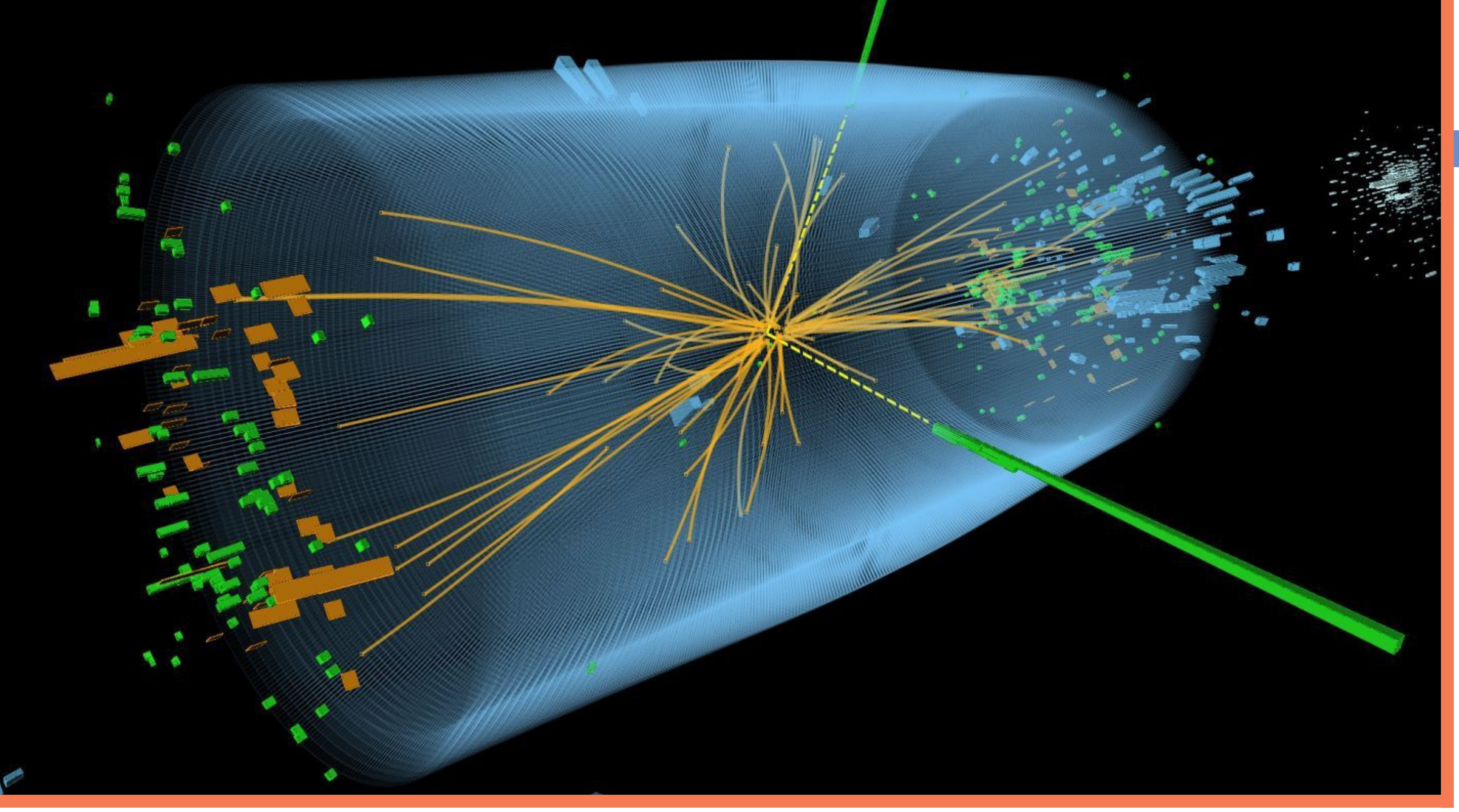
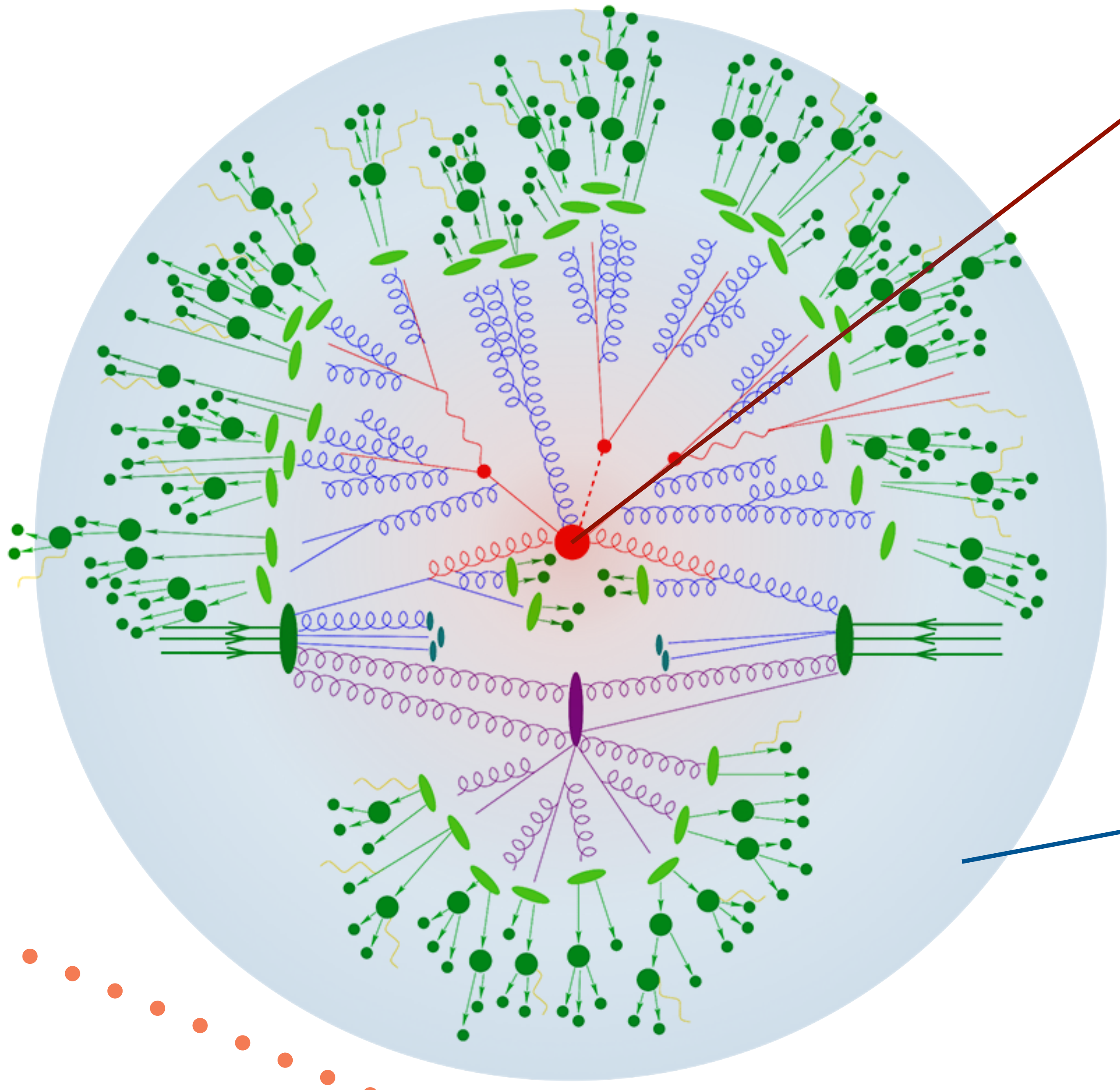
FIXED-ORDER CALCULATIONS*

* focus on *recent* results that are *representative* for on-going *progress* and *relevant* for phenomenology *(personal selection)*

Alexander Huss

CERN





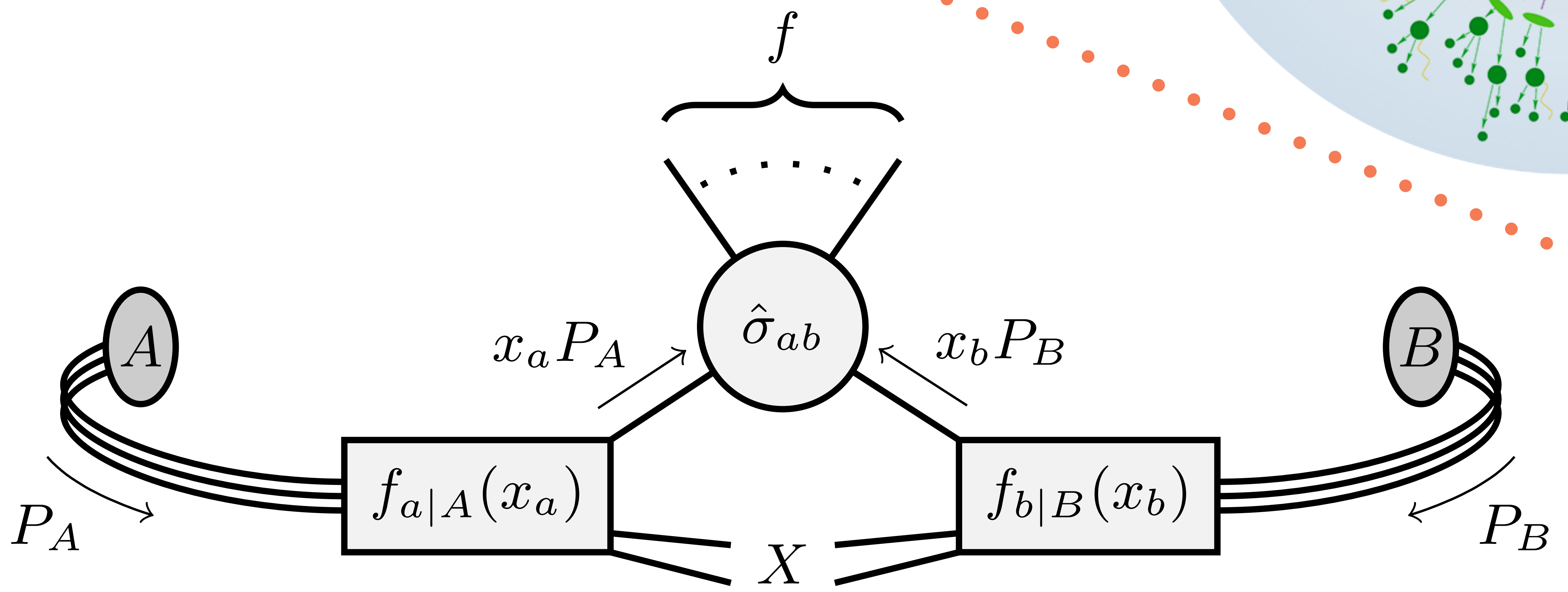
Short distance "hard"

- high scales: 10^2—10^3 GeV

evolution towards a physical observable state

Long distance "soft"

- low scales: $\mathcal{O}$(few GeV)



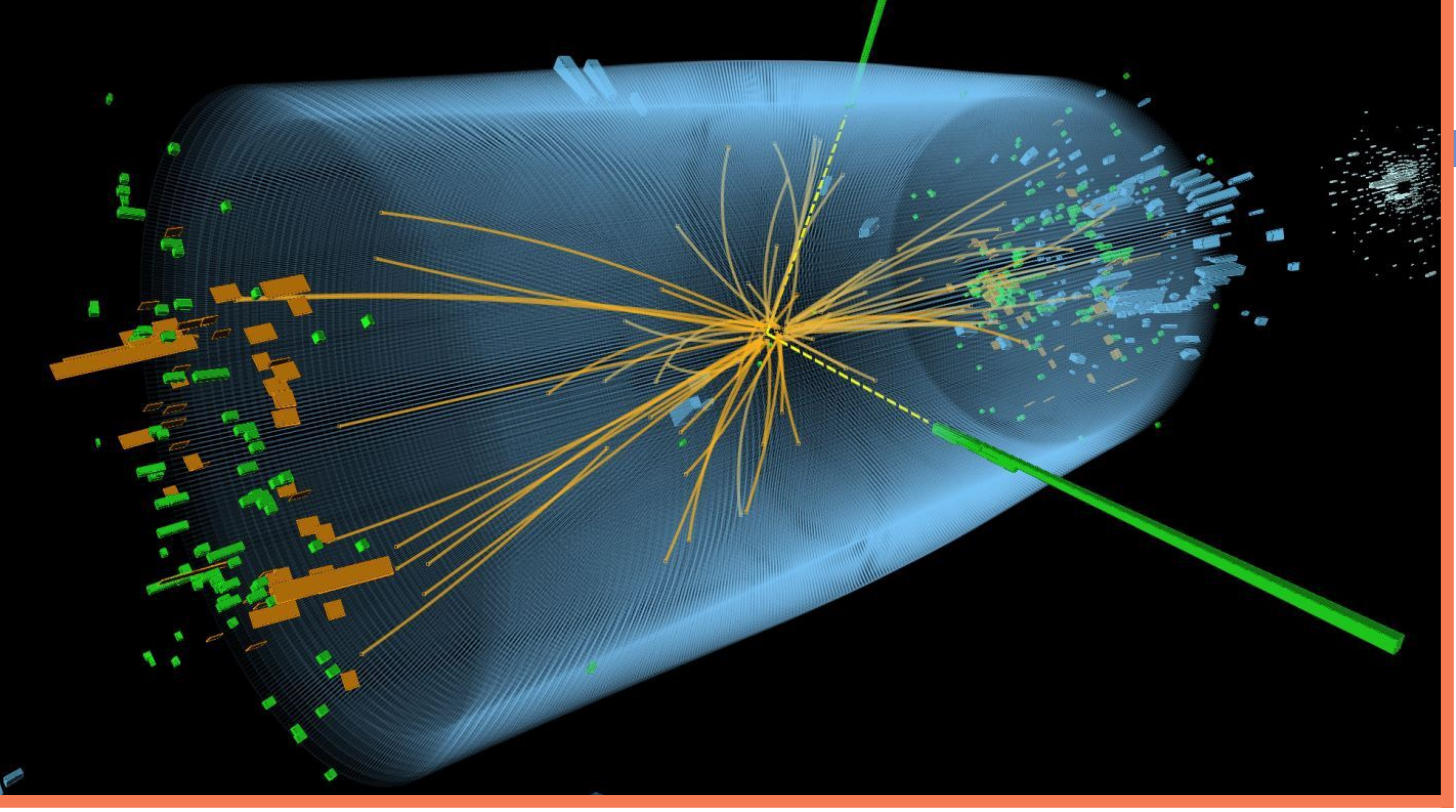
Focus:
high momentum transfer
& clean signatures

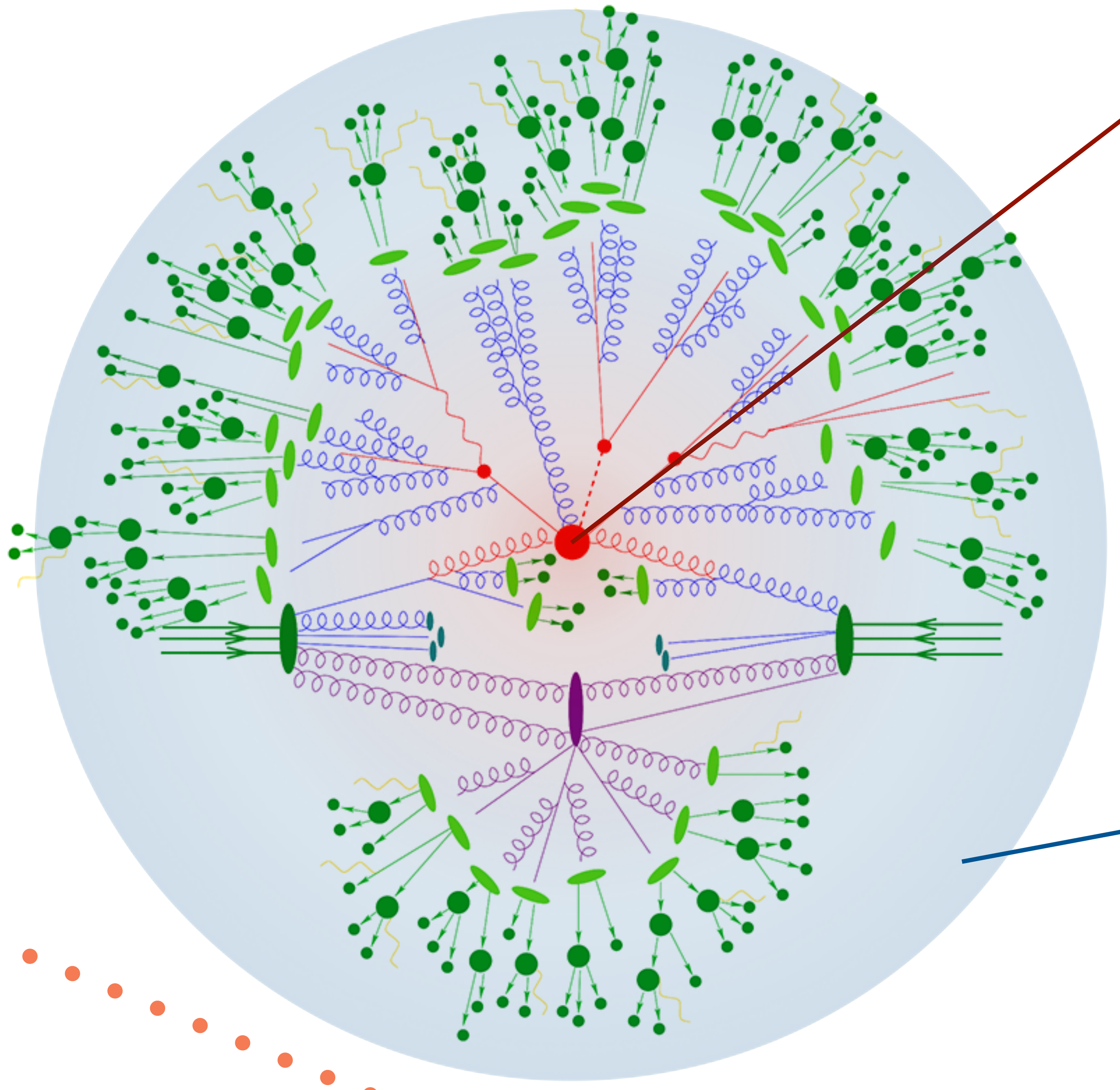
$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_{a|A}(x_a)\, f_{b|B}(x_b)\; \hat{\sigma}_{ab}(x_a, x_b)\; (1+\mathcal{O}(\Lambda_{\mathrm{QCD}}^p/Q^p))$$

parton distribution functions (PDFs)
(non-perturbative, universal)

hard scattering
(perturbation theory)

non-perturbative effects
(power suppressed)





Short distance "hard"

- high scales: 10^2—10^3 GeV

evolution towards a physical observable state

Long distance "soft"

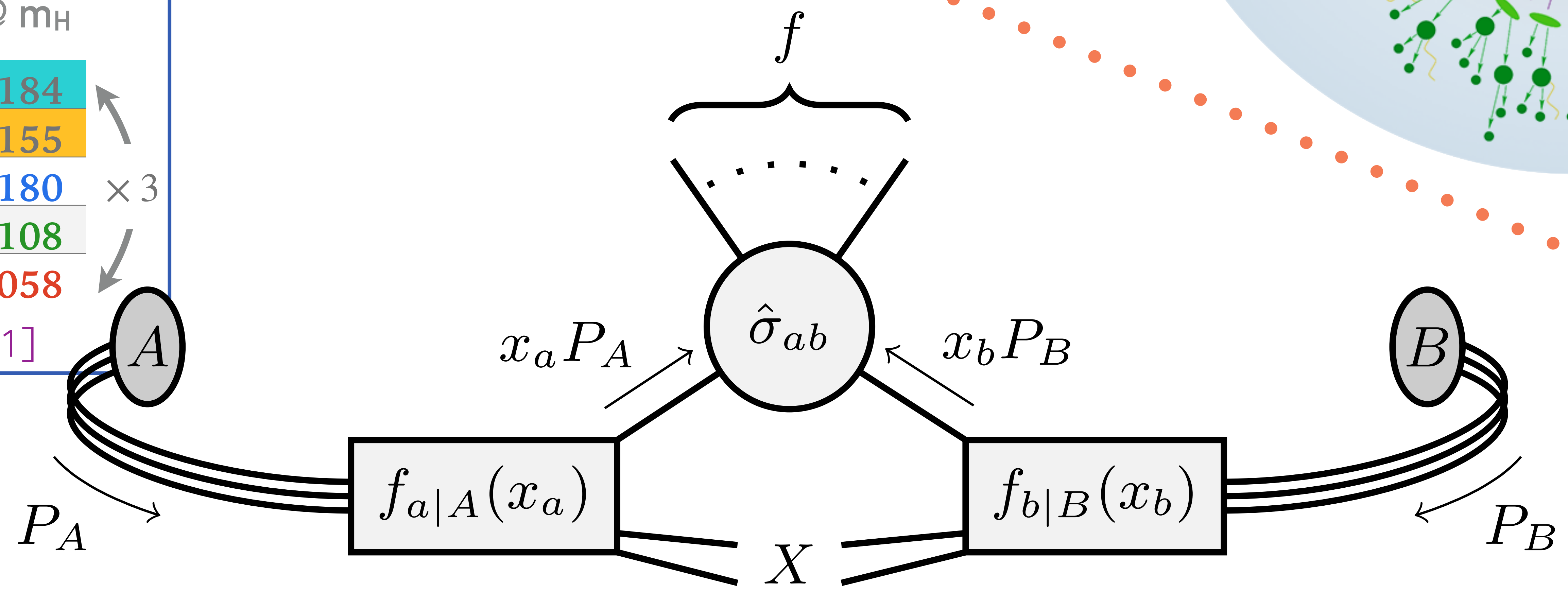
- low scales: $\mathcal{O}$(few GeV)

gg-lumi, ratio to PDF4LHC15 @ m_H

PDF4LHC15	1.0000	±	0.0184
PDF4LHC21	0.9930	±	0.0155
CT18	0.9914	±	0.0180
MSHT20	0.9930	±	0.0108
NNPDF40	0.9986	±	0.0058

× 3

[from slide by G.Salam—Higgs21]



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(non-perturbative, universal)

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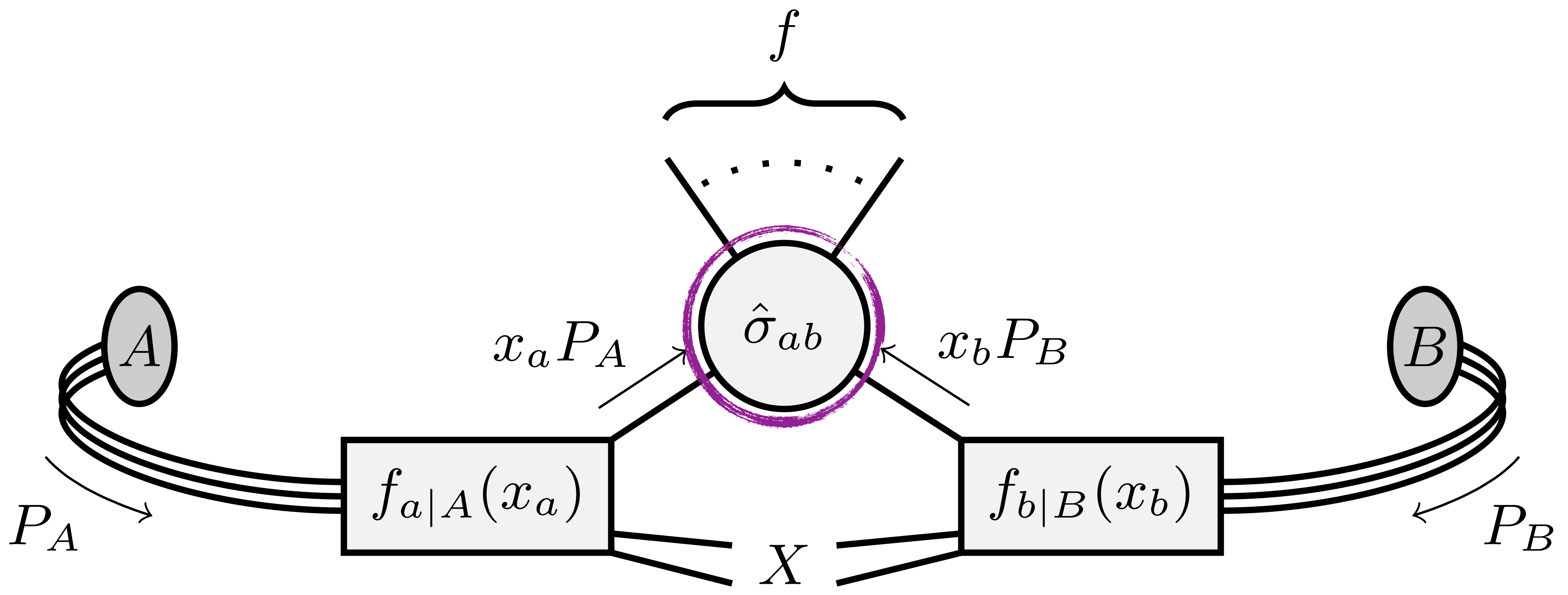
non-perturbative effects
(power suppressed)

What is the power p?
[Ferrario Ravasio, Limatola, Nason '20; +Caola, Melnikov '21]

NLO — Conceptually Solved?

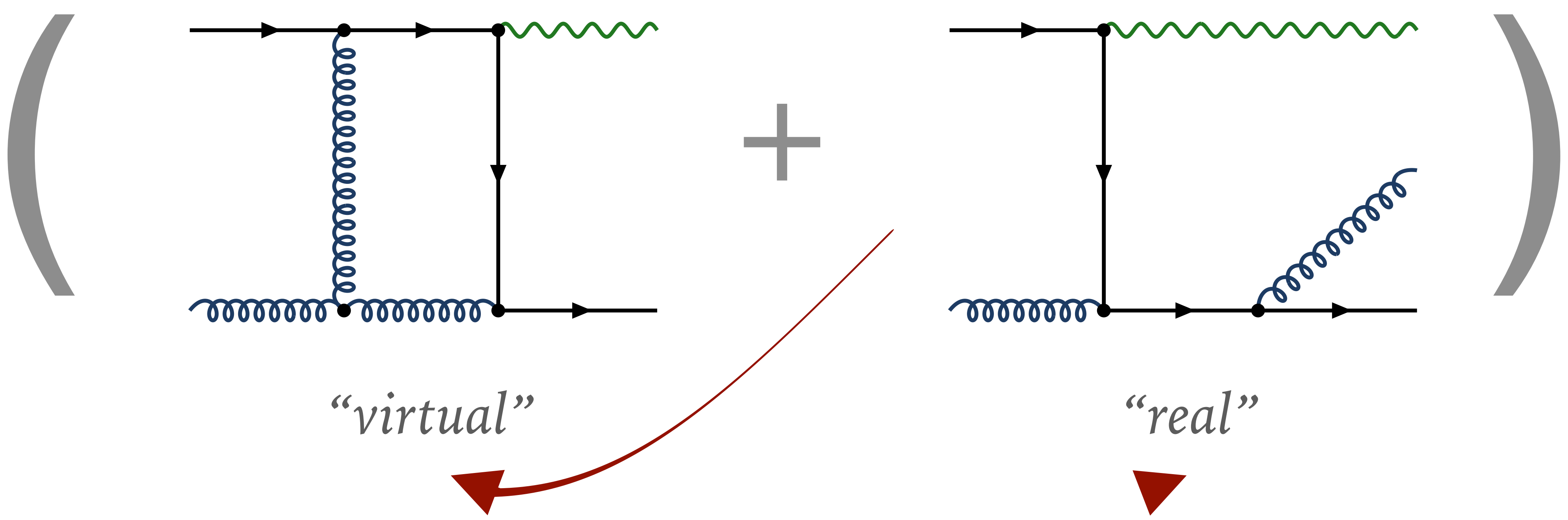
one-loop amplitudes
(all master integrals known, well understood: log, Li_2*)*



Trend: off-shell
↪ high-multiplicity
- $2 \to 8$ $(t\bar{t}W)$ NLO QCD+EW [Denner, Pellicioli, Schwan '22]
- $2 \to 9$ $(t\bar{t}W + j)$ NLO QCD [Bi, Kraus, Reinartz, Worek '23]

More frontiers:
- loop-induced
- polarization
- …

automated 1-loop providers
- Gosam [Chiesa et al. '14]
- MadGraph5_aMC@NLO [Frixione et al. '18]
- NLOX [Honeywell et al. '18]
- OpenLoops [Pozzorini et al. '19]
- Recola [Actis et al. '16]
- …



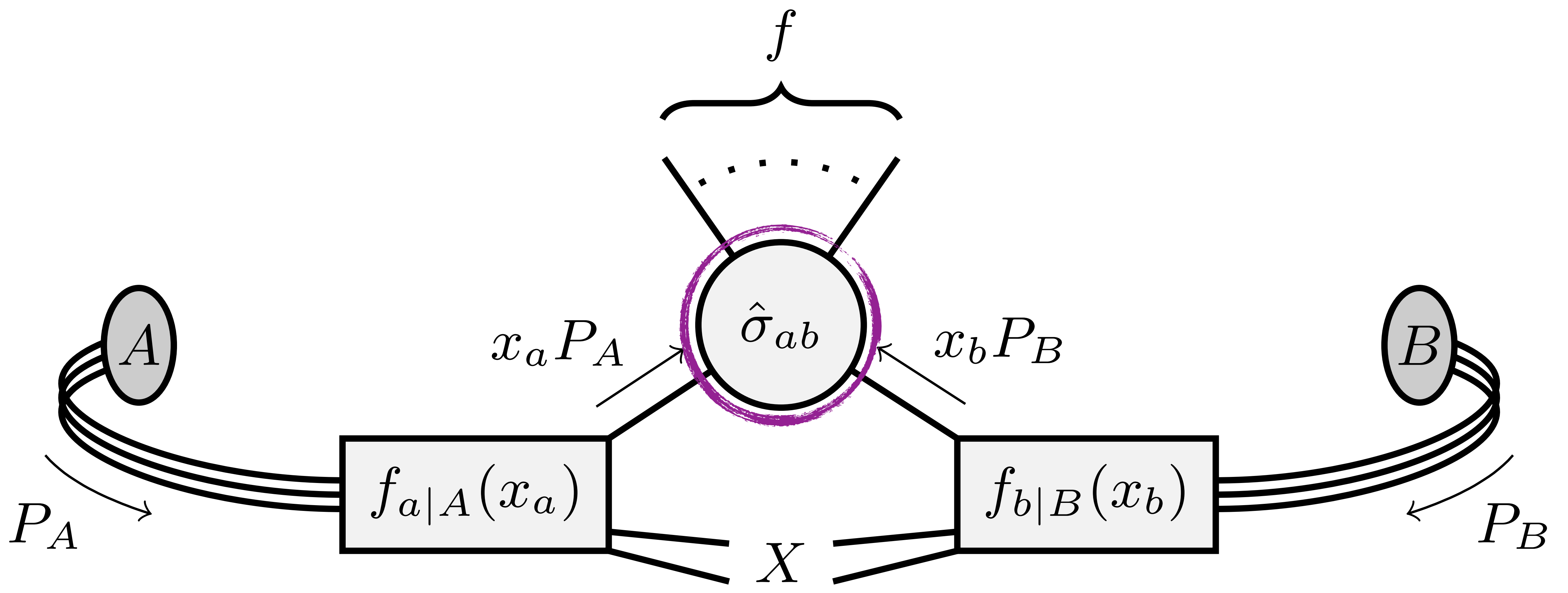
automated NLO subtraction
- dipoles [Catani, Seymour '96]
- FKS [Frixione, Kunszt, Signer '96]
- …

IR subtraction
(fully automated & efficient)

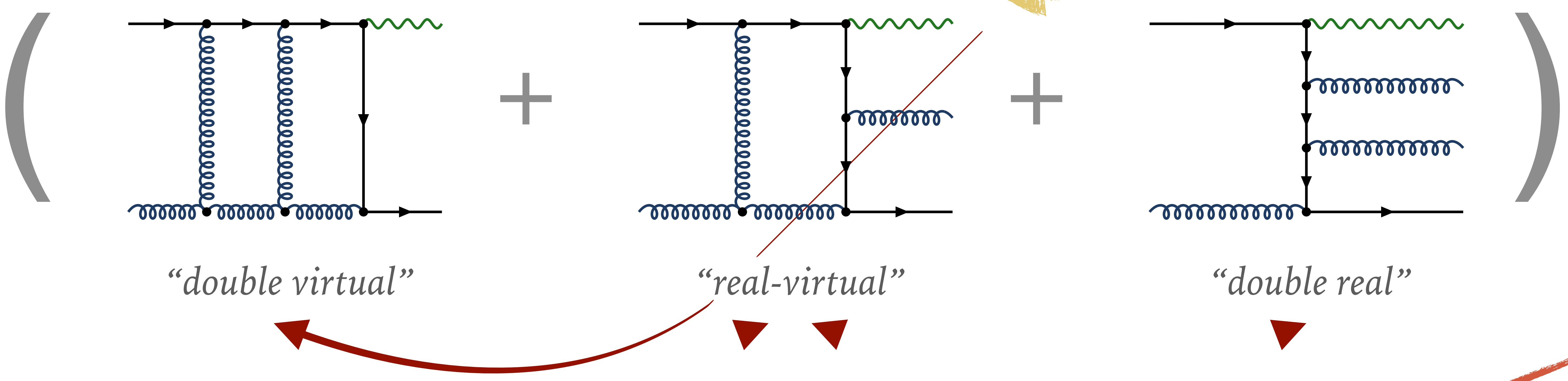
infrared singularities

NNLO – The Building Blocks & Challenges

two-loop amplitudes
(new class of functions, combinatoric & algebraic complexity)



one-loop amplitudes
(evaluation in singular & unstable regions)



IR subtraction
(involved IR structure, numerical stability, construction)

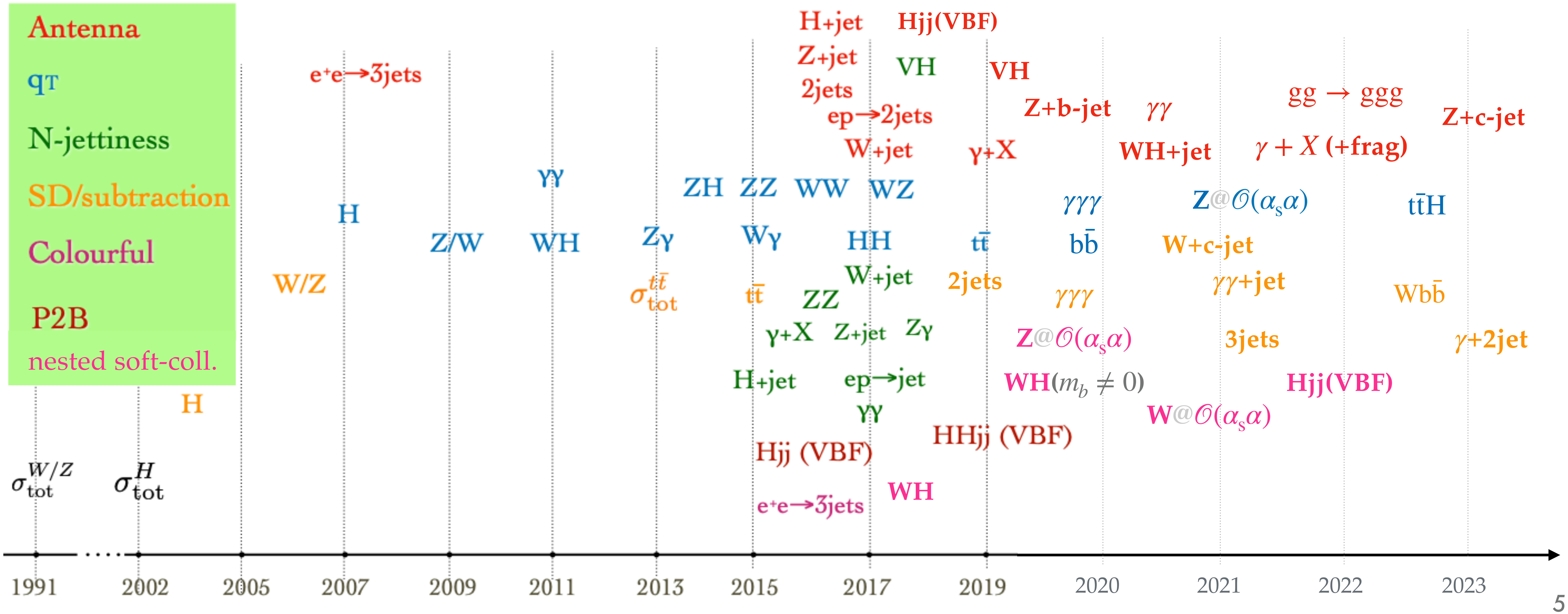
infrared singularities

WHAT CAN WE DO TODAY? — THE NNLO TIMELINE

[adapted from slide by M. Grazzini]

Tremendous progress in the past ~ 10 years!

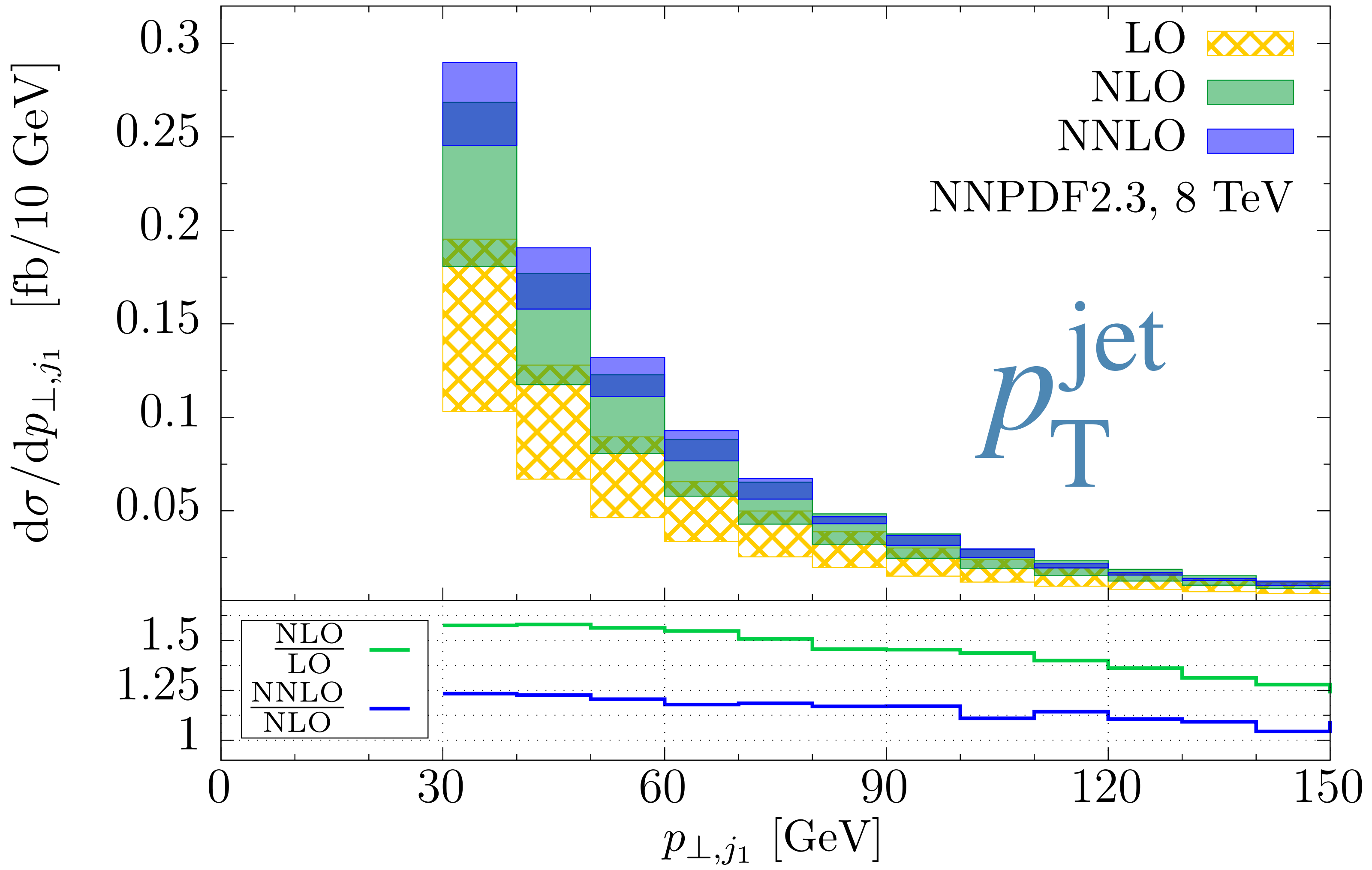
↪ $2 \to 2$ under good control; $2 \to 3$ steady progress



NNLO Reaching Maturity

"Standard" $2 \to 2$ well established ⇝ independent calculations *(validation!)*

H+jet × 3



residue subtraction

[Caola, Melnikov, Schulze '15]

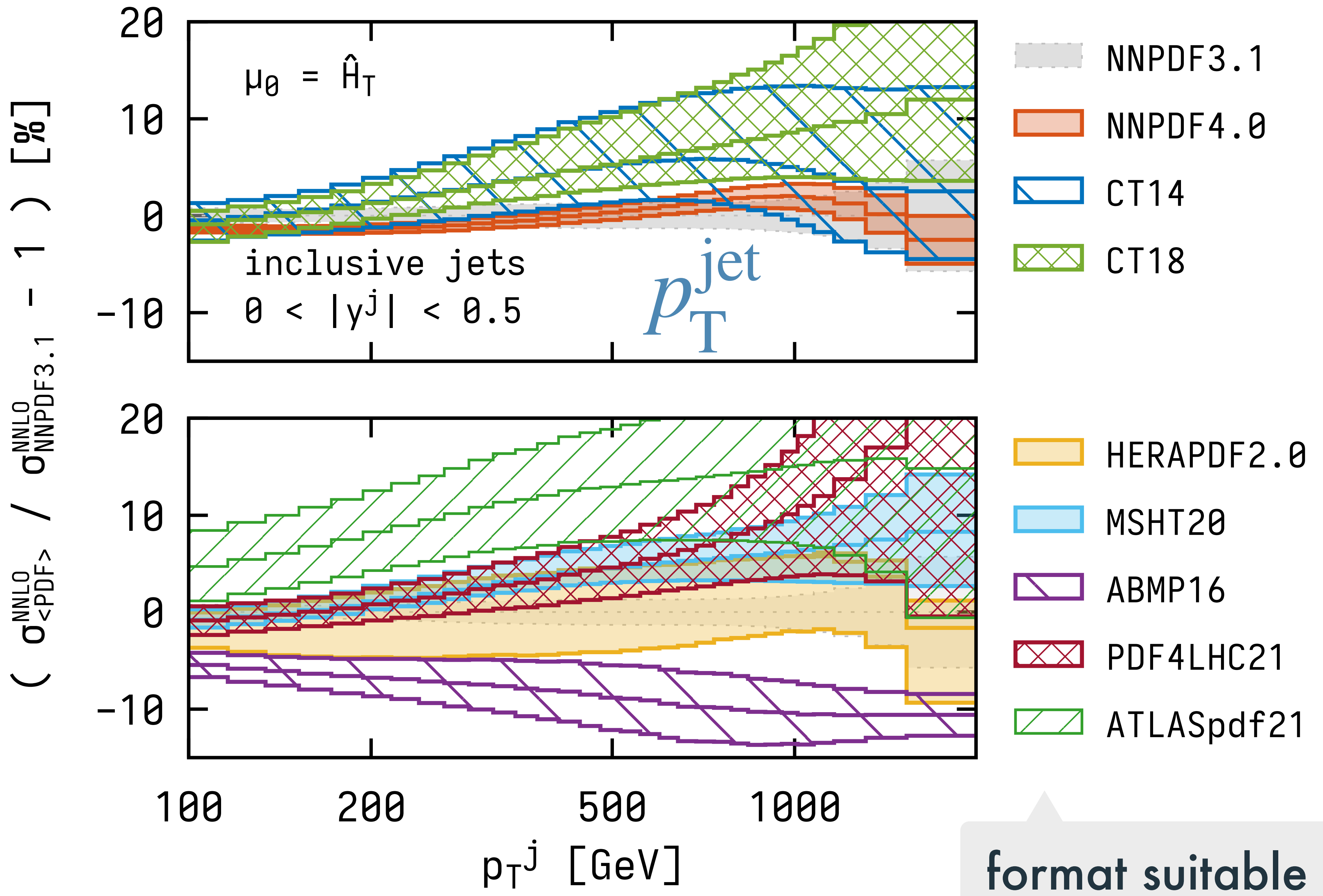
τ_1 jettiness subtraction

[Boughezal, Focke, Giele, Liu, Petriello '15]
[Campbell, Ellis, Seth '19]

antenna subtraction

[Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '16]

jets × 2



format suitable for PDF fits
[NNLOJET+APPLfast '22]

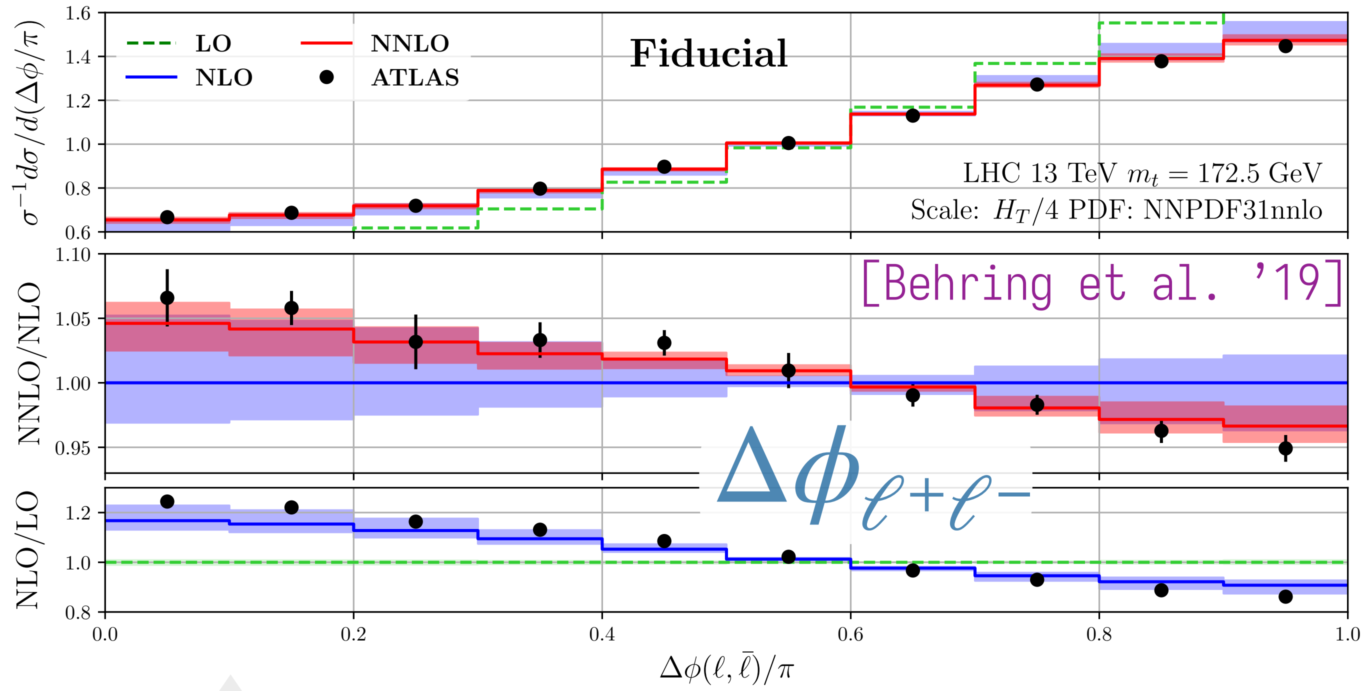
antenna subtraction

[Currie, Glover, Pires '16] (LC)
[Chen, Gehrmann, Glover, AH, Mo '22]

Stripper

[Czakon, van Hameren, Mitov, Poncelet '19]

Top Pairs – $t\bar{t}$ × 2



comparison in fiducial volume essential for agreement

Stripper

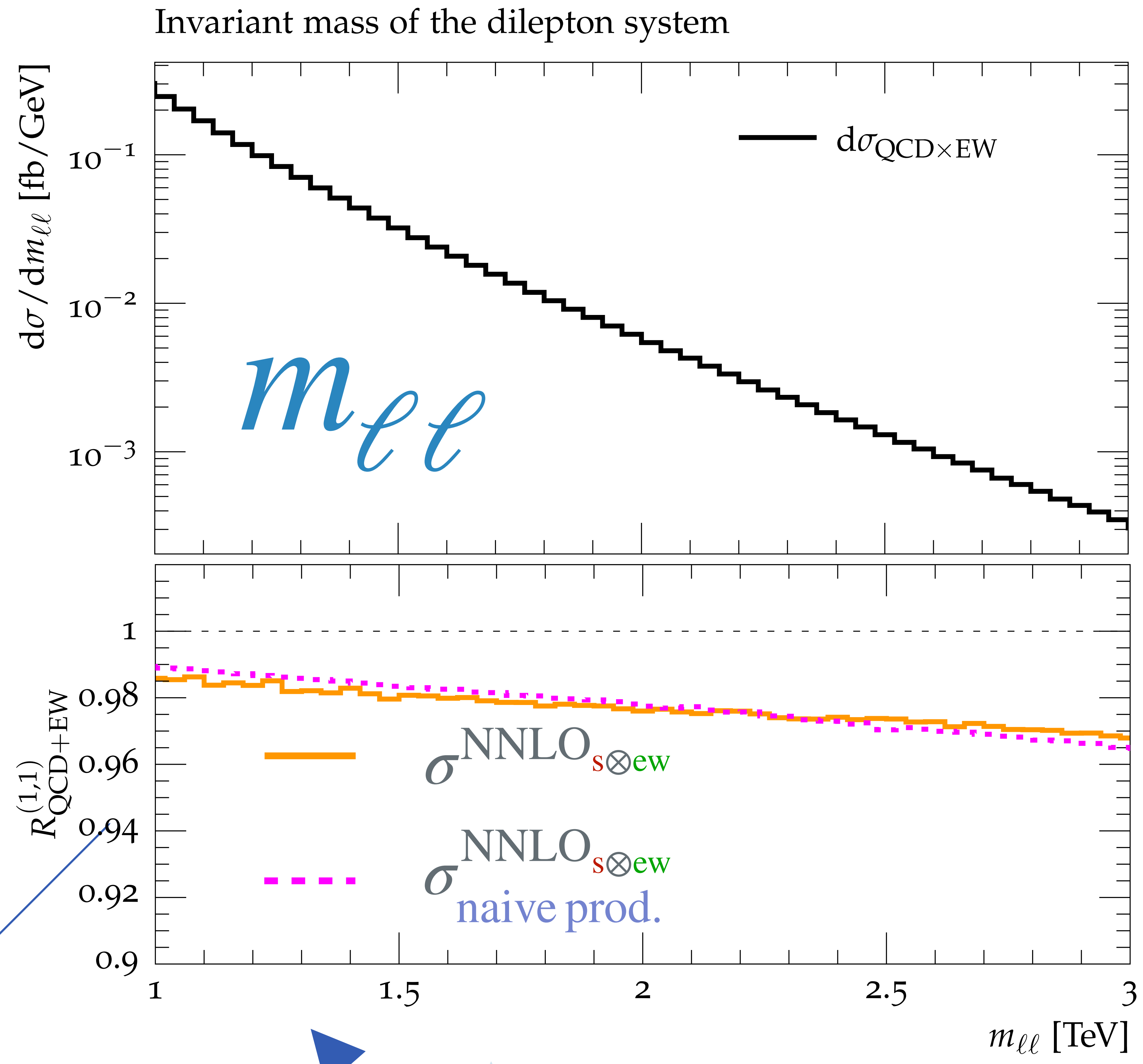
[Czakon, Heymes, Mitov '15]

q_T subtraction

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli '19]

Beyond "Standard" 2 → 2 Calculations

- adding flavour (also: $Wb\bar{b}$)
  - Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Majer '20]
  - W+c-jet [Czakon, Mitov, Pellen, Poncelet '20,'23]
  - Z+c-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Garcia, Stagnitto '23]
- adding masses
  - $pp \to WH\ (H \to b\bar{b})$ [Behring, Bizoń, Caola, Melnikov, Röntsch '20]
  - $pp \to b\bar{b}$ [Catani, Devoto, Grazzini, Kallweit, Mazzitelli '21]
- identified particles / fragmentation
  - hadron fragmentation [Czakon, Generet, Mitov, Poncelet '21,'22]
  - isolated photons [Gehrmann, Schürmann '22; + Chen, Glover, Höfer, AH '22]
- mixed QCD×EW
  - full off-shell Drell-Yan
    - W [Buonocore, Grazzini, Kallweit, Savoini, Tramontano '21];
    - Z [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '21]; [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Rontsch, Signorile-Signorile '22]
- beyond approximations
  - non-factorizable corrections
    - VBF [Liu, Melnikov, Penin '19]; [Dreyer, Karlberg, Tancredi '20]; [... '23]
    - single-t [Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile, Wang '22]
  - Higgs beyond HTL ($m_t \to \infty$) [Czakon, Harlander, Klappert, Niggetiedt '20]



$NNLO_{s\otimes ew} \sim -1\,\%$ on σ^{fid}

naive product can't capture kinematic features (resonance/shoulder)

but works remarkably well in high-energy tails (Sudakov logs fact.)

Beyond "Standard" $2 \to 2$ Calculations

- [2] **adding flavour** (also: $Wb\bar{b}$)
  - Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Majer '20]
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- **beyond approximations**
  - [1] non-factorizable corrections
    - VBF [Liu, Melnikov, Penin '19]; [Dreyer, Karlberg, Tancredi '20]; [... '23]
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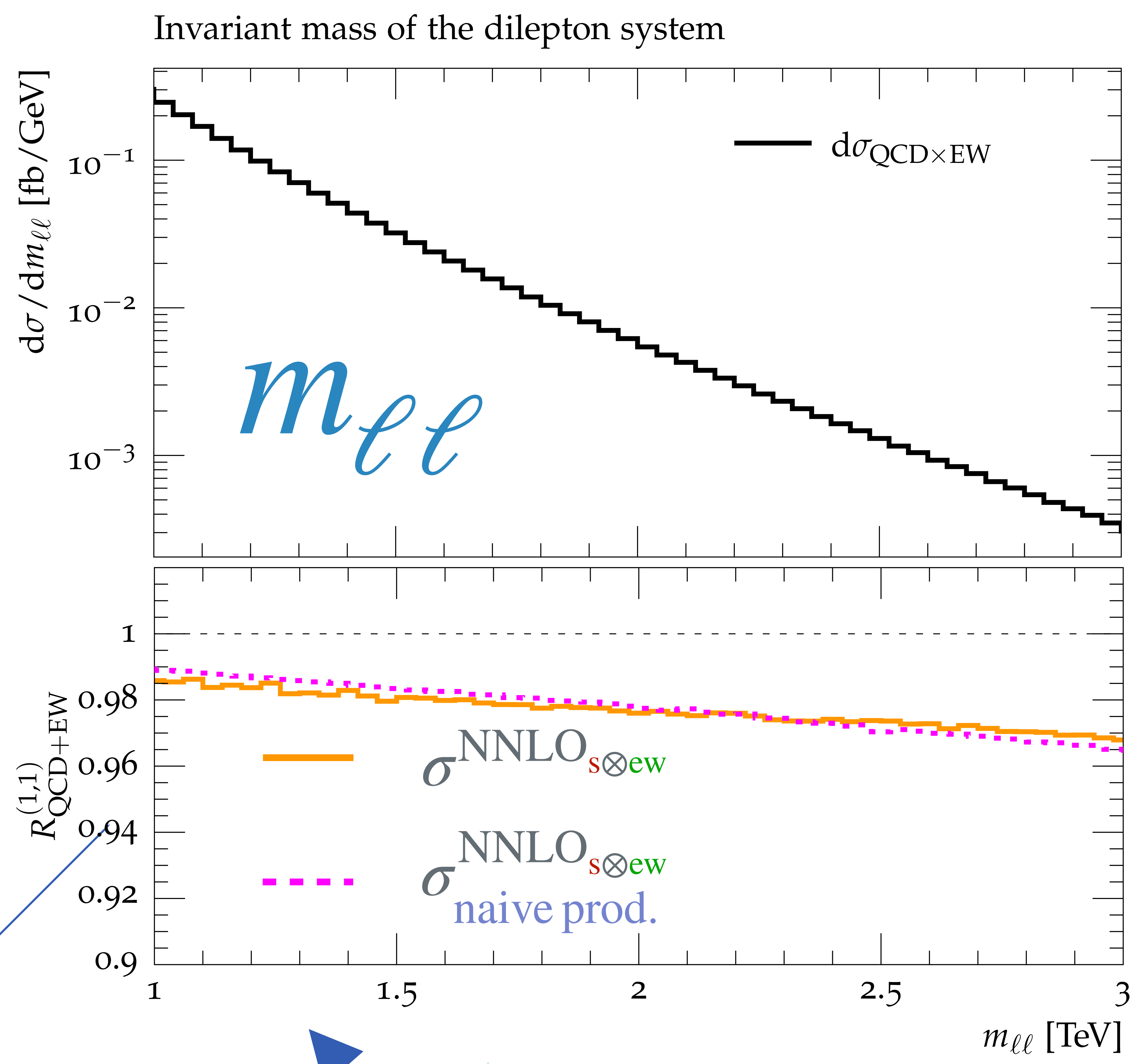
  - Higgs beyond HTL ($m_t \to \infty$) [Czakon, Harlander, Klappert, Niggetiedt '20]



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but works remarkably well in high-energy tails (Sudakov logs fact.)

Non-Factorizable Corrections

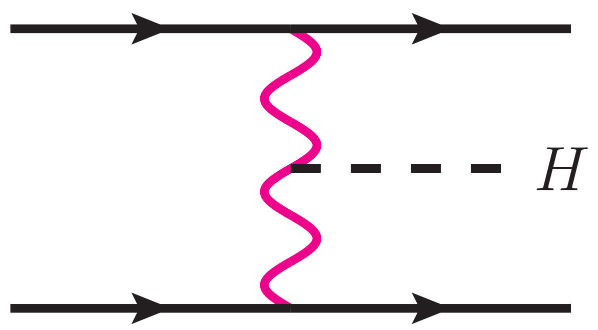
<u>NLO</u>: vanishes <u>NNLO</u>: $\times (N_c^2 - 1)^{-1}$

↪ assumed small but can be π^2 enhanced?

- only Abelian gluons; UV finite
- no collinear sing. (only soft)

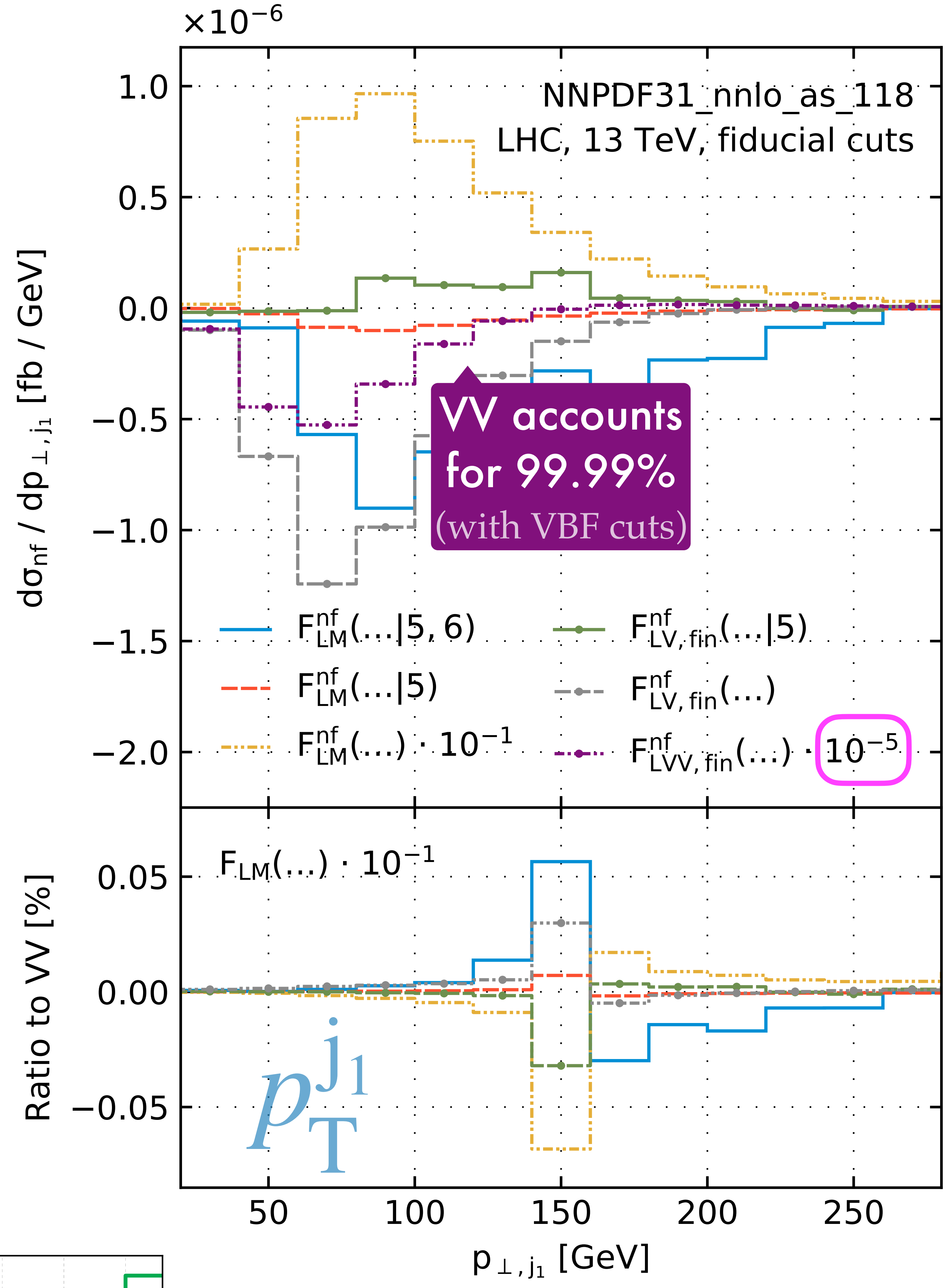
VBF

$q\,Q \to q'\,Q' + H$



Full RR, RV,
VV expanded in $p_T^j/\sqrt{s}$

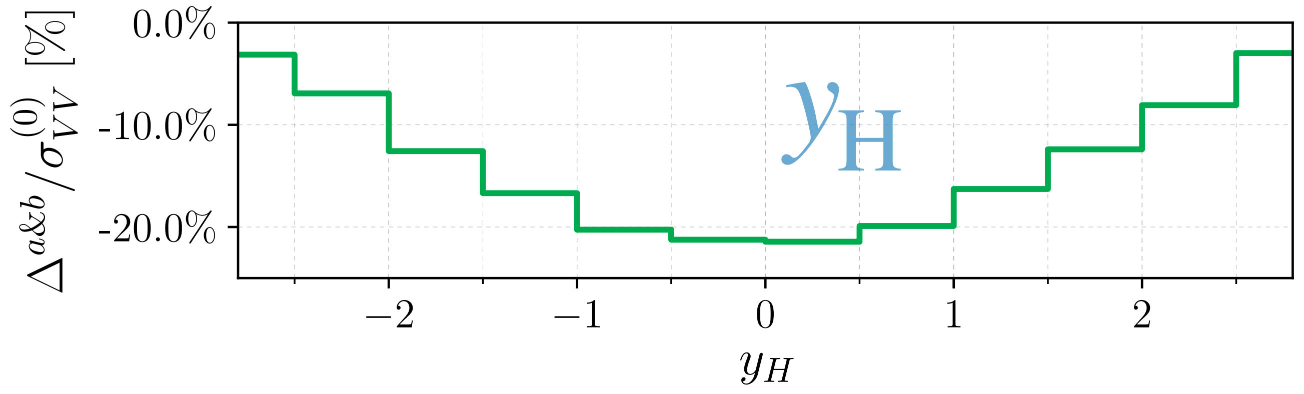
- $\sigma_{\text{nf}}^{\text{NNLO}} \sim -0.4\,\%\ \sigma^{\text{LO}}$
- $\mathcal{O}(1\%)$ on distributions



[Asteriadis, Brønnum-Hansen, Melnikov '23]

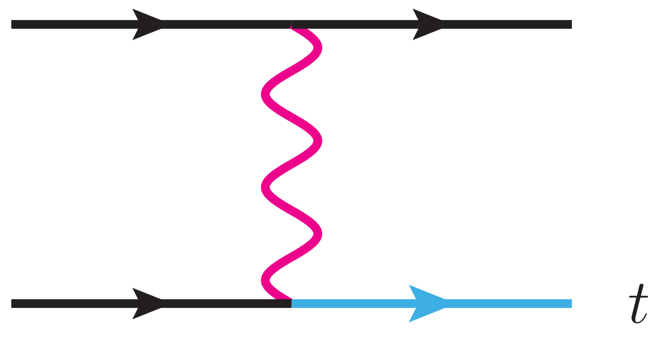
[Long, Melnikov, Quarroz '23]

next-to-eikonal correction
⇝ reduces VV by $\mathcal{O}(20\%)$



Single top

$q\,b \to q'\,t$



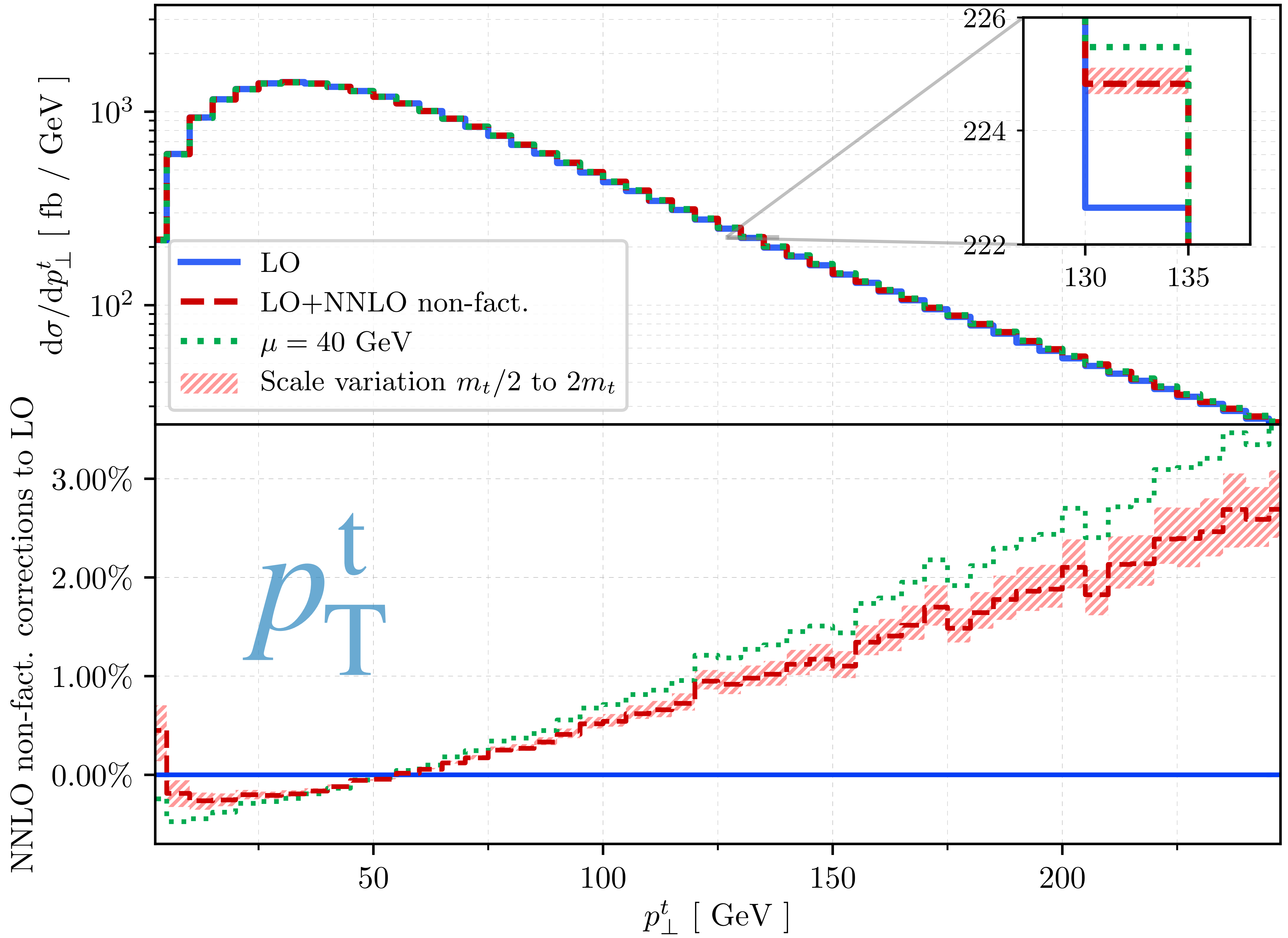
Complete non-factorizable NNLO

- $\sigma_{\text{nf}}^{\text{NNLO}} \sim +0.3\,\%\ \sigma^{\text{LO}}$
- $\mathcal{O}(1\%)$ on distributions

corrections not flat ⇝ increase to high-p_T

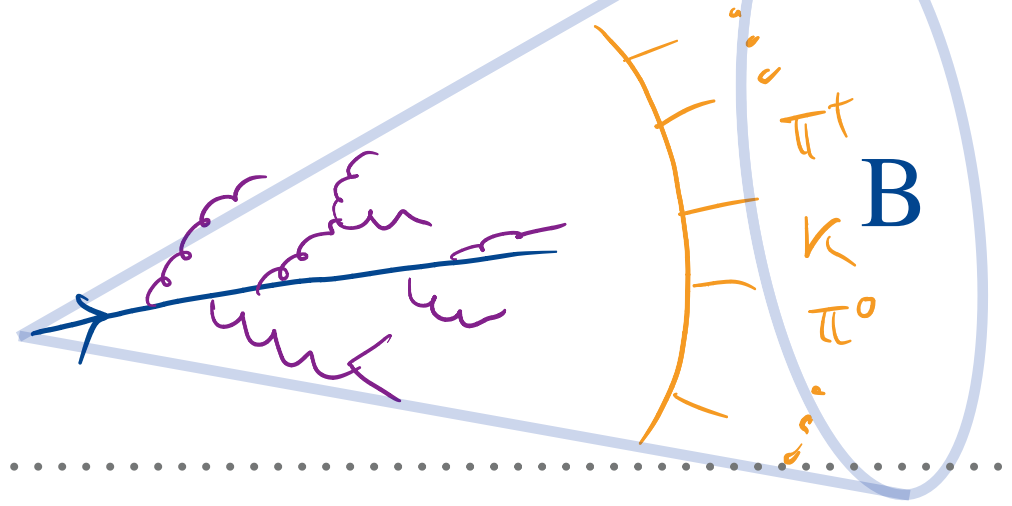
fact. ≃ 2–10× non-fact.

<u>but:</u> peak region ⇝ fact. ≃ non-fact.



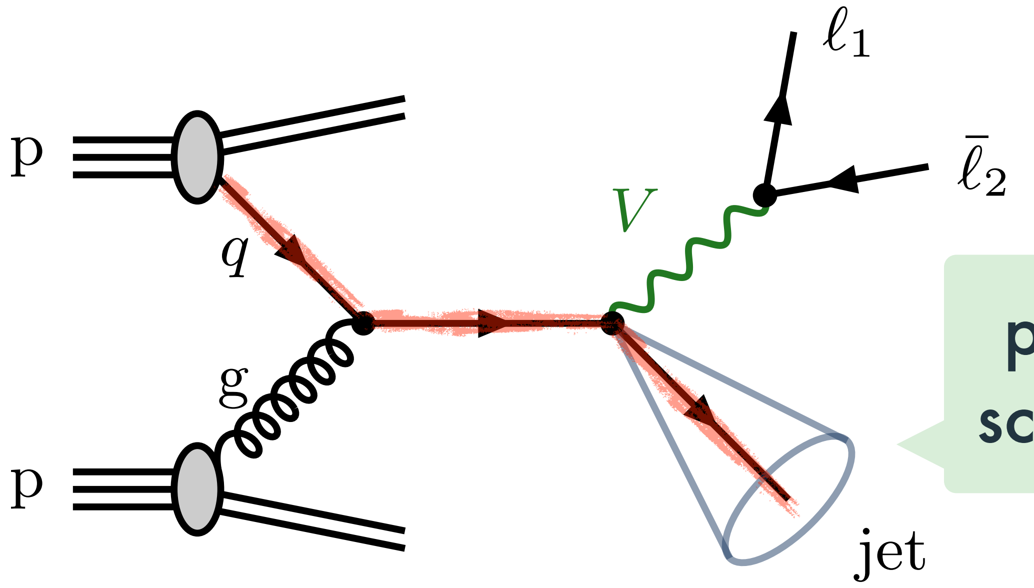
[Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile, Wang '22]

Jet Flavour ⇝ Proxy for q_f



Flavoured jets are everywhere:

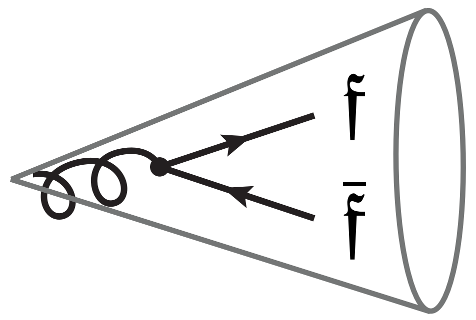
- Higgs physics ⇝ couplings
  (60% $H \to b\bar{b}$)
- top physics ⇝ PDFs, α_s, BSM
  ($|V_{tb}| \sim 1$)
- f-jet + E_T^{miss} ⇝ BSM
- $V + f$-jet ⇝ PDFs, α_s, BG



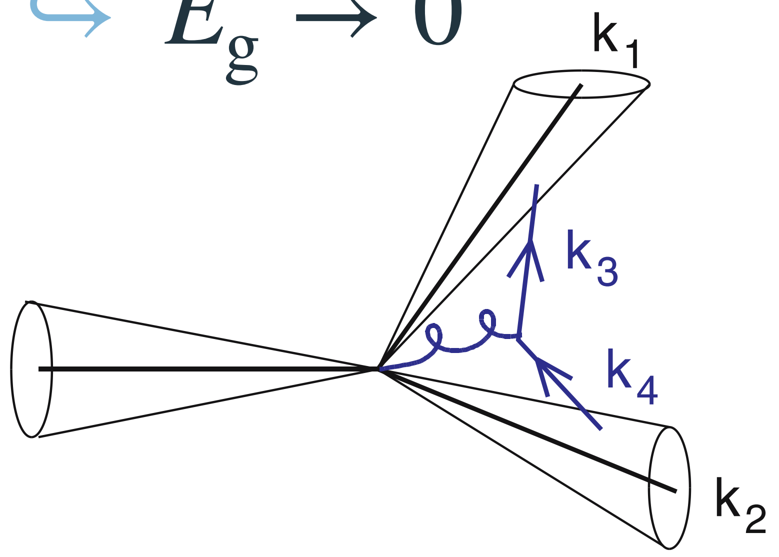
pinpoint specific scattering process

Theory: ill-defined/divergent ($m_q \equiv 0$)

- collinear (NLO)
  ↪ $g \leftrightarrow (\mathfrak{f} \parallel \bar{\mathfrak{f}})$



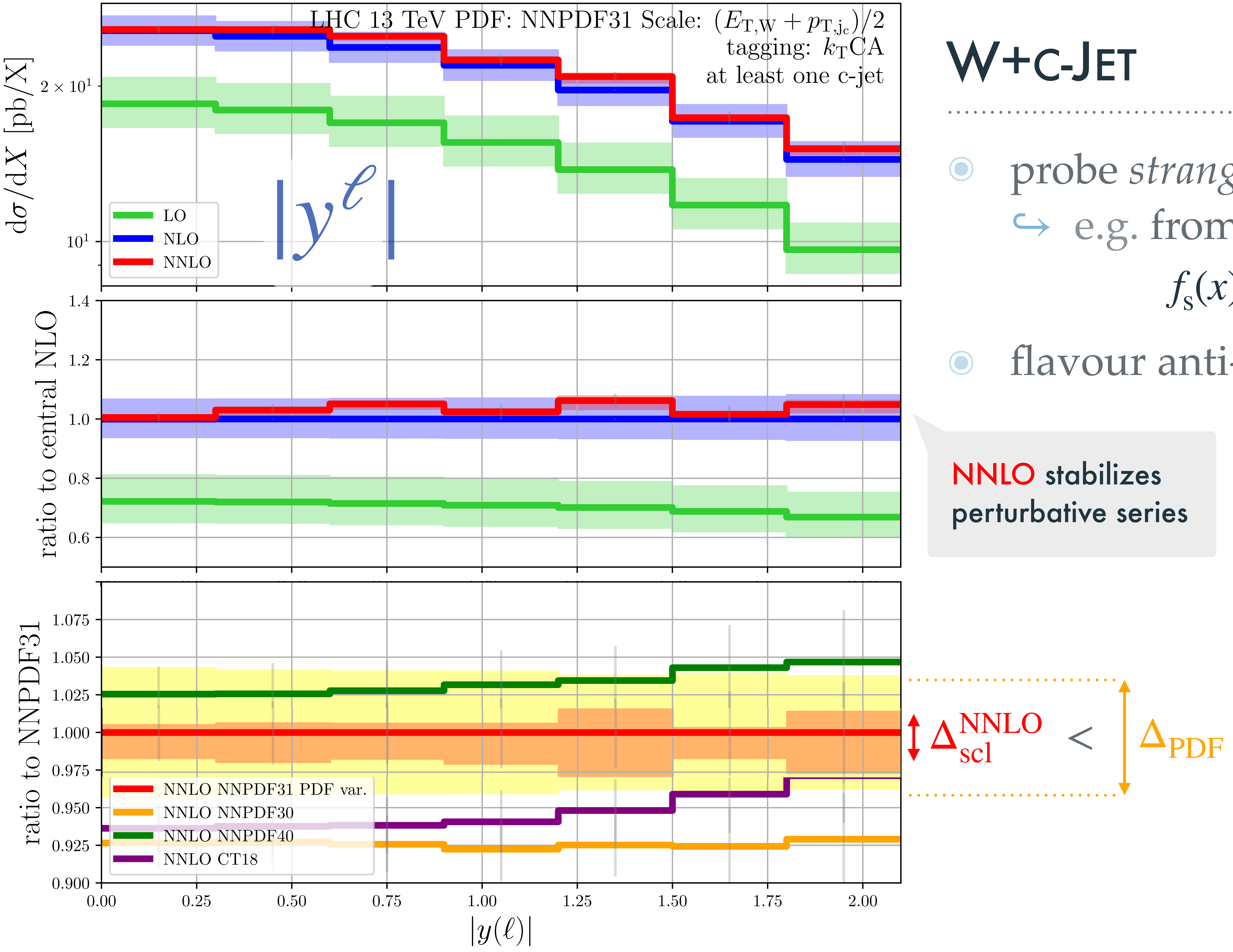
- soft (NNLO)
  ↪ $E_g \to 0$



not IR safe!

※ LHC experiments: naive anti-k_T

- *original solution:* flavour-k_T [Banfi, Salam, Zanderighi '06]
  ↪ (un-)folding, e.g. Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Majer '20]
- *new ideas* in the past year (⇝ compatibility with anti-k_T)
  - flavoured anti-k_T [Czakon, Mitov, Poncelet '22]
  - SoftDrop grooming [Caletti, Larkoski, Marzani, Reichelt '22]
  - Winner-Take-All axis [Caletti, Larkoski, Marzani, Reichelt '22]
  - flavour dressing [Gauld, AH, Stagnitto '22]
  - … [1507.00508; 1512.05265; 1702.02947; 2104.06920; 2202.05082]



W+c-Jet

[Czakon, Mitov, Pellen, Poncelet '23]

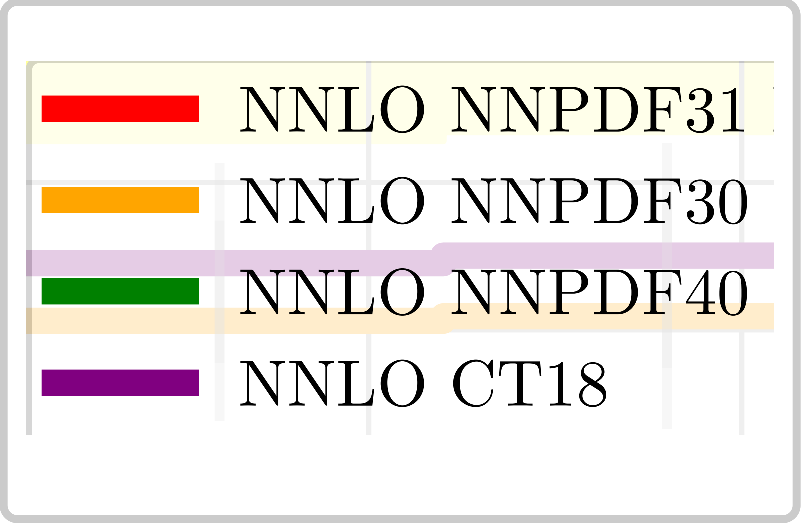
- probe *strange* content of proton
  - ↪ e.g. from 3-loops: [Catani et al. '04]

$$f_s(x) \neq f_{\bar{s}}(x)$$

- flavour anti-k_T algorithm

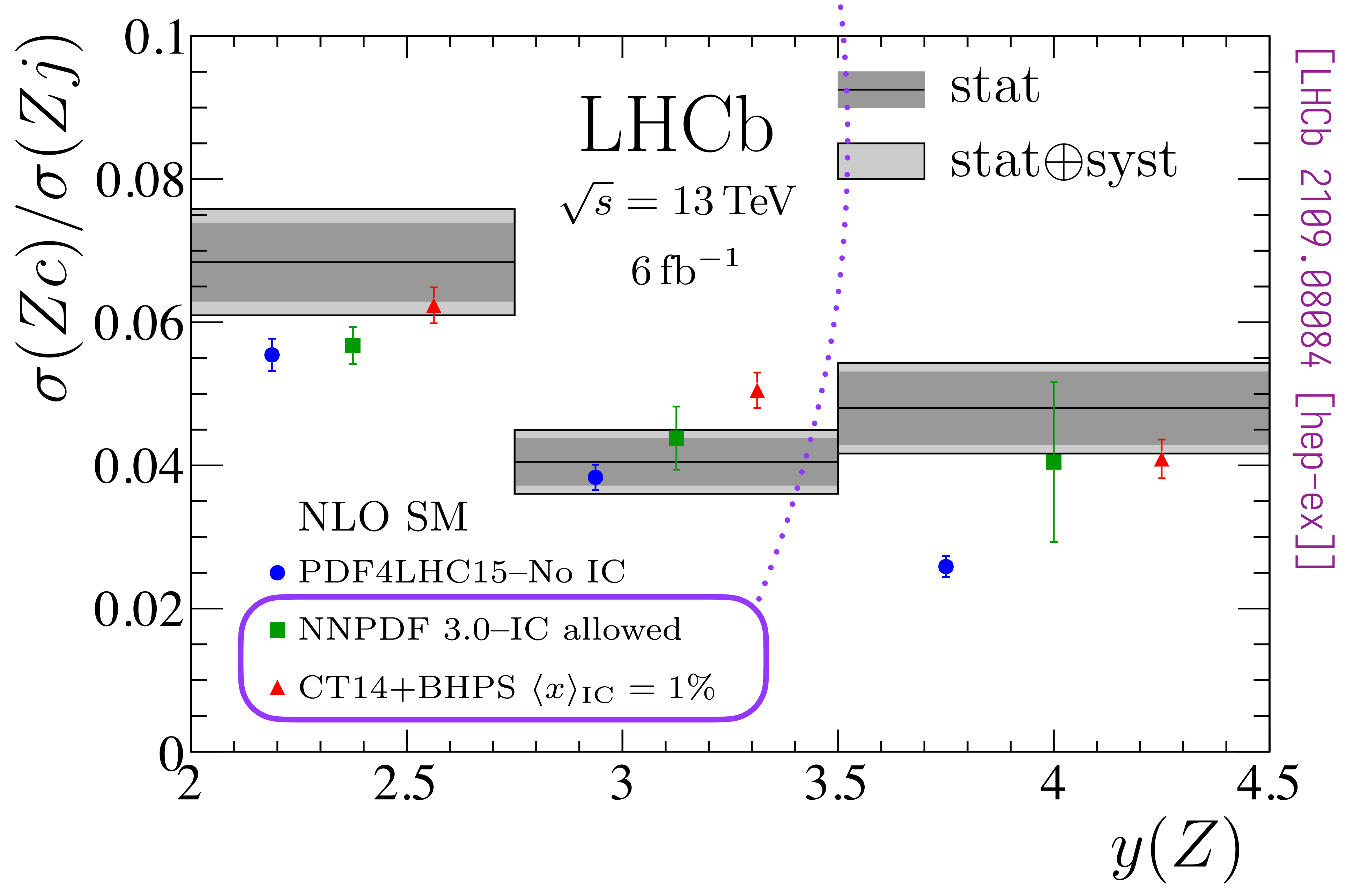
NNLO stabilizes perturbative series

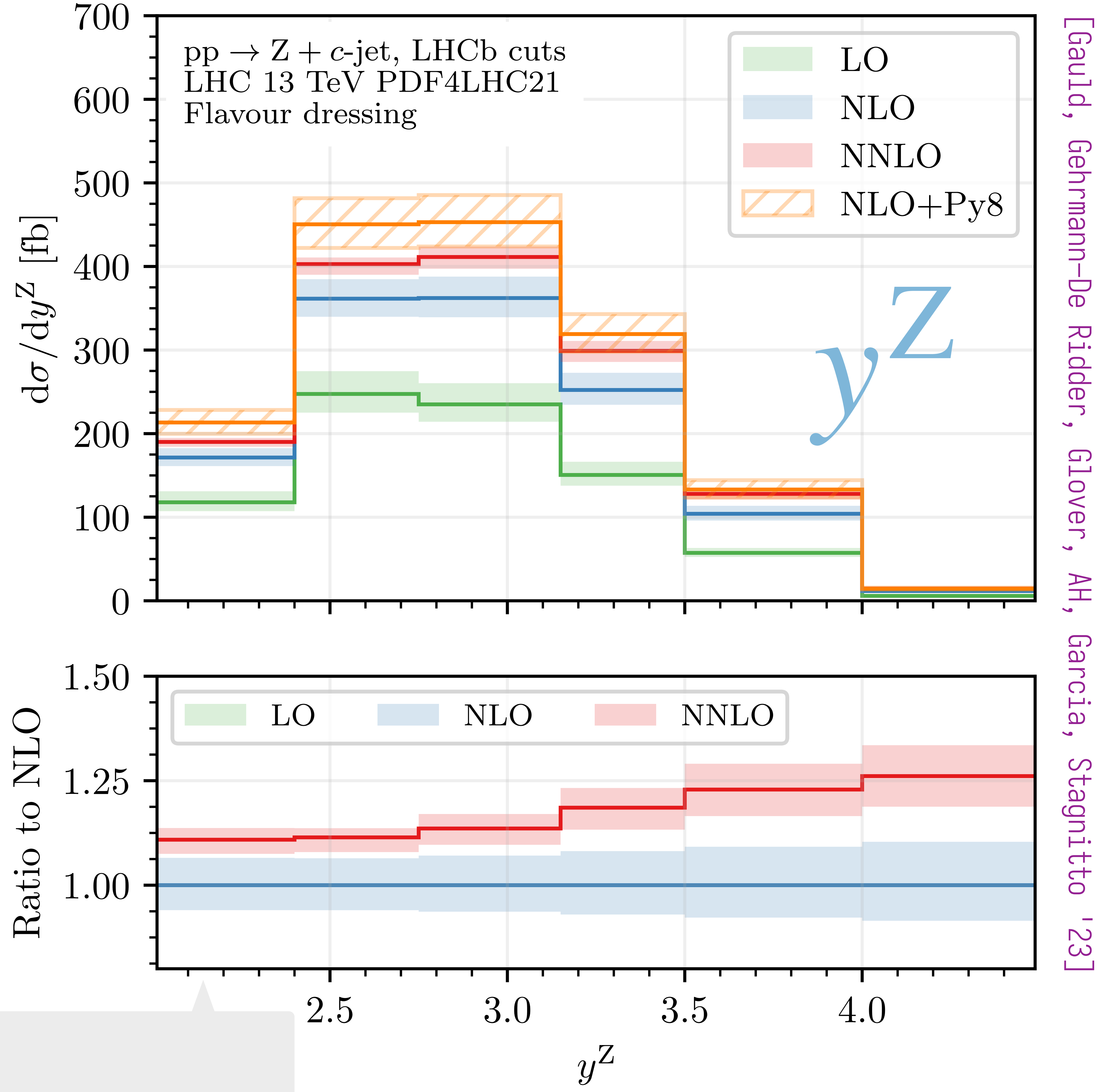
sensitivity to constrain PDFs



Z+c-Jet

- *"is there an intrinsic charm (IC) component in the proton?"*
  ↪ evidence (3σ) [NNPDF Nature 608 (2022)]
- LHCb kinematics (very forward)
  ⇝ sensitivity to IC





- NNLO 10–20%
- outside of NLO
- affects *shape* (flavour-dressing)

NNLO between Py8 & Hw7

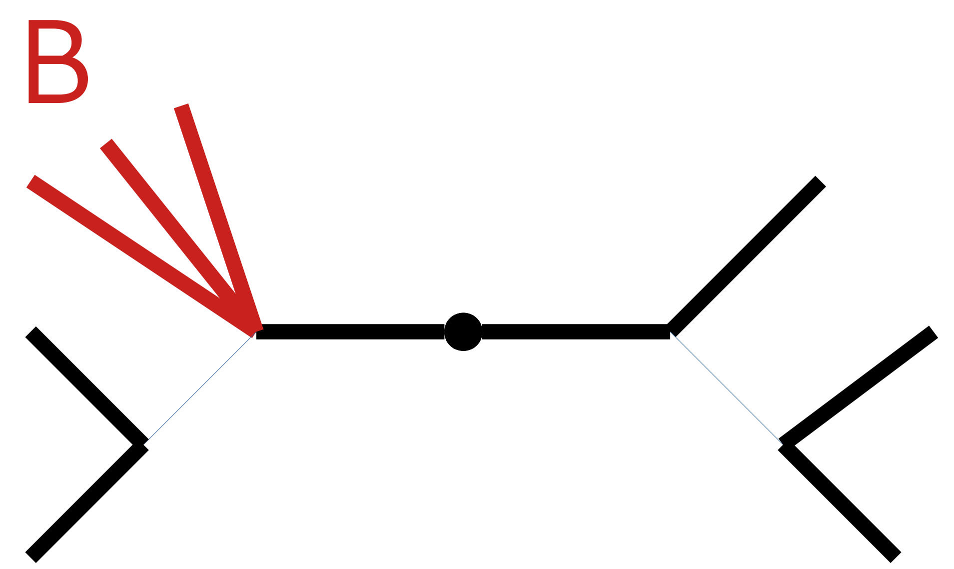
↪ Hw7 ≃ NNLO; Py8 $\xrightarrow{y^Z \gg 0}$ NNLO

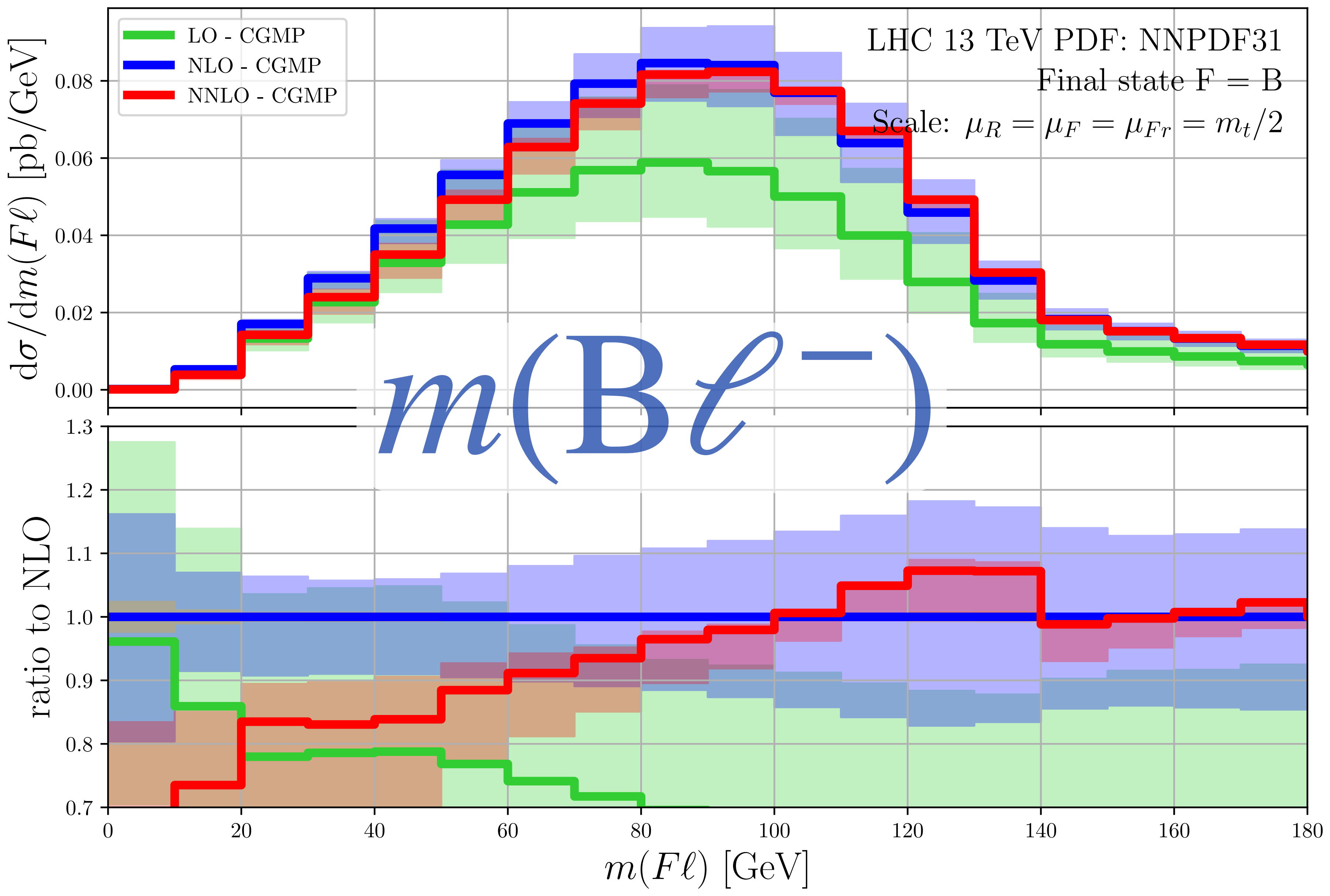
Caution: different c-jet definitions! additional IR safety issues in EXP:

- tag if *at least* one c-hadron ($g \nrightarrow c\bar{c}$)
- p_T(c-hadron) > 5 GeV ($c \nrightarrow cg$)

B-Hadron in $t\bar{t}$

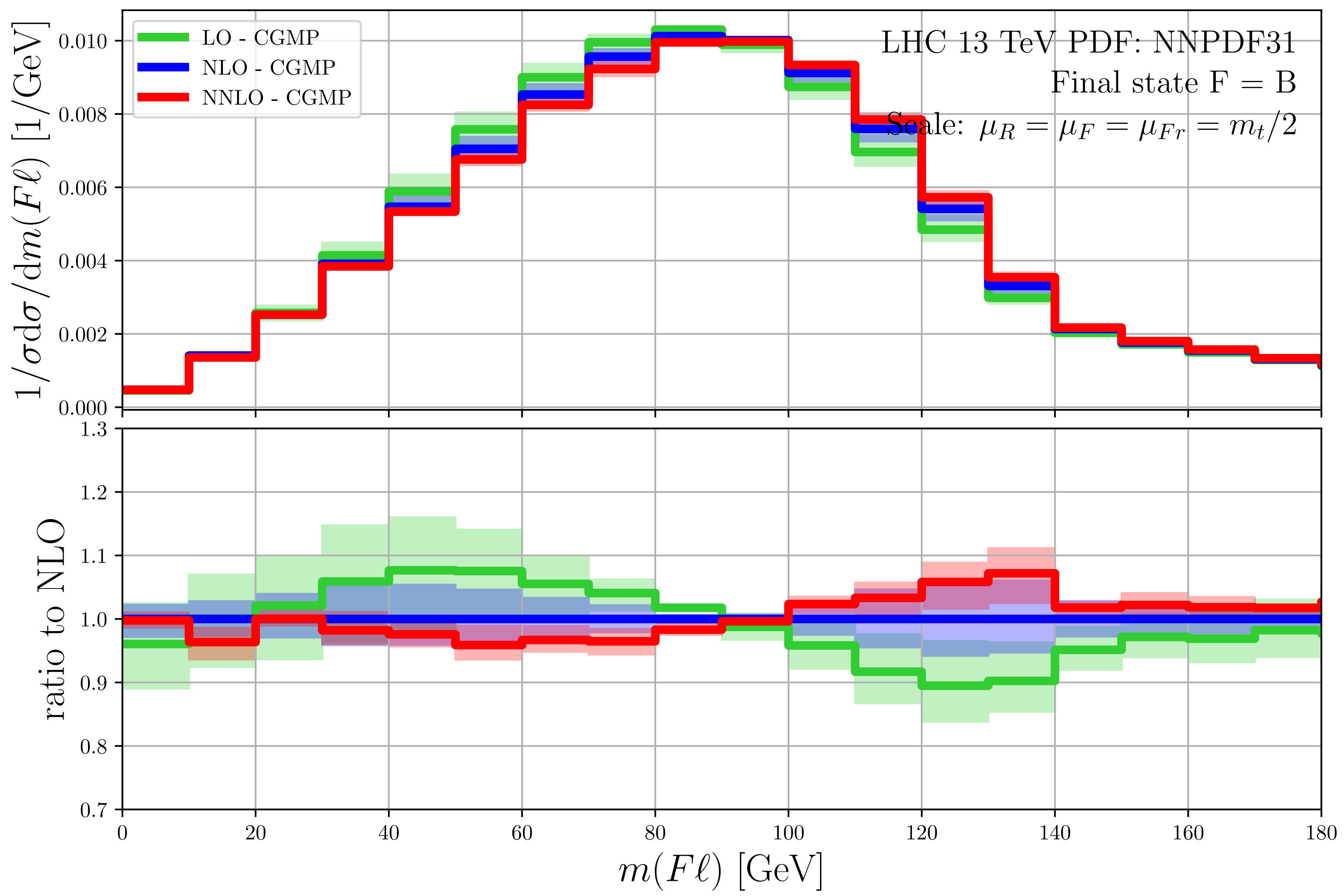
[Czakon, Generet, Mitov, Poncelet '21,'22]





non-overlap
$m(B\ell) \lesssim 50\,\mathrm{GeV}$

normalized
⊖ jet cuts



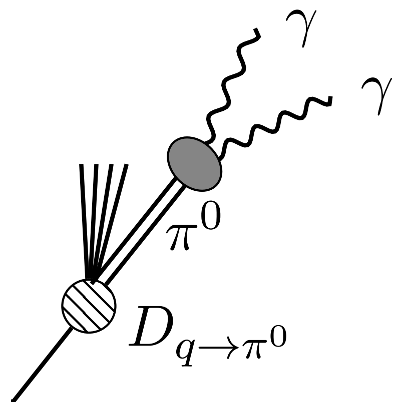
- $t\bar{t}$ ⇝ high purity & statistics
- B-hadrons measured precisely
  ↪ precise m_t extraction?
- $m_t \gg m_b$
  ↪ small power corrections
- extract $D_{i\to B}$ from e^+e^- data

shape sensitive to m_t

$\delta_{\rm NNLO}$ shape distortion
⇝ $\Delta m_t \sim 1\,\mathrm{GeV}$

Isolated Photons $\gamma + \text{jet}$

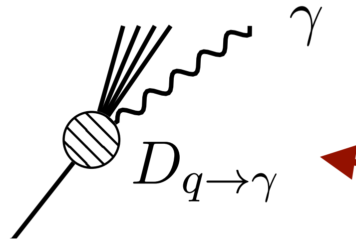
EXP: require *photon isolation* to eliminate



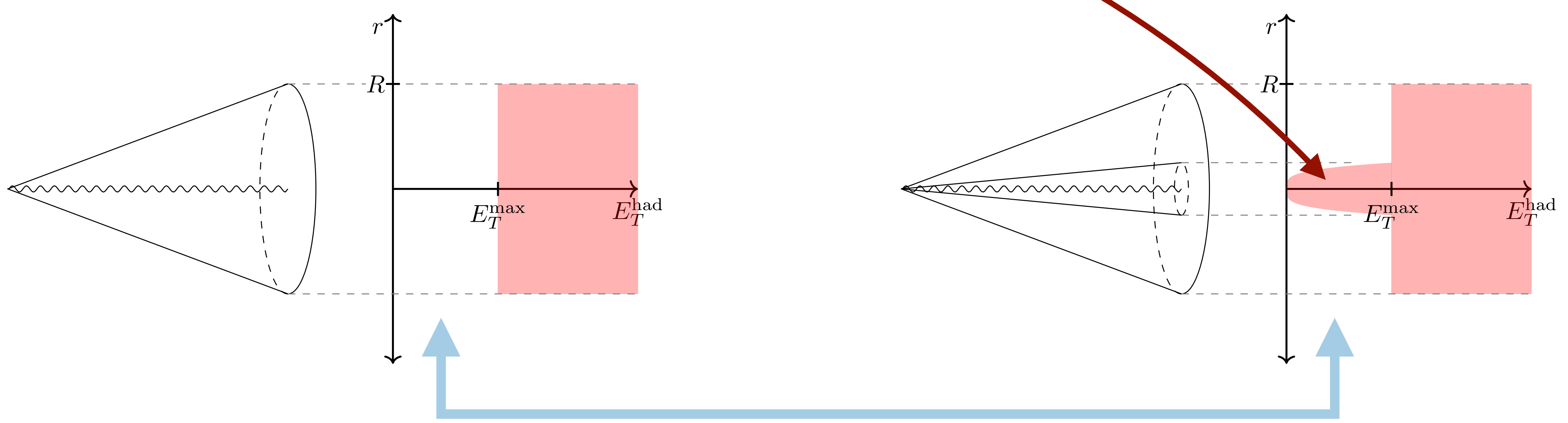
overwhelming background
from hadronization

TH: so far relied on *idealized isolations*

[Frixione '98]

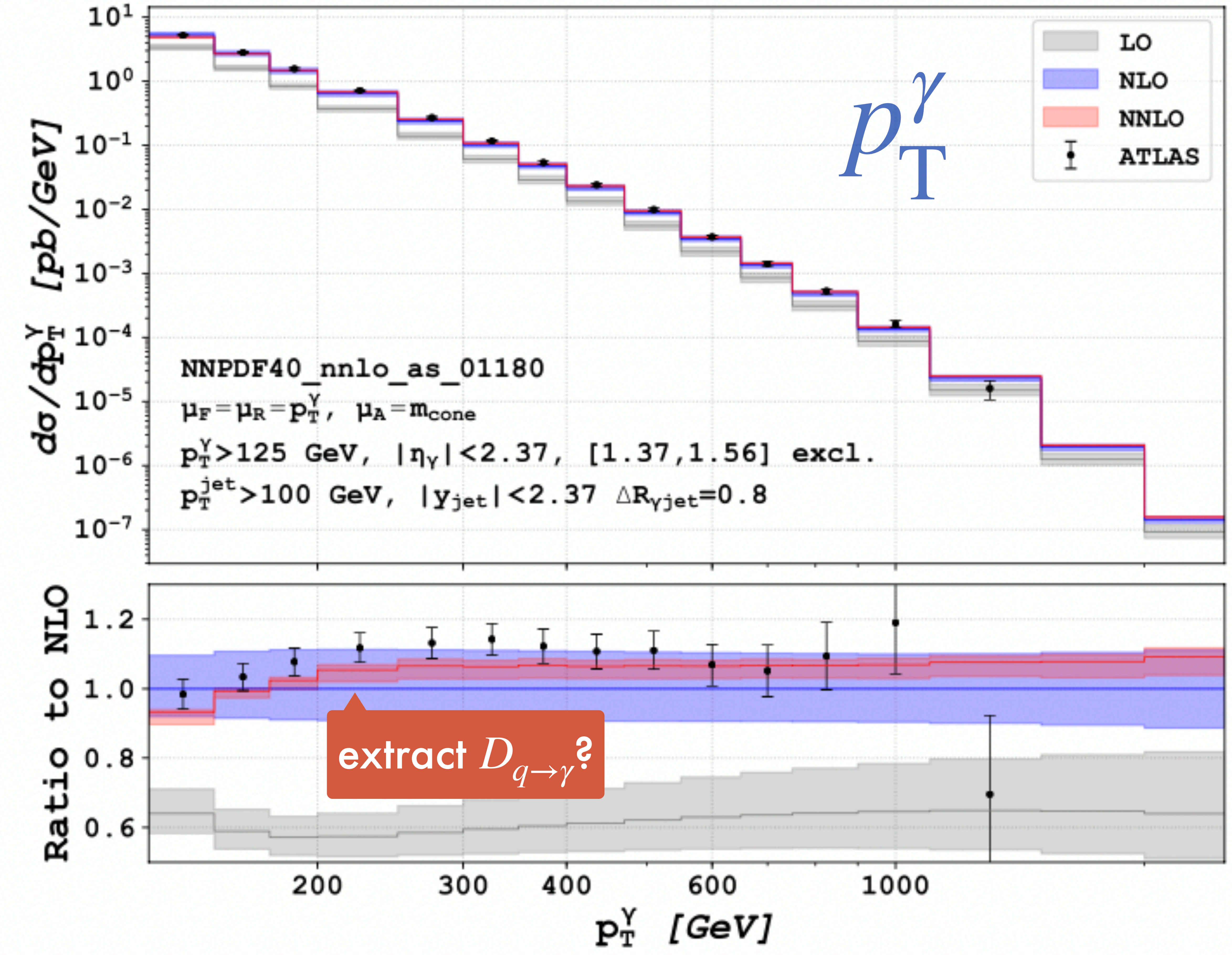


remove complex
fragmentation component



mismatch: few–10% [LH '13 '15]

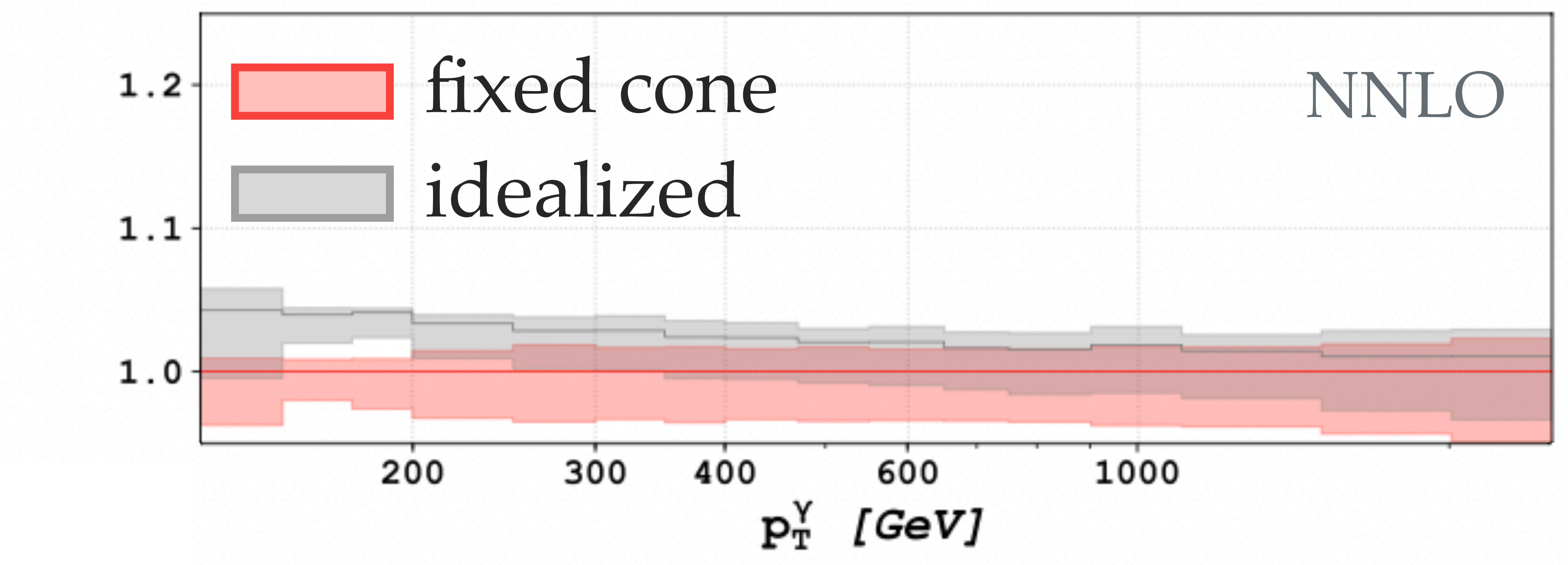
$\sim \mathcal{O}(\Delta_{\text{exp}}, \Delta_{\text{scl}}^{\text{NNLO}})$



[Chen, Gehrmann, Glover, Höfer, AH, Schürmann '22]

matters

at NNLO!



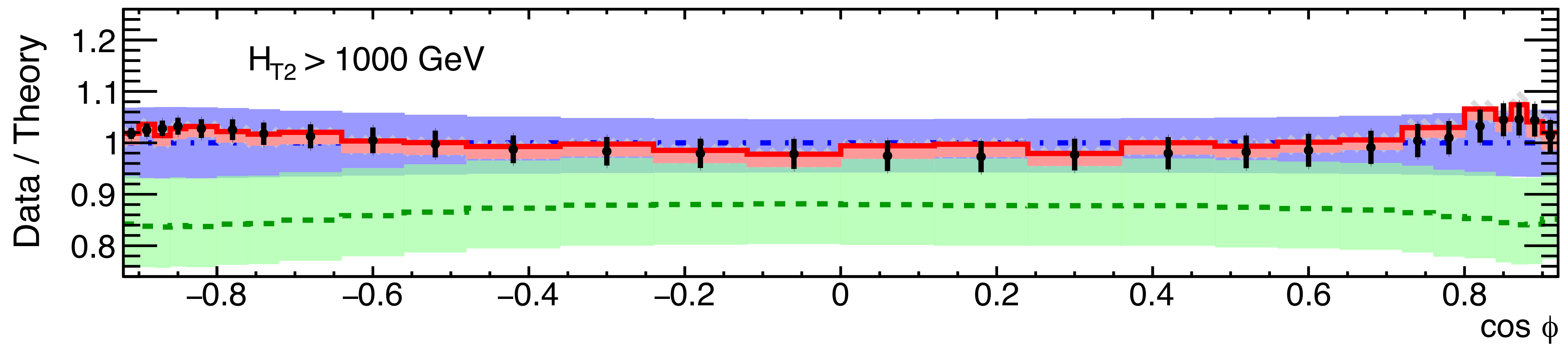
THE 2 → 3 FRONTIER:

- pp → $\gamma\gamma\gamma$
  [Chawdhry, Czakon, Mitov, Poncelet '19]
  [Kallweit, Sotnikov, Wiesemann '20]
- pp → $\gamma\gamma + j$
  [Chawdhry, Czakon, Mitov, Poncelet '21]
  (gluon-fusion @ NLO ≃ N³LO)
  ↪ [Badger, Gehrmann, Marcoli, Moodie '21]
- pp → jjj
  [Czakon, Mitov, Poncelet '21]
  ($gg \to ggg$; antenna automation)
  ↪ [Chen, Gehrmann, Glover, Huss, Marcoli '22]
- pp → $Wb\bar{b}$
  [Hartanto, Poncelet, Popescu, Zoia '22]
  [Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini '22] (approx. m_b)
- pp → $t\bar{t}H$
  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]
- pp → $\gamma + jj$
  [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]

among the most complex NNLO calculations

100M CPU hours!!!

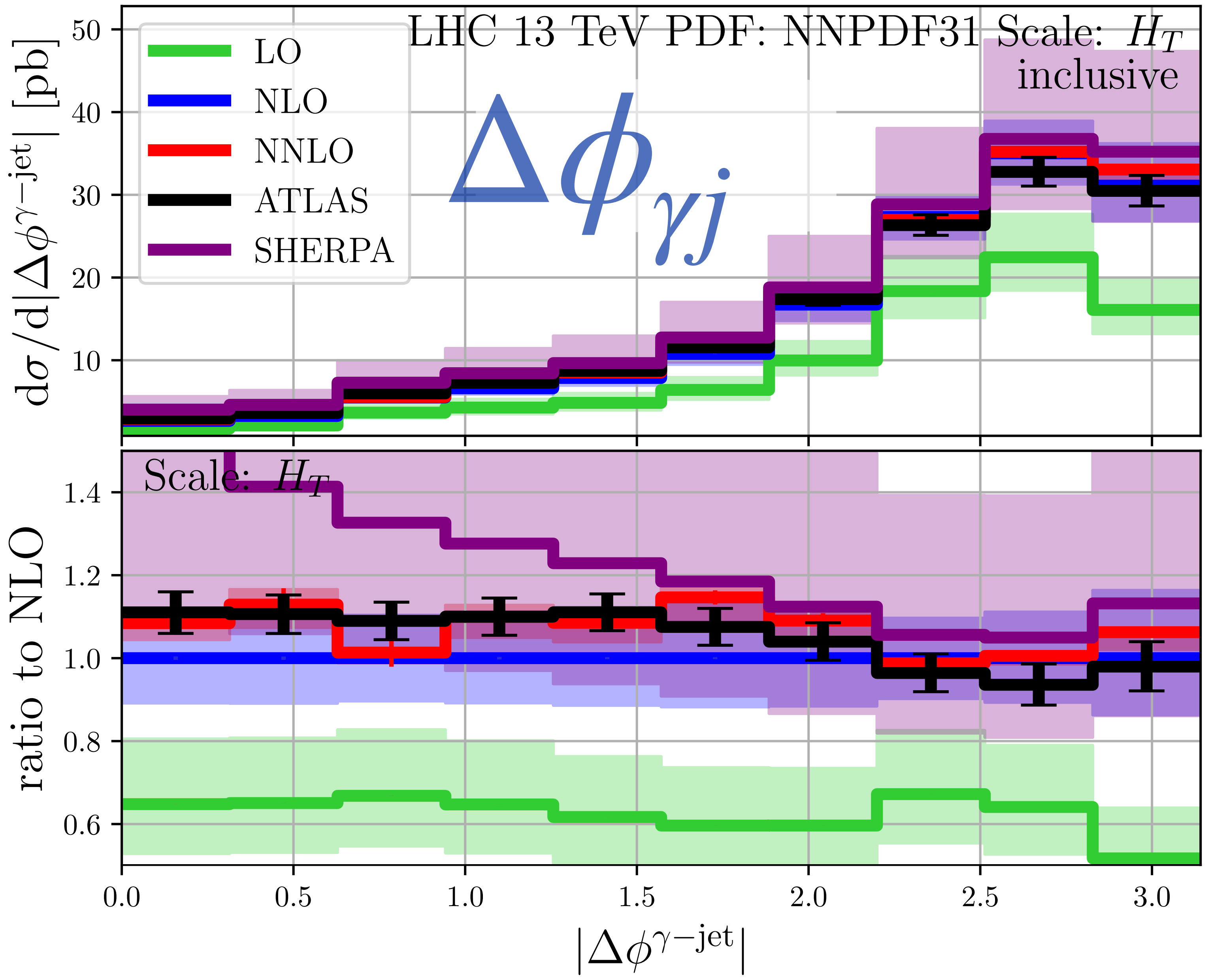
$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{\sigma}\int \sum_{i,j} d\sigma \frac{k_{T,i}\,k_{T,j}}{\sum_k k_{T,k}}\,\delta(\cos\phi - \cos\phi_{ij})$$



⇝ $\alpha_s(m_Z) = 0.1175 \pm 0.0006\ (\text{exp.})^{+0.0034}_{-0.0017}\ (\text{theo.})$

[ATLAS arXiv:2301.09351]

LO → NLO (× 1.6)
NNLO ⇝ pert. stabilisation
& good description of shape
Sherpa (≃ NLO)
⇝ large uncertainties (?!)
overshoots at low $\Delta\phi_{\gamma j}$



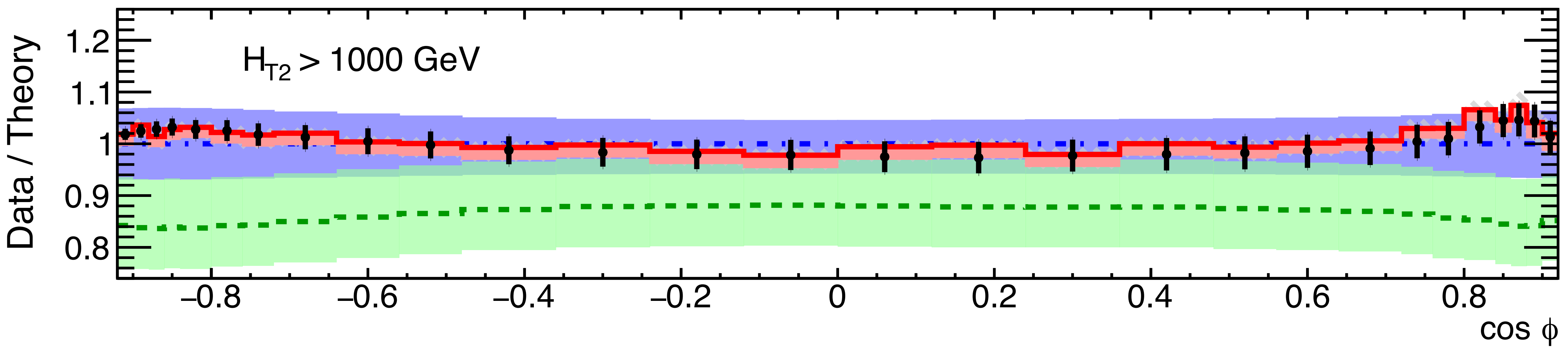
The $2 \to 3$ Frontier:

among the most complex NNLO calculations

100M CPU hours!!!

- $pp \to \gamma\gamma\gamma$
  [Chawdhry, Czakon, Mitov, Poncelet '19]
  [Kallweit, Sotnikov, Wiesemann '20]
- $pp \to \gamma\gamma + j$
  [Chawdhry, Czakon, Mitov, Poncelet '21]
  (gluon-fusion @ NLO ≃ N³LO)
  ↪ [Badger, Gehrmann, Marcoli, Moodie '21]
- $pp \to jjj$
  [Czakon, Mitov, Poncelet '21]
  ($gg \to ggg$; antenna automation)
  ↪ [Chen, Gehrmann, Glover, Huss, Marcoli '22]
- 1 $pp \to Wb\bar{b}$
  [Hartanto, Poncelet, Popescu, Zoia '22]
  [Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini '22] (approx. m_b)
- 2 $pp \to t\bar{t}H$
  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]
- $pp \to \gamma + jj$
  [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]

$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{\sigma}\int \sum_{i,j} d\sigma \, \frac{k_{T,i}\,k_{T,j}}{\sum_k k_{T,k}} \, \delta(\cos\phi - \cos\phi_{ij})$$



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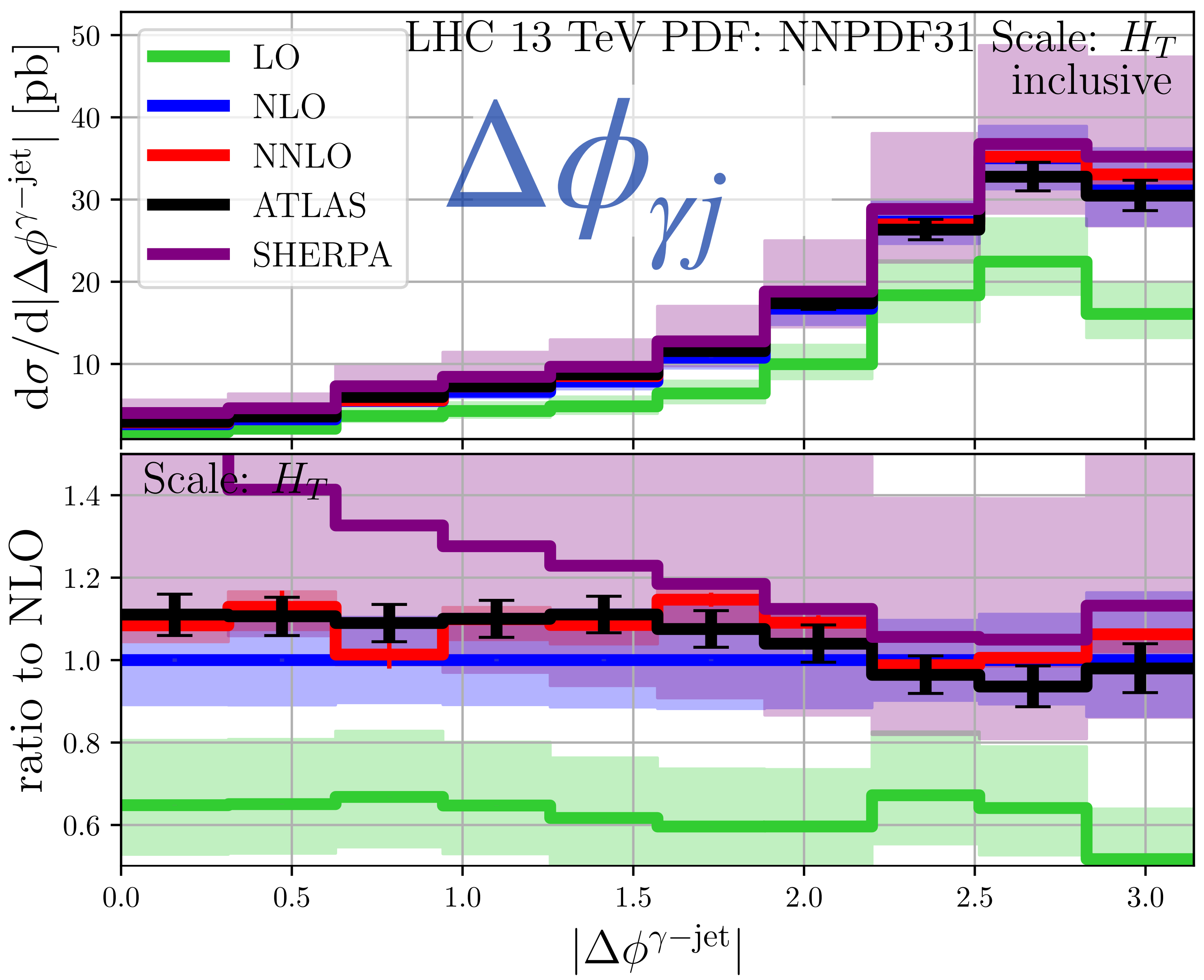
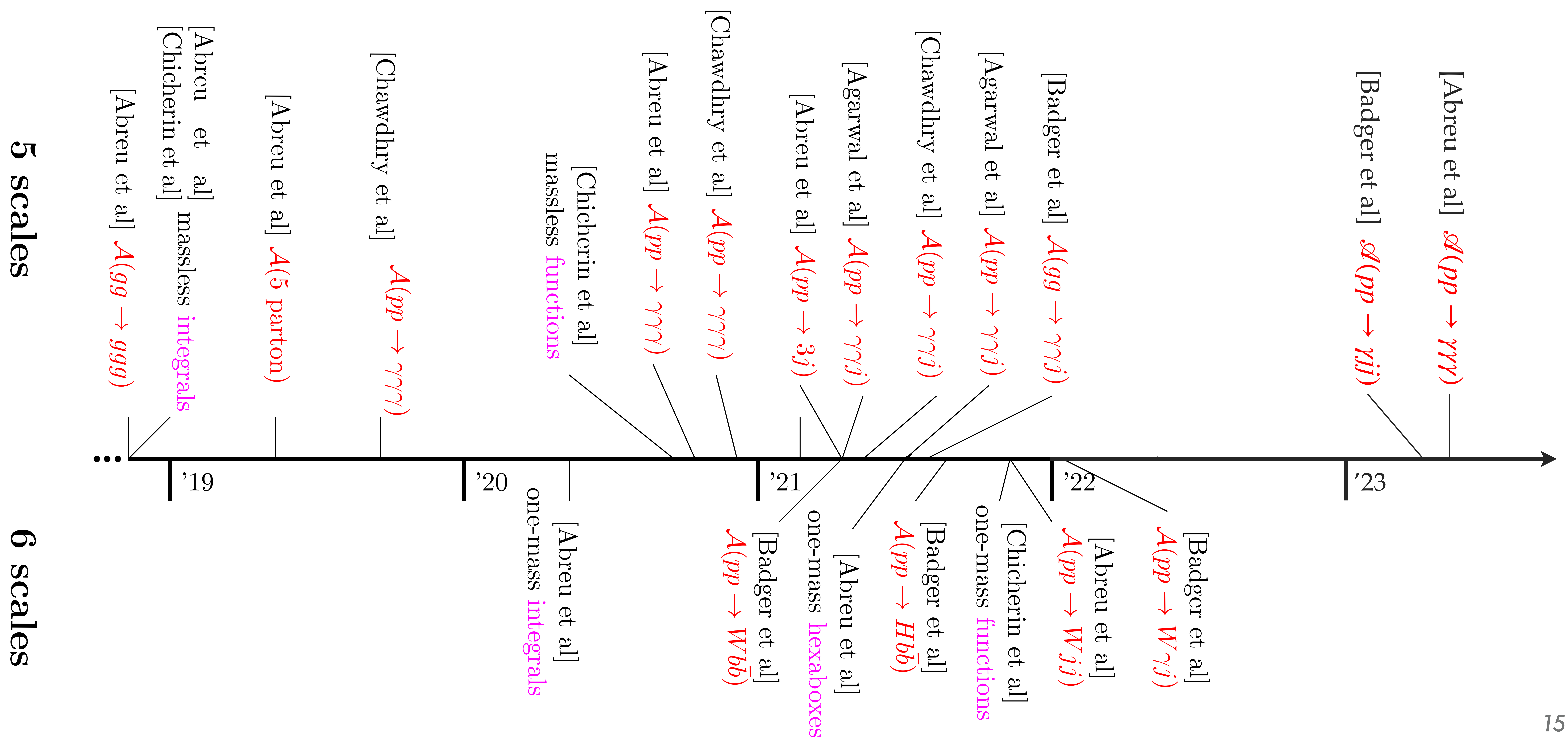


Two-Loop 2 → 3 Timeline

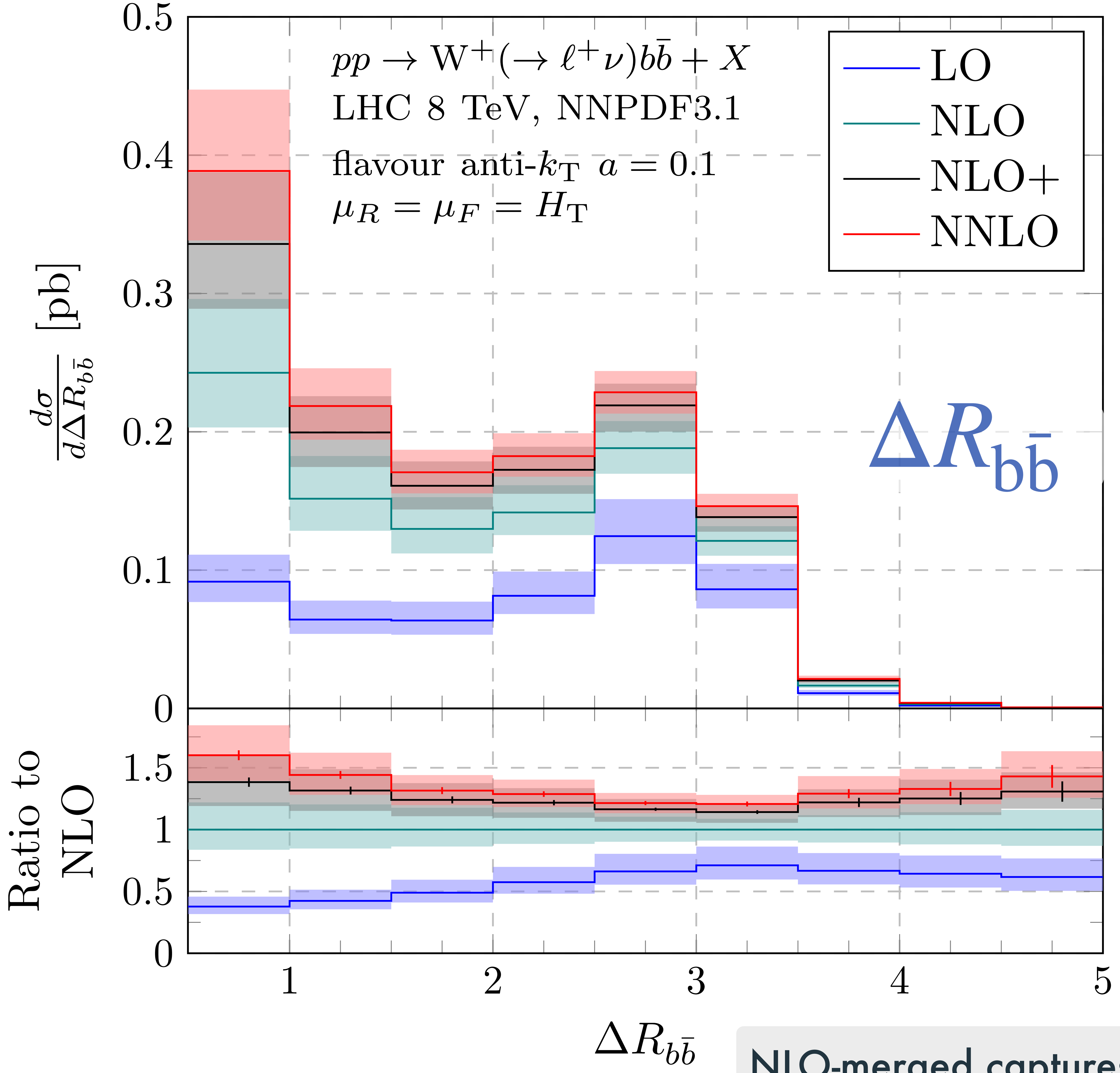
[adapted from slide by B. Page — HP2 '22]



W+b$\bar{\text{b}}$

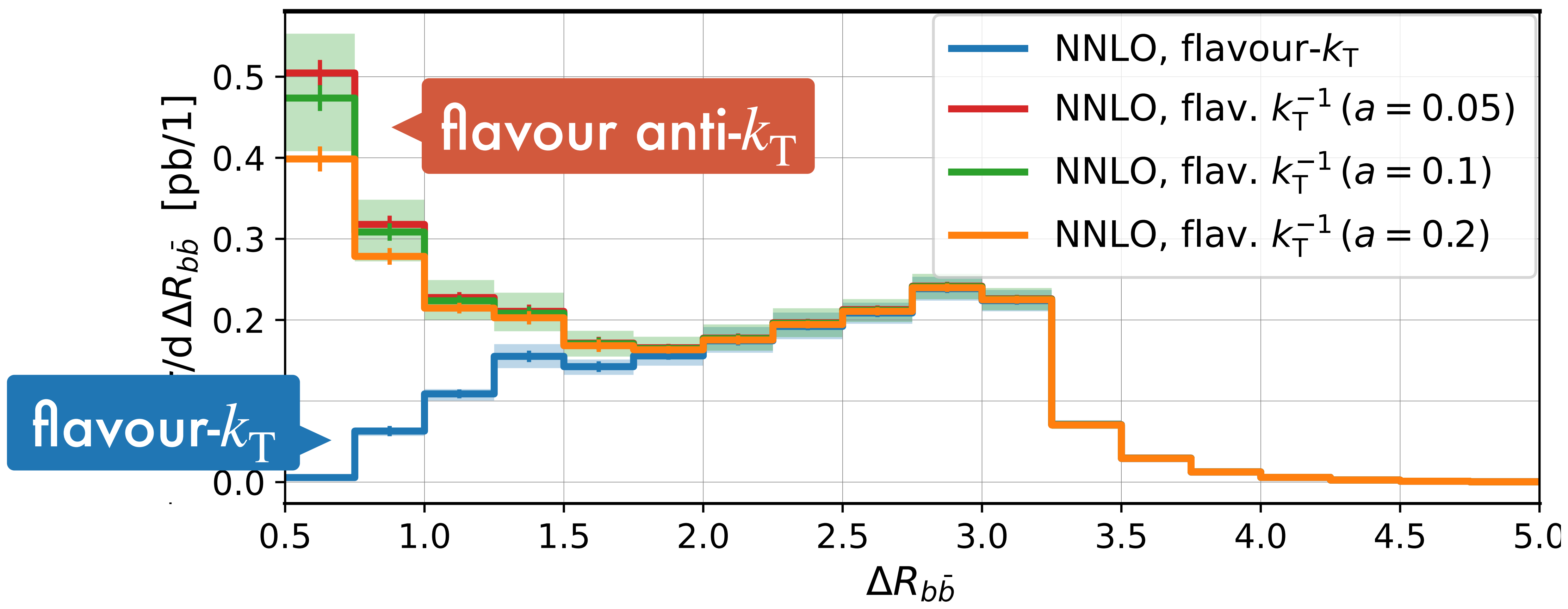
[Hartanto, Poncelet, Popescu, Zoia '22]

- first NNLO 2 → 3 w/ external mass
  - ↭ *challenge:* 2-loop amplitude [Abreu et al. '22]
- irreducible background to
  - ↪ *VH*, single top, BSM searches
- test perturbative QCD
  - ↪ large NLO corrections, 4FS vs. 5FS, modelling of *flavoured jets*



$$\sigma^{\text{NLO+}}_{\text{Wb}\bar{\text{b}},\,\text{inc.}} \equiv \sigma^{\text{NLO}}_{\text{Wb}\bar{\text{b}},\,\text{exc.}} + \sigma^{\text{NLO}}_{\text{Wb}\bar{\text{b}}j,\,\text{inc.}}$$

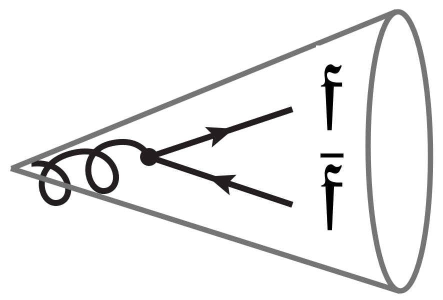
NLO-merged captures significant part of NNLO corrections



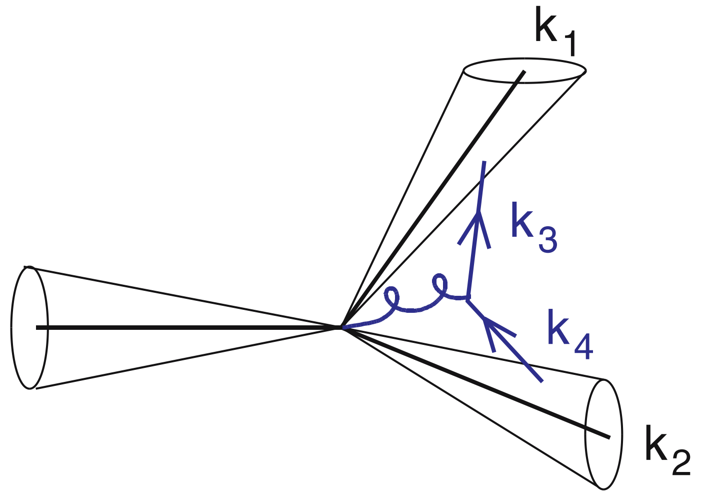
W+b$\bar{\text{b}}$ — Masses as Regulators

[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini '22]

- use massive bottom quarks (4FS)



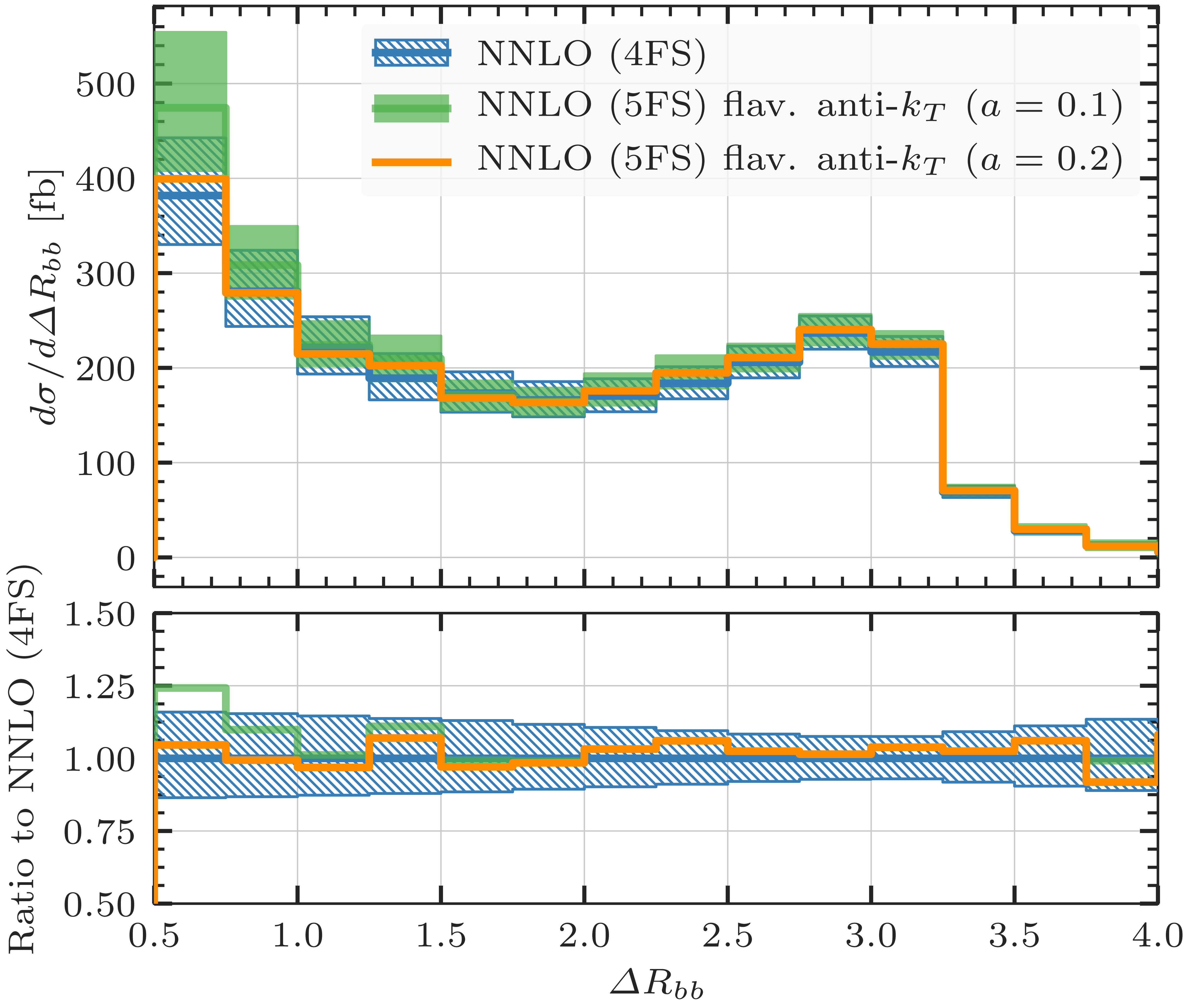
⇝ $\alpha_s \log(Q^2/m_f^2)$



⇝ $\alpha_s^2 \log(Q^2/m_f^2)$

- 2-loop amplitude
  unknown: 2 → 3 (w/ 2 masses)
  ⇝ "massifivation" [Mitov, Moch '07]
  ⇒ approximate up to $\mathcal{O}(m_b^2/Q^2)$

- overall good agreement between 4FS (—) & 5FS (— , —)



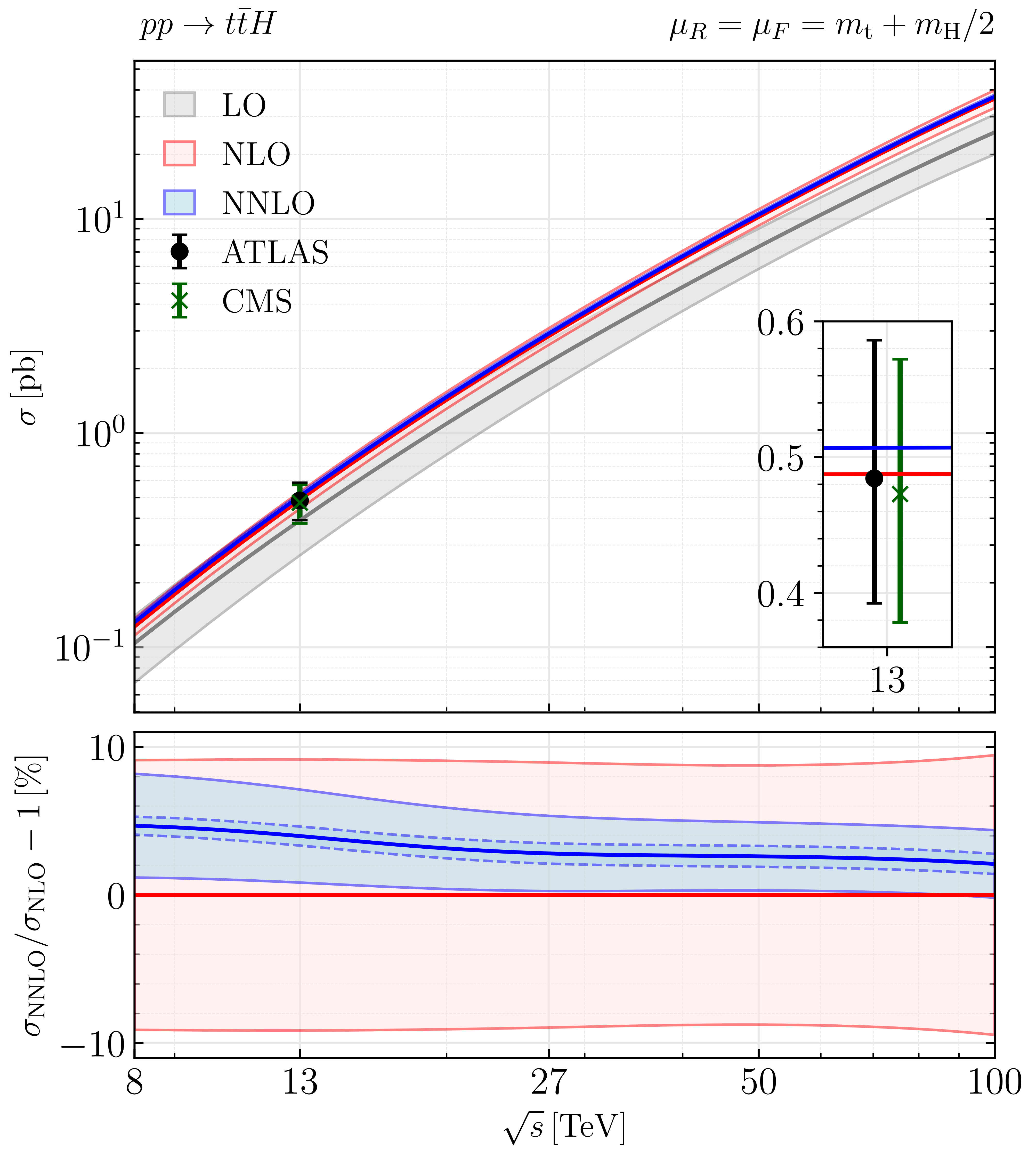
$t\bar{t}H$ — An Eikonal Higgs?!

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]

- a direct probe of the top Yukawa
  ↪ HL-LHC projection (exp): $\mathcal{O}(2\%)$
- missing ingredient: 2-loop amplitude
  ⇝ $2 \to 3$ (+ 2 masses): current frontier
- *apply:* soft Higgs approximation

$$\mathcal{M}^{t\bar{t}H}(p_t, p_{\bar{t}}, p_H) \simeq F(\alpha_s; m_t/\mu_R)\, J(p_H)\, \mathcal{M}^{t\bar{t}}(p_t, p_{\bar{t}})$$

- $\Delta^{\rm NLO}_{\rm scl} \simeq \pm 9\,\% \gg \Delta^{\rm NNLO}_{\rm scl} \simeq \pm 3\,\%$
- error estimate for approximation
  ⇝ $\pm 0.6\,\%$ (▭) on NNLO

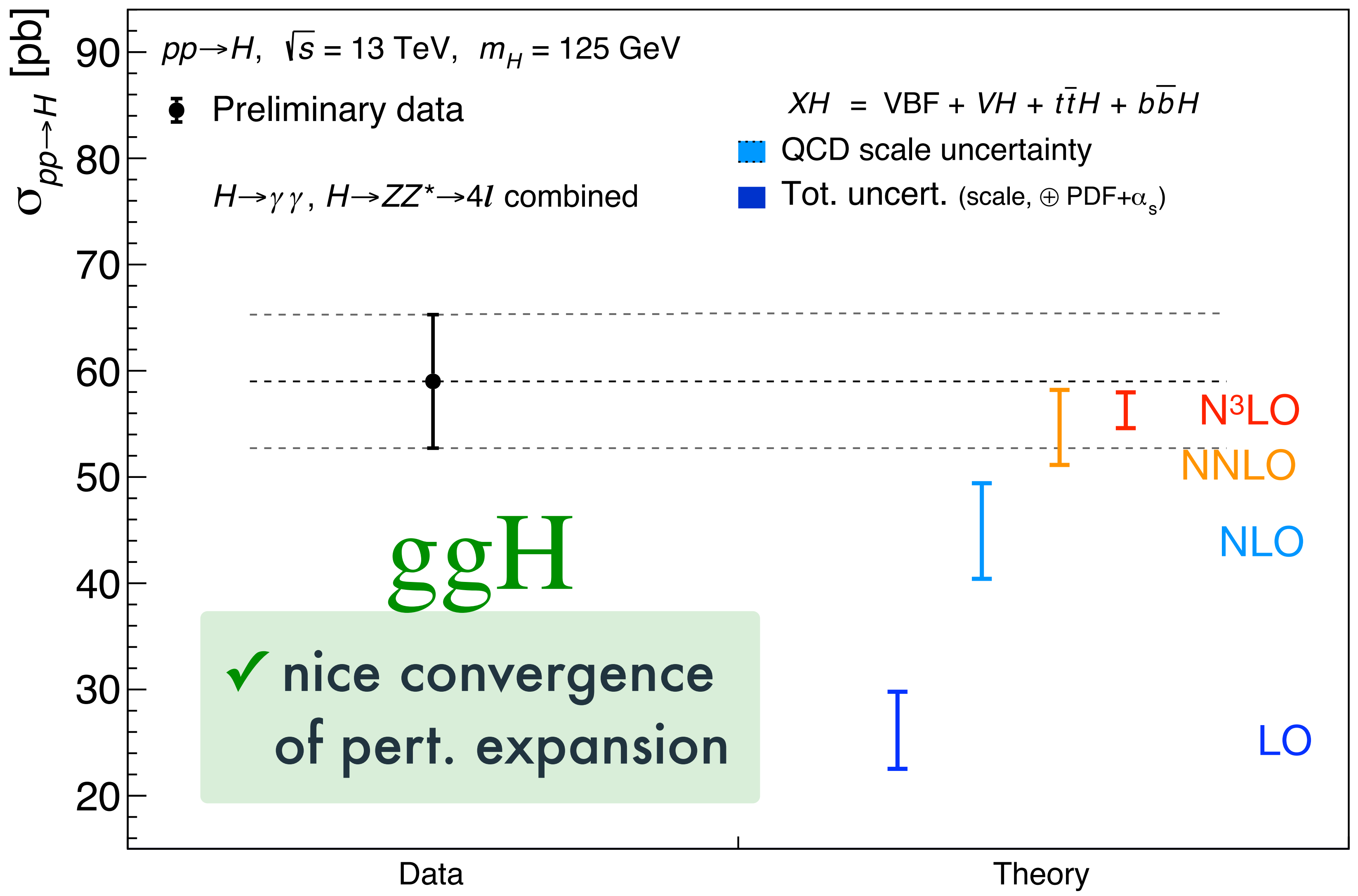


future: valid approximation also for $t\bar{t}Z$ & $t\bar{t}W^{\pm}$?

Going Beyond ⇝ N^3LO

Some processes require us to even push to the next order:

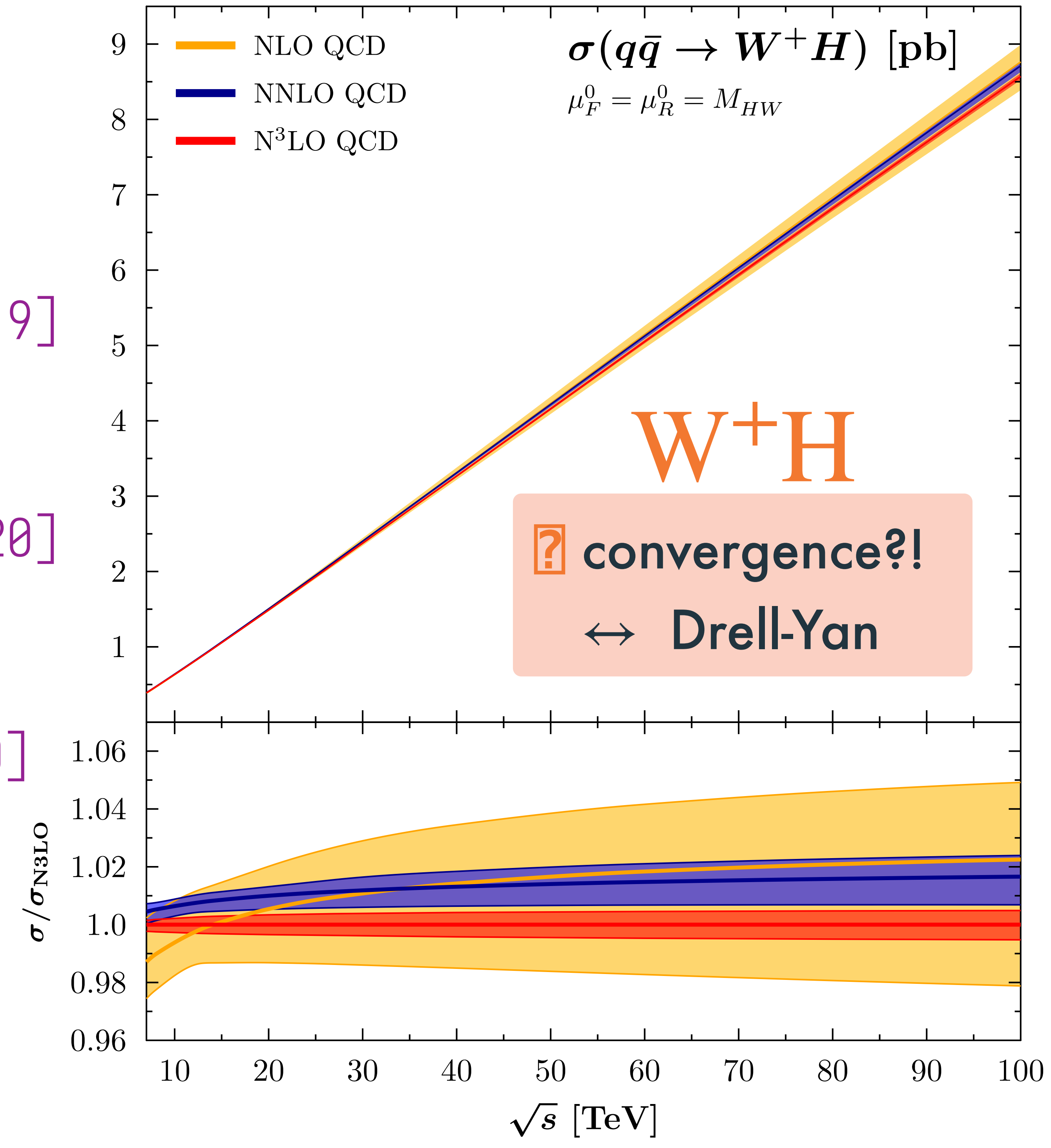
- "Standard candles"
  - ↪ very precisely measured
- slow perturbative convergence
  - ↪ $pp \to \gamma\gamma$
  - ↪ Higgs production $(gg \to H)$



[Anastasiou et al. '15]

Fully Inclusive calculations ⇜⇝ σ_{tot}

- $gg \to H$ ✓
  [C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger '15]
- VBF-H ✓, VBF-HH ✓
  [F. Dreyer, A. Karlberg '16, '18]
- $b\bar{b} \to H$ ✓
  [C. Duhr, F. Dulat, B. Mistlberger '19]
- $pp \to \gamma^*$?, $pp \to W^\pm$?
  [C. Duhr, F. Dulat, B. Mistlberger '20]
- $gg \to HH$ ✓
  [L. Chen, H. Li, H. Shao, J. Wang '20]
- $pp \to \gamma^*/Z$?
  [C. Duhr, B. Mistlberger '21]
- $pp \to VH$?
  [J. Baglio, C. Duhr, B. Mistlberger, R. Szafron '22]



FULLY DIFFERENTIAL ggH @ N^3LO

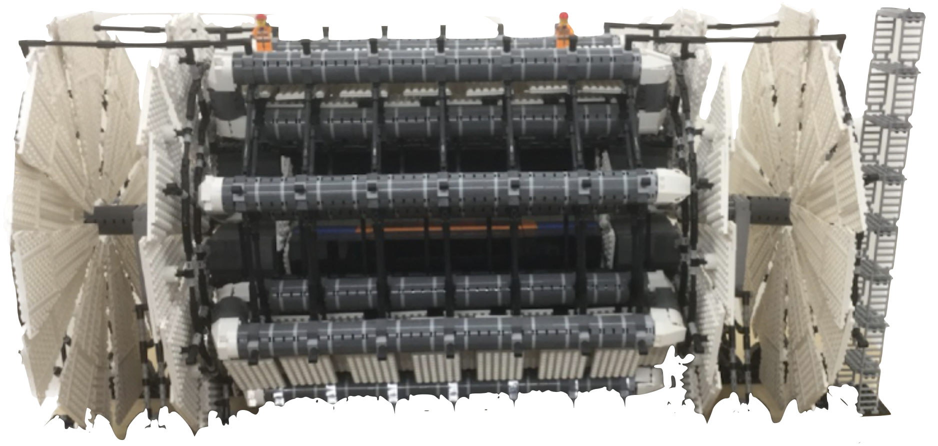
[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

FULLY INCLUSIVE

✗ limited to σ^{tot}

✓ very efficient $\mathcal{O}$(sec)

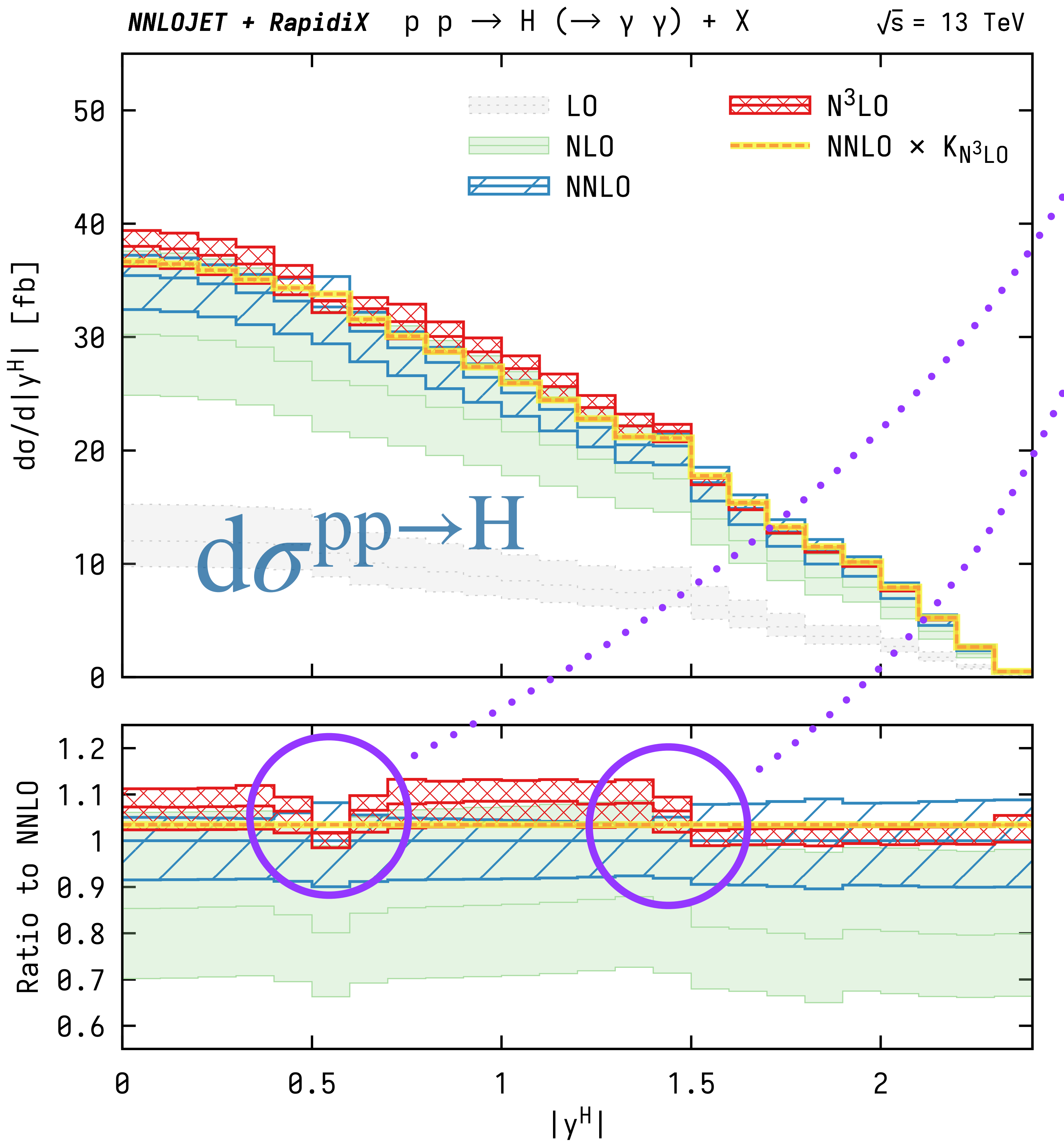
FULLY DIFFERENTIAL



✓ dσ ⇝ fiducial cuts, arbitrary distributions, …

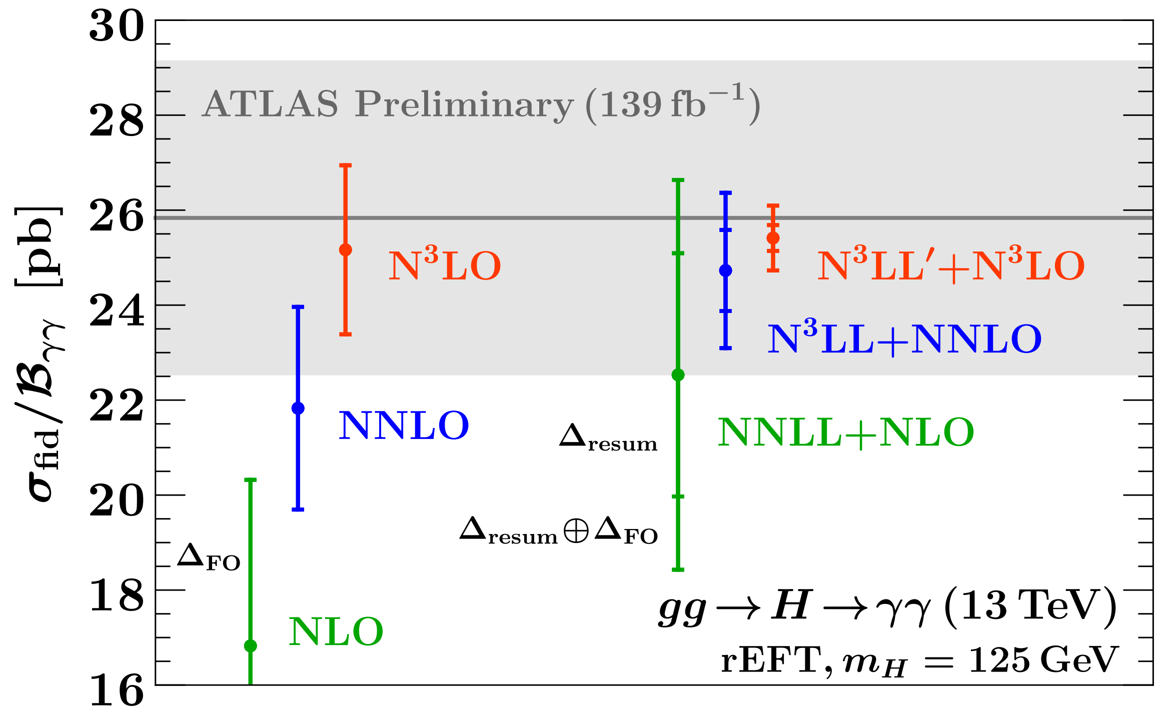
✗ computationally expensive $\mathcal{O}(10^5\text{-}10^6)$ h

ATLAS CUTS



linear fiducial power corrections ⇝ *instabilities*

[Billis, Dehnadi, Ebert, Michel, Tackmann '21]



⊕ can be cured by resummation

⊖ hard $\sigma^{\text{fid.}}$ should not need resummation?

Fully Differential ggH @ N^3LO

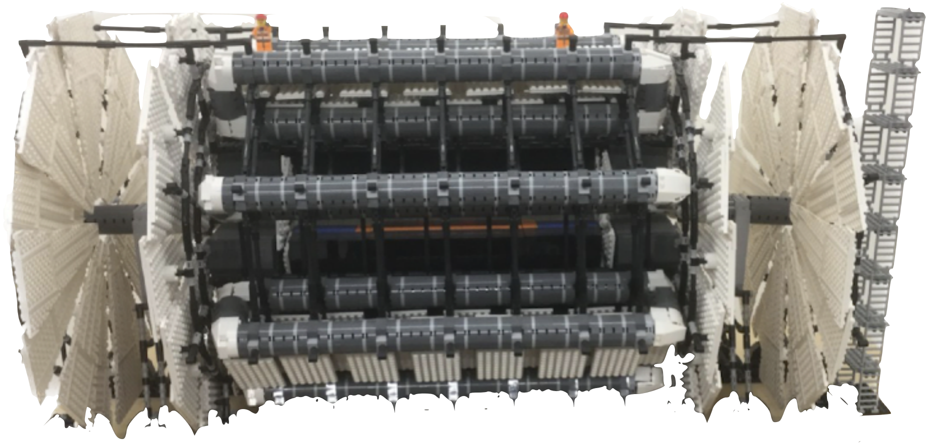
[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

Fully Inclusive

✗ limited to σ^{tot}

✓ very efficient $\mathcal{O}$(sec)

Fully Differential

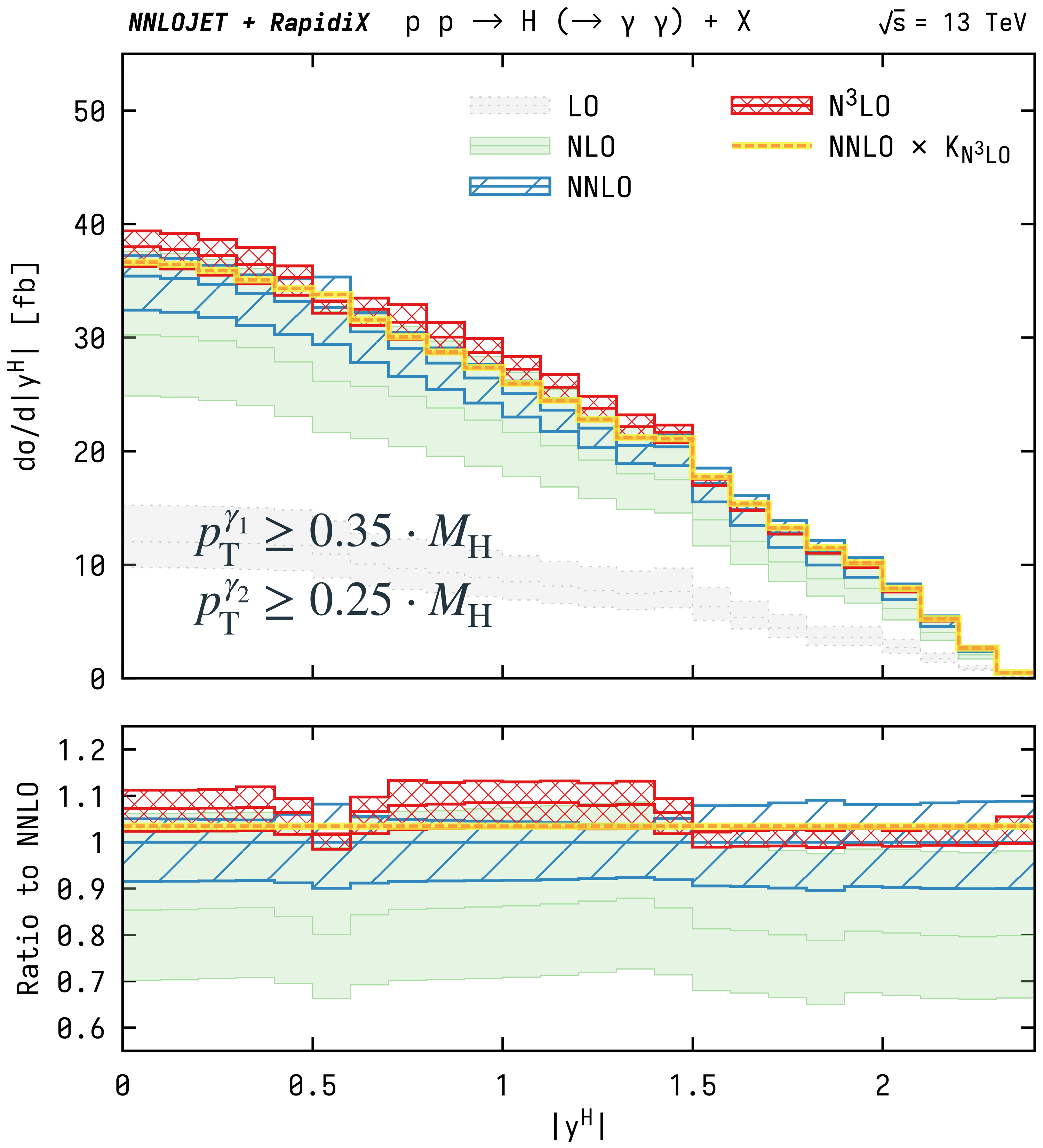


✓ dσ ⇝ fiducial cuts, arbitrary distributions, …

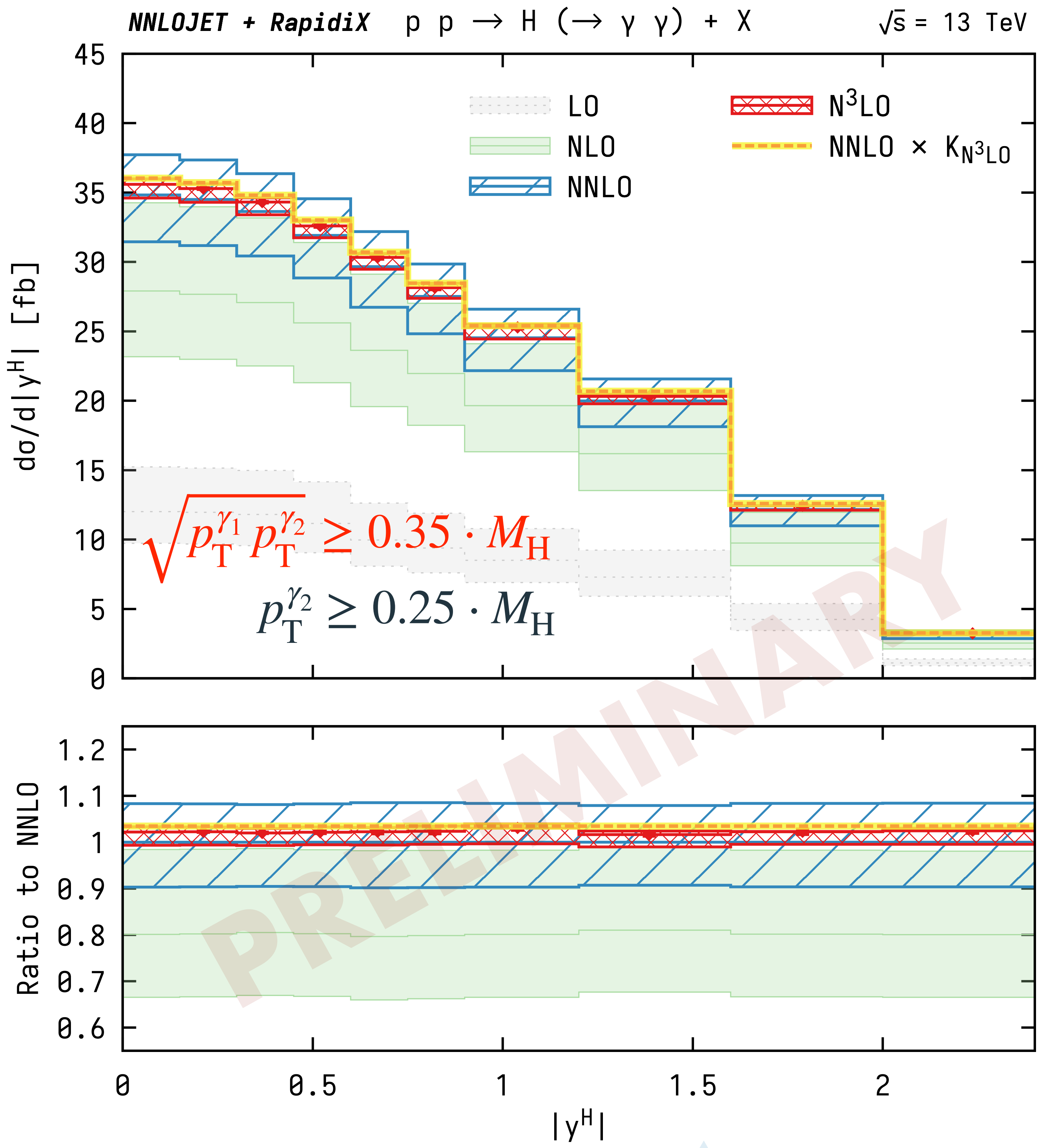
✗ computationally expensive $\mathcal{O}(10^5\text{-}10^6)$ h

~~lin. fid. power corr.~~
[Salam, Slade '21]

ATLAS Cuts



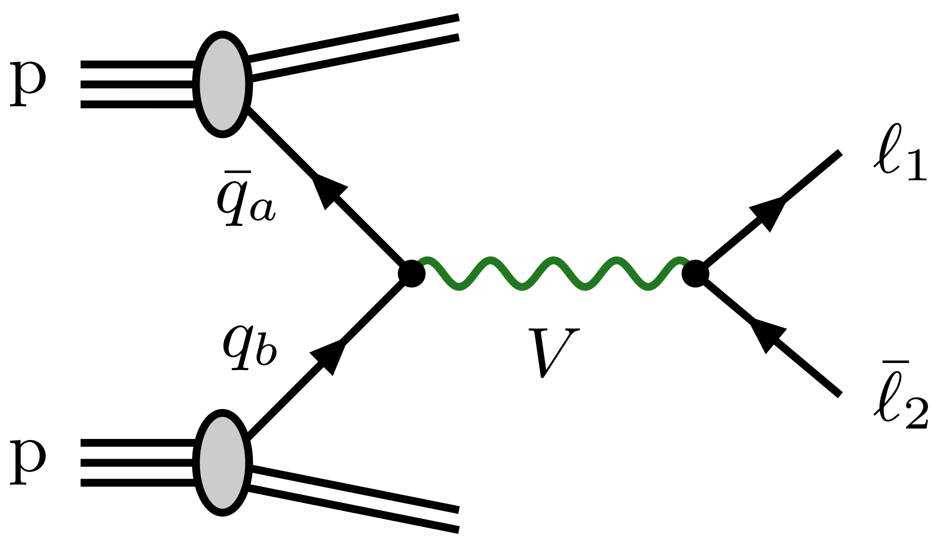
Product Cuts



no instabilities & flat K-factor: $N^3LO \simeq NNLO \times K_{N^3LO}$

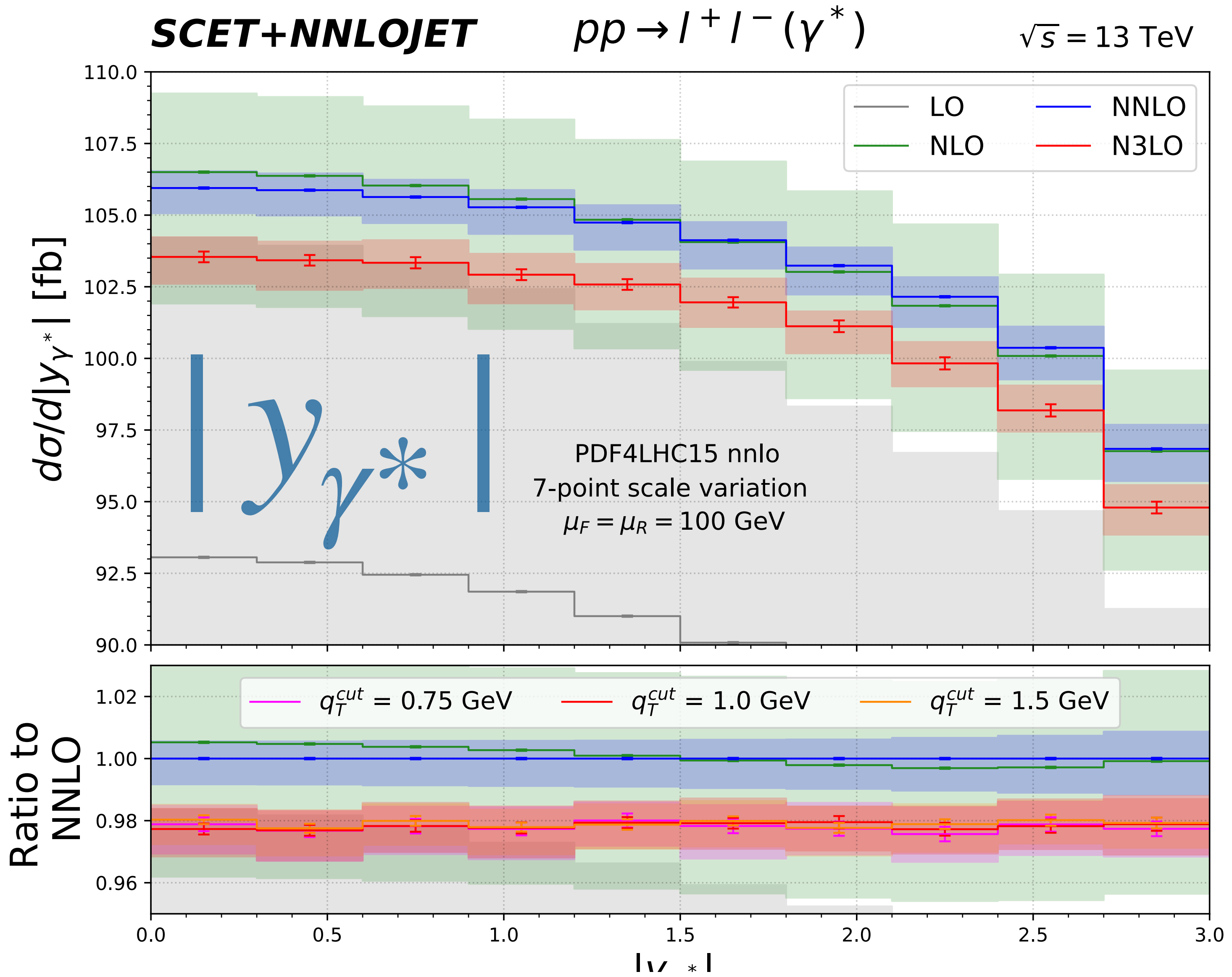
Drell Yan — a Standard Candle

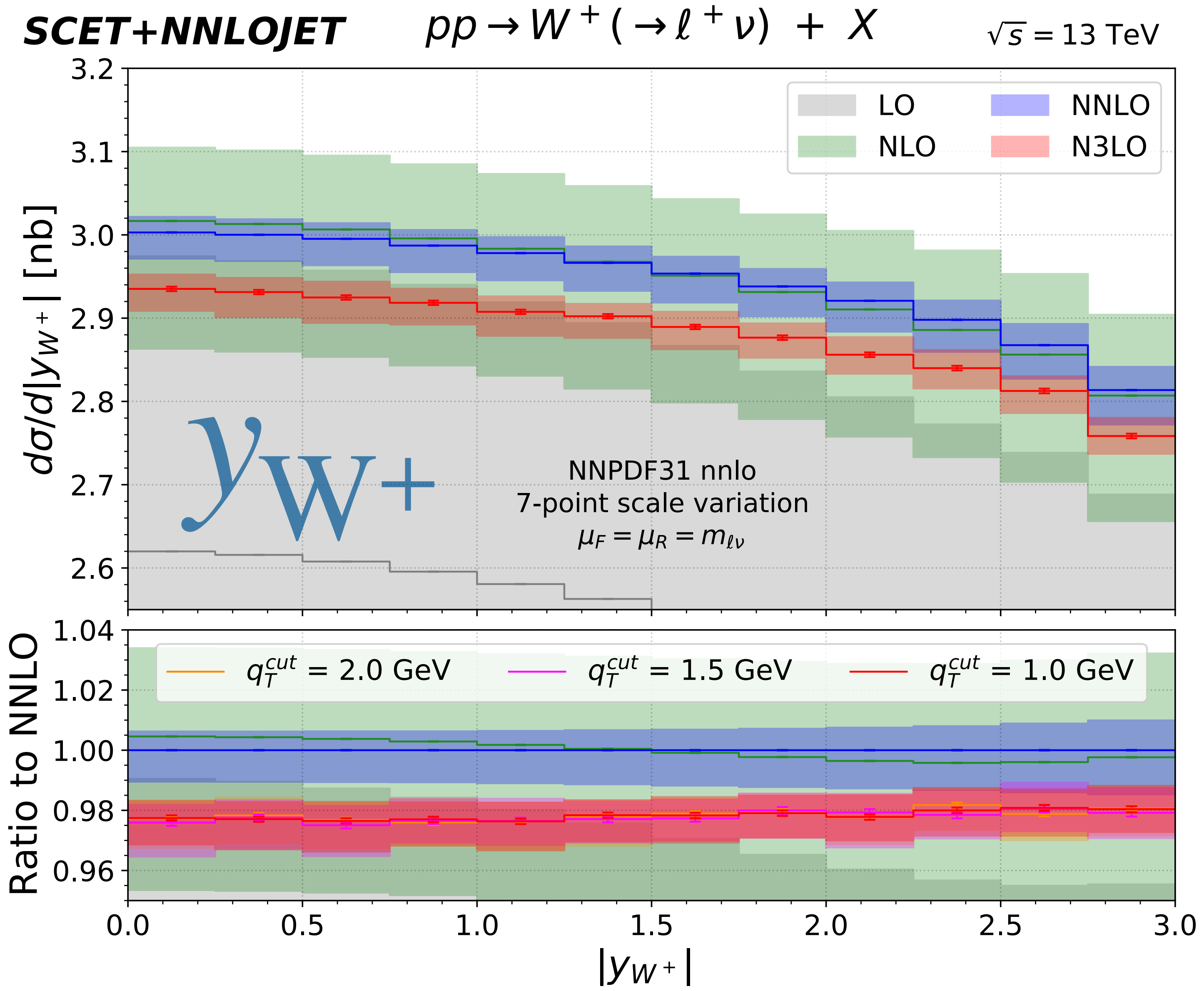
[Chen, Gehrmann, Glover, AH, Yang, Zhu '21,'22]

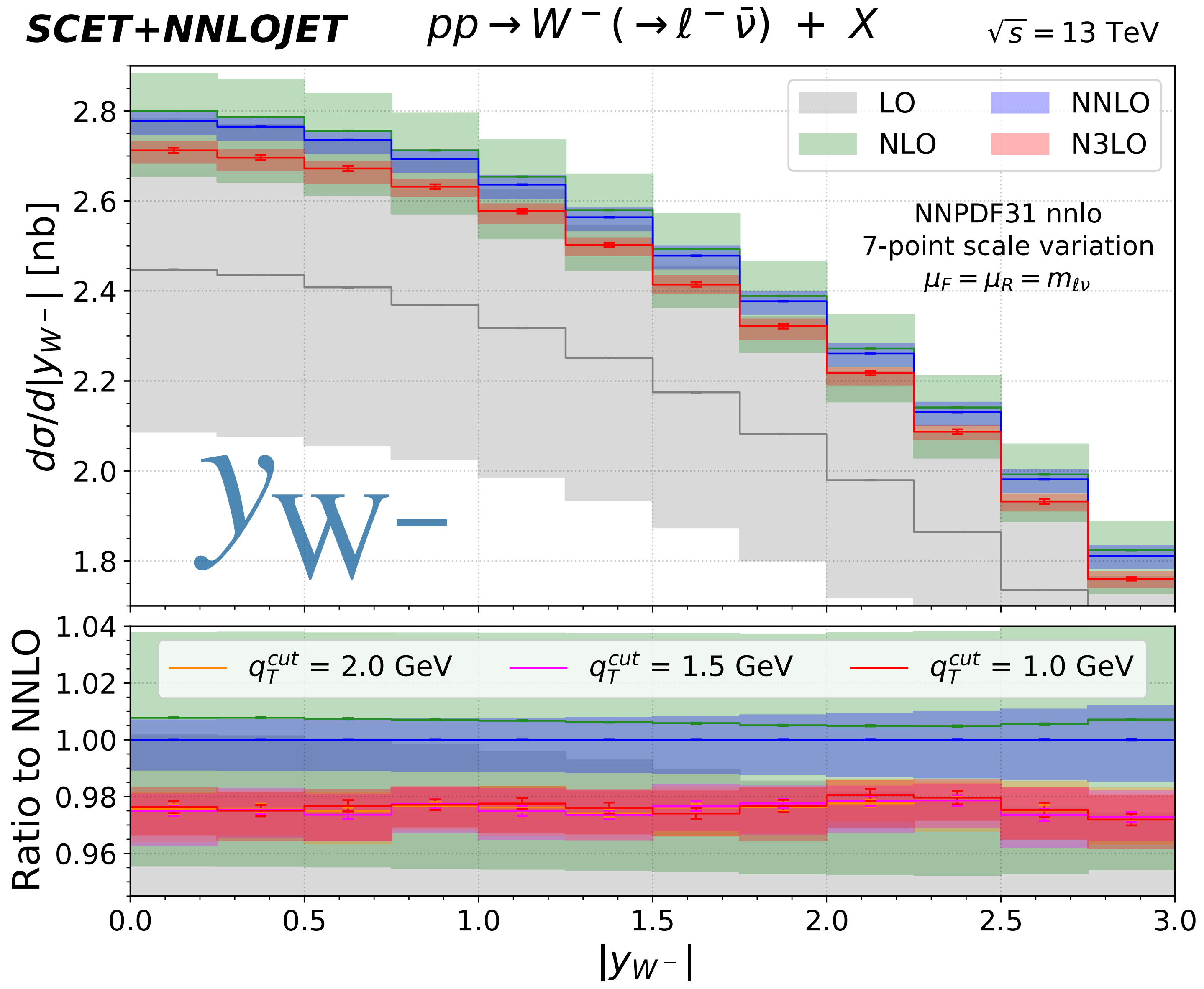


- clean signature $(\ell^{\pm}, E_{\mathrm{T}}^{\mathrm{miss}})$
  & large cross section:
  $(\sim 1000\ \mathrm{Z}\ \ \&\ \ \sim 4000\ \mathrm{W}^{\pm})$ / sec *
- detector calibration, BSM searches, luminosity monitor, PDFs, …
- precision measurements:
  ↪ $\sin^2(\theta_{\mathrm{w}}),\ M_{\mathrm{W}}$

- almost universal NNLO → N³LO corrections!
- NC & CC$^{\pm}$ probe different parton content





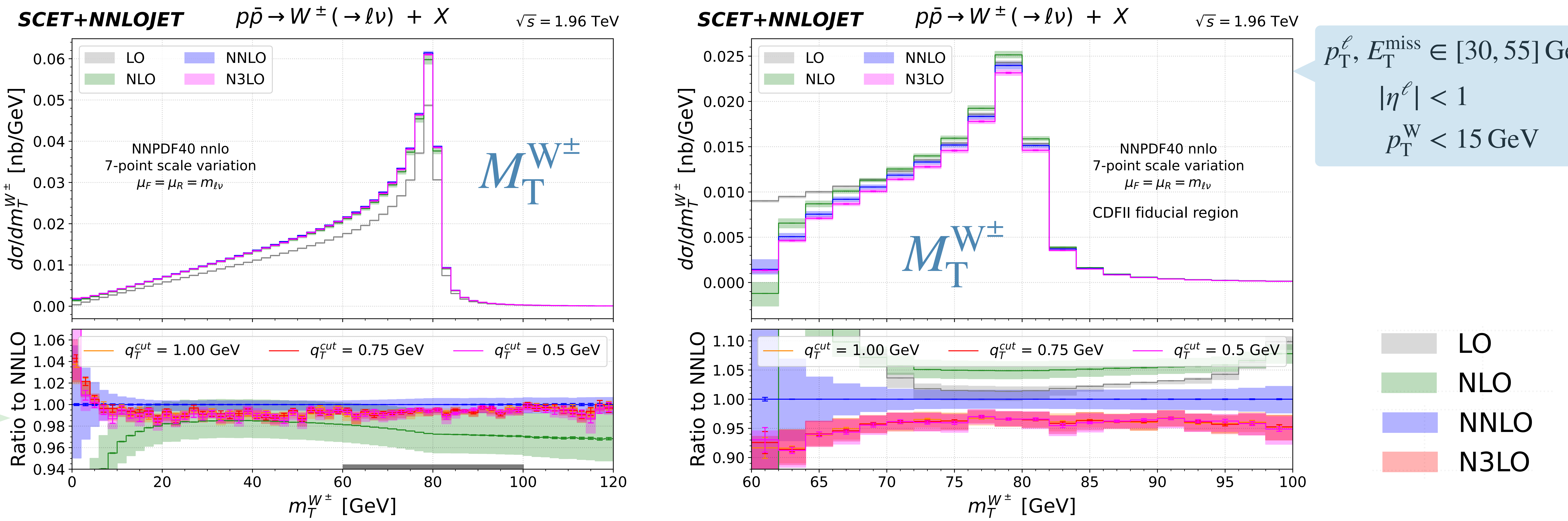


* $\mathscr{L} = 2 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

W Production — Absolute Spectrum

$$M_T^W \equiv \sqrt{E_T^\ell \, E_T^\nu \, (1 - \cos\Delta\varphi_{\ell\nu})}$$

Inclusive | Fiducial (CDF II)



$p_T^\ell,\ E_T^{\text{miss}} \in [30, 55]$ GeV
$|\eta^\ell| < 1$
$p_T^W < 15$ GeV

1.96TeV – better convergence as seen in σ^{tot}

- remain largely flat around peak; larger corrections at low M_T^W
- fiducial cuts impact pattern of radiative corrections
- larger N³LO corrections ($-1\,\%$ [inc.] vs. $-4\,\%$ [fid.])

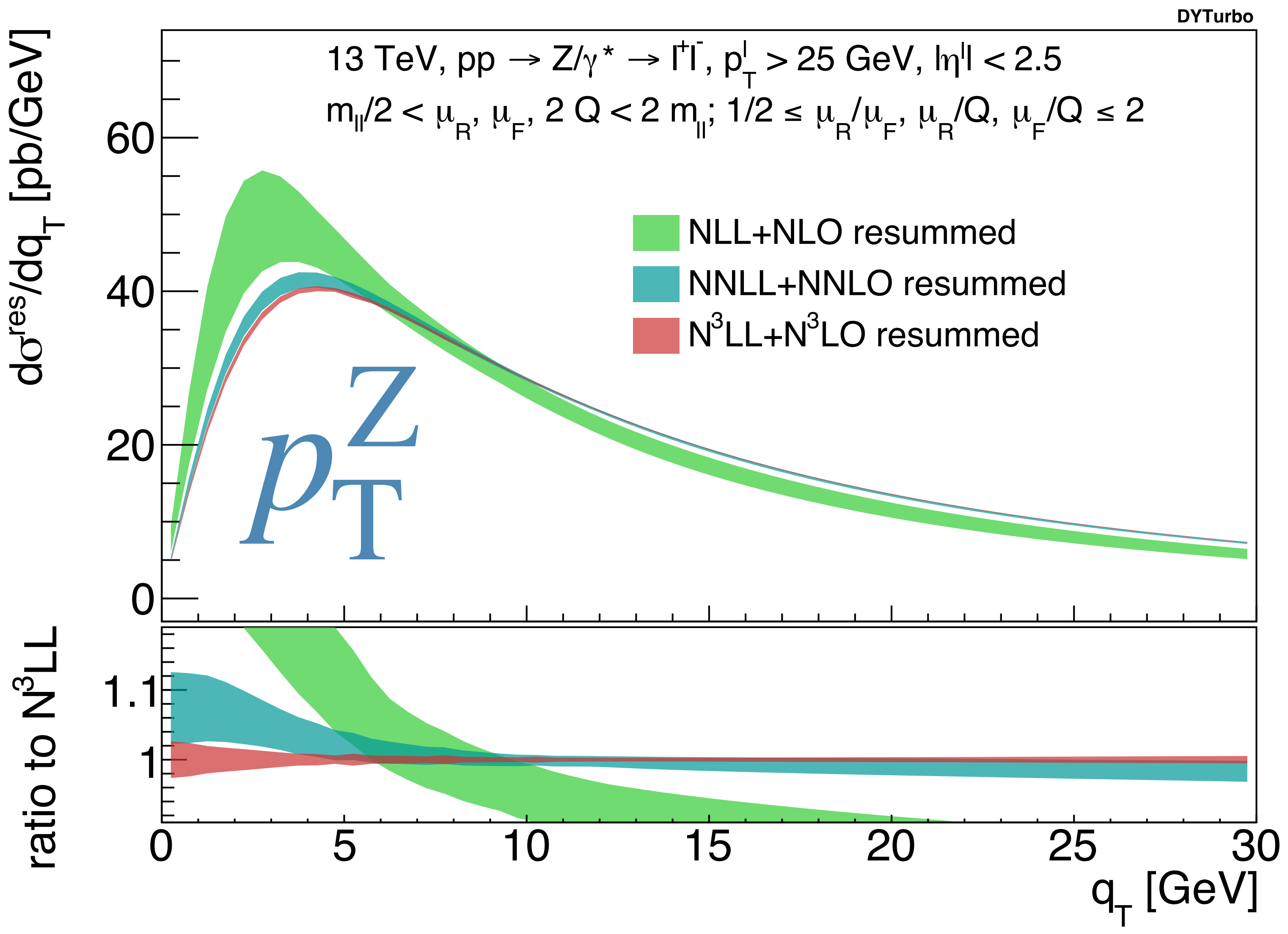
fiducial power corrections?

N³LO + Resummation

improved convergence ⇝ uncertainties: *few %*

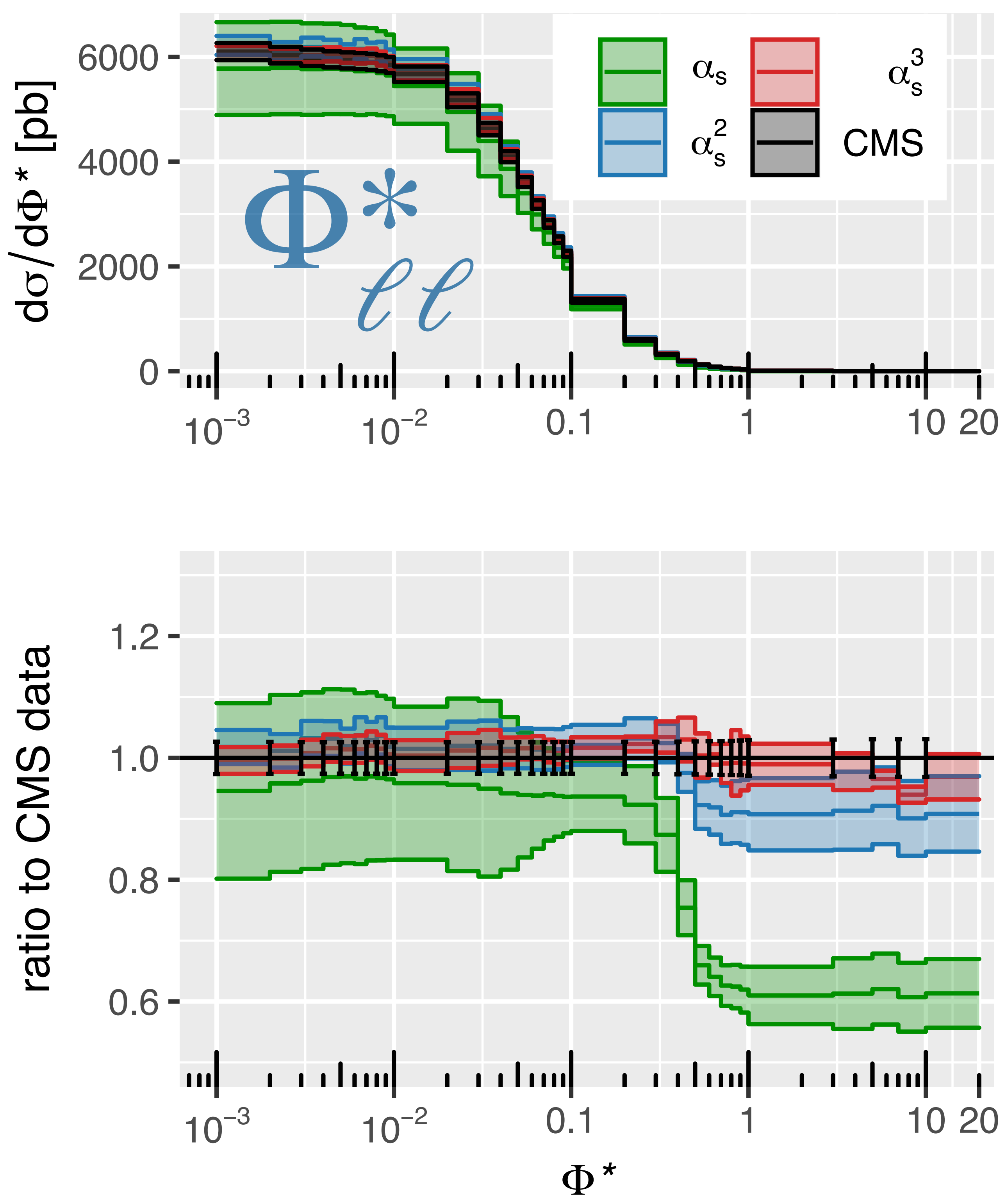
DYTurbo [Camarda, Cieri, Ferrera '22]

- more robust & reduced uncertainties



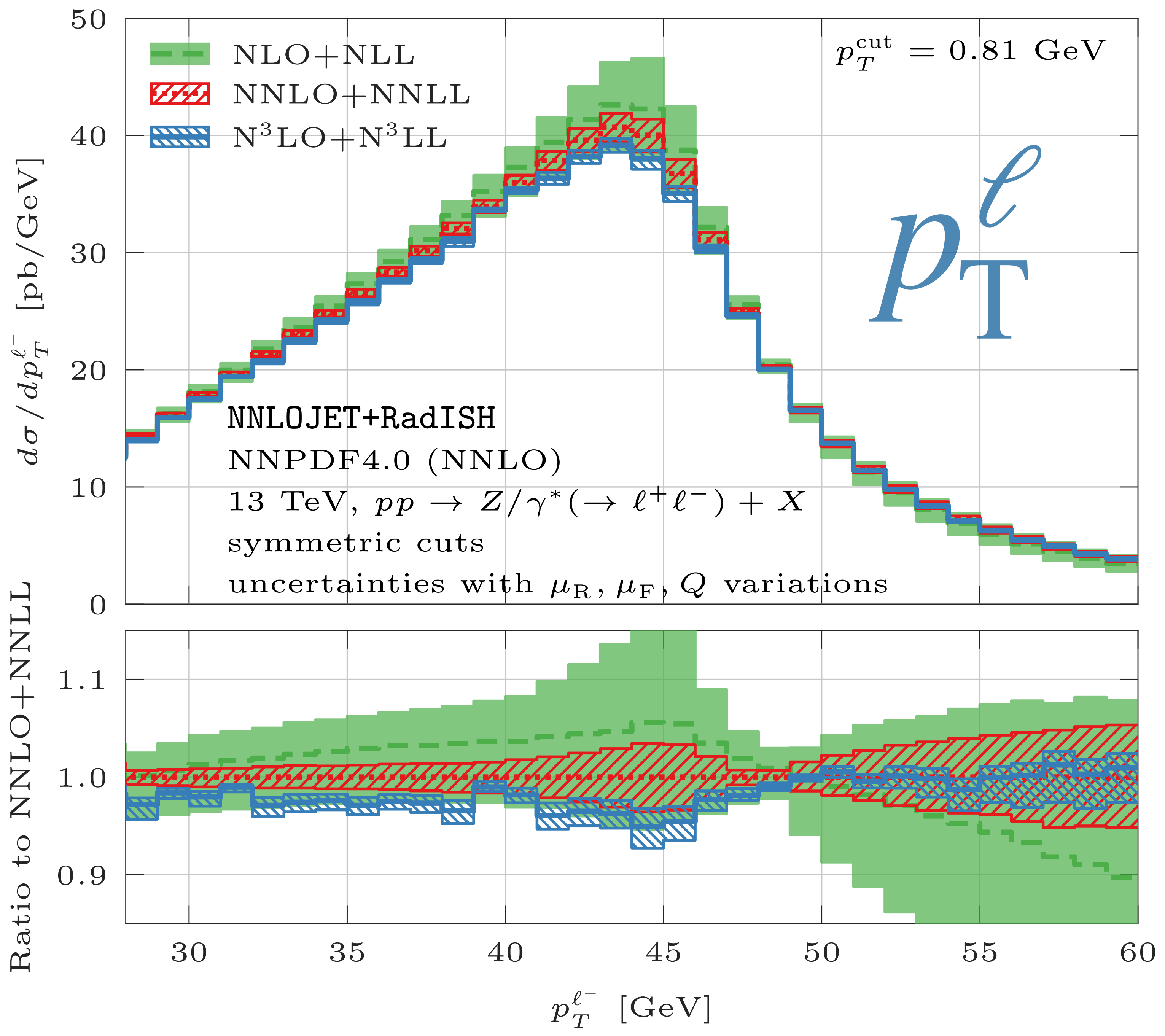
CuTe-MCFM [Neumann, Campbell '22]

- good agreement with data



NNLOjet+RadISH

[Chen, Gehrmann, Glover, AH, Monni, Rottoli, Re, Torrielli '22]



- some shape distortion ⇝ impact on M_W for CC$^\pm$?

Conclusions & Outlook

- perturbative calculations *crucial* to scrutinise the Standard Model
- NNLO in good shape (reduced uncertainties & improved TH-data comparison)
  - $2 \to 2$ largely done, steady progress for $2 \to 3$ ⇝ performance increasingly an issue
  - tying up loose ends ⇝ flavour, fragmentation, non-fact., mass effects, …
  - *loop amplitudes* becoming a bottleneck again ⇝ approximations in the interim
- N^3LO computation of *inclusive* $2 \to 1$ processes mature
  - first differential $pp \to$ ”colour neutral” ⇝ $pp \to \gamma\gamma$, $pp \to VH$ within reach
  - towards $2 \to 2$: massless 3-loop amplitudes, first steps for subtraction, …
- percent-level phenomenology: *everything becomes relevant*
  ⇝ PDFs (+N^3LO evolution), parametric, QCD×EW, non-perturbative, …

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Thank you!

BACKUP

Mixed QCD–EW Corrections for Drell–Yan

$$d\sigma = d\sigma_{LO}\left(1 + \left(\frac{\alpha_s}{2\pi}\right)\delta^{(1,0)} + \left(\frac{\alpha}{2\pi}\right)\delta^{(0,1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\delta^{(2,0)} + \left(\frac{\alpha_s}{2\pi}\right)\left(\frac{\alpha}{2\pi}\right)\delta^{(1,1)} + \cdots\right)$$

notation

$$\sigma^{\mathrm{NLO}_{s\oplus ew}} \sim 1,\ \delta^{(1,0)},\ \delta^{(0,1)}$$

$$\sigma^{\mathrm{NNLO}_{s\otimes ew}} \sim 1,\ \delta^{(1,0)},\ \delta^{(0,1)},\ \delta^{(1,1)}$$

$$\sigma^{\mathrm{NNLO}_{s\otimes ew}}_{\text{naive prod.}} \sim 1,\ \delta^{(1,0)},\ \delta^{(0,1)},\ \delta^{(1,0)}\times\delta^{(0,1)}$$

resonant / on-shell

- pole expansion [Dittmaier, Huss, Schwinn '14,'15]
- on-shell Z (QCD×QED) [Delto, Jaquier, Melnikov, Röntsch '19]
- σ_Z^{tot} [Bonciani, Buccioni, Rana, Vicini '20]
- on-shell
  [Buccioni, Caola, Delto, Jaquier, Melnikov, Roentsch '20]
  [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

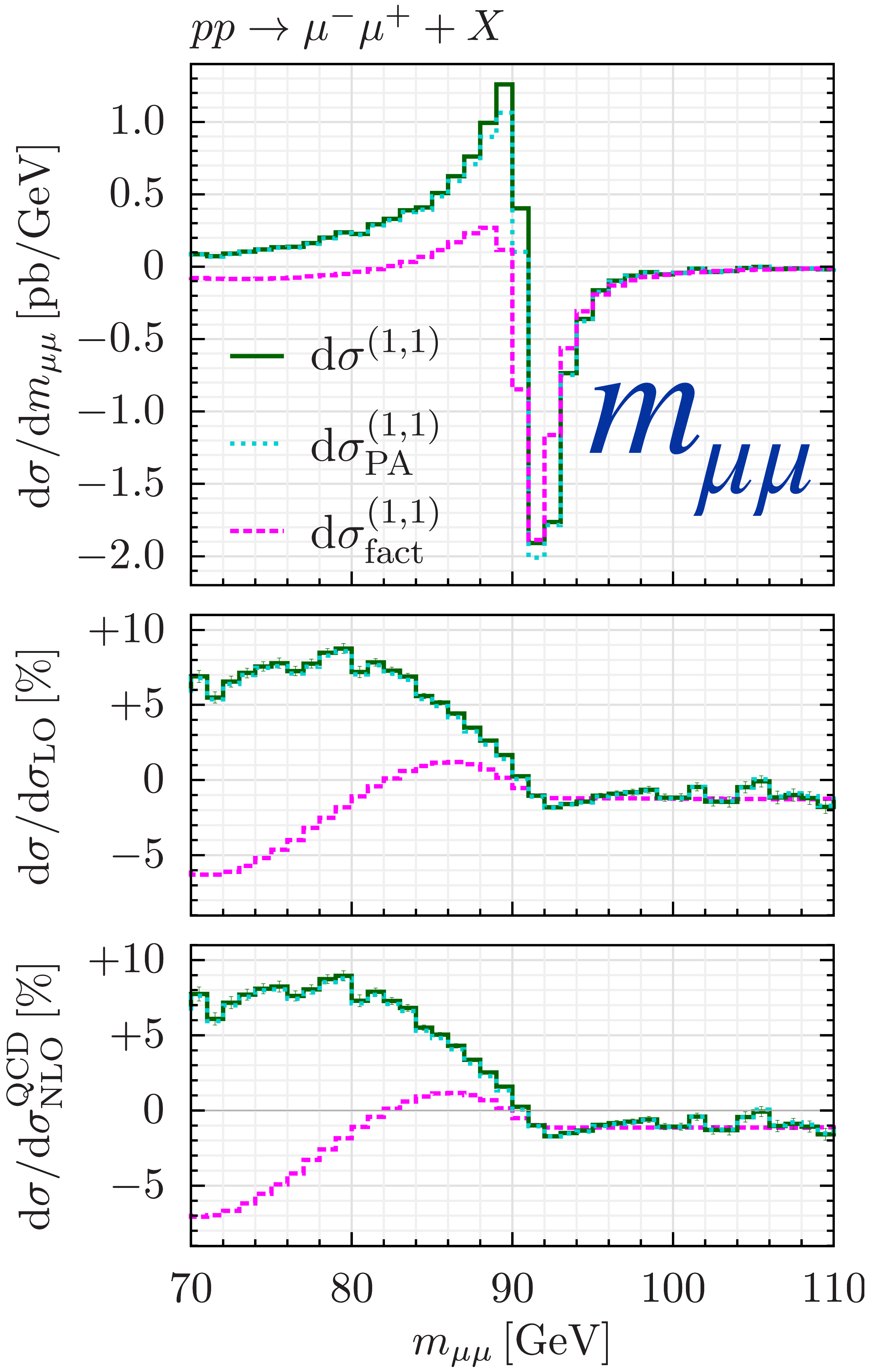
off-shell

- W
  [Buonocore, Grazzini, Kallweit, Savoini, Tramontano '21]
- Z
  [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '21]
  [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Rontsch, Signorile-Signorile '22]

$\mathcal{O}(\alpha_s\,\alpha)$ — Resonance Region

bare muons
("dressing" ⇝ × 1/2)

[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '21]

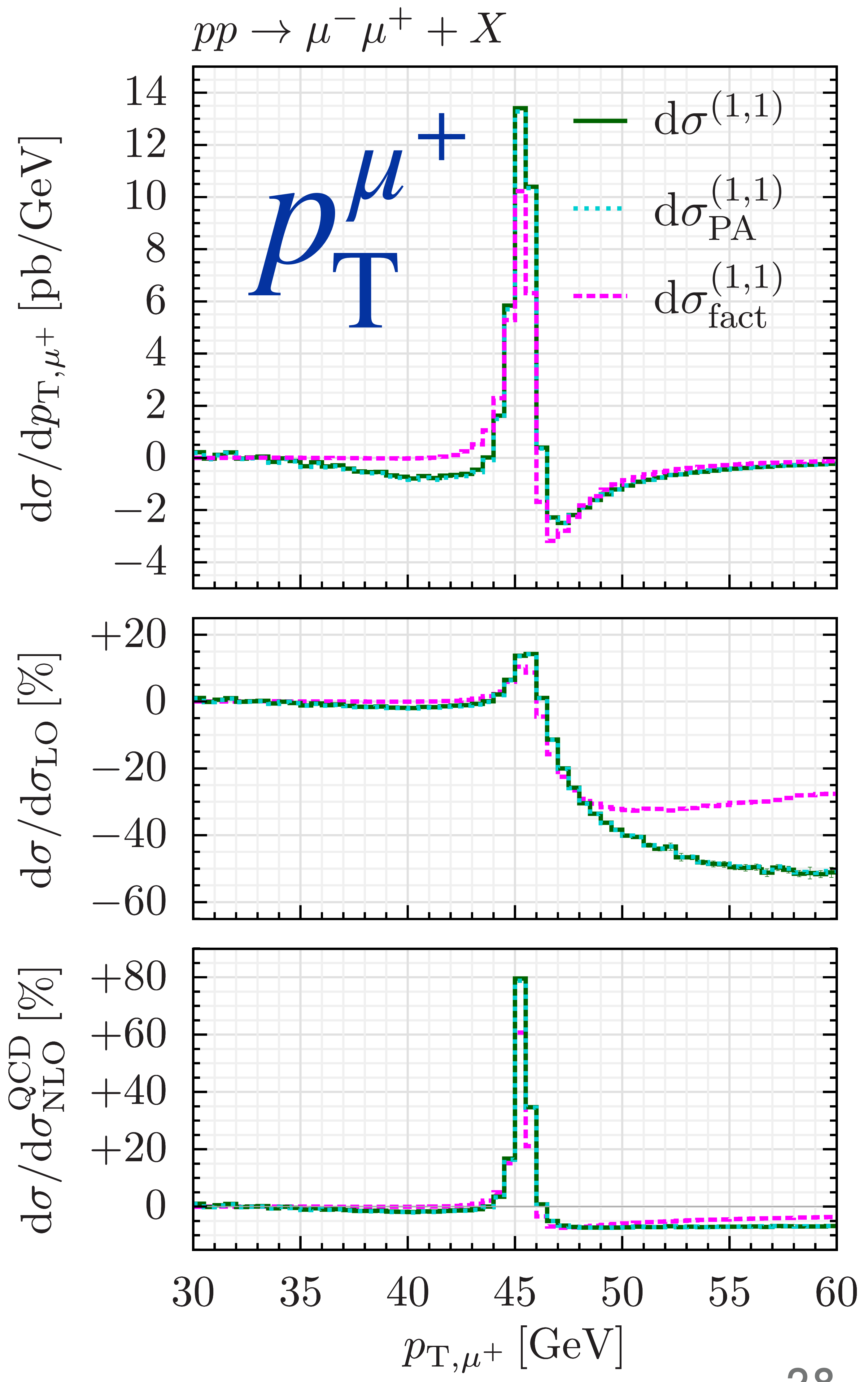


$\sigma^{\mathrm{NNLO}}_{\mathrm{s}\otimes\mathrm{ew}}$

$\sigma^{\mathrm{NNLO}_{\mathrm{s}\otimes\mathrm{ew}}}_{\mathrm{PA}}$

$\sigma^{\mathrm{NNLO}_{\mathrm{s}\otimes\mathrm{ew}}}_{\text{naive prod.}}$

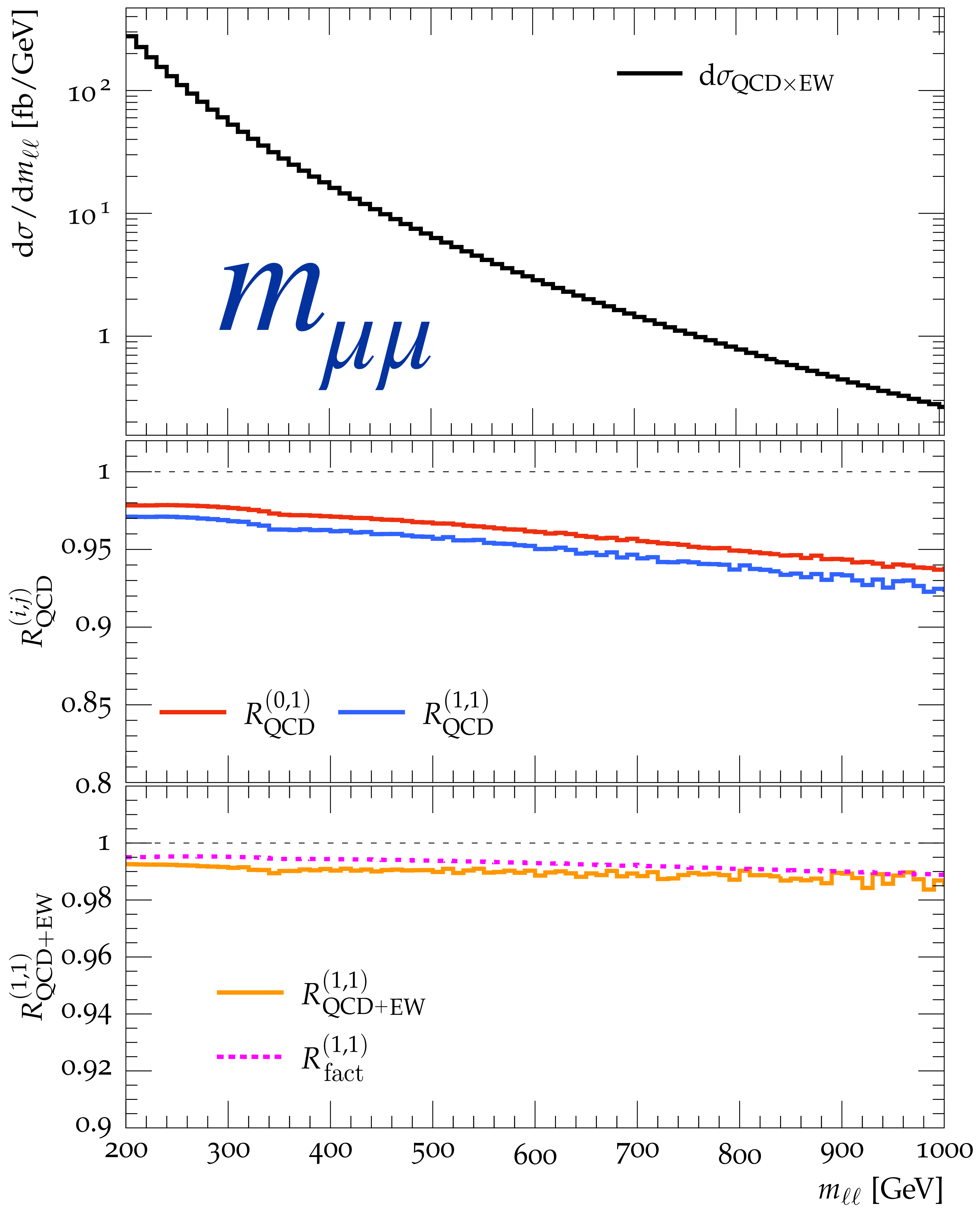
- naive product not able to capture kinematic effects
  - ↪ fails below resonance ($m_{\ell\ell}$)
  - ↪ fails away from shoulder (p_T^μ)
- pole approximation (PA)
  - ↪ well-captures full result here



$\mathcal{O}(\alpha_s\,\alpha)$ — High-Energy Tails

[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Rontsch, Signorile-Signorile '22]

Invariant mass of the dilepton system



- naive product
  - ↪ works well at high-$m_{\ell\ell}$
  - ↪ differences in p_T^μ spectrum
- tails ⇝ Sudakov (non flat)
  - ↪ QCD×EW ~ −3 %

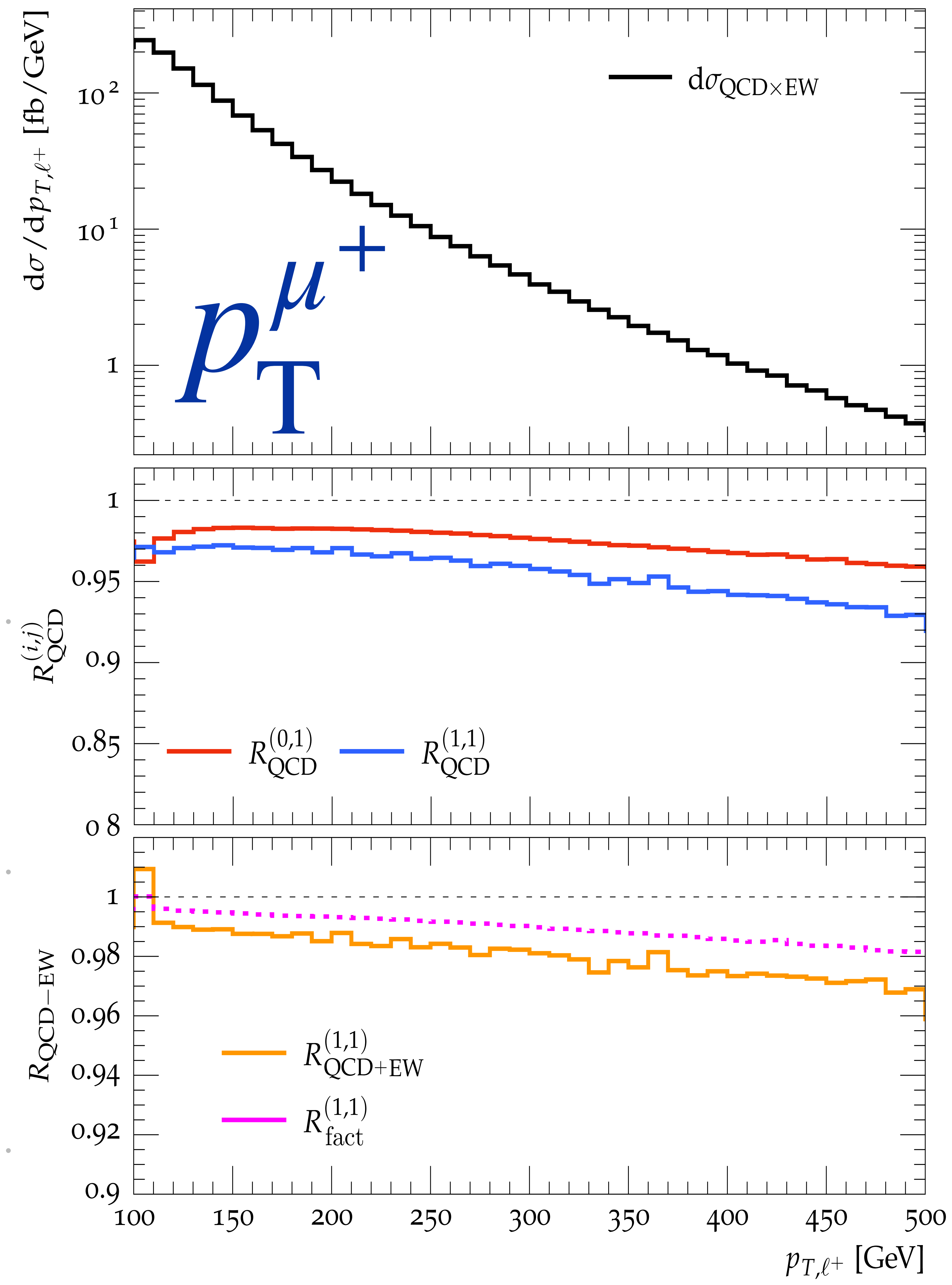
(blue line) $\sigma^{NNLO}_{s\otimes ew}$

(red line) $\sigma^{NLO}_{s\oplus ew}$

(orange line) $\sigma^{NNLO}_{s\otimes ew}$

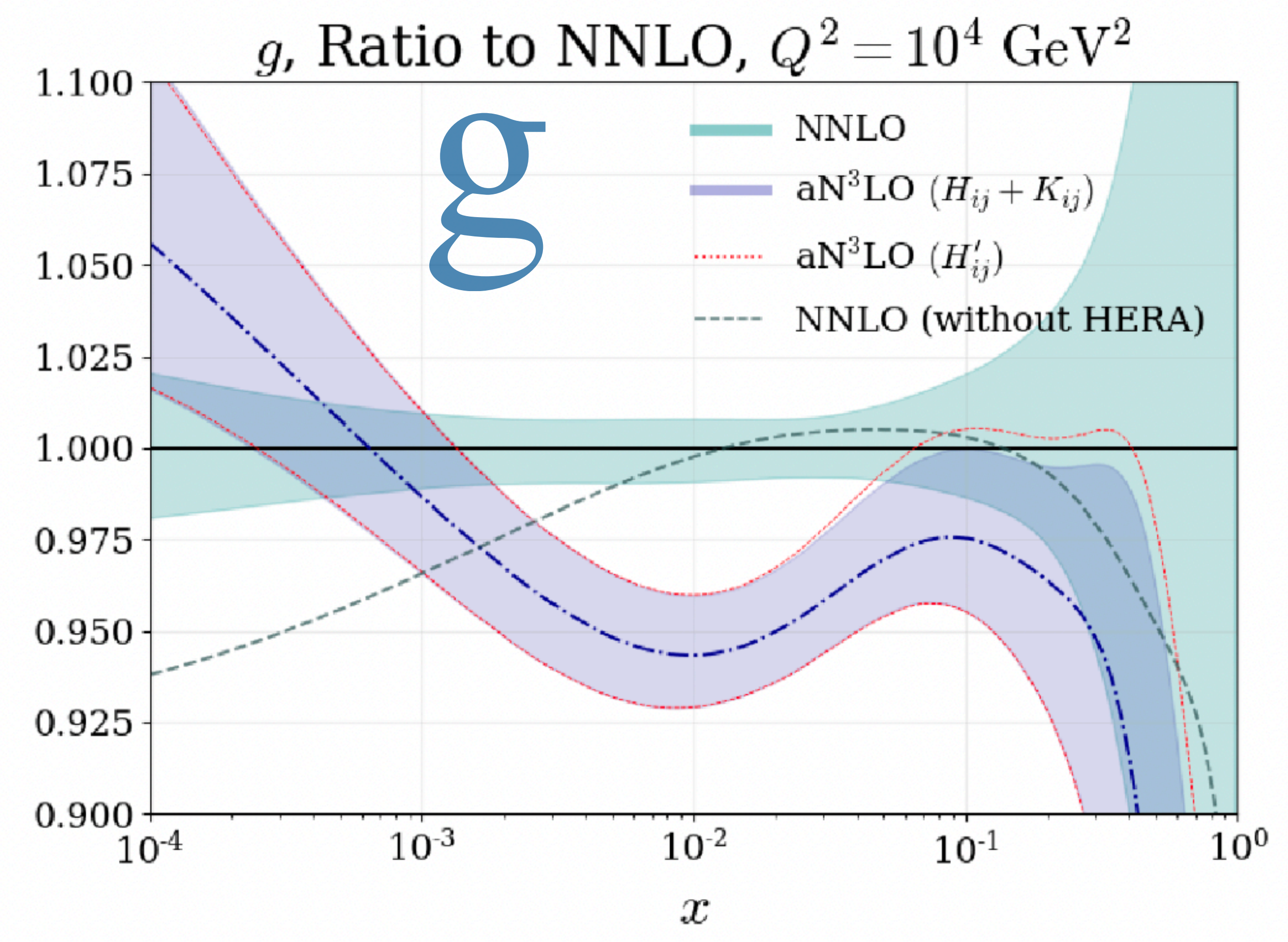
(magenta dashed line) $\sigma^{NNLO}_{s\otimes ew}$ naive prod.

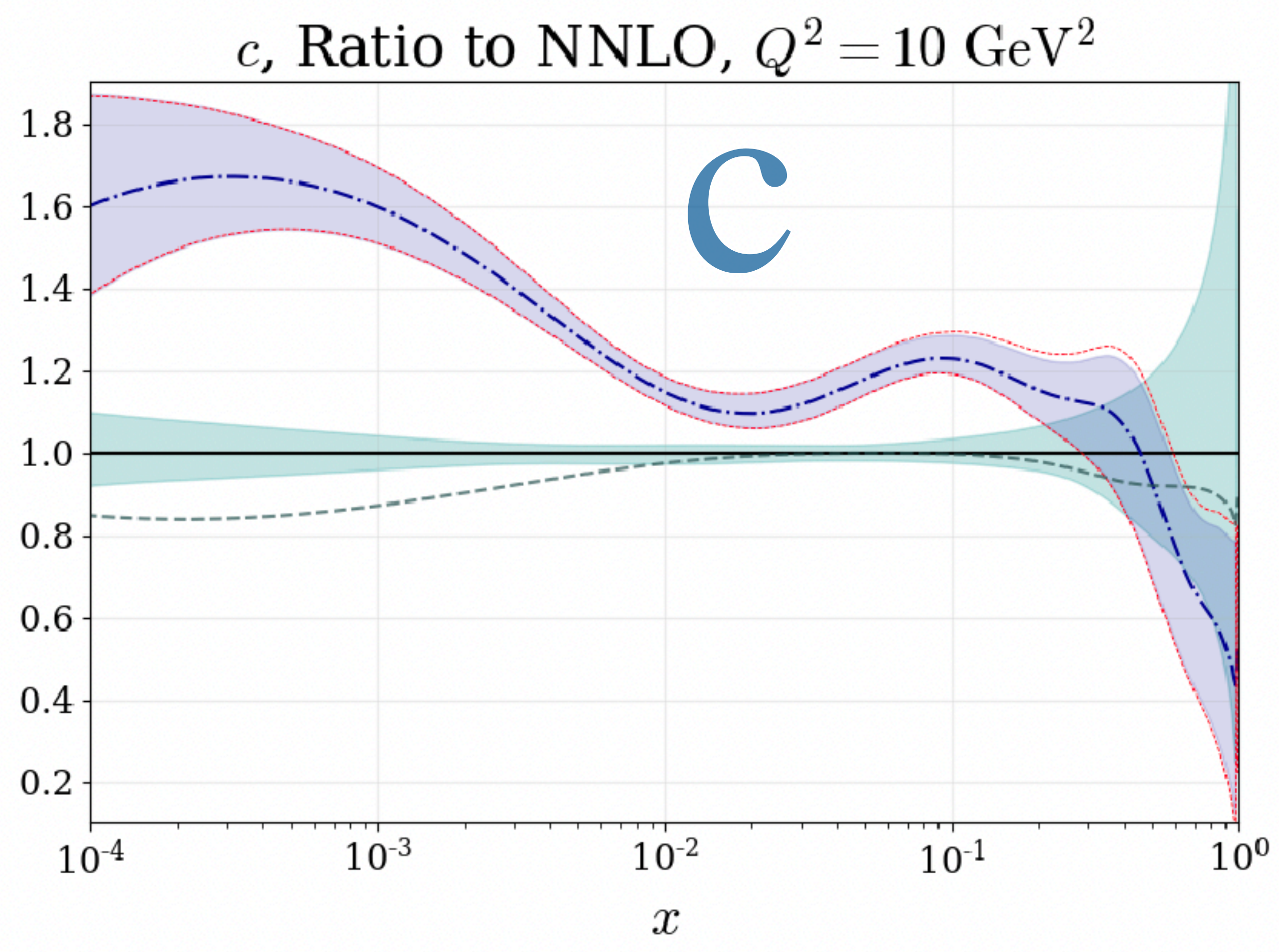
Transverse momentum of the positron



N^3LO Parton Distribution Functions

- N3LO evolution
  - ⇝ 4-loop splitting functions
    [Moch, Ruijl, Ueda, Vermaseren, Vogt '17,'18,'22];
    [Herzog, Falcioni, Moch, Vogt '23], in progress…
- aN3LO PDFs (MSHT)
  [McGowan, Cridge, Harland-Lang, Thorne '22]

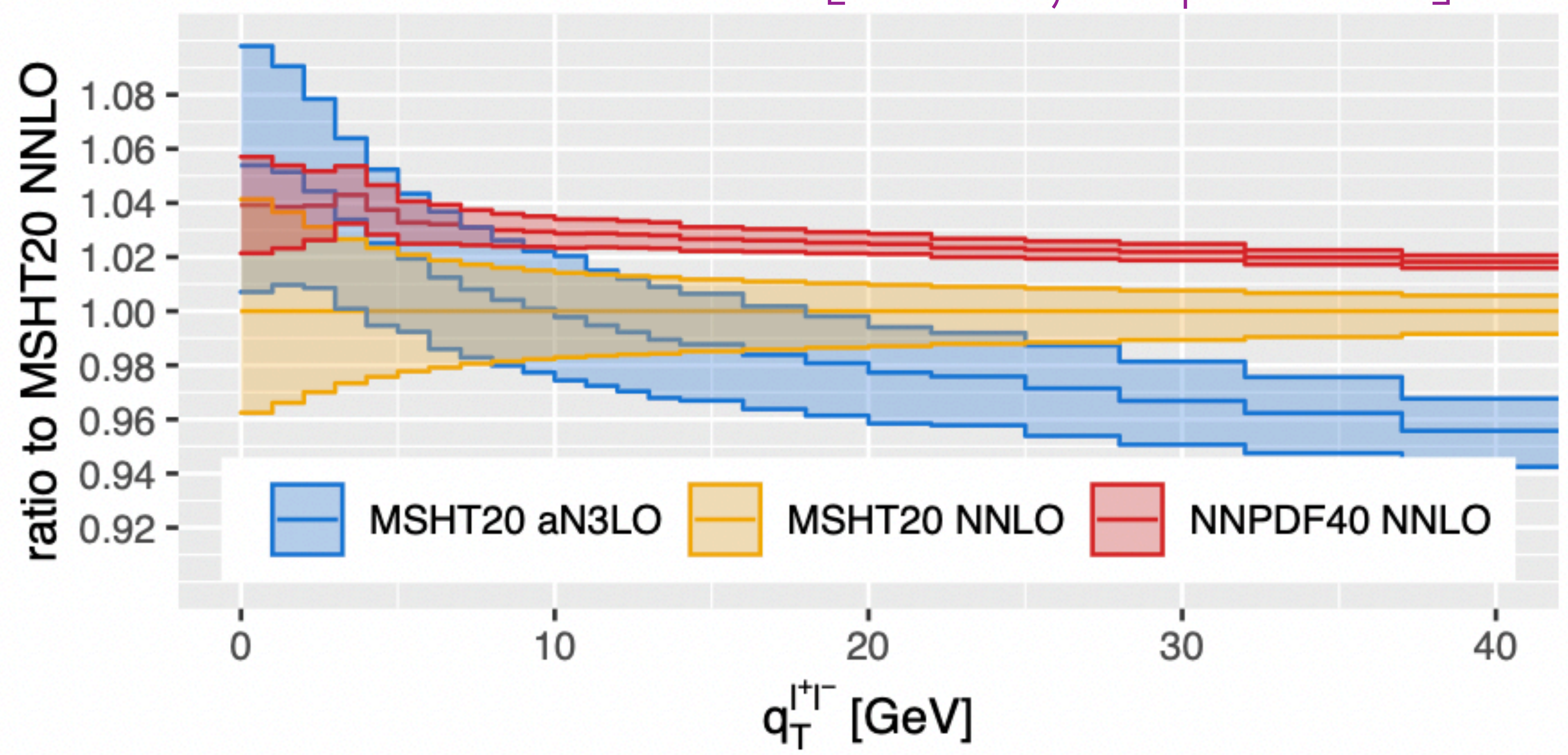




ggH: $\delta\sigma^{N^3LO}$ ↘ VBF: $\delta\sigma^{N^3LO}$ ↗

- purely resummed p_T^Z spectrum
  - ⇝ PDF uncertainties

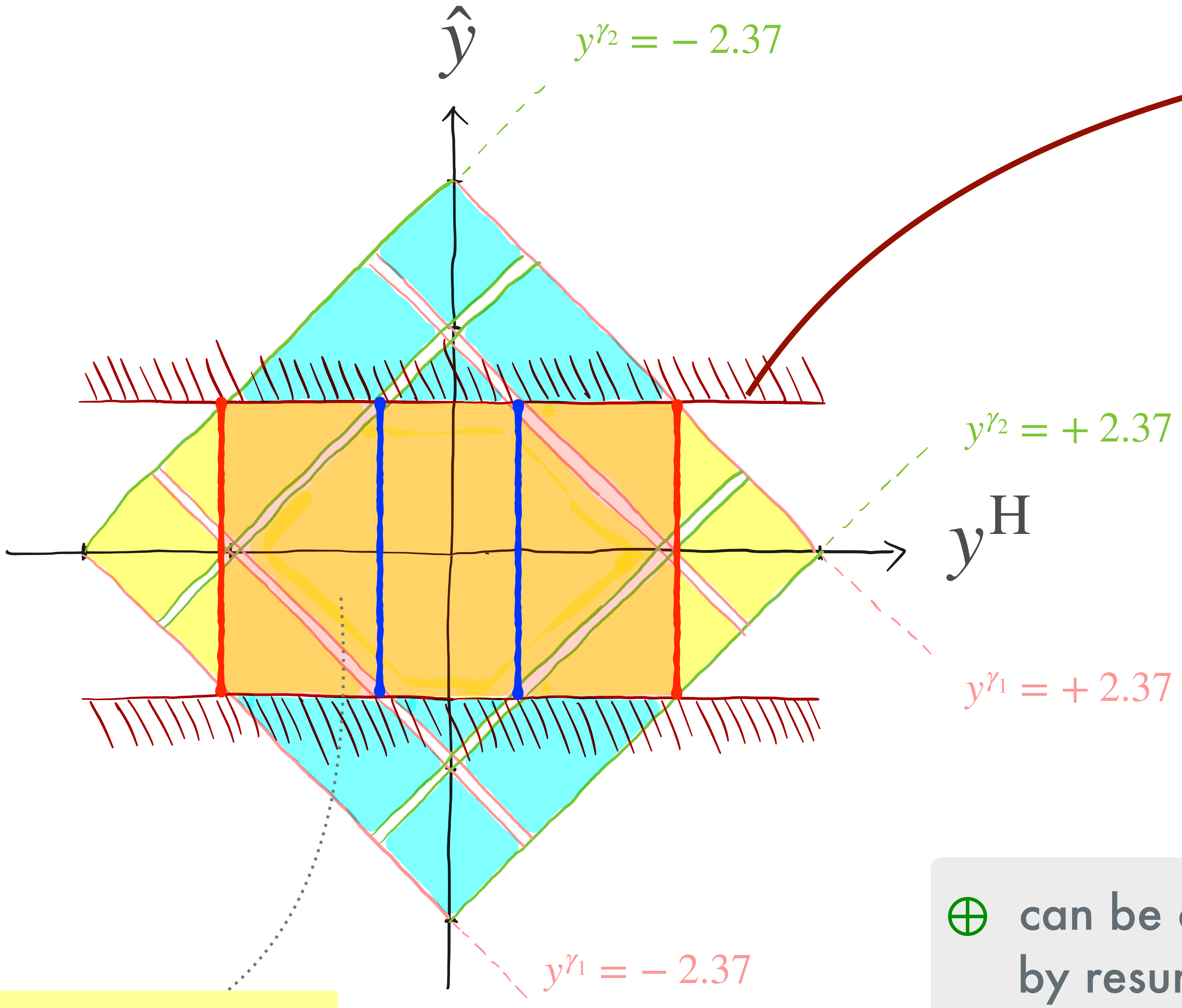
[Neumann, Campbell '22]



syst. differences between PDFs

PDF (NNLO → N^3LO) $\delta\sigma^{N^3LO}$ ↗ (?)

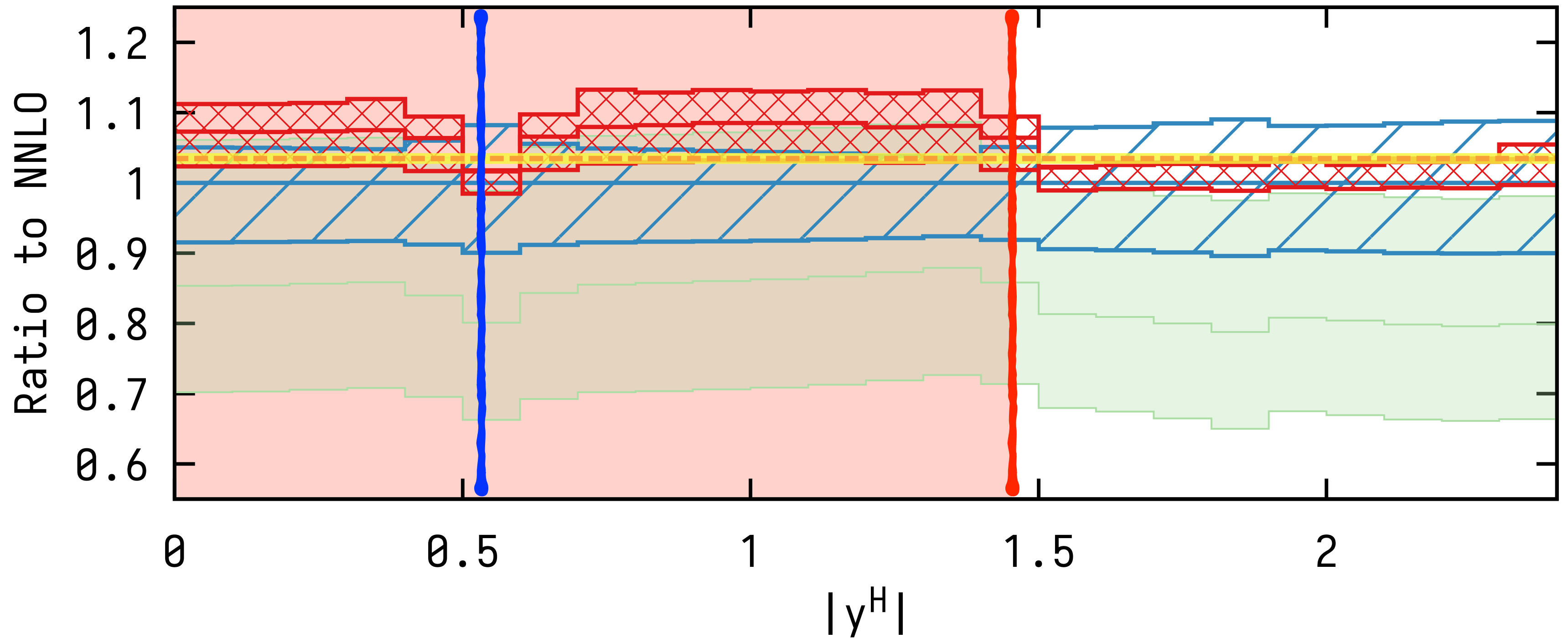
FIDUCIAL ACCEPTANCES & y_H



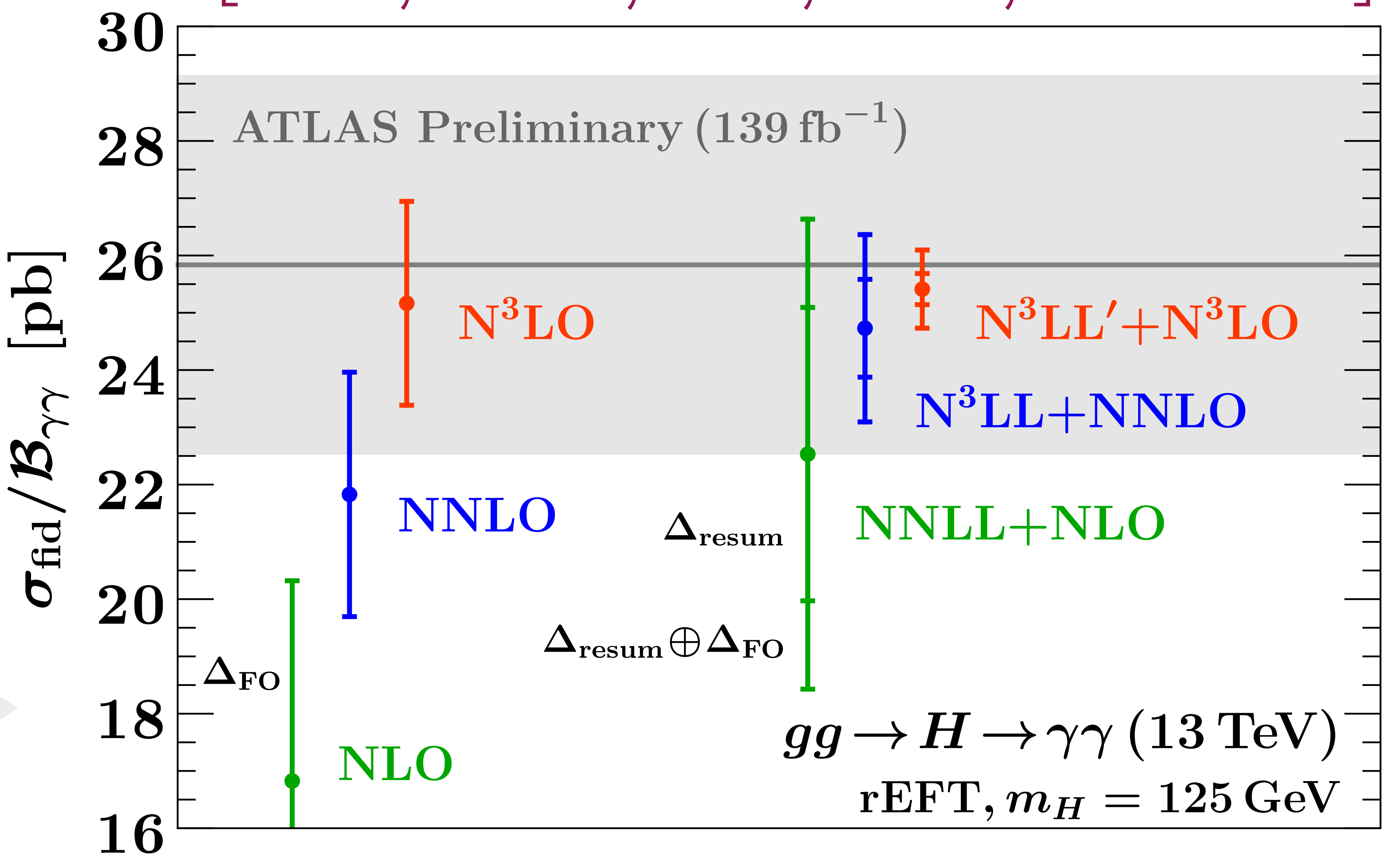
Born acceptance

$$\hat{y} \sim \frac{1}{2}\,|\,\Delta y(\gamma_1, \gamma_2)\,| \leq \cosh^{-1}\left(\frac{M_H}{2p_T^{\min}}\right) \approx 0.9$$

- linear fiducial power corrections!



[Billis, Dehnadi, Ebert, Michel, Tackmann '21]

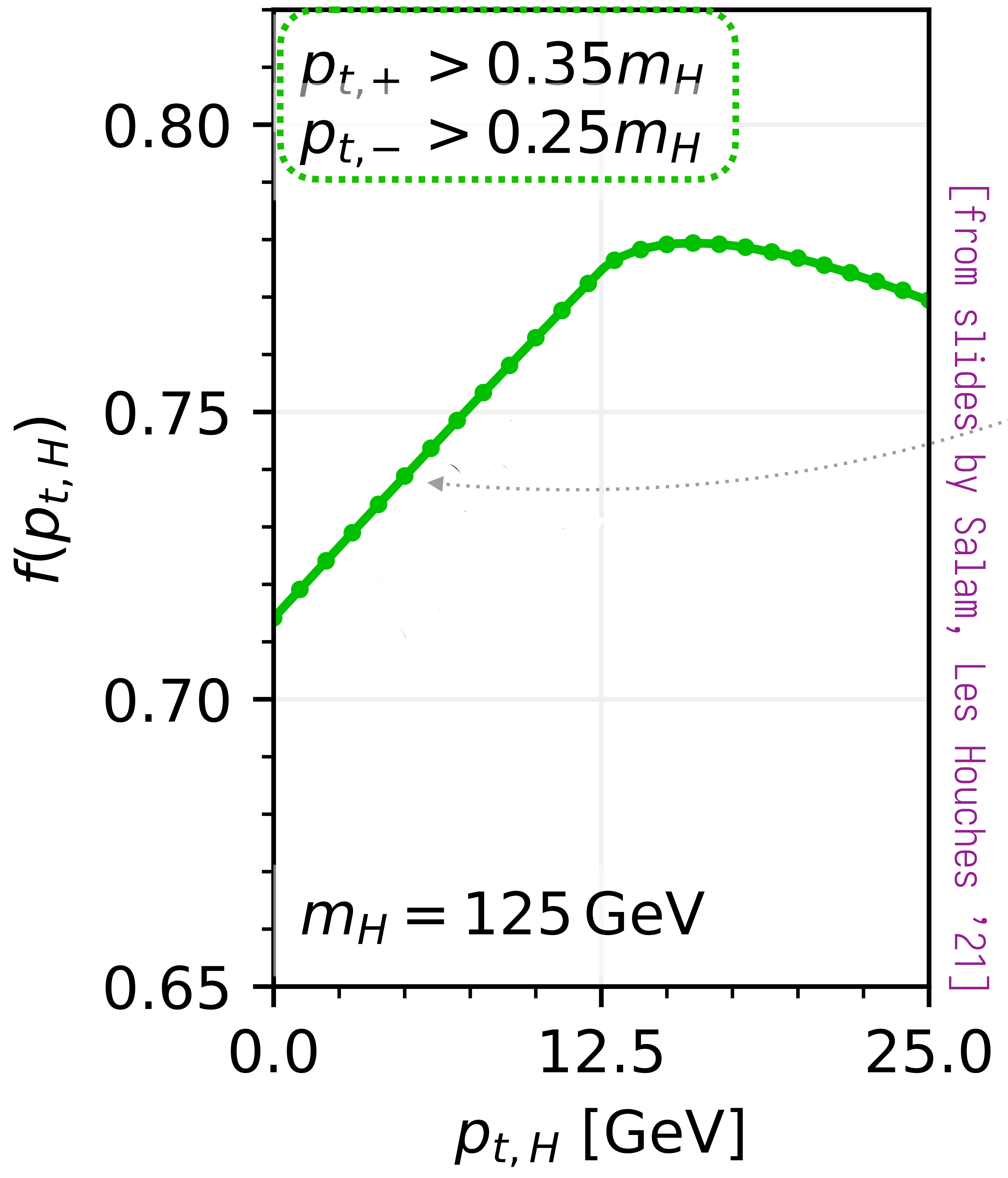


⊕ can be cured by resummation

⊖ hard $\sigma^{\rm fid.}$ should not need resummation

ACCEPTANCE $f(p_T^H)$

“with cuts” / “no cuts”



[from slides by Salam, Les Houches '21]

$$f(p_T^H) = f_0 + f_1 \cdot p_T^H + \mathcal{O}\left((p_T^H)^2\right)$$

[Frixione, Ridolfi '97; Ebert, Tackmann '19 + Michel, Stewart '21; Alekhin et al. '21]

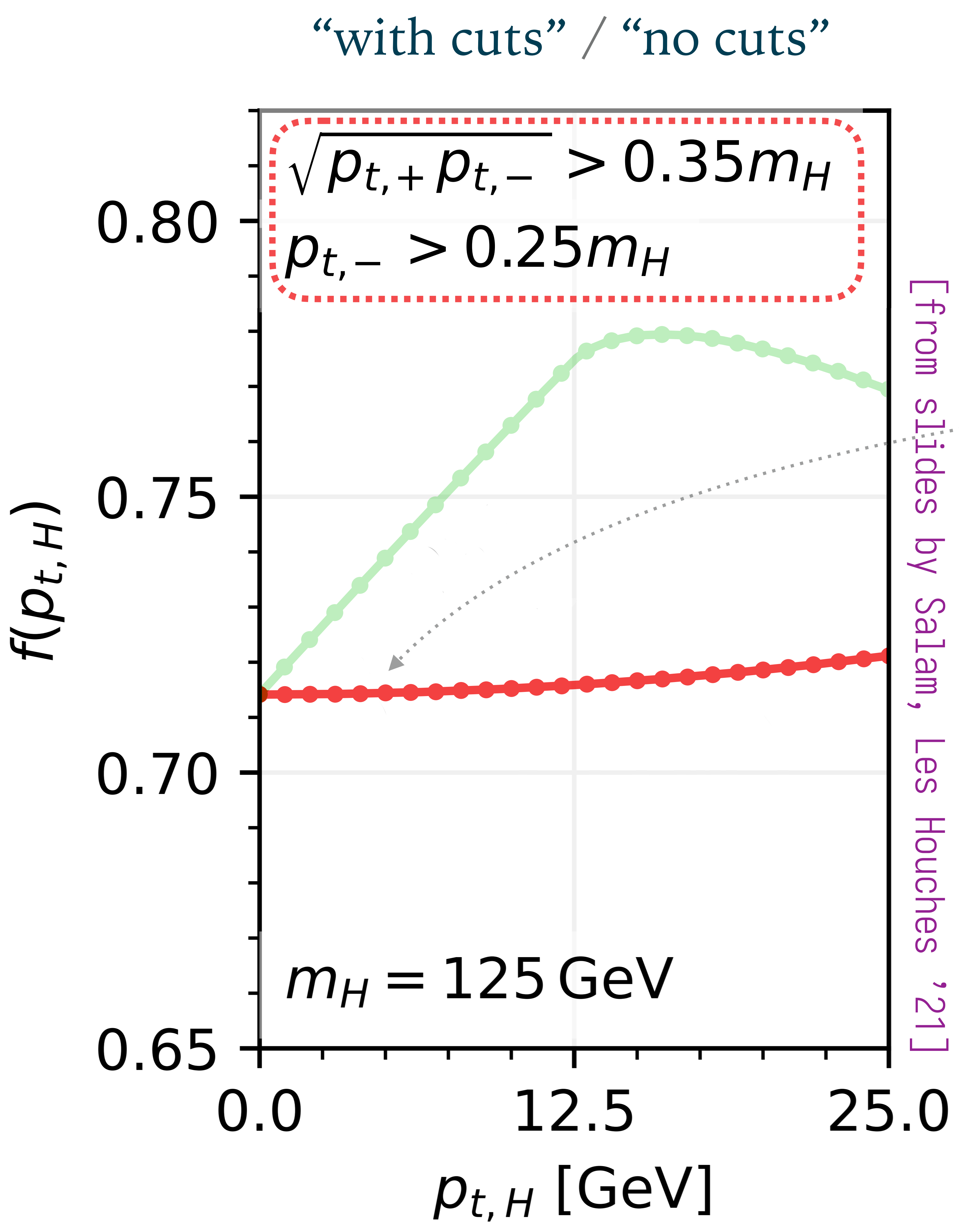
- Linear p_T^H dependence
  - **factorial growth** for fixed-order
  - ***sensitivity*** to very low p_T^H

$$\frac{\sigma_{\text{asym}} - f_0\,\sigma_{\text{inc.}}}{\sigma_0\, f_0} \simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.31_{\alpha_s^3} + \ldots$$

$$\simeq 0.12 \text{ @ N}^3\text{LL}$$

[Salam, Slade '21]

ACCEPTANCE $f(p_T^H)$



$$f(p_T^H) = f_0 + f_1 \cdot p_T^H + f_2 \cdot (p_T^H)^2 + \mathcal{O}\big((p_T^H)^3\big)$$

- Quadratic p_T^H dependence
  - *suppress* factorial growth
  - fixed order $\simeq$ resummation ✓

$$\frac{\sigma_{\text{prod}} - f_0\,\sigma_{\text{inc.}}}{\sigma_0\, f_0} \simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots$$

$$\simeq 0.006 \text{ @ N}^3\text{LL}$$

[Salam, Slade '21]

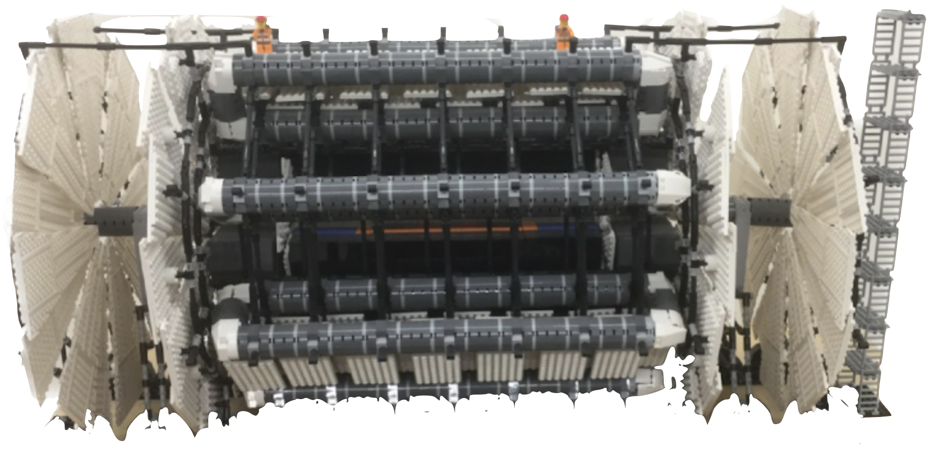
Going Differential @ N^3LO — q_T Subtraction

Fully Inclusive

✗ limited to σ^{tot}

✓ very efficient $\mathcal{O}(\text{sec})$

Fully Differential

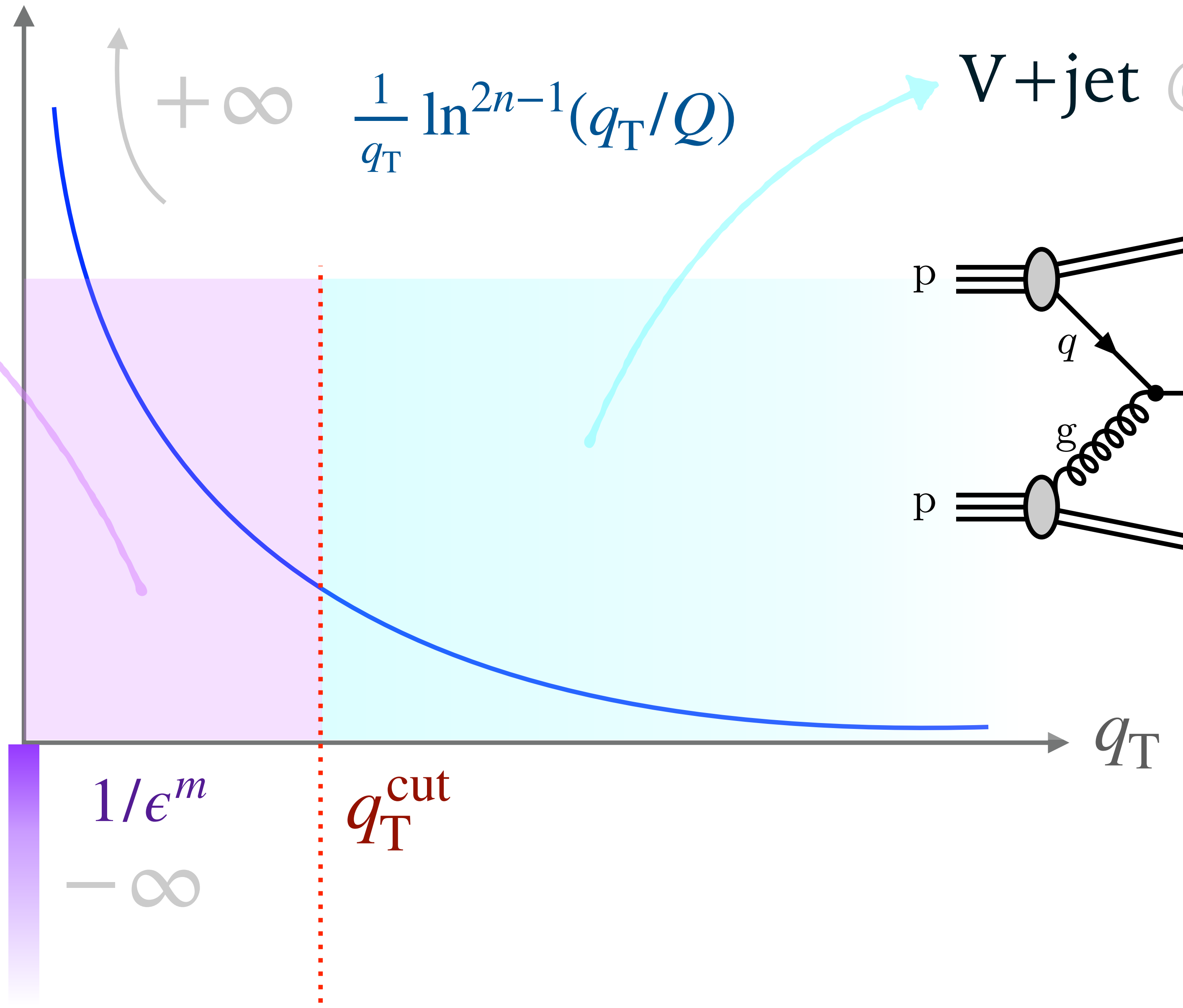


✓ $d\sigma$ ⇝ fiducial cuts, arbitrary distributions, …

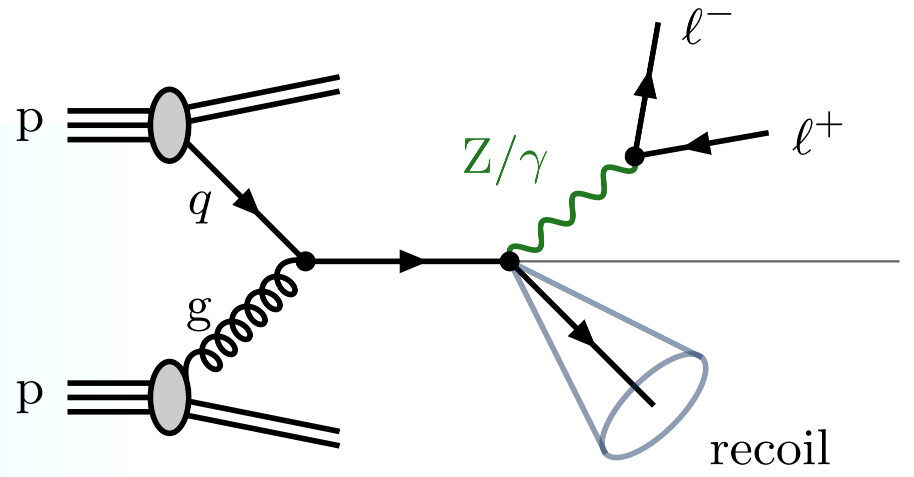
✗ computationally expensive $\mathcal{O}(10^5\text{-}10^6)$ h

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$ [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$ [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$ [Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, Vita '20]



V+jet @ NNLO



[Catani, Grazzini '07]

$$d\sigma^V_{N^3LO} = d\sigma^V_{N^3LO}\Big|_{q_T<q_T^{cut}} + d\sigma^V_{N^3LO}\Big|_{q_T>q_T^{cut}}$$

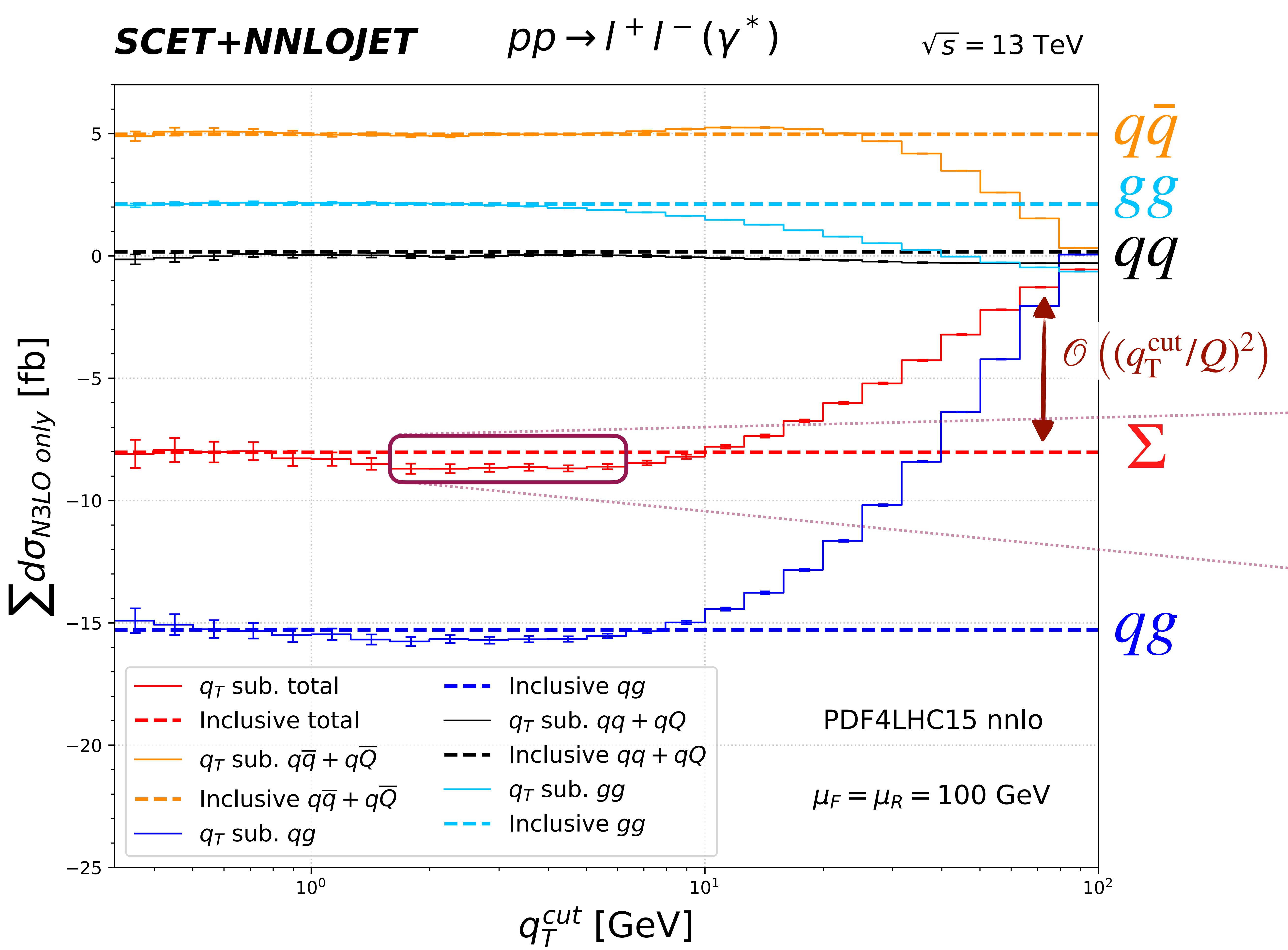
$$= \mathcal{H}^V_{N^3LO} \otimes d\sigma^V_{LO} + \left[d\sigma^{V+\text{jet}}_{NNLO} - d\sigma^{V,CT}_{N^3LO}\right]_{q_T>q_T^{cut}} + \mathcal{O}\left((q_T^{cut}/Q)^n\right)$$

slicing error

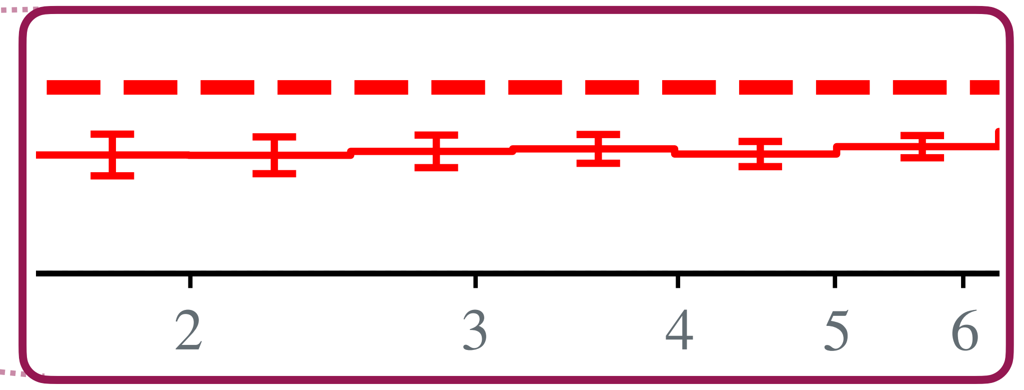
q_T^{cut} as small as possible ↪ suppress power corrections ⇜⇝ q_T^{cut} as large as possible ↪ numerical stability & efficiency

VALIDATION

[Chen, Gehrmann, Glover, AH, Yang Zhu ‘21, ‘22]



- *fully independent* calculation of the inclusive cross section
- ---- ⇝ analytic result [Duhr, Dulat, Mistlberger ’20]
- “fake” plateau: $q_T^{cut} \in [2, 5]$ GeV
  ↪ 12% error on δN³LO!



- converges to correct result for $q_T^{cut} \lesssim 1$ GeV
- fit & extrapolate?
  ⇝ marginal gains for potentially uncontrolled systematics

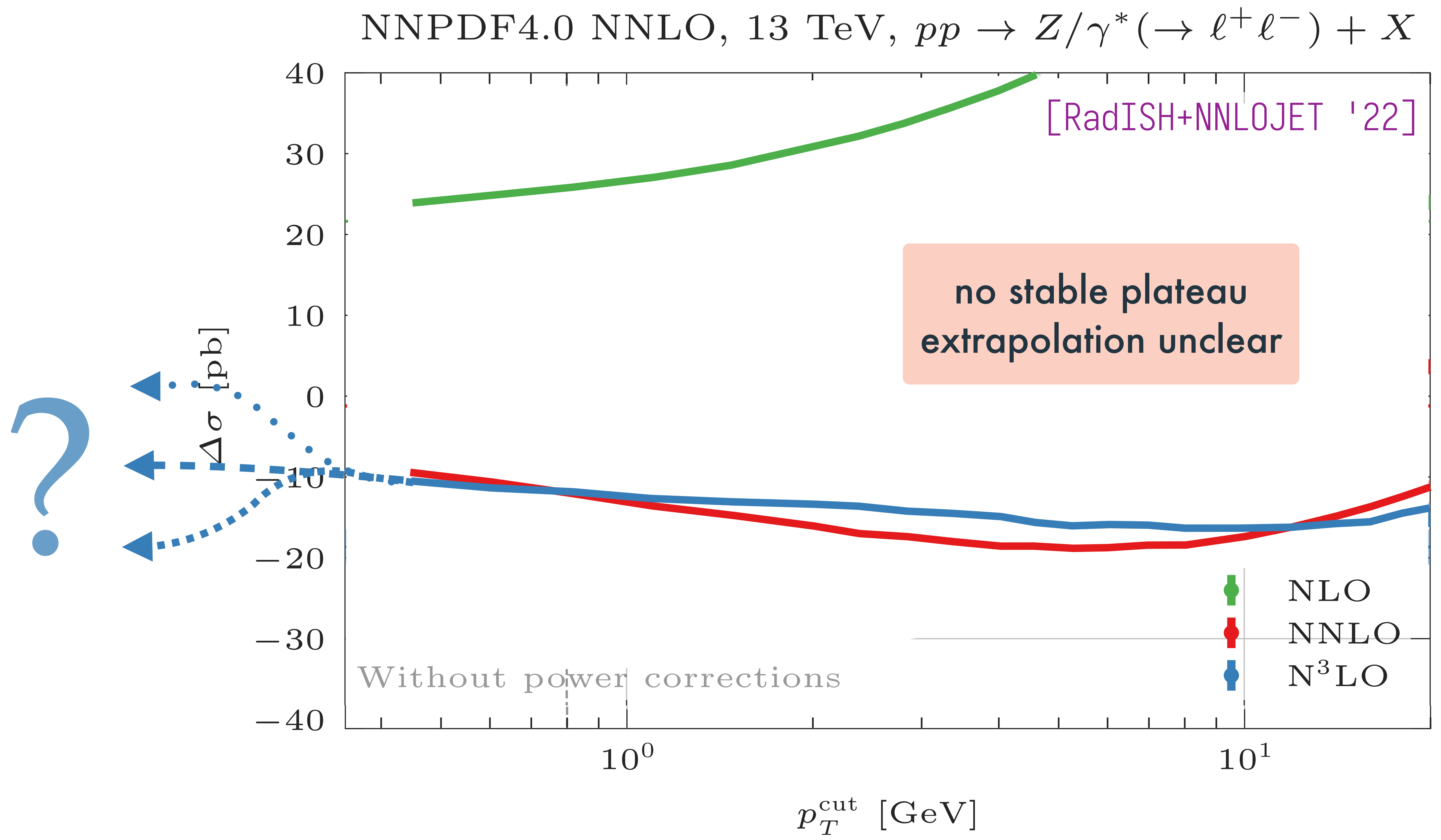
Fiducial Cuts and Linear Power Corrections — N^{3}LO Slicing

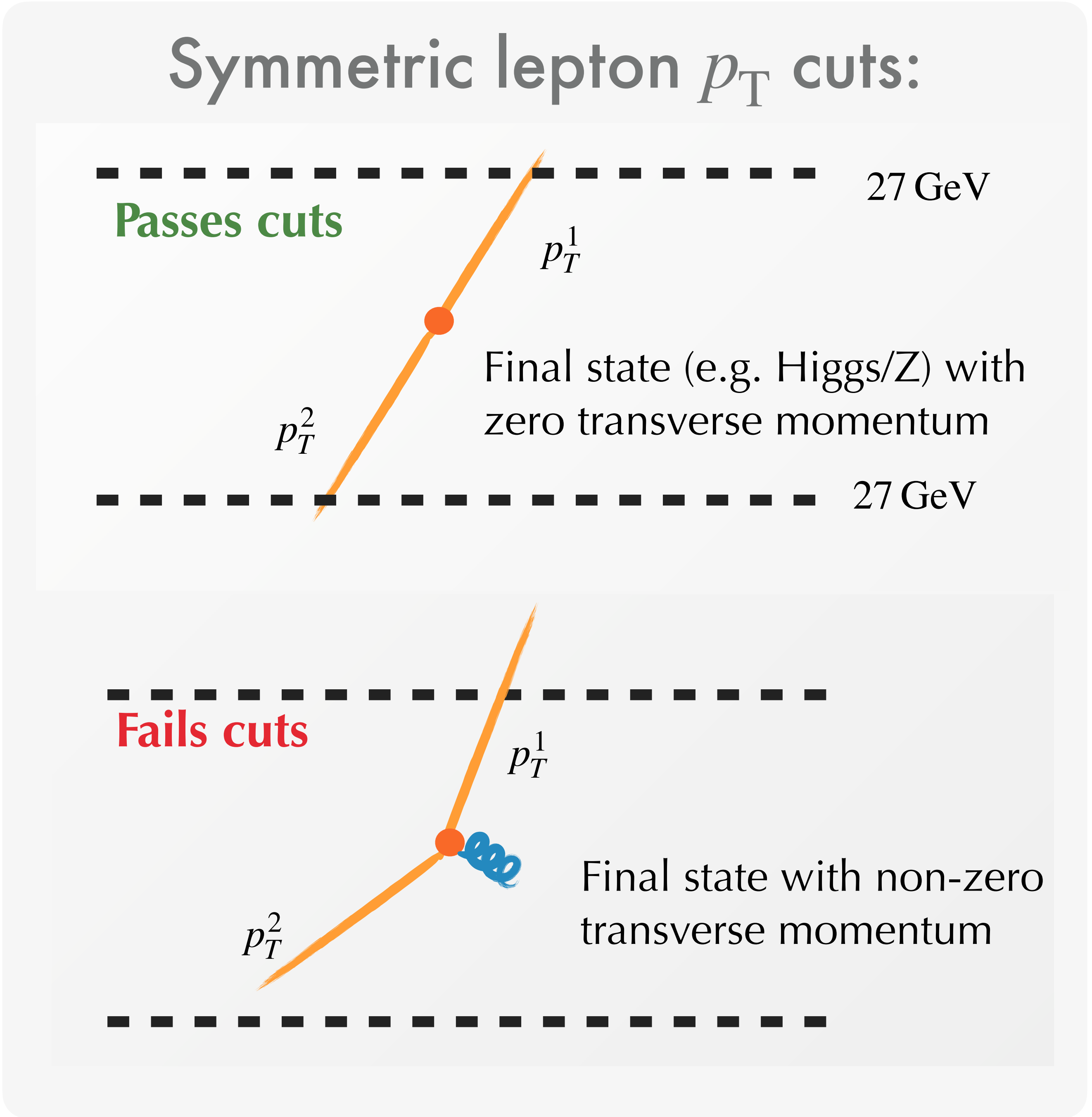
- fiducial cuts ⇝ can induce linear power corrections

  [Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]

  - can jeopardise q_T slicing $\mathcal{O}\left((q_T^{cut}/Q)^2\right)$ ⇝ $\mathcal{O}\left(q_T^{cut}/Q\right)$

    [$q_T^{cut} \lesssim 1\,\text{GeV}$] [$q_T^{cut} \lesssim 10^{-2}\,\text{GeV}$?!]





[sketch by L. Rottoli — SM@LHC '22]

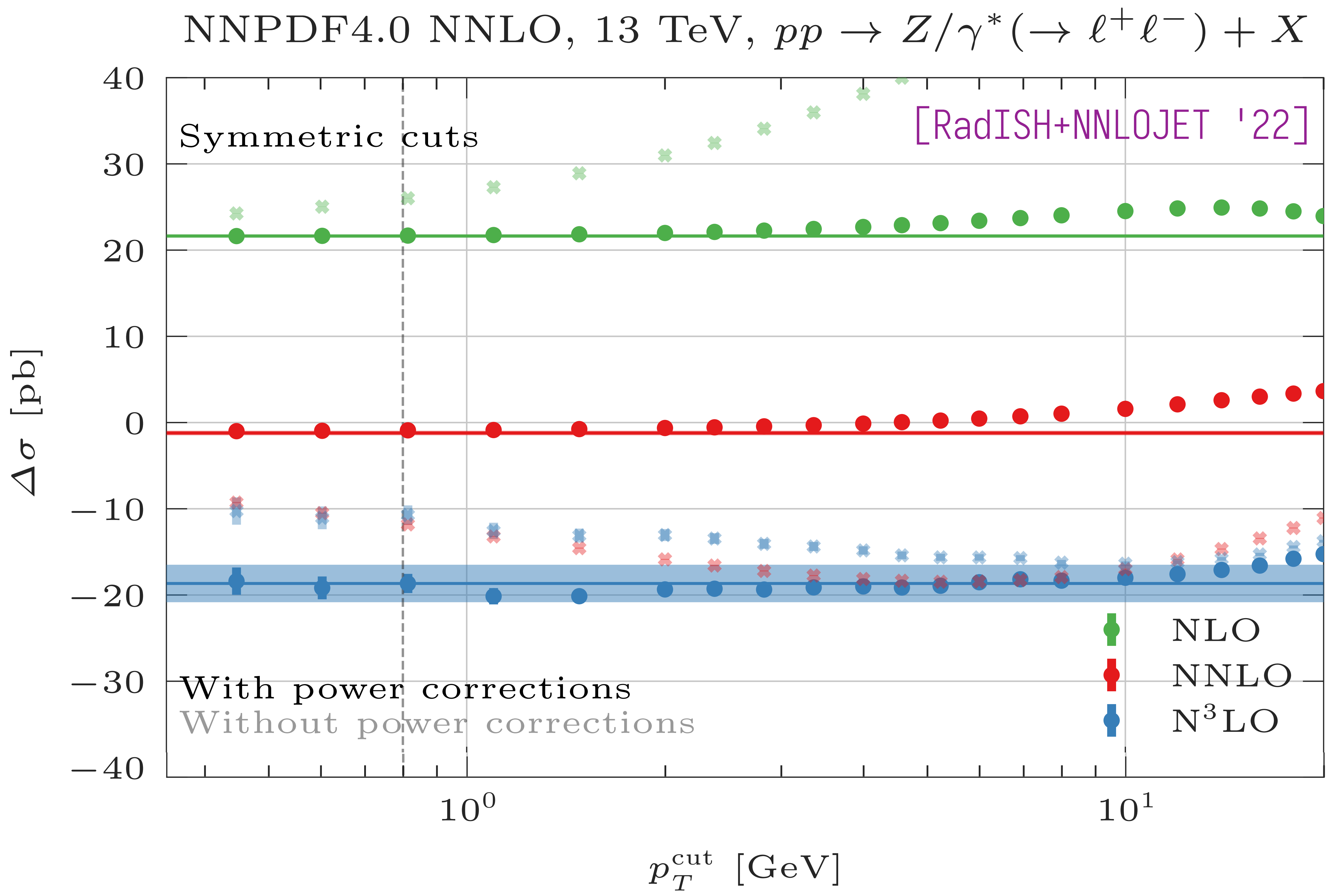
Fiducial Cuts and Linear Power Corrections — N3LO Slicing

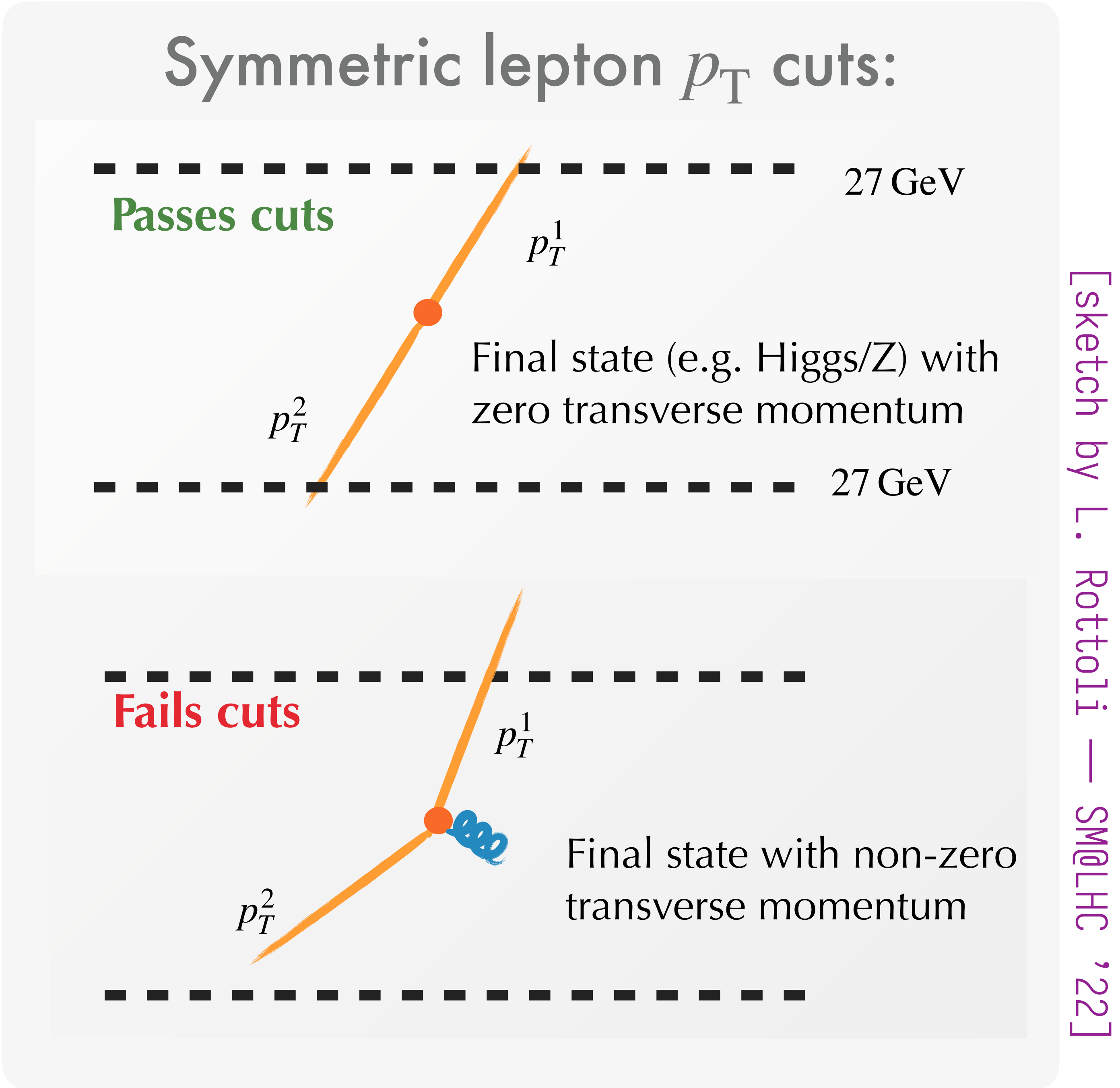
- fiducial cuts ⇝ can induce linear power corrections

  [Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]

  - can jeopardise q_T slicing $\mathcal{O}\left((q_T^{\text{cut}}/Q)^2\right)$ ⇝ $\mathcal{O}\left(q_T^{\text{cut}}/Q\right)$

    $[q_T^{\text{cut}} \lesssim 1\,\text{GeV}]$ $[q_T^{\text{cut}} \lesssim 10^{-2}\,\text{GeV ?!}]$





can *compute* & *subtract* the linear term:

↪ simple boost of $V \to \ell\bar{\ell}$ system

(pure kinematics & acceptance effect)

[Catani, de Florian, Ferrera, Grazzini '15]
[Ebert, Michel, Stewart, Tackmann '21]