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Progress report

IN-SITU ACCEPTANCE DETERMINATION FOR DIPHOTON EVENTS AT FCC-EE

Follow-up of this presentation on 17/04/23

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Gratefully acknowledging discussions with
Michael Benedikt, Alain Blondel, Mogens Dam, Christophe Grojean, Katsunobu Oide, Emmanuel Perez

Scientific motivation

- Well described in the 1st and 3rd case studies submitted to Snowmass 2021

See https://www.overleaf.com/read/nknybgrqqwbp

3 Towards an ultimate measurement of the Z peak cross section

The Z peak cross section can be measured at FCC-ee with hadronic and dimuon events produced at $\sqrt{s} = 91.2\,\mathrm{GeV}$, with a potential relative statistical precision of $\mathcal{O}(10^{-6})$. Together with the ratio R_ℓ (Section 1), this quantity allows the determination of the number of light neutrino types.

A limiting systematic uncertainty comes from the absolute determination of the integrated luminosity. A determination with low-angle Bhabha scattering is likely to be limited by a relative theoretical precision of $\mathcal{O}(10^{-4})$ [6], but that might not be the case for large angle diphoton production, $e^+e^- \to \gamma\gamma$ (to be checked with actual full two-loop calculations [7]). The requirements on the detector design to measure the absolute luminosity with diphoton events, and in particular to separate these events from the large angle Bhabha background, will be studied.

1 Towards an ultimate measurement of $R_\ell = \frac{\sigma(\mathrm{Z}\to\mathrm{hadrons})}{\sigma(\mathrm{Z}\to\mathrm{leptons})}$

The ratio R_ℓ can be measured at FCC-ee with an event sample of 5×10^{12} Z produced at $\sqrt{s} = 91.2\,\mathrm{GeV}$, and therefore benefits from a potential relative statistical precision of $\mathcal{O}(3 \times 10^{-6})$ for each lepton type. It is a key quantity [4] that serves – in conjunction with the total Z decay width and the peak hadronic cross section – as input to several fundamental quantities:

i) the measurement of the leptonic Z partial width $\Gamma_{\ell\ell}$, a very clean electroweak observable whose relation to the Z mass is the ρ (or T) parameter, and unaffected by $\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2)$, with a 10^{-5} relative precision;

ii) the measurement of the strong coupling constant $\alpha_{\mathrm{S}}(m_{\mathrm{Z}}^2)$ with an absolute experimental uncertainty below 0.0001 (Section 5);

iii) the measurement of the number of light neutrinos with a precision of 0.0004.

Experience from LEP showed that a limiting systematic uncertainty comes from the knowledge of the geometrical acceptance for lepton pairs. The requirements on the detector design to match the statistical precision will be studied in the full context of the constraints from the interaction region layout. As a by-product, the determination of the geometrical acceptance for the $e^+e^- \to \gamma\gamma$ process (which may be used for the measurement of the absolute luminosity, potentially with a statistical precision of a few 10^{-5}) will be investigated – see also Section 3. The knowledge of the acceptance for the more abundant hadronic Z decays, a much easier problem at LEP, will need to be verified at the same level of precision.

Lumi measurement with $e^+e^- \to \gamma\gamma$

Acceptance determination for $e^+e^- \to \ell^+\ell^-$

- A case study is aimed at detector requirements and theory precision constraints
  - To match systematic uncertainty to the FCC-ee expected statistical precision
    - → Just pick your case study !

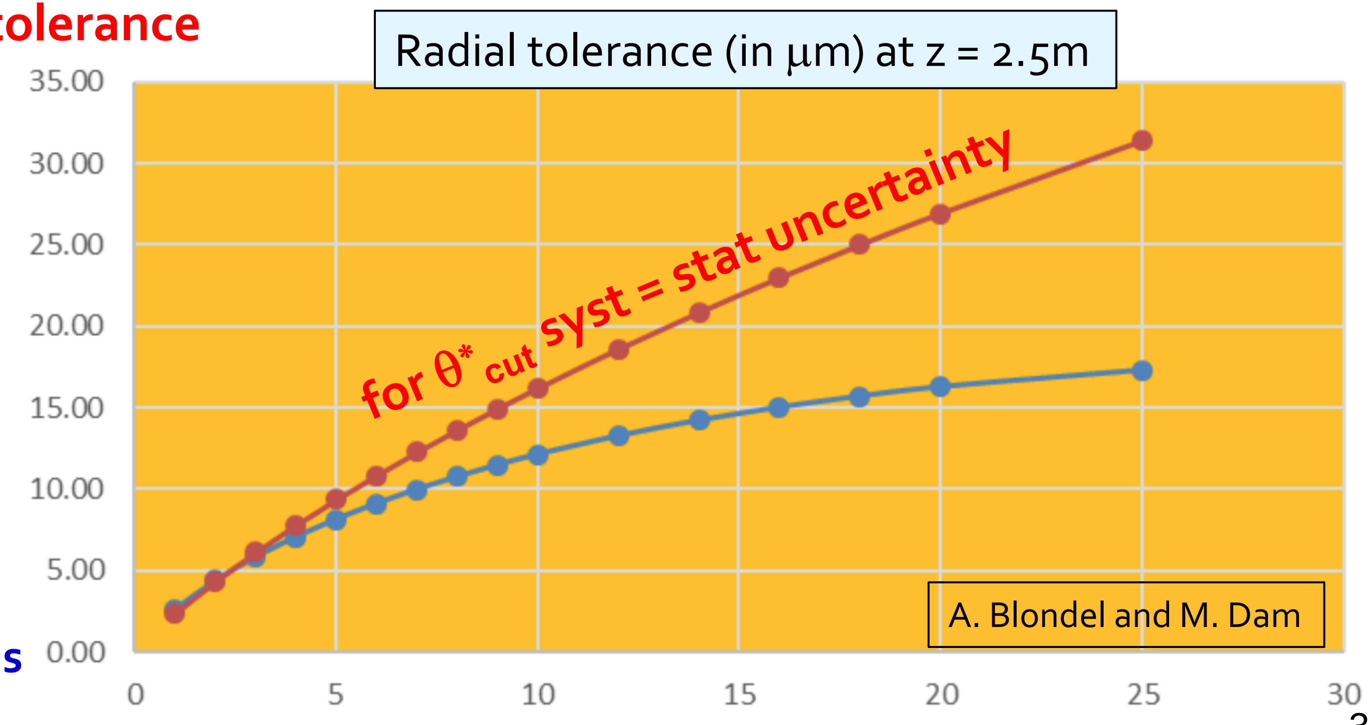
Luminosity measurement with $e^+e^- \to \gamma\gamma$ events

- At the Z pole, σ_{tot} = 60 (40) pb for θ^*_{cut} = 10° (20°) – Total luminosity = 45 ab^{-1} / expt
  - Total of 3 (2) 10^9 $e^+e^- \to \gamma\gamma$ events / expt $\Delta L/L \sim 2.10^{-5}$ stat
    - Cross section is strongly peaked forward/backward
      - ➔ Major systematic uncertainty : θ^*_{cut} accuracy

$$\frac{d^2\sigma}{d\cos\theta^* d\phi^*} = \frac{\alpha^2_{em}}{s}\frac{1+\cos^2\theta^*}{\sin^2\theta^*}$$

- Challenging detector design tolerance
  - For θ^*_{cut} = 20°
    - $\Delta r \ll 27$ µm & $\Delta z \ll 75$ µm
  - For θ^*_{cut} = 10°
    - $\Delta r \ll 16$ µm & $\Delta z \ll 90$ µm
  - Even smaller for dileptons!
    - Δr (Δz) $\ll$ 8 (22) µm at 20°

Also relative between the two endcaps



- Angular cut accuracy (10/20°)
  - $\Delta\theta^* \ll 6.5$ (10) µrad for $\gamma\gamma$
  - $\Delta\theta^* \ll 5$ (3) µrad for dileptons

NB. The detector is not in the collision rest frame

- Crossing angle α_0 in the horizontal plane for crab waist collisions: $\alpha_0 \simeq 30$ mrad

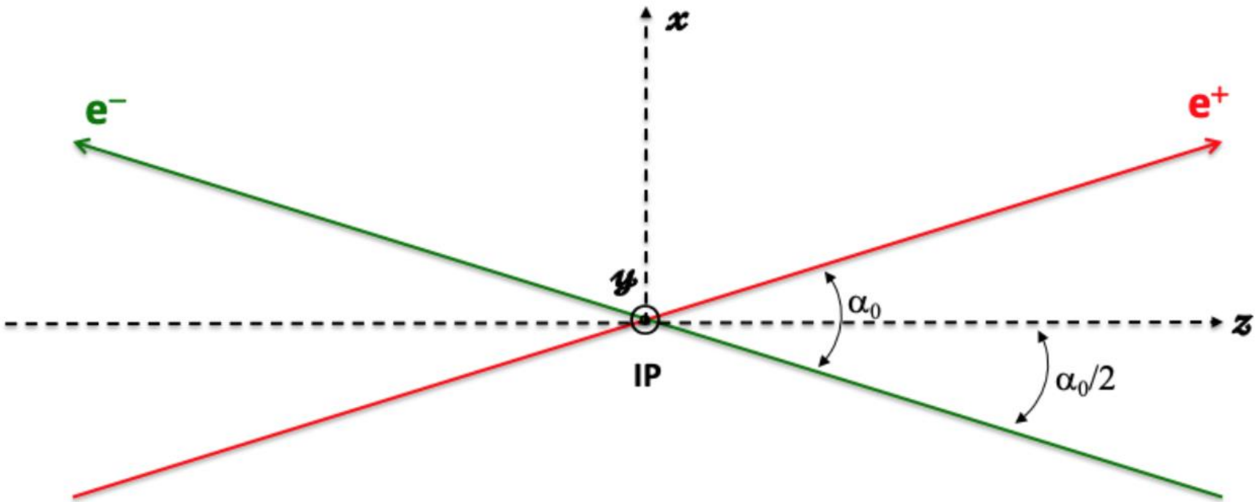


Fig. 1 At FCC-ee, the beams cross with an angle α_0 in the horizontal plane (x, z) (the plane of this figure). The vertical axis y, perpendicular to this paper sheet - i.e., to the horizontal plane – and the interaction point (IP) are also represented.

- Longitudinal boost ε_0 due to uneven placement of RF along the ring: $\varepsilon_0 \simeq 0.02\%$

$$\boxed{E_e^{\pm} = E(1 \pm \varepsilon_0)}$$

NB. The detector is not in the collision rest frame

- **It originally appeared as an additional burden**
  - Lorentz transforms are needed in order to go from the laboratory to the CM frame and back

$$\cos\theta = \frac{\cos\theta^* \cos\frac{\alpha}{2}\sqrt{1-\varepsilon^2} + \varepsilon\cos\frac{\alpha}{2}\left(1 + A^*\cos\frac{\alpha}{2}\sqrt{1-\varepsilon^2}\right)}{1 + A^*} \quad \text{with} \quad A^* = \sin\frac{\alpha}{2}\sin\theta^*\cos\phi^* + \varepsilon\cos\theta^*$$

$$\cos\theta^* = \frac{\cos\theta\cos\frac{\alpha}{2}\sqrt{1-\varepsilon^2} + \varepsilon\cos\frac{\alpha}{2}\left(1 - A\cos\frac{\alpha}{2}\sqrt{1-\varepsilon^2}\right)}{1 - A} \quad \text{with} \quad A = \sin\frac{\alpha}{2}\sin\theta\cos\phi + \varepsilon\cos\theta$$

  - A polar angle (θ*) cut in the collision rest frame depends on θ and ϕ in the laboratory frame

    $\longrightarrow$ Detector design tolerance on θ AND ϕ !

- **Reality may actually be significantly brighter**
  - The large crossing angle, if known precisely, provides an absolute angle scale to each event
    - The crossing angle "propagates" to final state particles through energy/momentum conservation

      Fundamental for an absolute in situ determination of θ^*_{cut} with $e^+e^- \to \gamma\gamma$ events
  - The dependence of θ* on ϕ is not too large
    - Tolerance on ϕ is 2/α ~65 times looser than on θ, i.e. ~ 450 μrad at 10°

      450 μrad at 10° corresponds to a ~200 μm design precision in the ϕ direction at ϕ = π/2

$$\boxed{\frac{\partial\theta^*}{\partial\phi} \approx \frac{\alpha}{2}\cos\theta\sin\phi}$$

Reminder: Energy-Momentum conservation

- **Total energy-momentum conservation applied to two-body final states (+ one ISR γ)**
  - **For example, for dilepton and diphoton events, $e^+e^- \to e^+e^-(\gamma)$, $\mu^+\mu^-(\gamma)$, $\tau^+\tau^-(\gamma)$, $\gamma\gamma(\gamma)$**

$$\boxed{\begin{array}{lllll} E^+\sin\theta^+\cos\phi^+ & + E^-\sin\theta^-\cos\phi^- & + |p_z^\gamma|\tan\alpha/2 & = & \sqrt{s/(1-\varepsilon^2)}\tan\alpha/2 \\ E^+\sin\theta^+\sin\phi^+ & + E^-\sin\theta^-\sin\phi^- & & = & 0 \quad , \\ E^+\cos\theta^+ & + E^-\cos\theta^- & + \; p_z^\gamma & = & \varepsilon\sqrt{s/(1-\varepsilon^2)} \quad , \\ E^+ & + E^- & + \; |p_z^\gamma|/\cos\alpha/2 & = & \sqrt{s/(1-\varepsilon^2)}/\cos\alpha/2 \end{array}}$$

- Where $E^\pm$ are the measured energies of the outgoing $e^\pm$, $\mu^\pm$, $\tau^\pm$, or the forward/backward γ
- Where $\theta^\pm$ are measured with respect to the z axis in this "FCC-ee" frame,
- Where $\phi^\pm$ are measured with respect to the x axis in the plane transverse to the z axis,
- Where α, ε, and the x, y, z axes have been defined previously
- Where $\sqrt{s}$ is the centre-of-mass energy of the collision

$$\sqrt{s} = 2E\sqrt{1-\epsilon^2}\cos\frac{\alpha}{2} \quad \text{and} \quad E_e^\pm = E(1\pm\epsilon)$$

Solve E, p conservation for α and ε

- See this didactic presentation at the 2nd EPOL Workshop for a step-by-step proof
  - Crossing angle (valid even with an ISR photon)

$$\boxed{\alpha = 2\arcsin\left[\frac{\sin\left(\phi^+ - \phi^-\right)\sin\theta^+\sin\theta^-}{\sin\phi^+\sin\theta^+ - \sin\phi^-\sin\theta^-}\right]}$$

  - Longitudinal boost (here, the ISR photon is absorbed in the ε spread)

$$\boxed{\varepsilon = \frac{x_+\cos\theta^+ + x_-\cos\theta^-}{\cos\alpha/2},}$$

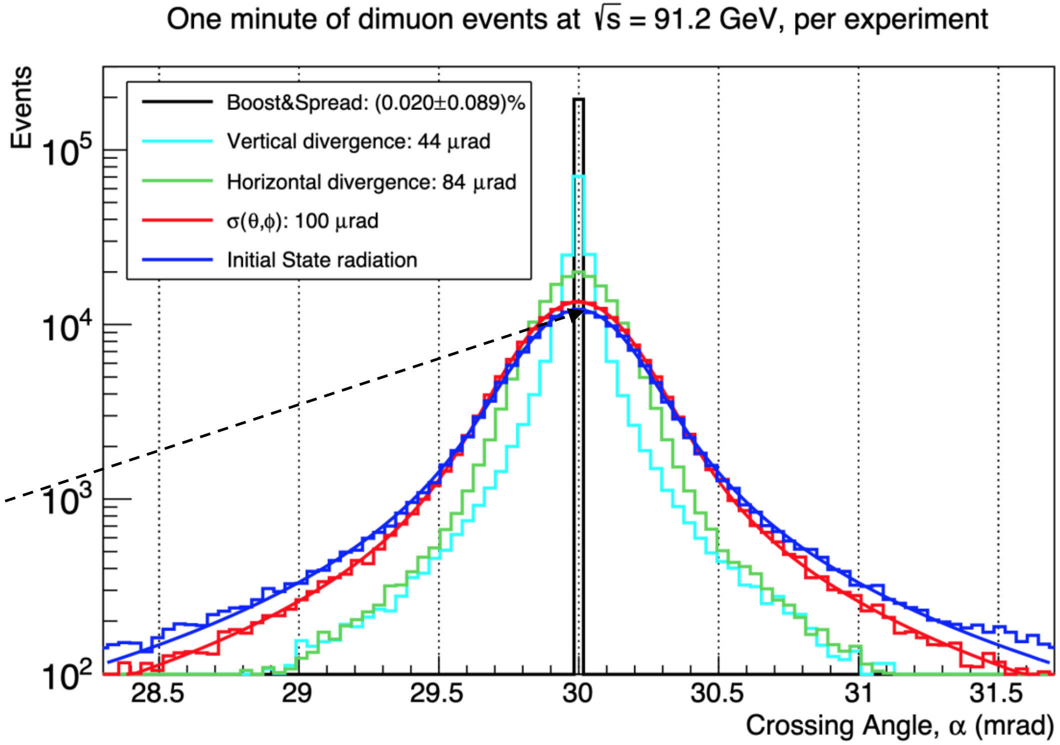
  - Reduced lepton/photon energies:

$$\boxed{x_\pm = \frac{\mp\sin\theta^\mp\sin\phi^\mp}{\sin\theta^+\sin\phi^+ - \sin\theta^-\sin\phi^-}.}$$

$$x_\pm = E^\pm\cos(\alpha/2)\sqrt{s/(1-\varepsilon^2)}$$

Crossing angle after one minute at the Z pole

- **Angles measured from dimuons in the tracker (assumed to be perfectly aligned)**



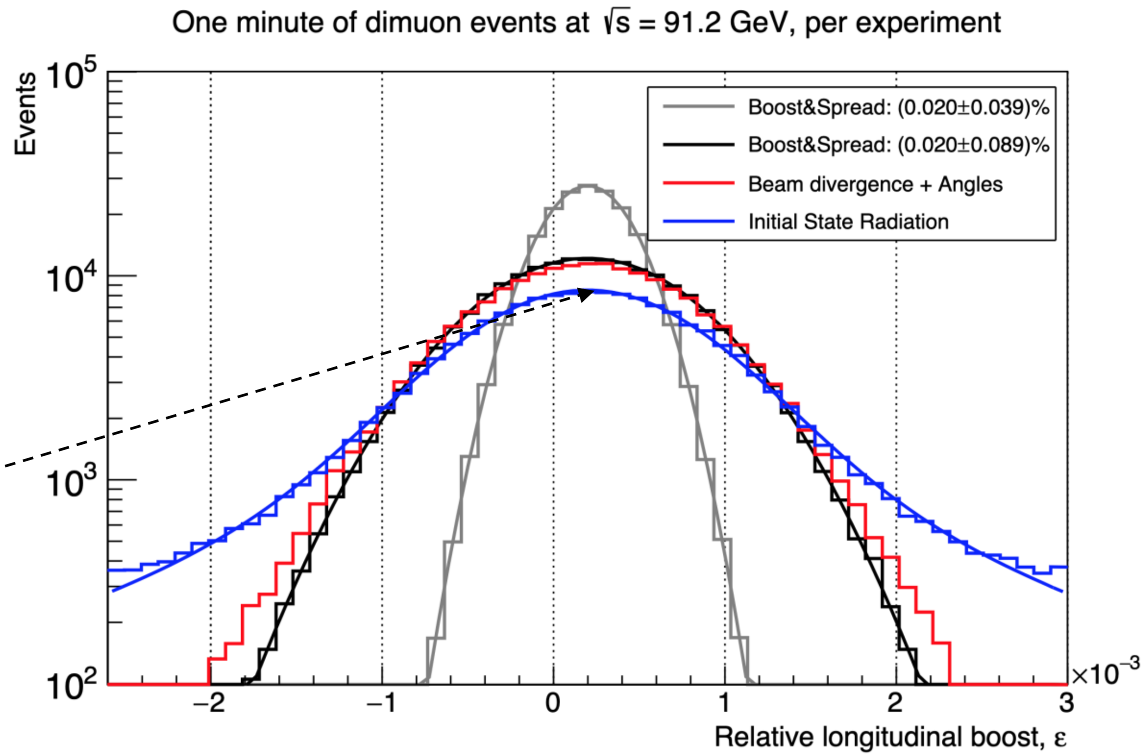
Mean value = nominal crossing angle α_0

$$\boxed{\alpha_0 = 29.9999 \pm 0.0006\,\mathrm{mrad}.}$$

- **The spread of the distribution is dominated by the muon angular resolution (here 0.1 mrad)**
  - **Horizontal beam divergence ($= [\varepsilon^*_x / \beta^*_x]^{1/2} = 84\ \mu$rad) is the next-to-largest contributor**
    - ➔ **Almost insensitive to ISR, and totally insensitive to the longitudinal boost and its spread $(0.02 \pm 0.089)\%$**

Longitudinal boost after one minute at the Z pole

- Angles measured from dimuons in the tracker (assumed to be perfectly aligned)



Mean value = nominal boost ε_0

$$\boxed{\varepsilon_0 = (1.98 \pm 0.02) \times 10^{-4}}$$

- The spread is dominated by the natural beam energy spread
  - Initial state radiation is the next-to-largest contributor
    - ➔ Marginal contribution from the muon angular resolution

In-situ acceptance determination for $e^+e^- \to \gamma\gamma$ events

- **In a first step, let's assume the following**
  - The values of α_0 and ε_0 are known a priori from a perfectly aligned tracker
  - The photon azimuthal angle tolerance is much better than 0.45 (0.65) mrad at 10° (20°)
    - This corresponds – for example – to a 200 (600) μm design tolerance at 10° (20°)
    - For each endcap separately (cell-to-cell), and for one endcap with respect to the other endcap
  - As a consequence, ignore for now the photon azimuth in the acceptance determination
    - Only consider polar angle biases $\Delta\theta^+$ & $\Delta\theta^-$ in this first step

- **Minimize the following χ^2 with respect to $\Delta\theta^+$ and $\Delta\theta^-$**

$$\chi^2 = \sum_{i=1}^{N_{evt}} \frac{(\alpha_i - \alpha_0)^2}{\sigma_\alpha^2} + \frac{(\varepsilon_i - \varepsilon_0)^2}{\sigma_\varepsilon^2}$$

  - In each bin of θ^* and ϕ^*
    - In my talk in April, minimization was done "manually" with a fast simulation
    - Today, we'll do everything analytically (and check if we find the same result before moving)
  - With $\alpha_\imath$ and $\varepsilon_\imath$ measured event-by-event from the photon angles:

$$\alpha = 2 \arcsin\left[\frac{\sin(\phi^+ - \phi^-)\sin\theta^+\sin\theta^-}{\sin\phi^+\sin\theta^+ - \sin\phi^-\sin\theta^-}\right]$$

$$\varepsilon = \frac{x_+\cos\theta^+ + x_-\cos\theta^-}{\cos\alpha/2},$$

$$x_\pm = \frac{\mp\sin\theta^\mp\sin\phi^\mp}{\sin\theta^+\sin\phi^+ - \sin\theta^-\sin\phi^-}.$$

Analytical expression of the χ^2

$\Delta\alpha$ $\qquad$ $\Delta\varepsilon$

$$\chi^2 = \sum_{i=1}^{N_{evt}} \frac{(\alpha_i - \alpha_0)^2}{\sigma_\alpha^2} + \frac{(\varepsilon_i - \varepsilon_0)^2}{\sigma_\varepsilon^2}$$

$$\Delta\alpha(\theta^+,\theta^-,\phi^+,\phi^-) = \frac{\partial\alpha}{\partial\theta^+}\Delta\theta^+ + \frac{\partial\alpha}{\partial\theta^-}\Delta\theta^- + \cancel{\frac{\partial\alpha}{\partial\phi^+}\Delta\phi^+ + \frac{\partial\alpha}{\partial\phi^-}\Delta\phi^-}$$

$$\Delta\varepsilon(\theta^+,\theta^-,\phi^+,\phi^-) = \frac{\partial\varepsilon}{\partial\theta^+}\Delta\theta^+ + \frac{\partial\varepsilon}{\partial\theta^-}\Delta\theta^- + \cancel{\frac{\partial\varepsilon}{\partial\phi^+}\Delta\phi^+ + \frac{\partial\varepsilon}{\partial\phi^-}\Delta\phi^-}$$

$$\Delta\alpha = \frac{1}{2}\left(\underbrace{\frac{\partial\alpha}{\partial\theta^+} + \frac{\partial\alpha}{\partial\theta^-}}_{f_1}\right)(\Delta\theta^+ + \Delta\theta^-) + \frac{1}{2}\left(\underbrace{\frac{\partial\alpha}{\partial\theta^+} - \frac{\partial\alpha}{\partial\theta^-}}_{f_2}\right)(\Delta\theta^+ - \Delta\theta^-)$$

$$\Delta\varepsilon = \frac{1}{2}\left(\underbrace{\frac{\partial\varepsilon}{\partial\theta^+} + \frac{\partial\varepsilon}{\partial\theta^-}}_{g_1}\right)(\Delta\theta^+ + \Delta\theta^-) + \frac{1}{2}\left(\underbrace{\frac{\partial\varepsilon}{\partial\theta^+} - \frac{\partial\varepsilon}{\partial\theta^-}}_{g_2}\right)(\Delta\theta^+ - \Delta\theta^-)$$

$\mathcal{O}(\alpha^2, \varepsilon^2, \alpha\varepsilon)$

$$f_1 = \alpha\left[-\frac{\alpha}{2}\frac{\cos^3\theta^*\cos\phi^*}{\sin^2\theta^*} + \varepsilon\frac{1+\cos^2\theta^*}{\sin\theta^*}\right],$$

$$\boxed{f_2 = \alpha\frac{\cos\theta^*}{\sin\theta^*}, \quad g_1 = -\frac{1}{\sin\theta^*},}$$

f_2: $\Rightarrow \boldsymbol{\Delta\theta^+ - \Delta\theta^-}$

g_1: $\Rightarrow \boldsymbol{\Delta\theta^+ + \Delta\theta^-}$

$$g_2 = \frac{\alpha}{2}\frac{\cos 2\theta^*\cos\phi^*}{\sin^2\theta^*} - \varepsilon\frac{\cos\theta^*}{\sin\theta^*},$$

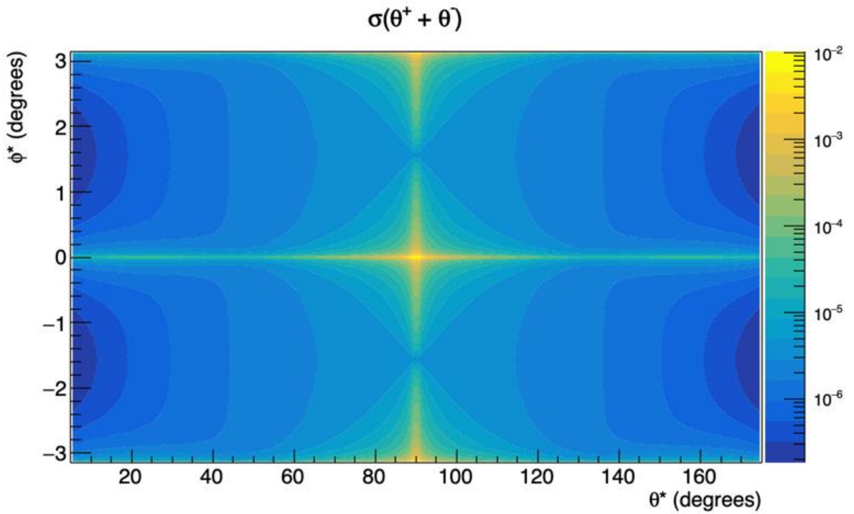
$$\cos\theta^* = (\cos\theta^+ - \cos\theta^-)/2,$$
$$\cos\phi^* = (\cos\phi^+ - \cos\phi^-)/2,$$
$$\sin\theta^* = (\sin\theta^+ + \sin\theta^-)/2,$$
$$\sin\phi^* = (\sin\phi^+ - \sin\phi^-)/2.$$

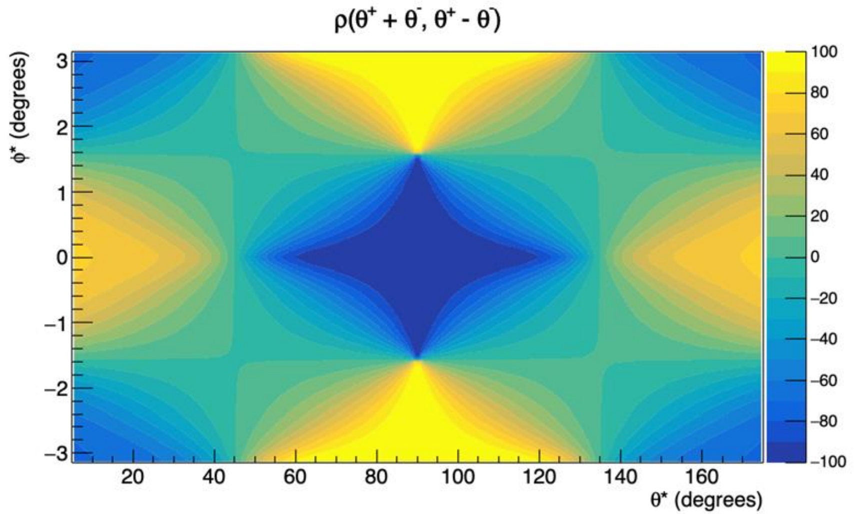
In each (θ^*, ϕ^*) bin, $\chi^2 = \Delta^T V \Delta$ (V^{-1} = 2×2 error matrix)

$$V_{ij}(\theta^*,\phi^*) = N(\theta^*,\phi^*)\left[\frac{f_i f_j}{\sigma_\alpha^2} + \frac{g_i g_j}{\sigma_\varepsilon^2}\right] \qquad \Delta = \frac{1}{2}\begin{bmatrix}\Delta\theta^+ + \Delta\theta^- \\ \Delta\theta^+ - \Delta\theta^-\end{bmatrix}$$

$\sigma_{\alpha,\varepsilon}$ obtained from error propagation ⊕ beam divergence (for α) ⊕ energy spread (for ε)

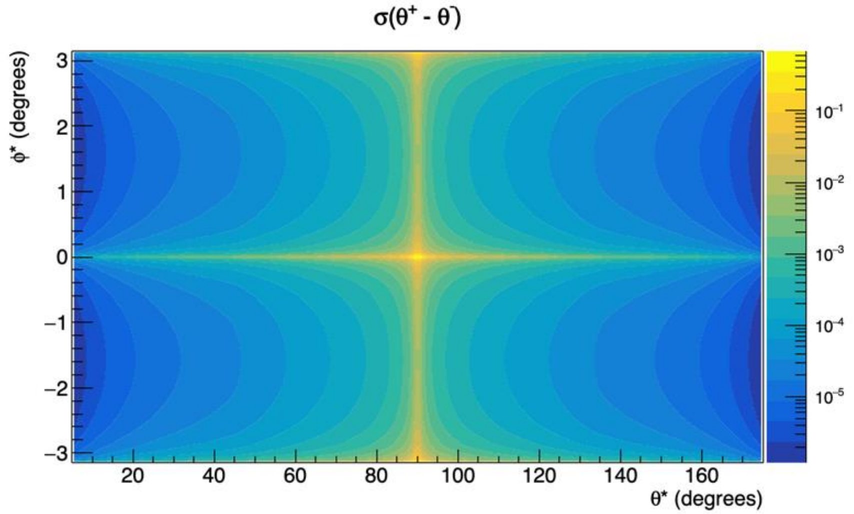
Error matrix with 45 ab^{-1} @ Z pole for $e^+e^- \to \gamma\gamma$ events





- **Bin size : 1°×1°**
  - Commensurate with CLD/IDEA/crystal calo readout cell
- **Photon position resolution $\sigma_{x,y}$ = 0.5 mm**
  - Typical of CLD / IDEA / crystal calorimeter
    - Preshower in front of the endcaps : $\sigma_{x,y}$ = 0.075 mm

In the endcaps: $\boxed{\sigma_\theta = \frac{\sigma_{x,y}\cos^2\theta}{z}, \ \sigma_\phi = \frac{\sigma_{x,y}}{z\tan\theta}}$

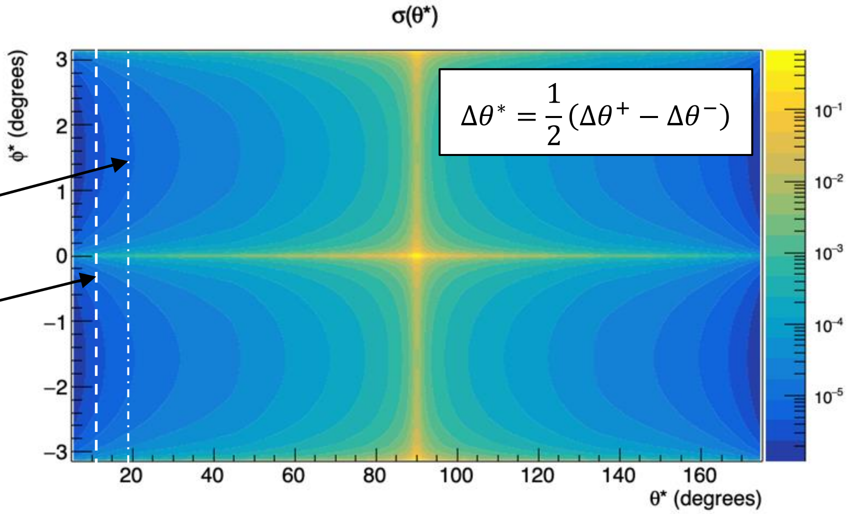


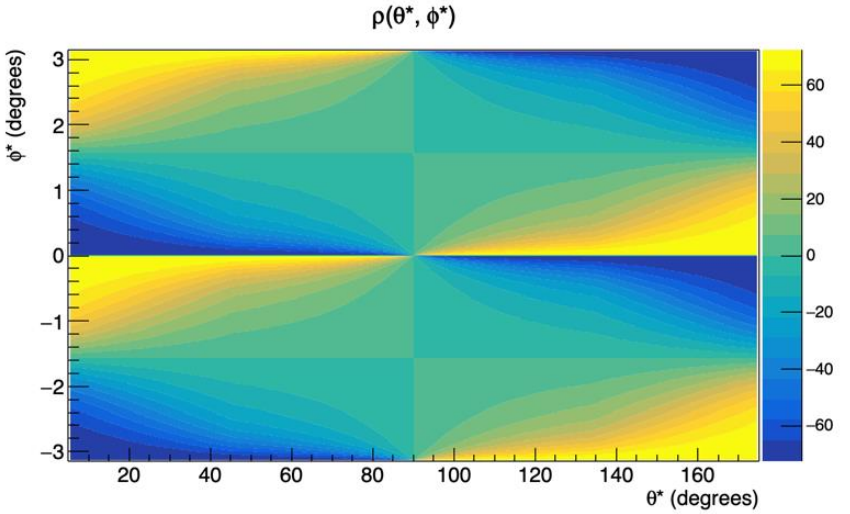
Error matrix in the collision frame with 45 ab^{-1} @ Z pole

Polar angle cut

At 20^{o}

At 10^{o}

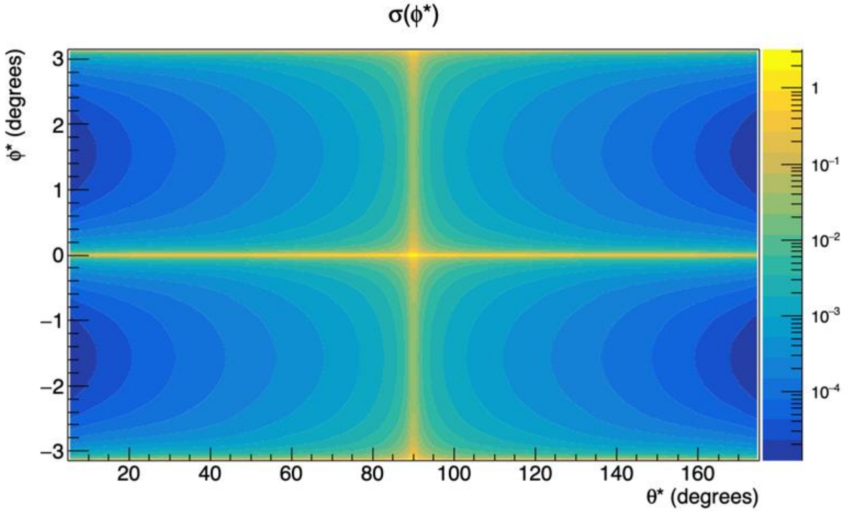




- **Transfer matrix from collision rest frame to lab frame**

$$\frac{1}{2}\begin{pmatrix} \Delta\theta^+ + \Delta\theta^- \\ \Delta\theta^+ - \Delta\theta^- \end{pmatrix} = \begin{pmatrix} -\frac{\alpha}{2}\sin\theta^*\cos\phi^* - \varepsilon\cos\theta^* & -\frac{\alpha}{2}\cos\theta^*\sin\phi^* \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \Delta\theta^* \\ \Delta\phi^* \end{pmatrix} \quad @\ \mathcal{O}(\alpha^2, \varepsilon^2, \alpha\varepsilon)$$

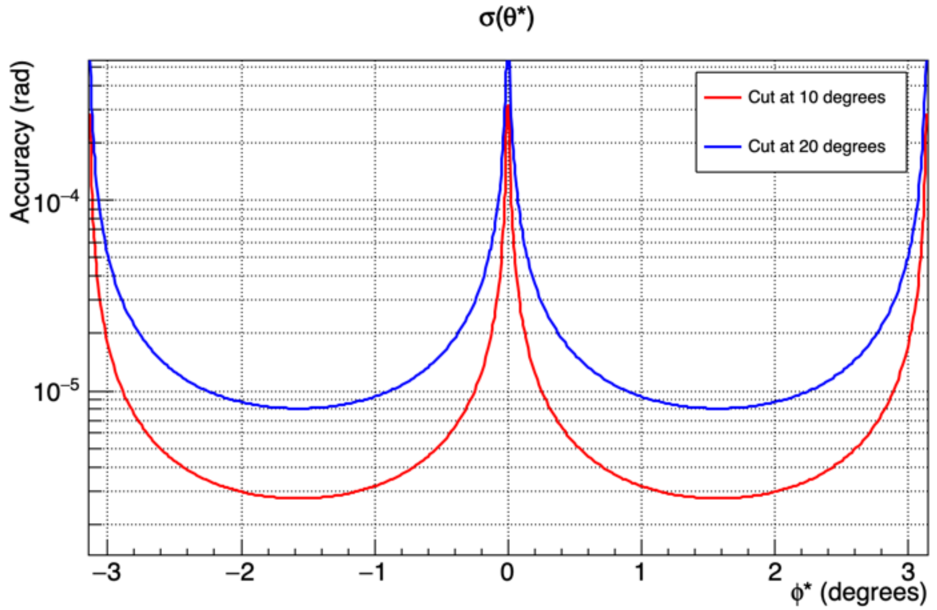
T



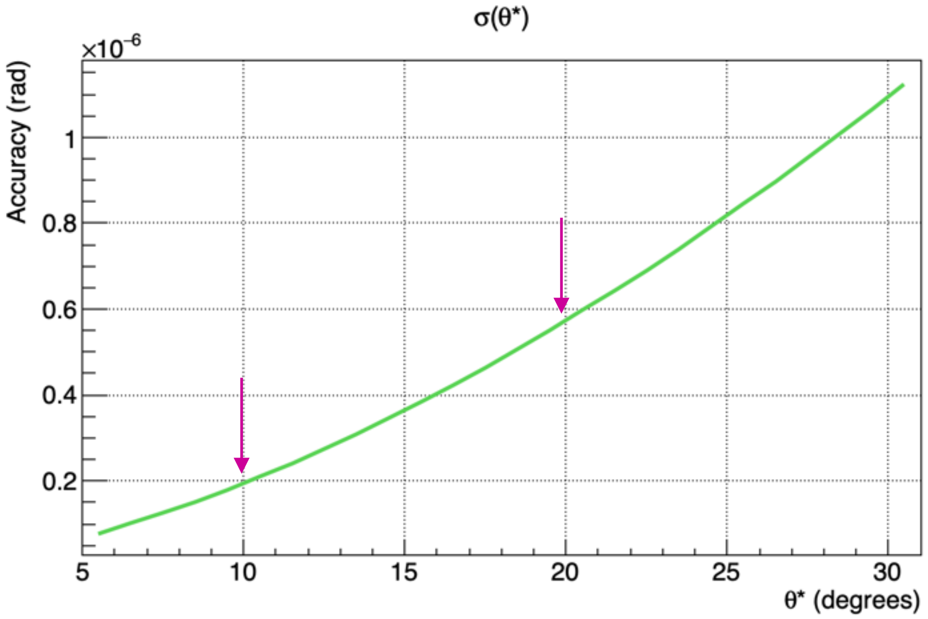
$$V_{\mathrm{cm}} = T^T\, V_{\mathrm{lab}}\, T$$

Precision of acceptance determination

θ* accuracy along ϕ* at 10° (20°)



Integration (of V) over ϕ*



- If the crossing angle, longitudinal boost, and azimuthal angles are known perfectly
  - Acceptance accuracy can be determined in situ with a sub-mrad precision
    - ➔ 0.2 μrad with a polar angle θ* cut at 10° (was 0.25 μrad in April with the simulation)
    - ➔ 0.6 μrad with a polar angle θ* cut at 20° (was 0.75 μrad in April with the simulation)
  - Reminder: Tolerance estimated to be 6.5 μrad

ISR, Selection cuts, Actual α_0 & ε_0 smearing

Simultaneous fit of α and ε with $e^+e^- \to \gamma\gamma$ events

- For each event, fit α, ε, θ* and ϕ* – not just the last two
  - Need to use the four angle measurements θ^+, θ^-, ϕ^+, ϕ^- in the χ^2

$$\Delta\alpha = \frac{1}{2}\left(\frac{\partial\alpha}{\partial\theta^+}+\frac{\partial\alpha}{\partial\theta^-}\right)(\Delta\theta^+ + \Delta\theta^-) + \frac{1}{2}\left(\frac{\partial\alpha}{\partial\theta^+}-\frac{\partial\alpha}{\partial\theta^-}\right)(\Delta\theta^+ - \Delta\theta^-)$$
$$\boxed{+\frac{1}{2}\overbrace{\left(\frac{\partial\alpha}{\partial\phi^+}+\frac{\partial\alpha}{\partial\phi^-}\right)}^{f_3}(\Delta\phi^+ + \Delta\phi^-) + \frac{1}{2}\overbrace{\left(\frac{\partial\alpha}{\partial\phi^+}-\frac{\partial\alpha}{\partial\phi^-}\right)}^{f_4}(\Delta\phi^+ - \Delta\phi^-)}$$
$$\Delta\varepsilon = \frac{1}{2}\left(\frac{\partial\varepsilon}{\partial\theta^+}+\frac{\partial\varepsilon}{\partial\theta^-}\right)(\Delta\theta^+ + \Delta\theta^-) + \frac{1}{2}\left(\frac{\partial\varepsilon}{\partial\theta^+}-\frac{\partial\varepsilon}{\partial\theta^-}\right)(\Delta\theta^+ - \Delta\theta^-)$$
$$\boxed{+\frac{1}{2}\underbrace{\left(\frac{\partial\varepsilon}{\partial\phi^+}+\frac{\partial\varepsilon}{\partial\phi^-}\right)}_{g_3}(\Delta\phi^+ + \Delta\phi^-) + \frac{1}{2}\underbrace{\left(\frac{\partial\varepsilon}{\partial\phi^+}-\frac{\partial\varepsilon}{\partial\phi^-}\right)}_{g_4}(\Delta\phi^+ - \Delta\phi^-)}$$

$\Delta\phi^+ + \Delta\phi^-$

$$f_3 = -\alpha\frac{\cos\phi^*}{\sin\phi^*},\quad g_3 = -\frac{\alpha}{2}\frac{\cos\theta^*}{\sin\theta^*\sin\phi^*},$$
$$f_4 = -2\frac{\sin\theta^*}{\sin\phi^*},\quad g_4 = -\frac{\cos\theta^*\cos\phi^*}{\sin\phi^*},$$

$\Delta\phi^+ - \Delta\phi^-$

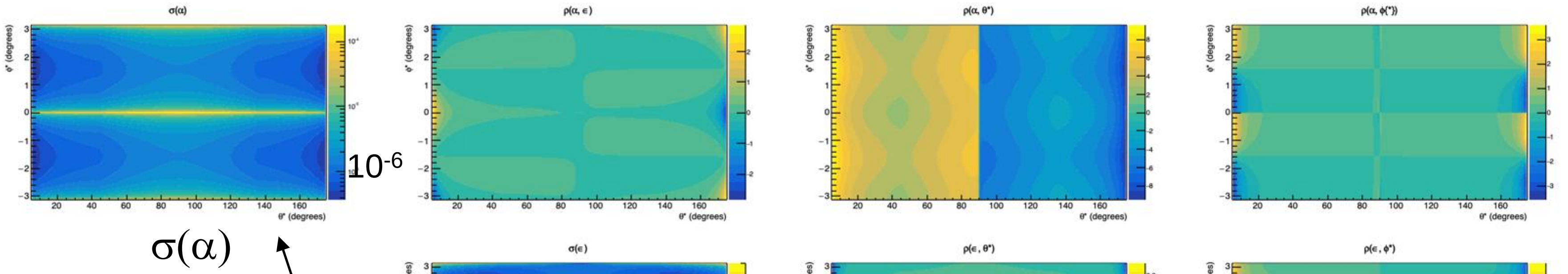
  - In each (θ*, ϕ*) bin, $\chi^2 = \Delta^T V \Delta$ (4×4 not regular matrix)

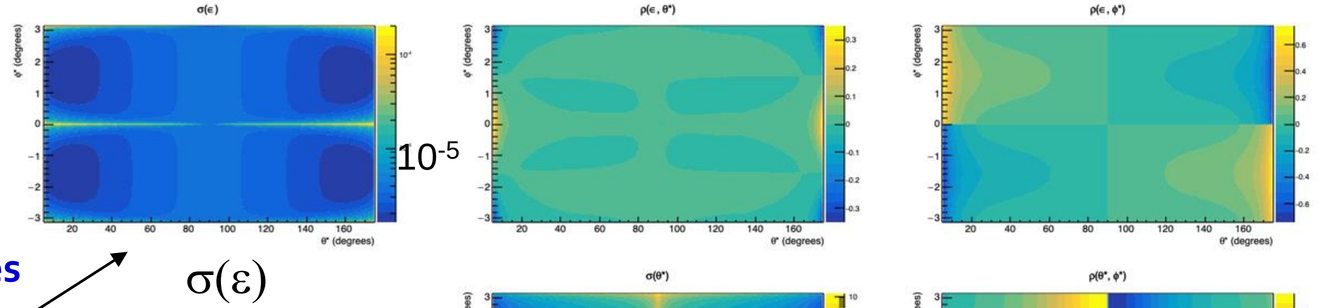
$$V_{ij}(\theta^*,\phi^*) = N(\theta^*,\phi^*)\left[\frac{f_i f_j}{\sigma_\alpha^2} + \frac{g_i g_j}{\sigma_\varepsilon^2}\right]$$

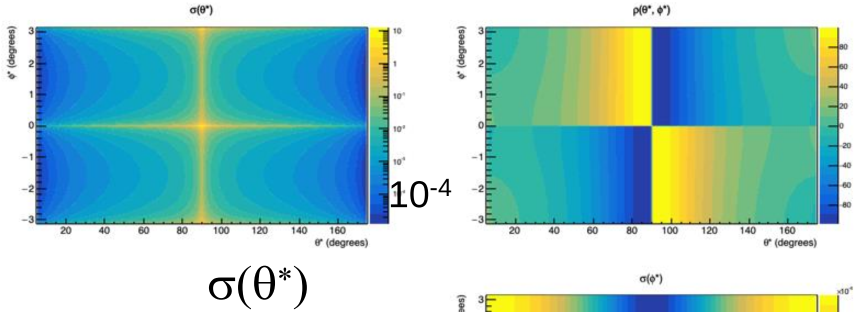
$$\Delta = \frac{1}{2}\begin{bmatrix}\Delta\theta^+ + \Delta\theta^- \\ \Delta\theta^+ - \Delta\theta^- \\ \Delta\phi^+ + \Delta\phi^- \\ \Delta\phi^+ - \Delta\phi^-\end{bmatrix}$$

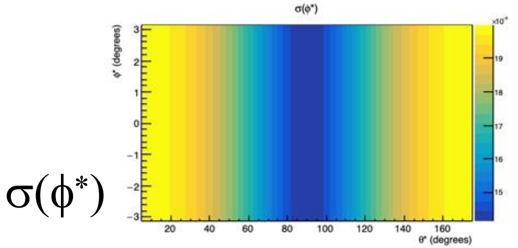
  - θ*/ϕ correlation: Add a 100 μm constraint on rϕ in the χ^2 (i.e., in V)
    - For each endcap separately (cell-to-cell) $\quad (r\Delta\phi^+/10^{-4})^2 + (r\Delta\phi^-/10^{-4})^2$
    - For one endcap wrt the other endcap $\quad (\Delta(\phi^+ + \phi^-)/4\times10^{-5})^2 + (\Delta(\phi^+ - \phi^-)/4\times10^{-5})^2$

Error matrix with 45 ab^{-1} @ Z pole for $e^+e^- \to \gamma\gamma$ events









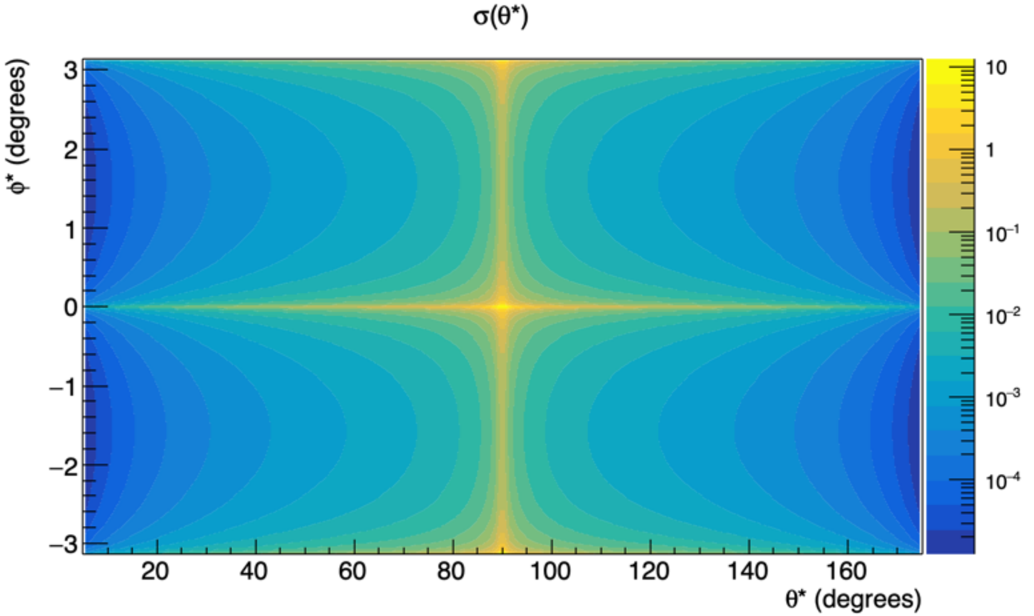
- **After integration on θ*, ϕ***
  - α known to 1 μrad in 10 minutes
  - ε known to 2×10^{-6} in 10 minutes
- **Error matrix shown in the ($\Delta\alpha_0$, $\Delta\varepsilon_0$, $\Delta\theta^*$, $\Delta\phi^*$) basis**
  - With a moderate (?) 100 μm constraint on rϕ
- **See backup slides for the transfer matrix**

  From the $(\Delta\alpha_0, \Delta\varepsilon_0, \Delta\theta^*, \Delta\phi^*)$ basis

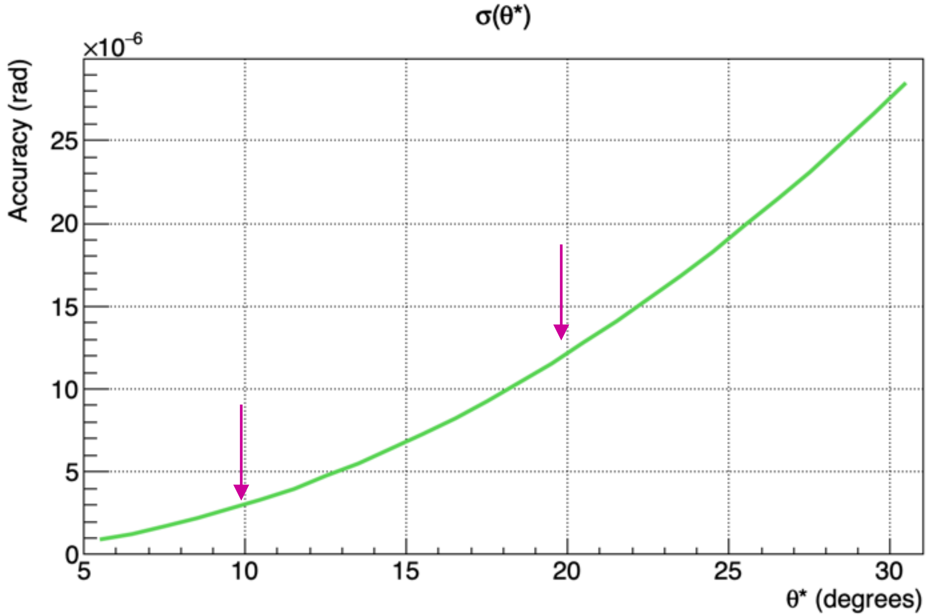
  To the $(\Delta\theta^+ + \Delta\theta^-, \Delta\theta^+ - \Delta\theta^-, \Delta\phi^+ + \Delta\phi^-, \Delta\phi^+ - \Delta\phi^-)$ basis

Precision of acceptance determination

θ^* uncertainty in the (θ^*, ϕ^*) plane



Integration over ϕ^*



- Acceptance precision in the right ball park at 10° and 20°

- **Bottom line**

This condition is needed anyway

- Traded a tolerance of 10 μm in radius for a tolerance of 100 μm in the ϕ direction
  - No dependence in the relative distance between the two endcaps (while θ is Δz-dependent)
  - Measurements in the ϕ direction are mostly relative (total is 2π !)

This is far from being the end of the story

- One (two) additional constraint(s), not used so far, can be added to the χ^2

$$\boxed{x^{+} - x^{-} = A^{*} \quad \left(= \sin\frac{\alpha}{2}\sin\theta^{*}\cos\phi^{*} + \varepsilon\cos\theta^{*}\right)}$$

$$\boxed{x_{\pm} = \frac{\mp\sin\theta^{\mp}\sin\phi^{\mp}}{\sin\theta^{+}\sin\phi^{+} - \sin\theta^{-}\sin\phi^{-}}.}$$

  - Constraining A^{*} in situ with the angle measurements will reduce the θ^{*}/ϕ correlation

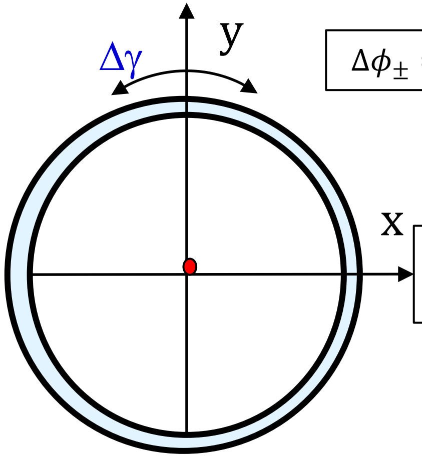
- In this presentation, the acceptance cut was defined as a straight cut on θ^{*}
  - This cut corresponds to apply the same cut to the two photons
  - In real life, the cut will be applied on only one of the two photons
    - Changing side at each event (forward – backward – forward – backward – etc.)
  - This trick reduces the sensitivity of the acceptance cut
    - In particular to the relative misalignment of the two endcaps in the ϕ direction
      - ➔ Due either to a global rotation around z, or a global translation in the (x,y) plane
  - This trick will in turn loosen the tolerance of this relative misalignment

This is far from being the end of the story

- **A global relative (x,y) misalignment of the two endcaps is measurable in situ as well !**
  - **Rotation around z**

Translation in the x,y plane

$$\Delta\phi_\pm = \Delta\gamma^z_\pm$$

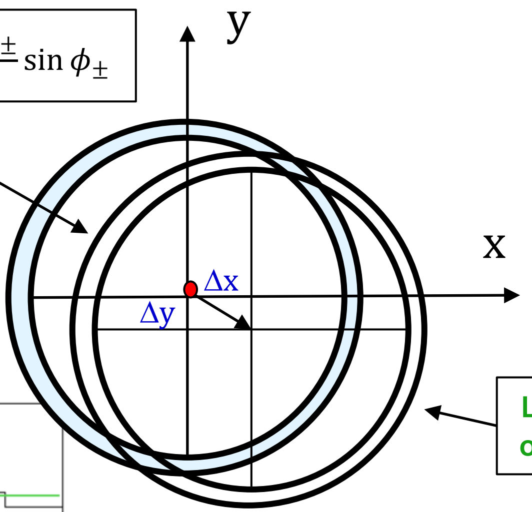


$$\frac{\partial\theta^*}{\partial\phi} \approx \frac{\alpha}{2}\cos\theta\sin\phi$$

More events on this side

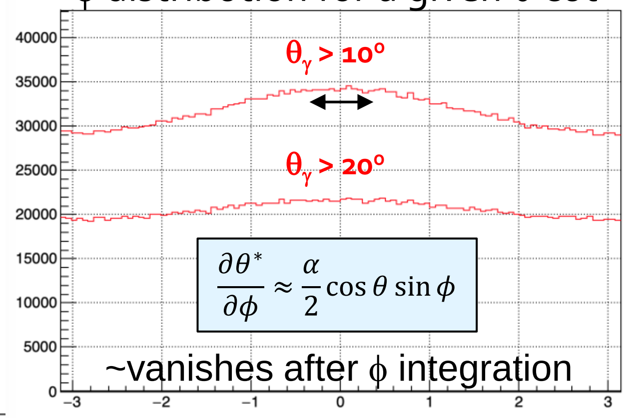
$$\Delta\phi_\pm = \frac{\Delta y_\pm}{r}\cos\phi_\pm - \frac{\Delta x_\pm}{r}\sin\phi_\pm$$

More events on this side

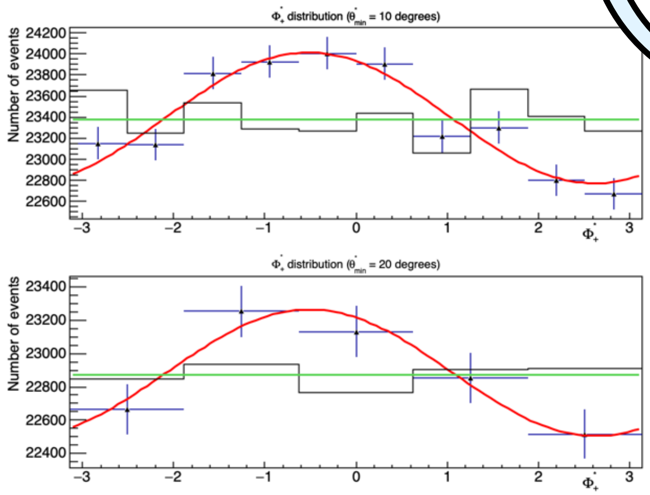


Less events on that side

ϕ distribution for a given θ cut



~vanishes after ϕ integration



~vanishes after ϕ integration

+ cosmics?

+ $e^+e^- \rightarrow e^+e^-$?

ϕ^* distribution for a given θ^*

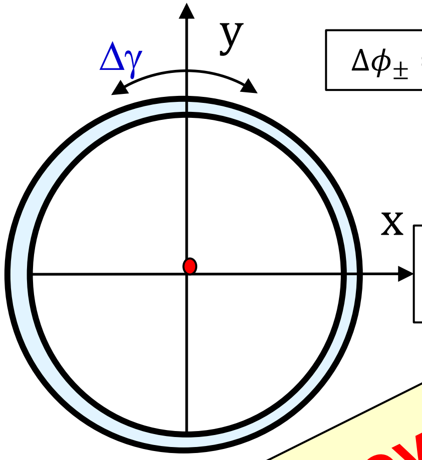
0.01% of the total statistics at 91.2 GeV

1 cm displacement

This is far from being the end of the story

- A global relative (x,y) misalignment of the two endcaps is measurable in situ as well !
  - Rotation around z
  - Translation in the x,y plane

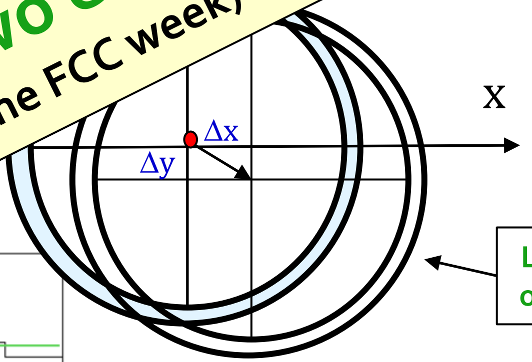
$$\Delta\phi_\pm = \Delta\gamma^z_\pm$$



More events on this side

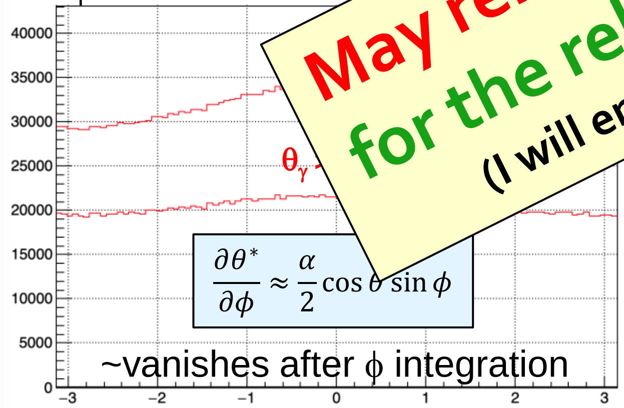
$$\frac{\partial\theta^*}{\partial\phi} \approx \frac{\alpha}{2}\cos\theta\sin\phi$$

$$\Delta\phi_\pm = \frac{\Delta y_\pm}{r}\cos\phi \ldots$$



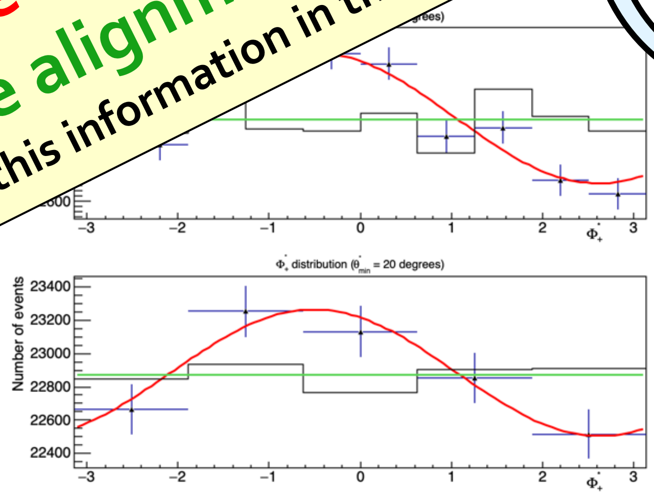
Less events on that side

ϕ distribution for a ...



$$\frac{\partial\theta^*}{\partial\phi} \approx \frac{\alpha}{2}\cos\theta\sin\phi$$

~vanishes after ϕ integration



~vanishes after ϕ integration

May remove the necessity of a precise survey for the relative alignment of the two endcaps

(I will enter all this information in the χ^2 after the FCC week)

+ cosmics?

+ $e^+e^- \to e^+e^-$?

ϕ^* distribution for a given θ^*

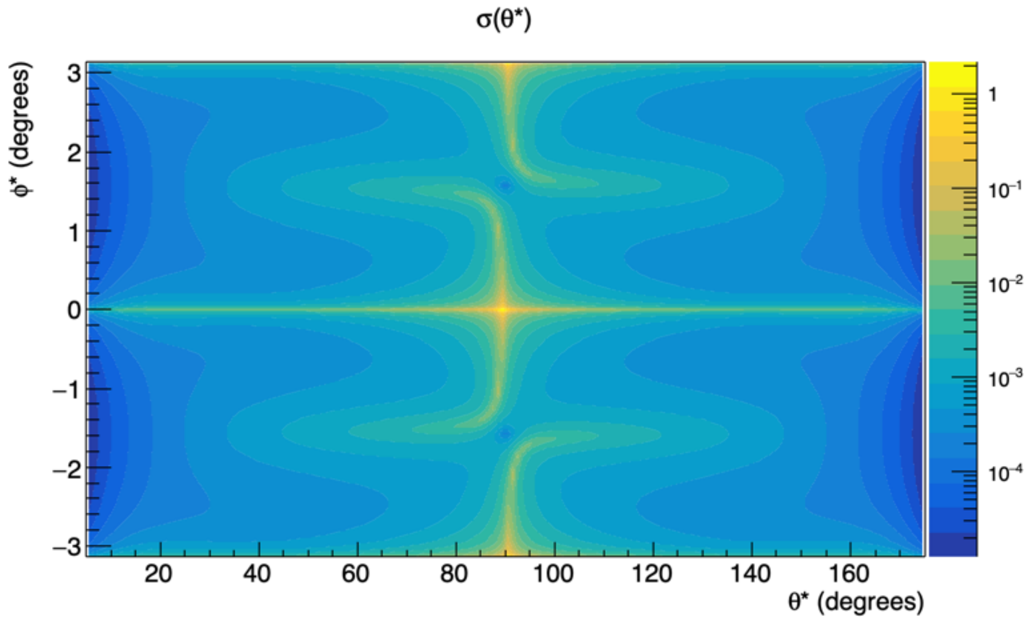
0.01% of the total statistics at 91.2 GeV

1 cm displacement

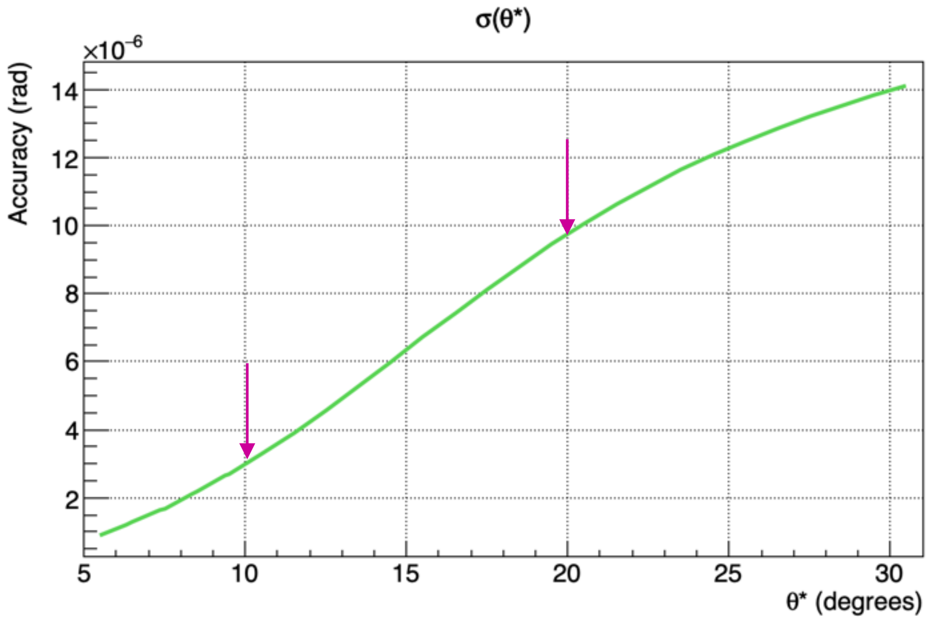
Bonus slides

Including $x^+ - x^- = A^*$

θ^* uncertainty in the (θ^*, ϕ^*) plane



Integration over ϕ^*



Lorentz transform

- **From the collision frame to the laboratory frame**

$$
\begin{aligned}
p_x &= p_x^* + \sin\frac{\alpha}{2}\left[\gamma E^* + \frac{\gamma-1}{\beta^2}\left(p_x^*\sin\frac{\alpha}{2} + p_z^*\varepsilon\cos\frac{\alpha}{2}\right)\right],\\
p_y &= p_y^*,\\
p_z &= p_z^* + \varepsilon\cos\frac{\alpha}{2}\left[\gamma E^* + \frac{\gamma-1}{\beta^2}\left(p_x^*\sin\frac{\alpha}{2} + p_z^*\varepsilon\cos\frac{\alpha}{2}\right)\right]\\
E &= \gamma\left[E^* + p_x^*\sin\frac{\alpha}{2} + p_z^*\varepsilon\cos\frac{\alpha}{2}\right].
\end{aligned}
$$

Exact expressions

$\mathcal{O}(\alpha^2, \varepsilon^2, \alpha\varepsilon)$

$$
\begin{aligned}
E^+\sin\theta^+\cos\phi^+ &= E^*\left(\sin\theta^*\cos\phi^* + \sin\alpha/2\right),\\
E^+\sin\theta^+\sin\phi^+ &= E^*\sin\theta^*\sin\phi^*,\\
E^+\cos\theta^+ &= E^*\left(\cos\theta^* + \varepsilon\right),\\
E^+ &= E^*\left(1 + \sin\alpha/2\sin\theta^*\cos\phi^* + \varepsilon\cos\theta^*\right),\\
\\
E^-\sin\theta^-\cos\phi^- &= -E^*\left(\sin\theta^*\cos\phi^* - \sin\alpha/2\right),\\
E^-\sin\theta^-\sin\phi^- &= -E^*\sin\theta^*\sin\phi^*,\\
E^-\cos\theta^- &= -E^*\left(\cos\theta^* - \varepsilon\right),\\
E^- &= E^*\left(1 - \sin\alpha/2\sin\theta^*\cos\phi^* - \varepsilon\cos\theta^*\right).
\end{aligned}
$$

Lorentz transform

- Relations between measured angles in the detector and α, ε, θ*, ϕ*
  - Still to be cross-checked by Emmanuel

$$\frac{\cos\theta^+ + \cos\theta^-}{2} = -A\cos\theta^* + \varepsilon + \mathcal{O}(\alpha^3, \alpha^2\varepsilon, \alpha\varepsilon^2, \varepsilon^3),$$

$$\frac{\cos\theta^+ - \cos\theta^-}{2} = \left(1 + A^2 - \frac{\alpha^2}{8} - \frac{\varepsilon^2}{2}\right)\cos\theta^* - \frac{A\varepsilon}{2} + \mathcal{O}(\alpha^3, \alpha^2\varepsilon, \alpha\varepsilon^2, \varepsilon^3).$$

$$\frac{\cot\phi^+ + \cot\phi^-}{2} = \cot\phi^* + \frac{\alpha A}{4\sin\theta^*\sin\phi^*} + \mathcal{O}(\alpha^3, \alpha^2\varepsilon, \alpha\varepsilon^2, \varepsilon^3).$$

$$\frac{\cot\phi^+ - \cot\phi^-}{2} = \frac{\alpha}{2\sin\theta^*\sin\phi^*} + \mathcal{O}(\alpha^3, \alpha^2\varepsilon, \alpha\varepsilon^2, \varepsilon^3)..$$

With $A = \sin\frac{\alpha}{2}\sin\theta^*\cos\phi^* + \varepsilon\cos\frac{\alpha}{2}\cos\theta^*.$

Lorentz transform

- Transfer matrix for small α, ε, θ^*, ϕ^* deviations
  - Still to be cross-checked by Emmanuel

$$\frac{\Delta\theta^+ + \Delta\theta^-}{2} = -A\Delta\theta^* - \frac{\alpha}{2}\cos\theta^*\sin\phi^*\Delta\phi^* + \frac{1}{2}\cos\theta^*\cos\phi^*\Delta\alpha - \sin\theta^*\Delta\varepsilon, \tag{C103}$$

$$\begin{aligned}\frac{\Delta\theta^+ - \Delta\theta^-}{2} = {} & \Delta\theta^* \\ & + \left[\left(-2A\sin\theta^* - \xi\cos\theta^*\right)\frac{\cos\phi^*\cos\theta^*}{2\sin\theta^*} + \frac{\alpha}{4}\frac{\cos\theta^*}{\sin\theta^*} + \frac{\varepsilon}{4}\cos\phi^*\right]\Delta\alpha \\ & + \left[\left(-2A\cos\theta^* + \xi\sin\theta^*\right)\frac{\cos\theta^*}{\sin\theta^*} + \frac{\alpha}{4}\cos\phi^* + 2\varepsilon\frac{\cos\theta^*}{\sin\theta^*}\right]\Delta\varepsilon,\end{aligned} \tag{C104}$$

$$\begin{aligned}\frac{\Delta\phi^+ + \Delta\phi^-}{2} = {} & \Delta\phi^* \\ & + \left(\frac{\alpha}{2}\frac{\cos\phi^*\sin\phi^*}{\sin^2\theta^*} - \frac{\alpha}{4}\cos\phi^*\sin\phi^* - \frac{\varepsilon}{4}\frac{\cos\theta^*\sin\phi^*}{\sin\theta^*}\right)\Delta\alpha \\ & - \frac{\alpha}{4}\frac{\cos\theta^*\sin\phi^*}{\sin\theta^*}\Delta\varepsilon,\end{aligned} \tag{C106}$$

$$\frac{\Delta\phi^+ - \Delta\phi^-}{2} = \frac{\alpha}{2}\frac{\cos\theta^*\sin\phi^*}{\sin^2\theta^*}\Delta\theta^* - \frac{\alpha}{2}\frac{\cos\phi^*}{\sin\theta^*}\Delta\phi^* - \frac{1}{2}\frac{\sin\phi^*}{\sin\theta^*}\Delta\alpha. \tag{C107}$$