Understanding medium modification of jet substructure through energy correlators

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IGFAE, Universidade de Santiago de Compostela

TH Heavy Ion Coffee
CERN
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C. Andres, FD, R. K. Elayavalli, J. Holguin, C. Marquet, I. Moult, arXiv:2209.11236

Instituto Galego de Física de Altas Enerxías

UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

XUNTA DE GALICIA

Jet modification in HIC

- Energy loss through medium-induced radiation
  - Widely studied in terms of suppression of jets and high-p_T hadrons
- Color coherence effects
  - Breaking of angular ordering
  - Expected to modify jet inner structure
  - Understood in terms of simplified calculations, but not yet unequivocally seen in observables
- Medium response
  - Recoils from the medium which become part of the jet
  - Needed to properly describe some jet observables
  - You first need to understand how the parton shower is modified before understanding how the medium responds to it

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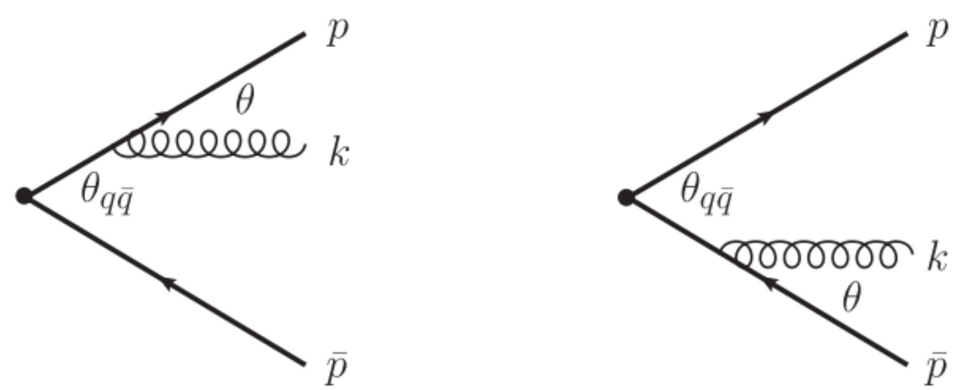
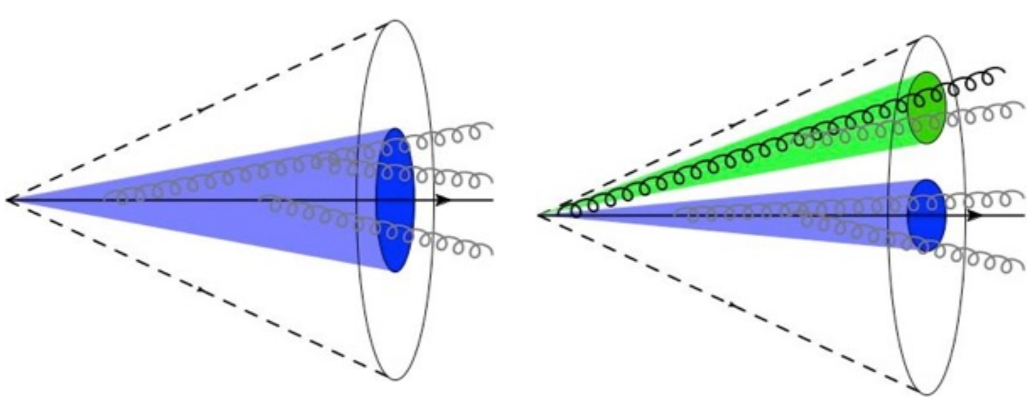
Color coherence in jet quenching

- Antenna calculations show that medium interactions can break angular ordering

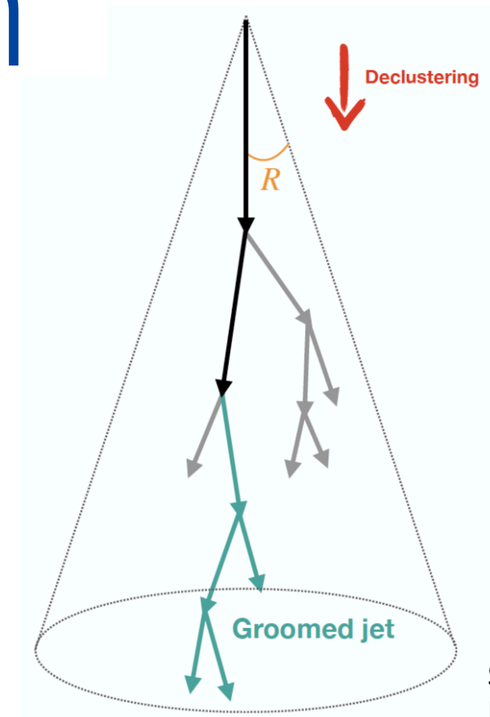


- Emergence of a resolution scale



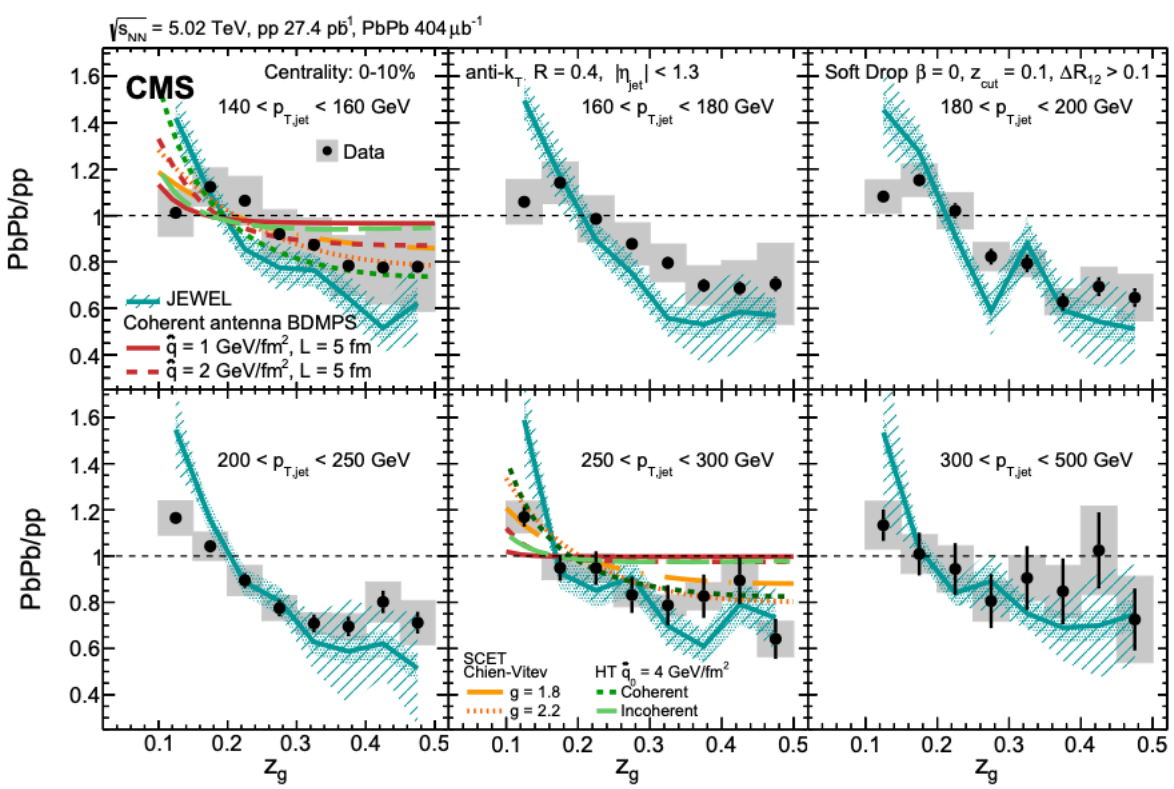
- Can this be seen at the level of one splitting?

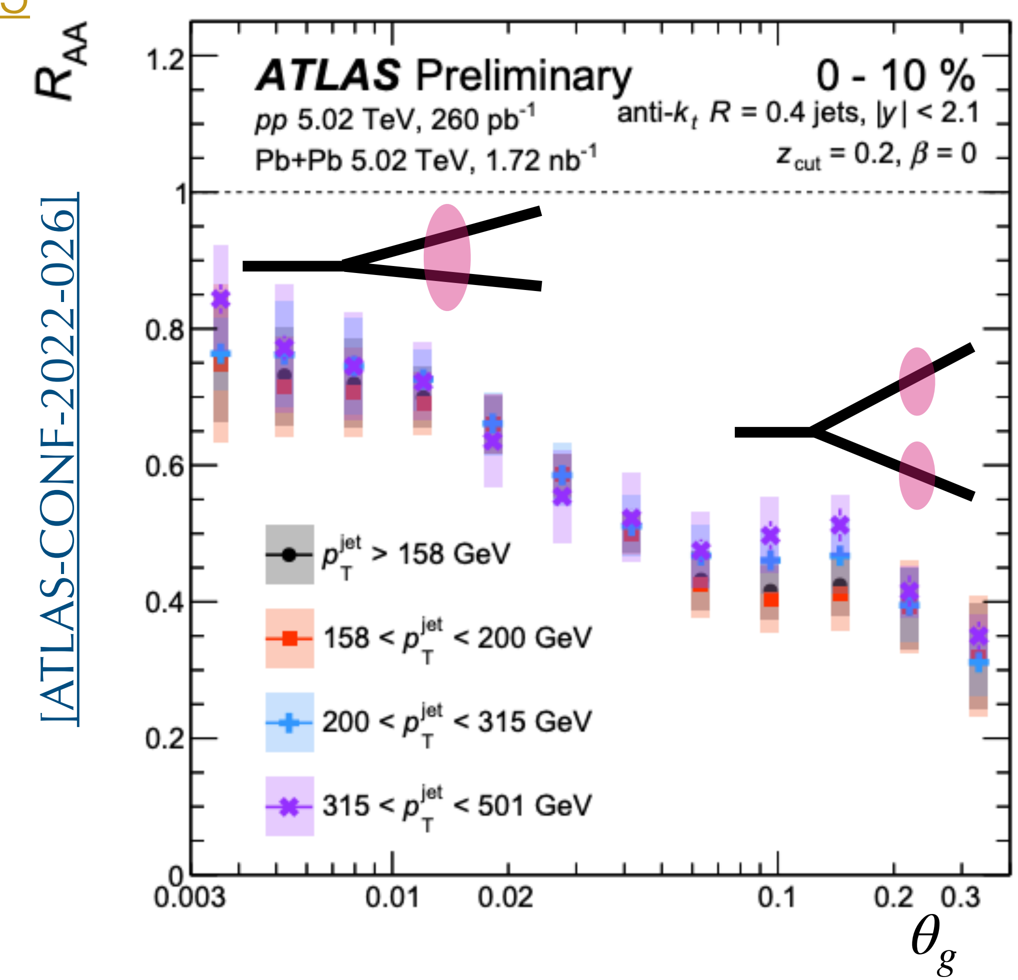
Measurements of jet substructure modification



- Recent emphasis on grooming techniques
- Extract angle and energy fraction of the hardest splitting θ_g, z_g
- Issues with having a robust angular variable from grooming
  J. Mulligan, M. Ploskon 2006.01812
- Proposed grooming procedure for HIC

Y. Mehtar-Tani, A. Soto-Ontoso, K. Tywoniuk 1911.00375

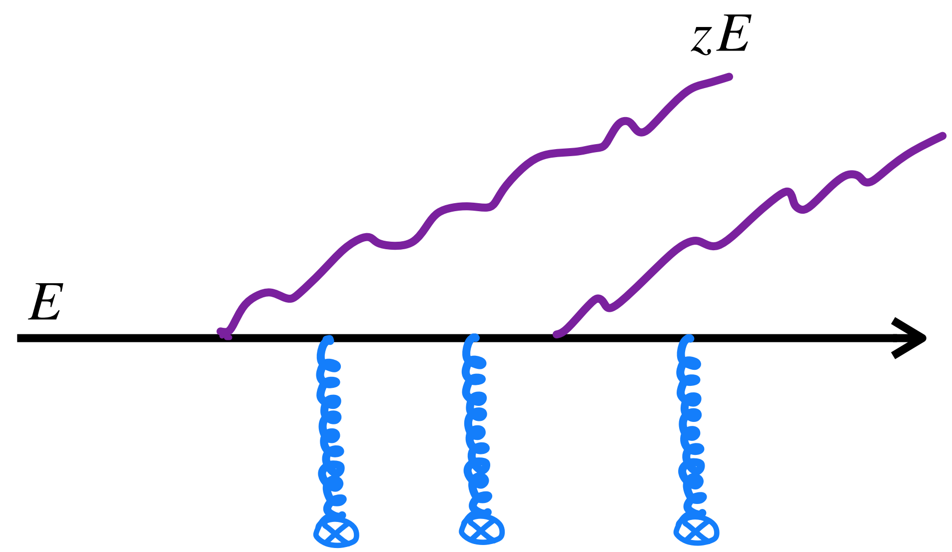




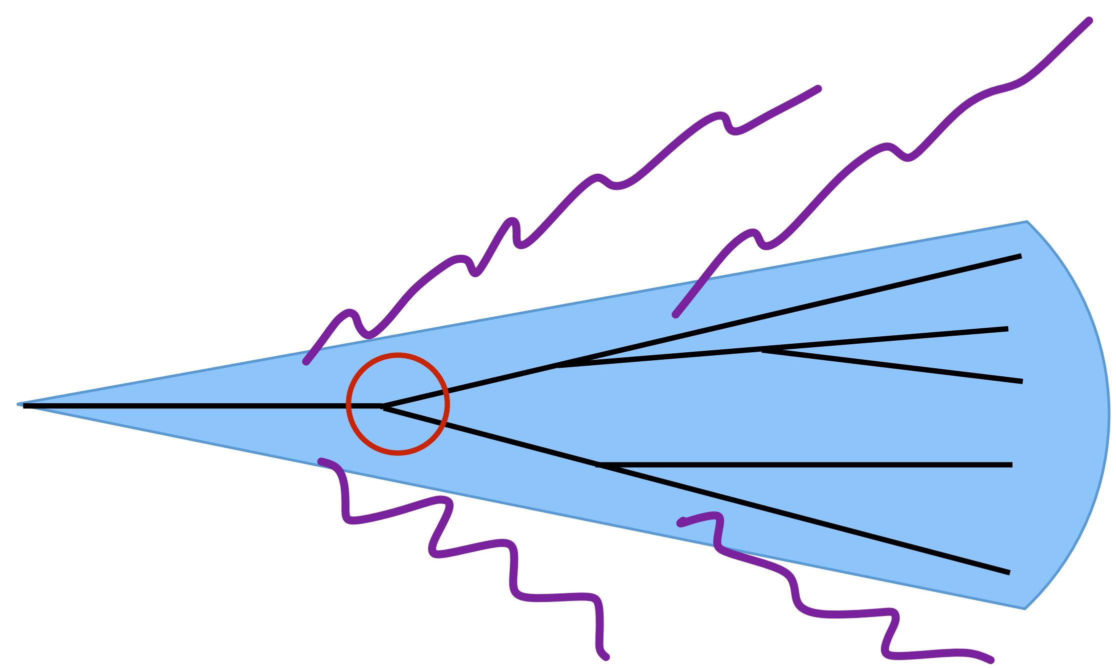
[ATLAS-CONF-2022-026]

From energy loss to jet substructure

- For energy loss calculation we only need the soft limit $z \ll 1$
  - ✦ Soft divergence of the vacuum vertex



- For jet substructure
  - ✦ Emissions from multiple sources
  - ✦ Harder vertices



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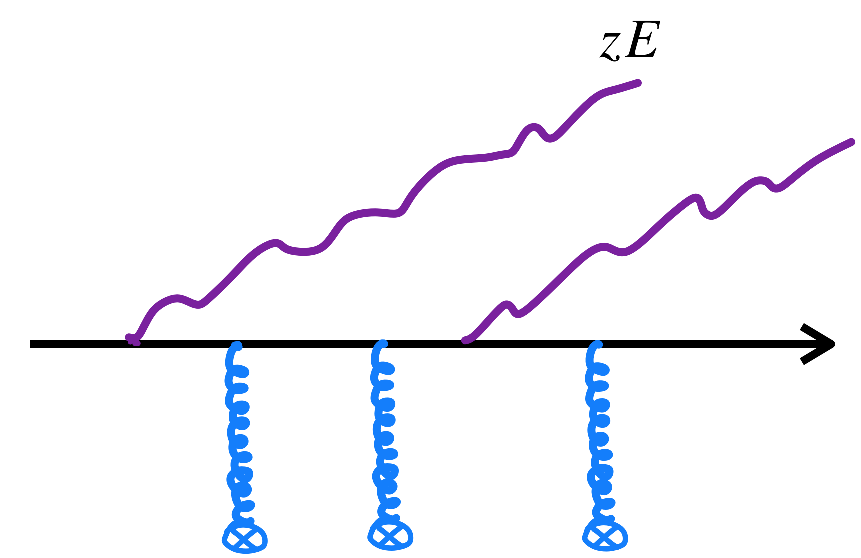

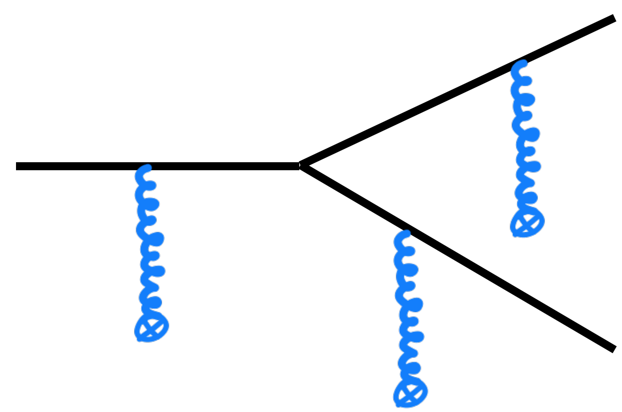

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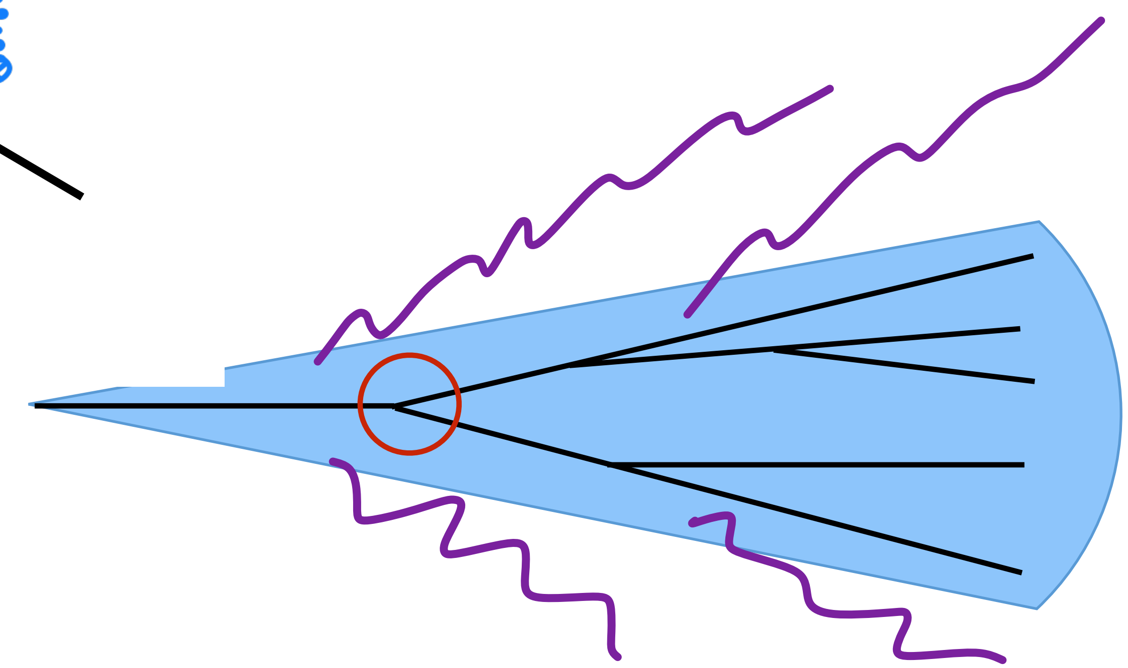
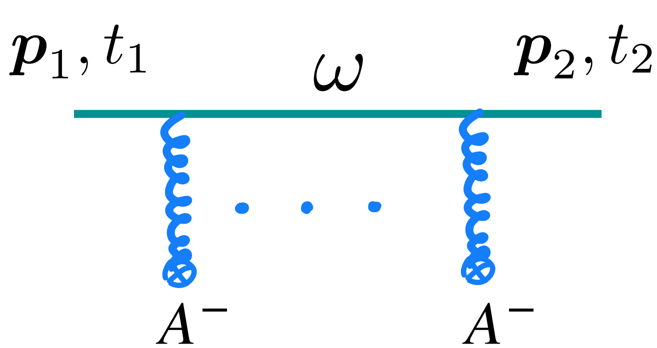


Formalism

- All particles have a large longitudinal momentum compared to their transverse momenta and therefore there is a decoupling between transverse and longitudinal dynamics

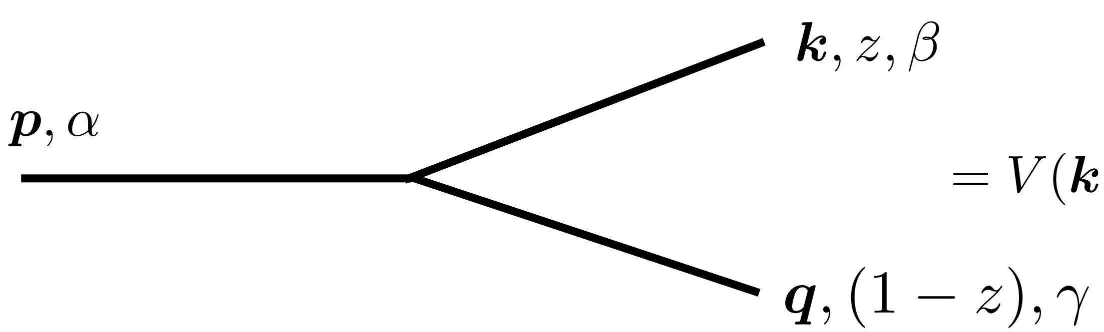
- We work in a mixed representation $(\boldsymbol{p}, t)$ with momentum coordinates in the transverse direction and "time" (+ coordinate) in the longitudinal direction.

- Multiple scatterings resumed through propagators in a background field



$$= \mathcal{G}_R(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1; \omega)$$

- Vacuum vertices

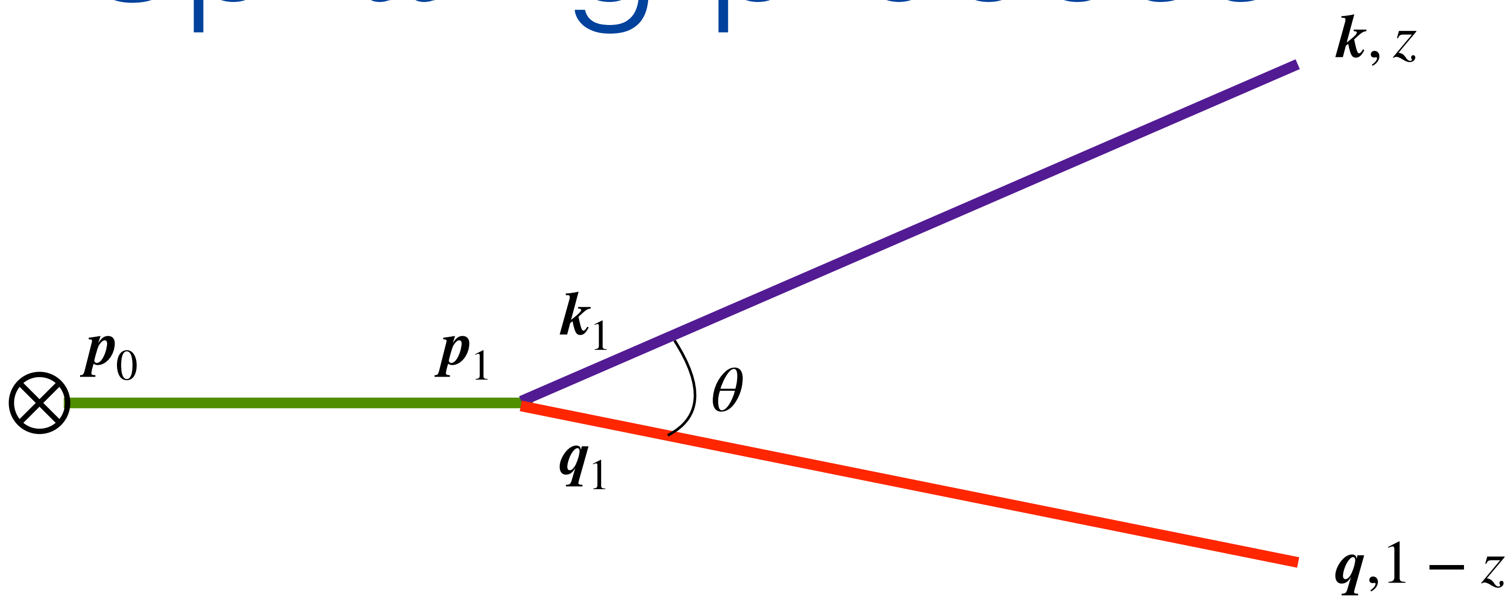


$$= V(\boldsymbol{k} - z\boldsymbol{p}, z) T^{\alpha\beta\gamma} (2\pi)^2 \delta^{(2)}(\boldsymbol{p} - \boldsymbol{k} - \boldsymbol{q})$$

- Background field averaged at the level of the cross section

$$\left\langle A^{a-}(\boldsymbol{q}_1, t_1) A^{b-\dagger}(\boldsymbol{q}_2, t_2) \right\rangle = \delta^{ab} \delta(t_2 - t_1) \delta^{(2)}(\boldsymbol{q}_1 - \boldsymbol{q}_2) v(\boldsymbol{q}_1)$$

Splitting process



$$\mathcal{M}^{\alpha\beta} = \frac{1}{2E}\int_{\boldsymbol{p}_0\boldsymbol{p}_1\boldsymbol{k}_1\boldsymbol{q}_1}\int_{t_0}^{\infty} dt_1\,(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1-\boldsymbol{k}_1-\boldsymbol{q}_1)\mathcal{G}_{R_b}^{\alpha\alpha_1}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)$$
$$\times\,\mathcal{G}_{R_c}^{\beta\beta_1}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\,V(\boldsymbol{k}_1-z\boldsymbol{p}_1,z)T^{\alpha_1\beta_1\gamma_1}\,\mathcal{G}_{R_a}^{\gamma_1\gamma}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\,\mathcal{M}_0^{\gamma}(E,\boldsymbol{p}_0)$$

Where each of the in-medium propagators is of the form:

$$\mathcal{G}_R(t_2,\boldsymbol{x}_2;t_1,\boldsymbol{x}_1;\omega) = \int_{\boldsymbol{x}_1}^{\boldsymbol{x}_2}\mathcal{D}\boldsymbol{r}\,\exp\left\{\frac{i\omega}{2}\int_{t_1}^{t_2}d\xi\;\dot{\boldsymbol{r}}^2(\xi)\right\}\underbrace{\mathrm{P}\exp\left\{ig\int_{t_1}^{t_2}d\xi\;A_R^-(\xi,\boldsymbol{r}(\xi))\right\}}_{V_R(t_2,t_1;[\boldsymbol{r}])}$$

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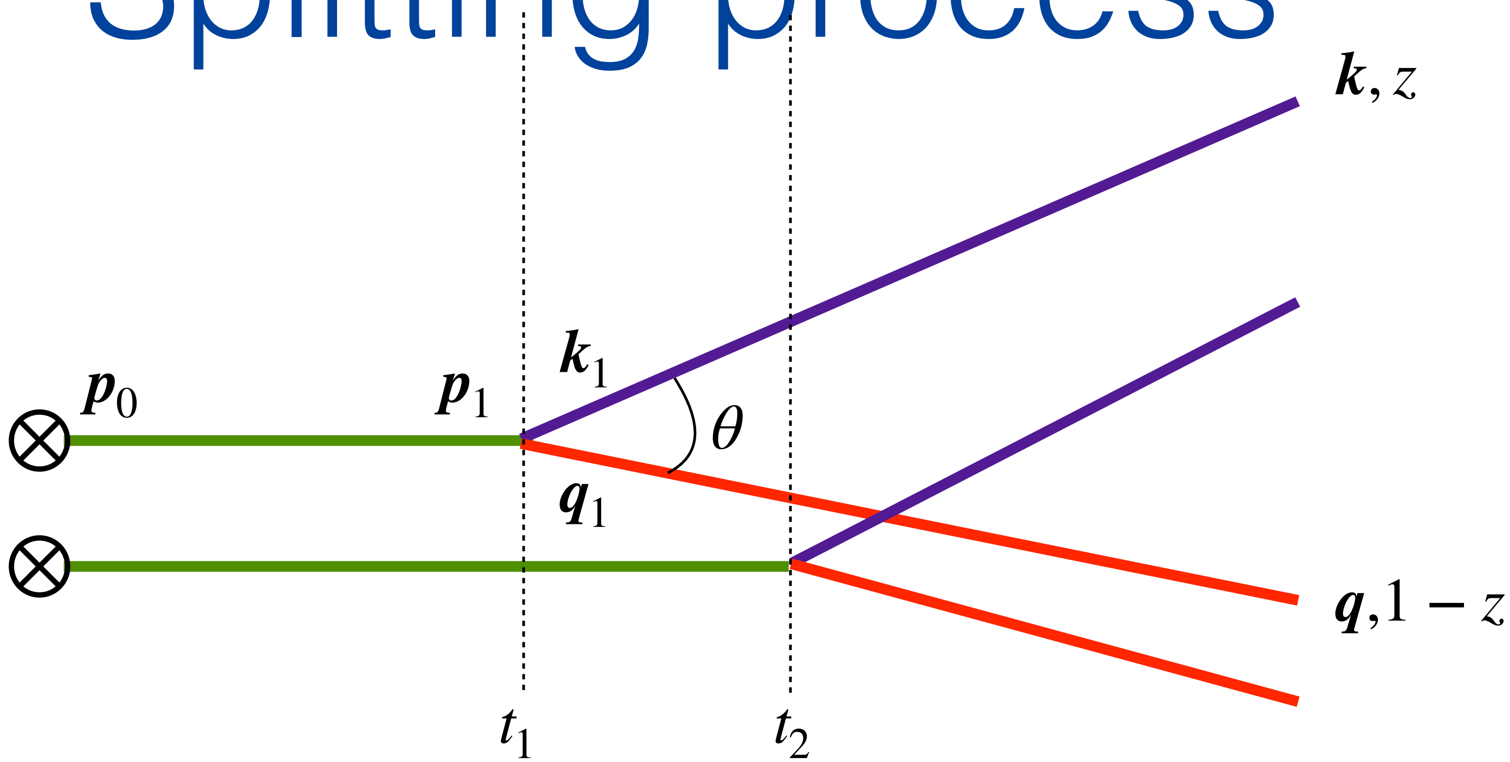

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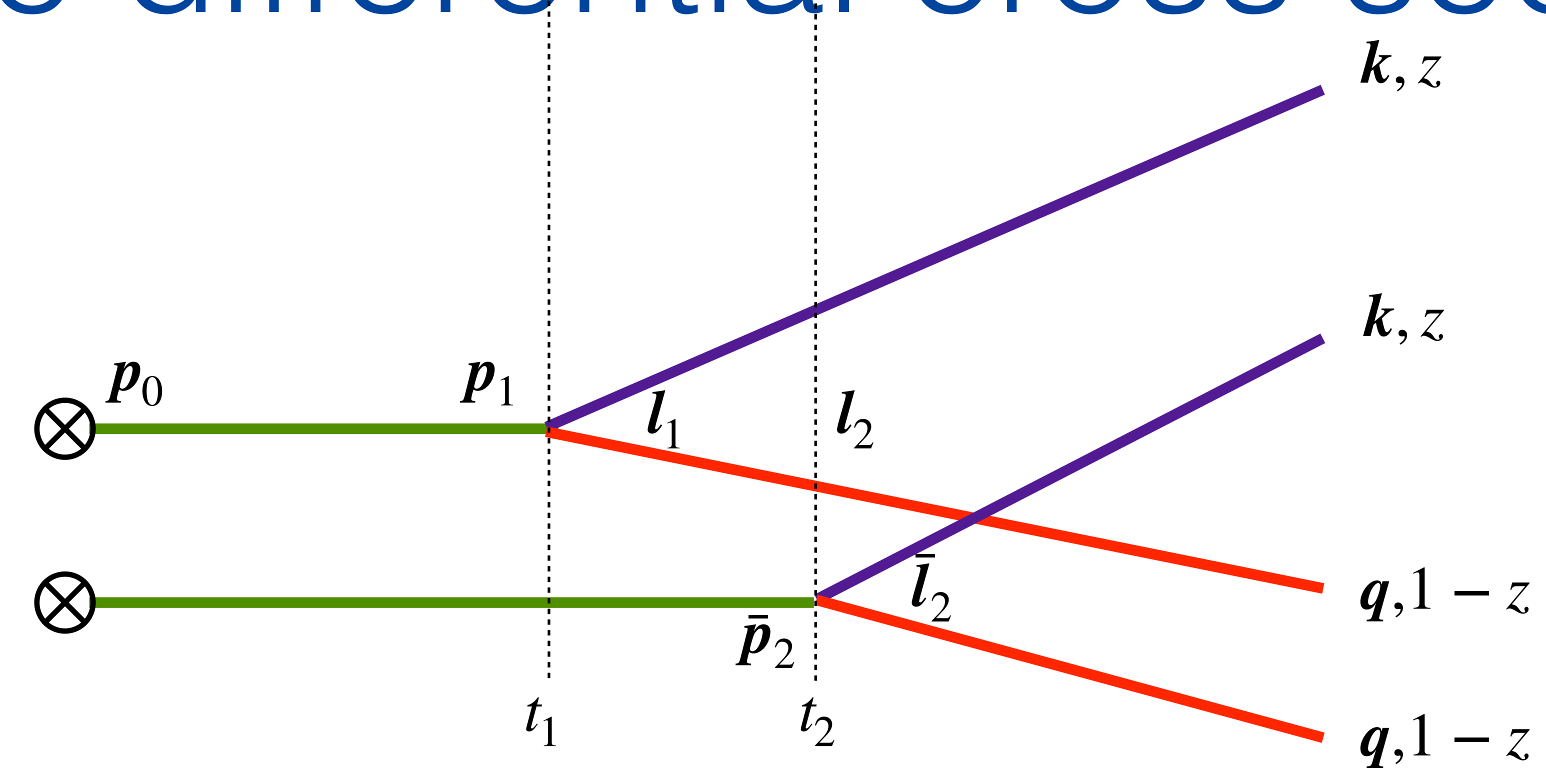
$$\left\langle|\mathcal{M}|^2\right\rangle \propto \Big\langle\mathcal{G}_{R_b}^{\alpha\alpha_1}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)\mathcal{G}_{R_c}^{\beta\beta_1}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\mathcal{G}_{R_b}^{\dagger\bar{\alpha}_2\alpha}(\bar{\boldsymbol{k}}_2,t_2;\boldsymbol{k},L;zE)$$
$$\times\mathcal{G}_{R_c}^{\dagger\bar{\beta}_2\beta}(\bar{\boldsymbol{q}}_2,t_2;\boldsymbol{q},L;(1-z)E)\mathcal{G}_{R_a}^{\gamma_1\gamma}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\mathcal{G}_{R_a}^{\dagger\bar{\gamma}\bar{\gamma}_2}(\bar{\boldsymbol{p}}_0,t_0;\bar{\boldsymbol{p}}_2,t_2;E)\Big\rangle$$

Double differential cross section



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re} \int_{\boldsymbol{p}_0 \boldsymbol{p}_1 \bar{\boldsymbol{p}}_2 \boldsymbol{l}_1 \boldsymbol{l}_2 \bar{\boldsymbol{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1 \cdot \bar{\boldsymbol{l}}_2)$$
$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k} + \boldsymbol{q} - \bar{\boldsymbol{p}}_2, z)$$
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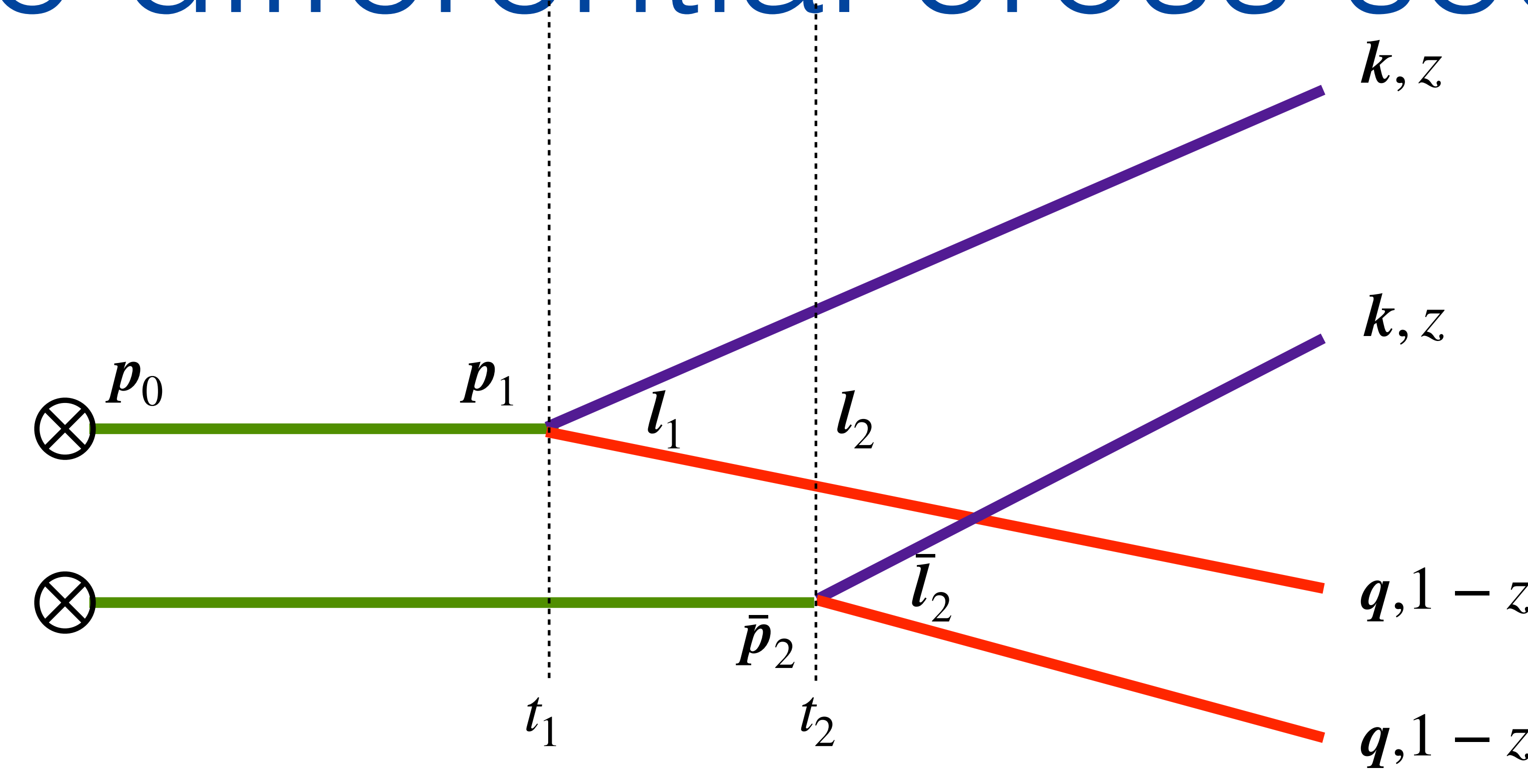
Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section



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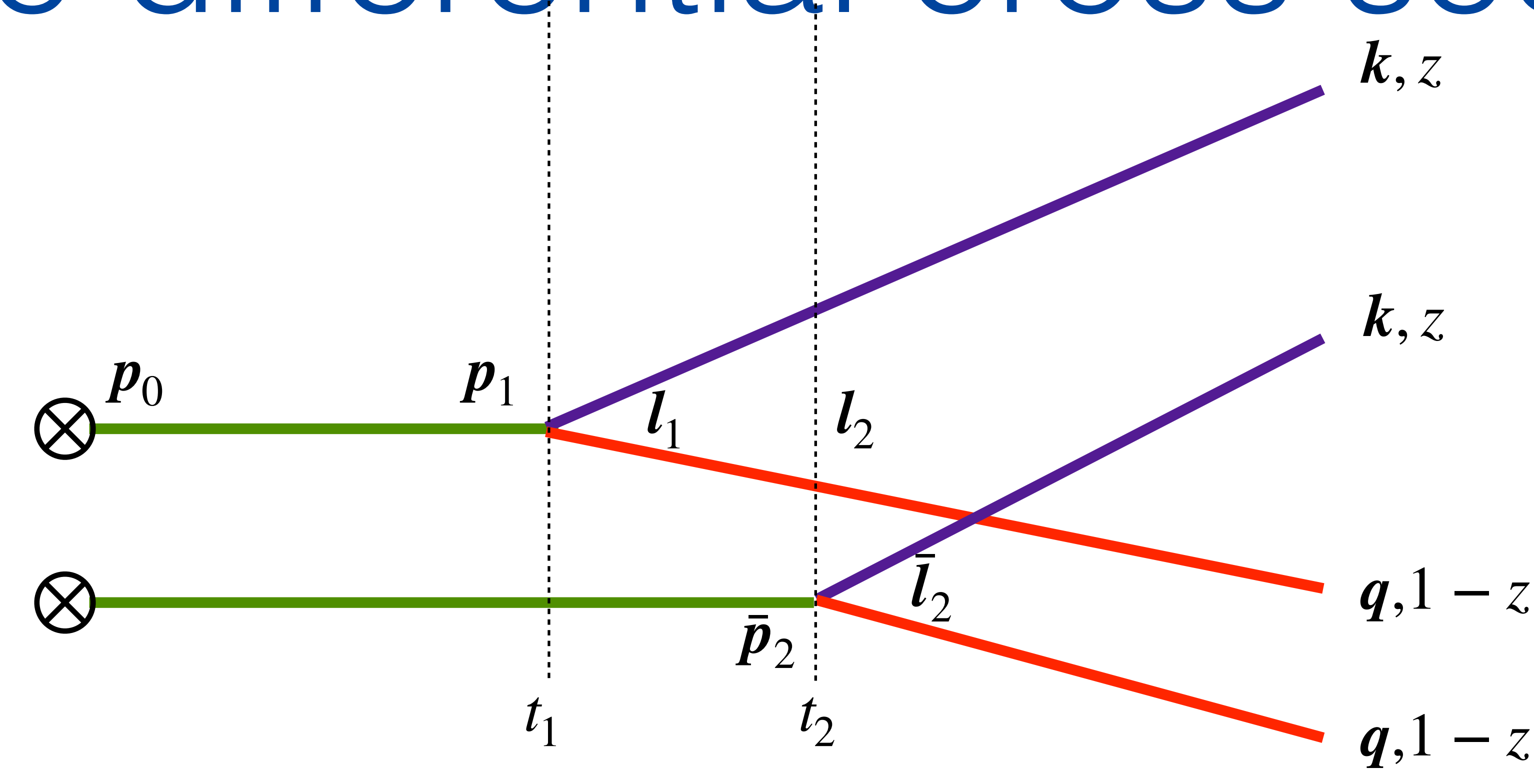

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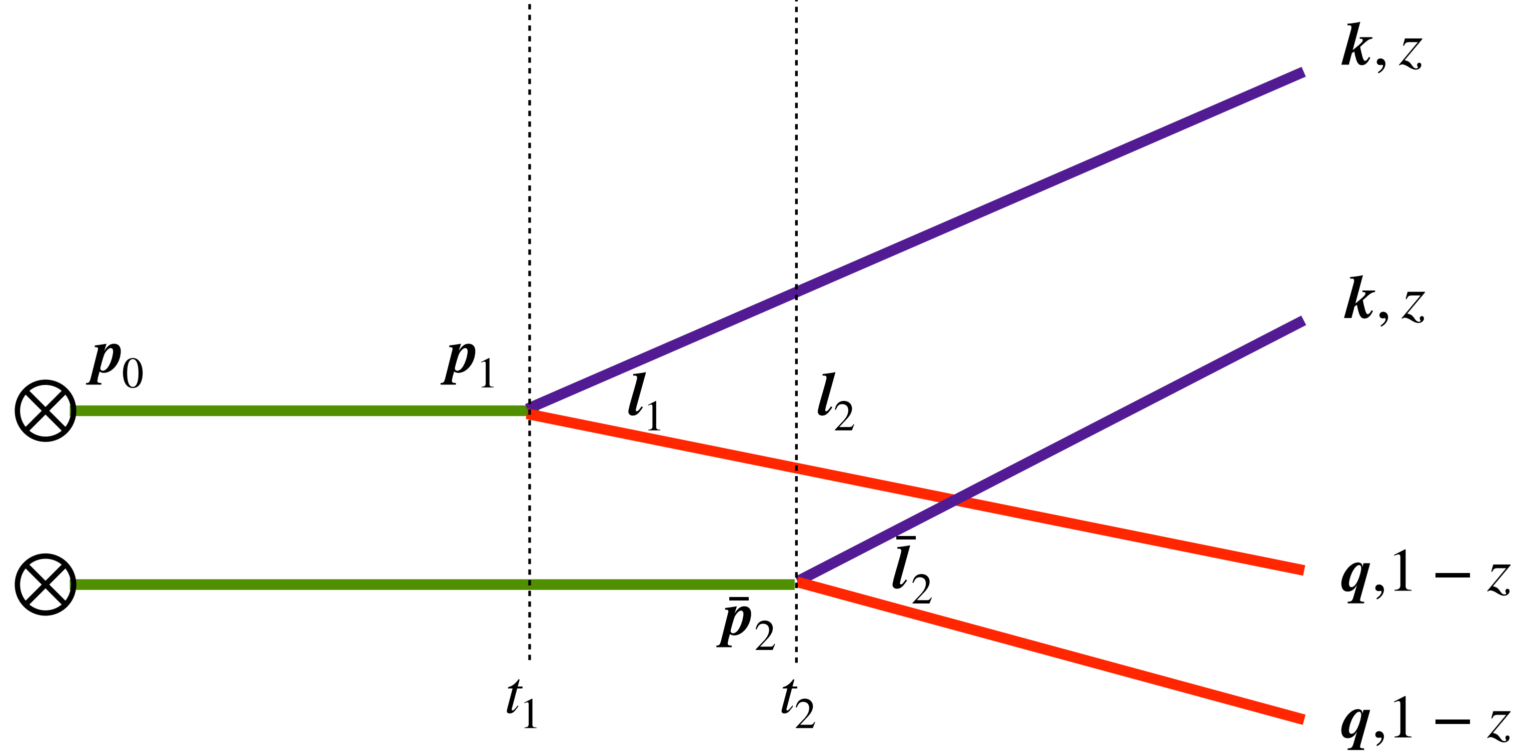

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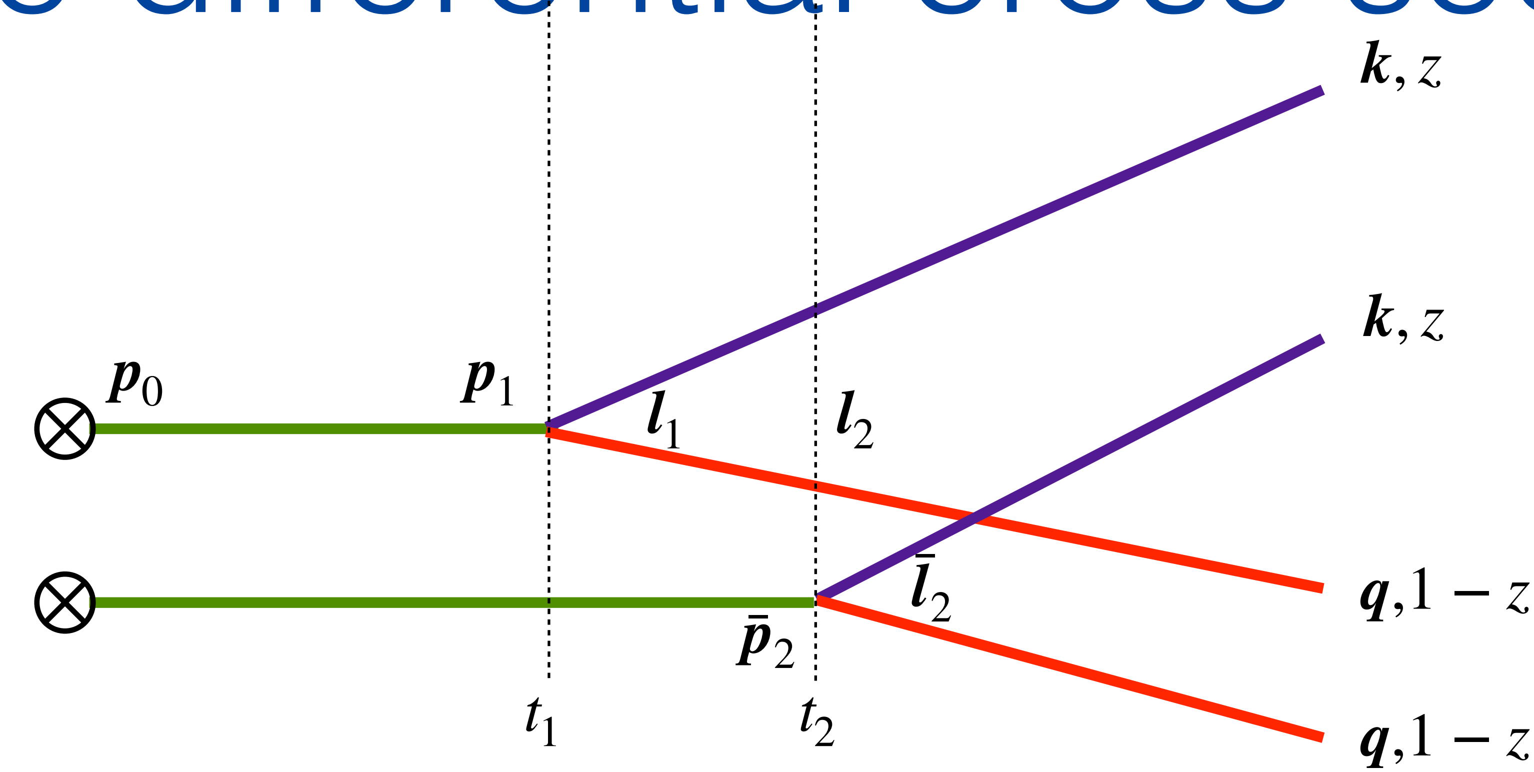

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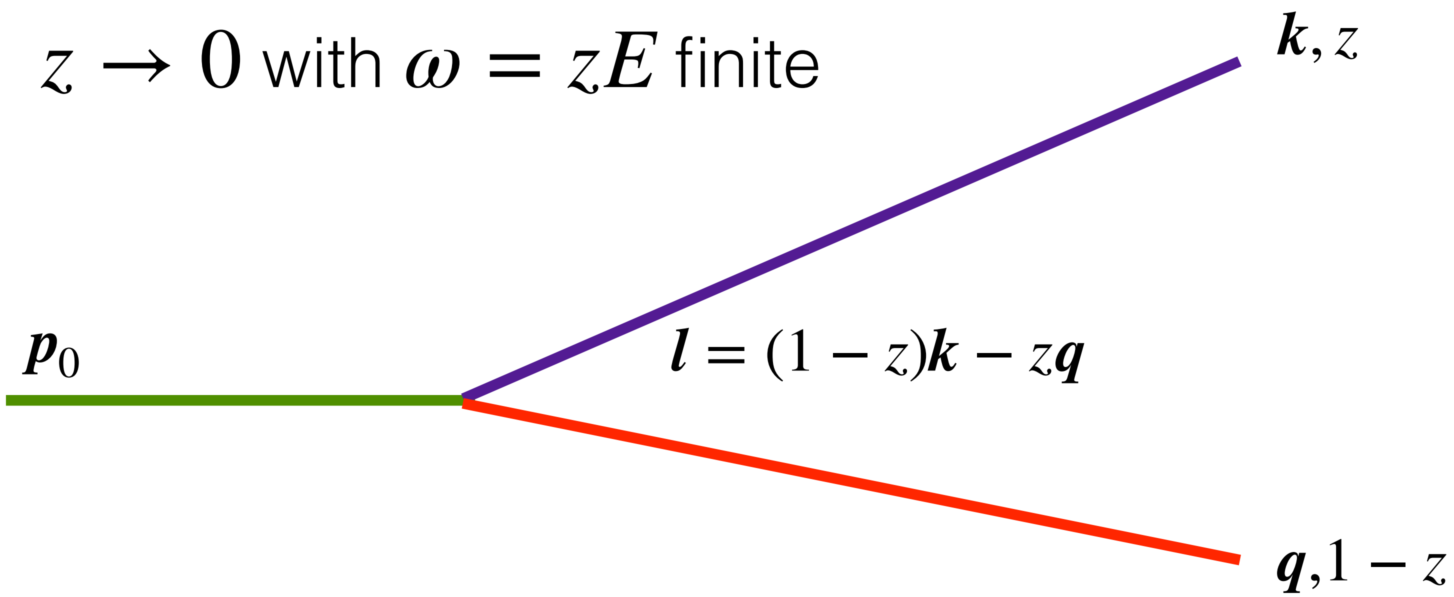
Soft limit

$z \to 0$ with $\omega = zE$ finite



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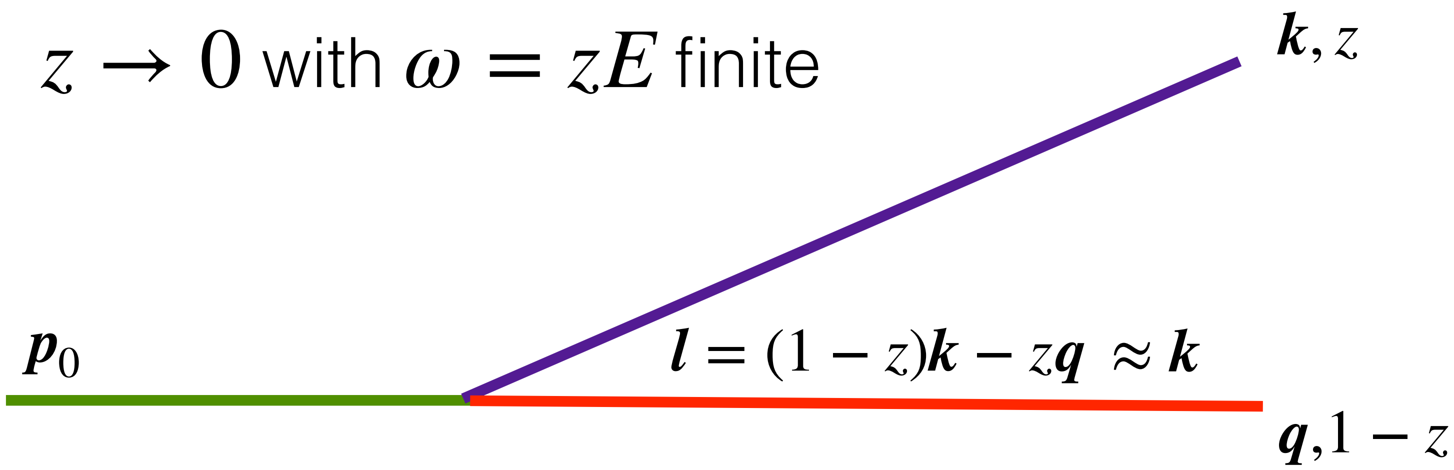

Angle of emission depends only on transverse momentum of the soft particle

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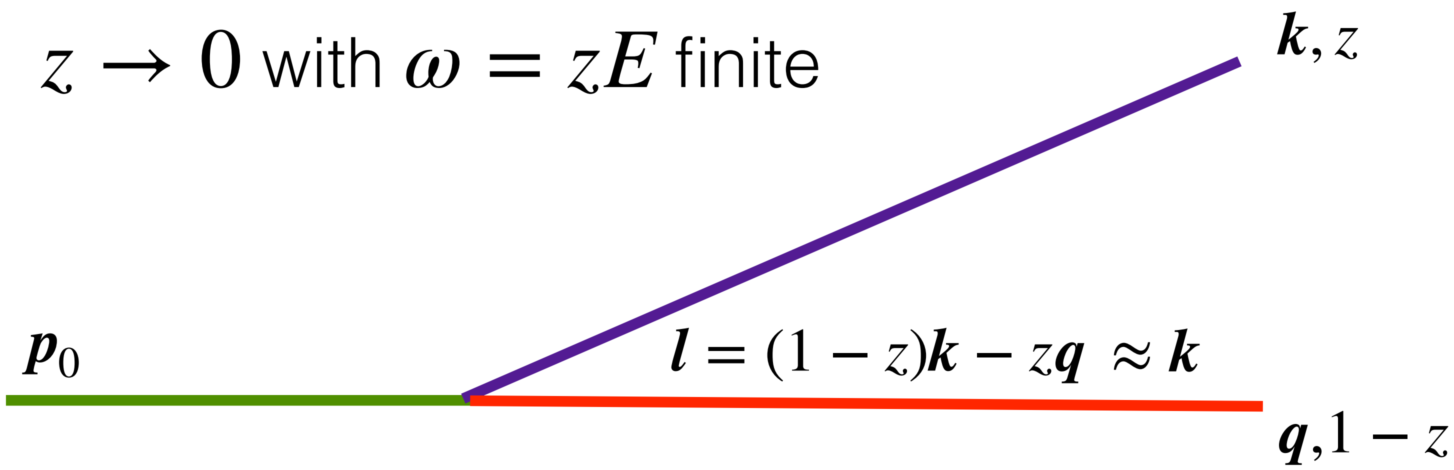

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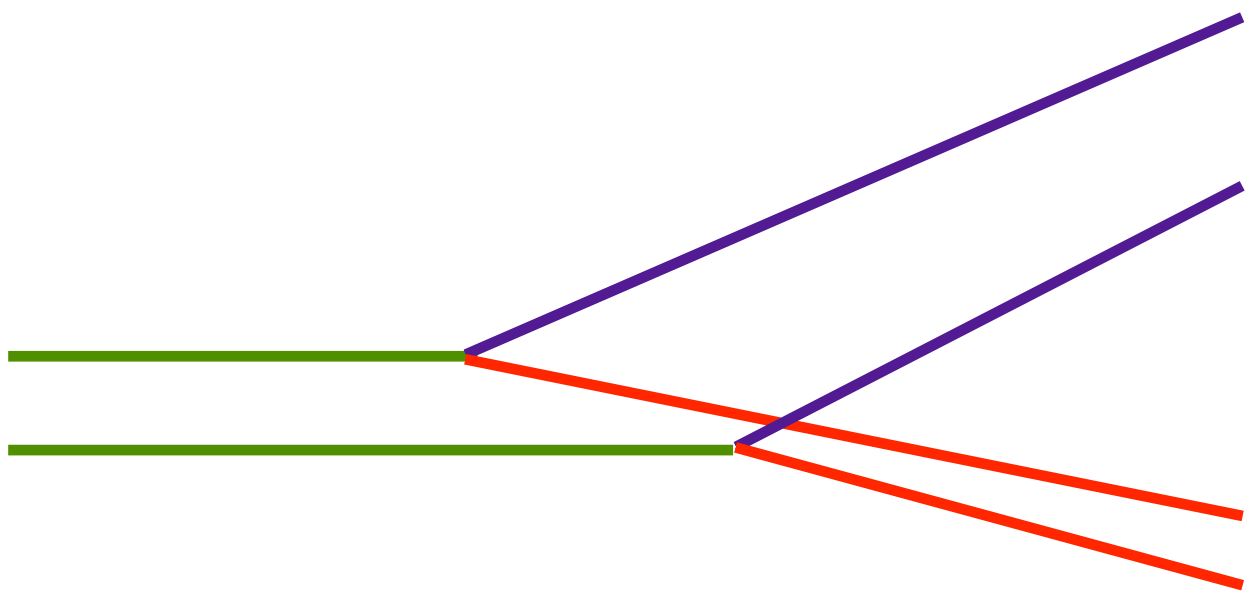

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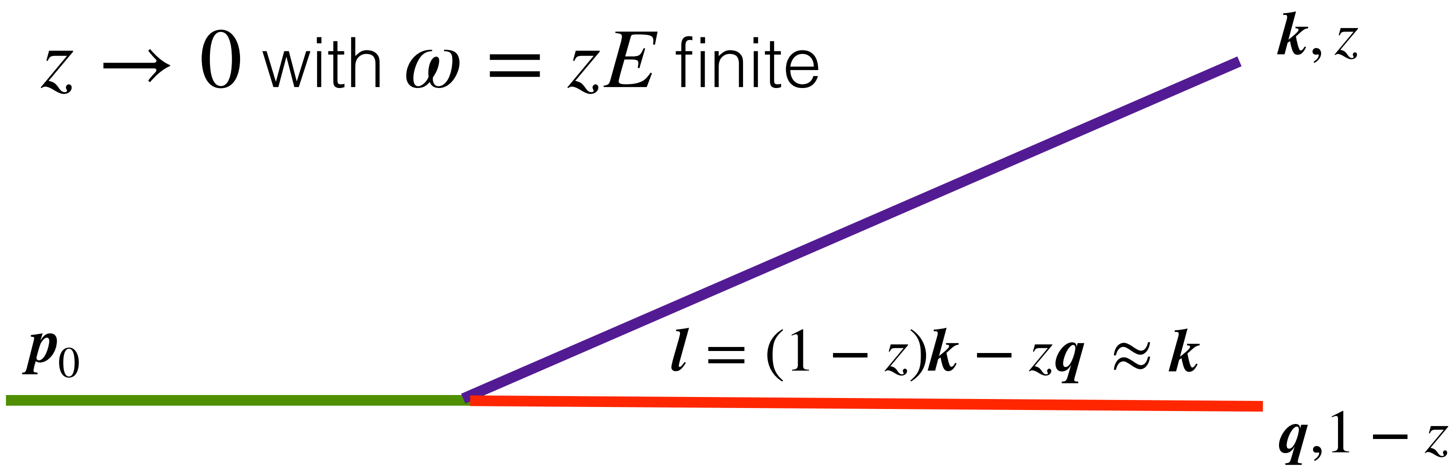

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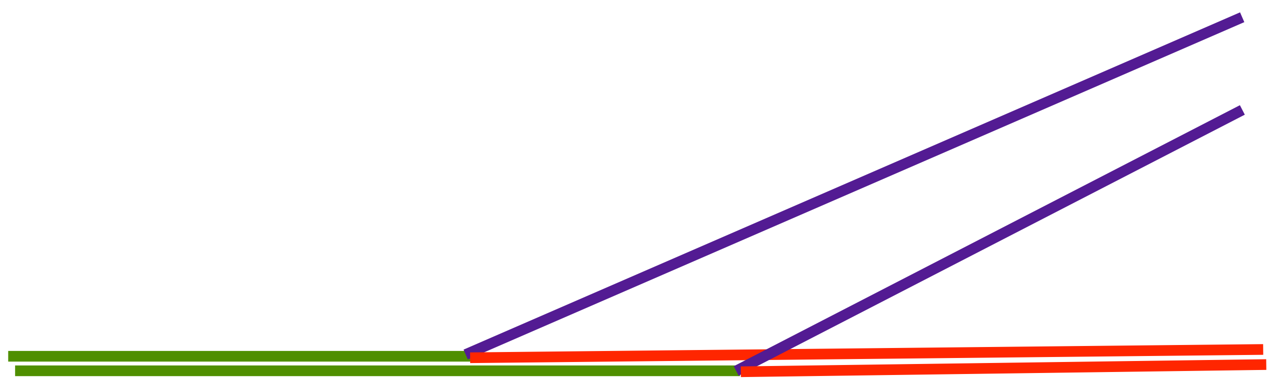

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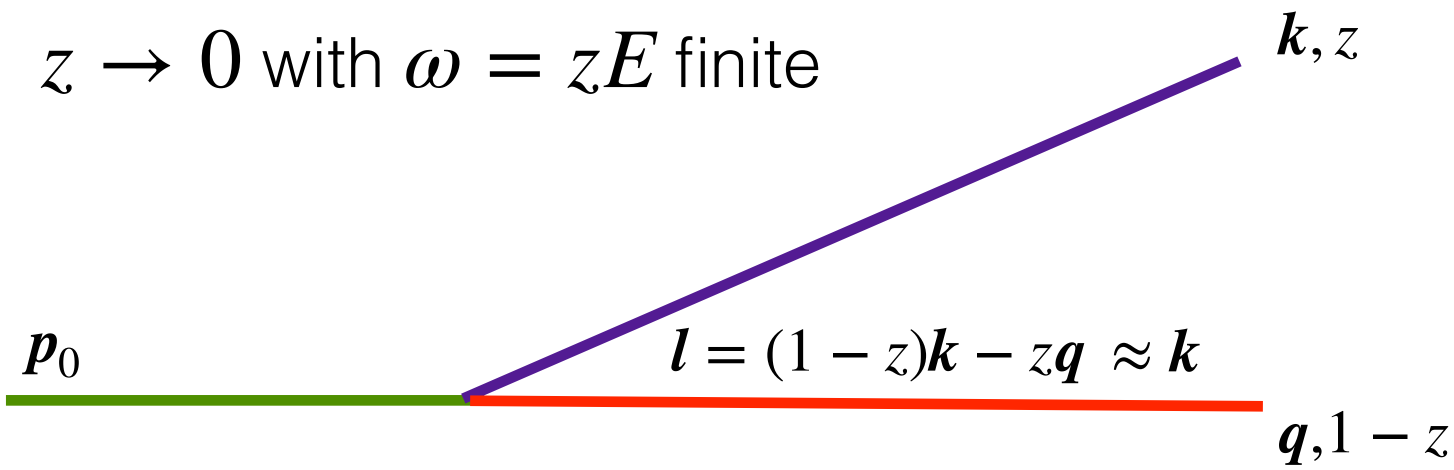

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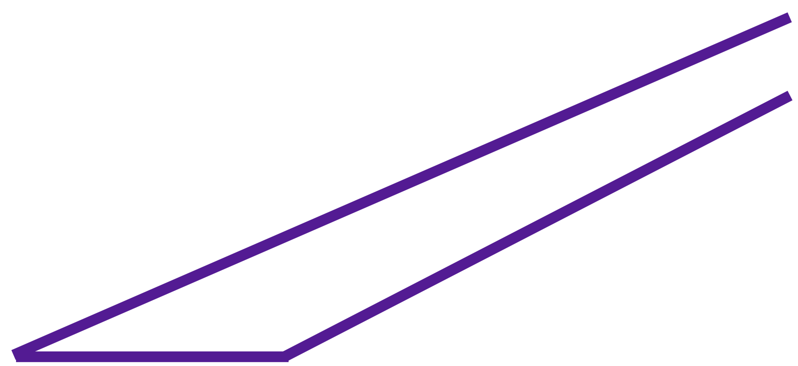

Initial and final broadening of the hard particle cancels out

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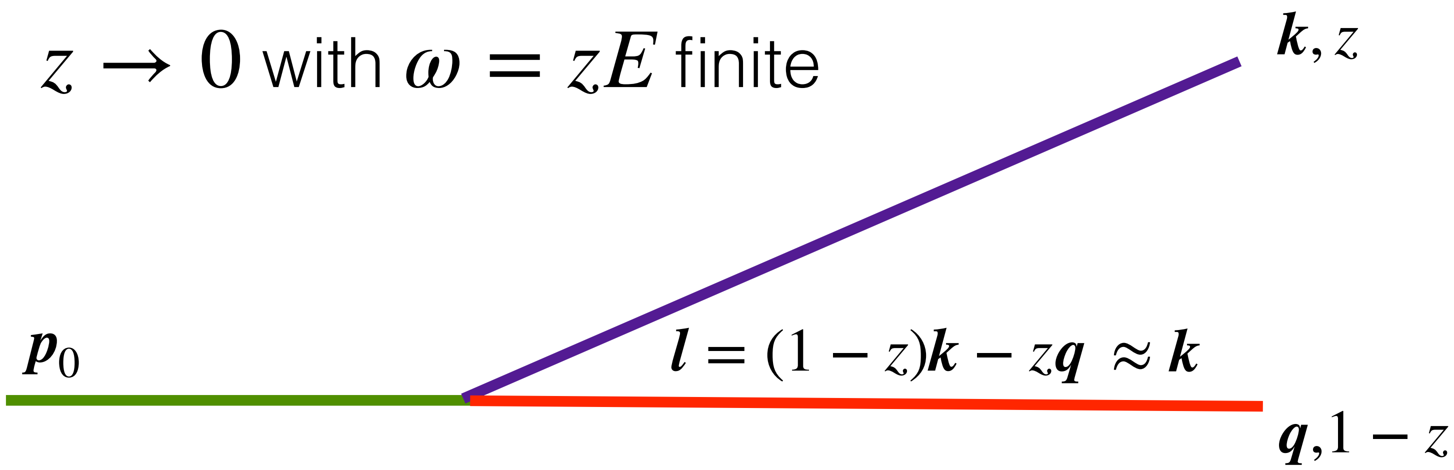

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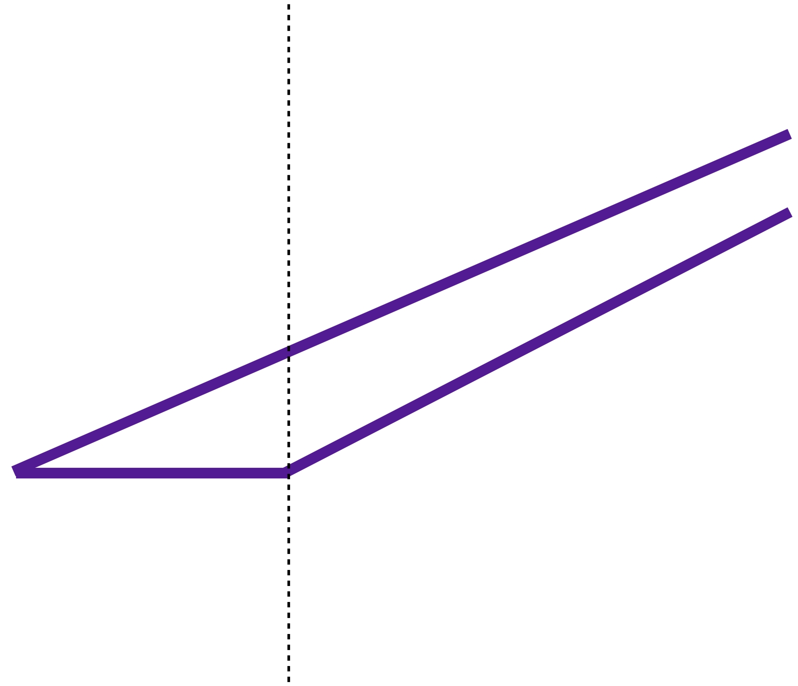

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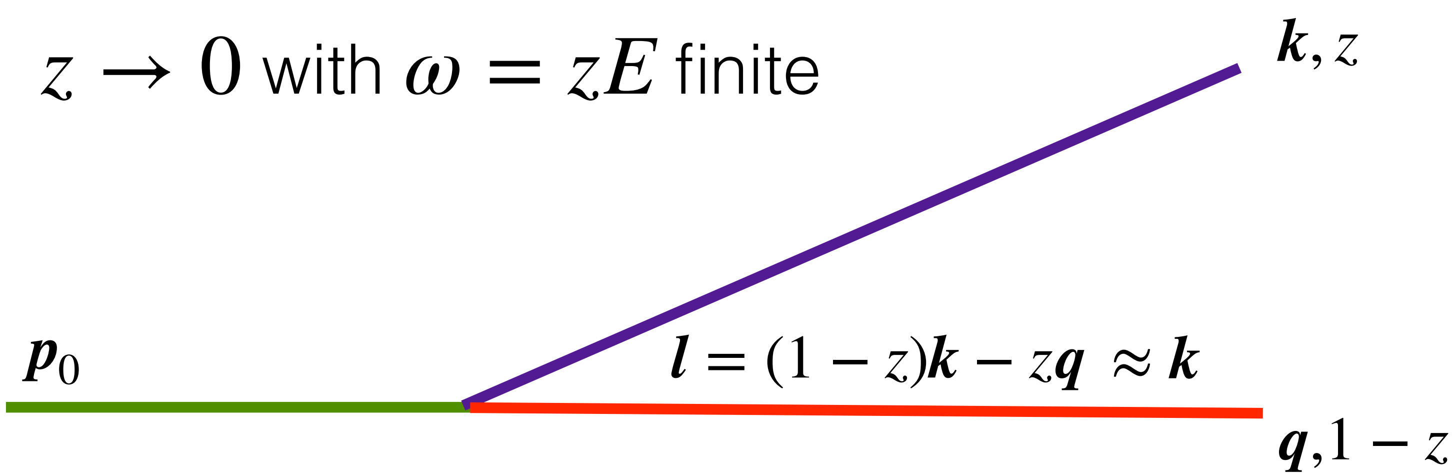

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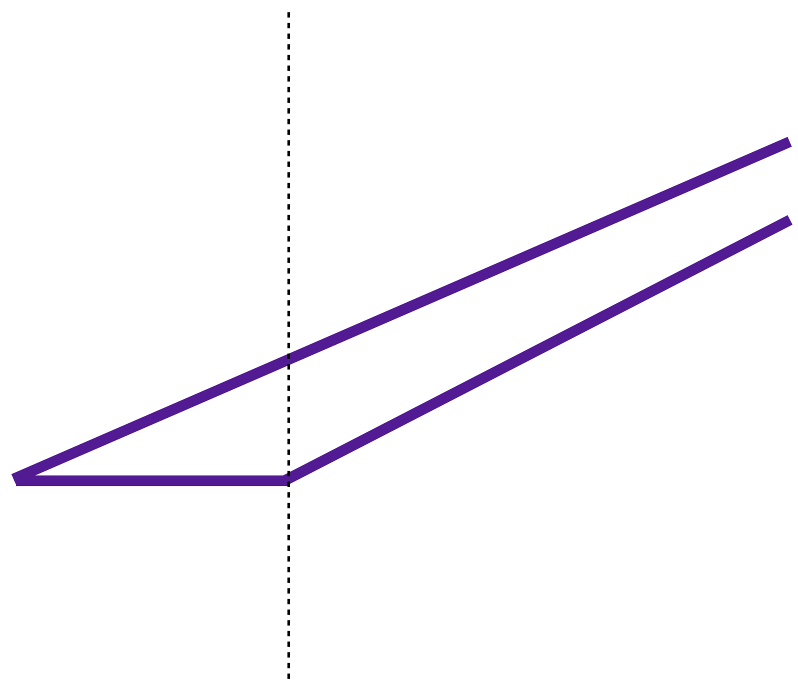

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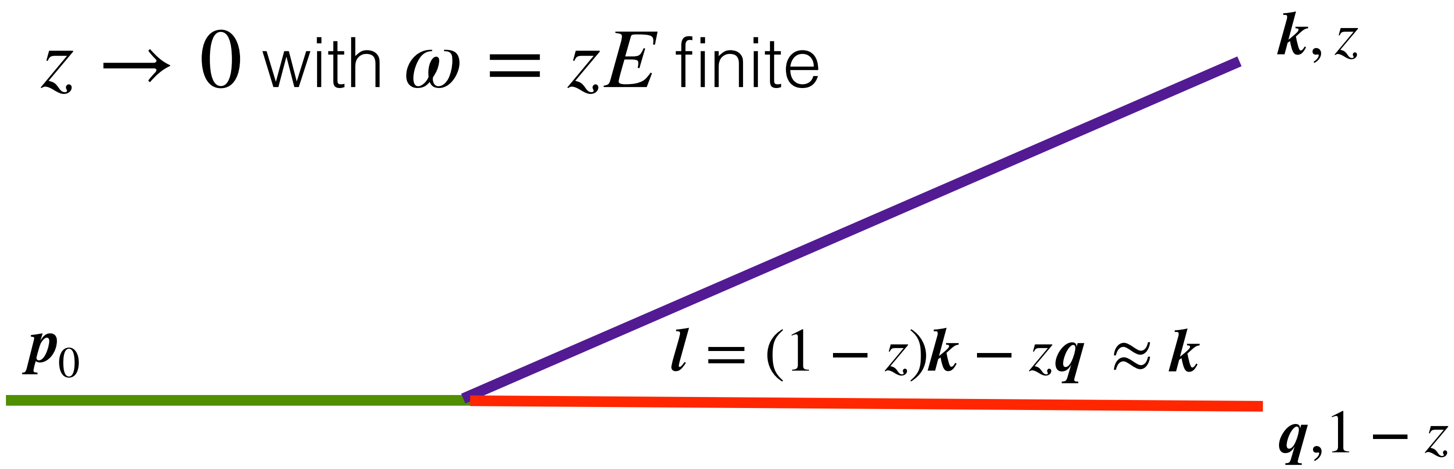

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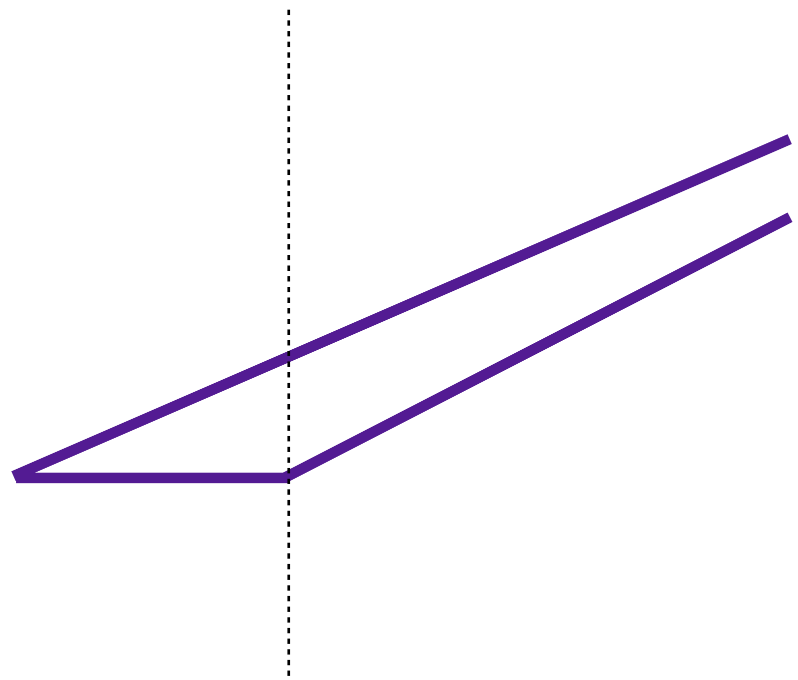

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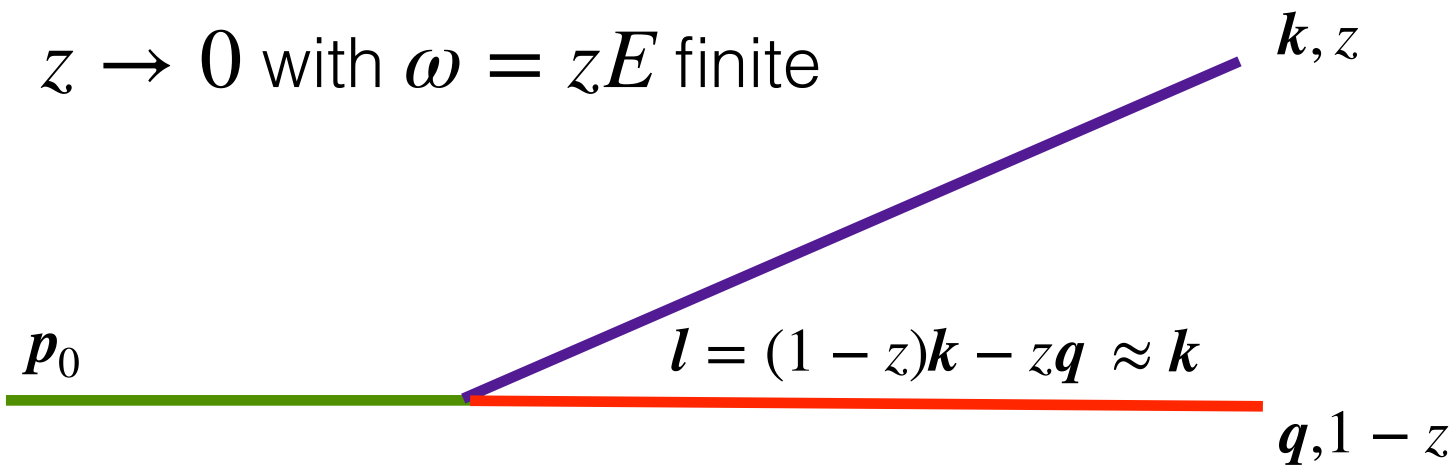
Recently evaluated numerically with multiple scatterings and realistic interaction

Andres, Apolinario, FD 2002.01517
Andres, FD, Gonzalez Martinez 2011.06522

Soft limit

$z \to 0$ with $\omega = zE$ finite



Angle of emission depends only on transverse momentum of the soft particle

Useful for energy loss, less so for jet substructure

the hard particle cancels out

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t', \boldsymbol{q}; t, \boldsymbol{p}) \mathcal{P}(\infty, \boldsymbol{k}; t', \boldsymbol{q})$$

Recently evaluated numerically with multiple scatterings and realistic interaction

Andres, Apolinario, FD 2002.01517
Andres, FD, Gonzalez Martinez 2011.06522

Evaluation of in-medium splittings

- Full evaluation keeping z and θ not yet implemented

- Two available approximations:
  - Opacity expansion ($N = 1$) Sievert, Vitev 1807.03799
    - Unitarity problems can lead to negative cross sections
    - Recursive formulas to generate all orders (not yet implemented numerically)
  - "Tilted" Wilson lines
    - Resums multiple scatterings in the eikonal approximation
    - Assumes semi-hard splittings (z not too small)

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653
Isaksen, Tywoniuk 2107.02542

Tilted Wilson lines

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653
Isaksen, Tywoniuk 2107.02542

- Use high-energy limit of propagators: vacuum propagator times a Wilson line in the classical trajectory

$$\mathcal{G}_R(t_2,\boldsymbol{p}_2;t_1,\boldsymbol{p}_1;\omega) \to (2\pi)^2\delta^{(2)}(\boldsymbol{p}_2-\boldsymbol{p}_1)\, e^{-i\frac{p_2^2}{2\omega}(t_2-t_1)} V_R(t_2,t_1;[\boldsymbol{n}t])$$

- Calculate averages of Wilson lines in the large-N_c limit (calculations also available for finite N_c). All averages can be expressed in terms of fundamental dipoles and quadrupoles

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\, V_1 V_2^\dagger \right\rangle = S_{12} \qquad \frac{1}{N_c}\left\langle \mathrm{Tr}\, V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger \right\rangle = Q$$

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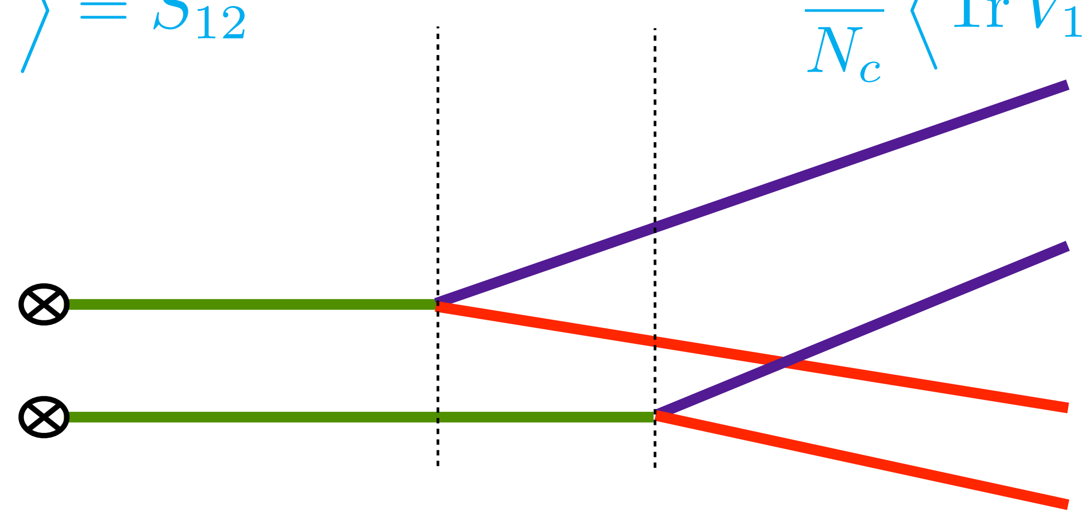

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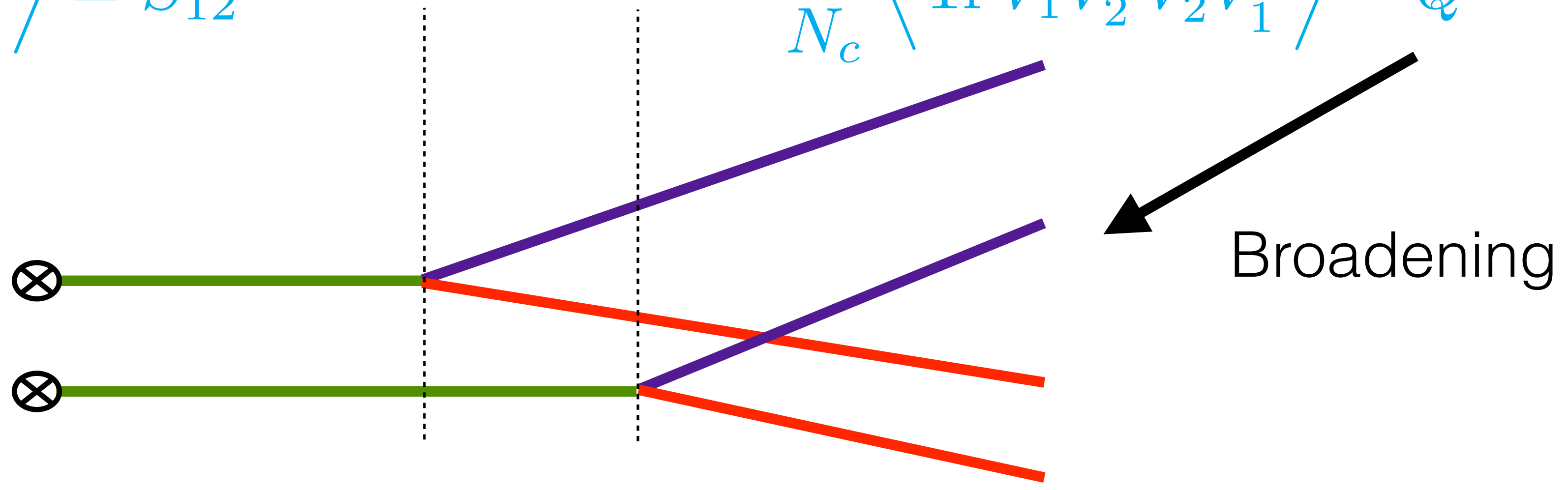

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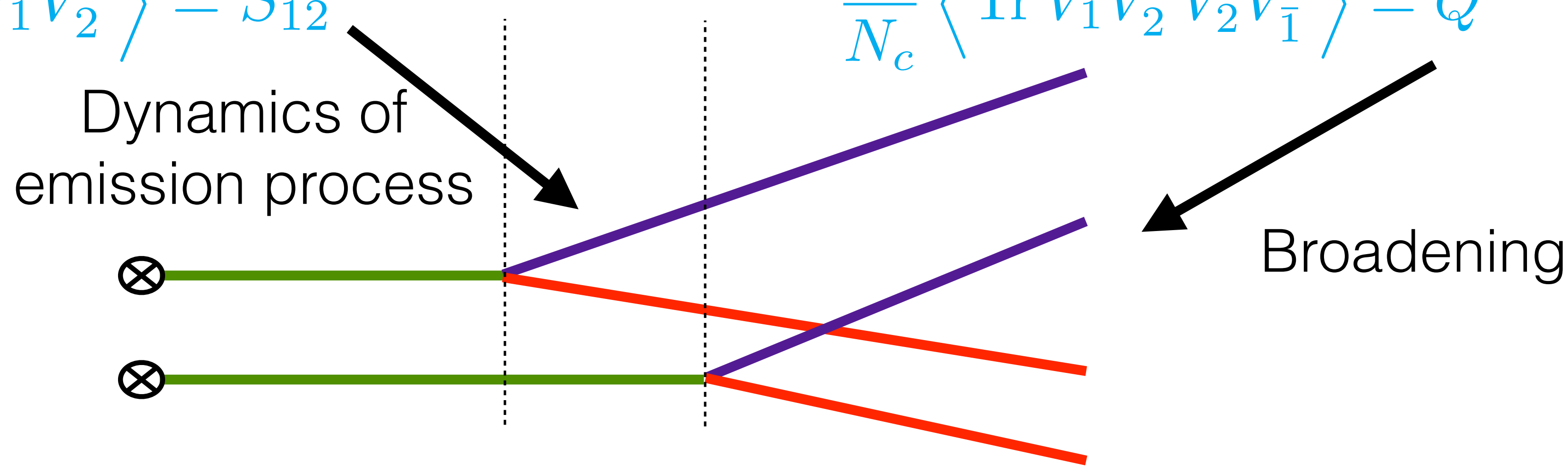


Time and angular scales

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653

- For a static medium of length L within the harmonic approximation one can read off the relevant scales directly from the formulas
  - (Vacuum) formation time:

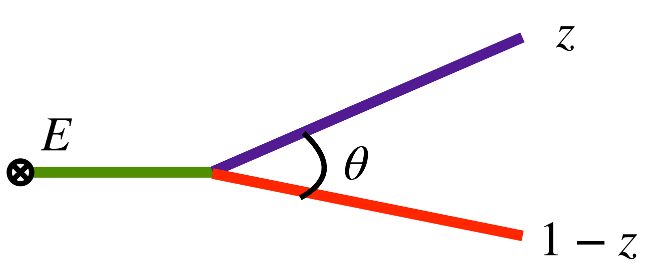
$$t_f = \frac{2}{z(1-z)E\theta^2}$$

$$\theta_L \sim (EL)^{-1/2}$$

Below θ_L all emissions have a formation time larger than L

  - Decoherence time:

$$S_{12}(\tau) = e^{-\frac{1}{12}\hat{q}(1+z^2)\theta^2\tau^3} \qquad t_d \sim (\hat{q}\theta^2)^{-1/3} \qquad \theta_c \sim (\hat{q}L^3)^{-1/2}$$



Below θ_c splittings do not color decohere and the medium does not resolve them

If $\theta_L > \theta_c$ then θ_c becomes irrelevant

Time and angular scales

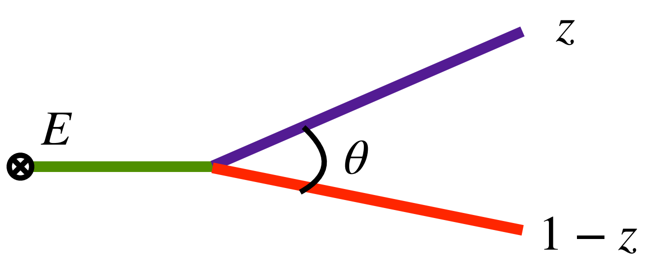
FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653

- For a static medium of length L within the harmonic approximation one can read off the relevant scales directly from the formulas

Can be extended to include a more realistic interaction or expanding media, but then we would not know the scales directly from the equations

  - ✦ Decoherence time:

$$S_{12}(\tau) = e^{-\frac{1}{12}\hat{q}(1+z^2)\theta^2\tau^3} \qquad t_d \sim (\hat{q}\theta^2)^{-1/3} \qquad \theta_c \sim (\hat{q}L^3)^{-1/2}$$



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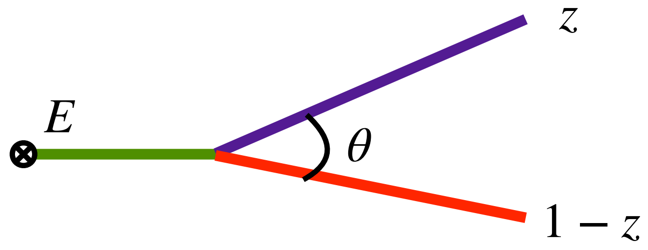
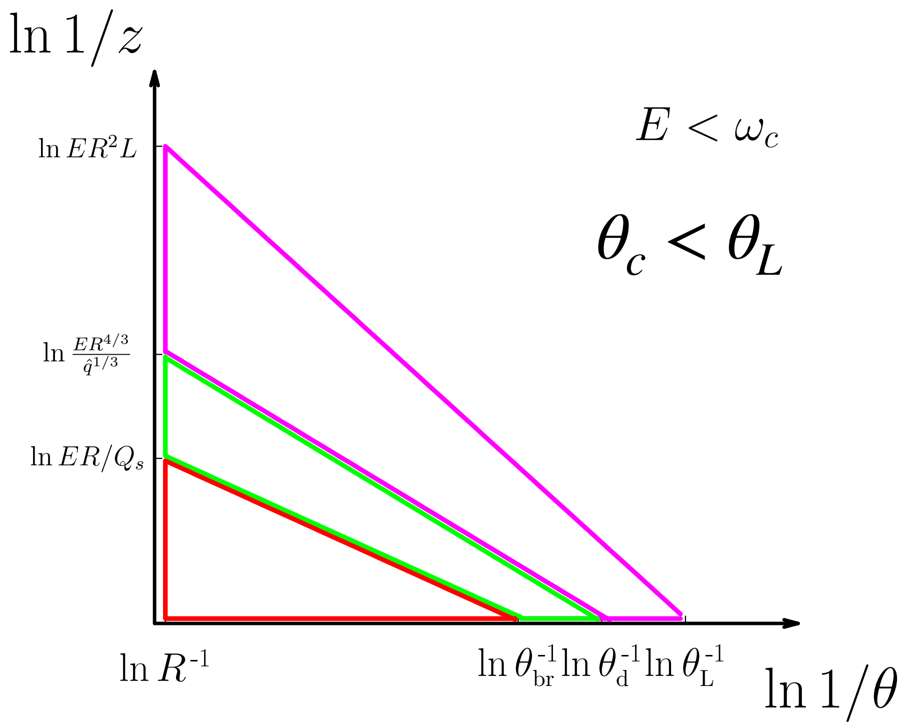
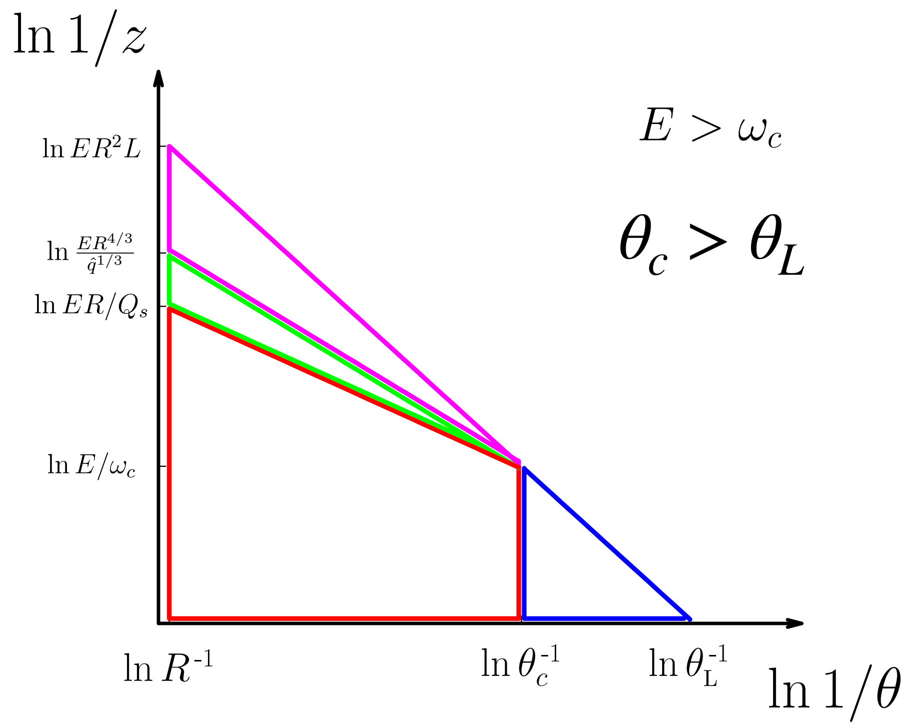


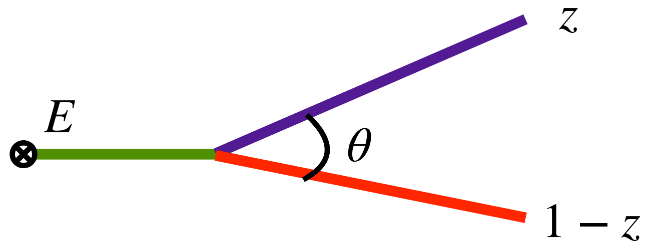
Lund plane

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653





$\omega_c \sim \hat{q}L^2$



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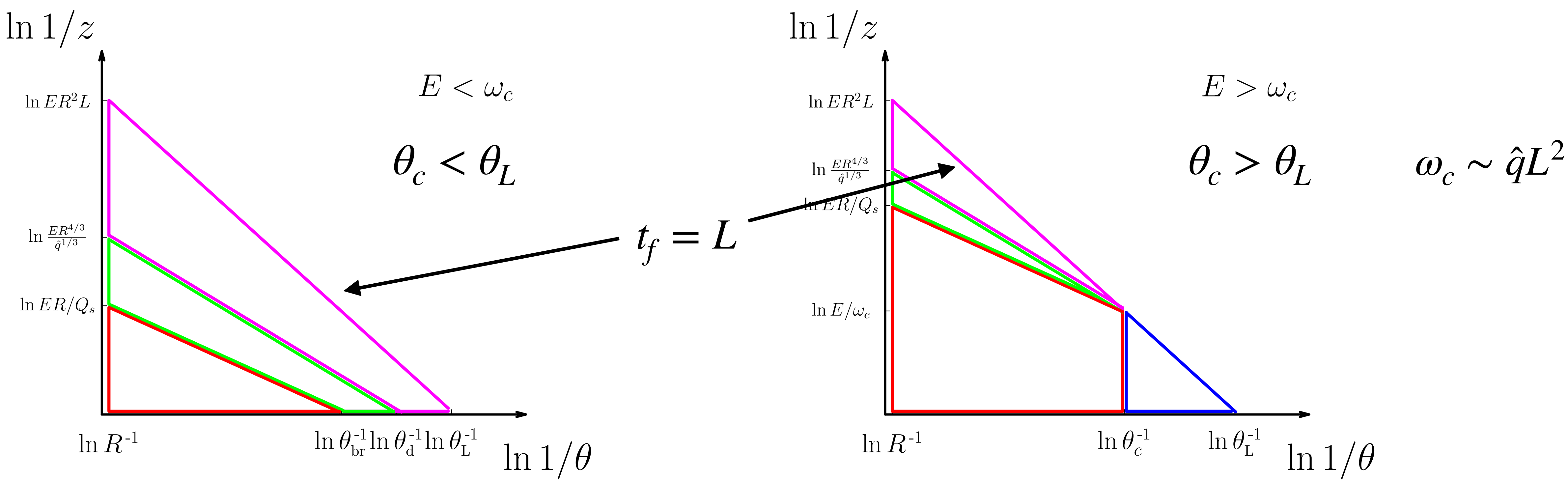
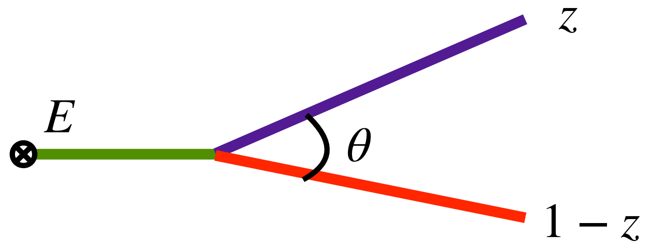
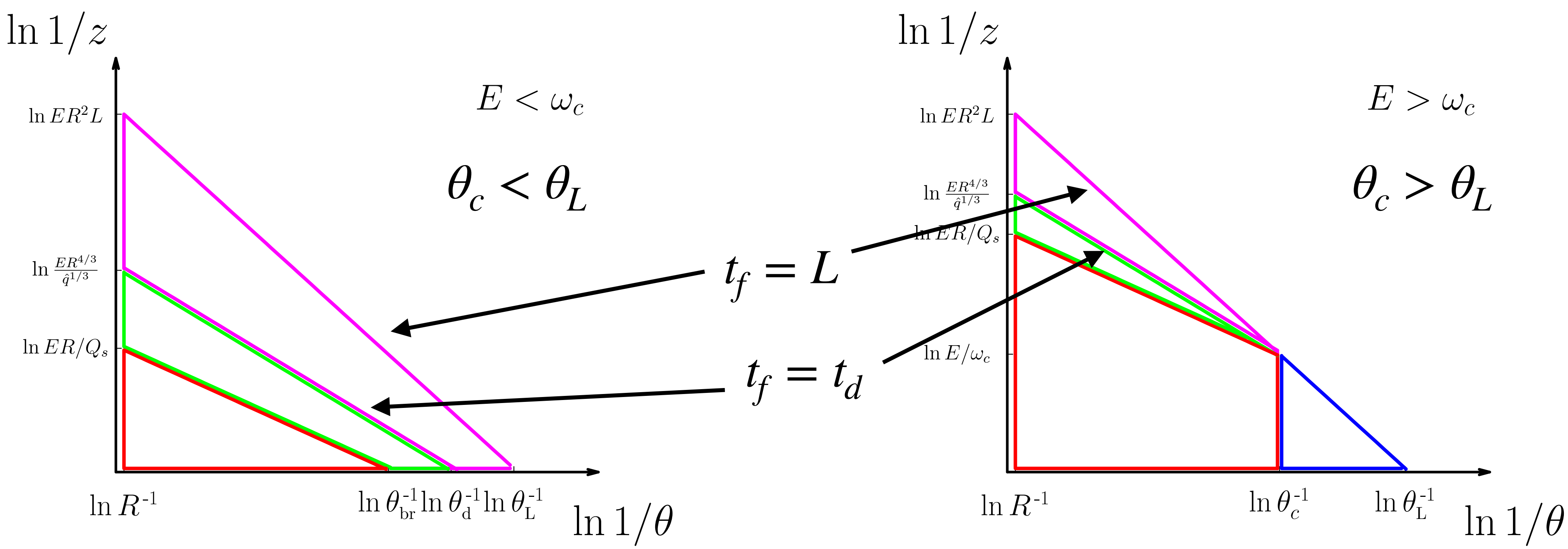
FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653





Lund plane

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653



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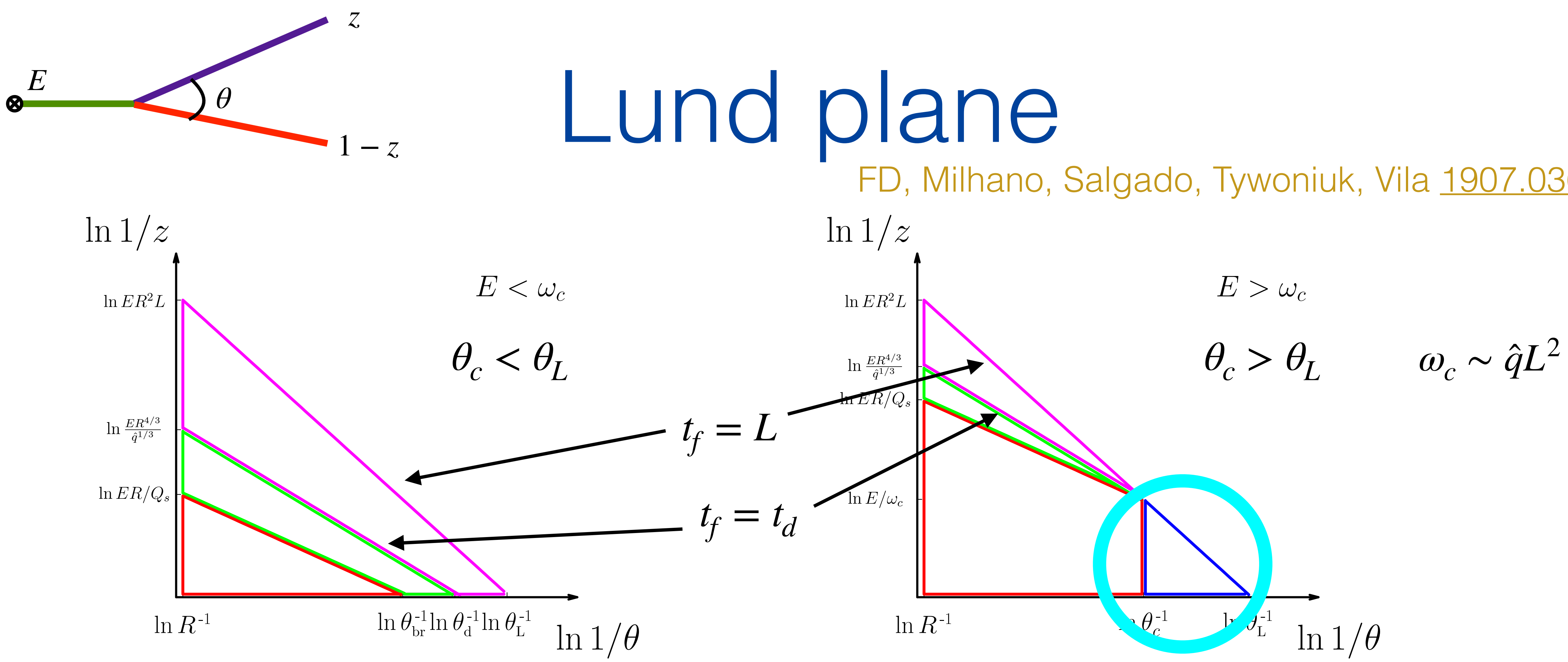

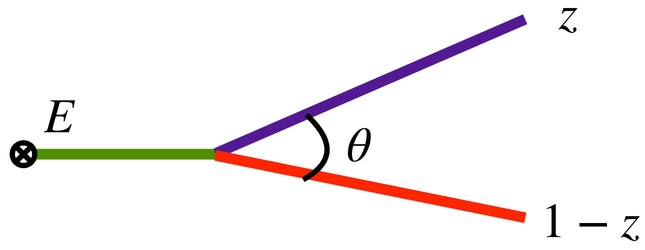

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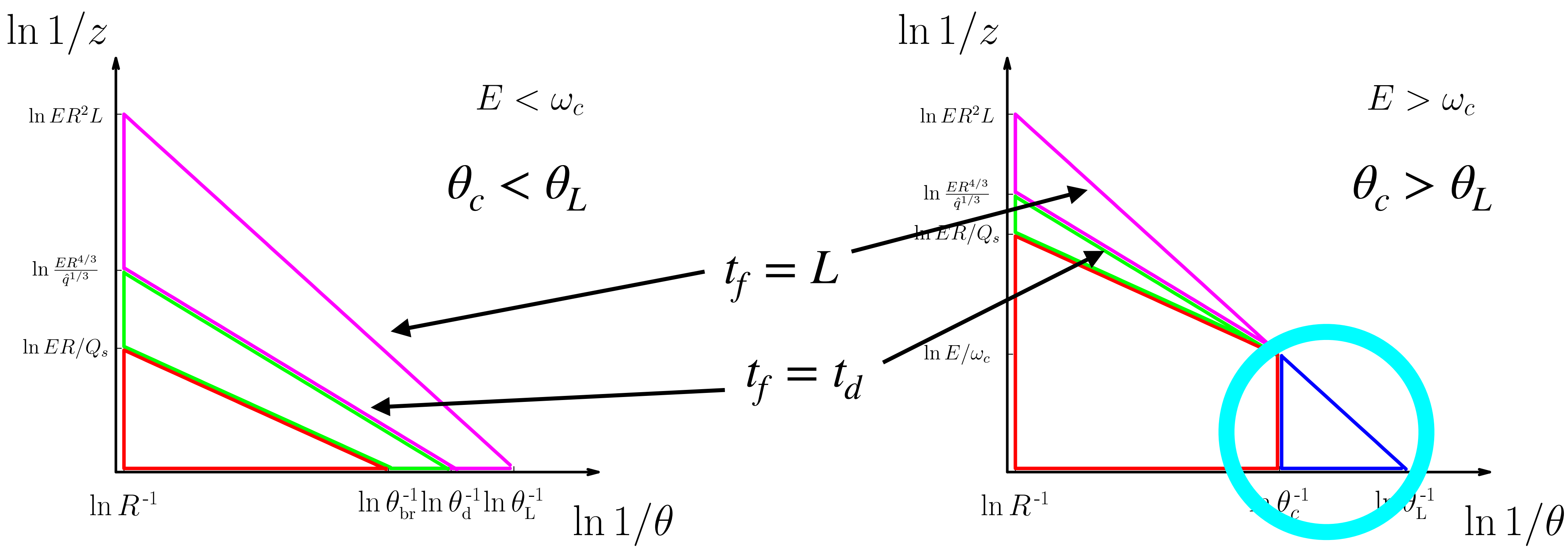
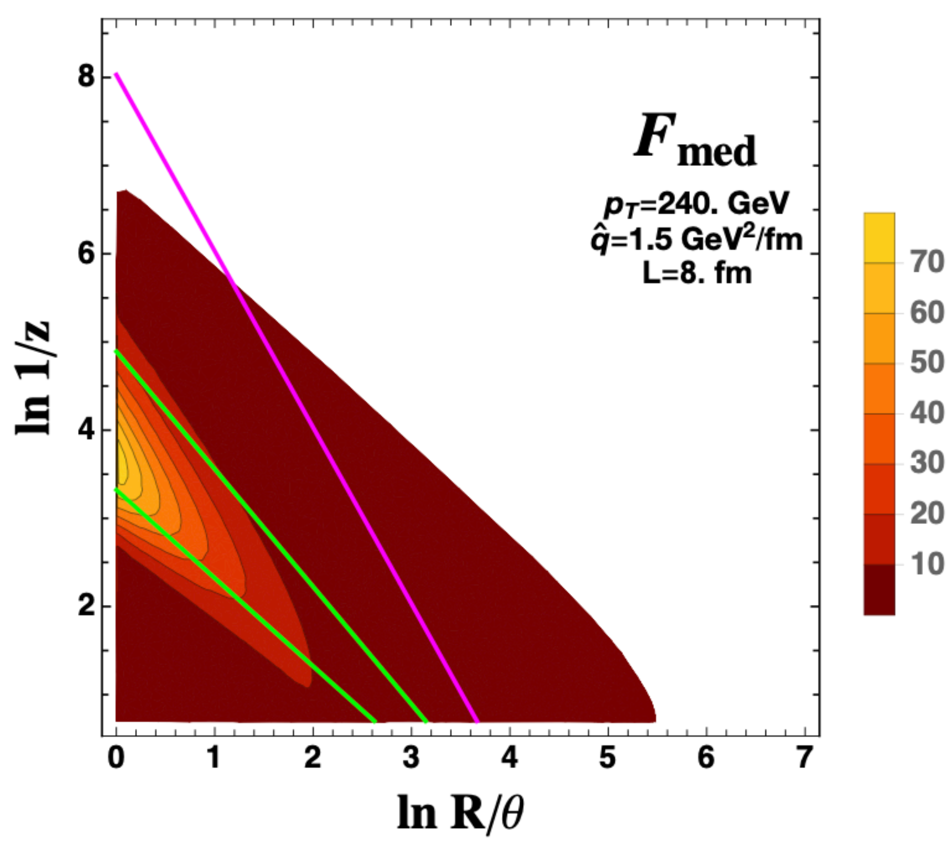
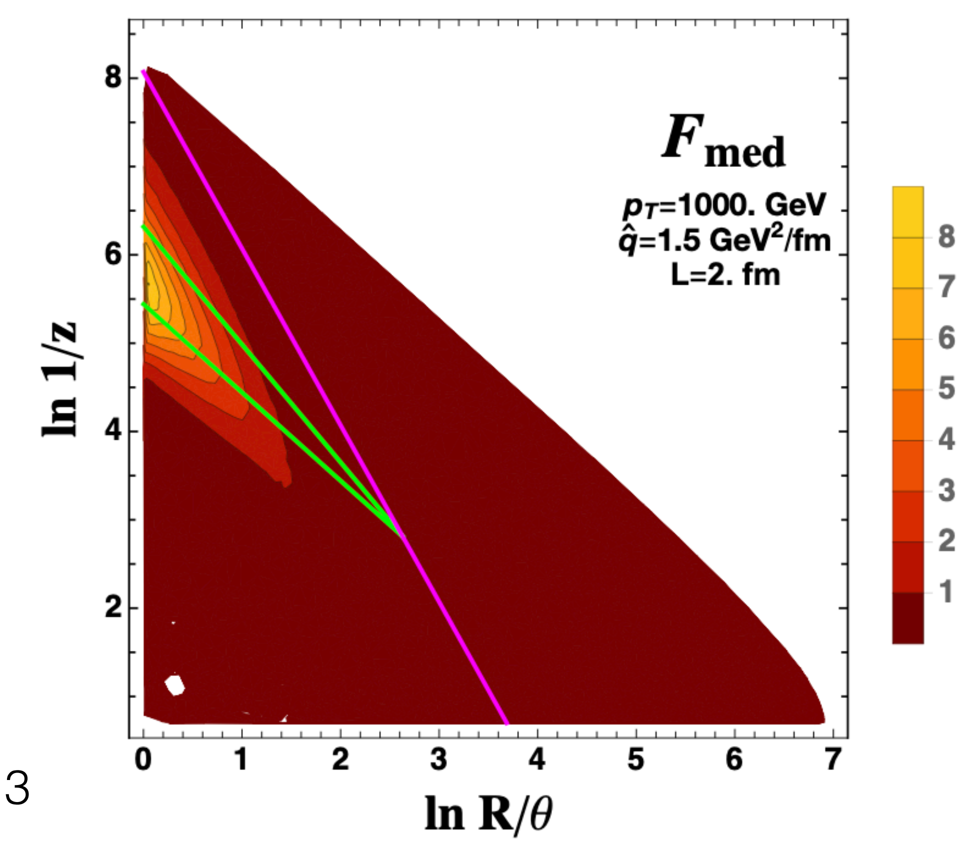


$\omega_c \sim \hat{q}L^2$

$$\frac{d\sigma_{qg}}{d\theta dz} = (1 + F_{\text{med}}(z,\theta))\,\frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz}$$





$\gamma \to q\bar{q}$

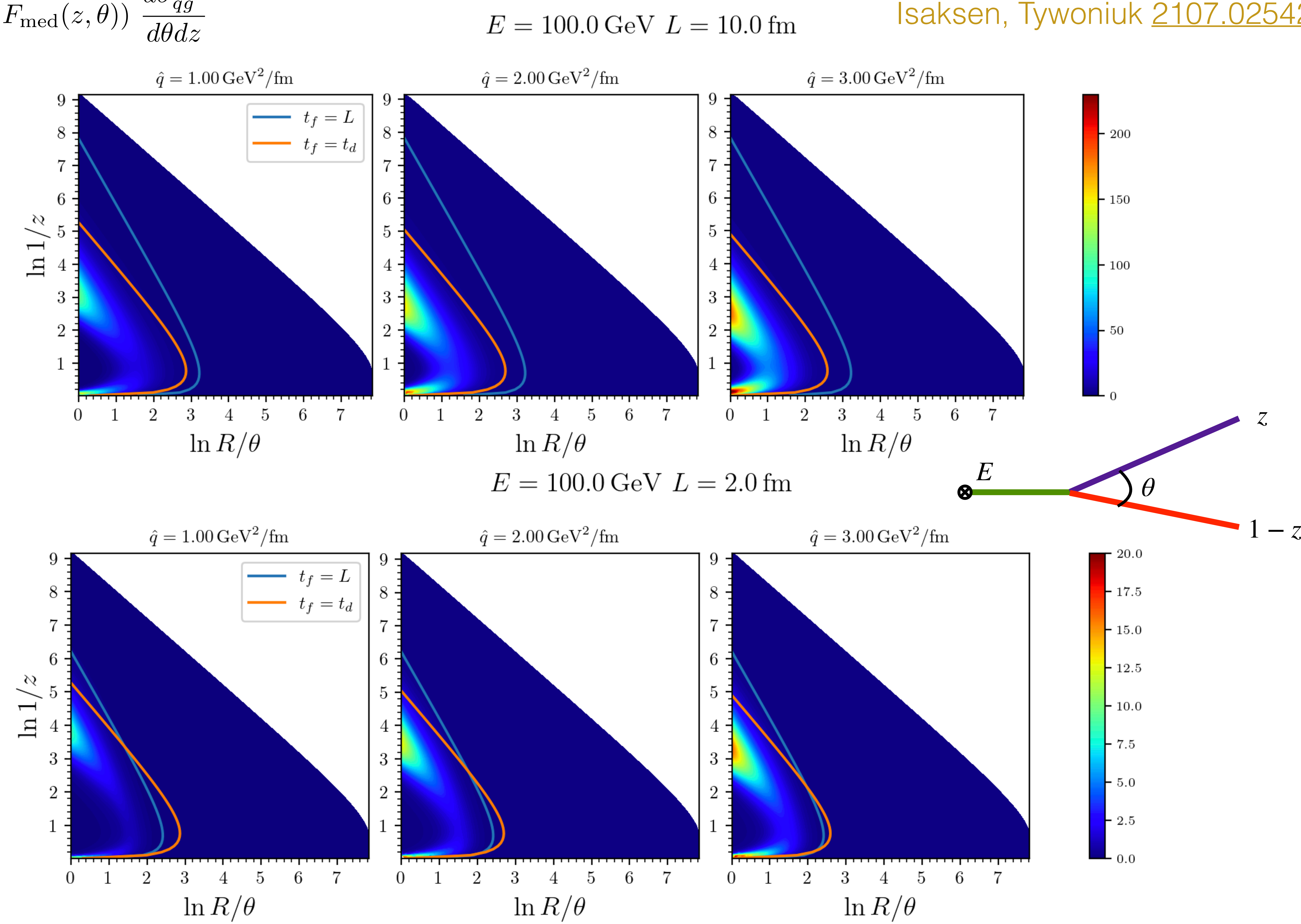
Lund planes $q \to qg$

$$\frac{d\sigma_{qg}}{d\theta dz} = (1 + F_{\mathrm{med}}(z,\theta))\ \frac{d\sigma_{qg}^{\mathrm{vac}}}{d\theta dz}$$

Isaksen, Tywoniuk 2107.02542

$\theta_c < \theta_L$

$\theta_c > \theta_L$



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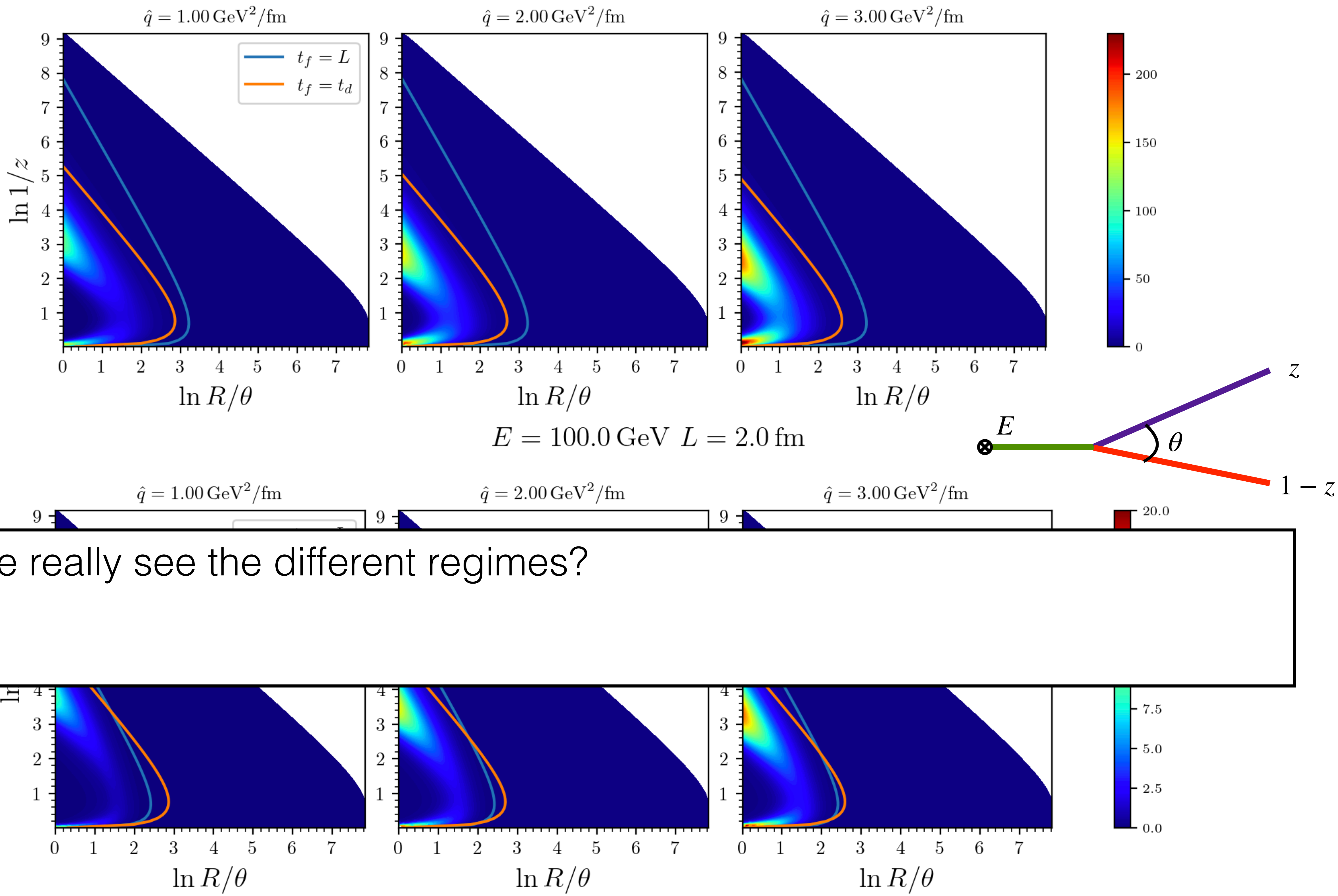
Isaksen, Tywoniuk 2107.02542

$E = 100.0\,\mathrm{GeV}\;\; L = 10.0\,\mathrm{fm}$

$\theta_c < \theta_L$



- Can we really see the different regimes?

$\theta_c > \theta_L$

Lund planes $q \to qg$

$$\frac{d\sigma_{qg}}{d\theta dz} = (1 + F_{\rm med}(z,\theta))\ \frac{d\sigma^{\rm vac}_{qg}}{d\theta dz}$$

Isaksen, Tywoniuk [2107.02542](https://arxiv.org/abs/2107.02542)

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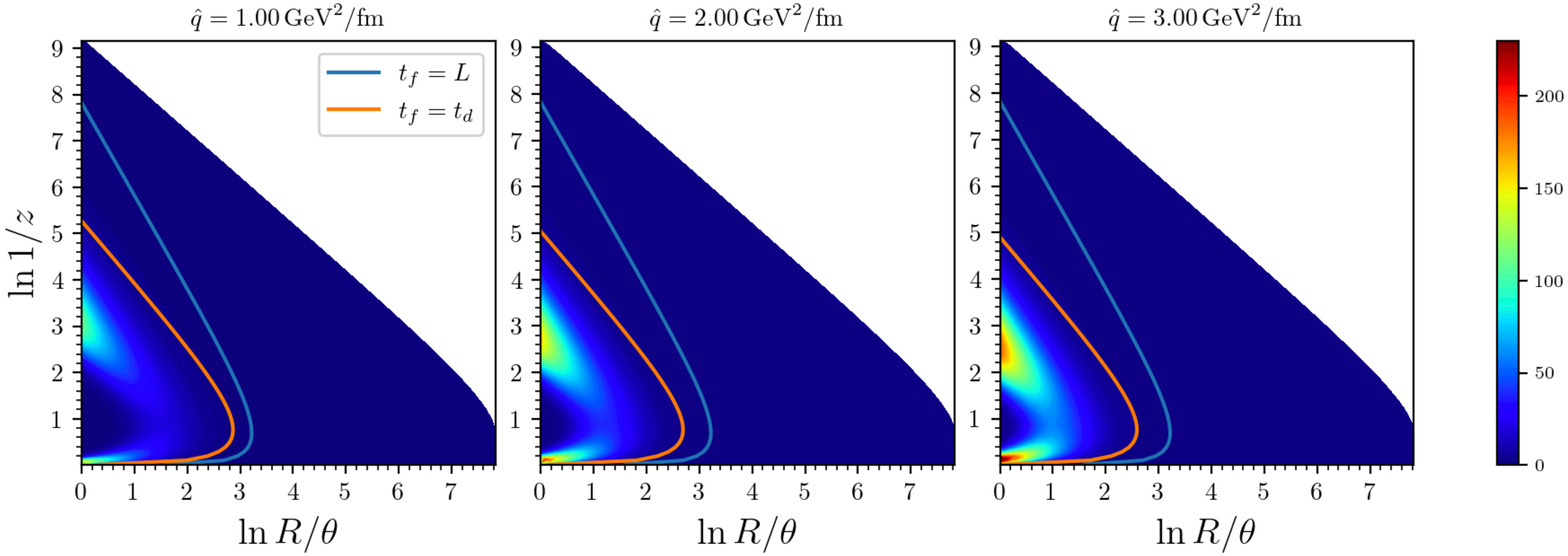
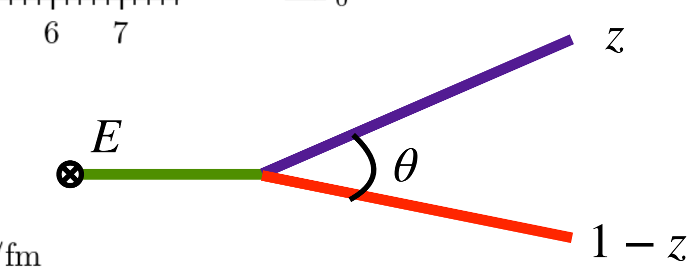
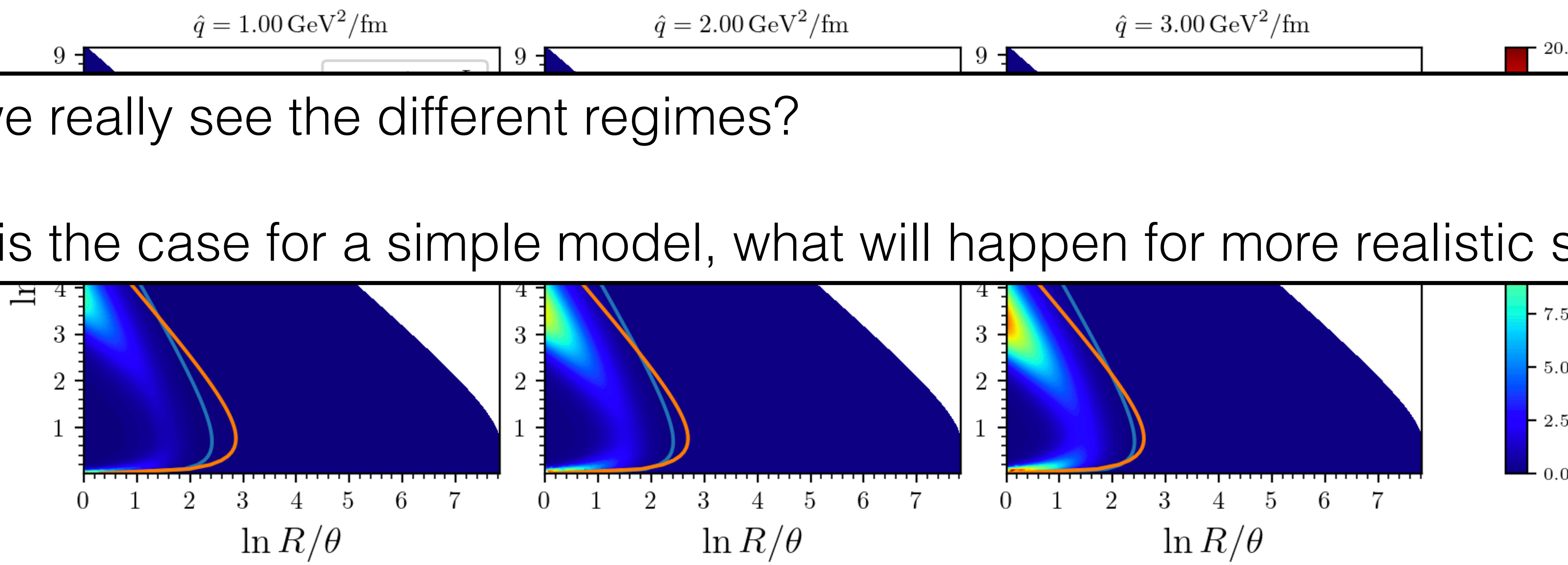




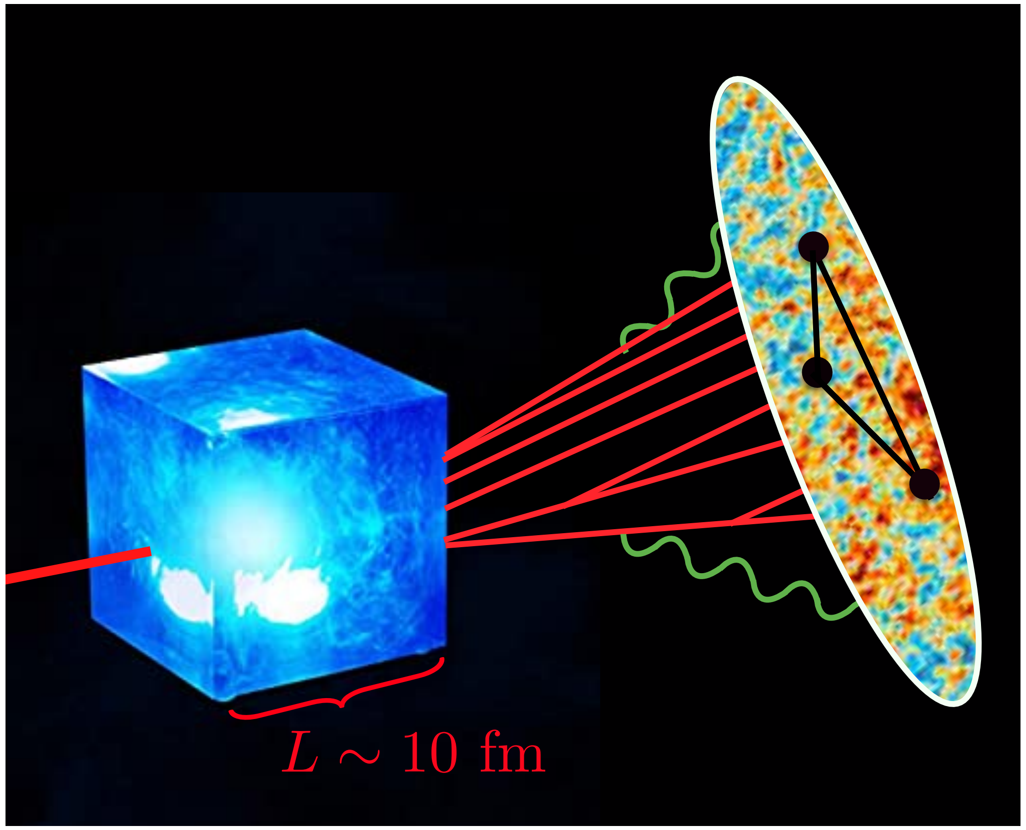
$E = 100.0\,\mathrm{GeV}\ \ L = 2.0\,\mathrm{fm}$

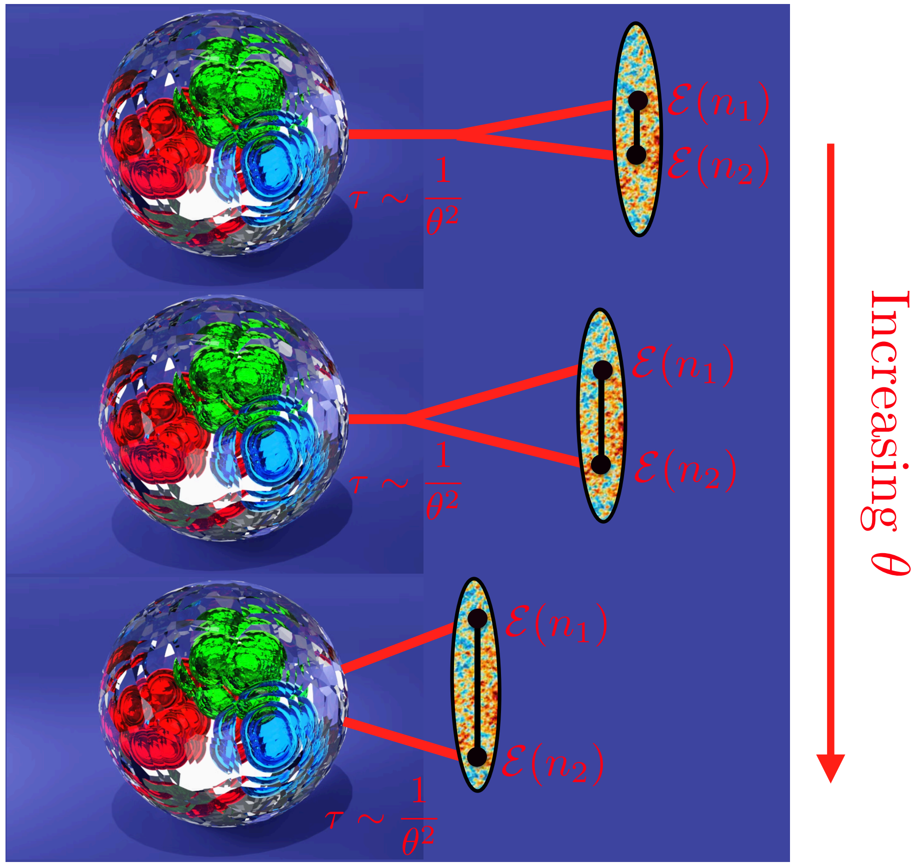
$\theta_c > \theta_L$



- Can we really see the different regimes?
- If this is the case for a simple model, what will happen for more realistic setups?

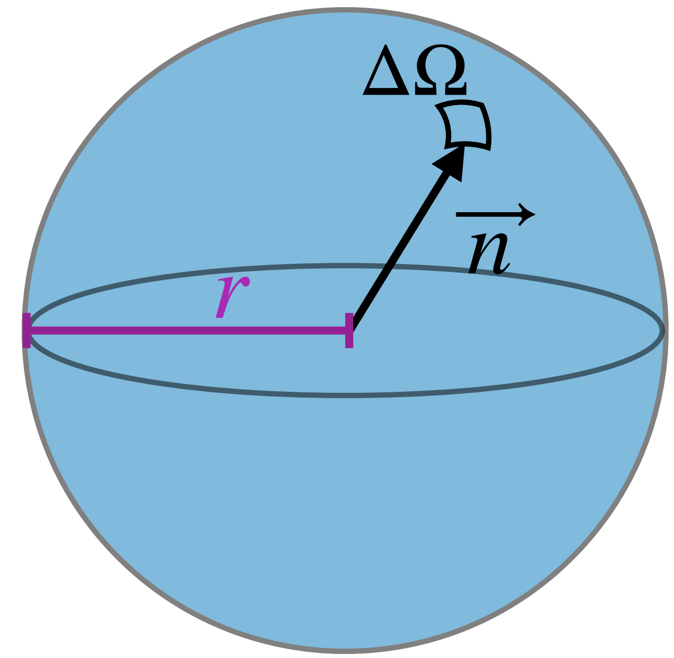
Energy Correlators





Energy flux operators

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\, r^2 n^i T_{0i}(t, r\vec{n})$$



- The 1-point function measures the total energy flux through an area element

$$\langle \mathcal{E}(\vec{n}) \rangle \propto \sum_i E_i$$

Sum over all particles going through $\boldsymbol{\Delta\Omega}$

- Energy weighting naturally removes soft physics without grooming

D. Hoffman, J. Maldacena 0803.1467

Energy correlators

$$\frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{d\sigma_{ij}}{d\vec{n}_i d\vec{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\vec{n}_i - \vec{n}_1)\delta^{(2)}(\vec{n}_j - \vec{n}_2)$$

Energy correlators

- 2-point function

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Inclusive cross section to produce two particles

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Hard scale of the process

Energy correlators

- 2-point function

Inclusive cross section to produce two particles

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Hard scale of the process

- As a function of the relative angle only

$$\frac{d\Sigma^{(n)}}{d\theta} = \int d\vec{n}_{1,2}\frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}}\delta(\vec{n}_2\cdot\vec{n}_1-\cos\theta)$$

Energy correlators

- 2-point function

Inclusive cross section to produce two particles

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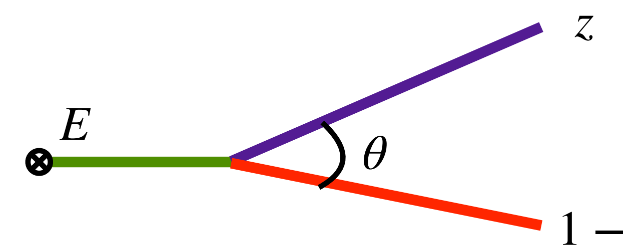
  - Infrared and collinear safe for $n = 1$

Energy correlators



- For a quark jet at first order, $Q = E$ the energy of the jet

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \frac{d\sigma_{qg}}{d\theta dz} z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$

μ_s a softer scale over which the cross section is inclusive

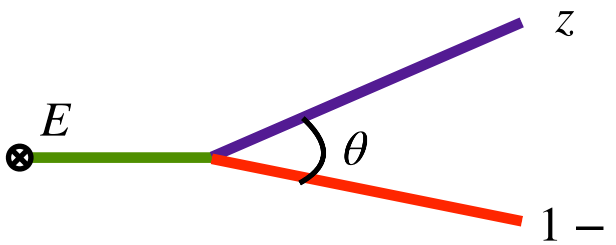
- qq and gg contributions are higher order

- Additional energy loss ($E_q + E_g \neq E$) is also subleading

$$z = \frac{E_g}{E}$$

Energy correlators in vacuum

D. Hoffman, J. Maldacena 0803.1467
H. Chen , I. Moult, J. Sandor, H. X. Zhu 2202.04085



- At leading order

$$\frac{d\sigma_{qg}^{\mathrm{vac}}}{d\theta dz} = \frac{\alpha_s C_{\mathrm{F}} \sigma}{\pi} \frac{1+(1-z)^2}{z\ \theta} \quad \Rightarrow \quad \frac{d\Sigma^{(1)}}{d\theta} \sim \frac{1}{\theta}$$

- Collinear emissions can be resummed using CFT techniques changing the scaling only by an anomalous dimension

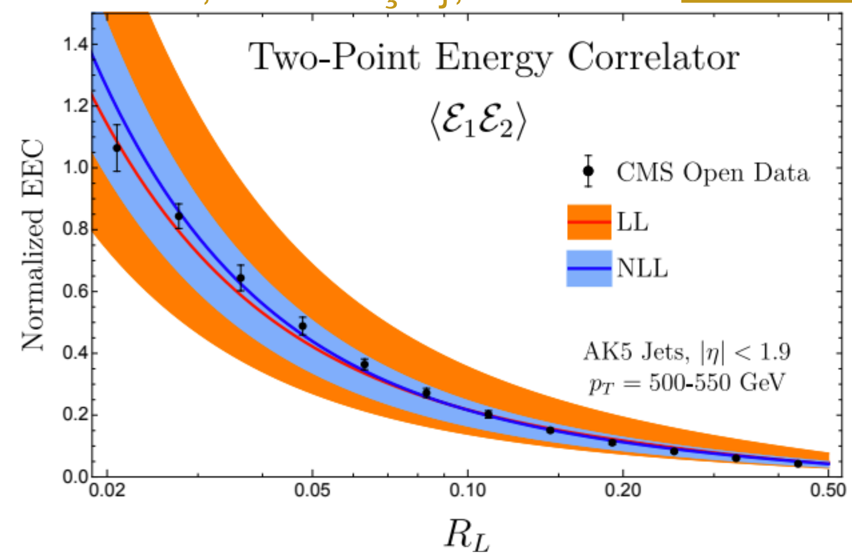
$$\frac{d\Sigma^{(1)}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

$\gamma(3)$ is the twist-2 spin-3 QCD anomalous dimension

- Higher-orders, soft physics, quark/gluon ratios can change the overall normalization but not the power-law behavior

Energy correlators in vacuum

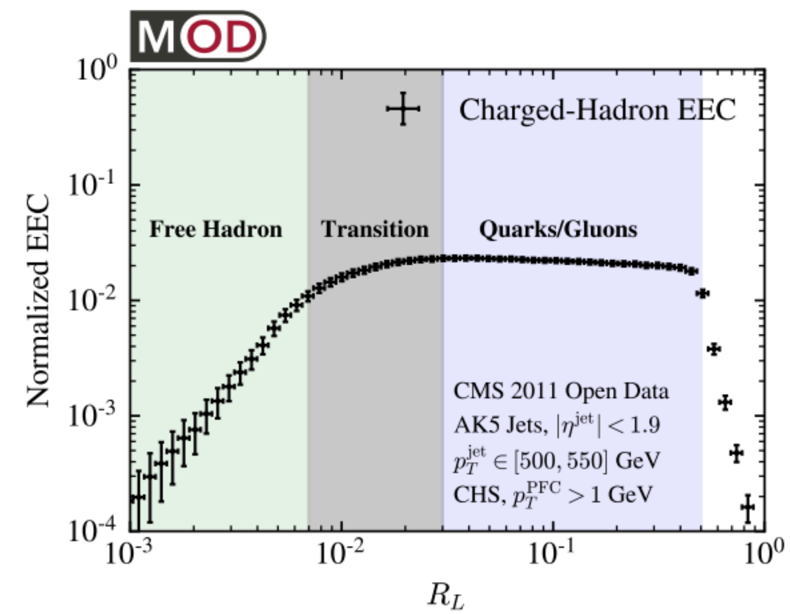
K. Lee, B. Meçaj, I. Moult 2205.03414



- Have not yet been measured

- Analyses done by theorist with CMS open data

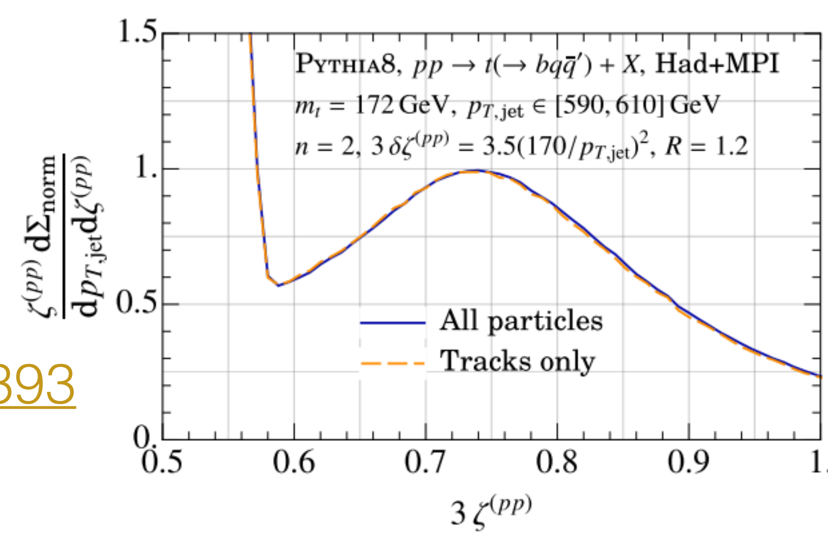
P. T. Komiske, I. Moult, J. Thaler, H. X. Zhu 2201.07800



- Sensitivity to hadronization transition

- Sensitivity to top mass in the 3-point function

J. Holguin, J. Thaler, A. Pathak, M. Procura 2201.08393

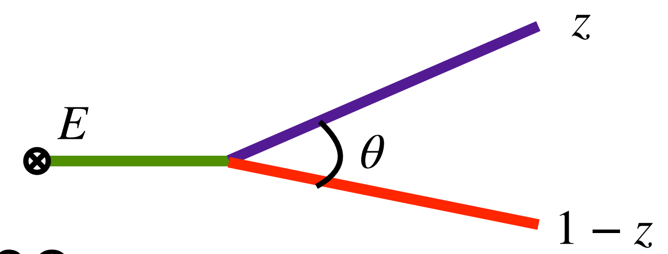


Energy correlators in HIC

- Background is expected to be less of an issue
  - ✦ Energy weighting removes most of the soft physics
  - ✦ Uncorrelated background does not affect the shape of the correlations, only the normalization

- Observables are not event-by-event
  - ✦ Fluctuations are less important
  - ✦ Requires large statistics
  - ✦ Cannot be used to tag events

Energy correlators in HIC



- We take the cross section from the tilted Wilson line evaluation

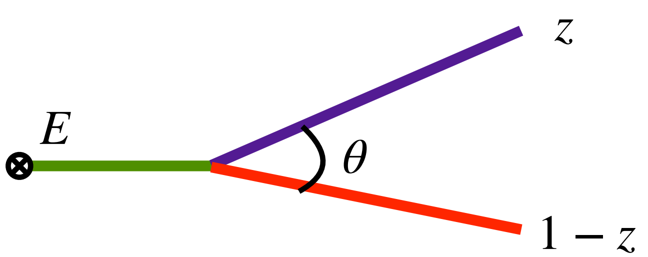
$$\frac{d\sigma_{qg}}{d\theta dz} = (1 + F_{\mathrm{med}}(z,\theta))\,\frac{d\sigma_{qg}^{\mathrm{vac}}}{d\theta dz} \qquad\qquad F_{\mathrm{med}}(z,\theta) \xrightarrow{\theta<\theta_L} 0$$

- We do not expect medium modification at small angles, thus vacuum collinear resummation should still be valid

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \left(g^{(n)}(\theta,\alpha_s) + F_{\mathrm{med}}(z,\theta)\right) \frac{d\sigma_{qg}^{\mathrm{vac}}}{d\theta dz}$$
$$\times\, z^n (1-z)^n \left(1 + \mathcal{O}\left(\alpha_s \ln \theta_L^{-1},\ \frac{\mu_s}{zE}\right)\right) + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$

$$g^{(1)} = \theta^{\gamma(3)} + \mathcal{O}(\theta)$$

Recap of the model



- Three parameters $E, \hat{q}, L$

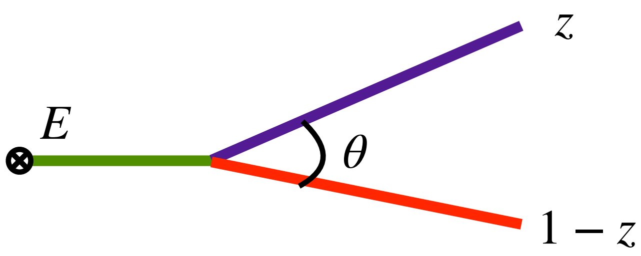
- Two competing angular scales

$$\theta_L \sim (EL)^{-1/2} \qquad\qquad \theta_c \sim (\hat{q}L^3)^{-1/2}$$

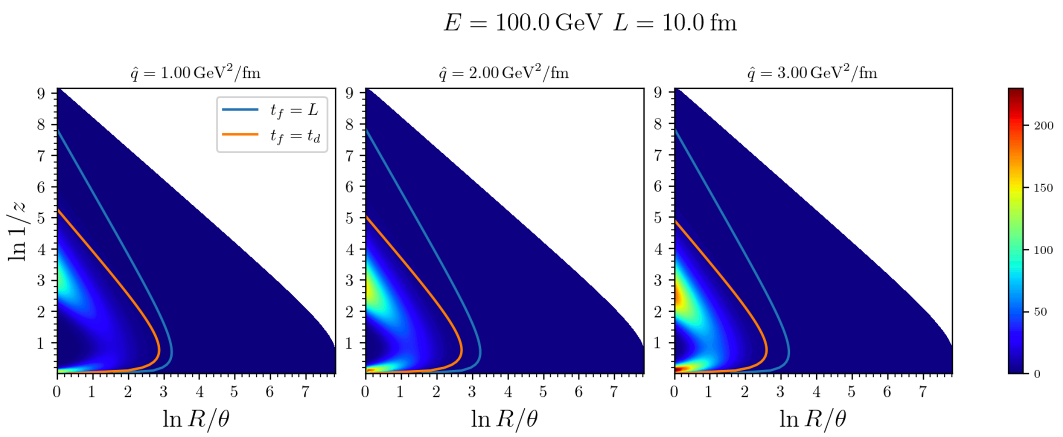
- For $\theta < \theta_L$, splitting occurs outside of the medium, no medium modification is expected

- For $\theta < \theta_c$, the medium does not resolve the splitting, small medium-modification expected

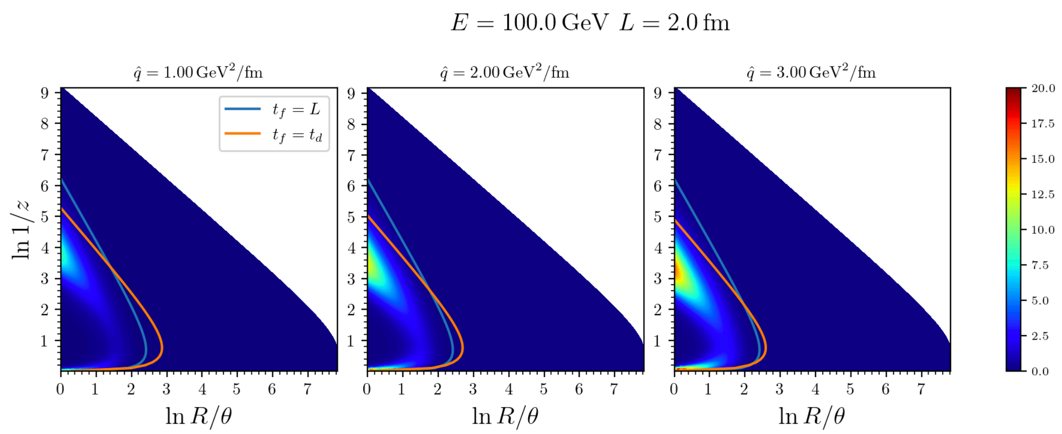
Results



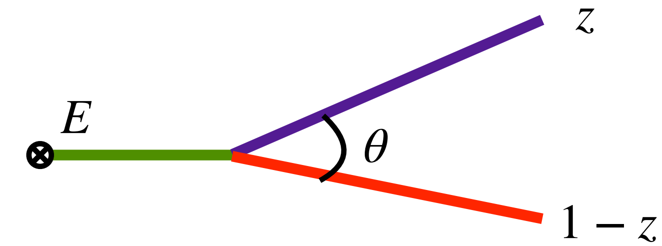
$\theta_c < \theta_L$



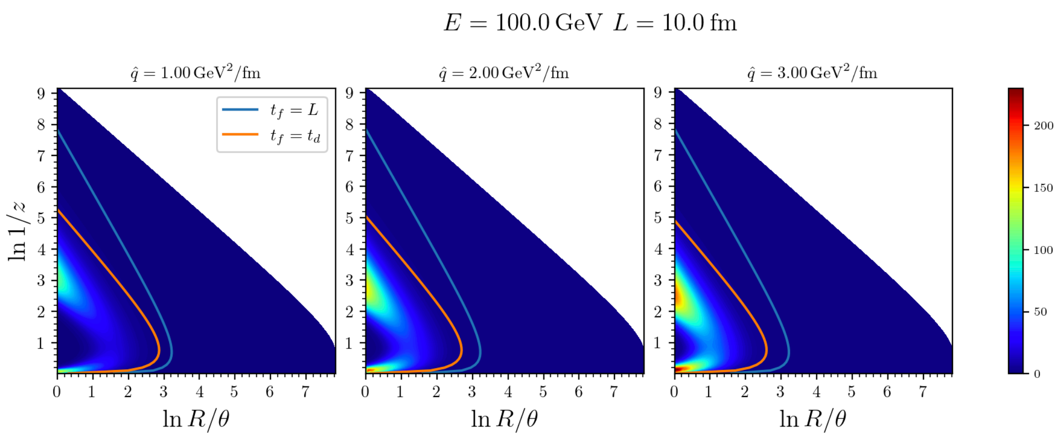
$\theta_c > \theta_L$

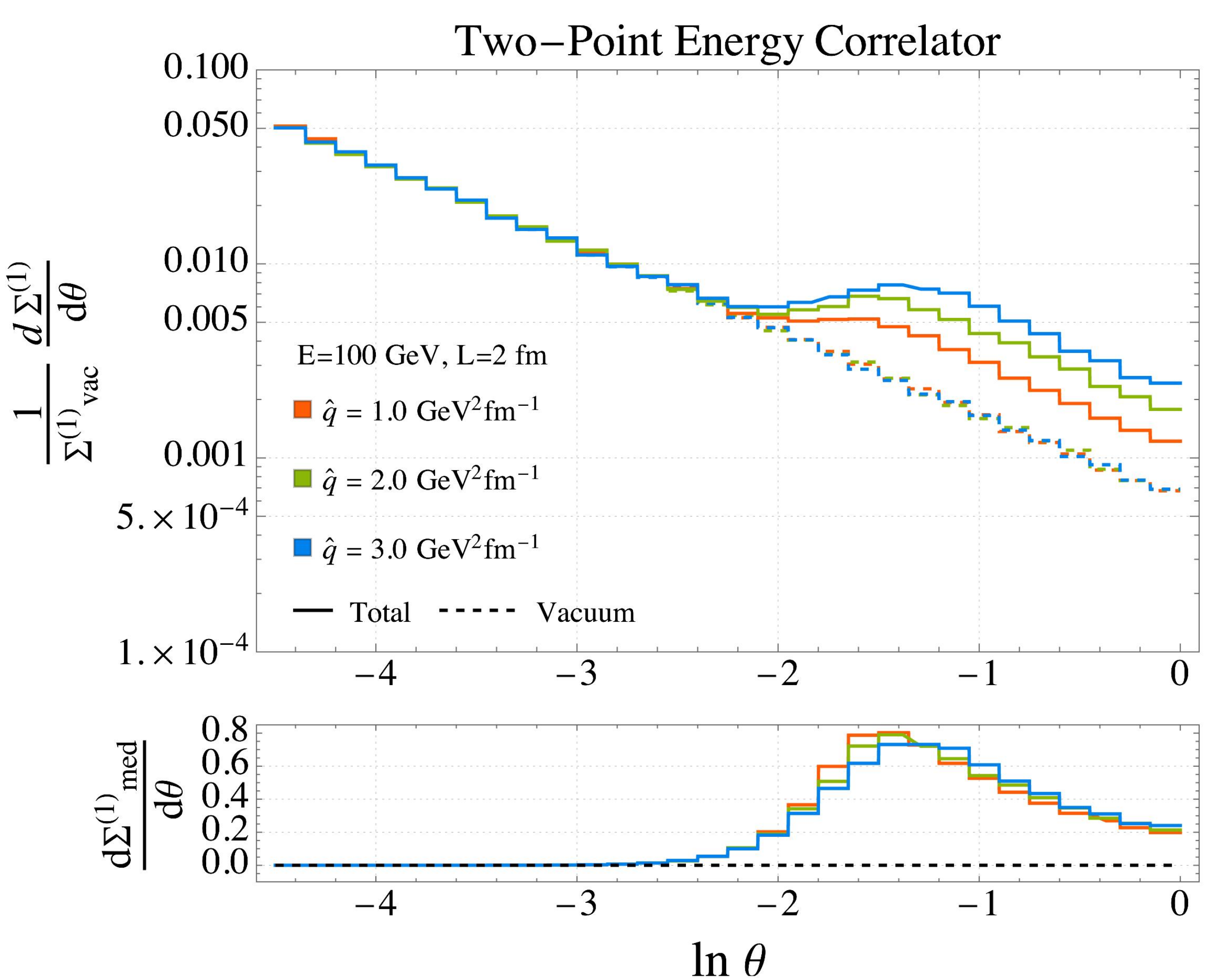


Results

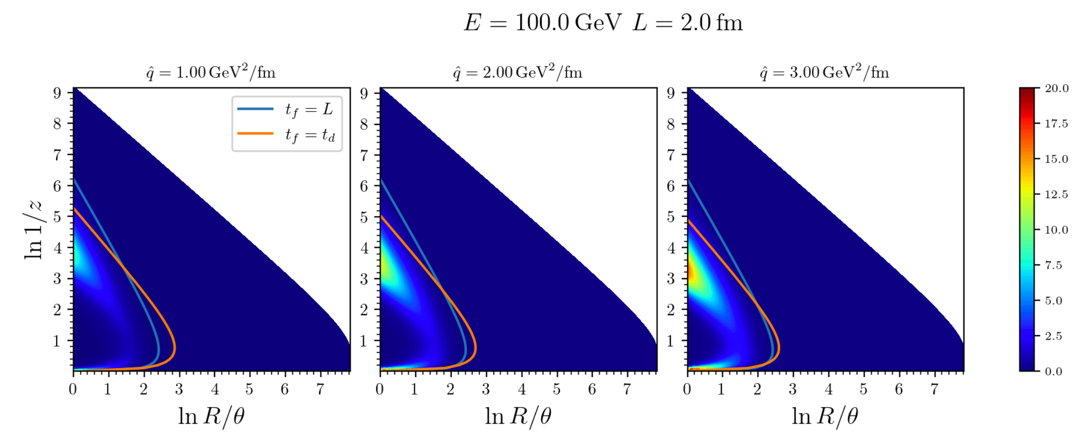


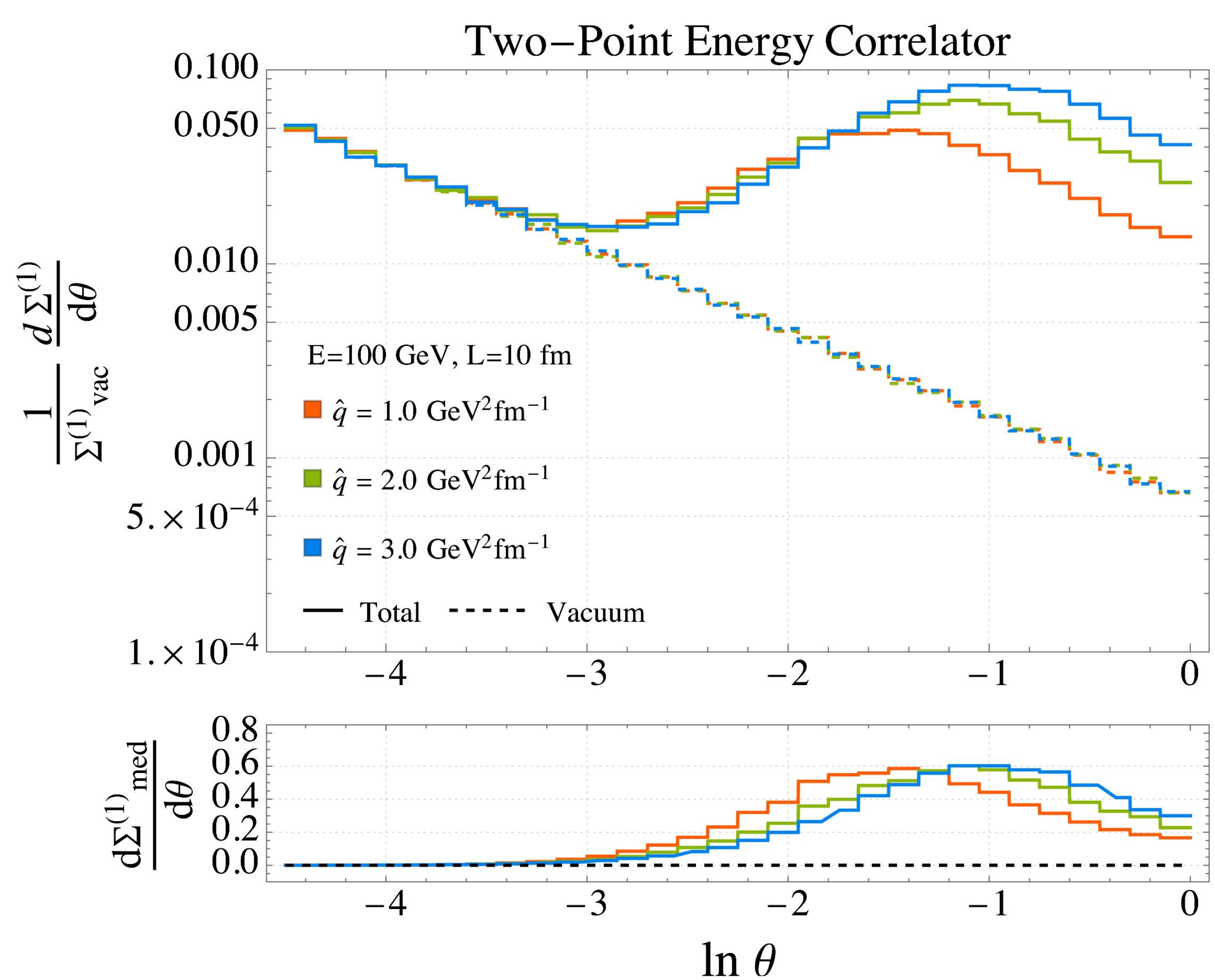
$\theta_c < \theta_L$



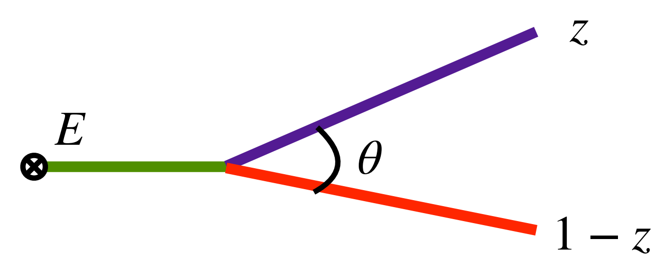


$\theta_c > \theta_L$

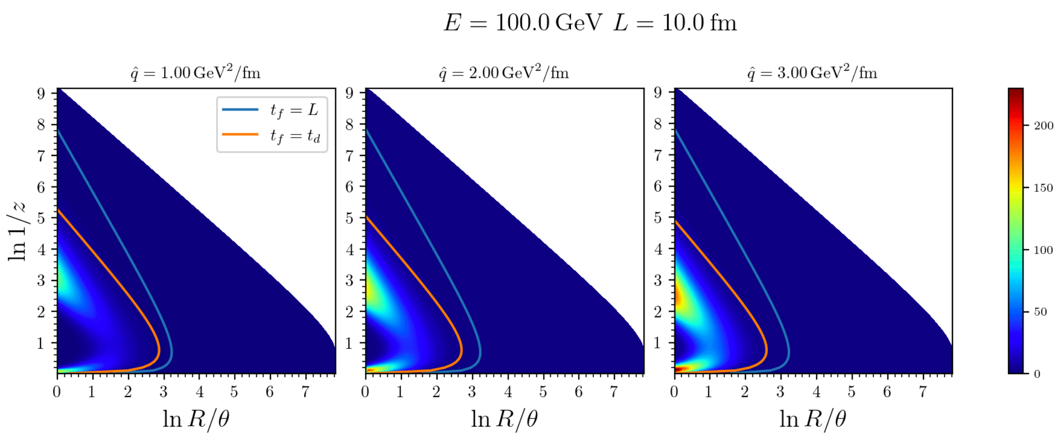


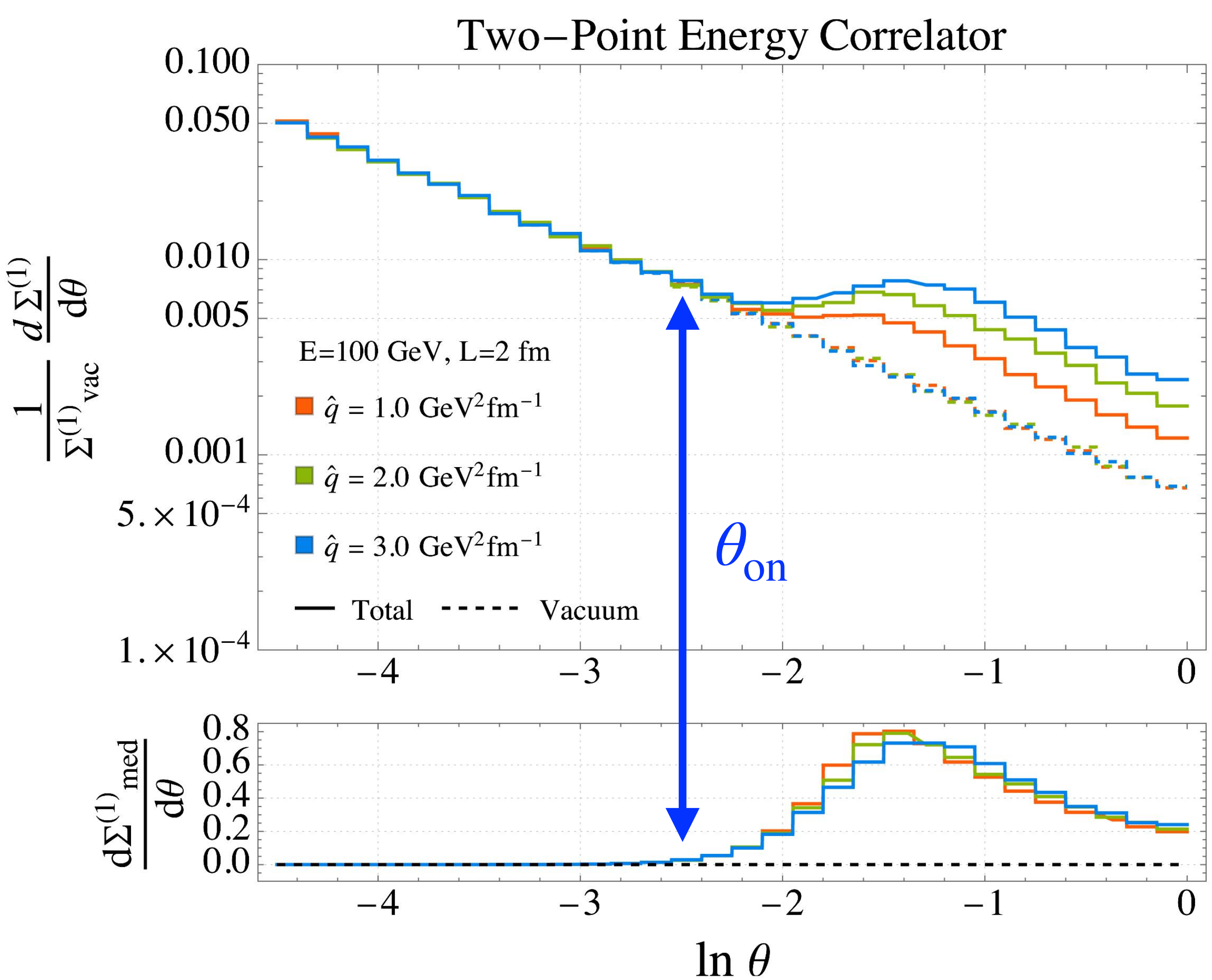


Results

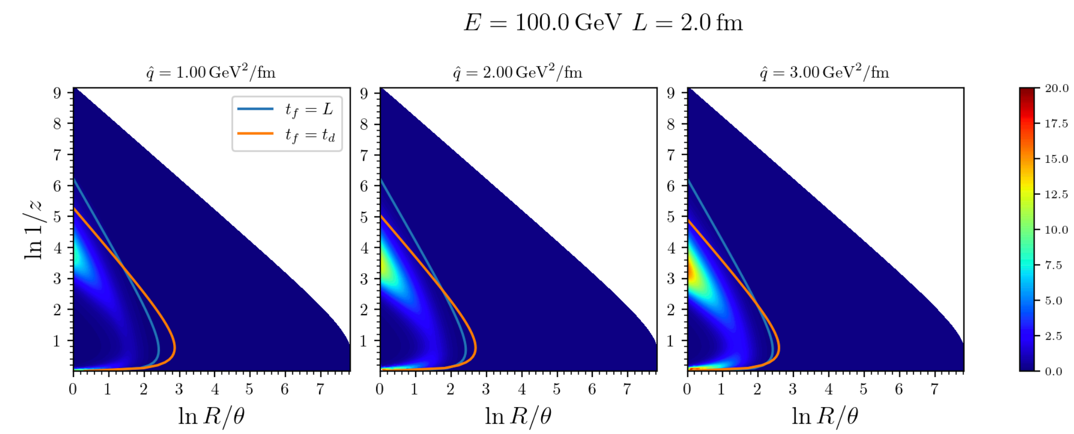


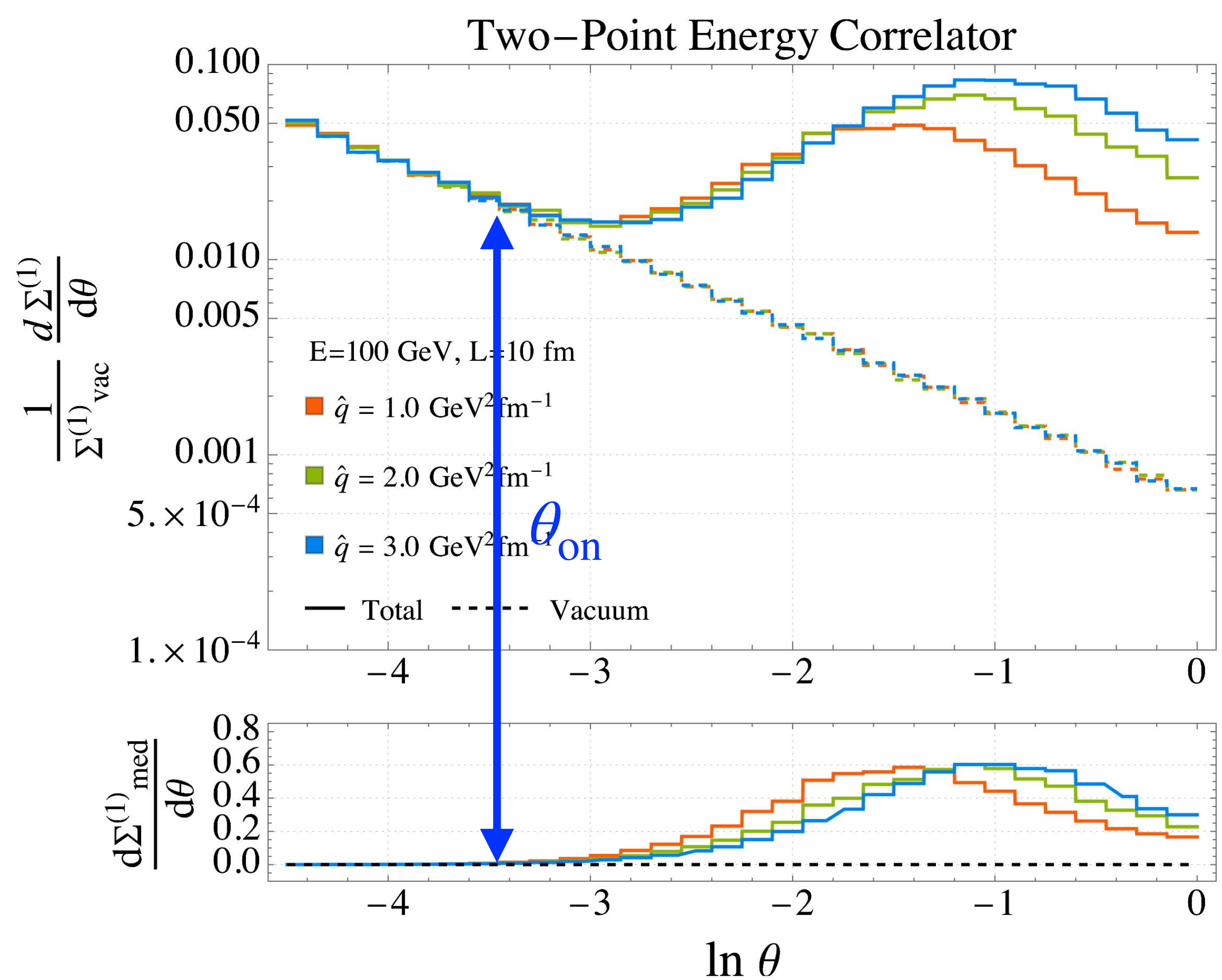
$\theta_c < \theta_L$



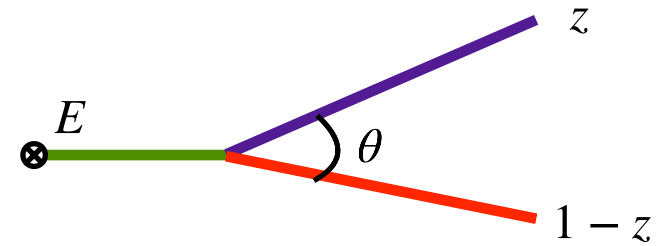


$\theta_c > \theta_L$



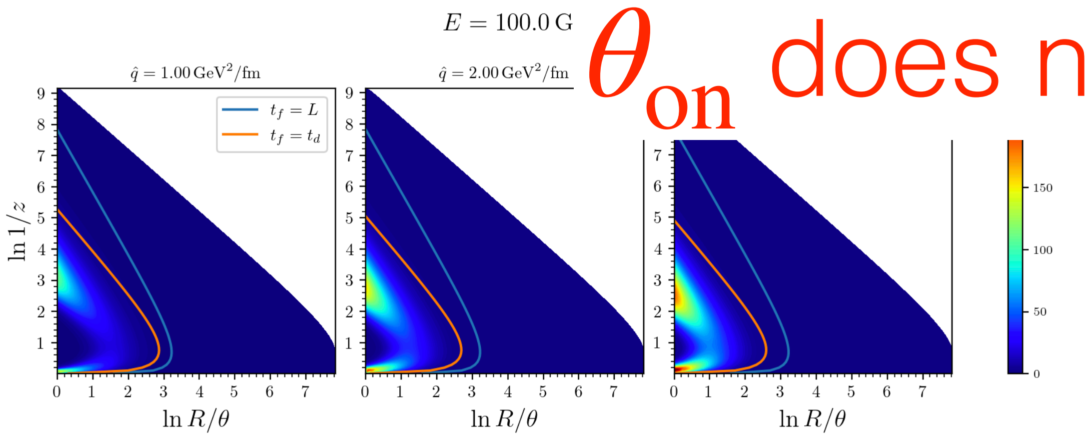


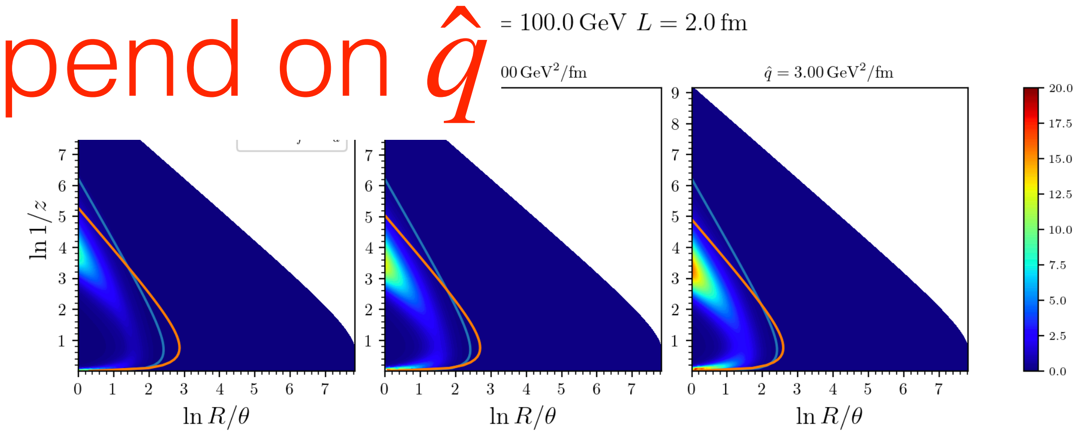
Results

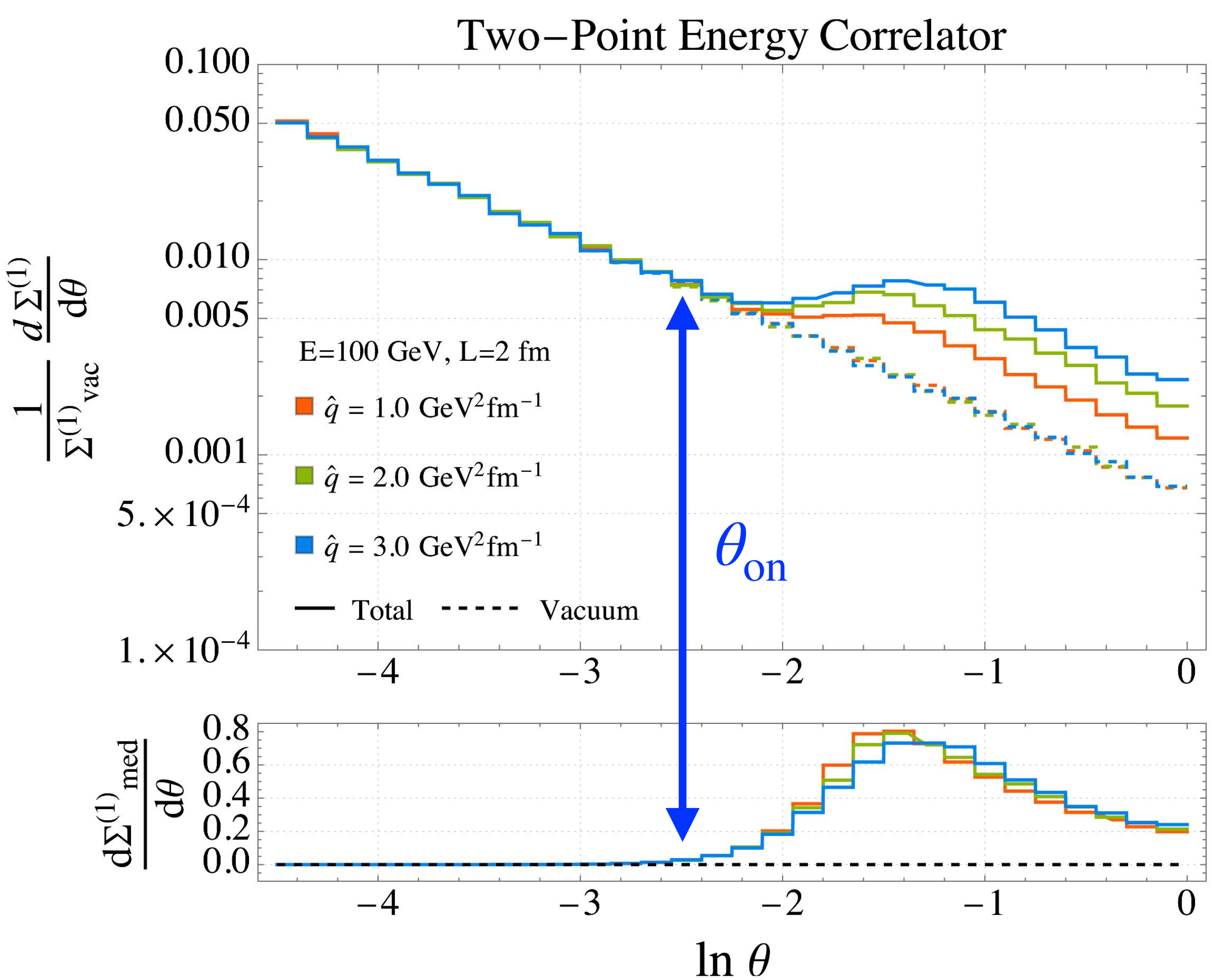


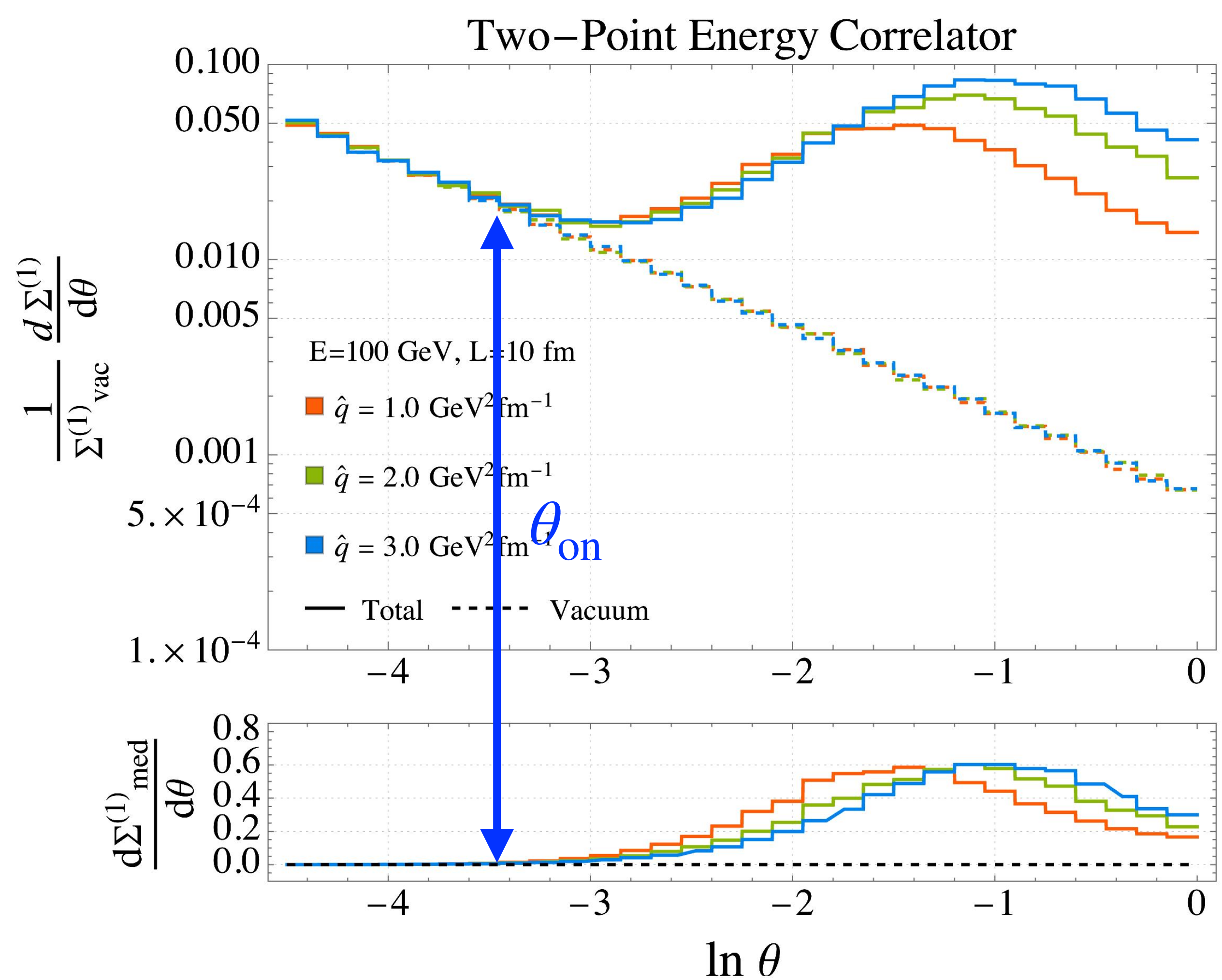
$\theta_c < \theta_L$ | $\theta_c > \theta_L$

θ_{on} does not depend on $\hat{q}$

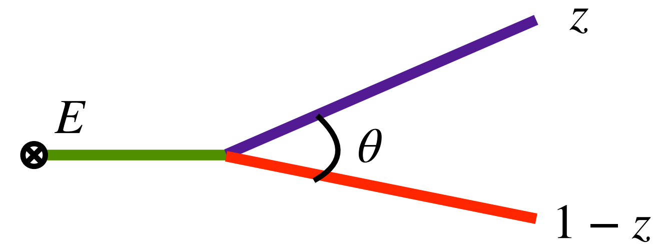






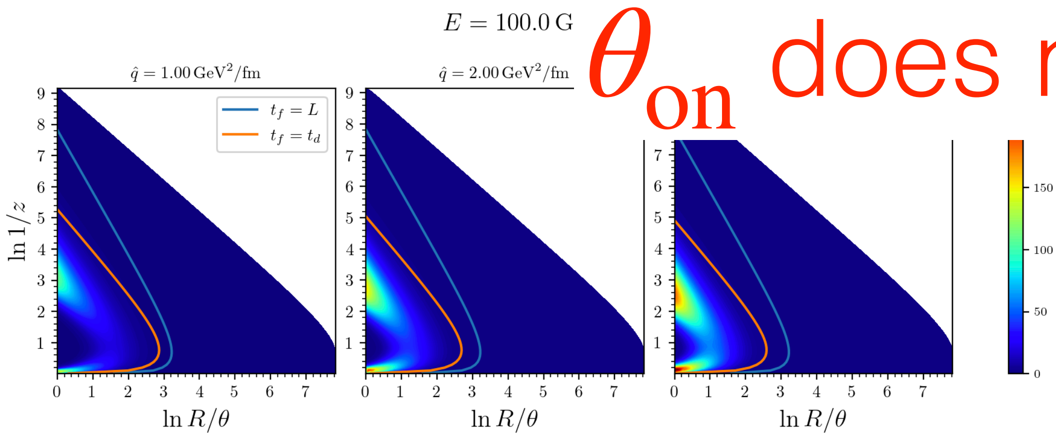


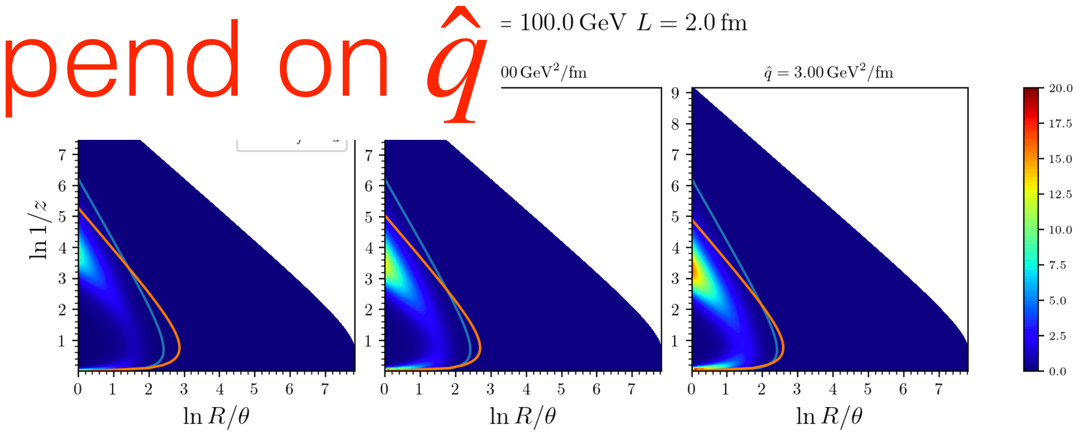
Results

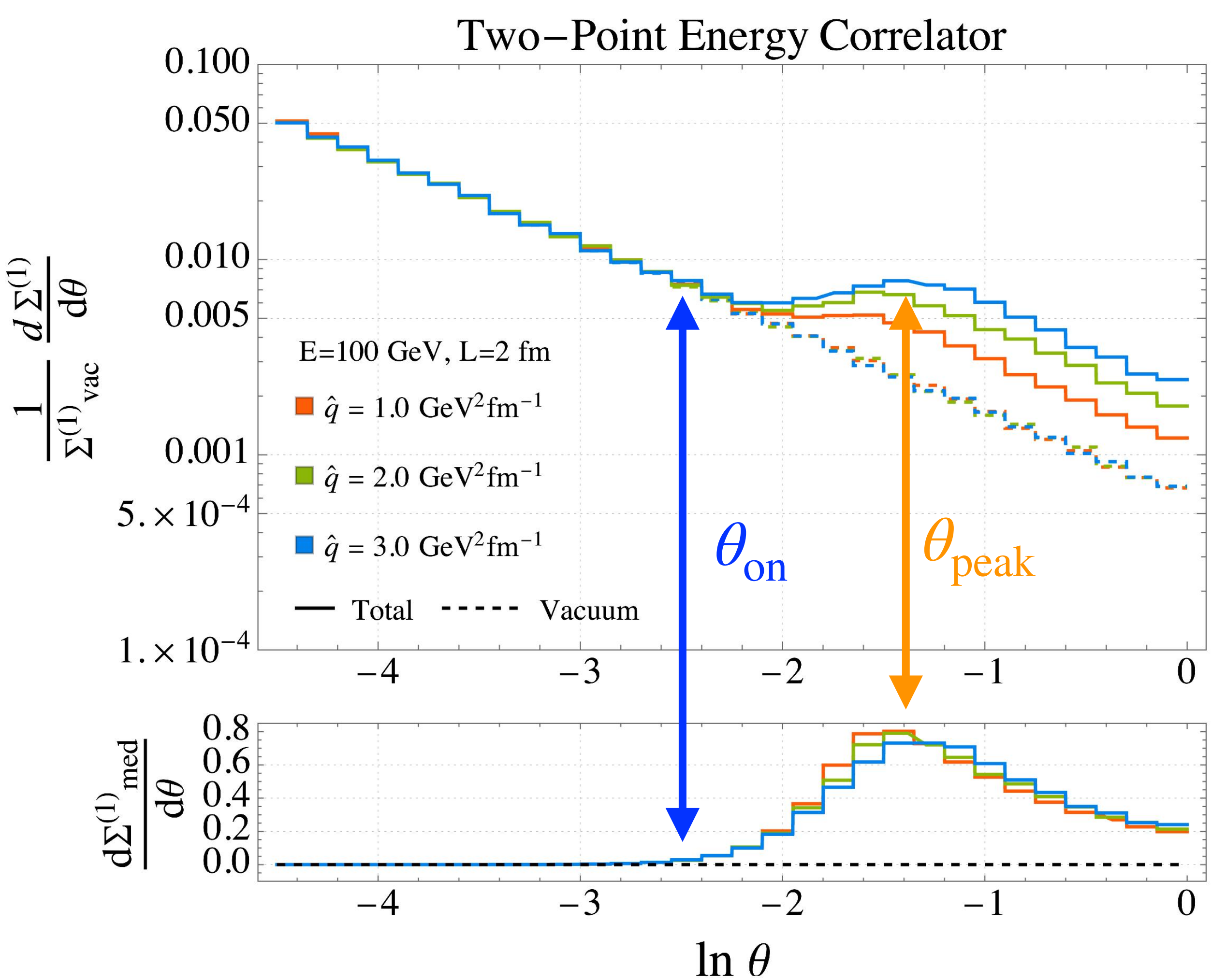


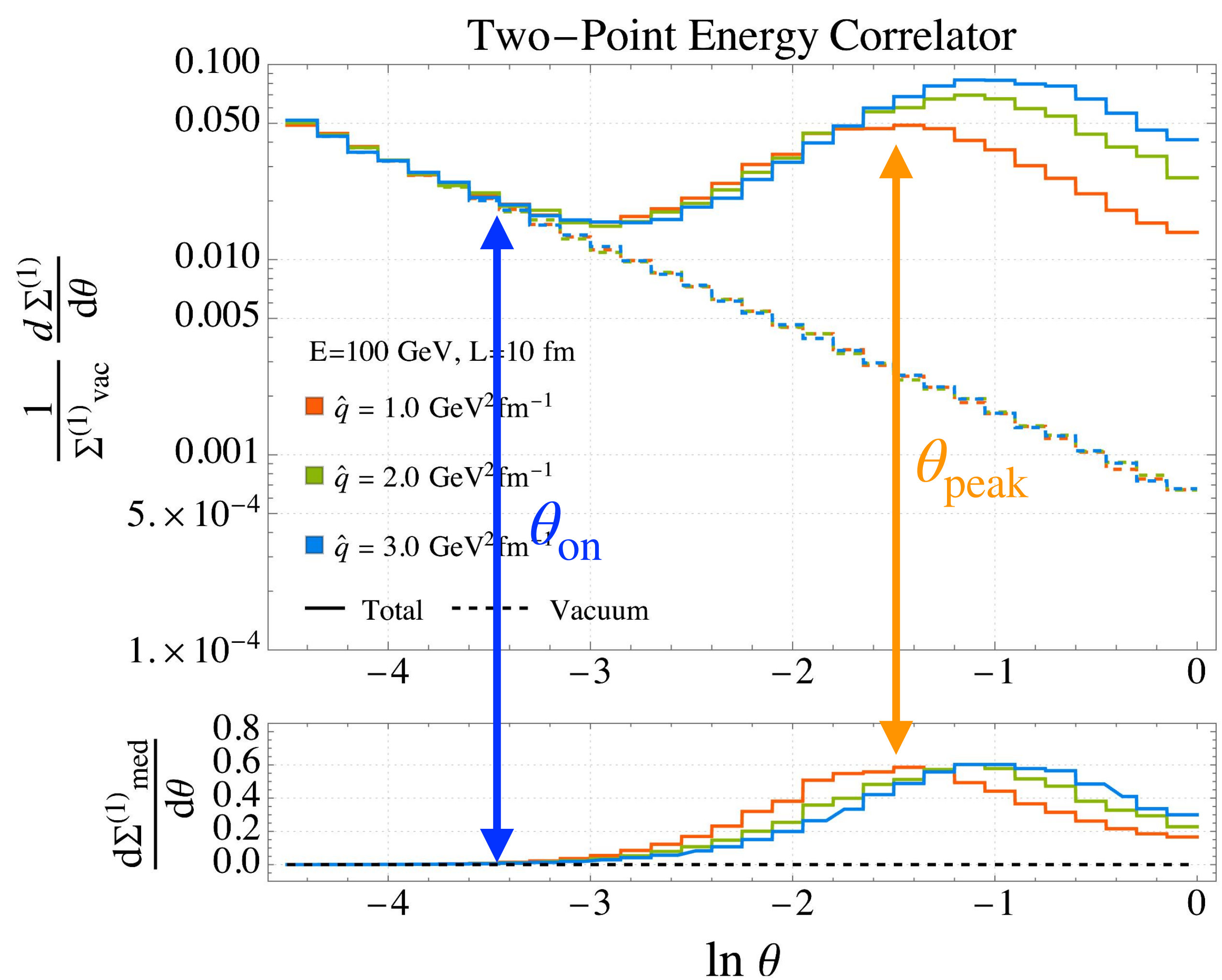
$\theta_c < \theta_L$ $\theta_c > \theta_L$

θ_{on} does not depend on $\hat{q}$









Extracting the behavior of $\theta_{\rm on}$ and $\theta_{\rm peak}$

- Generated the EEC for 248 sets of parameters with $E \in [50{,}700]$ GeV, $L \in [0.2{,}10]$ fm, $\hat{q} \in [1{,}3]$ GeV^2/fm

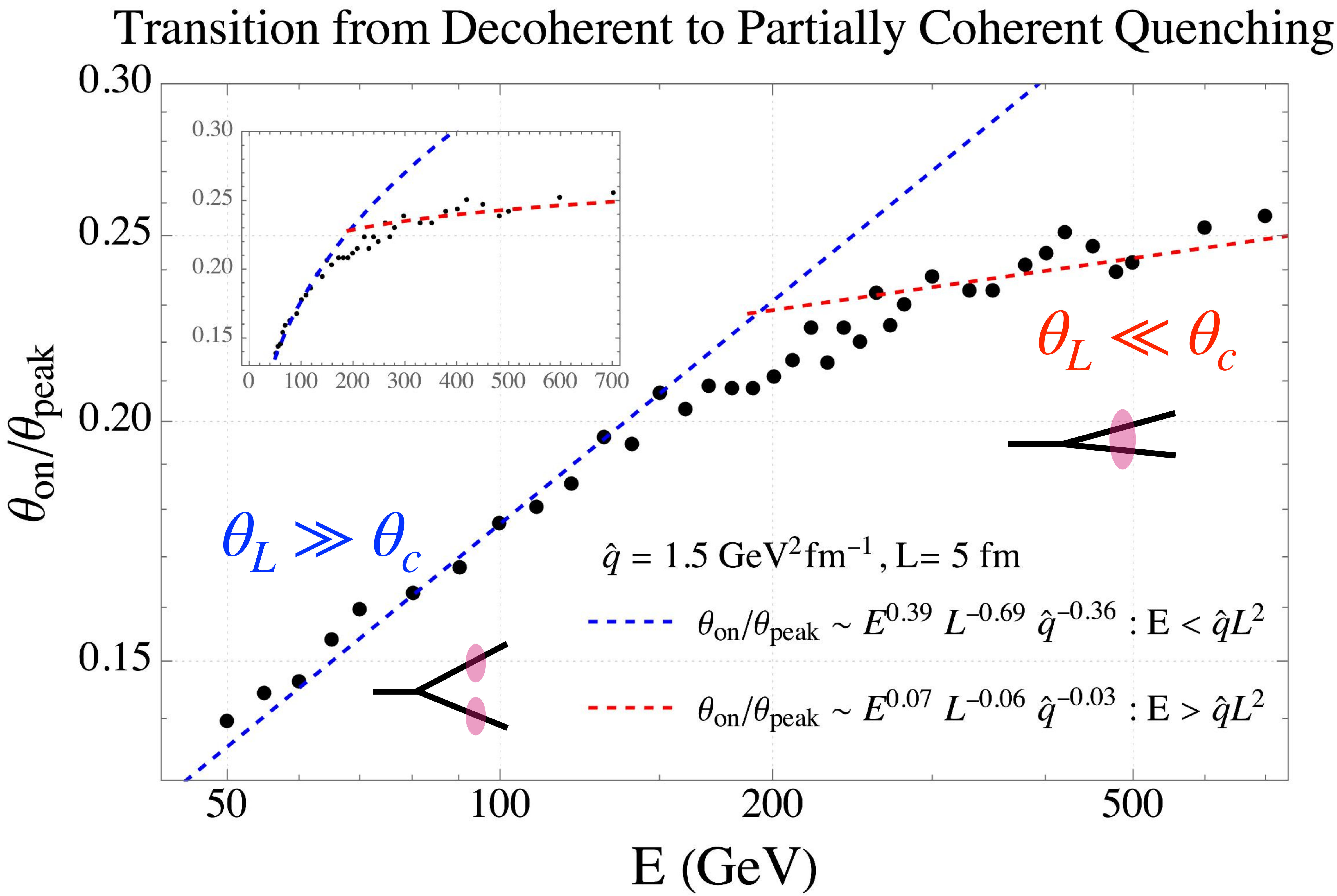
- Extracted scaling behavior of $\theta_{\rm on}$ and $\theta_{\rm peak}$ in terms of the three parameters

- In all regions the onset angle exhibits the same behavior

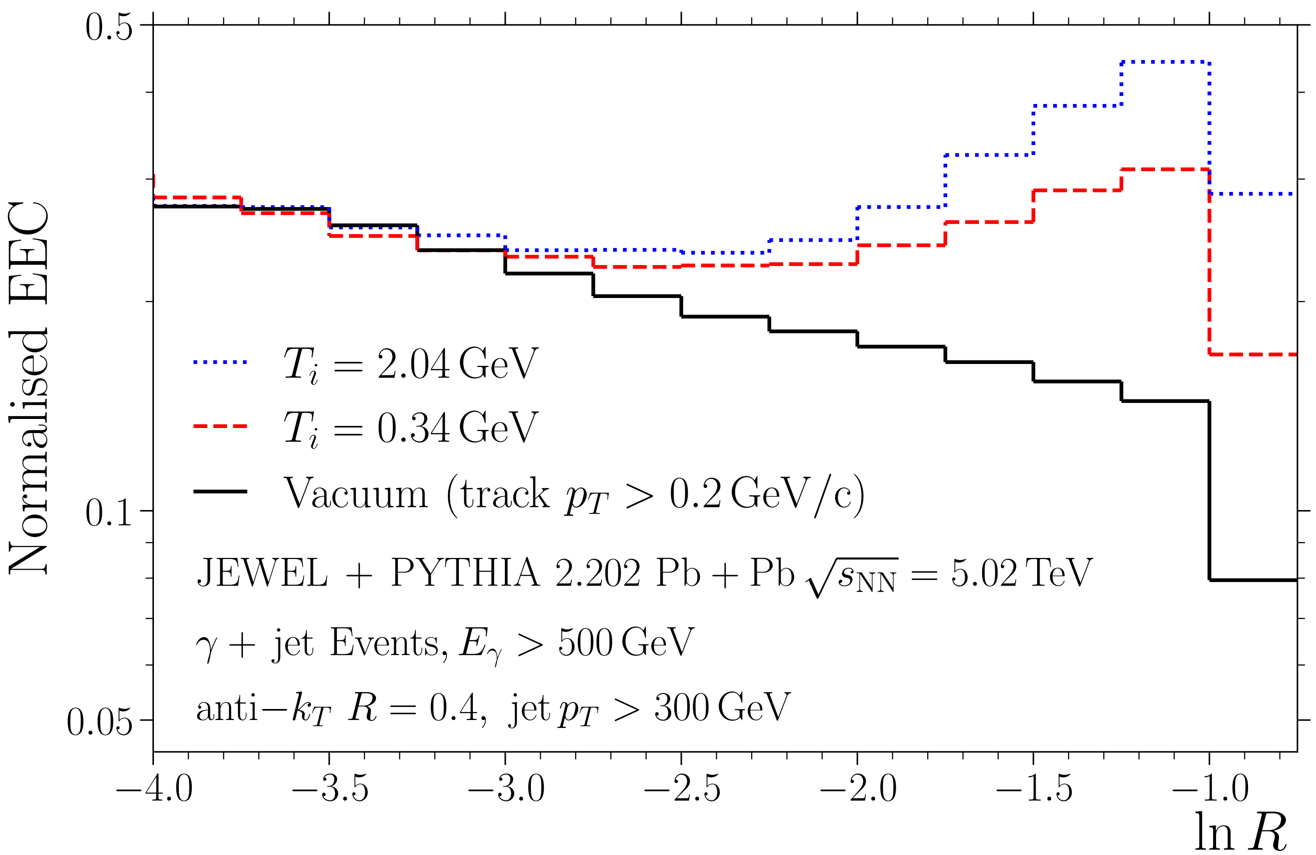
$$\theta_{\rm on} \sim \theta_L^{1\pm0.1}$$

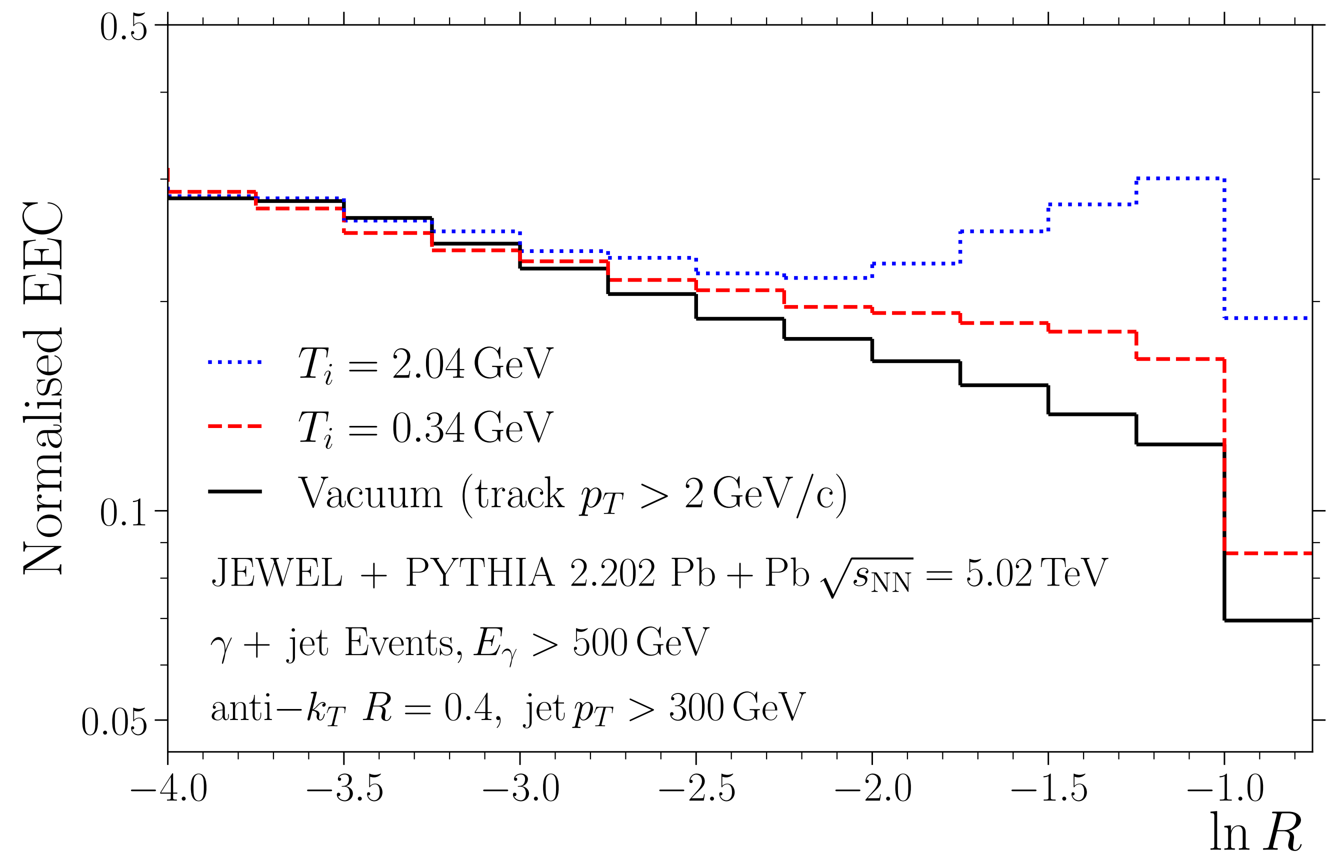
- The peak angle has different behaviors in the two different regimes
  - For $\theta_L > \theta_c$: $\theta_{\rm peak}^{\rm DC} \sim E^{-0.86\pm0.1} L^{0.21\pm0.1} \hat{q}^{0.36\pm0.1} \sim \theta_d^{1.4\pm0.1} \theta_L^{-0.4\pm0.1}$
  - For $\theta_L < \theta_c$: $\theta_{\rm peak}^{\rm PC} \sim E^{-0.54\pm0.1} L^{-0.31\pm0.1} \hat{q}^{0.09\pm0.1} \sim \theta_c^{-0.2\pm0.1} \theta_L^{1.1\pm0.1}$

EECs and color coherence



EECs with JEWEL





Peak structure resilient to p_T cuts

Conclusions

- Energy correlators provide a powerful tool for understanding jets in HIC
  - ✦ Experimentally accesible
  - ✦ Can be calculated perturbatively thanks to insensitivity to soft physics and uncorrelated background
  - ✦ Characteristic features of the calculation for in-medium splittings are clearly imprinted in the observables
- Energy correlators provide a robust angular variable which can be used to probe color coherence in jets in the QGP

Outlook

- Lots of new exciting developments!
- Test other models for the in-medium splitting calculation
  - GLV: Onset angle is not defined as sharply as in the multiple scattering case. Could be used to show the importance of the LPM regime
  - Tilted Wilson lines with Yukawa potential: Onset of coherence is NOT a feature of the harmonic approximation
- Expanding media
  - Using energy correlators to find the relevant angular scales
- Heavy quarks
  - Can be used to measure the dead-cone (calculation in pp coming out very soon)
- Monte Carlo studies
  - Test resilience to background
  - Test the effects of having the full parton shower

Thank you!

In-medium propagator

- Can be formally written in coordinate space in terms of a path integral

$$\mathcal{G}_R(t_2,\boldsymbol{x}_2;t_1,\boldsymbol{x}_1;\omega)=\int_{\boldsymbol{x}_1}^{\boldsymbol{x}_2}\mathcal{D}\boldsymbol{r}\exp\left\{\frac{i\omega}{2}\int_{t_1}^{t_2}d\xi\;\dot{\boldsymbol{r}}^2(\xi)\right\}\mathrm{P}\exp\left\{ig\int_{t_1}^{t_2}d\xi\;A_R^-(\xi,\boldsymbol{r}(\xi))\right\}$$

- Satisfies the following Schwinger-Dyson type equation

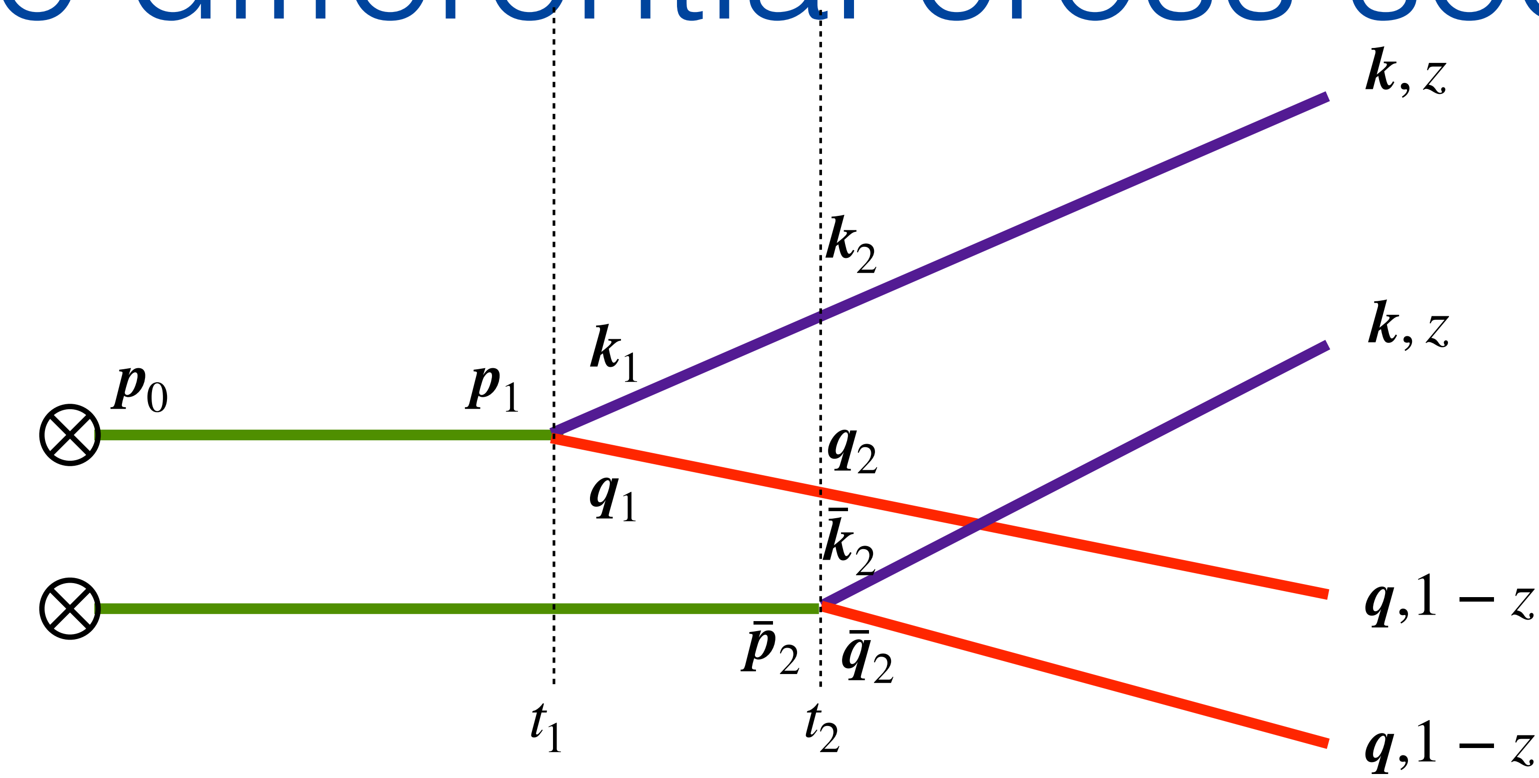
$$\begin{aligned}\mathcal{G}_R(\boldsymbol{p}_2,t_2;\boldsymbol{p}_1,t_1;\omega)&=(2\pi)^2\delta^{(2)}(\boldsymbol{p}_2-\boldsymbol{p}_1)e^{-i\frac{p_2^2}{2\omega}(t_2-t_1)}\\&\quad+ig\int_{t_1}^{t_2}ds\;e^{-i\frac{p_2^2}{2\omega}(t_2-s)}\int_{\boldsymbol{p}'}A_R^-(s,\boldsymbol{p}_2-\boldsymbol{p}')\mathcal{G}_R(\boldsymbol{p}',s;\boldsymbol{p}_1,t_1;\omega)\end{aligned}$$

- And convolution relations

$$\int_{\boldsymbol{p}_2}\mathcal{G}_R(\boldsymbol{p}_3,t_3;\boldsymbol{p}_2,t_2;\omega)\mathcal{G}_R(\boldsymbol{p}_2,t_2;\boldsymbol{p}_1,t_1;\omega)=\mathcal{G}_R(\boldsymbol{p}_3,t_3;\boldsymbol{p}_1,t_1;\omega)$$

$$\int_{\boldsymbol{p}_2}\mathcal{G}_R^\dagger(\bar{\boldsymbol{p}}_1,t_1;\boldsymbol{p}_2,t_2;\omega)\mathcal{G}_R(\boldsymbol{p}_2,t_2;\boldsymbol{p}_1,t_1;\omega)=(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1-\bar{\boldsymbol{p}}_1)$$

Double differential cross section

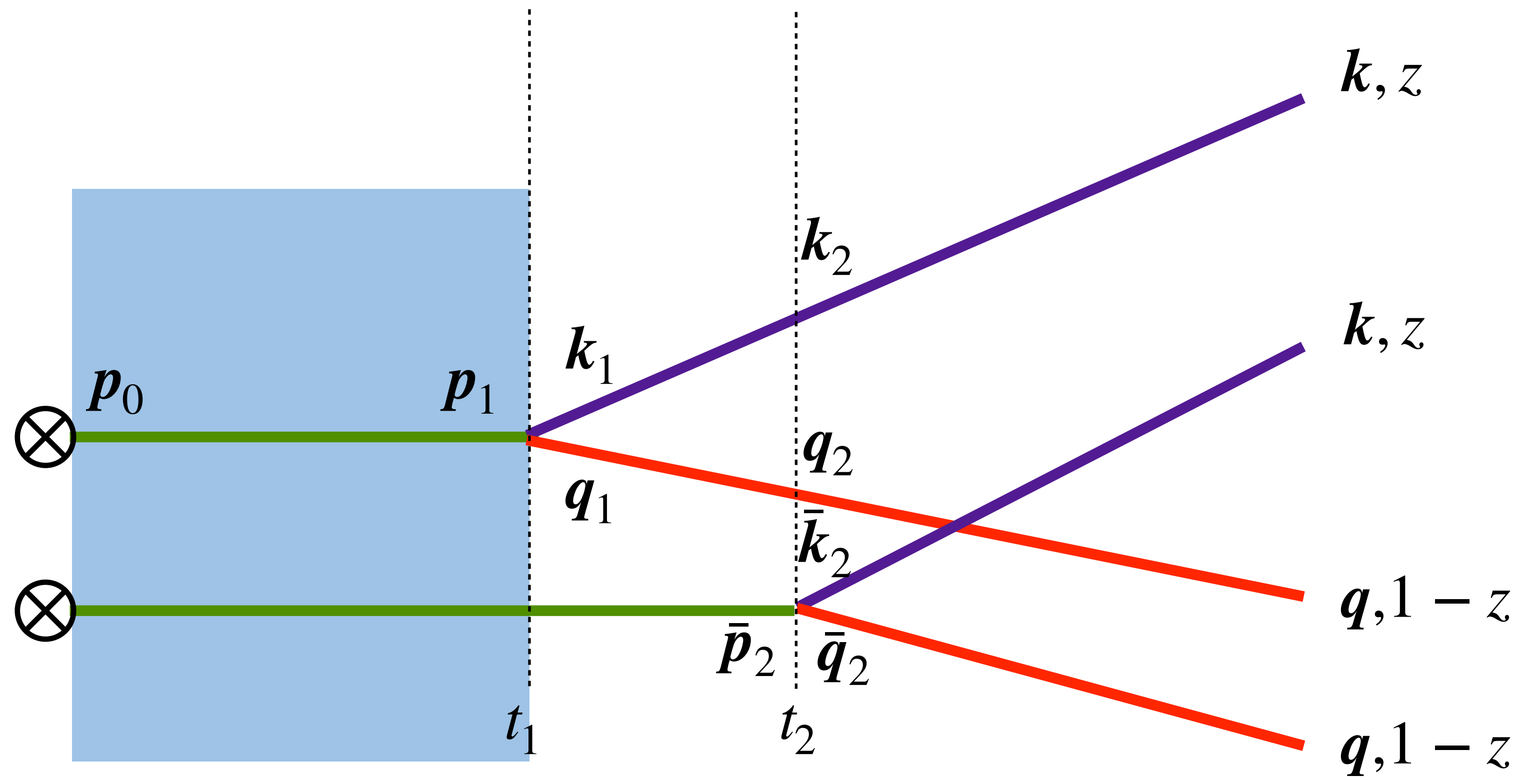


- The locality of the medium averages $\langle A^-(t)A^-(t')\rangle \propto \delta(t-t')$ implies that at any given time:
    - Averages can be factored into regions with constant number of particles
    - The sum of all momenta in the amplitude is equal to the sum of all momenta in the conjugate amplitude
    - When considering the ensemble of all particles in the amplitude and conjugate amplitude, the overall color state is always a singlet

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585

Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section



Average depends on total momentum transfer only

$$= \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0)$$

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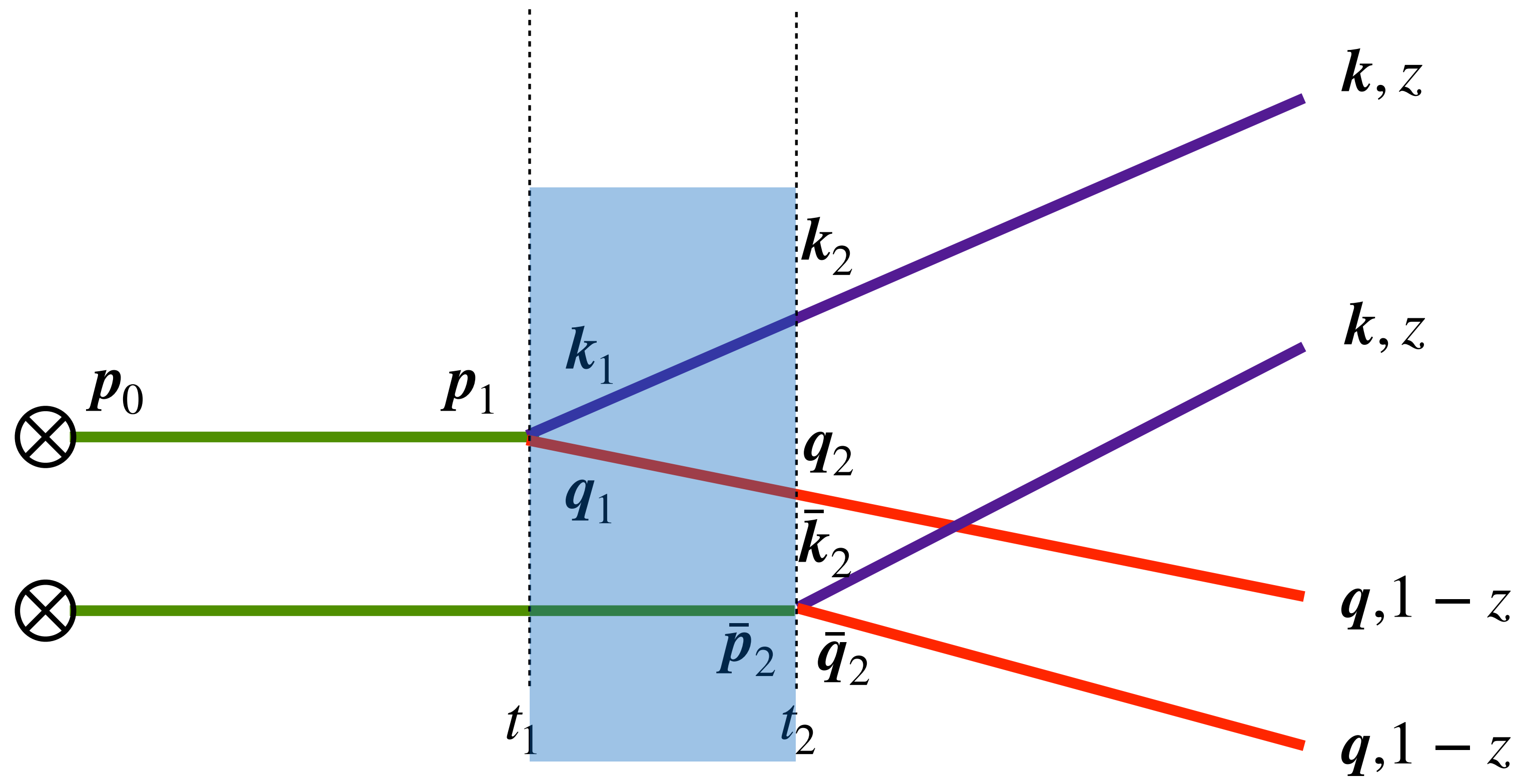
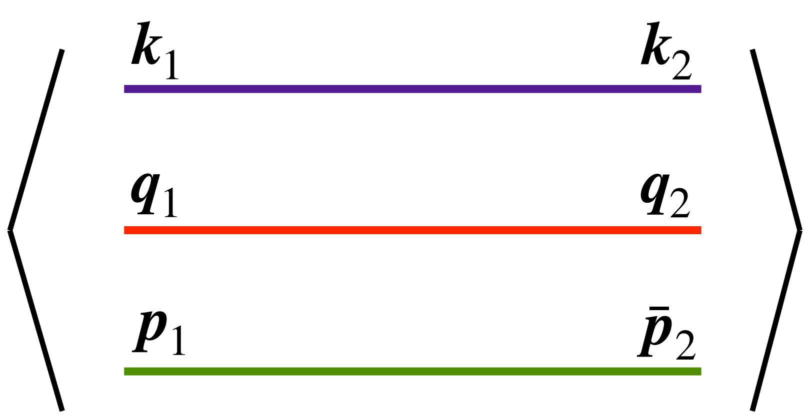
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section





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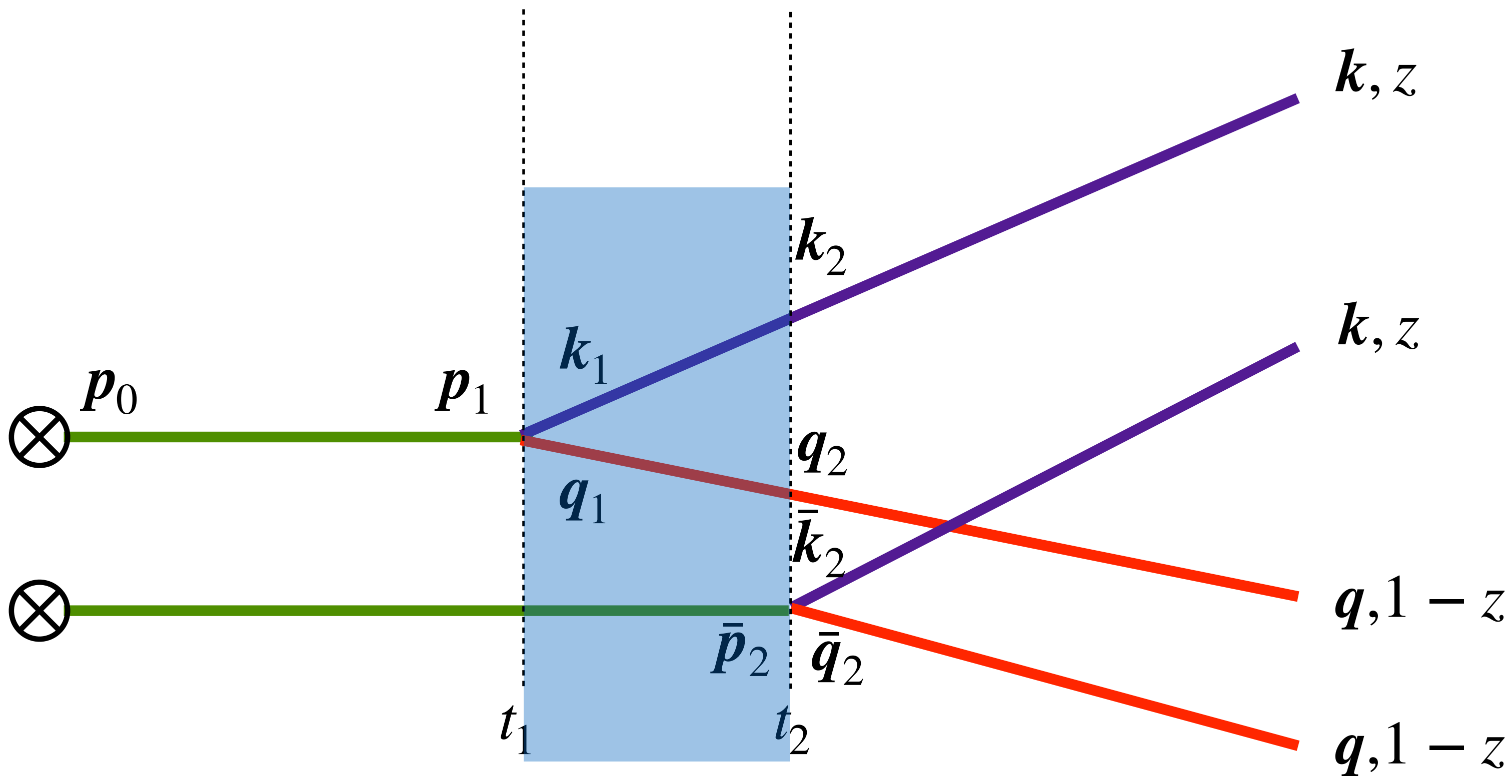
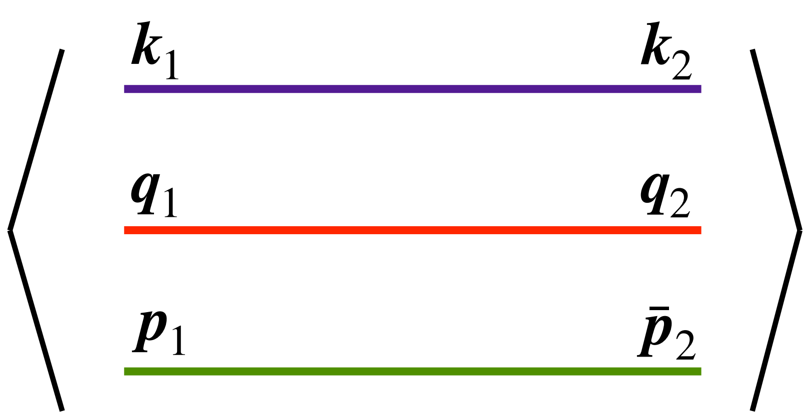
Double differential cross section



$$\boldsymbol{k}_1 + \boldsymbol{q}_1 = \boldsymbol{p}_1 \qquad \boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{p}}_2$$



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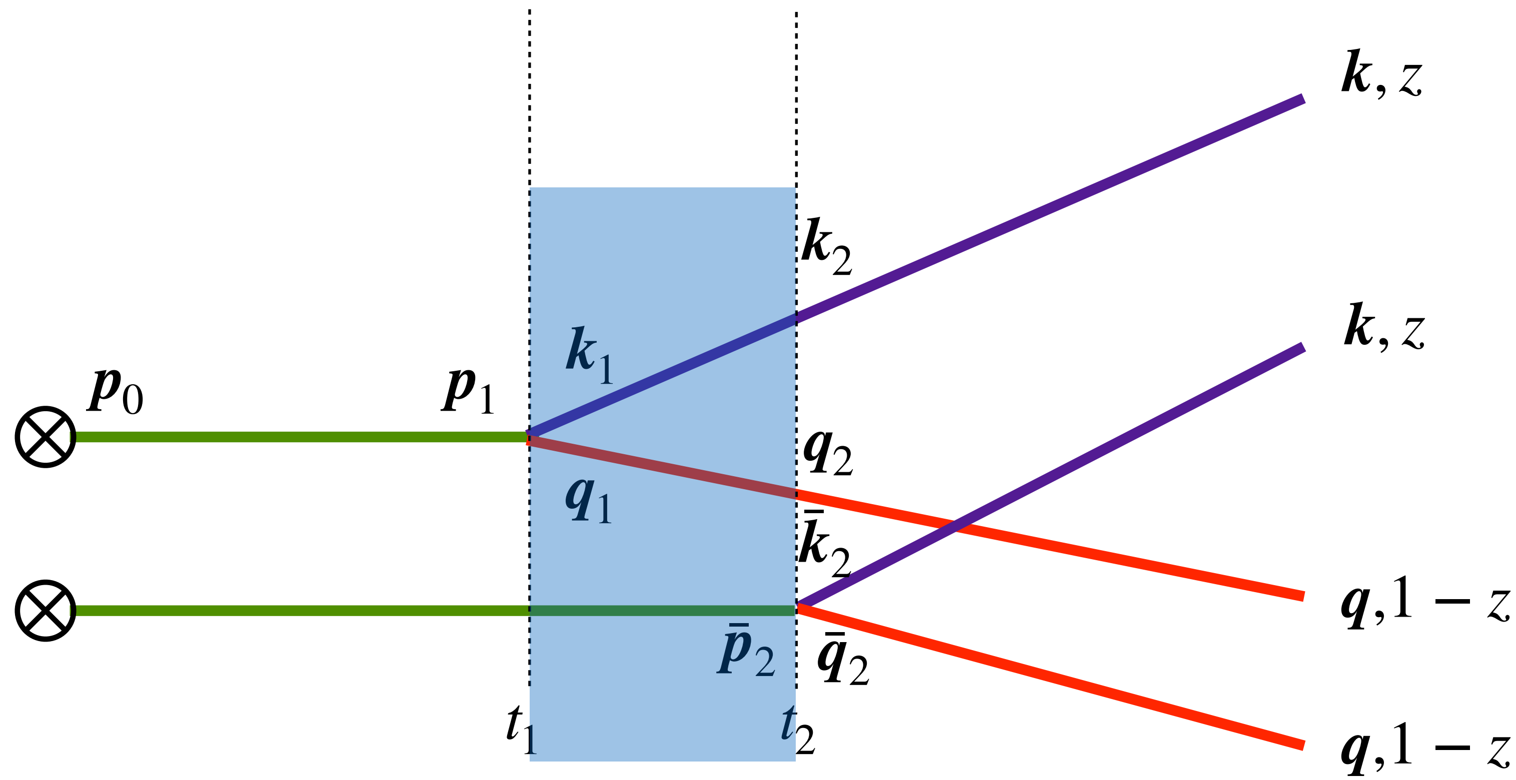
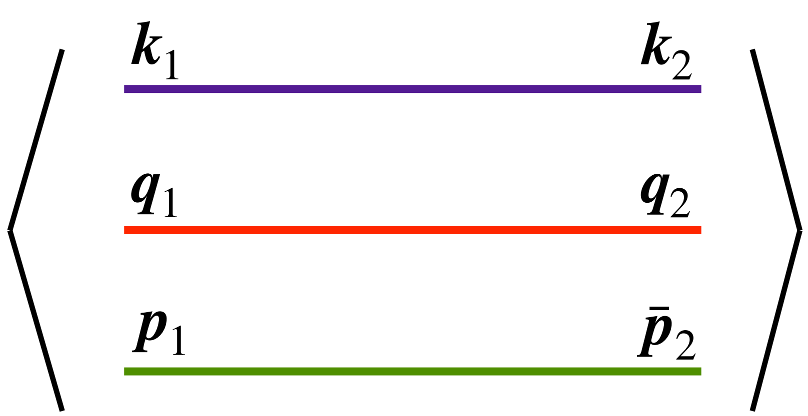
Double differential cross section



Same as vertex factors

$$\boldsymbol{k}_1 + \boldsymbol{q}_1 = \boldsymbol{p}_1 \qquad \boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{p}}_2 \qquad \boldsymbol{l}_1 = (1 - z)\boldsymbol{k}_1 - z\boldsymbol{q}_1 \qquad \boldsymbol{l}_2 = (1 - z)\boldsymbol{k}_2 - z\boldsymbol{q}_2$$



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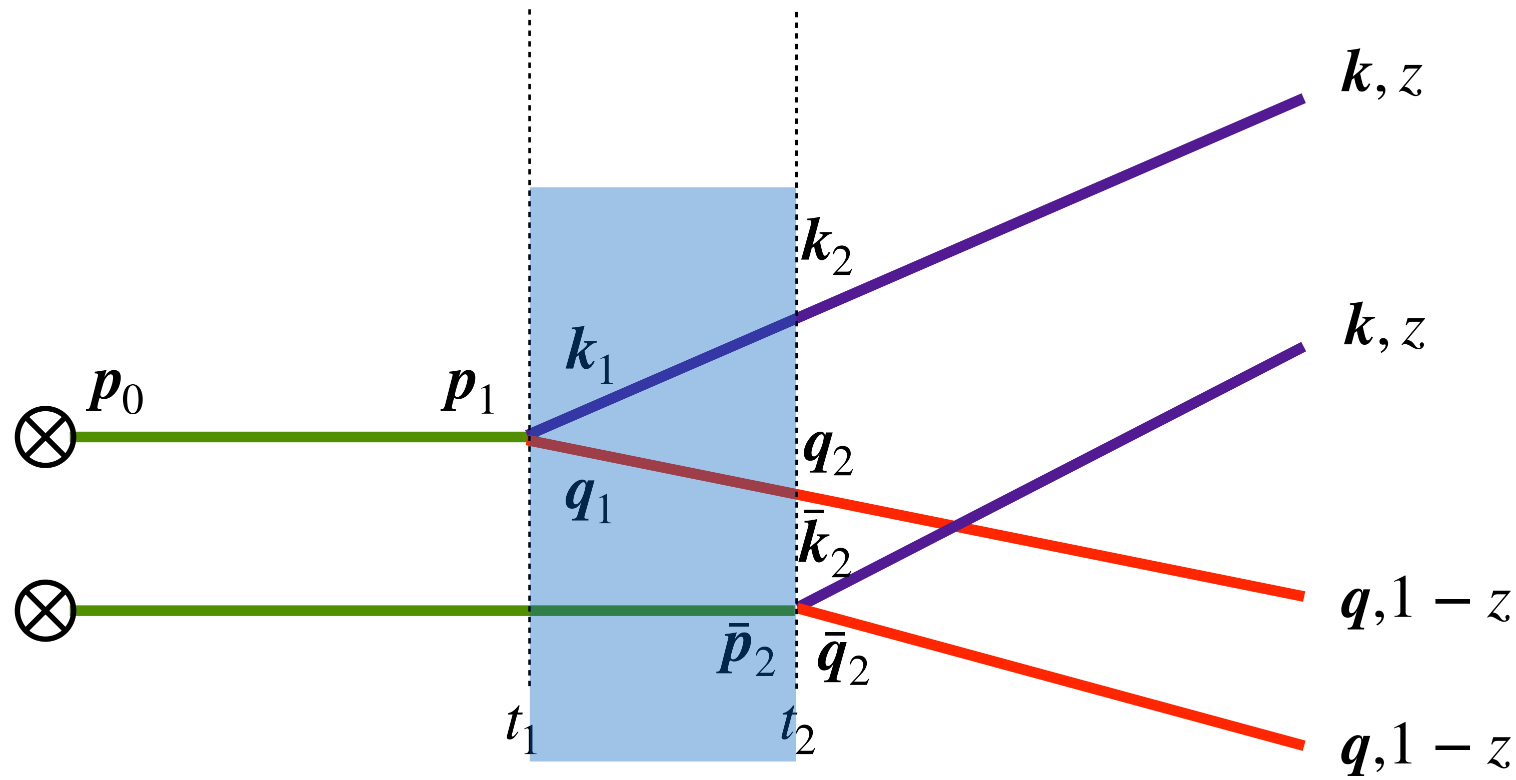
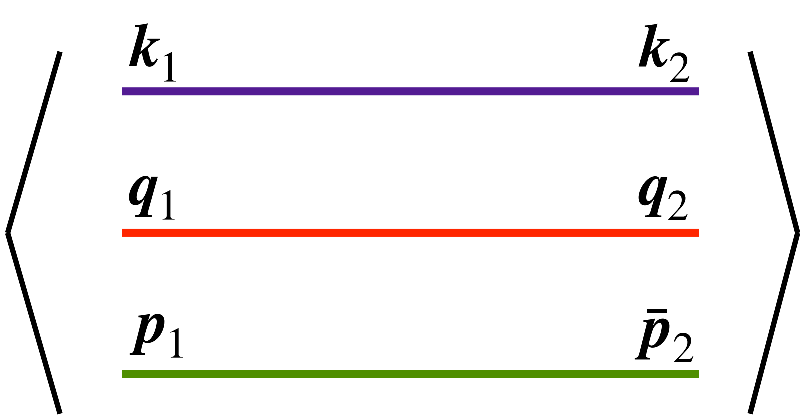
Double differential cross section



Same as vertex factors

$\boldsymbol{k}_1 + \boldsymbol{q}_1 = \boldsymbol{p}_1 \qquad \boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{p}}_2 \qquad \boldsymbol{l}_1 = (1-z)\boldsymbol{k}_1 - z\boldsymbol{q}_1 \qquad \boldsymbol{l}_2 = (1-z)\boldsymbol{k}_2 - z\boldsymbol{q}_2$



Average depends on $\boldsymbol{l}_1, \boldsymbol{l}_2, \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1$

$$\mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)$$

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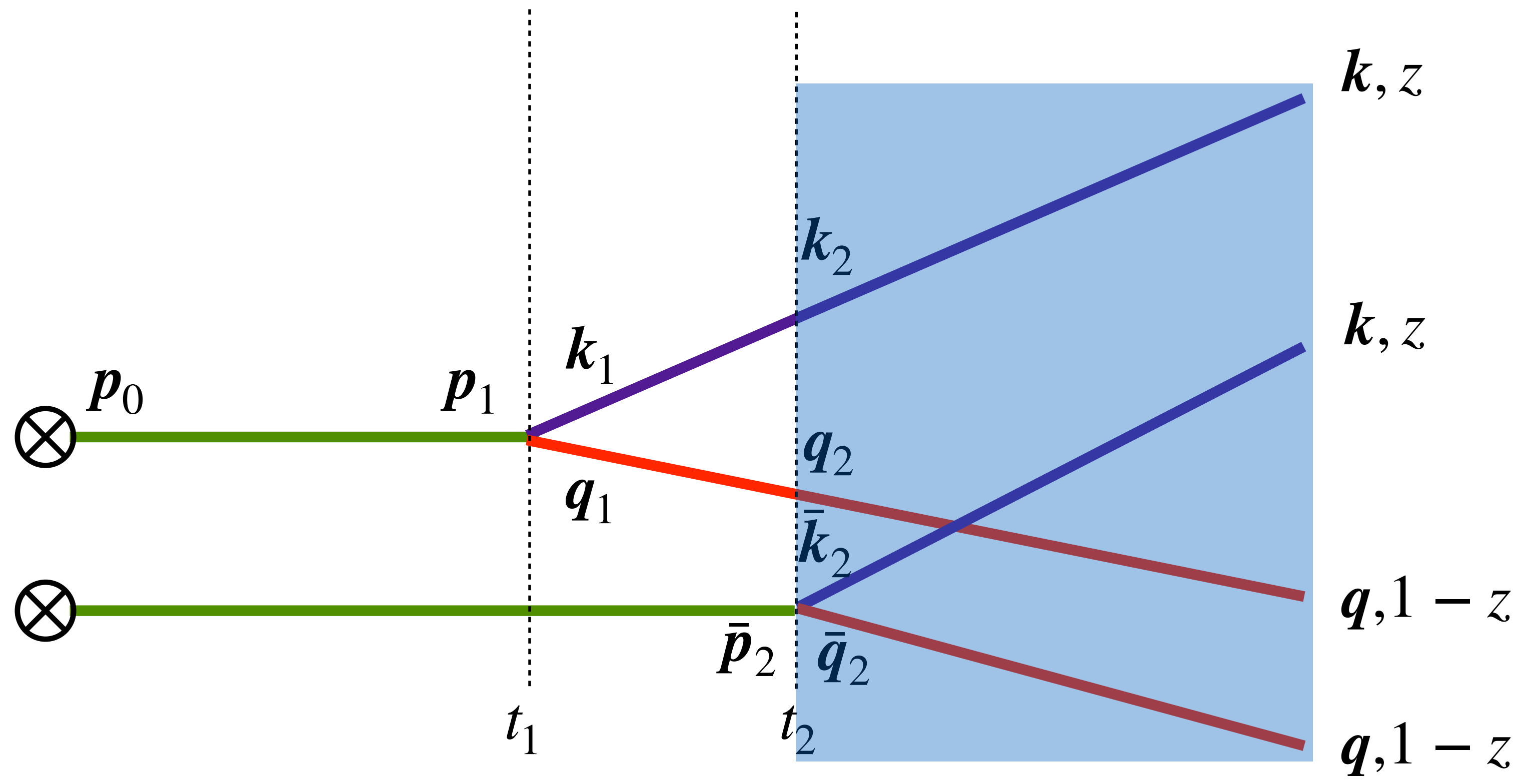
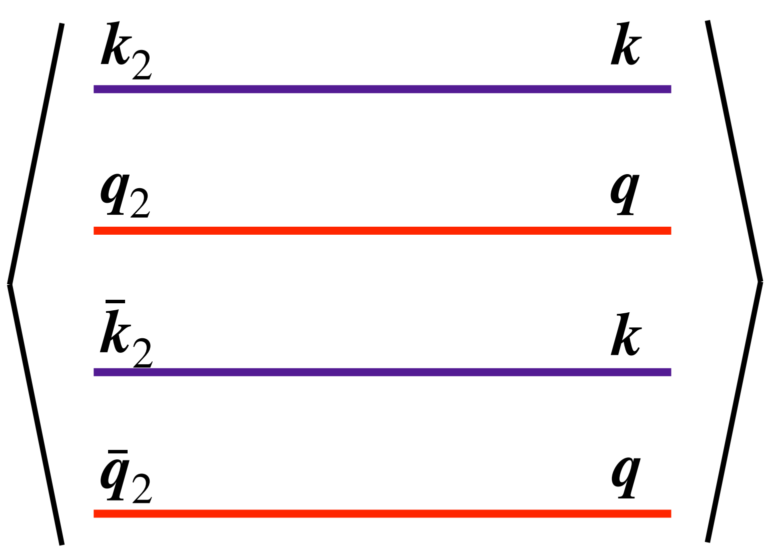
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section





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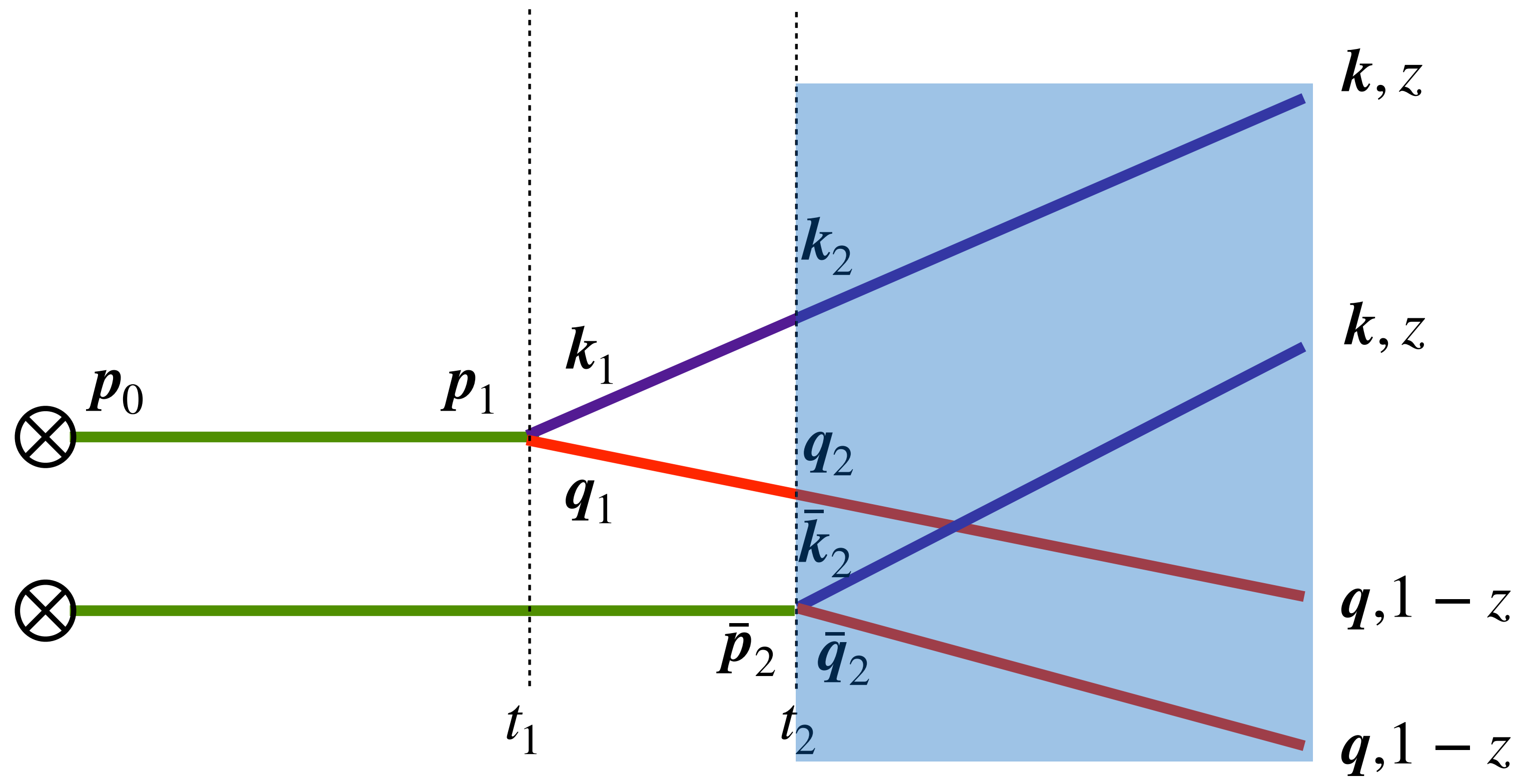
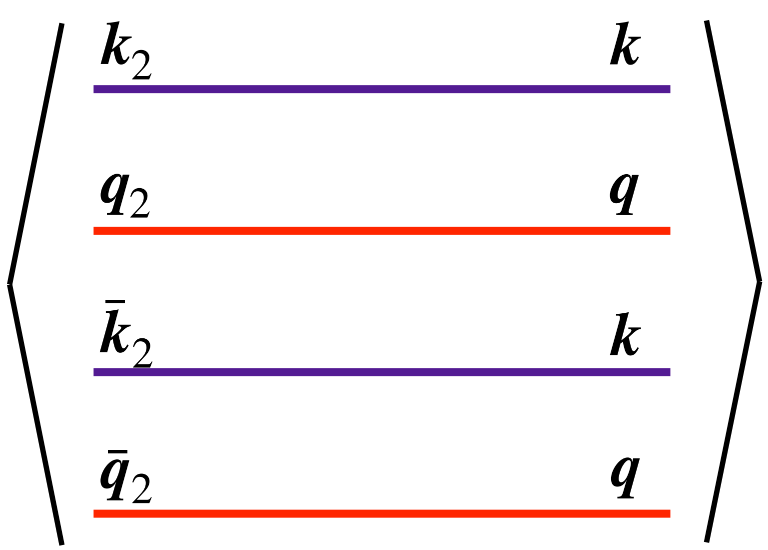


$$k_2 + q_2 = \bar{k}_2 + \bar{q}_2$$



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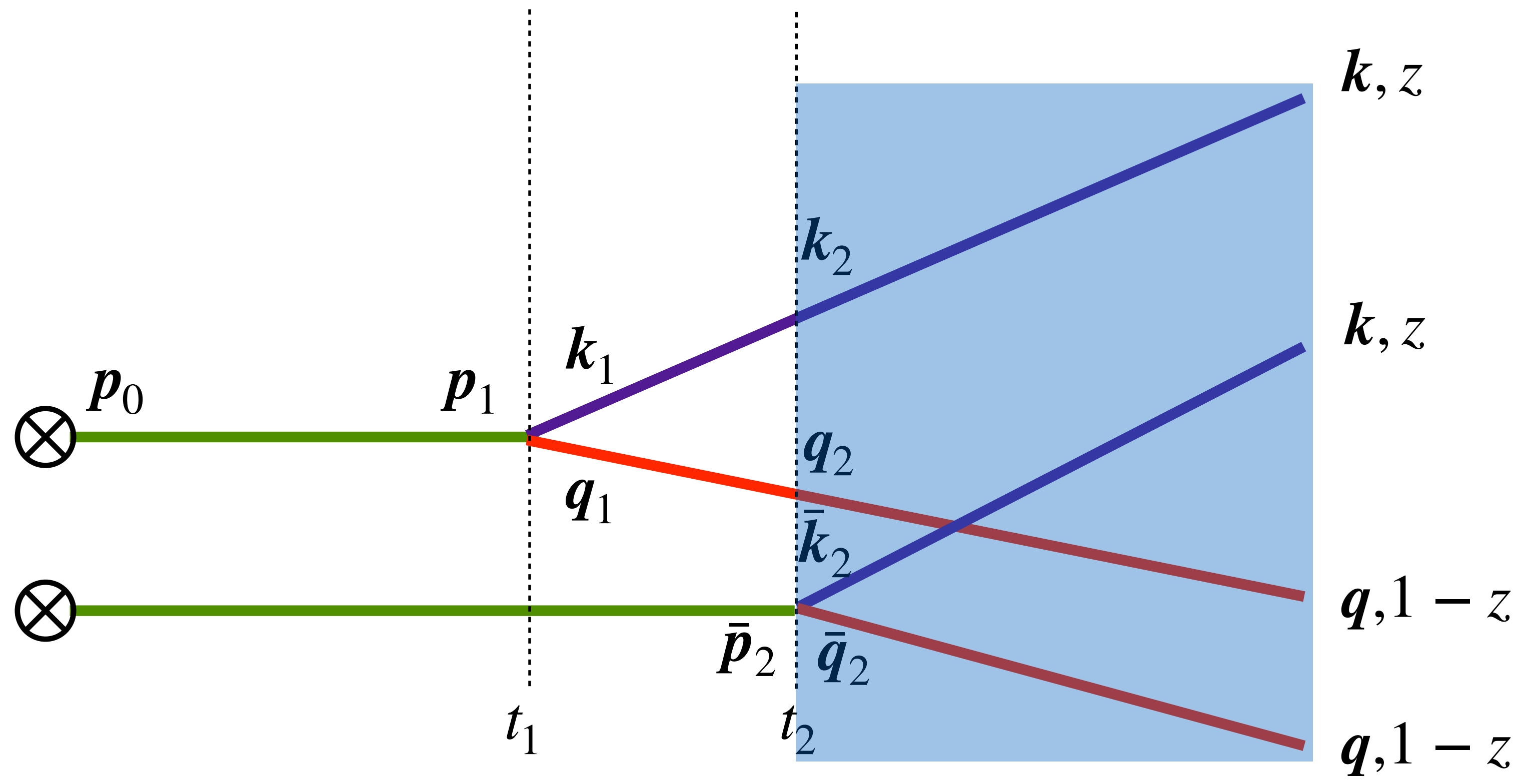
Double differential cross section

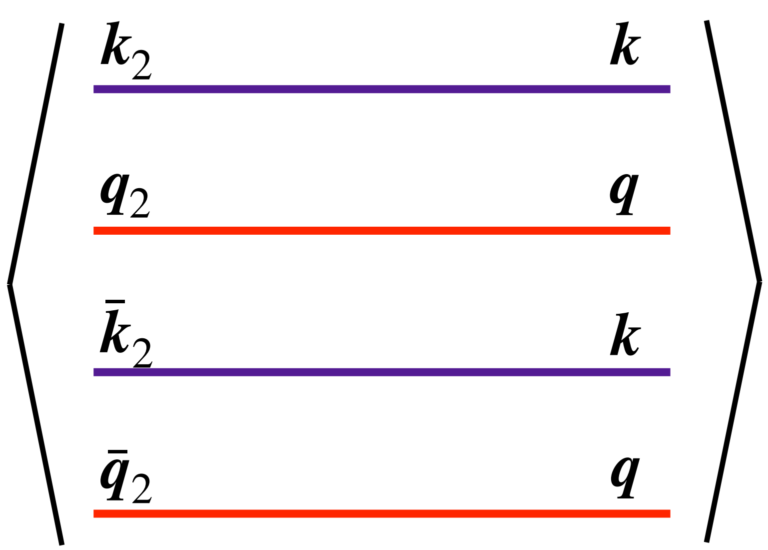


$$\boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{k}}_2 + \bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l}_2 = (1-z)\boldsymbol{k}_2 - z\boldsymbol{q}_2$$

$$\bar{\boldsymbol{l}}_2 = (1-z)\bar{\boldsymbol{k}}_2 - z\bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l} = (1-z)\boldsymbol{k} - z\boldsymbol{q}$$



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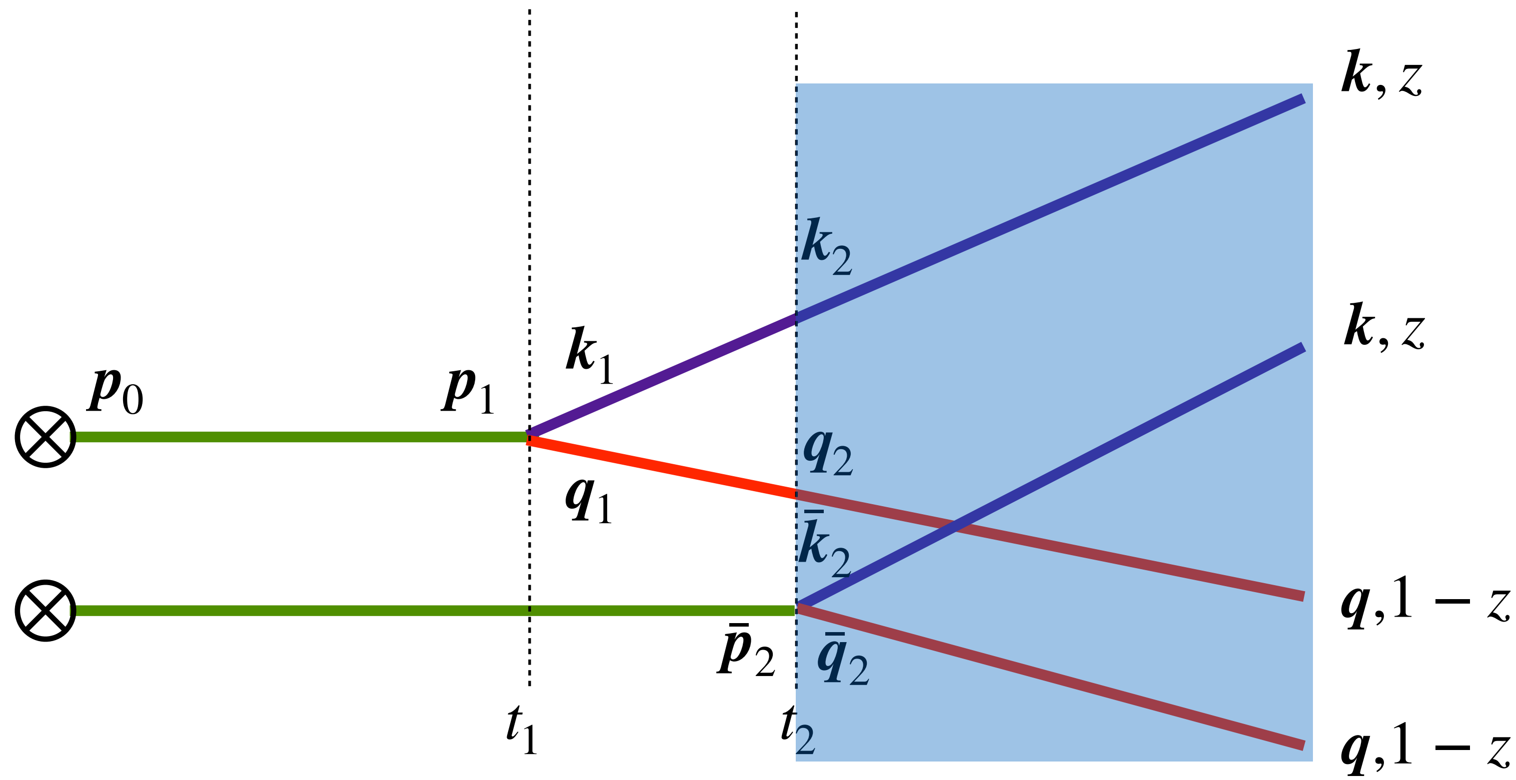
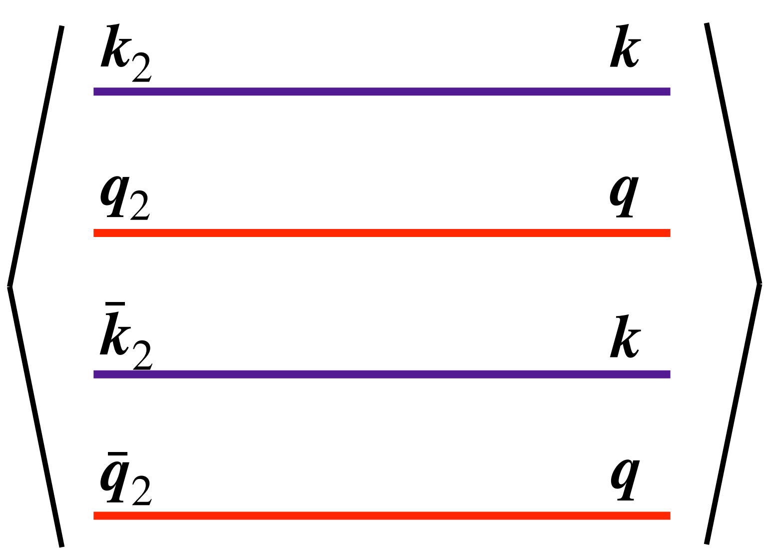


$$\boldsymbol{k}_2 + \boldsymbol{q}_2 = \bar{\boldsymbol{k}}_2 + \bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l}_2 = (1-z)\boldsymbol{k}_2 - z\boldsymbol{q}_2$$
$$\bar{\boldsymbol{l}}_2 = (1-z)\bar{\boldsymbol{k}}_2 - z\bar{\boldsymbol{q}}_2$$

$$\boldsymbol{l} = (1-z)\boldsymbol{k} - z\boldsymbol{q}$$

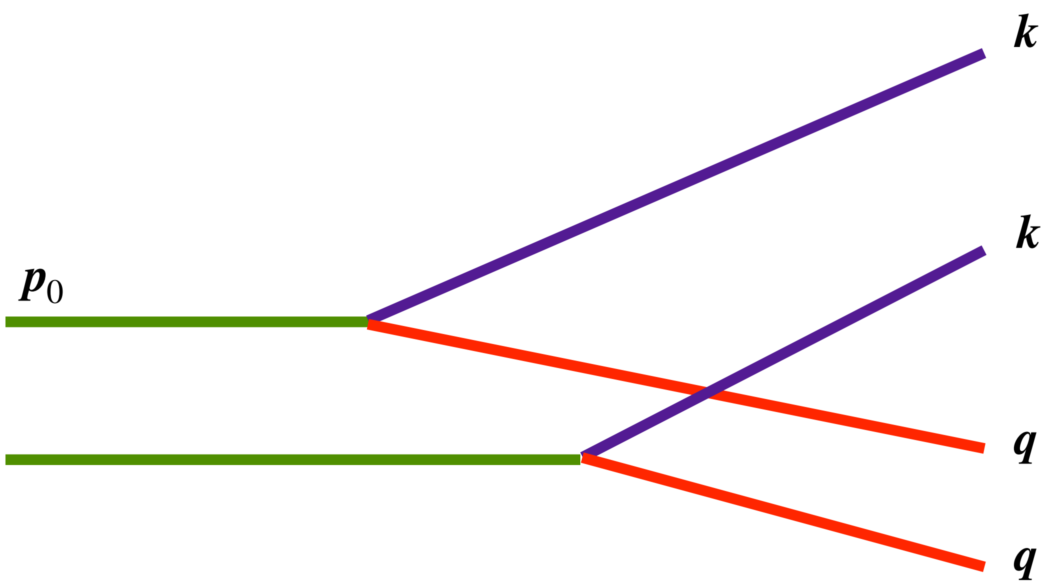


Average depends on $\boldsymbol{l}, \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2$, and $\boldsymbol{k} + \boldsymbol{q} - \boldsymbol{k}_2 - \boldsymbol{q}_2$

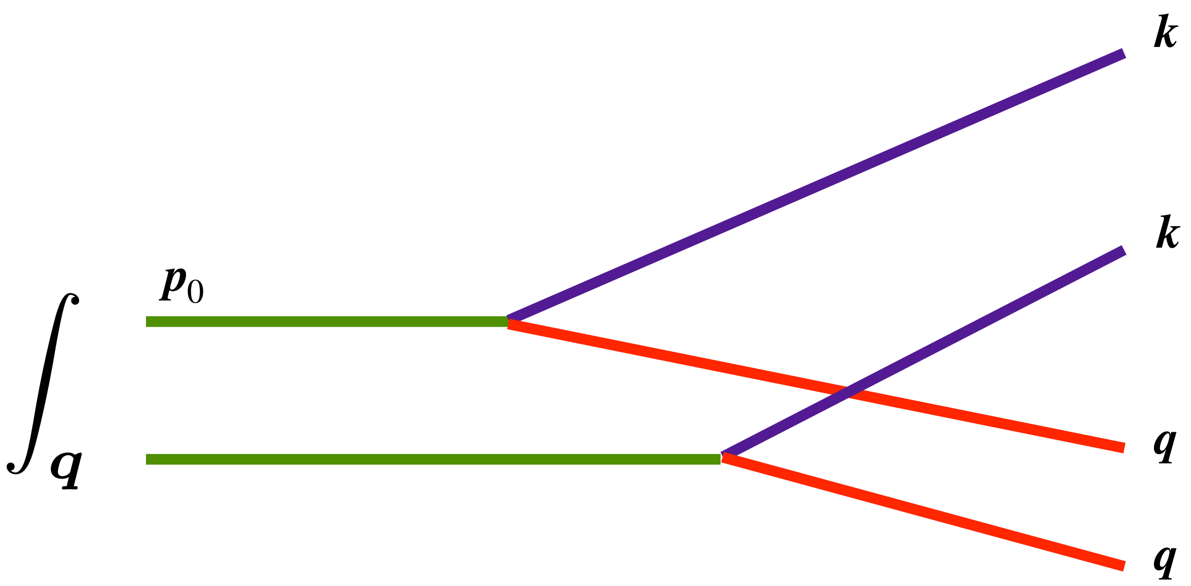
$$\mathcal{S}^{(4)}(\boldsymbol{l}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k} + \boldsymbol{q} - \boldsymbol{k}_2 - \boldsymbol{q}_2, z)$$

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

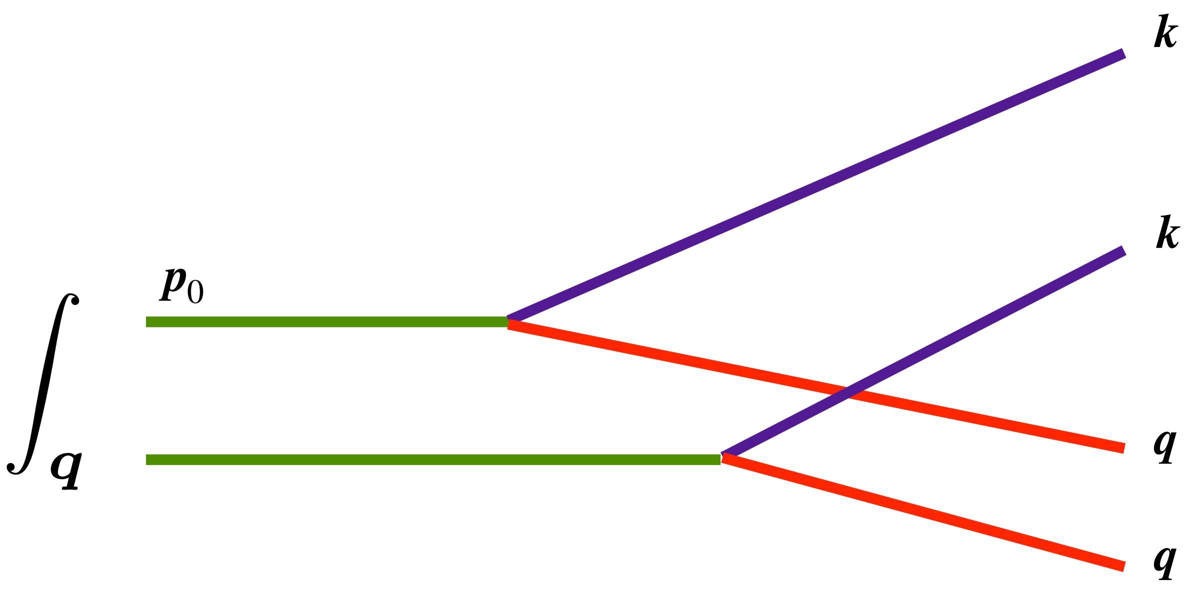
Integrate over final particles



Integrate over final particles

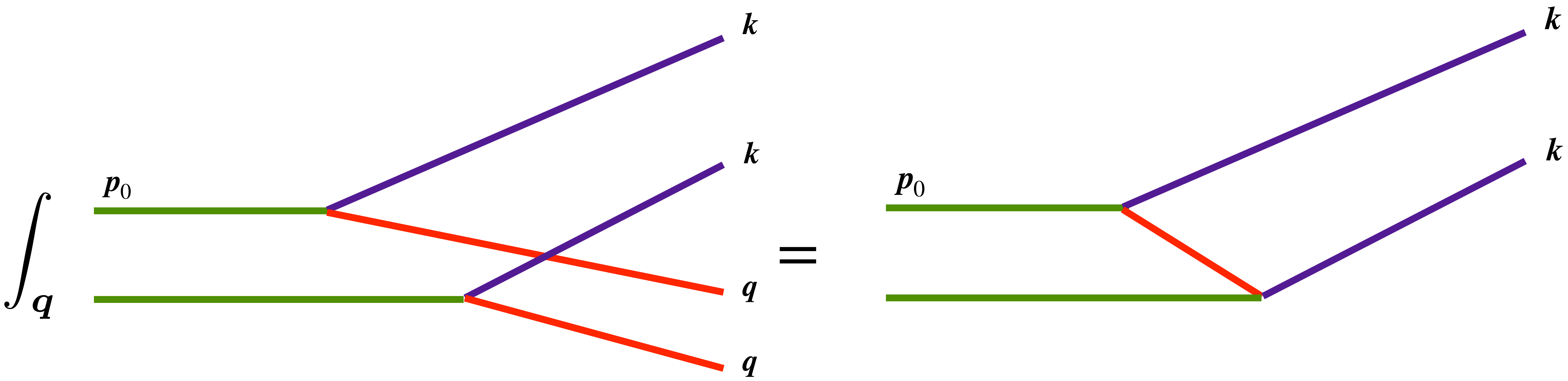


Integrate over final particles



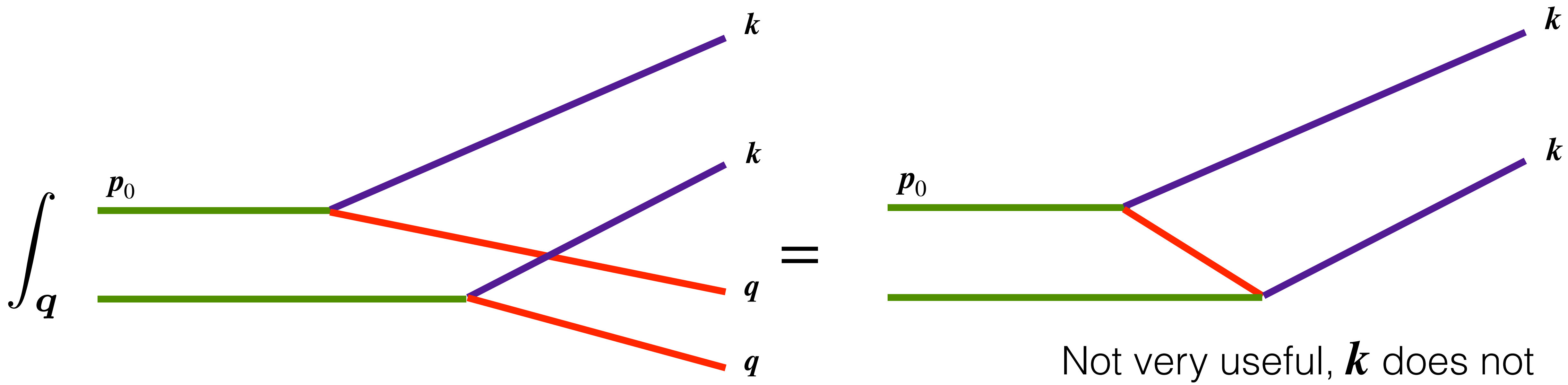
$$\int_{\boldsymbol{q}} \mathcal{G}^{\dagger}_{R_c}(\bar{\boldsymbol{q}}_2, t_2; \boldsymbol{q}, L; (1-z)E)\mathcal{G}_{R_c}(\boldsymbol{q}, L; \boldsymbol{q}_2, t_2; (1-z)E) = (2\pi)^2\delta^{(2)}(\boldsymbol{q}_2 - \bar{\boldsymbol{q}}_2)$$

Integrate over final particles



$$\int_{\boldsymbol{q}} \mathcal{G}^{\dagger}_{R_c}(\bar{\boldsymbol{q}}_2, t_2; \boldsymbol{q}, L; (1-z)E)\,\mathcal{G}_{R_c}(\boldsymbol{q}, L; \boldsymbol{q}_2, t_2; (1-z)E) = (2\pi)^2\delta^{(2)}(\boldsymbol{q}_2 - \bar{\boldsymbol{q}}_2)$$

Integrate over final particles

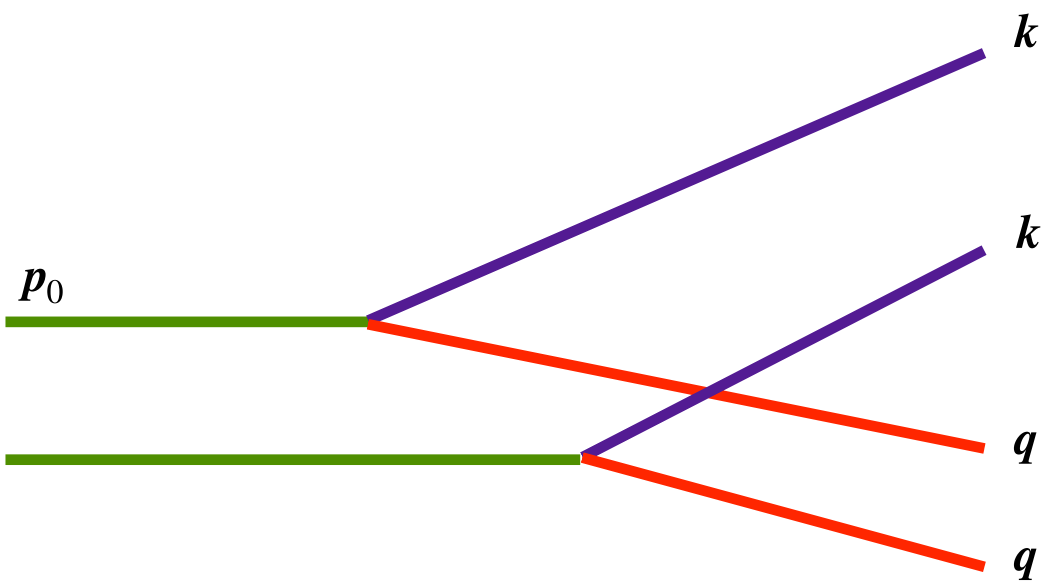


Not very useful, $\boldsymbol{k}$ does not provide information about the splitting

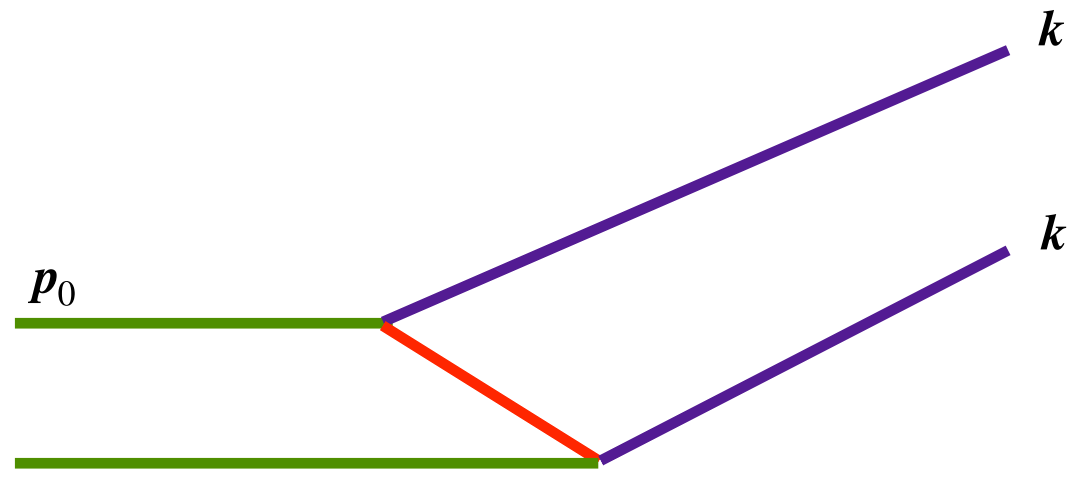
$$\int_{\boldsymbol{q}} \mathcal{G}^{\dagger}_{R_c}(\bar{\boldsymbol{q}}_2, t_2; \boldsymbol{q}, L; (1-z)E)\mathcal{G}_{R_c}(\boldsymbol{q}, L; \boldsymbol{q}_2, t_2; (1-z)E) = (2\pi)^2\delta^{(2)}(\boldsymbol{q}_2 - \bar{\boldsymbol{q}}_2)$$

Integrate over final particles

$\int_{\boldsymbol{k}} \int_{\boldsymbol{q}}$



$= \int_{\boldsymbol{k}}$

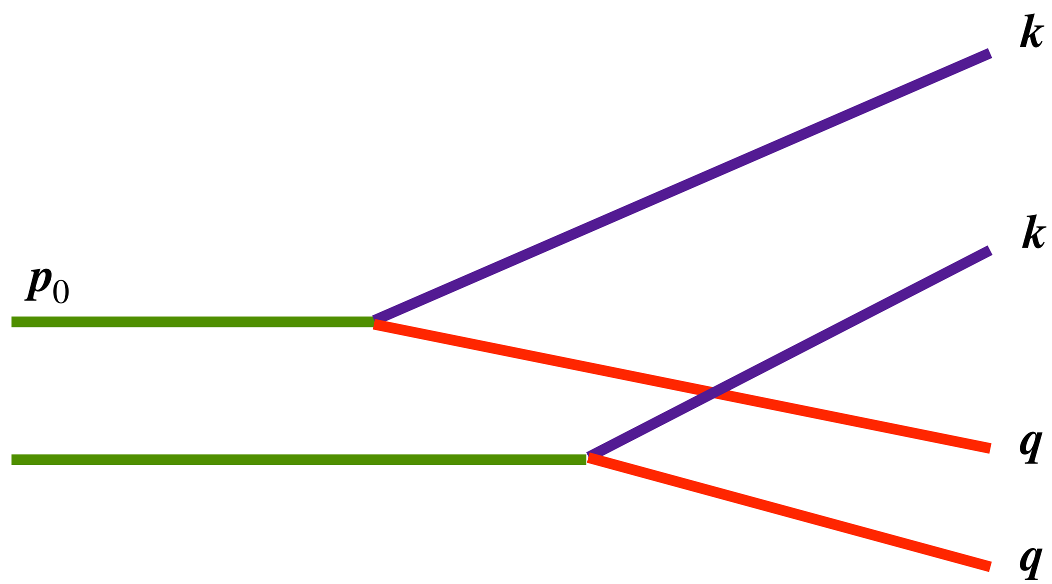


Not very useful, $\boldsymbol{k}$ does not provide information about the splitting

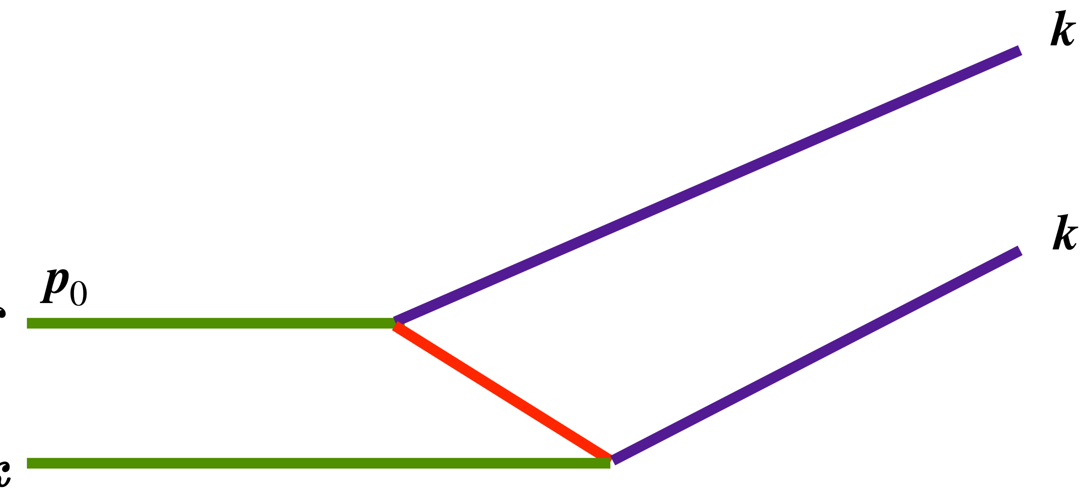
$$\int_{\boldsymbol{q}} \mathcal{G}^{\dagger}_{R_c}(\bar{\boldsymbol{q}}_2, t_2; \boldsymbol{q}, L; (1-z)E)\mathcal{G}_{R_c}(\boldsymbol{q}, L; \boldsymbol{q}_2, t_2; (1-z)E) = (2\pi)^2\delta^{(2)}(\boldsymbol{q}_2 - \bar{\boldsymbol{q}}_2)$$

Integrate over final particles

$\int_{\boldsymbol{k}} \int_{\boldsymbol{q}}$



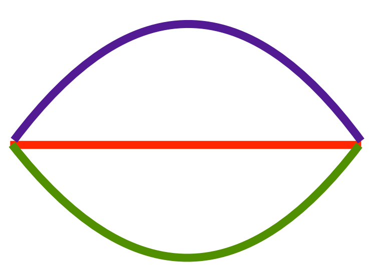
$= \int_{\boldsymbol{k}}$



Not very useful, $\boldsymbol{k}$ does not provide information about the splitting

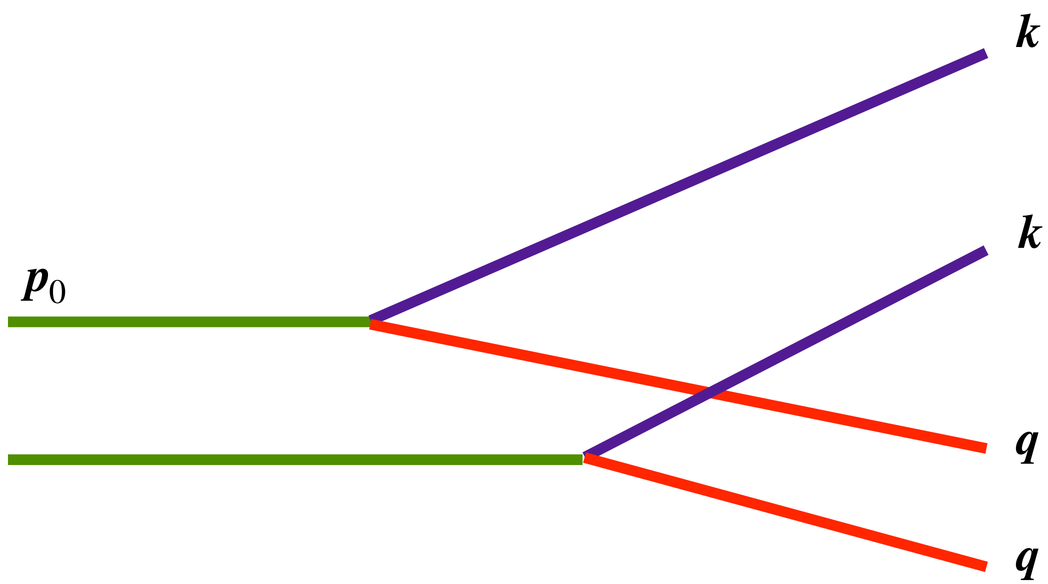
$$\int_{\boldsymbol{q}} \mathcal{G}^{\dagger}_{R_c}(\bar{\boldsymbol{q}}_2, t_2; \boldsymbol{q}, L; (1-z)E)\mathcal{G}_{R_c}(\boldsymbol{q}, L; \boldsymbol{q}_2, t_2; (1-z)E) = (2\pi)^2\delta^{(2)}(\boldsymbol{q}_2 - \bar{\boldsymbol{q}}_2)$$

$=$

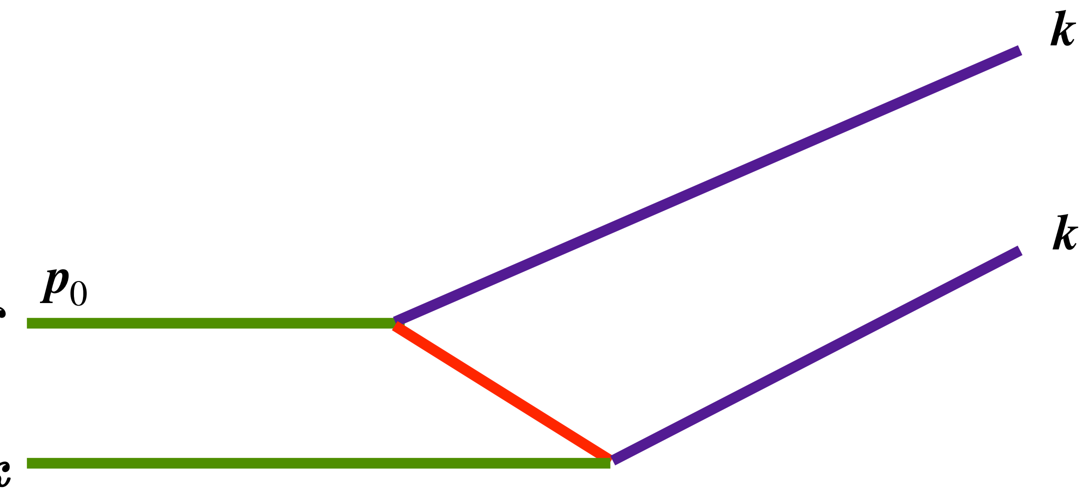


Integrate over final particles

$\int_{\boldsymbol{k}} \int_{\boldsymbol{q}}$



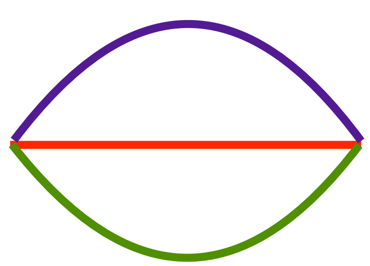
$= \int_{\boldsymbol{k}}$



$$\int_{\boldsymbol{q}} \mathcal{G}^{\dagger}_{R_c}(\bar{\boldsymbol{q}}_2, t_2; \boldsymbol{q}, L; (1-z)E)\mathcal{G}_{R_c}(\boldsymbol{q}, L; \boldsymbol{q}_2, t_2; (1-z)E) = (2\pi)^2\delta^{(2)}(\boldsymbol{q}_2 - \bar{\boldsymbol{q}}_2)$$

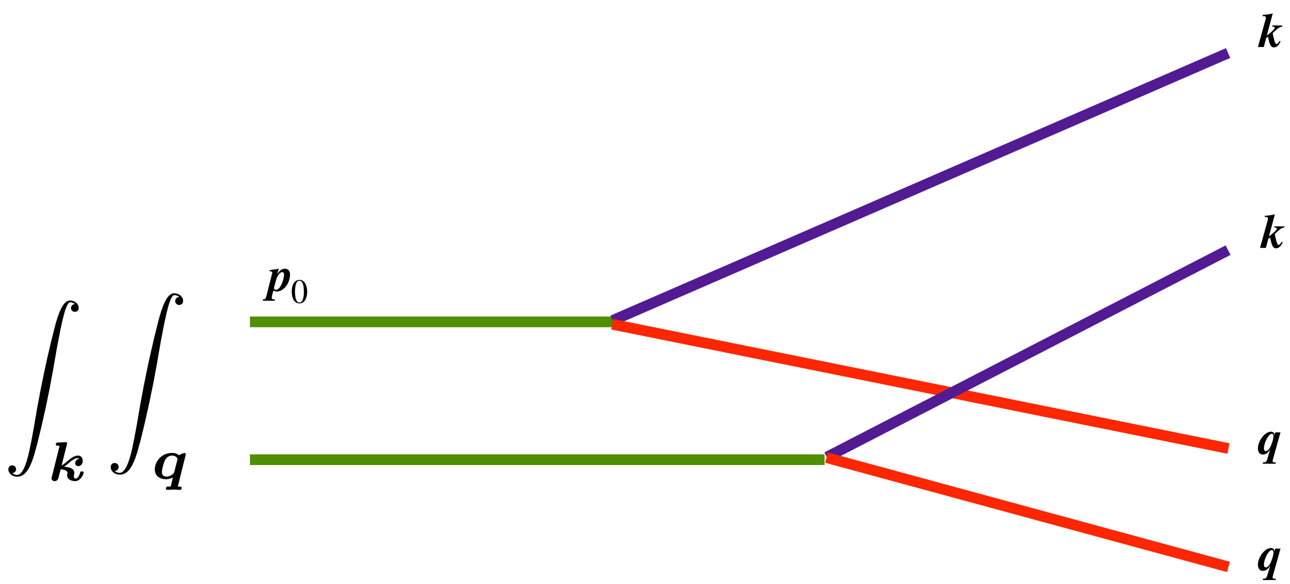
Not very useful, $\boldsymbol{k}$ does not provide information about the splitting

$=$



Gives the energy spectrum $\dfrac{dI}{dz}$ where all transverse information has been lost

Integrate over final particles



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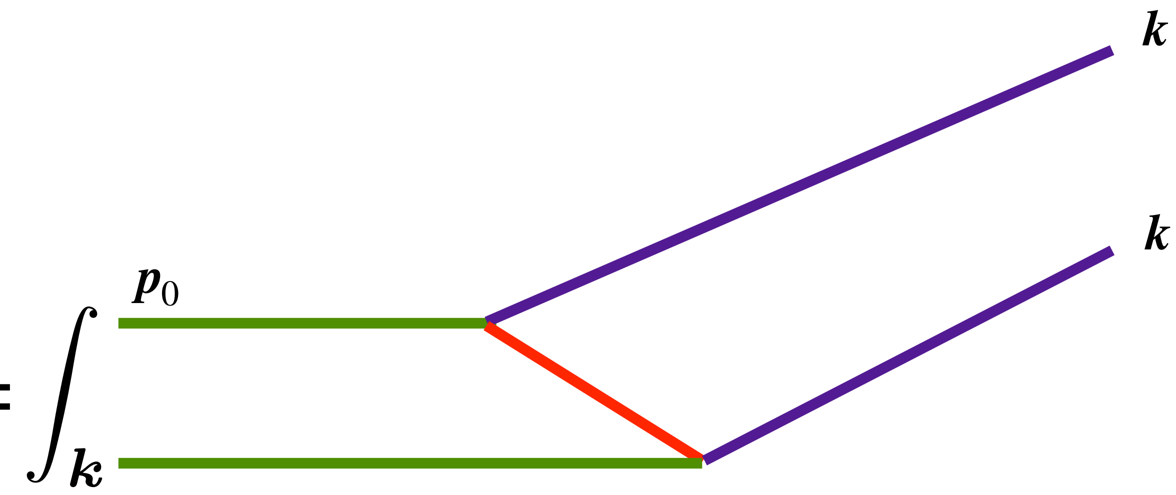
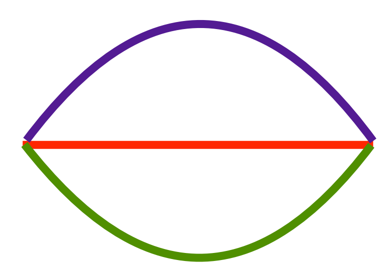


$$\int_{\boldsymbol{q}} \mathcal{G}^{\dagger}_{R_c}(\bar{\boldsymbol{q}}_2, t_2; \boldsymbol{q}, L; (1-z)E)\mathcal{G}_{R_c}(\boldsymbol{q}, L; \boldsymbol{q}_2, t_2; (1-z)E) = (2\pi)^2\delta^{(2)}(\boldsymbol{q}_2 - \bar{\boldsymbol{q}}_2)$$

Not very useful, $\boldsymbol{k}$ does not provide information about the splitting

$=$ 

Gives the energy spectrum $\dfrac{dI}{dz}$ where all transverse information has been lost

None of these simplifications allows us to keep track of the splitting angle while reducing the complexity of the calculation