FCC Physics Case

FCC Week, London, June 5th 2023

Matthew McCullough
CERN

The World as We've Known It

1 H																		2 He
3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	**	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
		*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb		
		**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No		

Wikipedia

The World as We've Known It

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f'_2(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
$a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
$a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(?^{-+})$
$X(1840)$	$?^?(?^{??})$
• $\phi_3(1850)$	$0^-(3^{--})$
$\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
• $f_2(1950)$	$0^+(2^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $a_4(2040)$	$1^-(4^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$a_6(2450)$	$1^-(6^{++})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K^*_0(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K^*_0(1430)$	$1/2(0^+)$
• $K^*_2(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K^*_3(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K^*_0(1950)$	$1/2(0^+)$
$K^*_2(1980)$	$1/2(2^+)$
• $K^*_4(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K^*_5(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D^*_0(2400)^0$	$1/2(0^+)$
$D^*_0(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D^*_2(2460)^0$	$1/2(2^+)$
• $D^*_2(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D^\pm_s$	$0(0^-)$
• $D^{*\pm}_s$	$0(?^?)$
• $D^*_{s0}(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D^*_{s1}(2700)^\pm$	$0(1^-)$
$D^*_{sJ}(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I(J^P)$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B^*_J(5732)$	$?(?^?)$
• $B_1(5721)^0$	$1/2(1^+)$
• $B^*_2(5747)^0$	$1/2(2^+)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

	$I(J^P)$
• B^0_s	$0(0^-)$
• B^*_s	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B^*_{s2}(5840)^0$	$0(2^+)$
$B^*_{sJ}(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

	$I(J^P)$
• $B^\pm_c$	$0(0^-)$

$c\bar{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(2P)$	$0^+(0^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4250)^\pm$	$?(?^?)$
• $X(4260)$	$?^?(1^{--})$
$X(4350)$	$0^+(?^{?+})$
• $X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
$X(4430)^\pm$	$?(1^+)$
• $X(4660)$	$?^?(1^{--})$

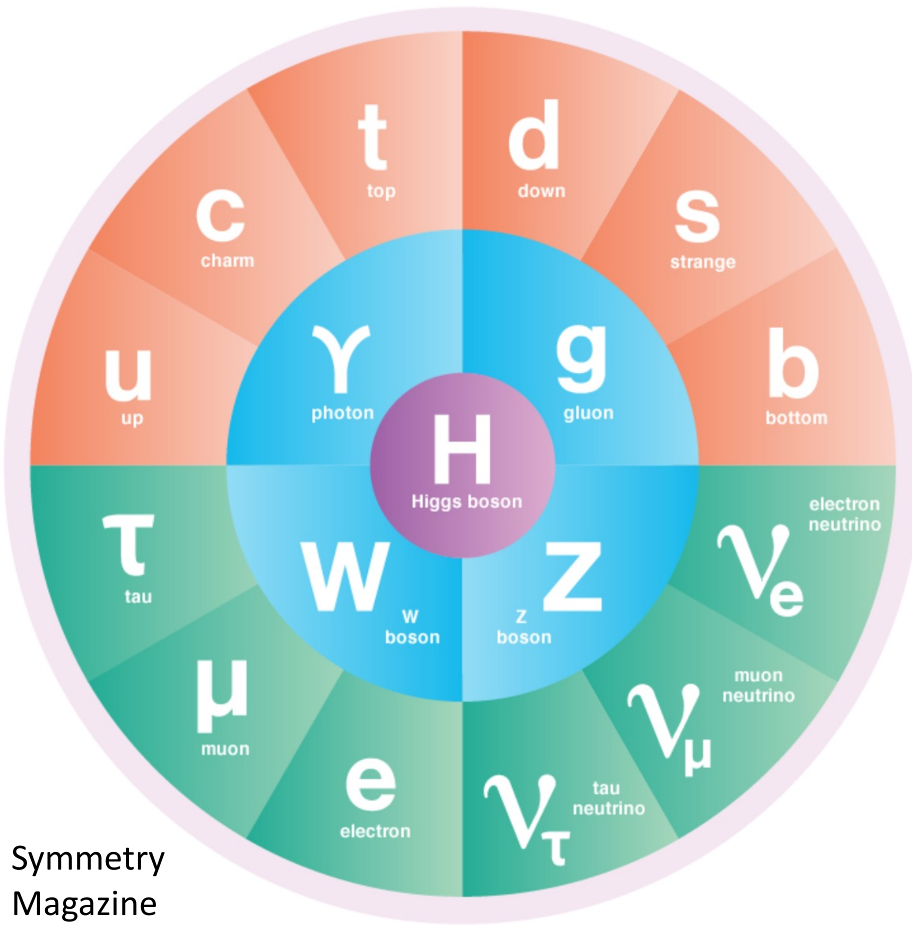
$b\bar{b}$

	$I^G(J^{PC})$
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_b(3P)$	$?^?(?^{?+})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

PDG

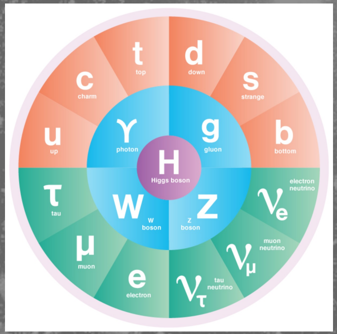
Don't even ask about the baryons...

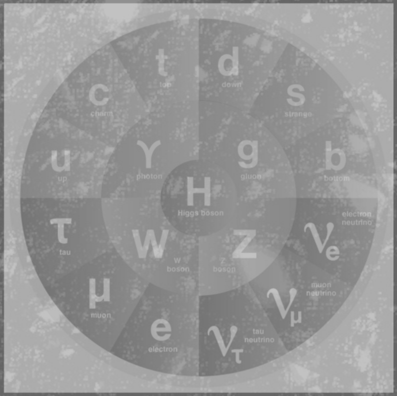
The World as We've Known It



Symmetry
Magazine

Our World as We've Known It





Question: With all this in mind, where, and how, might we make progress?

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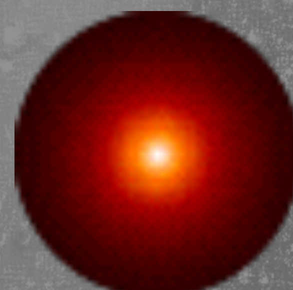
Answer:

With unbiased exploration of the dark sector(s).

Visible Matters

Visible, baryonic matter makes up 16% of all the matter in our Universe. 73% of visible matter is Hydrogen. 25% is Helium. All the rest: 2%.

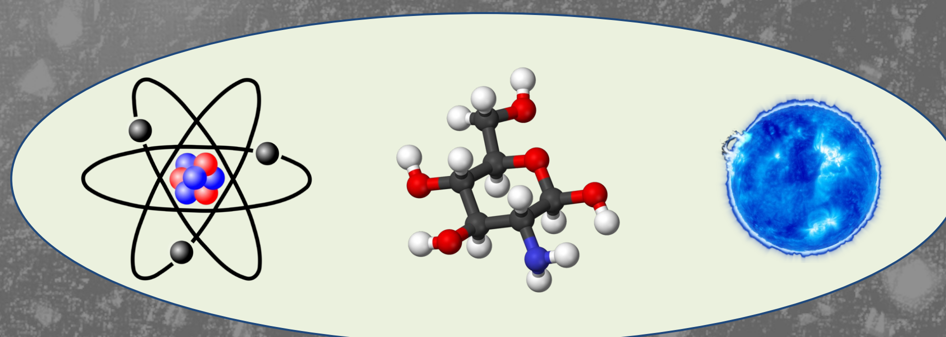


So, to a good approximation, the majority of visible matter is in relatively uninteresting atoms. Particularly hydrogen.

But the phenomenology of visible matter is not, to a good approximation, the phenomenology of hydrogen!

$e \quad u \quad d \quad Z \quad h \quad \mu \quad c \quad s \quad g \quad \tau \quad t \quad b \quad \gamma \quad W$

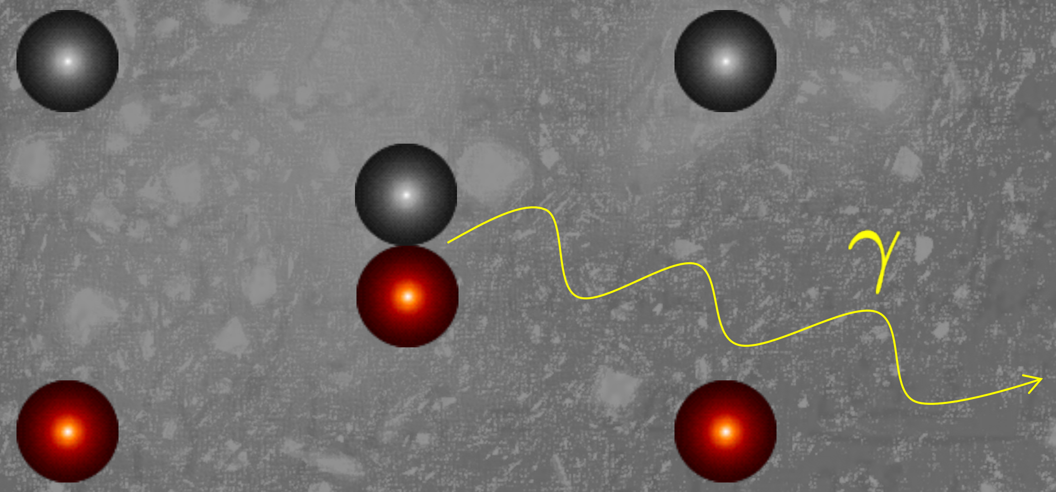
Within that visible 16% we observe extraordinary complexity.



The visible sector is rich, whichever length scale you view it at.

Dark Matters

Often assumed the phenomenology of the dark sector, including interactions with us, is also dominated by a single state.



Consider the rich phenomenology of the visible sector. Why should the dark sector be simple?

Our Dark Future

Would not be surprised if first dark world discovery is of light states which are not necessarily the dark matter itself, but are connected with dark sectors.

Our Dark Future

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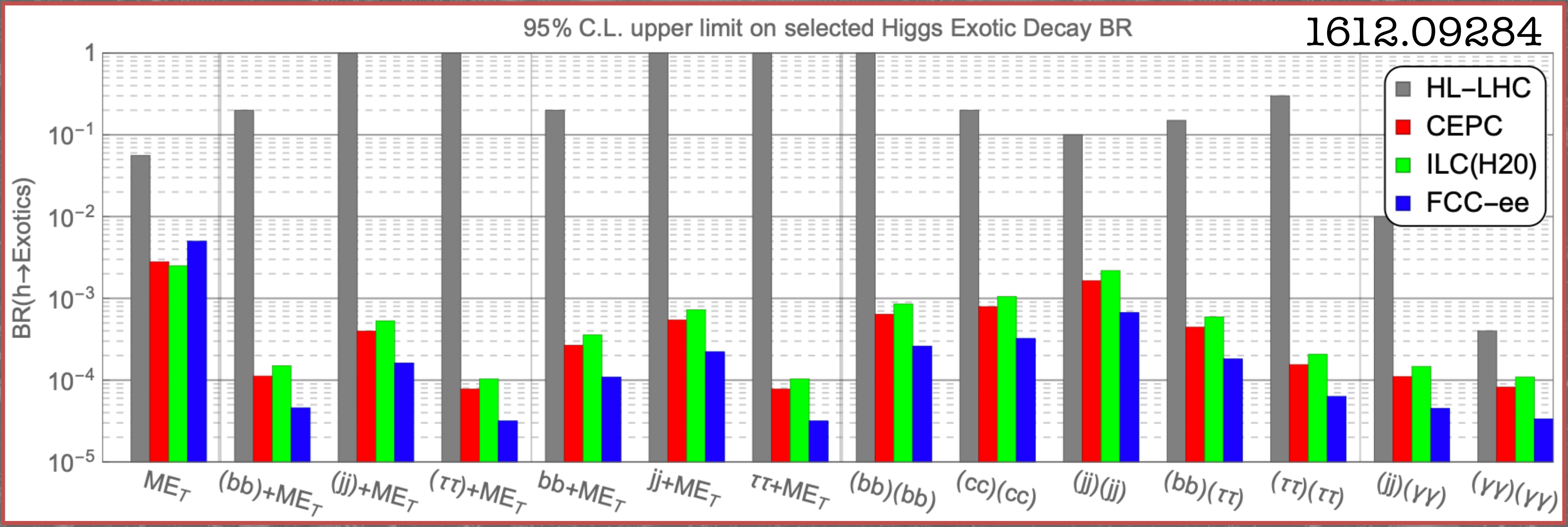
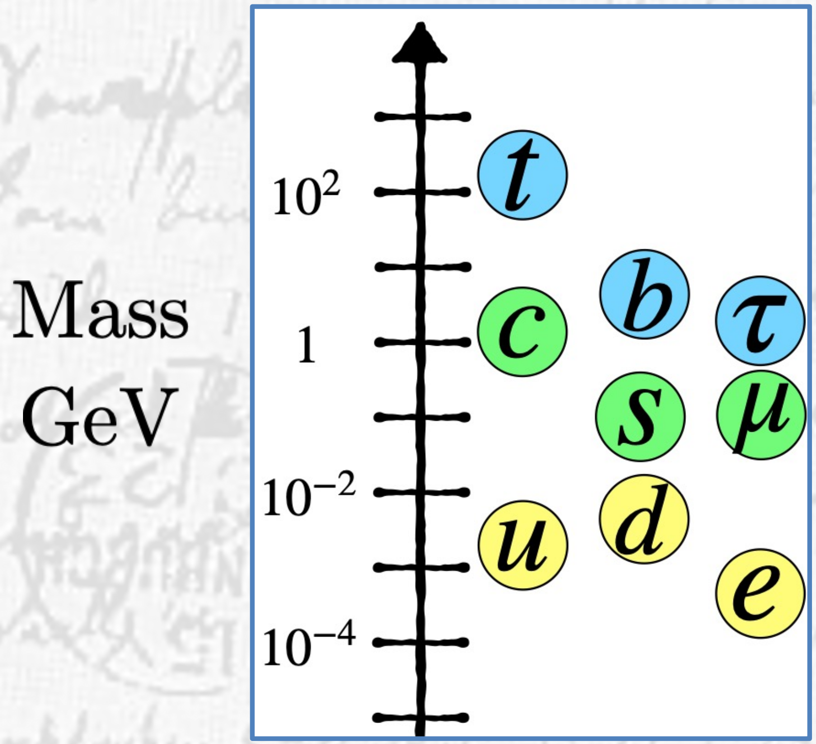


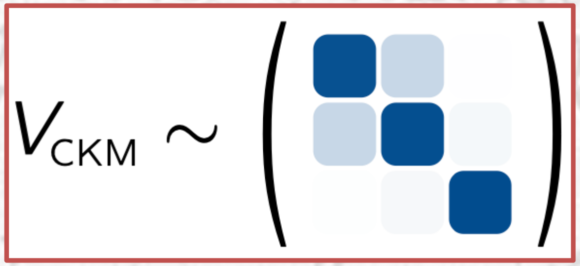
FCC-ee enables us to explore the richness of the dark sector with unprecedented breadth.

Question: With all this in mind, where, and how, might we make progress?

Answer:
By indirectly searching deep into the UV with unprecedented precision.

Clearly something lurking in UV, should show up in small modifications of Standard Model processes…





Figures borrowed from A. Greljo.

Indirect requires precision, thus statistics…

How about five trillion Z’s?

EW Factory Physics

...a quantum leap in our understanding of electroweak physics...

Observable	Present value	±	error	FCC-ee (statistical)	FCC-ee (systematic)
m_Z (keV/c^2)	91 186 700	±	2200	5	100
Γ_Z (keV)	2 495 200	±	2300	8	100
R_ℓ^Z ($\times 10^3$)	20 767	±	25	0.06	1
$\alpha_s(m_Z)$ ($\times 10^4$)	1196	±	30	0.1	1.6
R_b ($\times 10^6$)	216 290	±	660	0.3	<60
σ^0_{had} ($\times 10^3$) (nb)	41 541	±	37	0.1	4
N_ν($\times 10^3$)	2991	±	7	0.005	1
$\sin^2\theta_W^{eff}$($\times 10^6$)	231 480	±	160	3	2–5
$1/\alpha_{QED}(m_Z)$($\times 10^3$)	128 952	±	14	4	Small
$A_{FB}^{b,0}$ ($\times 10^4$)	992	±	16	0.02	<1
$A_{FB}^{pol,\tau}$ ($\times 10^4$)	1498	±	49	0.15	<2
m_W (keV/c^2)	803 500	±	15 000	600	300

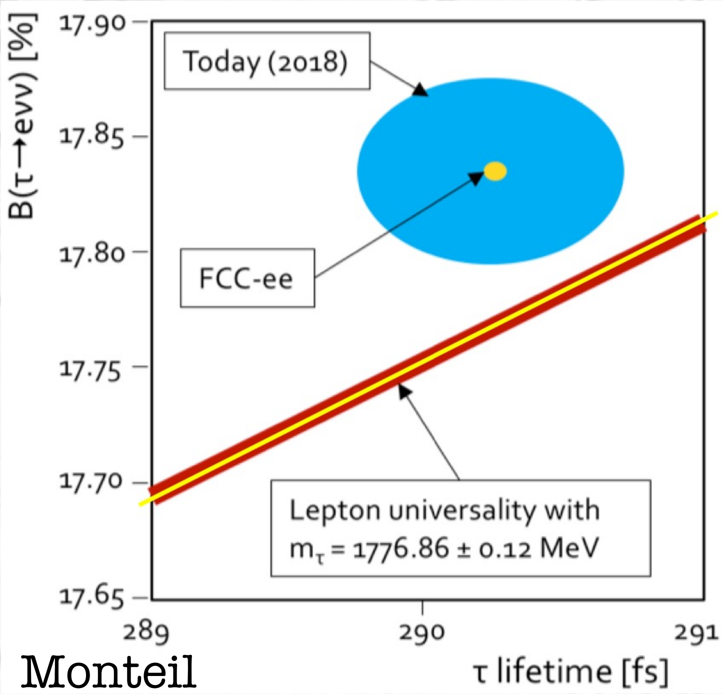
Compare these columns.

Flavour-Factory Physics

An unparalleled probe of flavour physics!

Particle production (10^9)	B^0 / $\overline{B}^0$	B^+ / B^-	B^0_s / $\overline{B}^0_s$	Λ_b / $\overline{\Lambda}_b$	$c\overline{c}$	τ^-/τ^+
Belle II	27.5	27.5	n/a	n/a	65	45
FCC-ee	300	300	80	80	600	150

For example...

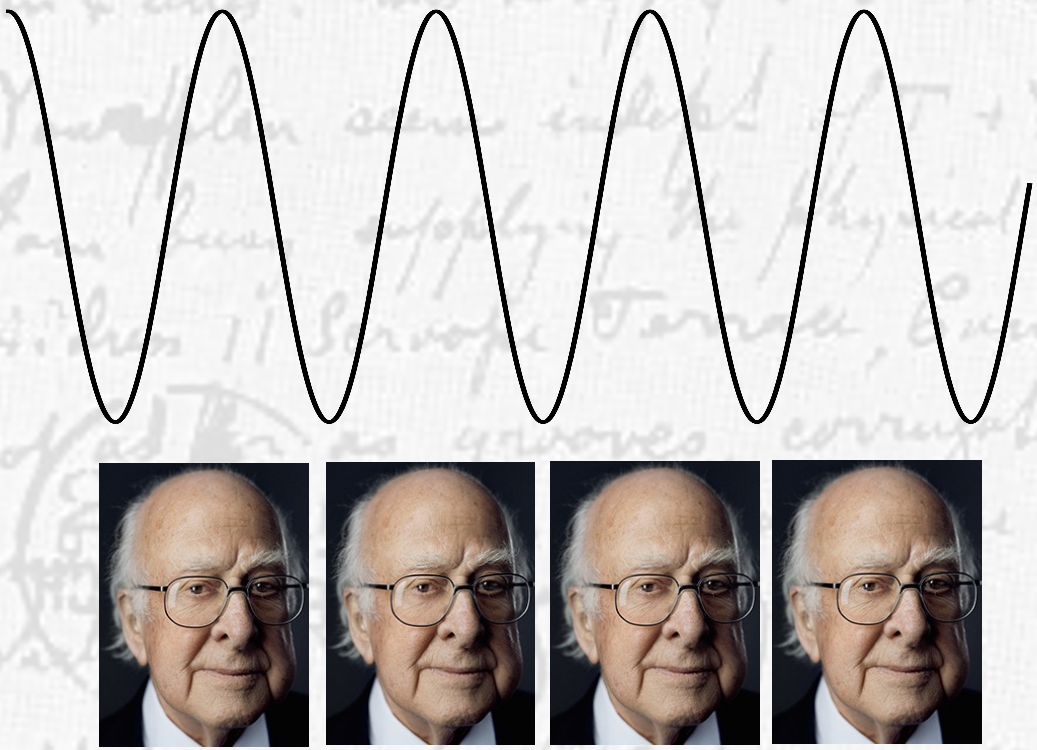


Question: With all this in mind, where, and how, might we make progress?

Answer:
By putting the Higgs under a microscope.

Is the Higgs Fundamental?

The Higgs boson has a size/wavelength. What's inside?



Precision measurements are different ways of probing the "compositeness of the Higgs".

$\lambda_h \approx 10^{-17}$ m

$\lambda_{10\ \text{TeV}} \approx 10^{-19}$ m

Composite Higgs - Basics

Explicit global symmetry breaking generates the composite Higgs potential:

$$V(h) = \epsilon f^2 \Lambda^2 F\left(h/f\right)$$

How much symmetry breaking

How the symmetry is broken...

Assumption until now has been that the global symmetry is broken in the most "simple" way, hence the structure of the Higgs potential is "simple".

Beyond Minimality

Relaxing this assumption to a completely general explicit breaking, hence general Higgs potential:

$$V_\epsilon = \frac{\lambda}{f^{n-4}} \epsilon_{a_1, a_2, \ldots, a_n} \phi^{a_1} \phi^{a_2} \ldots \phi^{a_n}$$

How the symmetry is broken...

Turns out the fully general family of Higgs potentials is comprised of so-called:

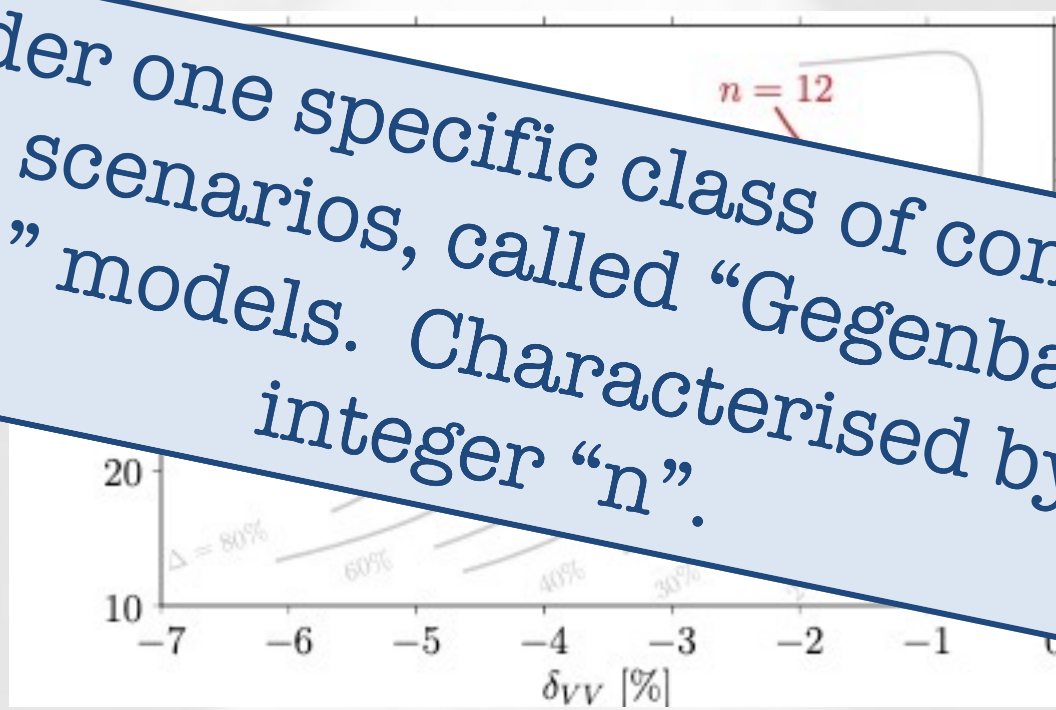
$$V = \epsilon m_\rho^2 f^2 \, G_n^{(N-1)/2}(\cos \Pi / f)$$

"Gegenbauer" functions!

Durieux, MM, Salvioni. 2021

Composite Higgs (GT)

Modifications to self-interaction relative to other couplings are huge:



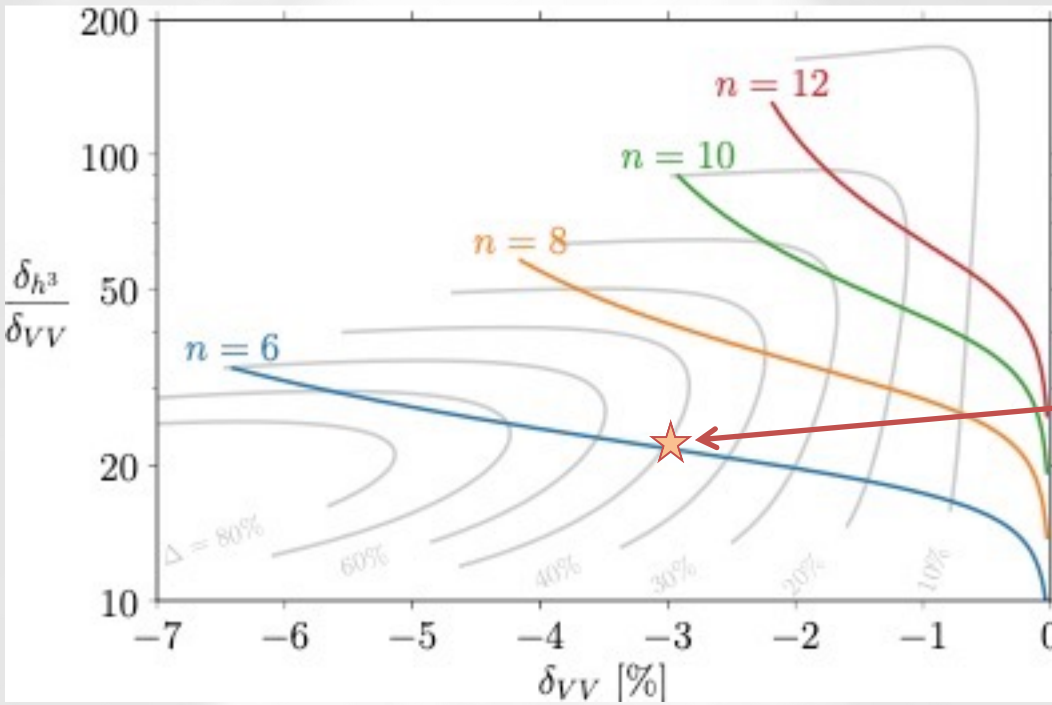
Consider one specific class of composite Higgs scenarios, called "Gegenbauer's Twin" models. Characterised by an integer "n".

Compositeness could show up in self-interaction!

Fine-tuning is small. The Higgs could still, naturally, be composite!

Composite Higgs (GT)

Modifications to self-interaction relative to other couplings are huge:



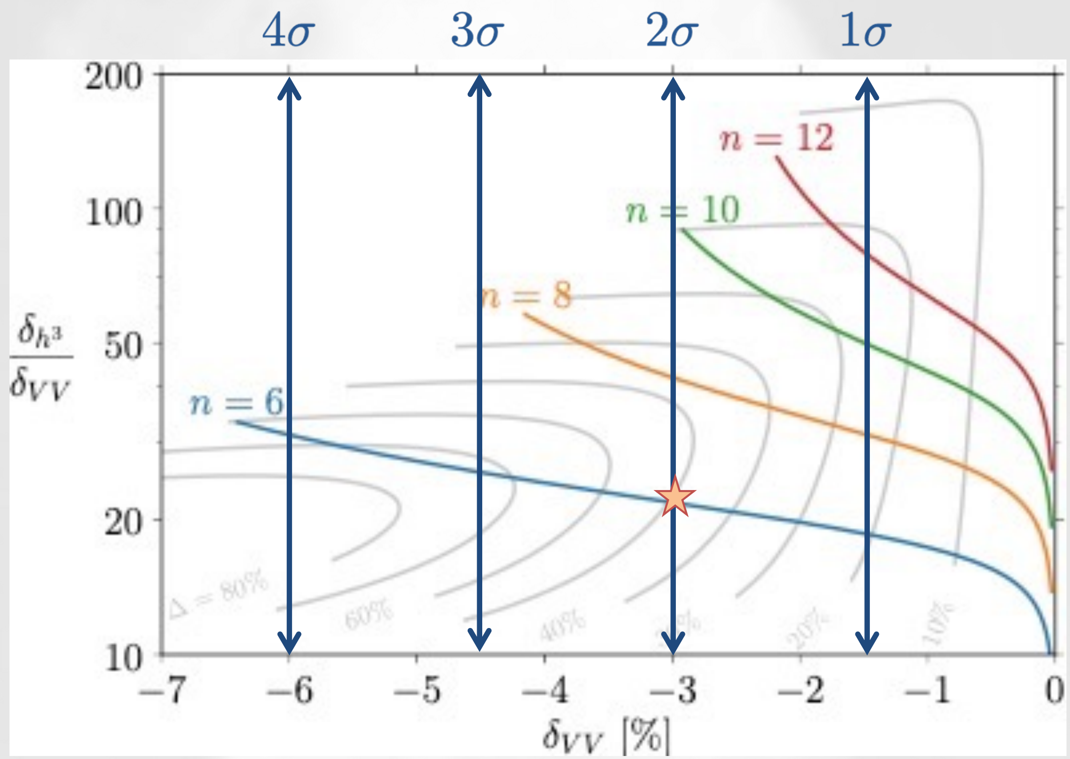
Compositeness could show up in self-interaction!

Example parameter point. 50% fine-tuning.

Fine-tuning is small. The Higgs could still, naturally, be composite!

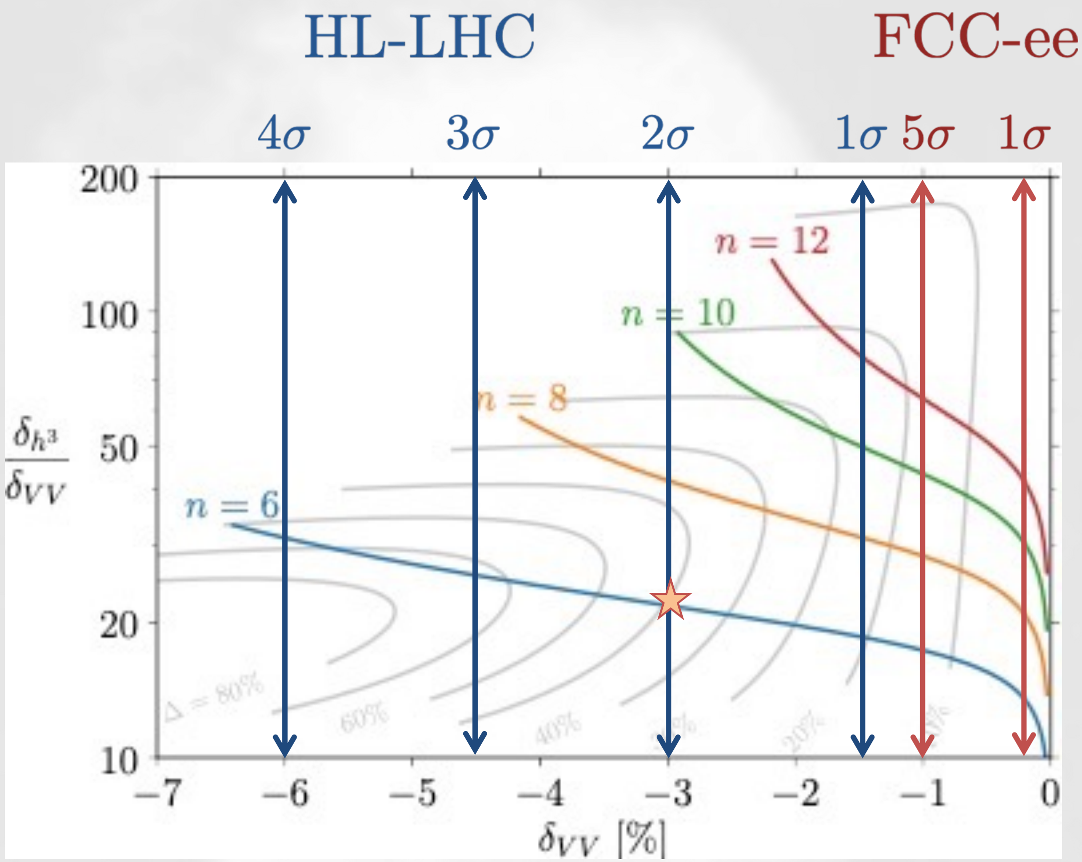
Composite Higgs (GT)

HL-LHC



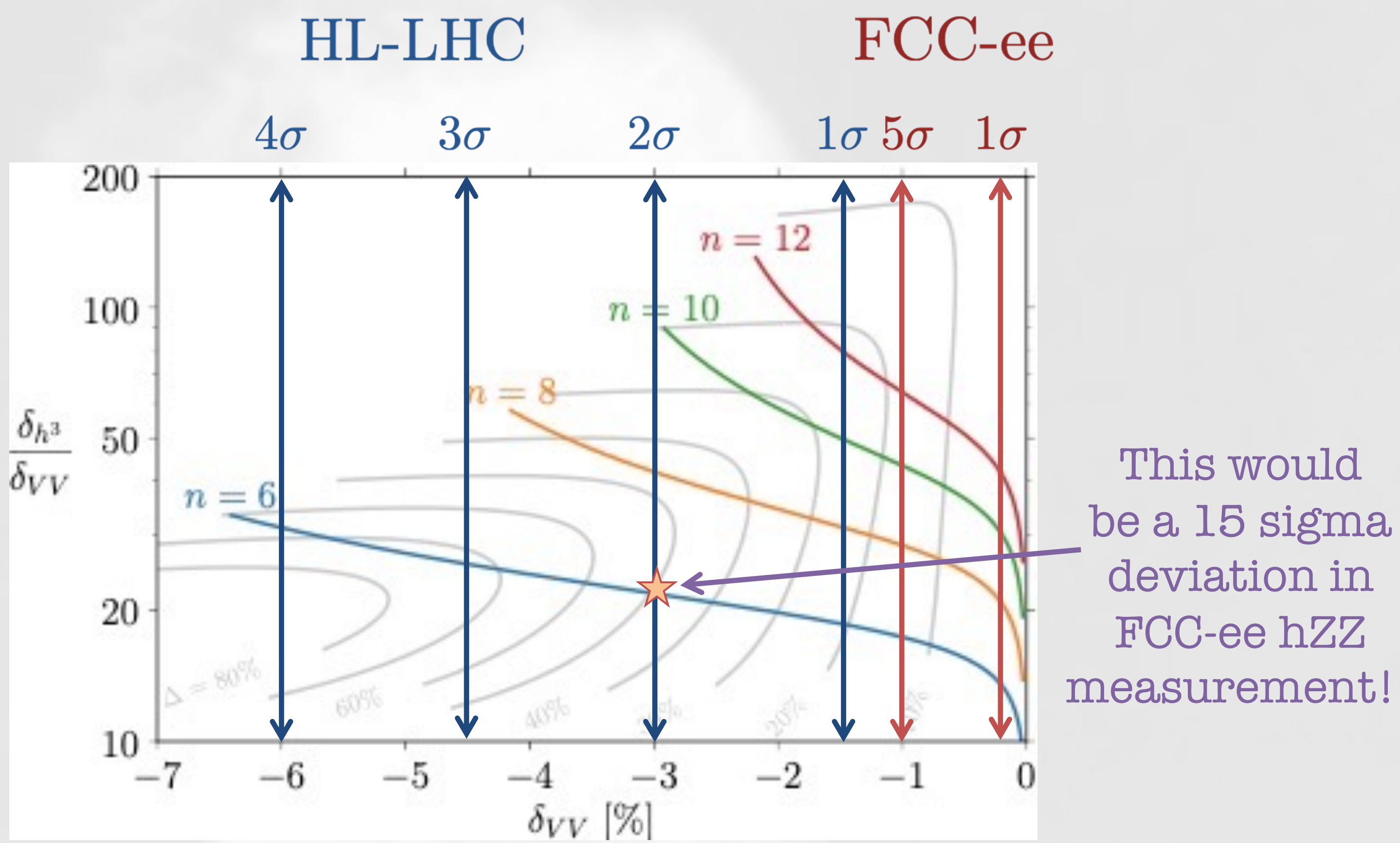
HL-LHC alone would leave a lot of natural parameter space largely unexplored.

Composite Higgs (GT)



What isn't even a 1σ deviation at HL-LHC can be a 5 σ discrepancy at FCC-ee! All natural parameter space conclusively probed!

Composite Higgs (GT)



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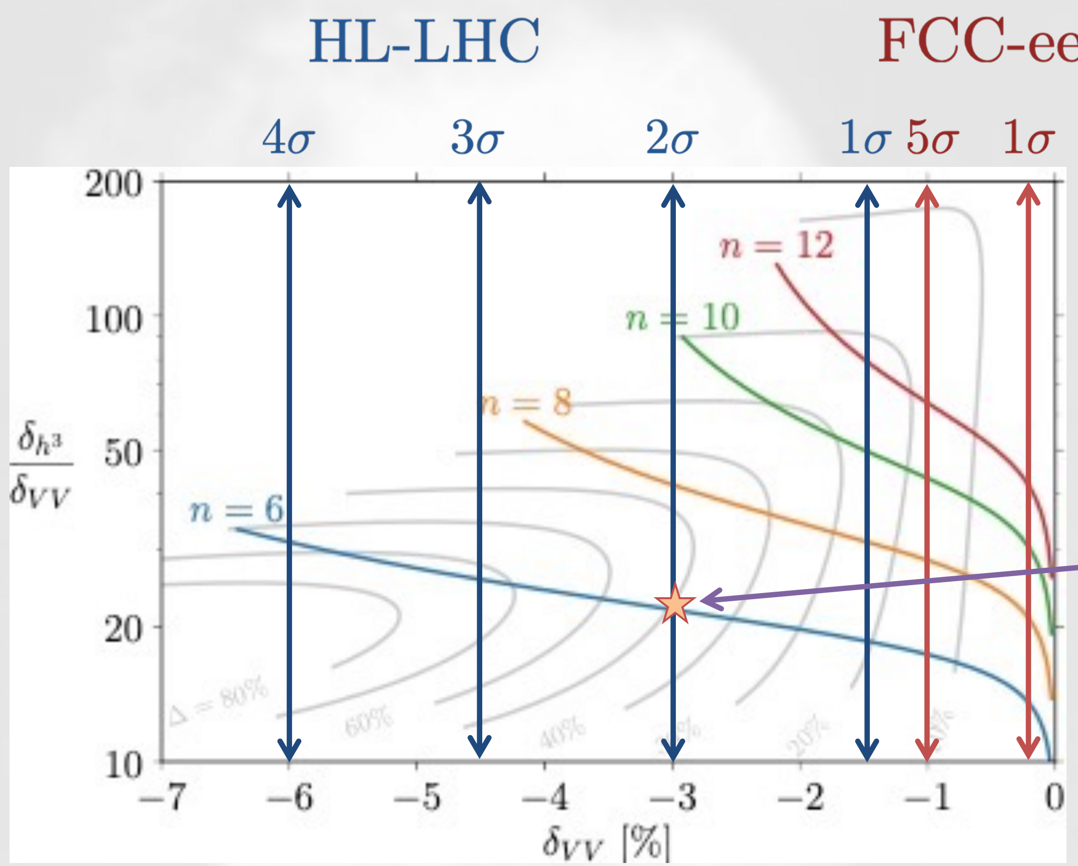
Composite Higgs (GT)



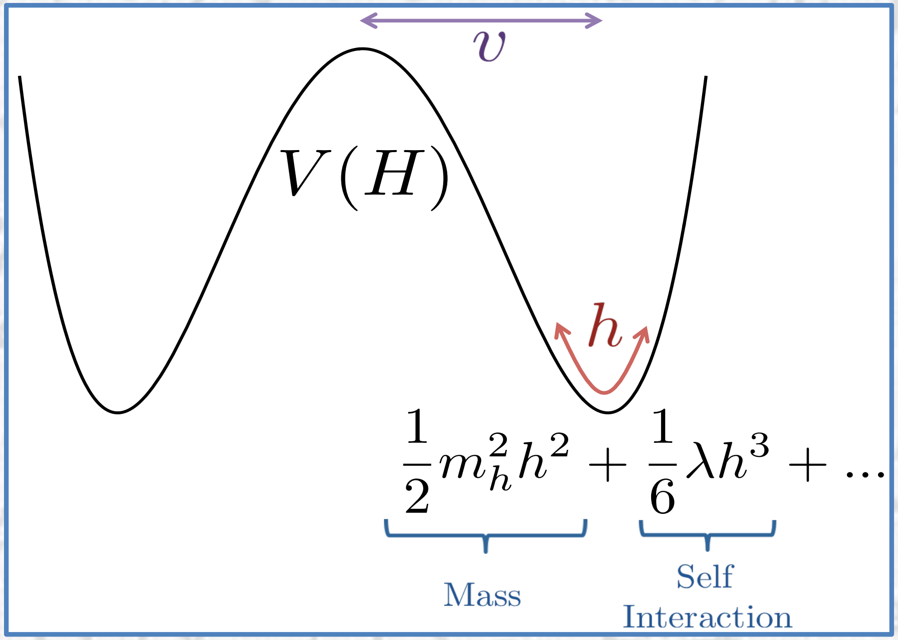
This would be at least a 13 sigma deviation in FCC-hh self-coupling measurement!!

What isn't even a 1σ deviation at HL-LHC can be a 5 σ discrepancy at FCC-ee! All natural parameter space conclusively probed!

Question: With all this in mind, where, and how, might we make progress?

Answer:
By being ambitious.

What is the Higgs Field Potential?



Important because it determines how the Universe froze in the EW sector, giving mass to gauge bosons, fermions, the Higgs…

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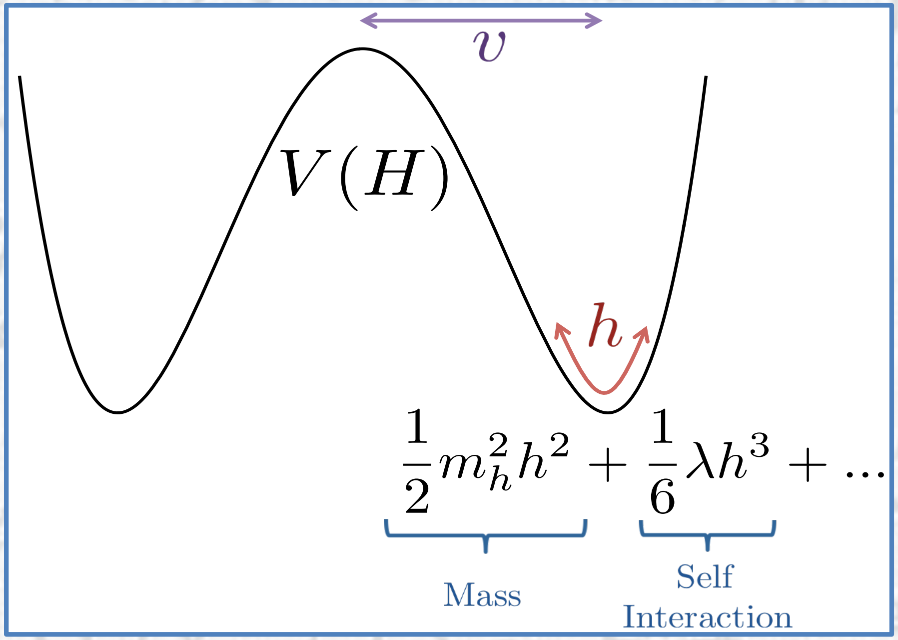
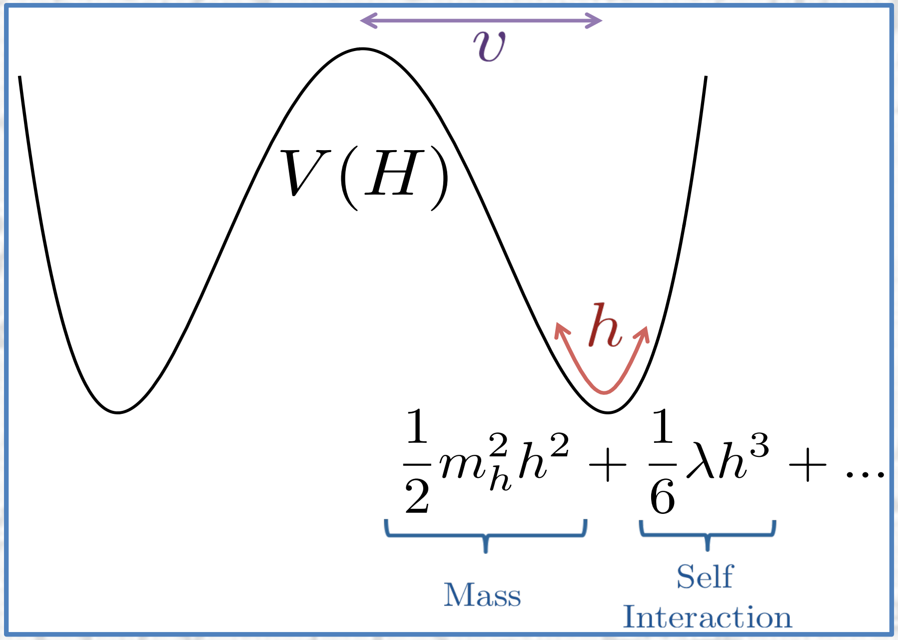


...because it determines how the Universe will end...

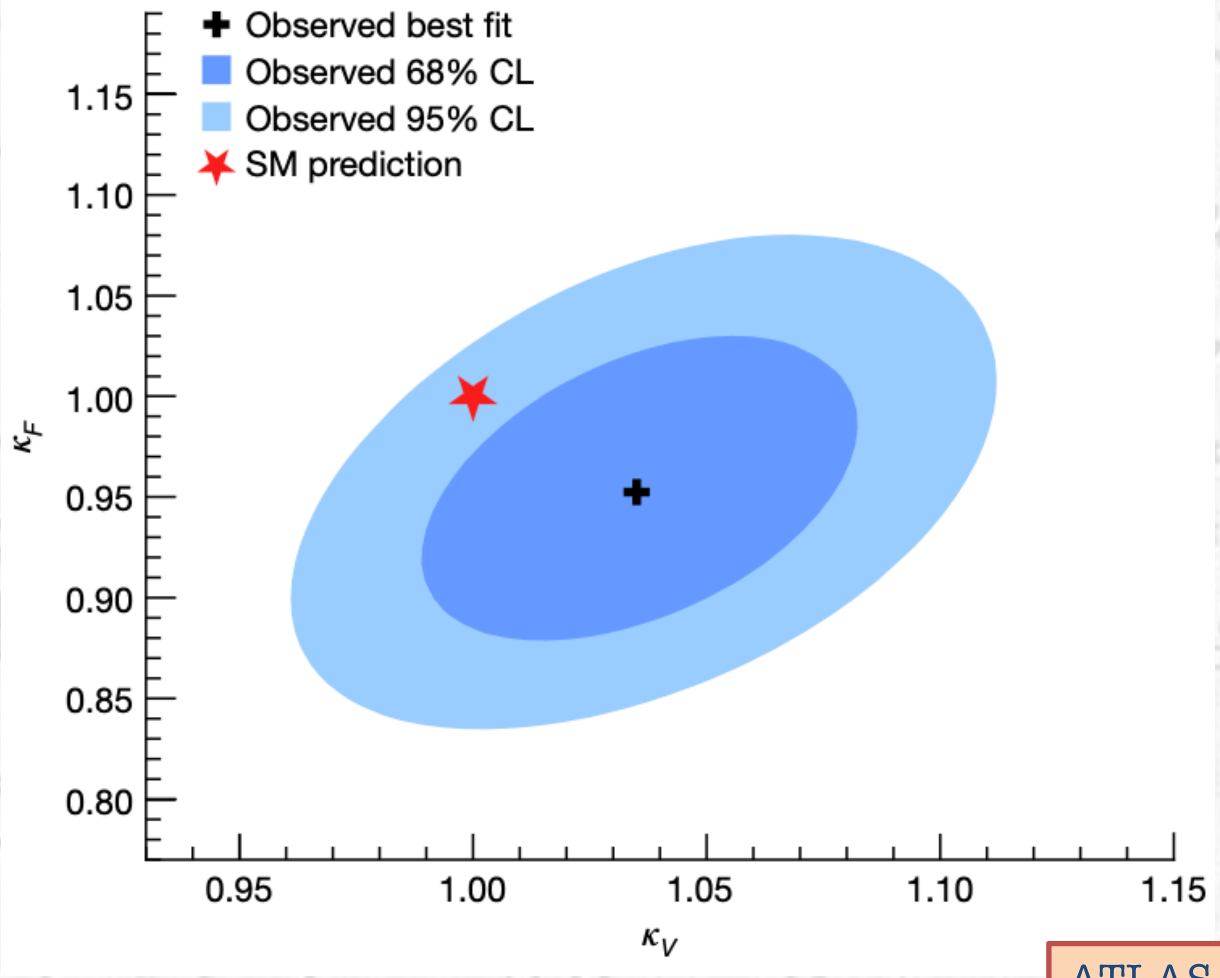
What is the Higgs Field Potential?



... but it is incredibly difficult to access...

Status of Higgs Couplings

What are experimental limits on modifications of couplings relative to Standard Model prediction?



ATLAS, Nature, 2022

Status of Higgs Couplings

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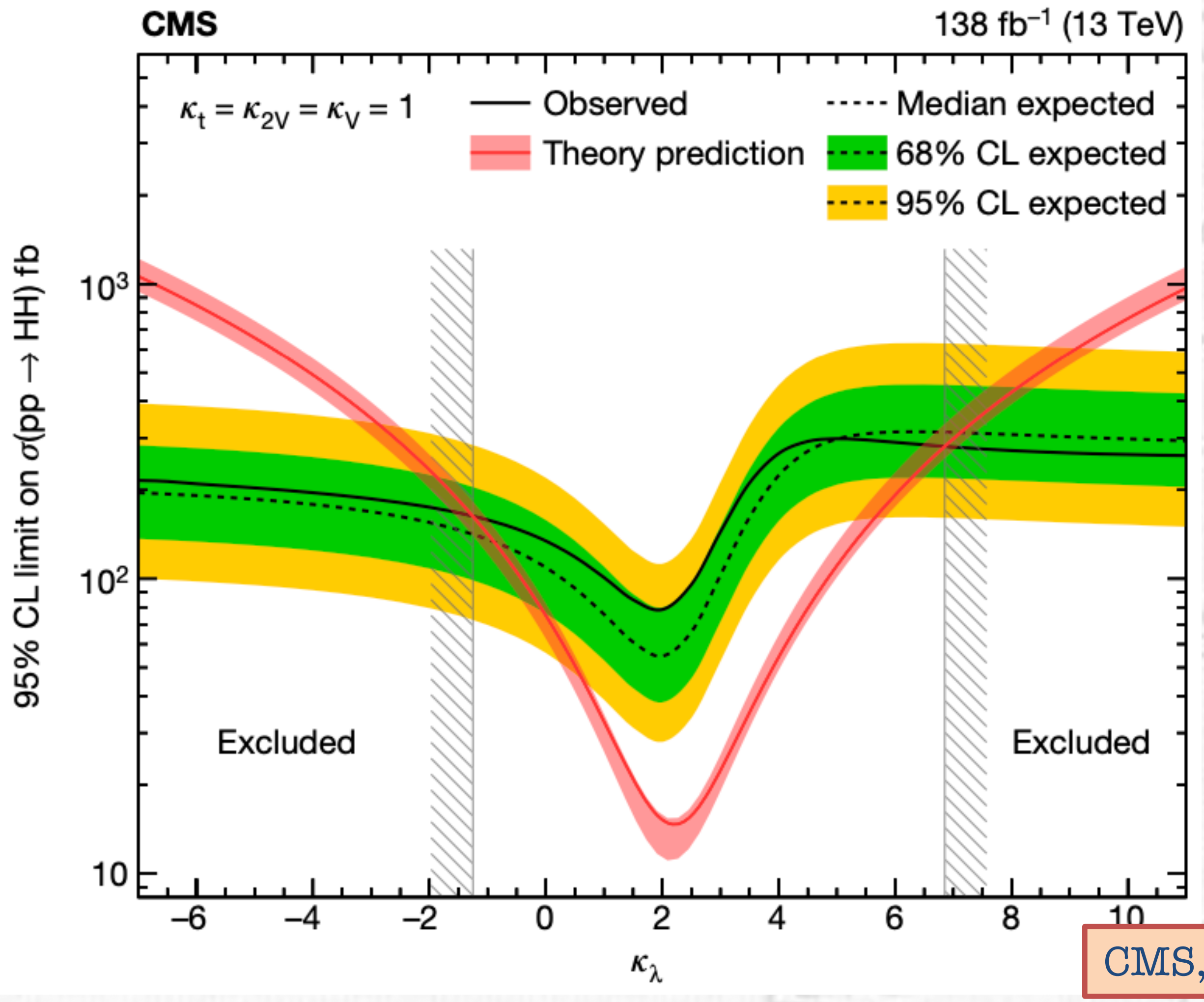


CMS, Nature, 2022

Self-Coupling Dominance?

No obstruction to having Higgs self-coupling modifications a "loop factor" greater than **all** other coupling modifications. Could have

$$\left|\frac{\delta_{h^3}}{\delta_{VV}}\right| \lesssim \min\left[\left(\frac{4\pi v}{m_h}\right)^2, \left(\frac{M}{m_h}\right)^2\right]$$

without fine-tuning any parameters, so as big as,

$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

Durieux, MM, Salvioni. 2022

Custodial Quadruplet

Ok, but in reality is there a viable scenario?

Yes: The custodial quadruplet scalar. Projecting the $(\mathbf{4},\mathbf{4})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ onto EW group we have

$$(\mathbf{4},\mathbf{4}) \quad \rightarrow \quad \mathbf{4}_{1/2} + \mathbf{4}_{3/2}$$

and including all possible couplings to the Higgs we have for scalar quadruplet

$$\mathcal{L}_{\mathrm{SO}(4)} = -\lambda\Big(H^* H^* (\epsilon H)\Phi + \frac{1}{\sqrt{3}} H^* H^* H^* \widetilde{\Phi}\Big) + \mathrm{h.c.}$$

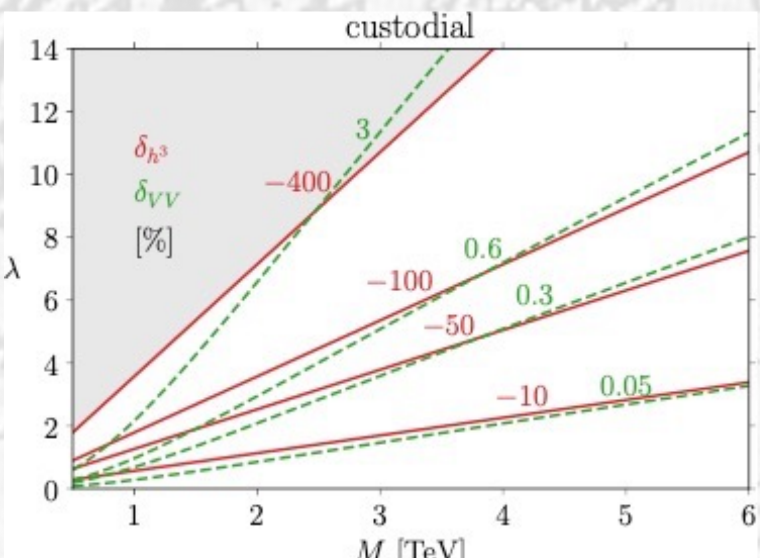
which has exactly the pattern described.

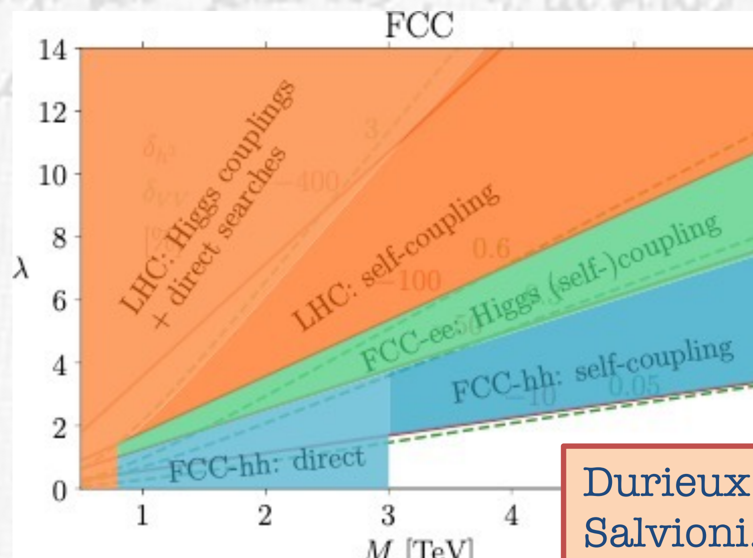
Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-level, all other couplings modified at dim-6 one-loop, or dim-8. All calculable, giving

$$-\frac{\delta_{VV}}{\delta_{h^3}} = 3\left(\frac{m_h}{4\pi v}\right)^2 + \left(\frac{m_h}{M}\right)^2 \approx \frac{1}{200} + \frac{1}{580}\left(\frac{3\ \mathrm{TeV}}{M}\right)^2$$

Remarkably close to NDA estimate!



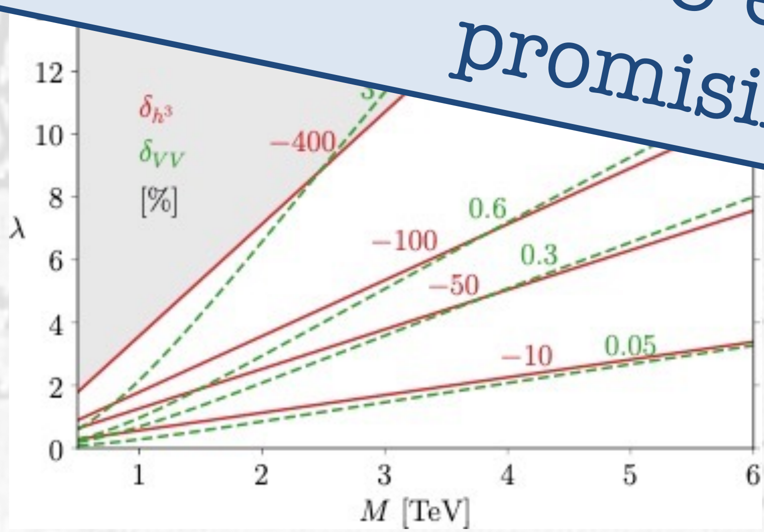


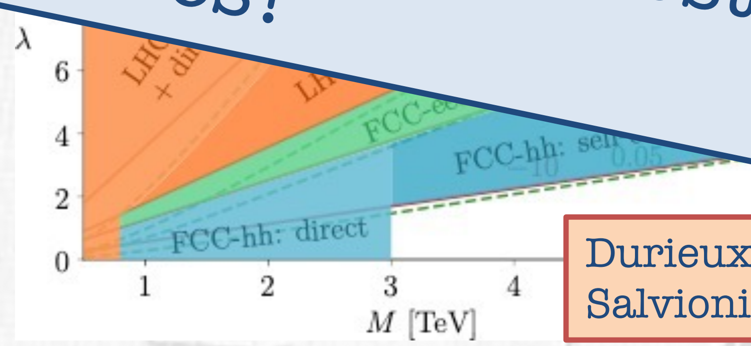
Durieux, MM, Salvioni. 2022

Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-lev... couplings modified at dim-6 one-loo... ble, giving

Punchline: Currently only know the self-interaction at the level of 100's %. There is plenty of room for enormous new physics effects to show up in the self-coupling and FCC-ee and hh are the most promising probes!



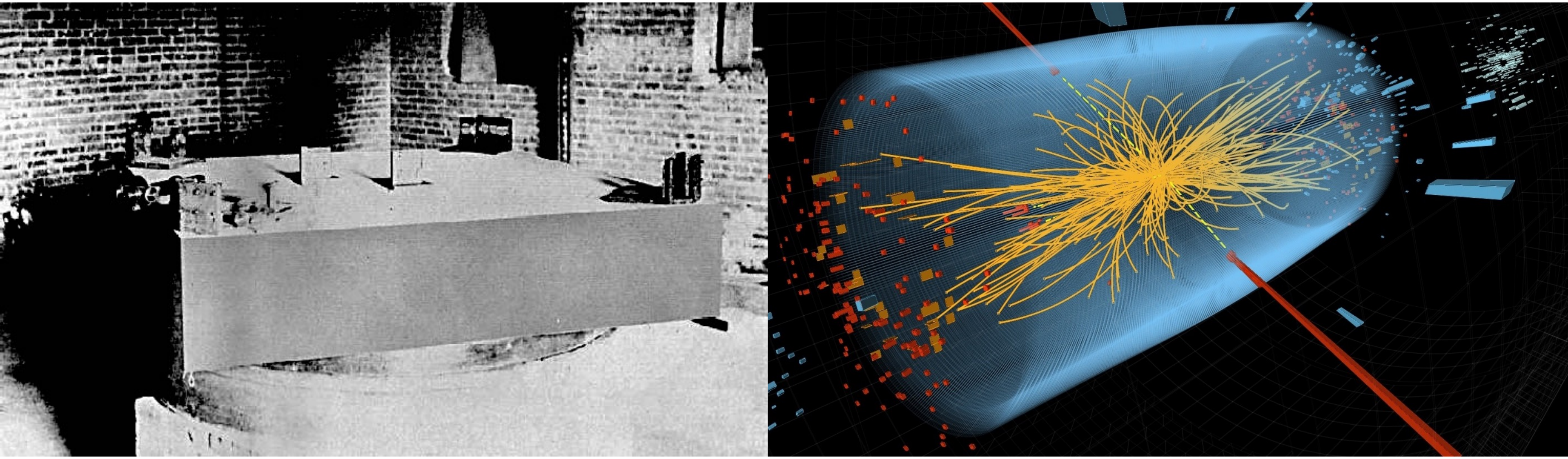


Durieux, MM, Salvioni. 2022

The physics case?

Delivering ambitious, unbiased, high precision exploration of the Higgs Boson and far beyond.

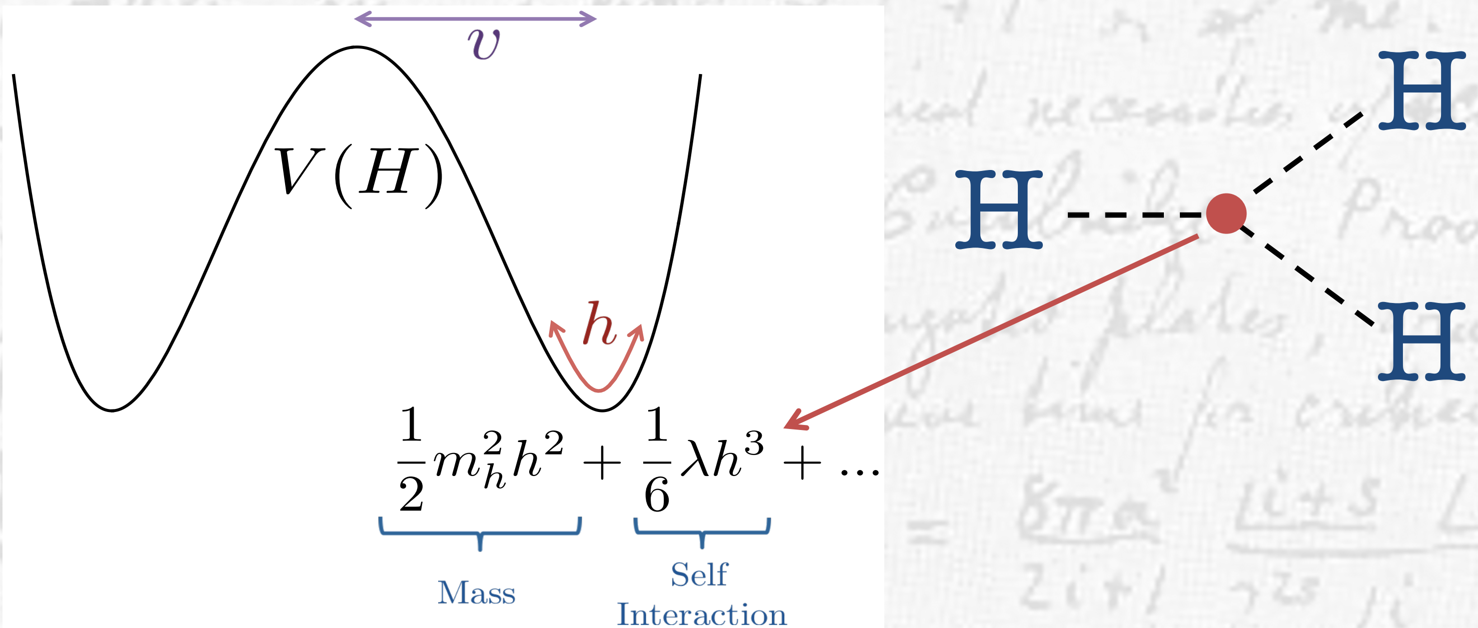
Where are we on our journey to the heart of nature?



"The future cannot be predicted, but futures can be invented" (Gabor, 1963)

Naïve Dimensional Analysis

It's known that O_6 contributes to Higgs self-interaction, etc.



But less-well appreciated are the theoretical aspects underlying it…

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a Higgs potential which looks like

"Compositeness" Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a "generic" function. The position of the minimum of the potential doesn't care about the prefactor:

$$V'(h) = 0 \Leftrightarrow F'(h/f) = 0$$

So, if minimum is to occur at $h = v \ll f$ then one has to fine-tune the contributions to the potential from the composite physics.

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a Higgs potential which looks like

Compositeness Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. However, it is typical that the operator

$$\mathcal{O}_H \sim \frac{1}{f^2}\left(\partial^\mu |H|^2\right)^2$$

is generated. This modifies all Higgs couplings by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a p... which looks like

Compositeness Scale

$$\ldots^2 F(h/f)$$

In vanilla scenarios, Higgs coupling measurements suggest that if the Higgs is composite then there must be some fine-tuning of parameters at least at the 10% or so level!

is generated. This modifies all Higgs ... by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$