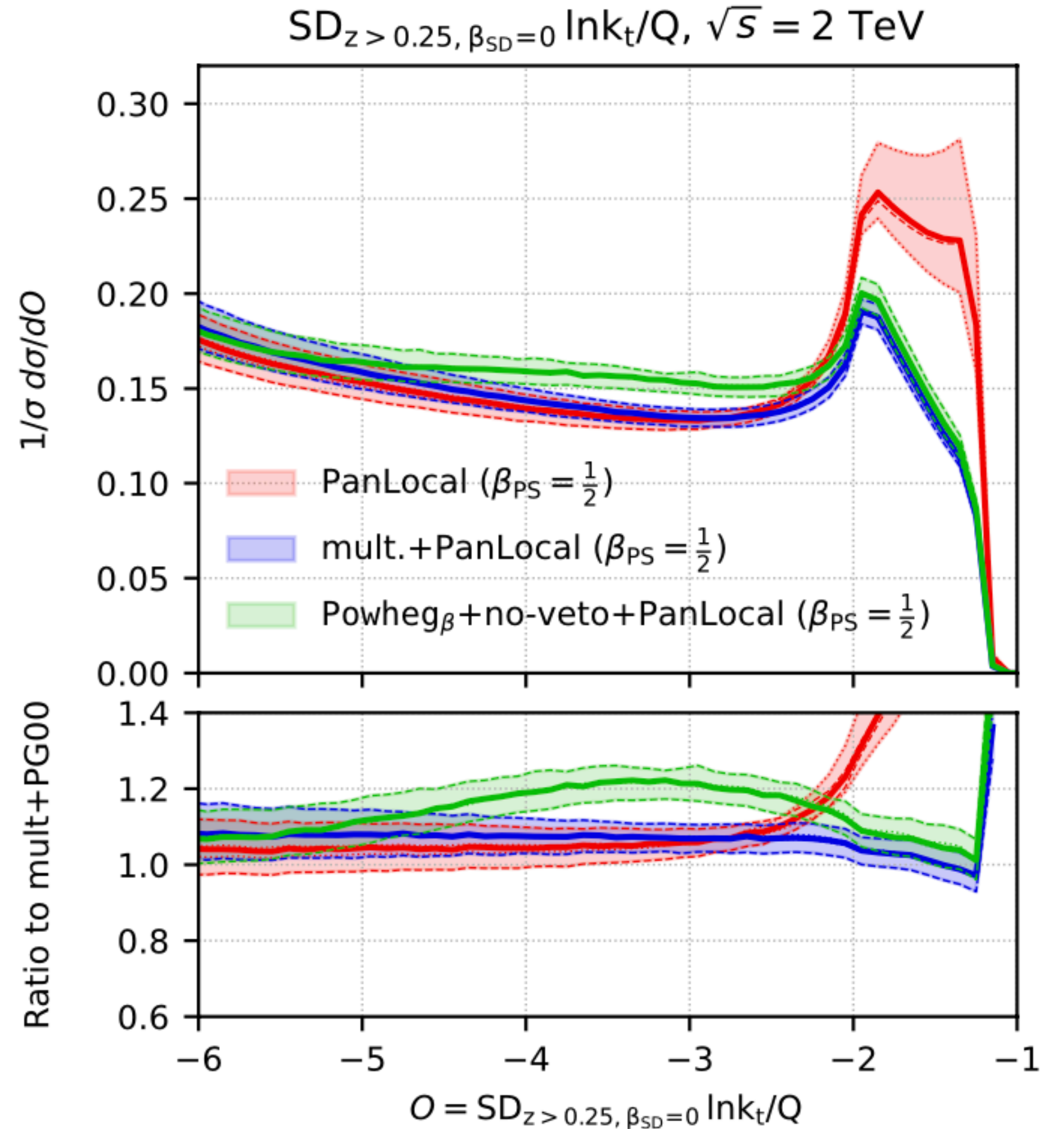
$$\bar{\alpha}\Delta = \frac{2C_F\alpha_s}{\pi} \cdot \frac{4\pi^2 - 15}{24}.$$

- Since  $\Delta$  is  $\mathcal{O}(1)$  this gives rise to a tower  $\propto \alpha_s (\bar{\alpha}_s L^2)^n$  in eq. (1), which breaks NNDL.

- Newly developed NLL showers constrain matching to N(N)L0
- Well known matching schemes may be affected by breaking of logarithmic accuracy in specific observables
- Further work needed to upgrade NNLO generators to NLL accuracy

[Hamilton, Karlberg, Salam, Scyboz, Verheyen 2023]

e.g. Soft-drop jets  $k_t$



# Treatment of heavy resonances

- Correct handling of resonances at higher orders essential at FCA
- Technology for resonance-aware NLO+PS already available in main MC generators.

- ▶ Logarithmic accuracy of matching with virtuality-preserving mappings

- MG5\_aMC@NLO [Frederix, Frixione, Papanastasiou, Prestel, Torrielli '16]

- ▶ Phase space remapping & prescription for including the resonance in LHE
- ▶  $pp \rightarrow tj$
- ▶ SMC: Herwig6, Pythia8<sup>†</sup>

- I focus on POWHEG BOX RES+Pythia8 and POWHEG BOX RES+Herwig7

[T. Ježo 2023]

- Whizard [Chokouf  Nejad, Kilian, Lindert, Pozzorini, Reuter, Weiss '16]

- ▶ Resonance-aware FKS with  $Z \rightarrow b\bar{b}$  and  $H \rightarrow b\bar{b}$  RH
- ▶ Fixed order  $e^+e^- \rightarrow t\bar{t} \ \& \ e^+e^- \rightarrow t\bar{t}H$

- Sherpa [H che, Liebschner, Siebert '18]

- ▶ Resonance-aware CS subtraction
- ▶ Fixed order  $e^+e^- \rightarrow t\bar{t} \ \& \ pp \rightarrow t\bar{t}$

- Must be analysed/revisited in light of recent and future NLL developments

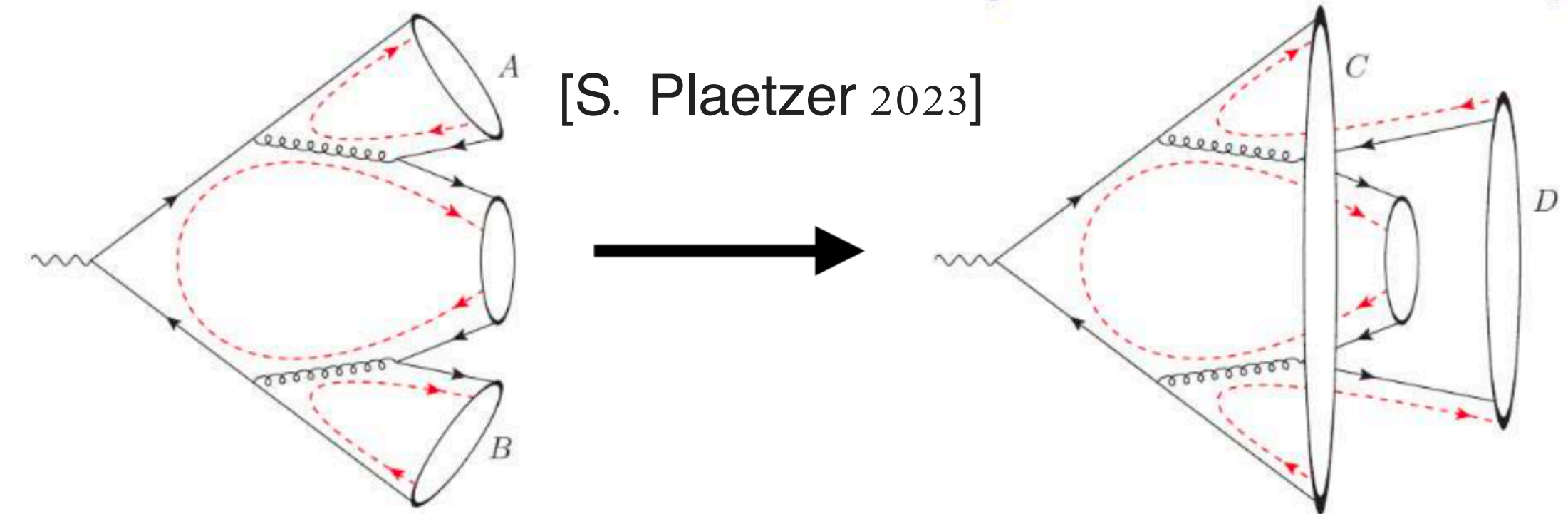
- ▶ Logarithmic accuracy of matching with virtuality-preserving mappings
- ▶ Higher order showers for reactions with massive quarks (e.g.  $t\bar{t}$ ,  $WW \rightarrow$  jets)
- ▶ Non-relativistic effects (NRQCD, unstable particle EFT), currently out of reach in MCs



# Non-perturbative QCD

- Modelling of NP effects is a crucial goal for precision programme

e.g. CR inspired by amplitude-level evolution in PS



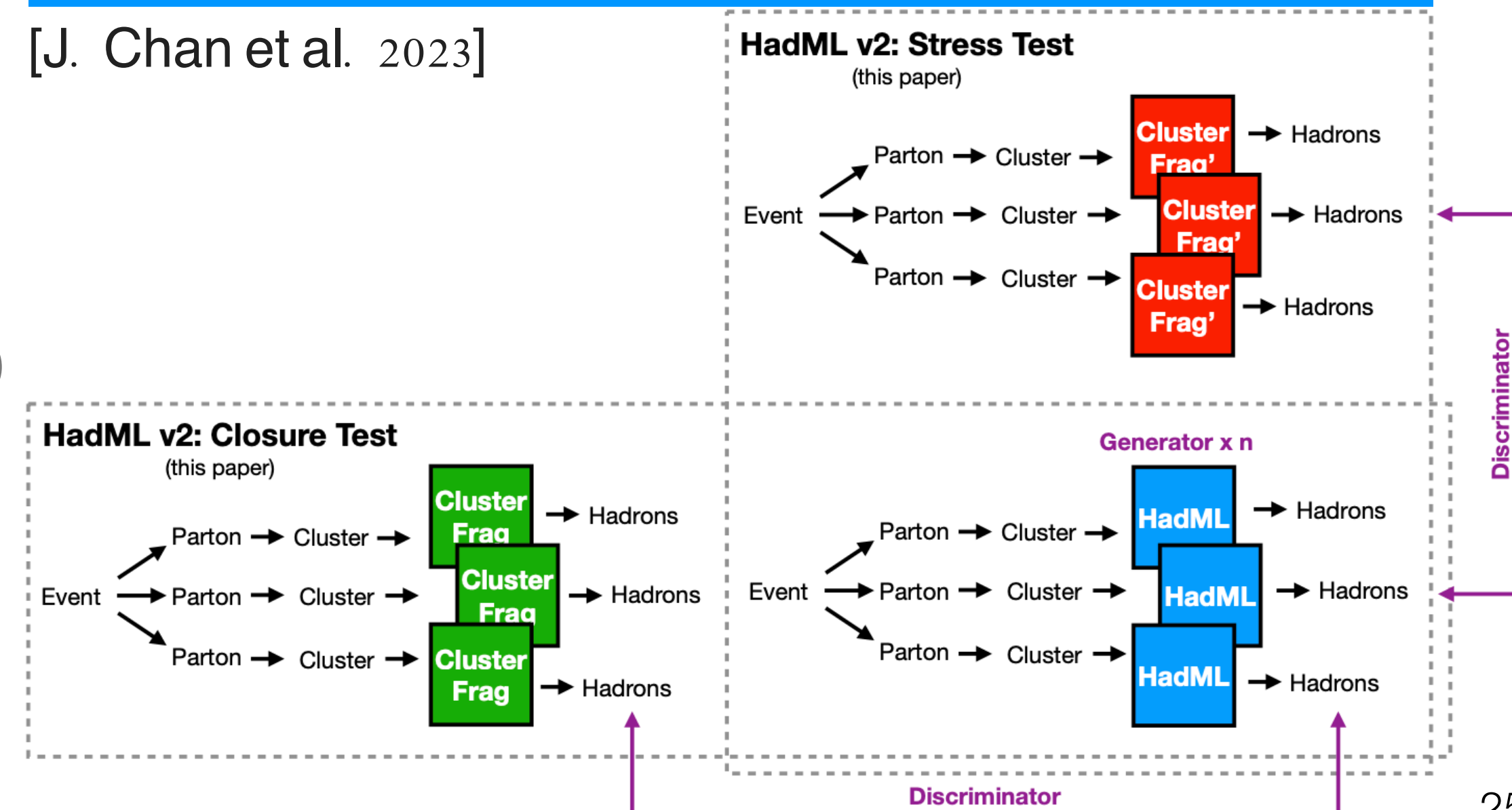
- ▶ Spectrum of old&new models of NP physics

- ▶ Input from FCCee is highly beneficial:

- Span of c.o.m. energies crucial for tuning, jointly w/ higher order MCs
- High-purity samples of gluon/heavy-quark jets beneficial for fragmentation models (used e.g. in jet tagging)
- Potential of cross-benefit between stages of FCCee (e.g. tunes in  $Z \rightarrow \text{jets}$  useful for  $ZH$ , CR at  $WW \rightarrow \text{jets}$ , ...)
- ▶ Crucial to explore implications of recent analytic calc<sup>ns</sup> (in large- $n_F$ ) for MC generators (e.g. mappings)

e.g. GANs as hadronisation model ( $\pi$ s only)

[J. Chan et al. 2023]



# First steps towards NLL QED (ISR) effects in parton showers

## QED Parton Shower

see for instance review in 0912.0749

[G. Stagnitto 2023]

Introduction of a cutoff  $x_+ = 1 - \epsilon$ , with  $\epsilon \ll 1$ , to regularise splitting kernels:

$$P_+(z) = \theta(x_+ - z)P(z) - \delta(1 - z) \int_0^{x_+} dx P(x)$$

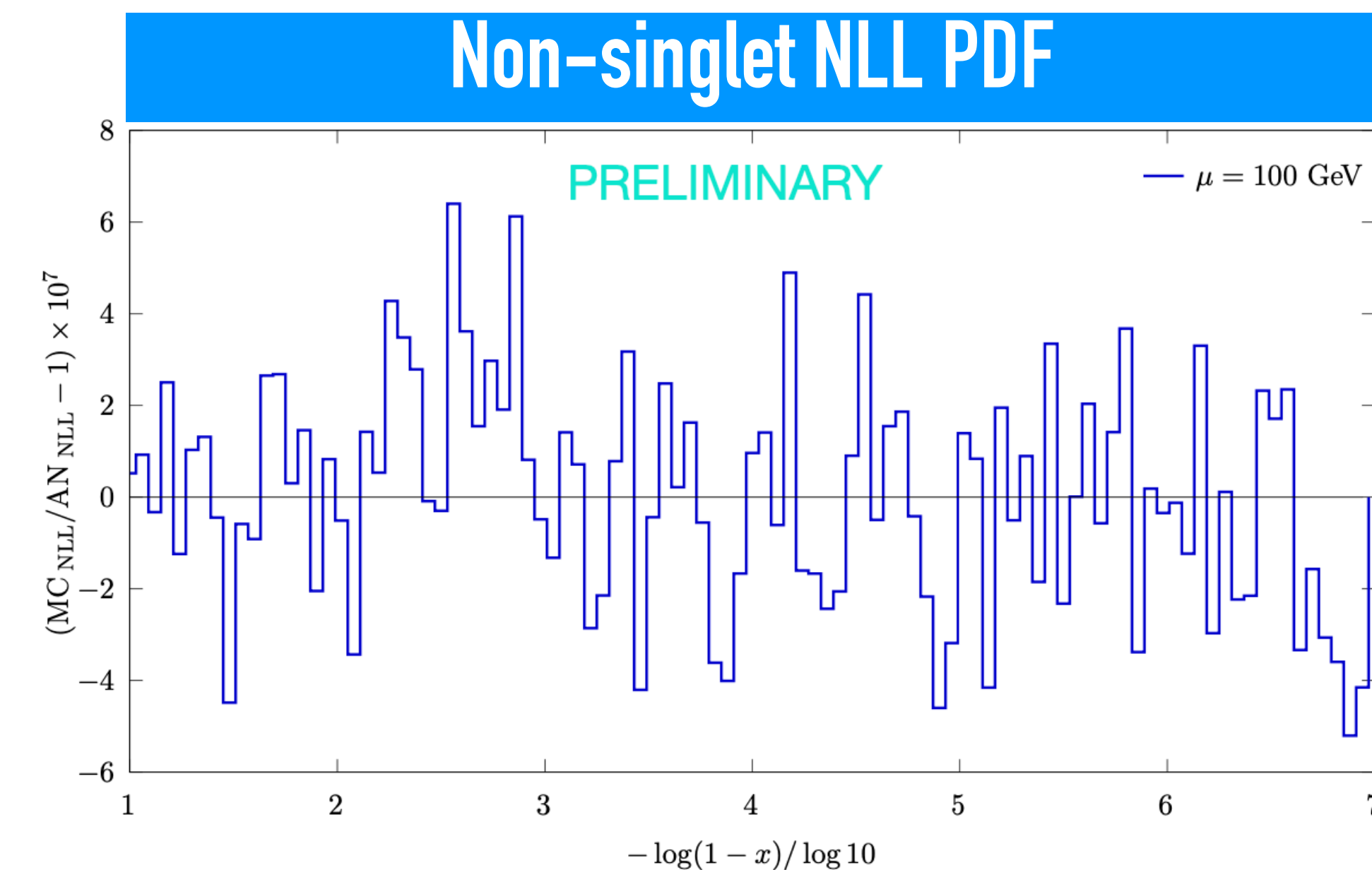
By introducing a Sukadov form factor:  $\Pi(s_1, s_2) = \exp \left( -\frac{\alpha}{2\pi} \int_{s_2}^{s_1} \frac{ds'}{s'} \int_0^{x_+} dz P(z) \right)$

one can recast the evolution equation in an iterative integral form:

$$D(x, s) = \sum_{n=0}^{\infty} \prod_{i=1}^n \left\{ \int_{m_e^2}^{s_{i-1}} \frac{ds_i}{s_i} \Pi(s_{i-1}, s_i) \frac{\alpha}{2\pi} \int_{x/(z_1 \dots z_{i-1})}^{x_+} \frac{dz_i}{z_i} P(z_i) \right\} \Pi(s_n, m_e^2) D \left( \frac{x}{z_1 \dots z_n}, m_e^2 \right)$$

which can be solved by means of a MC algorithm

- New extension of QED collinear factorisation to NLL provides the ingredients for a NLL (next-to-single-log) accurate evolution
- Currently inclusive treatment of radiation, more work needed for fully differential generator & interleaved QED $\otimes$ QCD PS



- Astounding experimental programme at FCCee, drastic reduction of statistical (and systematic) uncertainties: theory precision likely to be among the main bottlenecks
- Many (if not all) areas of theory calculations need to be involved (fixed order QCD + EW, resummations in QCD & QED, effective field theories, non-perturbative QCD, event generators , ...)
- Most challenges are technical in nature: hard calculations, currently beyond reach but likely to become achievable with the evolution of the field at the LHC in the coming decade(s), and substantial work
- Some deep conceptual issues, which need significant breakthroughs to improve their understanding: e.g. non-perturbative QCD (hadronisation, colour reconnection), currently a bottleneck in several studies
- Many new opportunities from high-quality experimental data, crucial to think of how to exploit it to improve on modelling aspects and theory uncertainties (e.g. heavy flavour & gluon fragmentation, hadronisation modelling, ...)