

# Theoretical calculation strategy

towards strong FCC-ee experimental demands

Janusz Gluza

FCC Week 2023

5 June 2023, London

'The reasonable man adapts himself to the world.  
The unreasonable one persists in trying to adapt the world to  
himself.  
Therefore all progress depends on the unreasonable man.'

— George Bernard Shaw, *Man and Superman*



# Backup slides

Staszek Jadach, inter alia: Petra, Hera, *LEP1*, *LEP2*, LHC, *FCC-ee*



LEP1 (1989) link: Z Physics at LEP1 : vol. 1 : Standard Physics  
LEP2 W-physics (2000), link: Reports of the WGs on Precision Calculations for LEP2 Physics

Standard Model Theory for the FCC-ee Tera-Z stage (2019): link

## Scheme of construction and the use of EWPO/EWPP at FCC-ee

### EXPERIMENT

**(A)**  
 Raw experimental **DATA**  
 including  
 cut-offs, efficiencies, QED

Removing detector  
 inefficiencies,  
 (simplifying cut-offs)

**(B)**  
 Experimental **DATA**  
 with idealised cut-offs  
 QED still present  
 (realistic observables)

Predicting realistic distributions

Fitting with MC, WT-diffs

Fitting using WT diff.,  
 MC programs  
 of KKMC class

**(C)**  
 EWPO's  
 or EWPP's  
 Parameters in  
 the effective Born,  
**QED subtracted**

### THEORY

**BSM Physics Models**  
 +SM without QED

**(D)**  
 SM calculations  
 1-2-3 EW loops  
 QED subtracted

non-MC filters  
 like ZFITTER/TOPAZ0

Scheme   DATA  $\longrightarrow$  Born  $\otimes$  QED  
 $\longrightarrow$   $\mathcal{O}(\alpha^1)^{\text{noQED}} \otimes \text{QED}$

- ⌘ Better Monte Carlo algorithm for phase space with very hard photons.  
Phase space generation in KKMC for extremely hard photos is inefficient.
- ⌘ Novel ideas for better incorporation of the collinear resummation within soft photon resummation, especially at the amplitude level (CEEX), main problems are loops.
- ⌘ Alternative methods of calculating spin amplitudes in CEEX,  
instead of Kleiss-Stirling, for massive particles?
- ⌘ Soft photon emission resummation from unstable charged particles like W boson.  
Outline is there but implementation nontrivial.
- ⌘ Subtraction of IR part from (gauge invariant) sets of multi-loop diagrams at the loop integrand level.
- ⌘ Fitting EWPOs to data using high statistics “MC templates”, weight differences,  
machine learning etc.
- ⌘ Effective methods of parametrising the virtual (loop) correction to be used in the  
matrix element in the MC generators.

In general: *Matching of higher order matrix elements with QED parton shower (IS, FS, exclusive).*

## TOOLS and methods, additions

---

Many groups present rapid progress:

- ▶ Analytical/**numerical** solutions for Master Integrals (MIs) by *differential equations DEs*;
- ▶ Sector decomposition (*SD*);
- ▶ Mellin-Barnes representations (*MB*);
- ▶ Reductions at the integrand level;
- ▶ Expansions by regions; Taylor expansion in Feynman parameters;
- ▶ Loop-tree duality (G. Rodrigo et al, Weinzierl et al.);
- ▶ Multi-loop amplitudes with numerical unitarity (Abreu et al.);
- ▶ Four-dimensional unsubtraction; Direct numerical evaluation of multi-loop integrals without contour deformation (R. Pittau et al.);
- ▶ Feynman parameters and dispersion relations (Song, Freitas);
- ▶ ...

## Recent exploratory methods

---

Feynman parameter integration through differential equations, Martijn Hidding, Johann Usovitsch, <https://arxiv.org/abs/2206.14790>

Feynman Integrals from Positivity Constraints, Mao Zeng  
<https://arxiv.org/abs/2303.15624>

News on IBPs:

Reduction to master integrals via intersection numbers and polynomial expansions Gaia Fontana, Tiziano Peraro  
<https://arxiv.org/pdf/2304.14336.pdf>

Macaulay Matrix for Feynman Integrals: Linear Relations and Intersection Numbers, Chestnov et al, <https://arxiv.org/abs/2204.12983>

Targeting multi-loop integrals with neural networks, R. Winterhalder et al  
<https://arxiv.org/abs/2112.09145>

## Input and calculated/measured parameters

---

Schemes:  $G_\mu$  vs  $M_W$ , ...

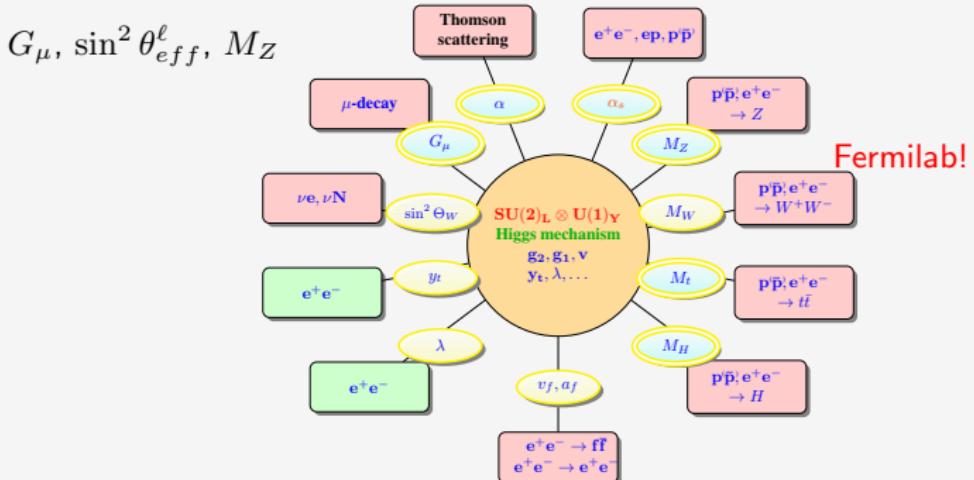


Fig. from the FCC-ee report ' $\alpha_{QED}$ ' by F. Jegerlehner in [1905.05078](#)

## Input, theoretical and parametric errors,

---

A. Freitas et al., "Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee", <https://arxiv.org/abs/1906.05379>

Quantity	FCC-ee	Current intrinsic error	Projected intrinsic error (at start of FCC-ee)
$M_W$ [MeV]	$0.5\text{--}1^\ddagger$	4	( $\alpha^3$ , $\alpha^2 \alpha_s$ )
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	0.6	4.5	( $\alpha^3$ , $\alpha^2 \alpha_s$ )
$\Gamma_Z$ [MeV]	0.1	0.4	( $\alpha^3$ , $\alpha^2 \alpha_s$ , $\alpha \alpha_s^2$ )
$R_b$ [ $10^{-5}$ ]	6	11	( $\alpha^3$ , $\alpha^2 \alpha_s$ )
$R_\ell$ [ $10^{-3}$ ]	1	6	( $\alpha^3$ , $\alpha^2 \alpha_s$ )

<sup>‡</sup>The pure experimental precision on  $M_W$  is  $\sim 0.5$  MeV.

Quantity	FCC-ee	future parametric unc.	Main source
$M_W$ [MeV]	0.5 – 1	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	0.6	2 (1)	$\delta(\Delta\alpha)$
$\Gamma_Z$ [MeV]	0.1	0.1 (0.06)	$\delta \alpha_s$
$R_b$ [ $10^{-5}$ ]	6	< 1	$\delta \alpha_s$
$R_\ell$ [ $10^{-3}$ ]	1	1.3 (0.7)	$\delta \alpha_s$

Important input parameter errors are  $\delta(\Delta\alpha) = 3 \cdot 10^{-5}$ ,  $\delta \alpha_s = 0.00015$ .  
 $\alpha_s$  —> see the talk by David d'Enterria.

## Input and renormalization schemes

---

E.g. the bosonic 2-loop corrections shift the value of  $\Gamma_Z$  by 0.51 MeV when using  $M_W$  as input and 0.34 MeV when using  $G_\mu$  as input.

Reminder:  $\delta\Gamma_{Z,\text{FCC-ee}} = 0.1 \text{ MeV}$

Dubovsky et al, <https://doi.org/10.1016/j.physletb.2018.06.037>

$\Gamma_i$ [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	$\Gamma_d, \Gamma_s$	$\Gamma_u, \Gamma_c$	$\Gamma_b$	$\Gamma_Z$
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_{\text{bos}}^2)$	<b>0.017</b>	<b>0.019</b>	<b>0.058</b>	<b>0.057</b>	<b>0.167</b>	<b>0.505</b>
$\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	0.038	0.059	0.191	0.170	<b>0.190</b>	1.20

\* Fixed values of  $M_W$

**Table 3** Measurement of selected precision measurements at FCC-ee, compared with present precision. Statistical errors are indicated in bold phase. The systematic uncertainties are initial estimates, aim is to improve down to statistical errors. This set of measurements, together with those of the Higgs properties, achieves indirect sensitivity to new physics up to a scale  $\Lambda$  of 70 TeV in a description with dim 6 operators, and possibly much higher in specific new physics (non-decoupling) models

Observable	Present value $\pm$ error	FCC-ee stat.	FCC-ee syst.	Comment and leading exp. error
$m_Z$ (keV)	$91186700 \pm 2200$	<b>4</b>	100	From Z line shape scan Beam energy calibration
$\Gamma_Z$ (keV)	$2495200 \pm 2300$	<b>4</b>	25	From Z line shape scan Beam energy calibration
$\sin^2 \theta_W^{\text{eff}} (\times 10^6)$	$231480 \pm 160$	<b>2</b>	2.4	from $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{\text{QED}}(m_Z^2) (\times 10^3)$	$128952 \pm 14$	<b>3</b>	Small	From $A_{FB}^{\mu\mu}$ off peak QED&EW errors dominate
$R_\ell^Z (\times 10^3)$	$20767 \pm 25$	<b>0.06</b>	0.2–1	Ratio of hadrons to leptons Acceptance for leptons
$\alpha_S(m_Z^2) (\times 10^4)$	$1196 \pm 30$	<b>0.1</b>	0.4–1.6	From $R_\ell^Z$ above
$\sigma_{\text{had}}^0 (\times 10^3)$ (nb)	$41541 \pm 37$	<b>0.1</b>	4	Peak hadronic cross section Luminosity measurement
$N_V (\times 10^3)$	$2996 \pm 7$	<b>0.005</b>	1	Z peak cross sections Luminosity measurement
$R_b (\times 10^6)$	$216290 \pm 660$	<b>0.3</b>	< 60	Ratio of bb to hadrons

Observable	present value $\pm$ error	FCC-ee <b>Stat.</b>	FCC-ee Syst.	Comment and leading exp. error
$m_Z$ (keV)	$91186700 \pm 2200$	<b>4</b>	100	From Z line shape scan Beam energy calibration
$\Gamma_Z$ (keV)	$2495200 \pm 2300$	<b>4</b>	25	From Z line shape scan Beam energy calibration
$\sin^2 \theta_W^{\text{eff}} (\times 10^6)$	$231480 \pm 160$	<b>2</b>	2.4	from $A_{\text{FB}}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{\text{QED}}(m_Z^2) (\times 10^3)$	$128952 \pm 14$	<b>3</b>	small	from $A_{\text{FB}}^{\mu\mu}$ off peak QED&EW errors dominate
$R_\ell^Z (\times 10^3)$	$20767 \pm 25$	<b>0.06</b>	0.2-1	ratio of hadrons to leptons <b>acceptance for leptons</b>
$\alpha_s(m_Z^2) (\times 10^4)$	$1196 \pm 30$	<b>0.1</b>	0.4-1.6	from $R_\ell^Z$ above
$\sigma_{\text{had}}^0 (\times 10^3)$ (nb)	$41541 \pm 37$	<b>0.1</b>	4	peak hadronic cross section luminosity measurement
$N_\nu (\times 10^3)$	$2996 \pm 7$	<b>0.005</b>	1	Z peak cross sections Luminosity measurement
$R_b (\times 10^6)$	$216290 \pm 660$	<b>0.3</b>	< 60	ratio of bb to hadrons stat. extrapol. from SLD
$A_{\text{FB},0}^b (\times 10^4)$	$992 \pm 16$	<b>0.02</b>	1-3	b-quark asymmetry at Z pole from jet charge
$A_{\text{FB}}^{\text{pol},\tau} (\times 10^4)$	$1498 \pm 49$	<b>0.15</b>	<2	$\tau$ polarization asymmetry $\tau$ decay physics
$\tau$ lifetime (fs)	$290.3 \pm 0.5$	<b>0.001</b>	0.04	radial alignment
$\tau$ mass (MeV)	$1776.86 \pm 0.12$	<b>0.004</b>	0.04	momentum scale
$\tau$ leptonic ( $\mu\nu_\mu\nu_\tau$ ) B.R. (%)	$17.38 \pm 0.04$	<b>0.0001</b>	0.003	e/ $\mu$ /hadron separation
$m_W$ (MeV)	$80350 \pm 15$	<b>0.25</b>	0.3	From WW threshold scan Beam energy calibration
$\Gamma_W$ (MeV)	$2085 \pm 42$	1.2	0.3	From WW threshold scan Beam energy calibration
$\alpha_s(m_W^2) (\times 10^4)$	$1170 \pm 420$	<b>3</b>	small	from $R_\ell^W$
$N_\nu (\times 10^3)$	$2920 \pm 50$	<b>0.8</b>	small	ratio of invis. to leptonic in radiative Z returns
$m_{\text{top}}$ (MeV/c <sup>2</sup> )	$172740 \pm 500$	<b>17</b>	small	From $t\bar{t}$ threshold scan QCD errors dominate
$\Gamma_{\text{top}}$ (MeV/c <sup>2</sup> )	$1410 \pm 190$	45	small	From $t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{\text{top}}/\lambda_{\text{top}}^{\text{SM}}$	$1.2 \pm 0.3$	<b>0.10</b>	small	From $t\bar{t}$ threshold scan QCD errors dominate
ttZ couplings	$\pm 30\%$	0.5 – 1.5%	small	From $\sqrt{s} = 365$ GeV run

*Estimated theoretical uncertainties from missing higher orders and the perturbative orders (QCD/elw.) of the results included in the analysis.*

Partial Width	QCD	Electroweak	Total	on-shell Higgs
$H \rightarrow b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$	$\sim 0.5\%$	$\text{N}^4\text{LO} / \text{NLO}$
$H \rightarrow \tau^+\tau^-/\mu^+\mu^-$	—	$\sim 0.5\%$	$\sim 0.5\%$	— / NLO
$H \rightarrow gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$	$\text{N}^3\text{LO} / \text{NLO}$
$H \rightarrow \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$	NLO / NLO
$H \rightarrow Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$	LO / LO
$H \rightarrow WW/ZZ \rightarrow 4f$	$< 0.5\%$	$\sim 0.5\%$	$\sim 0.5\%$	NLO/NLO

## Higgs boson decays: theoretical status

---

Projected intrinsic and parametric uncertainties for the partial and total Higgs-boson decay width predictions. The last column: the target of FCC-ee precisions.

decay	intrinsic	para. $m_q$	para. $\alpha_s$	para. $M_H$	FCC-ee prec. on $g_{HXX}^2$
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	0.6%	< 0.1%	–	$\sim 0.8\%$
$H \rightarrow c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	< 0.1%	–	$\sim 1.4\%$
$H \rightarrow \tau^+ \tau^-$	< 0.1%	–	–	–	$\sim 1.1\%$
$H \rightarrow \mu^+ \mu^-$	< 0.1%	–	–	–	$\sim 12\%$
$H \rightarrow gg$	$\sim 1\%$		0.5% (0.3%)	–	$\sim 1.6\%$
$H \rightarrow \gamma\gamma$	< 1%	–	–	–	$\sim 3.0\%$
$H \rightarrow Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	
$H \rightarrow WW$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	$\sim 0.4\%$
$H \rightarrow ZZ$	$\lesssim 0.3\%^{\dagger}$	–	–	$\sim 0.1\%$	$\sim 0.3\%$
$\Gamma_{\text{tot}}$	$\sim 0.3\%$	$\sim 0.4\%$	< 0.1%	< 0.1%	$\sim 1\%$

<sup>†</sup> From  $e^+e^- \rightarrow HZ$  production

Z-resonance: SM Theory for the *FCC-ee Tera-Z stage (2019)*: link

---

1. Z-resonance and  $\gamma, Z', \dots \rightarrow$  Laurent series,

$$\mathcal{M} = \frac{R}{s - s_0} + \sum_{n=0}^{\infty} (s - s_0)^n B^{(n)}, \quad s_0 = \bar{M}_Z^2 + i\bar{M}_Z\bar{\Gamma}_Z.$$

2. We want to extract EW Z-vertex couplings and definitions like  $\sin^2 \theta_{\text{eff}}^f$ , but in reality, we deal with complicated process

$$e^+ e^- \rightarrow f^+ f^- + \text{invisible } (n \gamma + e^+ e^- \text{ pairs} + \dots)$$

$$\sigma^{e^+ e^- \rightarrow f^+ f^- + \dots}(s) = \int dx \underbrace{\widehat{f(x)}}_{\sigma^{e^+ e^- \rightarrow f^+ f^-}(s')} \delta(x - s'/s)$$

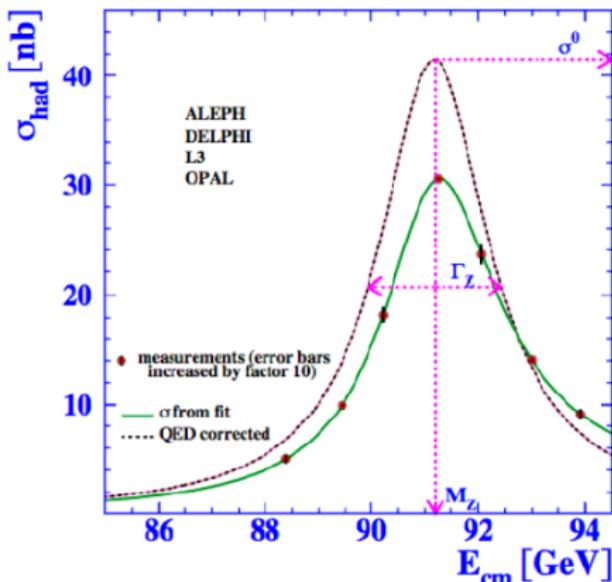
→ form factors, QED separation/deconvolution, non-factorizations,  
...

To determine the structure function/flux function kernels and hard scattering ansatz for data preparation or for unfolding is one of the challenges of FCC-ee-Z physics.

## QED unfolding

Altogether  $17 \cdot 10^6$  Z-boson decays at LEP

- Cross section : Z mass and width



- ◆ -30% QED corrections (ISR)

## How to unfold - rough scheme

---

We have to describe

$$e^+ e^- \rightarrow (\gamma, Z) \rightarrow f^+ f^-(\gamma),$$

S-matrix Ansatz in the complex energy plane

$$\begin{aligned} \mathcal{A}_{e^+ e^- \rightarrow b\bar{b}} &= \frac{R_Z}{s - s_Z} + \overbrace{\frac{R_\gamma}{s} + S + (s - s_Z)S'}^{Background} + \dots, \\ &\quad \underbrace{\gamma-Z \text{ interference}}_{s_Z = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z} \end{aligned}$$

- ▶  $R, S, S', \dots$  are individually gauge-invariant and UV-finite - **unitarity and analyticity of the S-matrix**. IR-finite, when soft and collinear real photon emission is added. [Willenbrock, Valencia, 1991] [Sirlin, 1991] [Stuart, 1991]

The term  $R_\gamma(s)/s$  is part of the background

---

- ▶ The poles of  $\mathcal{A}$  have complex residua  $R_Z$  and  $R_\gamma$ .
- ▶ There is only ONE pole in mathematics, while in physics we observe two of them: photon exchange at  $s = 0$ , Z exchange at  $s_0 = s_Z$ .  
Mathematicaly, the appearance of the photon pole is result of summing of part of background around  $Z$  pole,  $s_0 = s_Z$

[T. Riemann, APPB 2015]

$$\begin{aligned}\frac{R_\gamma(s)}{s} &= \frac{\sum_{n=0}^{\infty} R_n (s - s_0)^n}{s} \\ &= \frac{\sum_{n=0}^{\infty} R_n (s - s_0)^n}{s_0 - (s_0 - s)} \\ &= \sum_{n=0}^{\infty} R_n (s - s_0)^n \frac{1}{s_0} \frac{1}{1 - \frac{s_0 - s}{s_0}} \\ &= \sum_{n=0}^{\infty} R_n (s - s_0)^n \frac{1}{s_0} \left[ 1 + \frac{s_0 - s}{s_0} + \left( \frac{s_0 - s}{s_0} \right)^2 \dots \right];\end{aligned}$$

Consistent (gauge-invariant) theory setup:

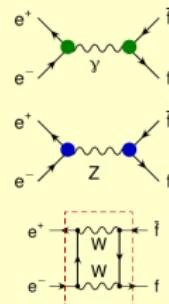
Expansion of  $\mathcal{A}[e^+e^- \rightarrow \mu^+\mu^-]$  about  $s_0 = M_Z^2 - iM_Z\Gamma_Z$ :

$$\mathcal{A}[e^+e^- \rightarrow f\bar{f}] = \frac{R}{s-s_0} + S + (s-s_0)T + \dots$$

$$R = g_Z^e(s_0)g_Z^f(s_0)$$

$$S = \left[ \frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^f + g_Z^e g_Z^f + S_{\text{box}} \right]_{s=s_0}$$

$g_V^f(s)$ : effective  $Vf\bar{f}$  couplings



At NNLO: Need  $R$  at  $\mathcal{O}(\alpha^2)$ ,  $S$  at  $\mathcal{O}(\alpha)$ , etc.

Current state of art: full one-loop for  $S, T$

- $\mathcal{O}(0.01\%)$  uncertainty within SM      see, e.g., Bardin, Grünwald, Passarino '99  
(improvements may be needed)
- Sensitivity to some NP beyond EWPO

## Z lineshape

6/18

Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \rightarrow f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85

Berends, Burgers, v. Neerven '88

Kniehl, Krawczyk, Kühn, Stuart '88

Beenakker, Berends, v. Neerven '89

Bardin et al. '91; Skrzypek '92

Montagna, Nicrosini, Piccinini '97

Soft photons (resummed) + collinear photons

$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^n h_{nm} \ln^m\left(\frac{s}{m_\theta^2}\right)$$

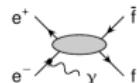
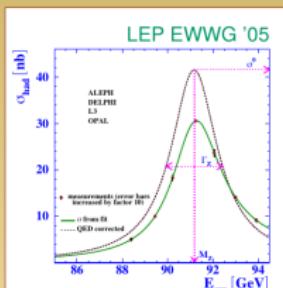
Universal ( $m=n$ ) logs known to  $n=6$ ,

also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

Exclusive description: MC tools

→ talk by Jadach



Factorization of massive and QED/QCD FSR:

$$\Gamma_f \approx \frac{N_c M_Z}{12\pi} \left[ (\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2) \frac{1}{1 + \text{Re } \Sigma'_Z} \right]_{s=M_Z^2}$$



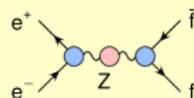
$\mathcal{R}_V^f, \mathcal{R}_A^f$ : Final-state QED/QCD radiation;

known to  $\mathcal{O}(\alpha_s^4), \mathcal{O}(\alpha^2), \mathcal{O}(\alpha\alpha_s)$  Kataev '92

Chetyrkin, Kühn, Kwiatkowski '96

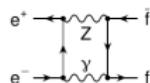
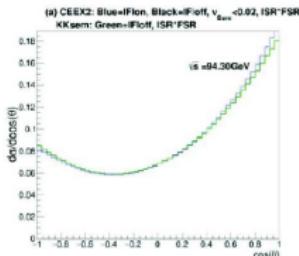
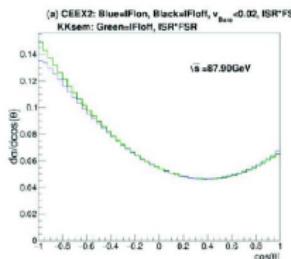
Baikov, Chetyrkin, Kühn, Rittinger '12

$g_V^f, g_A^f, \Sigma'_Z$ : Electroweak corrections



# QED unfolding, IFI, slide by A.Freitas, Snowmass 2020, pdf

- Interference between ISR and FSR suppressed by  $\Gamma_Z/M_Z$  on  $Z$  resonance
- Still relevant for high precision an off-resonance



Jadach, Yost '18

- Factorization from hard matrix element requires 4-variable convolution
- Soft-photon resummation can be included

Jadach, Yost '18  
Greco, Pancheri-Srivastava, Srivastava '75

EWPOs - refers to  $|M|^2$ ; EWPPs - refers to  $M$

---

Beyond Born level, one can write

$$\mathcal{M}_\gamma^{(0)}(e^- e^+ \rightarrow f^- f^+) = \frac{4\pi i \alpha_{em}(s)}{s} Q_e Q_f \gamma_\alpha \otimes \gamma^\alpha,$$

$$\begin{aligned} \mathcal{M}_Z^{(0)}(e^- e^+ \rightarrow f^- f^+) &= 4ie^2 \frac{\chi_Z(s)}{s} [M_{vv}^{ef} \gamma_\alpha \otimes \gamma^\alpha - M_{av}^{ef} \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \\ &\quad - M_{va}^{ef} \gamma_\alpha \times \gamma^\alpha \gamma_5 + M_{aa}^{ef} \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \gamma_5]. \end{aligned}$$

In the **pole scheme**, where  $\bar{M}_Z$  is defined as the real part of the pole of the S matrix, one has

$$\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2} 8\pi \alpha_{em}} K_Z(s) \simeq \frac{1}{1 + i \frac{\bar{\Gamma}_Z}{M_Z}} \frac{s}{s - \bar{M}_Z^2 + i \bar{M}_Z \bar{\Gamma}_Z} \simeq \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z(s)},$$

$$\Gamma_Z(s) = \frac{s}{M_Z^2} \Gamma_Z$$

SM Theory for the **FCC-ee Tera-Z** stage (2019): [link](#)

EWPOs - refers to  $|M|^2$ ; EWPPs - refers to  $M$

---

Definitions are related:

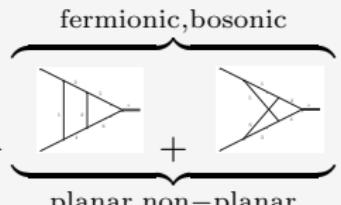
$$\bar{M}_Z \approx M_Z - \frac{1}{2} \frac{\Gamma_Z^2}{M_Z} \approx M_Z - 34 \text{ MeV},$$

$$\bar{\Gamma}_Z \approx \Gamma_Z - \frac{1}{2} \frac{\Gamma_Z^3}{M_Z^2} \approx \Gamma_Z - 0.9 \text{ MeV}.$$

- ▶ Known from LEP. One of examples why changing frameworks/assumptions/simplifications of calculations matter (!).
- ▶ However, at FCC-ee  $\delta\Gamma_Z \sim 0.1$  MeV. Non-factorization effects must be added properly beyond 1-loop.
- ▶ Is it necessary for FCC-ee accuracy to implement MC with radiative corrections calculated at the amplitudes level?
- ▶ At this precision it is important which parameters are taken as input parameters in schemes.

## EWPOs and Form Factors

---

$$V_\mu^{Zb\bar{b}} = \gamma_\mu [v_b(s) + a_b(s)\gamma_5] = \dots + \overbrace{\quad \quad \quad}^{\substack{\text{fermionic,bosonic} \\ \text{planar,non-planar}}} + \dots$$


Note approximate factorization of weak couplings

$$A_{FB} = \frac{\left[ \int_0^1 d\cos\theta - \int_{-1}^0 d\cos\theta \right] \frac{d\sigma}{d\cos\theta}}{-} \sim \overbrace{\frac{A_e}{2a_e v_e}}^{\sim 2} \overbrace{\frac{A_f}{2a_f v_f}}^{\sim 2} + \text{corrections}$$

$$A_f = \frac{2\Re e \frac{v_f}{a_f}}{1 + \left( \Re e \frac{v_f}{a_f} \right)^2} = \frac{1 - 4|Q_f|\sin^2 \theta_{\text{eff}}^f}{1 - 4|Q_f|\sin^2 \theta_{\text{eff}}^f + 8(Q_f \sin^2 \theta_{\text{eff}}^f)^2},$$

$$\sin^2 \theta_{\text{eff}}^f = F \left( \Re e \frac{v_f}{a_f} \right)$$

## EWPOs, Z pole

---

$$\begin{aligned}\sigma_{\text{had}}^0 &= \sigma[e^+e^- \rightarrow \text{hadrons}]_{s=M_Z^2}, \\ \Gamma_Z &= \sum_f \Gamma[Z \rightarrow f\bar{f}], \\ R_\ell &= \frac{\Gamma[Z \rightarrow \text{hadrons}]}{\Gamma[Z \rightarrow \ell^+\ell^-]}, \quad \ell = e, \mu, \tau, \\ R_q &= \frac{\Gamma[Z \rightarrow q\bar{q}]}{\Gamma[Z \rightarrow \text{hadrons}]}, \quad q = u, d, s, c, b.\end{aligned}$$

The remaining EWPOs are cross section asymmetries, measured at the  $Z$  pole, e.g., forward-backward asymmetry

$$A_{\text{FB}}^f = \frac{\sigma_f [\theta < \frac{\pi}{2}] - \sigma_f [\theta > \frac{\pi}{2}]}{\sigma_f [\theta < \frac{\pi}{2}] + \sigma_f [\theta > \frac{\pi}{2}]},$$

where  $\theta$  is the scattering angle between the incoming  $e^-$  and the outgoing  $f$ .

## 1. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

**Uncertainties of hadronic contributions to effective  $\alpha$  are a problem for electroweak precision physics:** besides top Yukawa  $y_t$  and Higgs self-coupling  $\lambda$

$\alpha$ ,  $G_\mu$ ,  $M_Z$  most precise input parameters  $\Rightarrow$  precision predictions  
 ↓  
 50% non-perturbative  $\sin^2 \Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \dots$   
 $\alpha(M_Z), G_\mu, M_Z$  best effective input parameters for VB physics (Z,W) etc.

$\frac{\delta\alpha}{\alpha}$	$\sim$	3.6	$\times$	$10^{-9}$
$\frac{\delta G_\mu}{G_\mu}$	$\sim$	8.6	$\times$	$10^{-6}$
$\frac{\delta M_Z}{M_Z}$	$\sim$	2.4	$\times$	$10^{-5}$
$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)}$	$\sim$	0.9 ÷ 1.6	$\times$	$10^{-4}$ (present : lost $10^5$ in precision!)
$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)}$	$\sim$	5.3	$\times$	$10^{-5}$ (FCC – ee/ILC requirement)

**LEP/SLD:**  $\sin^2 \Theta_{\text{eff}} = (1 - v_l/a_l)/4 = 0.23148 \pm 0.000017$

$$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta \sin^2 \Theta_{\text{eff}} = 0.00007 ; \quad \delta M_W/M_W \sim 4.3 \times 10^{-5}$$

affects most precision tests and new physics searches!!!

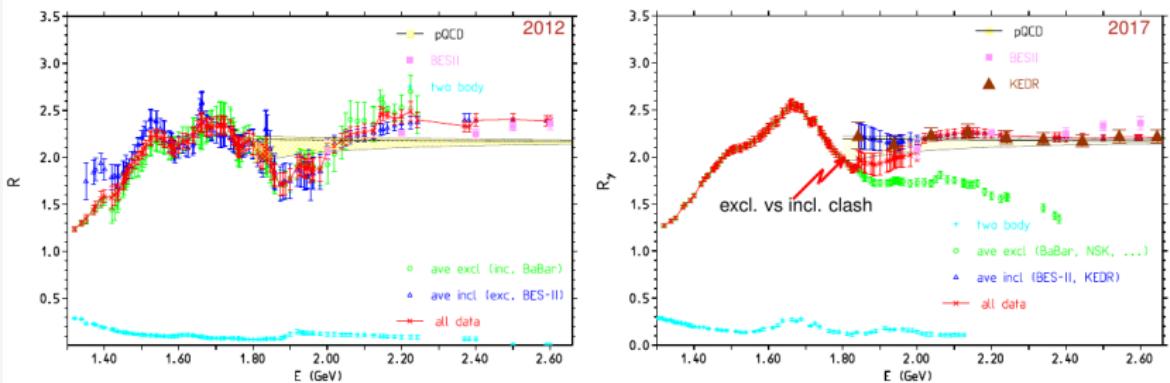
$$\frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}, \quad \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}, \quad \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}$$

For pQCD contributions very crucial: precise QCD parameters  $\alpha_s, m_c, m_b, m_t \Rightarrow$  Lattice-QCD

## SM precision parameters determination: $\alpha(M_Z^2)$

### ☐ Still an issue in HVP

- ☐ region 1.2 to 2 GeV data; test-ground exclusive vs inclusive  $R$  measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty



- illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

Three approaches should be further explored for better error estimate

---

Note: **theory-driven** standard analyses ( $R(s)$  integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in $\alpha$ :	present	direct	$1.7 \times 10^{-4}$
		Adler	$1.2 \times 10^{-4}$
	future	Adler QCD 0.2%	$5.4 \times 10^{-5}$
		Adler QCD 0.1%	$3.9 \times 10^{-5}$
	future	via $A_{FB}^{\mu\mu}$ off Z	$3 \times 10^{-5}$

- Adler function method is competitive with **Patrick Janot's** direct near  $Z$  pole determination via forward backward asymmetry in  $e^+e^- \rightarrow \mu^+\mu^-$

$$A_{FB}^{\mu\mu} = A_{FB,0}^{\mu\mu} + \frac{3}{4} \frac{a^2}{v^2} \frac{\mathcal{I}}{\mathcal{Z} + \mathcal{G}}$$

where

$\gamma$ -Z interference term

$$\mathcal{I} \propto \alpha(s) G_\mu$$

Z alone

$$\mathcal{Z} \propto G_\mu^2$$

$\gamma$  only

$$\mathcal{G} \propto \alpha^2(s)$$

v vector Z coupling

also depends on  $\alpha(s \sim M_Z^2)$  and  $\sin^2 \Theta_f(s \sim M_Z^2)$

a axial Z coupling

sensitive to  $\rho$ -parameter (strong  $M_t$  dependence)

- using  $v, a$  as measured at  $Z$ -peak

$$e^+ e^- \rightarrow \mu^+ \mu^- \text{ and } \alpha^2(s)$$

---

$\sigma_{\mu\mu}$ :

1. the photon-exchange term,  $\mathcal{G}$ , proportional to  $\alpha^2(s)$ ;
2. the Z-exchange term,  $\mathcal{Z}$ , proportional to  $G_F^2$  (where  $G_F$  is the Fermi constant);
3. the Z-photon interference term,  $\mathcal{I}$ , proportional to  $\alpha(s) \times G_F$

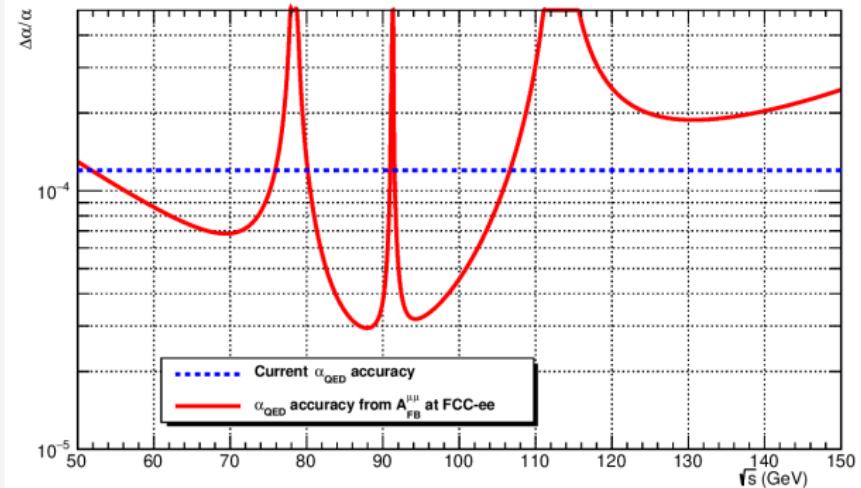
The muon forward-backward asymmetry,  $A_{FB}^{\mu\mu}$ , is maximally dependent on the interference term

$$A_{FB}^{\mu\mu} = A_{FB,0}^{\mu\mu} + \frac{3}{4} \frac{2}{2} \frac{\mathcal{I}}{\mathcal{G} + \mathcal{Z}},$$

varies with  $\alpha_{QED}(s)$  as follows:

$$\Delta A_{FB}^{\mu\mu} = \left( A_{FB}^{\mu\mu} - A_{FB,0}^{\mu\mu} \right) \times \frac{\mathcal{Z} - \mathcal{G}}{\mathcal{Z} + \mathcal{G}} \times \frac{\Delta \alpha}{\alpha}.$$

$$e^+ e^- \rightarrow \mu^+ \mu^- \text{ and } \alpha^2(s)$$



The best accuracy is obtained for one year of running either just below or just above the Z pole, at 87.9 and 94.3 GeV, respectively.

## Future: W, t, H

---

- ▶  $e^+e^- \rightarrow W^+W^-$  at 161 GeV:  $\delta m_W^{exp} = 0.5 \div 1$  MeV.

Challenge to get the same TH error:

$$\text{NNLO } e^+e^- \rightarrow 4f.$$

- ▶  $e^+e^- \rightarrow t\bar{t}$  at 350 GeV:  $\delta m_t^{exp} = 17$  MeV

Big challenge for theory, today  $> 100$  MeV, future projection  $\leq 50$  MeV:

~ 10 MeV unc. from mass def.;

~ 15 MeV from  $\alpha_s$  unc. to threshold mass def.;

~ 30 MeV - h. orders resummation

- ▶  $e^+e^- \rightarrow HZ$  at 240 GeV: Kinematic constraint fits with  $Z \rightarrow ll$  and  $H \rightarrow bb, \dots,$

$m_H = 125.35$  GeV  $\pm 150$  MeV [[link CMS](#)],  $\Gamma_H = 4.1_{4.0}^{5.1}$  MeV,  $\Gamma_H < 13$

MeV at 95 % C.L., [1901.00174](#)

$\delta m_H^{exp} = 10$  MeV; Theory errors subdominant.

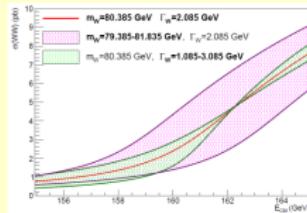
- High-precision measurement of  $M_W$  from  $e^+e^- \rightarrow W^+W^-$  at threshold

- a) Corrections near threshold enhanced by

$1/\beta$  and  $\ln \beta$

$$\beta \sim \sqrt{1 - 4 \frac{M_W^2 - i M_W \Gamma_W}{s}} \sim \sqrt{\Gamma_W / M_W}$$

- b) Non-resonant contributions are important



- Full  $\mathcal{O}(\alpha)$  calculation of  $e^+e^- \rightarrow 4f$

Denner, Dittmaier, Roth, Wieders '05

- EFT expansion in  $\alpha \sim \Gamma_W/M_W \sim \beta^2$

Beneke, Falgari, Schwinn, Signer, Zanderighi '07

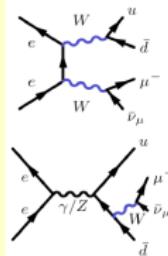
- NLO corrections with NNLO Coulomb correction

( $\propto 1/\beta^n$ ):  $\delta_{\text{th}} M_W \sim 3 \text{ MeV}$

Actis, Beneke, Falgari, Schwinn '08

- Adding NNLO corrections to  $ee \rightarrow WW$  and

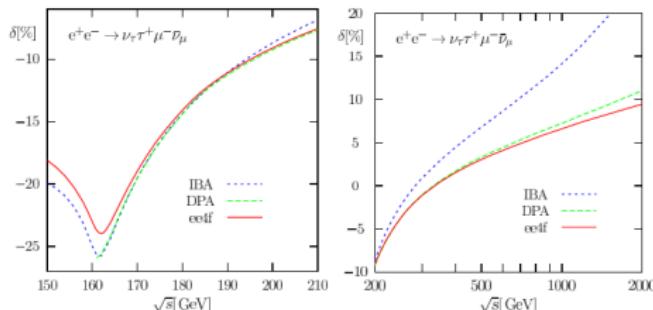
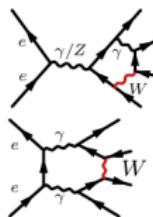
$W \rightarrow f\bar{f}$  and NNLO ISR:  $\delta_{\text{th}} M_W \lesssim 0.6 \text{ MeV}$



## Full NLO calculation for $e^+e^- \rightarrow 4f$

(Denner, Dittmaier, Roth, Wieders 05)

- More than 1000 1-loop diagrams, 5, 6-point loop integrals
- ⇒ pioneering methods for six-point diagrams  
now automated for LHC: RECOLA, OpenLoops, MadLoops
- complex mass scheme for  $W$  decay width
- fully differential calculation
- not easy to incorporate higher-order effects
- DPA not sufficient at threshold and for  $\sqrt{s} > 500$  GeV



# SM W-physics, FCC-ee-W

---



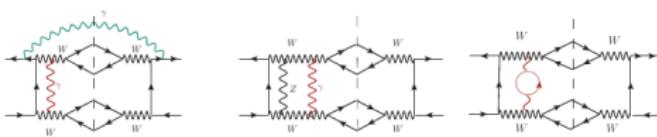
---

**EFT expansion in**  $\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta^2$       (Beneke/Falgari/CS/Signer/Zanderighi 07)

- systematically possible to include higher-order corrections
- limited to total cross section near threshold

## Leading NNLO corrections

- 2nd Coulomb correction  $\sim \alpha^2/\beta^2 \sim \alpha$       (Fadin et al. 95)
- Coulomb-enhanced corrections  $\sim \alpha^2/\beta \sim \alpha^{3/2}$       (Actis et al. 08)



- Numerical effect:  $\Delta\sigma_{WW} \sim 5\%$ ;  $[\delta M_W] \lesssim 3 \text{ MeV}$

$\sqrt{s}$ [GeV]	$\sigma(e^- e^+ \rightarrow e^- \bar{\nu}_\mu u d)(\text{fb})$			
	NLO <sub>EFT</sub>	NLO <sub>ee4f</sub> [DDRW]	$\Delta_{\text{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\text{NNLO}}(\alpha^2/\beta)$
161	117.5	118.77	0.44 (3.7%)	0.15 (1.3%)
170	397.8	404.5	0.25 (0.6%)	1.6 (3.9%)

### Implementation of state-of-the art calculations in public tools?

- **NLO-EW**  $e^-e^+ \rightarrow 4f$  now possible with standard tools  
(RECOLA, OpenLoops, MadLoops + SHERPA, MadGraph, WHIZARD...)  
but not (yet) optimized for  $e^-e^+$  (ISR, Beamstrahlung)
- **Two-loop Coulomb-enhanced corrections for differential observables doable;** (related:  $t\bar{t}$  with Coulomb resummation in WHIZARD)  
(no guarantee of formal accuracy for general distributions)

### Full NNLO in EFT for total cross section

- Soft  $\log \beta$  terms can be adapted from QCD results
- NNLO  $\log(m_e/M_W)$  terms doable (c.f. Bhabha scattering)
- two-loop hard non-logarithmic corrections  
(from amplitudes for  $e^+e^- \rightarrow W^+W^-$  at threshold: border of current capabilities)

resulting uncertainty from cross-section calculation

$$\Delta\sigma_{\text{hard}}^{(2)} = \left(\frac{\alpha}{2\pi}\right)^2 c^{(2)}\sigma^{(0)} \sim (1-2)\% \text{ for estimate } c^{(2)} = (c^{(1)})^2$$

### Full NNLO for $e^+e^- \rightarrow 4f$ : completely new methods needed

## Conclusions and outlook



- ▶ KoralW+YFSWW3: LEP2 precision is 0.5%.  
Factor of 20 ÷ 50 improvement is needed for FCCee
- ▶ Lesson from LEP2: be pragmatic, split into Double- and Single-Pole, pick only numerically dominant terms:
  - ▶  $\mathcal{O}(\alpha^1)$  for  $e^-e^+ \rightarrow 4f$  must be implemented in MC with explicit split into Double Pole and Single Pole. Calculations exist
  - ▶  $\mathcal{O}(\alpha^2)_{DP}$  calculations for the Double-Pole production and decay parts are needed! Feasible?
  - ▶  $\mathcal{O}(\alpha^2)_{SP}$  and  $\mathcal{O}(\alpha^3)$  seem to be negligible
- ▶ More detailed analysis at the threshold may be instrumental
  - ▶ EFT methods promising, but for now inclusive results only
  - ▶ Non-factorizable soft interferences can be exponentiated within YFS scheme. How much of the higher order corrs. would be reproduced this way?

The overall precision tag  $\sim 2 \times 10^{-4}$  feasible (?)

YFSWW3  $\oplus$  KoralW with new exponentiation

look like a good starting point

- $M_Z$ ,  $\Gamma_Z$ : From  $\sigma(\sqrt{s})$  lineshape
  - Main uncertainties:  $B$ -field calibration, QED
  - $\delta M_Z$ ,  $\delta \Gamma_Z \sim 0.1$  MeV could be achievable
- $m_t$ : Current status  $\delta m_t \sim 0.4$  GeV at LHC
  - Additional theory uncertainties?

PDG '18

Butenschoen et al. '16

Ferrario Ravasio, Nason, Oleari '18

From  $e^+e^- \rightarrow t\bar{t}$  at  $\sqrt{s} \sim 350$  GeV

today:

$$\begin{aligned}\delta m_t^{\overline{\text{MS}}} = & [ ]_{\text{exp}} \\ & \oplus [50 \text{ MeV}]_{\text{QCD}} \\ & \oplus [10 \text{ MeV}]_{\text{mass def.}} \\ & \oplus [70 \text{ MeV}]_{\alpha_s} \\ & > 100 \text{ MeV}\end{aligned}$$

future:

$$\begin{aligned}& [20 \text{ MeV}]_{\text{exp}} \\ & \oplus [30 \text{ MeV}]_{\text{QCD}} \quad (\text{h.o. resummation}) \\ & \oplus [10 \text{ MeV}]_{\text{mass def.}} \\ & \oplus [15 \text{ MeV}]_{\alpha_s} \quad (\delta \alpha_s \lesssim 0.0002) \\ & \lesssim 50 \text{ MeV}\end{aligned}$$

## Conclusions

- Top pair threshold scan allows precise mass determination
$$\Delta m_t < 100 \text{ MeV}$$
- Theory-dominated error,  $\sim 3\%$  QCD scale uncertainty
- Known corrections:
  - N<sup>3</sup>LO QCD + Higgs
  - N<sup>2</sup>LO electroweak + non-resonant
  - LL initial state radiation
- All corrections included in version 2 of QQbar\_threshold
  - [https://qqbARTHreshold.hepforge.org/](https://qqbarthreshold.hepforge.org/)

2019, pdf

## MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of  $m_t^{\text{MC}}$ ?

- picture of “top quark particle” does not apply (non-zero color charge)
- $m_t$  is a scheme-dependent parameter of a perturbative computation  
→ in which scheme do MC event generators calculate?
- relation of  $m_t^{\text{MC}}$  to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\text{MC}}$$

**pQCD contribution:**

- perturbative corrections
- depends on MC parton shower setup

**non-perturbative contribution:**

- effects of hadronization model
- may depend on parton shower setup

**Monte Carlo shift:**

- contribution arising from systematic MC uncertainties
- e.g. color reconnection, b-jet modelling, finite width,...