Theoretical calculation strategy

towards strong FCC-ee experimental demands

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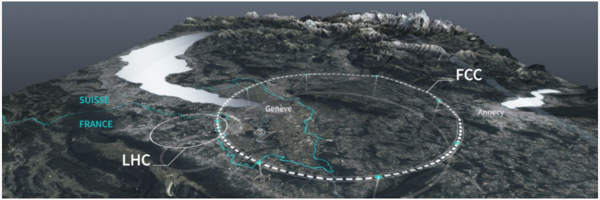
FCC Week 2023

5 June 2023, London

'The reasonable man adapts himself to the world.
The unreasonable one persists in trying to adapt the world to himself.
Therefore all progress depends on the unreasonable man.'

– George Bernard Shaw, Man and Superman



Parallel sessions on theoretical calculations, Tuesday

08:30 → 10:00 **PE&D: Physics Case + Theoretical calculations (I)**

- 08:30 **Precision electroweak**
  Speaker: Christoph Paus (Massachusetts Inst. of Technology (US))
- 09:00 **QCD & parton showers**
  Speaker: Pier Francesco Monni (CERN)
- 09:30 **BSM & physics case**
  Speaker: Sophie Alice Renner (University of Glasgow (GB))

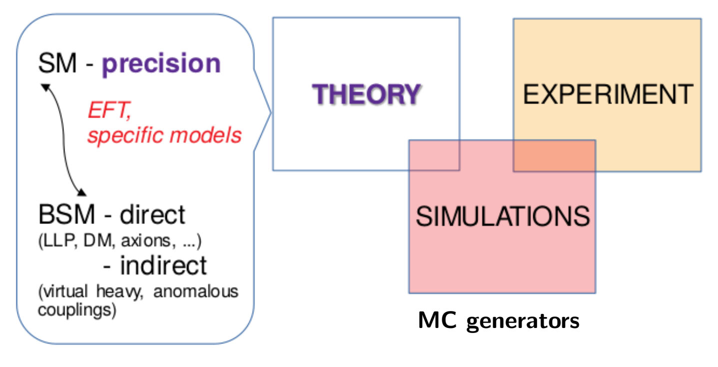
10:30 → 12:00 **PE&D: Physics Case + Theoretical calculations (II)**

- 10:30 **Higgs physics**
  Speaker: Jorge de Blas (Universidad de Granada (ES))
- 11:00 **Flavour physics**
  Speaker: Jernej F. Kamenik
- 11:30 **Latest developments in FCC Analysis**
  Speaker: Juraj Smiesko (CERN)

In memoriam and honour of Staszek Jadach (06.08.1947 – 23.02.2023)

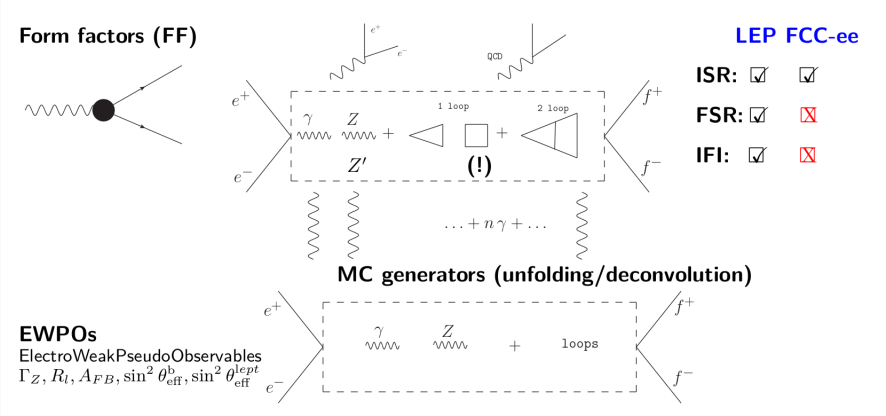
"FCC is a HEP project of the XXI century" (2014)

MC generators: Merging theory with experiment

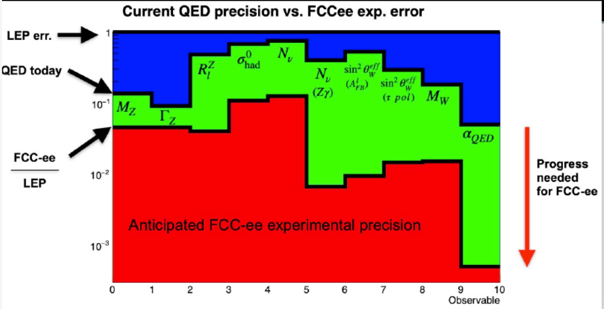


MC generators and theory (Z-pole)

Subtraction of γ-exchange, $\gamma - Z$ exchange, box (non-factorizable), non-resonance terms.
How to account for in MC generators (efficiency, particles ID,,,)?



QED challenges beyond LEP: the FCC-ee example



The present precision of QED theoretical predictions would severely limit the analysis of precise measurements at FCC-ee.

To properly confront the data with theoretical predictions of similar accuracy demands a huge progress in precision calculations!

Needed factor 6-200 improvement with respect to LEP.

arXiv:1903.09895

(Jadach&Skrzypek)

EW pseudo-observables EWPOs

Observable EWPO	Source LEP	Err.{QED} LEP	Stat[Syst] FCC-ee	LEP / FCC-ee	main development to be done
M_Z [MeV]	Z linesh.	2.1{0.3}	0.005[0.1]	$3\times3^{\star}$	light fermion pairs
Γ_Z [MeV]	Z linesh.	2.1{0.2}	0.008[0.1]	$2\times3^{\star}$	fermion pairs
σ^0_{had} [pb]	σ^0_{had}	37{25}	0.1[4.0]	$6\times3^{\star}$	better lumi MC
$R_l^Z\times10^3$	$\sigma(M_Z)$	25{12}	0.06[1.0]	$12\times3^{\star\star}$	better FSR
$N_\nu\times10^3$	$\sigma(M_Z)$	8{6}	0.005[1.0]	$6\times3^{\star\star}$	CEEX in lumi MC
$N_\nu\times10^3$	$Z\gamma$	150{60}	0.8[< 1]	$60\times3^{\star\star}$	$\mathcal{O}(\alpha^2)$ for $Z\gamma$
$\sin^2\theta_W^{eff}\times10^5$	$A_{FB}^{lept.}$	53{28}	0.3[0.5]	$55\times3^{\star\star}$	h.o. and EWPOs
$\sin^2\theta_W^{eff}\times10^5$	$\langle\mathcal{P}_\tau\rangle, A_{FB}^{pol,\tau}$	41{12}	0.6[< 0.6]	$20\times3^{\star\star}$	better τ decay MC
M_W [MeV]	mass rec.	33{6}	0.3[?.?]	$20\times3^{\star\star}$	$\mathcal{O}(\alpha)$, FSR_{exp}
M_W [MeV]	threshold	200{30}	0.5[0.3]	$100\times3^{\star\star\star}$	$\mathcal{O}(\alpha^2)$ at thresh.
$A_{FB,\mu}^{M_Z\pm3.5\text{GeV}}\times10^5$	$\frac{d\sigma}{d\cos\theta}$	2000{100}	1.0[0.3]	$100\times3^{\star\star\star}$	improved IFI

MC for FCC-ee (more in the FCC midterm report)

- Multi-purpose tools: Pythia, Herwig, Sherpa, MadGraph5 aMC@NLO, Whizard,...
- Dedicated: BabaYaga, RacoonWW, Racoon4f, KKMC-ee, KORAL[W/Z], BH[LUMI/WIDE], YSF[WW3/ZZ], ...

"Modern multi-purpose tools, such as PSMCs, generally poorly tested in e^+e^- high-energy environments" (S. Frixione talk, CERN July 2022 workshop, link).

Recent improvements (two examples):

- KKMCee - Jadach:2022mbe
  - Improvements for the simulation of fermion pair production.
  - Ongoing work to include additional collinear contributions in the YFS framework, beam spread parametrizations, and better descriptions of tau decays;
- Sherpa - Krauss:2022ajk
  - YFS resummation implemented in an automatic framework for ISR and FSR state QED.
  - ISR resummation can be applied to any e^+e^- process, FSR currently restricted to leptonic states.
  - Automated matching of the resummation (with collinear logs) to h.o. corrections. It is relatively simple to include new h.o. terms within the YFS resummation framework in Sherpa.

A few sample precision quantities of interest for the FCC-ee program

Quantity	Current precision	FCC-ee target precision	Required theory input	Available calc.	Needed theory improvement*
m_Z	2.1 MeV	0.1 MeV **0.1 MeV**	non-resonant $e^+e^- \to f\bar{f}$, initial-state radiation (ISR)	NLO, ISR logs up to 6th order	NNLO for $e^+e^- \to f\bar{f}$
Γ_Z	2.3 MeV	0.1 MeV **0.4** MeV			
$\sin^2 \theta^\ell_{\rm eff}$	1.6×10^{-4}	0.6×10^{-5} $\mathbf{4.5} \times 10^{-5}$			
m_W	12 MeV	0.4 MeV **4** MeV	lineshape of $e^+e^- \to WW$ near threshold	NLO ($ee \to 4f$ or EFT framework)	NNLO for $ee \to WW$, $W \to ff$ in EFT setup
HZZ coupling	—	0.2% **3 %**	cross-sect. for $e^+e^- \to HZ$	NLO + NNLO QCD	NNLO electroweak
m_t	>100 MeV	17 MeV **50** MeV	threshold scan $e^+e^- \to t\bar{t}$	N^3LO QCD, NNLO EW, resummations up to NNLL	Matching fixed orders with resummations, merging with MC, α_s (input)

Theory: 1906.05379, 2106.11802

Quantity	Required theory input	Available calc.	Needed theory improvement*
Γ_Z $\sin^2\theta^{\ell}_{\rm eff}$	vertex corrections for $Z \to f\bar{f}$	NNLO + partial higher orders	N^{3}LO EW + partial higher orders
m_W	SM corrections to the muon decay rate	NNLO + partial higher orders	N^{3}LO EW + partial higher orders

1956, 1-loop, Behrends, Finkelstein Sirlin

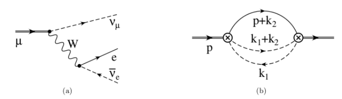
43 y ↓

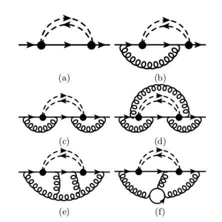
1999, 2-loops, van Ritbergen, Stuart

22 y ↓

2021, 3-loops

? ↓





Important (3-loop) step since 1999 (van Ritbergen & Stuart).

$$\Delta\tau_\mu(\alpha^3) \quad = \quad (9 \pm 1) \times 10^8 \;\; \mu s,$$

$$\tau_\mu^{exp} \quad = \quad 2.1969811 \pm 0.0000022 \; \mu s.$$

M. Fael, K. Schönwald, and M. Steinhauser, Third order corrections to the semileptonic $b \to c$ and the muon decays, PRD'2021, arXiv:2011.13654
M. Czakon, A. Czarnecki, and M. Dowling, Three-loop corrections to the muon and heavy quark decay rates, PRD'2021, arXiv:2104.05804

GRIFFIN, arxiv:2211.16272

Standard Model Theory for the *FCC-ee Tera-Z* stage *(2019)*: link

CERN-2019-003 (C4. by T.Riemman et al.)

In any case, we need to build a suitable theory framework. ZFITTER/DIZET will not be a useful basis for the FCC-ee, since it is structured to achieve consistent (1+1/2)-loop precision, but not beyond. No Laurent-series approach is foreseen in the kernel ZFITTER; but see Subsection C.4.5 on the SMATASY project and its applications to data. Further, later versions of the code lost modularity, owing to too-lazy additions concerning this item. We will have to begin developing a new program framework – probably object-oriented, e.g.,C++ – that is general enough to be extended to any loop order and to different assumptions about QED and inputs. All the future calculations, covering up to weak three loops and QCD four loops should be performed to fit into this new framework.

- In LEP/SLD era

  ZFITTER/DIZET(D. Bardin et al) ,TOPAZ0(G.Passarino et al), and BHM/WOH(W.Hollik et al, not public)..

- In future electron-positron colliders' era

  Formally gauge invariant setup .
  Extendability that accommodates higher precision
  and new physics.

→ Motivates this project! (GRIFFIN: **G**auge-**R**esonance-**I**n-**F**our-**F**ermion-**IN**teraction)

Lisong Chen, FCC workshop 2023 in Cracow, link

New EWPOs GRIFFIN C++ library for EW radiative corrections in fermion scattering and decay processes

Numerical Results:

$$|\rho_Z^f| = \frac{2\sqrt{2}F_A^f}{G_\mu M_Z^2}$$

	$\lvert\rho_Z^f\rvert$		$\sin^2\theta_{\text{eff}}^f$		$\Gamma_{Z\to f\bar{f}}$	
	DIZET 6.45	GRIFFIN	DIZET 6.45	GRIFFIN	DIZET 6.45	GRIFFIN
$\nu\bar{\nu}$	1.00800	1.00814	0.231119	NAN	0.167206	0.167197
$\ell\bar{\ell}$	1.00510	1.00519	0.231500	0.231534	0.083986	0.083975
$u\bar{u}$	1.00578	1.00573	0.231393	0.231420	0.299938	0.299958
$d\bar{d}$	1.00675	1.00651	0.231266	0.231309	0.382877	0.382846
$b\bar{b}$	0.99692	0.99420	0.232737	0.23292	0.376853	0.377432

	DIZET 6.45	GRIFFIN all orders	GRIFFIN $\mathcal{O}(\alpha, \alpha^2, \alpha_t\alpha_s, \alpha_t\alpha_s^2)$
Δr	3.63947×10^{-2}	3.68836×10^{-2}	3.63987×10^{-2}

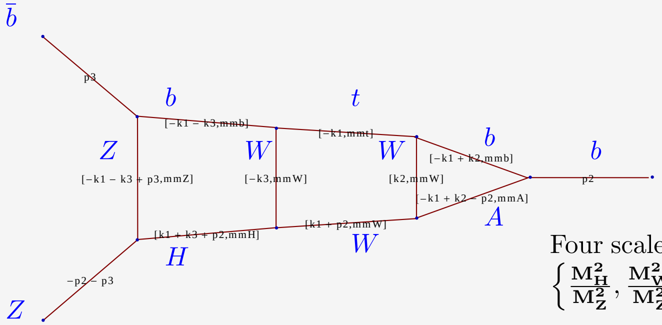
- Not a **one-one-one match.** (no leading N3LO implemented in dizet v.6.45)
- most numbers are in agreement up to at least **4-digit**. The actual discrepancy is in the realm of missing N3(4)LO.
- fictitious discrepancies stem from the input scheme/definition of the form factors/EWPOs.

Lisong Chen, FCC workshop 2023 in Cracow, link

Theoretical Calculation Strategy

	analytic	numerical
pole cancellation	exact	with numerical uncertainty
control of integrable singularities	analytic continuation	less straightforward
fast evaluation	yes	depends
extension to more scales/loops	difficult	*promising*
automation	difficult	*less difficult*

Adopted from G. Heinrich, 2009.00516



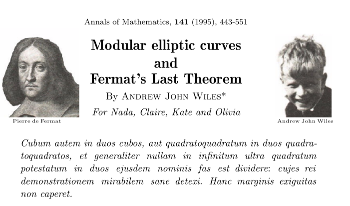
Four scales :

$$\left\{ \frac{\mathbf{M_H^2}}{\mathbf{M_Z^2}}, \frac{\mathbf{M_W^2}}{\mathbf{M_Z^2}}, \frac{\mathbf{m_t^2}}{\mathbf{M_Z^2}}, \frac{\mathbf{s+i}\varepsilon}{\mathbf{M_Z^2}} \right\}$$

Collider physics ... magic of the math world!

$$T = 2\pi\sqrt{\frac{l}{g}} \times {}_2F_1\left[\frac{1}{2}, \frac{1}{2}; 1; \sin^2\theta\right]$$

Analytic solutions for massive multiloop integrals, which describe scattering processes/decays, go beyound elliptic functions - how far?



Annals of Mathematics, **141** (1995), 443-551

Modular elliptic curves and Fermat's Last Theorem

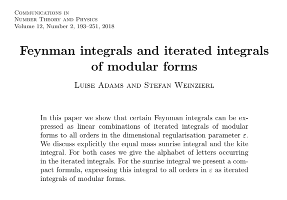
By Andrew John Wiles*

For Nada, Claire, Kate and Olivia

Pierre de Fermat

Andrew John Wiles

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatum in duos ejusdem nominis fas est dividere: cujes rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

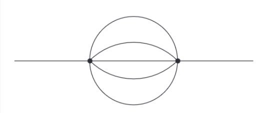


Communications in Number Theory and Physics
Volume 12, Number 2, 193–251, 2018

Feynman integrals and iterated integrals of modular forms

Luise Adams and Stefan Weinzierl

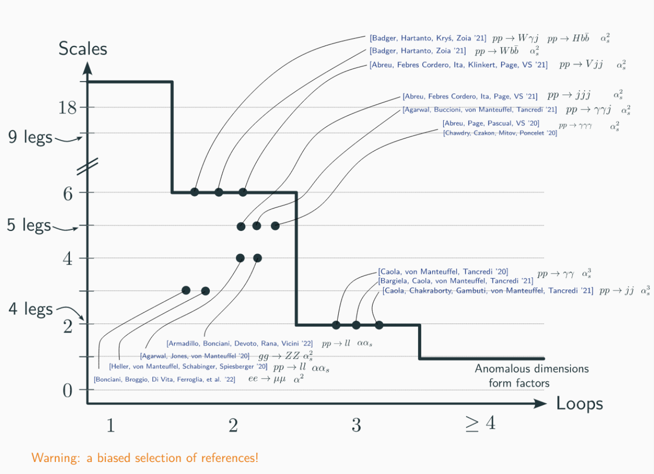
In this paper we show that certain Feynman integrals can be expressed as linear combinations of iterated integrals of modular forms to all orders in the dimensional regularisation parameter ε. We discuss explicitly the equal mass sunrise integral and the kite integral. For both cases we give the alphabet of letters occurring in the iterated integrals. For the sunrise integral we present a compact formula, expressing this integral to all orders in ε as iterated integrals of modular forms.



"Epsilon-factorized form" of banana 4-loop integrals $\rightarrow$ *fast evaluations to nearly arbitrary precision.*

'Taming Calabi-Yau Feynman Integrals: The Four-Loop Equal-Mass Banana Integral', S. Pögel, X. Wang, S. Weinzierl, Phys.Rev.Lett. 130 (2023)

Workshop: Precision calculations for future e^+e^- colliders: targets and tools, CERN 2022 TH workshop, link



From a talk by V. Sotnikov.

Direct numerical approach[1] (+ backup slides)

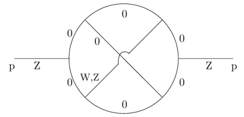
- Sector decomposition (SD) method:
  - FIESTA [2016], [A.V.Smirnov]
  - pySecDec, [Expansion by regions with pySecDec], *2022*
- The Mellin-Barnes (MB) method:
  - MB [M.Czakon, 2006]
  - MBnumerics [J.Usovitsch, I.Dubovyk, T.Riemann, 2015] – Minkowskian kinematics
- Differential equations (DEs) method:
  - `DiffExp` [F. Moriello, 2019; M. Hidding], *2021*
  - `AMFlow` [X. Liu, Y.-Q. Ma] *2022*
  - `SeaSyde` [T. Armadillo, R. Bonciani, S. Devoto, N. Rana, A. Vi] *2022*
- Integration-By-Parts (IBPs) - crucial for DEs
  - Kira [F. Lange, P. Maierhöffer, J. Usovitsch] *2021*
  - Reduze [C. Studerus, 2010]
  - FIRE [A. Smirnov, 2008]
  - LiteRed [R. Lee, 2014]

[1] All programs are public

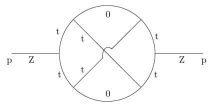
MIs with high accuracy, results*

* *Needed e.g. for 3-loop EWPOs,*

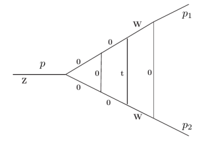
I. Dubovyk, A. Freitas, JG, K. Grzanka, M. Hidding, J. Usovitsch, 'Evaluation of multi-loop multi-scale Feynman integrals for precision physics', 2201.02576 (PRD'2022)



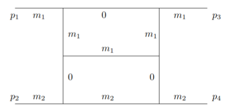
lhNp1



taNPl1



vtwPl



box2l

$$
\begin{aligned}
I_{\text{box2l}}[2,1,1,1,1,1,1,0,0,s,t,m_1^2,m_2^2] \quad &= \quad +0.000328707579/\epsilon^2 \\
&- \quad (0.0014129475 - 0.0020653306\, i)/\epsilon \\
&- \quad (0.005702737 - 0.000485980\, i) + \mathcal{O}(\epsilon), \\
& \quad\;\; 55 \text{ MIs}, s = 2, t = 5, m_1^2 = 4, m_2^2 = 16.
\end{aligned}
$$

MIs with high accuracy: AMFlow, CERN 2022,

https://indico.cern.ch/event/1140580/

Summary and Outlook

➢ What we have

- Auxiliary mass flow method fully automized the computation of boundary conditions for differential equations.
- → AMFlow is the first public tool which can compute arbitrary Feynman loop integrals, at arbitrary kinematic point, to arbitrary precision.

➢ What we need

- Powerful reduction techniques are urgently needed to construct differential equations, both for η and for dynamical variables.
- A guide for choosing better master integrals in general cases is needed, which may strongly simplify the differential equations.

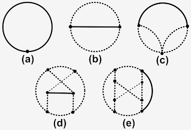
AMFlow method, $\eta = \infty \longrightarrow \eta = 0^+$ analytic continuation (auxiliary mass flow)

2. A set of Jan 27 2022 papers by Zhi-Feng Liu, Yan-Qin Ma and Xiao Liu:
https://inspirehep.net/literature/2020677, https://inspirehep.net/literature/2020676,
https://inspirehep.net/literature/2020880 and 1711.09572
https://inspirehep.net/literature/1639025.

$$\begin{aligned}
\widetilde{I}_{\vec{\nu}}(\eta) &= \int \left(\prod_{i=1}^{L} \frac{\mathrm{d}^D \ell_i}{i\pi^{D/2}} \right) \frac{\widetilde{\mathcal{D}}_{K+1}^{-\nu_{K+1}} \cdots \widetilde{\mathcal{D}}_N^{-\nu_N}}{\widetilde{\mathcal{D}}_1^{\nu_1} \cdots \widetilde{\mathcal{D}}_K^{\nu_K}}. \\
\widetilde{\mathcal{D}}_1 &= \ell_1^2 - m^2 + \mathrm{i}\eta \\
I_{\vec{\nu}} &= \lim_{\eta \to 0^+} \widetilde{I}_{\vec{\nu}}(\eta) \\
\mathrm{i}\frac{\partial}{\partial \eta} \vec{\widetilde{J}}(\eta) &= A(\eta) \widetilde{J}(\eta)
\end{aligned}$$

Key point: boundary conditions at $\eta \to \infty$ are single mass scale bubble integrals, solved iteratively.

MIs with high accuracy by AMFlow, results



$$I_{\vec{\nu}} = \int \left(\prod_{i=1}^{L} \frac{{}^D\ell_i}{\pi^{D/2}} \right) \frac{\mathcal{D}_{K+1}^{-\nu_{K+1}} \cdots \mathcal{D}_N^{-\nu_N}}{\mathcal{D}_1^{\nu_1} \cdots \mathcal{D}_K^{\nu_K}}, \quad \mathcal{D}_1 = \ell_1^2 - m^2 + 0^+$$

$$\widehat{I}_{\vec{\nu}'}(\ell_1^2) = \int \left(\prod_{i=2}^{L} \frac{{}^D\ell_i}{\pi^{D/2}} \right) \frac{\mathcal{D}_{K+1}^{-\nu_{K+1}} \cdots \mathcal{D}_N^{-\nu_N}}{\mathcal{D}_2^{\nu_2} \cdots \mathcal{D}_K^{\nu_K}}, \quad I_{\vec{\nu}} = \{\Gamma[\ldots]\}\widehat{I}_{\vec{\nu}'}(-m^2)$$

L-loop (↑ $I_{\vec{\nu}}$) **(L-1)-loop** (↑ $\widehat{I}_{\vec{\nu}'}$)

$$\begin{aligned} I[(e)] = &- 2.073855510286740\epsilon^{-2} - 7.812755312590133\epsilon^{-1} \\ &- 17.25882864945875 + 717.6808845492140\epsilon \\ &+ 8190.876448160049\epsilon^2 + 78840.29598046500\epsilon^3 \\ &+ 566649.1116484678\epsilon^4 + 3901713.802716081\epsilon^5 \\ &+ 23702384.71086095\epsilon^6 + 14214293.68205112\epsilon^7, \end{aligned}$$

10 orders in ϵ, 16-digit precision.

Such an exact boundary point can be transported by `DiffExp` *to any physical point*.

FCC-ee goals for theory, summary

From a bird's view, we need:

- *Improved unfolding techniques to go from observables to pseudo-observables* (QED interference effects, non-factorizable corrections, matching between EW corrections and radiated/resummed photons, ...).
- *Developing tools:* MC generators including NLO QCD and EW corrections (Bhabha, exclusive NNLO $e^+e^- \to f\bar{f}$), multiloop numerical, analytical programs.
- *Improved input parameters by roughly one order of magnitude* ($\alpha, \alpha_\mathrm{s}$, $m_W, m_H, m_t, \Delta\alpha_\mathrm{had}$, ...).
- *Full two-loop corrections* for $e^+e^- \to W^+W^-$, $e^+e^- \to ZH$ and $e^+e^- \to VVH$, $H \to ff(+\gamma)$.
- *Going beyond 2-loop corrections* (3-loop EW and mixed EW-QCD, leading 4-loop corrections for $Z \to 2f$ vertices).

The FCC-ee is a multi-decade project offering theoretical challenges on a comparable timescale!

The progress is great![2]

Thank you for your attention.

FCC hh ee he

NATIONAL SCIENCE CENTRE POLAND

[2] *'At each meeting it always seems to me that very little progress is made. Nevertheless, if you look ever any reasonable length of time, a few years say, you find a fantastic progress and it is hard to understand how that can happen at the same time that nothing is happening in anyone moment (Zeno's paradox).'* - R.P. Feynman

Backup slides

link