N-jettiness resummation for processes with coloured final states at the LHC

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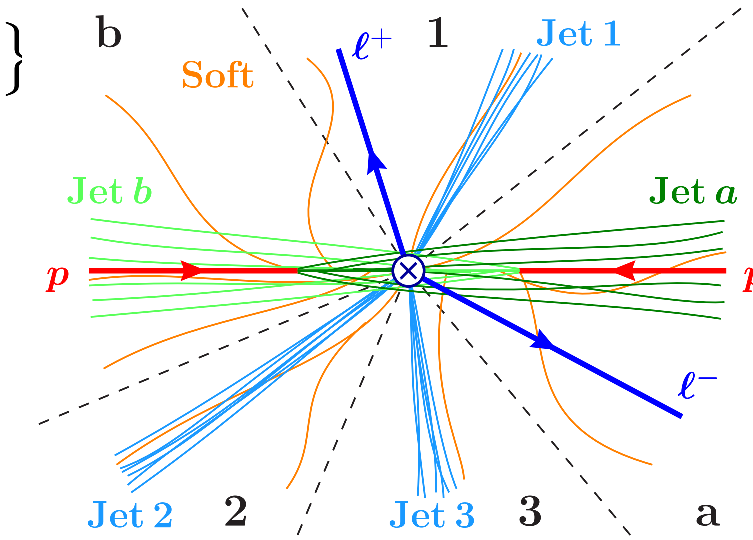
Overview of the talk

- Geneva Monte Carlo GENEVA http://geneva.physics.lbl.gov
- Recently implemented processes: single and double Higgs production
- Zero-jettiness resummation for top-quark pair production at the LHC
- One-jettiness resummation for Z+jet production at the LHC
- Conclusions & Outlook

N-Jettiness and Factorization

- N-jettiness resolution variables: given an M-particle phase space point with $M \geq N$

$$\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$$



- The limit $\mathcal{T}_N \to 0$ describes a N-jet event where the unresolved emissions be either soft or collinear to the final state jets or initial state beams

- Color singlet final state, relevant variable is 0-jettiness aka "beam thrust"

$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}|\, e^{-|\eta_k - Y|}$$

- Colour singlet case: cross section factorizes in the limit $\mathcal{T}_0 \to 0$ [Stewart, Tackmann,Waalewijn \`09,\`10], three different scales arise

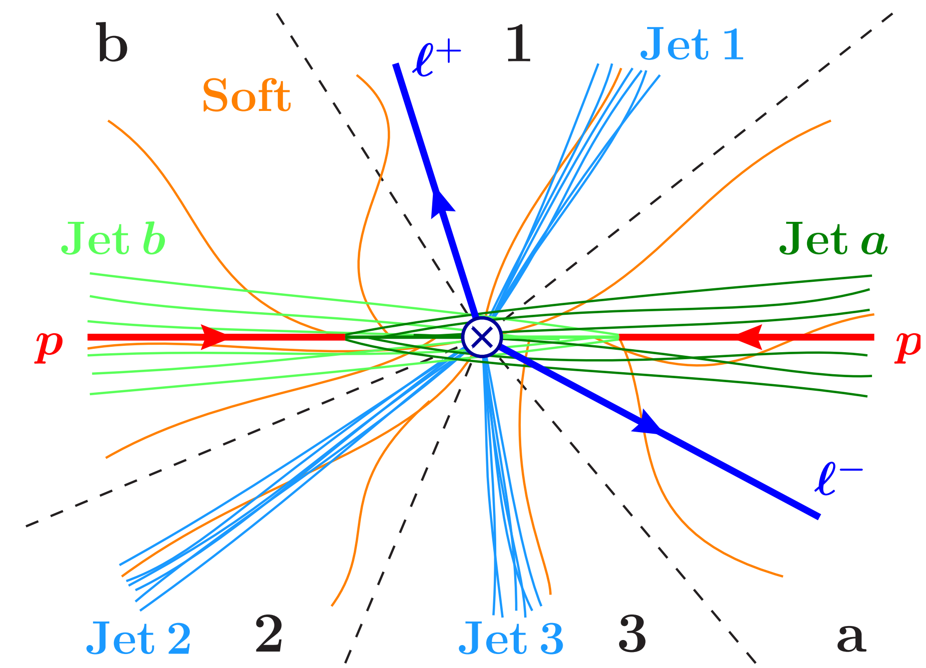
$$\mu_H = Q, \quad \mu_B = \sqrt{Q\mathcal{T}_0}, \quad \mu_S = \mathcal{T}_0$$

$$\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} = \sum_{ij} \underbrace{H_{ij}(Q^2, t, \mu_H)}_{\text{NNLO}} U_H(\mu_H, \mu) \{ [\underbrace{B_i(t_a, x_a, \mu_B)}_{\text{NNLO}} \otimes U_B(\mu_B, \mu)] \times [\underbrace{B_j(t_b, x_b, \mu_B)}_{\text{NNLO}} \otimes U_B(\mu_B, \mu)] \} \otimes [\underbrace{S(\mu_s)}_{\text{NNLO}} \otimes U_S(\mu_S, \mu)]$$

N-Jettiness and Factorization

- When an extra jet is present the relevant jet resolution variable is 1-jettiness

$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$



- Class of geometric measures $Q_i = \rho_i \, 2\, E_i$ (ρ_i dimensionless parameter), remove the dependence on the energies E_i and only depends on the directions $\hat{q}_i$. Introduce frame dependence.

- Choice of the ρ_i determines the frame in which the 1-jettiness is evaluated. We focus on 3 choices: Laboratory frame, Underlying Born (UB) frame ($Y_{Vj} = 0$), Color Singlet (CS) frame ($Y_V = 0$).

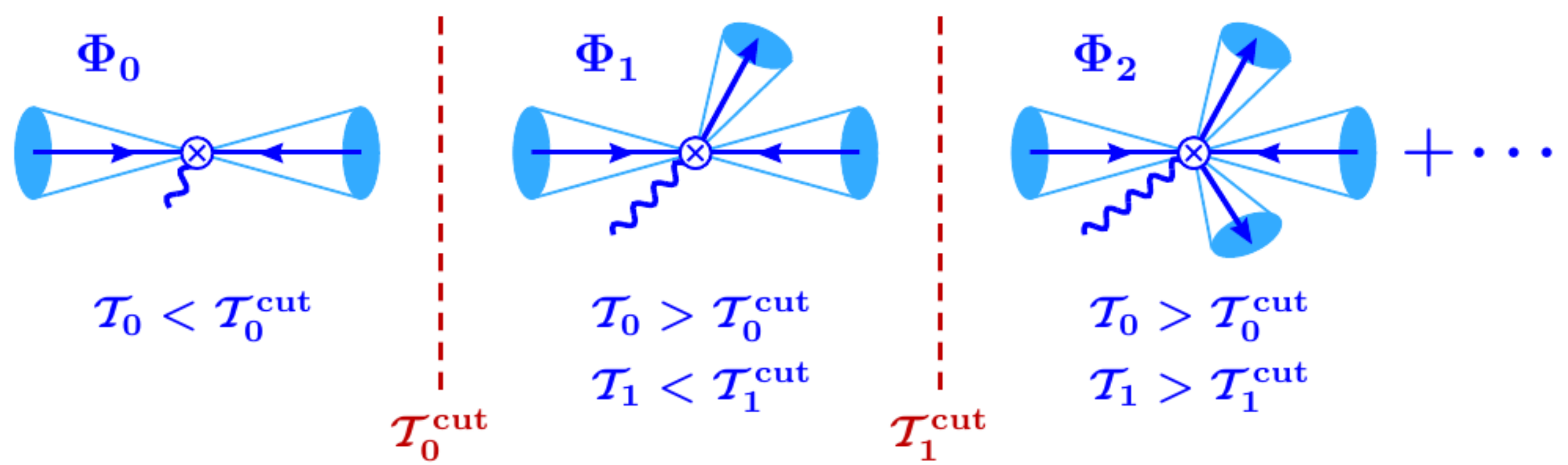
Monte Carlo implementation

- GENEVA [Alioli,Bauer,Berggren,Tackmann, Walsh `15], [Alioli,Bauer,Tackmann,Guns `16], [Alioli,Broggio,Lim, Kallweit,Rottoli `19],[Alioli,Broggio,Gavardi,Lim,Nagar,Napoletano,Kallweit,Rottoli `20-`21] combines 3 theoretical tools that are important for QCD predictions into a single framework
  - fully differential fixed-order calculations, up to NNLO via 0-jettiness or q_T subtraction
  - up to NNLL` resummation for 0-jettiness in SCET or N^{3}LL for q_T via RadISH for colour singlet processes
  - shower and hadronize events (PYTHIA8)
- IR-finite definition of events based on resolution parameters $\mathcal{T}_0^{\rm cut}$ and $\mathcal{T}_1^{\rm cut}$

$$\Phi_0 \text{ events:} \quad \frac{d\sigma_0^{\rm MC}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}),$$

$$\Phi_1 \text{ events:} \quad \frac{d\sigma_1^{\rm MC}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}; \mathcal{T}_1^{\rm cut}),$$

$$\Phi_2 \text{ events:} \quad \frac{d\sigma_{\geq 2}^{\rm MC}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}, \mathcal{T}_1 > \mathcal{T}_1^{\rm cut})$$

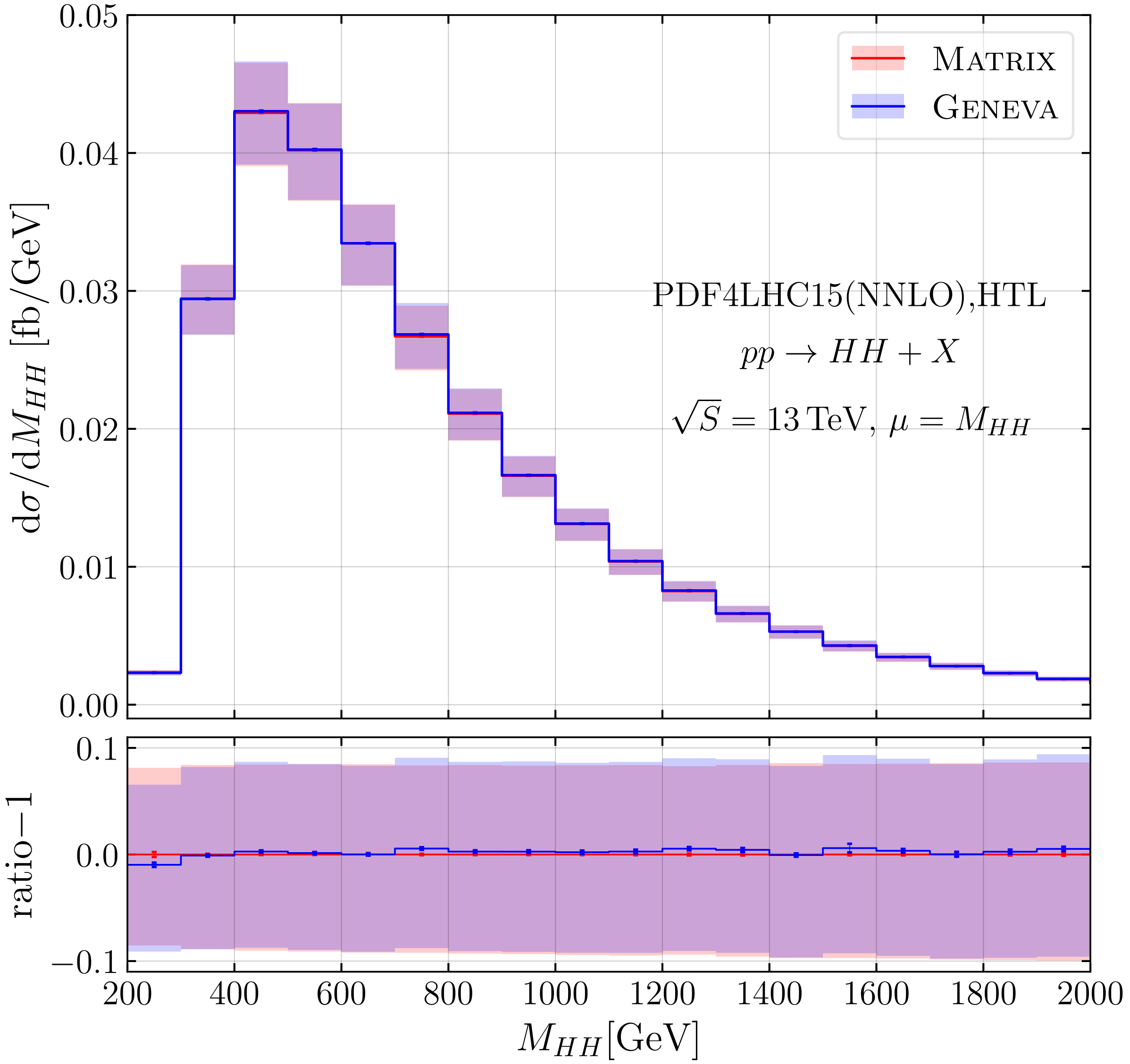


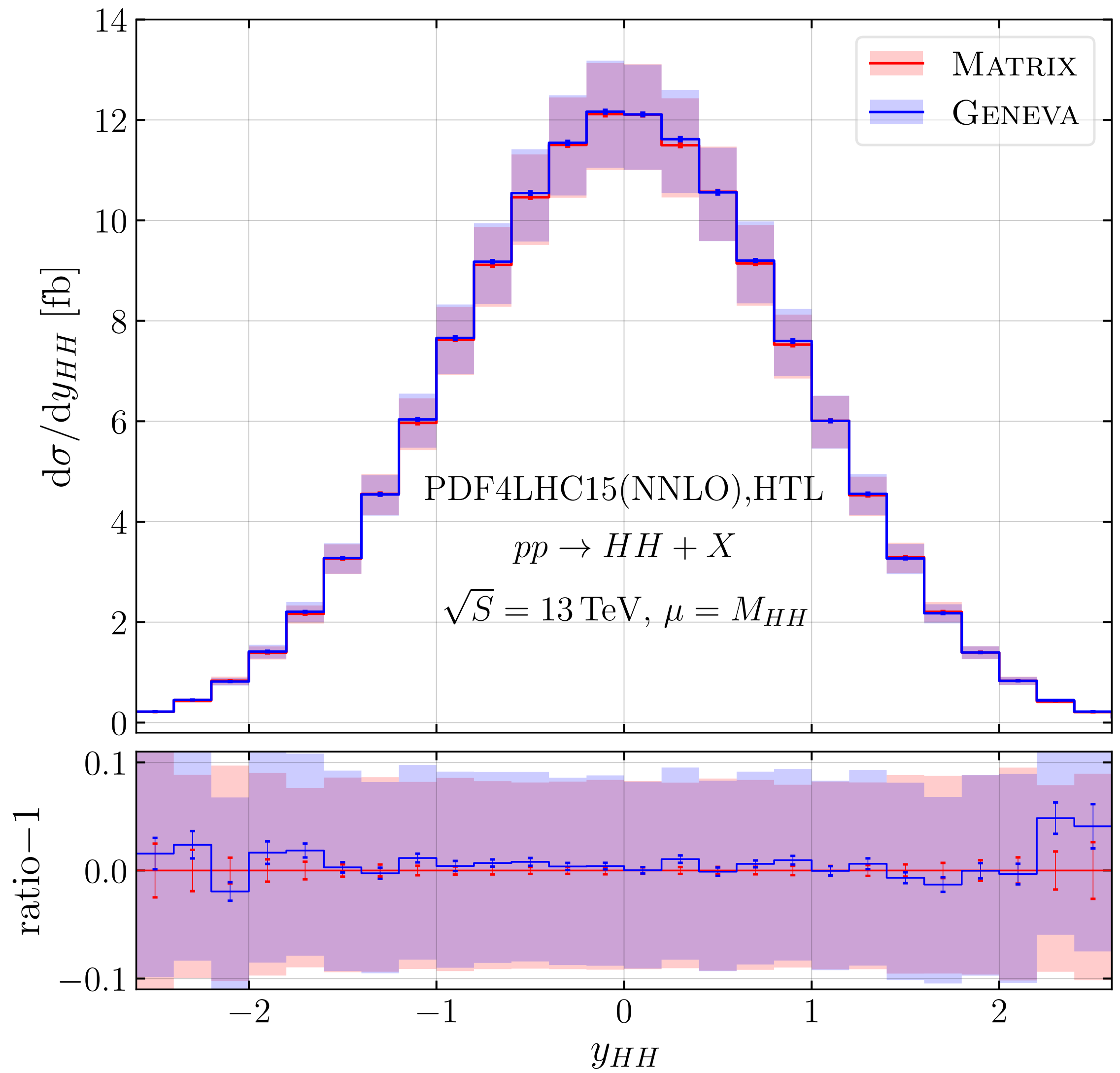
- When we take $\mathcal{T}_N^{\rm cut} \to 0$, large logarithms of $\mathcal{T}_N^{\rm cut}$, $\mathcal{T}_N$ appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum

Higgs Pair production

Based on arXiv:2212.10489, S. Alioli, G. Billis, AB, A. Gavardi, S. Kallweit, M.A. Lim, G. Marinelli, R. Nagar and D. Napoletano

Input parameters: $m_H = 125.09$ GeV, $\mathcal{T}_0^{\rm cut} = 1$ GeV, $\mathcal{T}_1^{\rm cut}$, $\mu_F = \mu_R = M_{HH}$

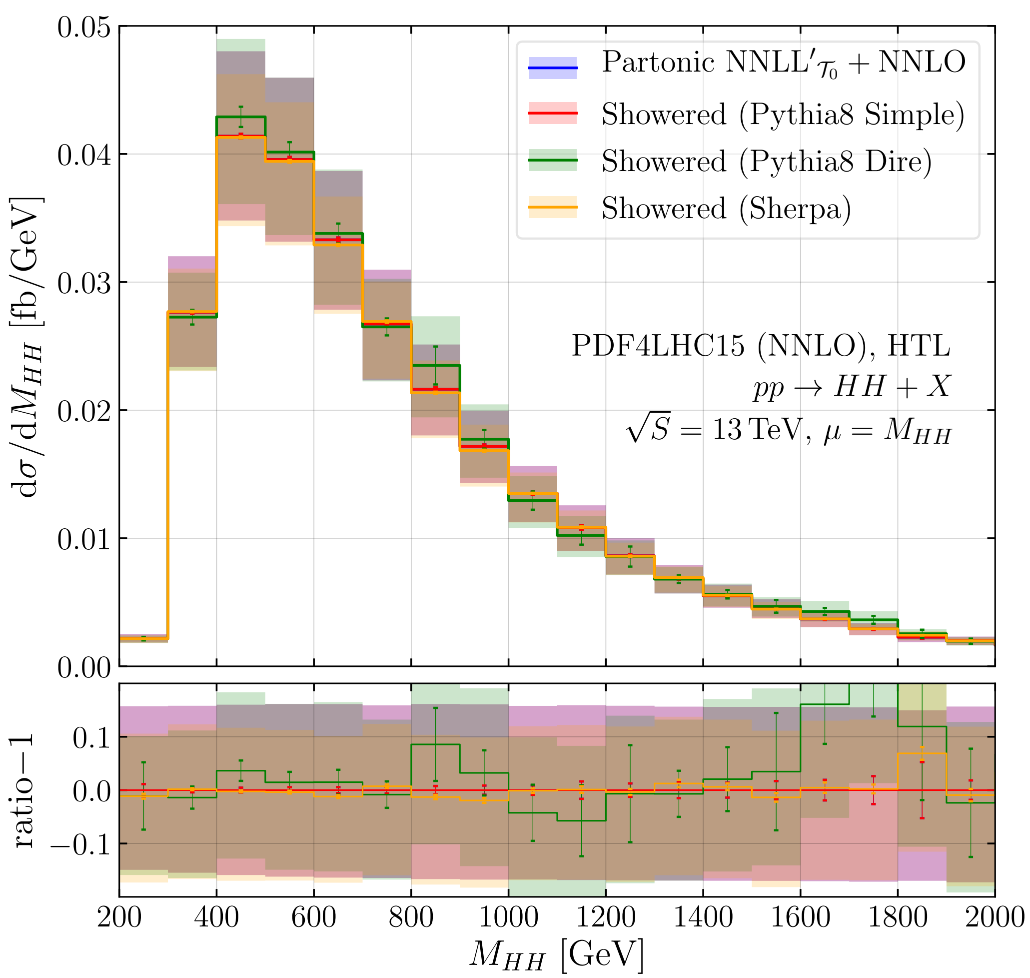




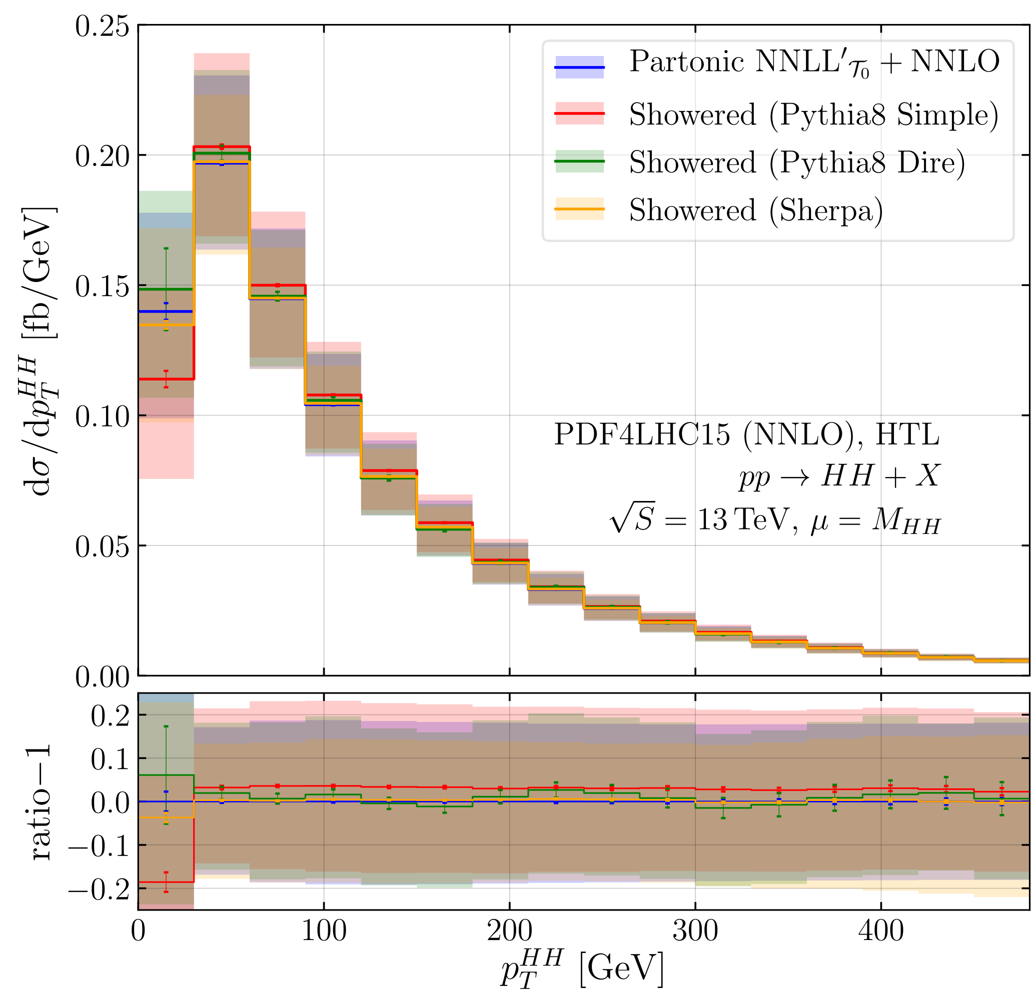
Validation against NNLO from Matrix

Higgs pair production

- Interface to three different showers:
  - Pythia 8
  - Pythia 8 Dire
  - Sherpa
- Need to include top mass corrections for phenomenology



Invariant mass of Higgs pair



Transverse momentum of Higgs pair

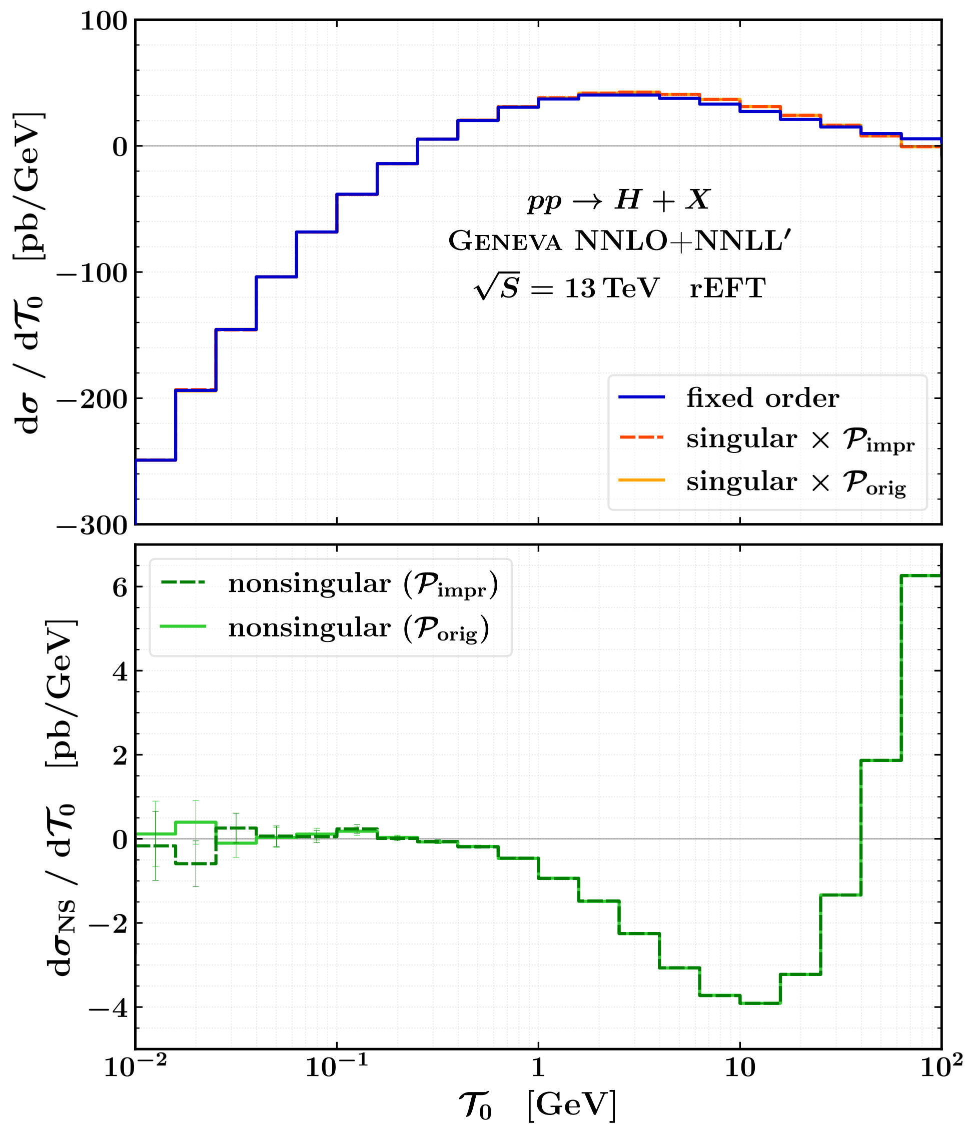
Higgs boson production via gluon fusion

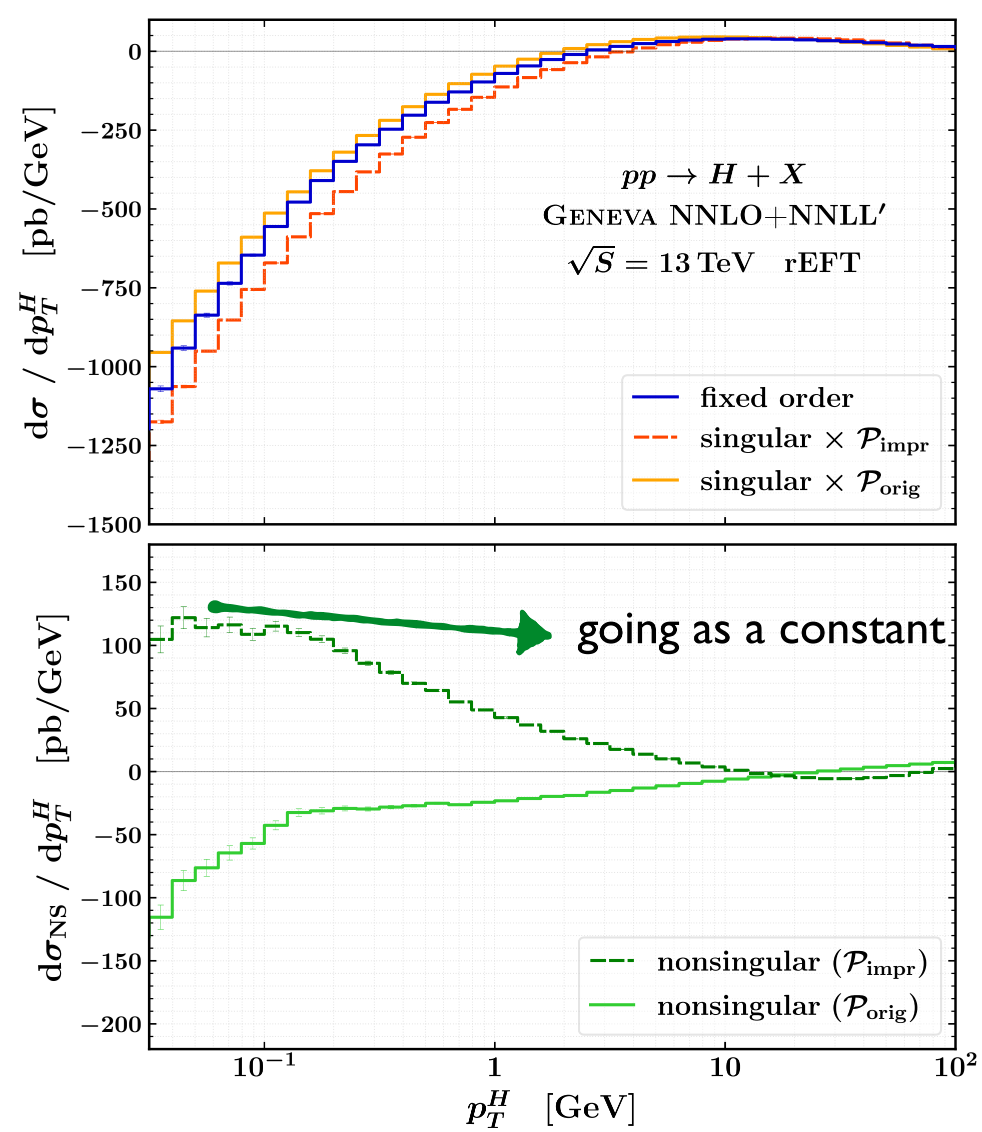
Based on arXiv:2301.11875 S. Alioli, G. Billis, AB, A. Gavardi, S. Kallweit, M.A. Lim, G. Marinelli, R. Nagar and D. Napoletano

- Calculation done in the Heavy Top Limit (HTL). Rescaling of HTL result by a factor equal to the ratio between the LO m_t-exact result and that obtained in pure EFT (rEFT)

- Improved splitting functions $$\frac{d\sigma_1^{\rm MC}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}; \mathcal{T}_1^{\rm cut}) = \Bigg\{ \Bigg[\frac{d\sigma^{\rm NNLL'}}{d\Phi_0 d\mathcal{T}_0} - \frac{d\sigma^{\rm NNLL'}}{d\Phi_0 d\mathcal{T}_0}\Bigg|_{\rm NLO_1} \Bigg] \mathcal{P}(\Phi_1) + \dots$$

$$\int \frac{d\Phi_{N+1}}{d\Phi_N d\mathcal{T}_N} \mathcal{P}(\Phi_{N+1}) = 1$$



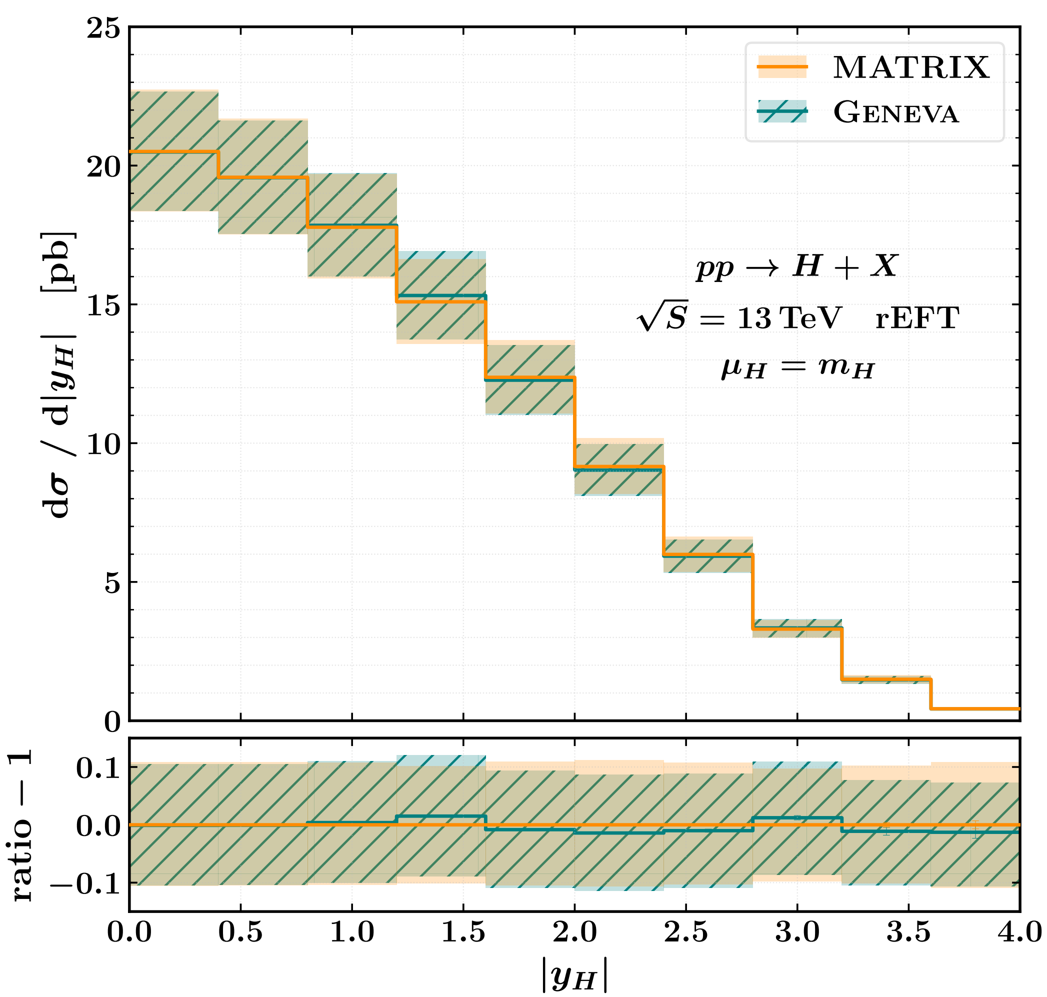


Fixed-order vs resummed expanded

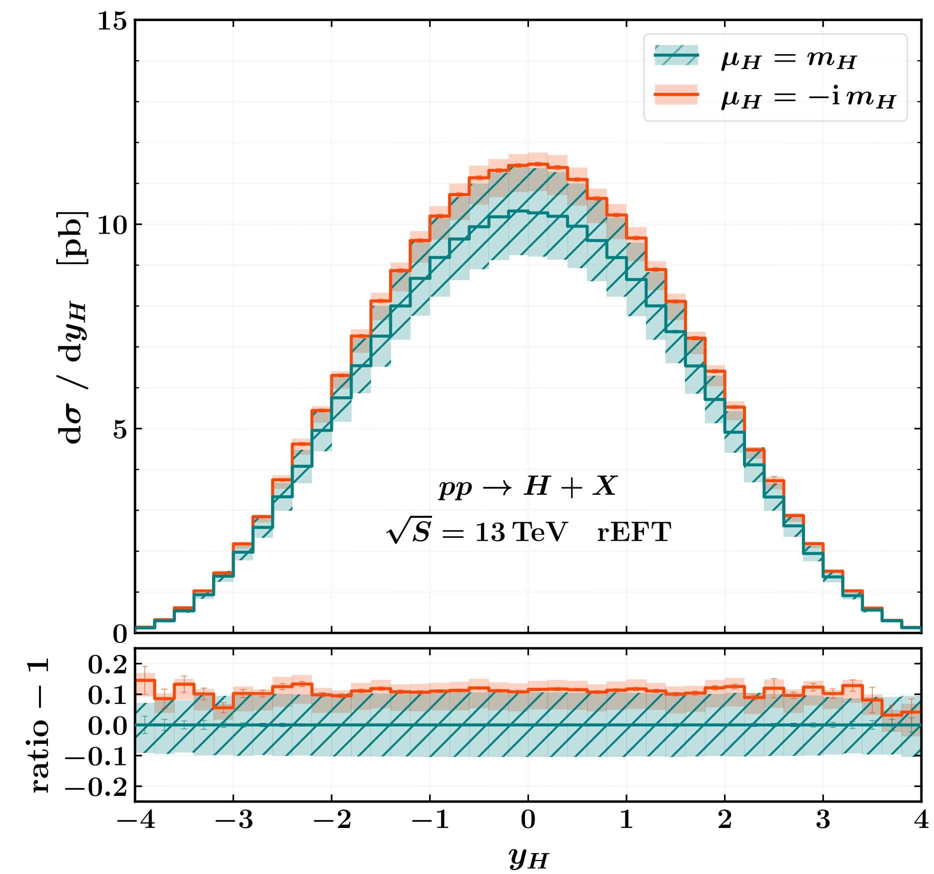
Higgs boson production via gluon fusion

NNLO validation

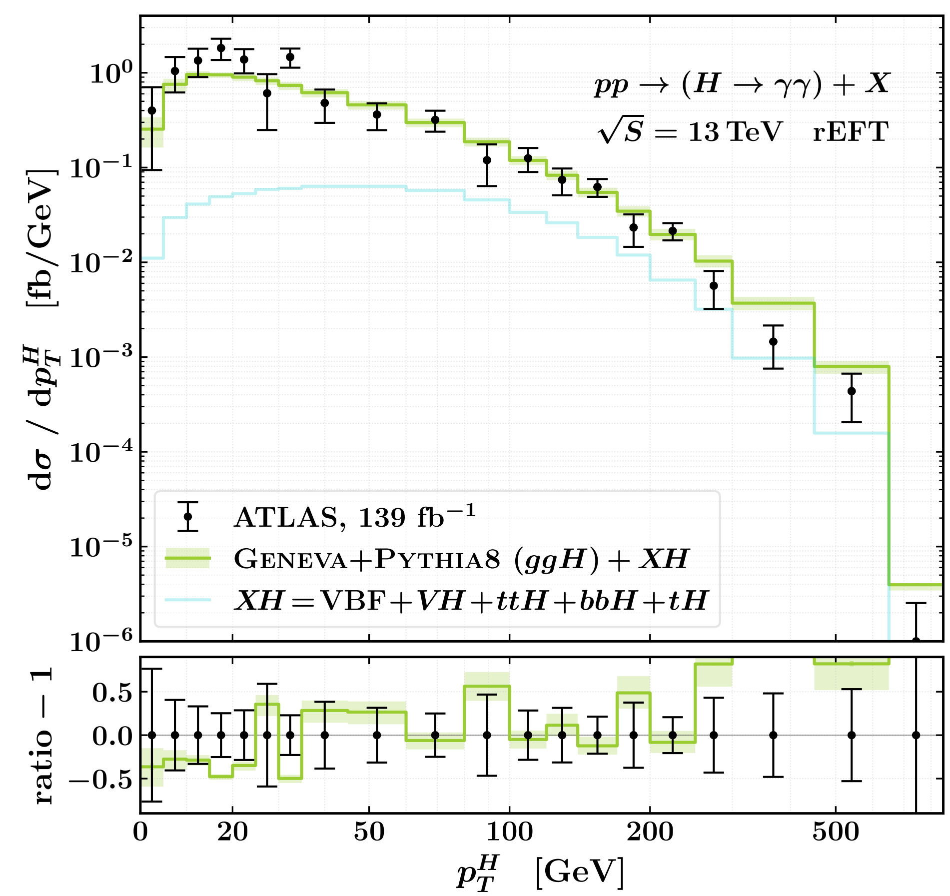
	GENEVA	ggHiggs	MATRIX
$\sigma_{gg\to H}^{\text{NNLO, rEFT}}$ [pb]	$42.33^{+4.39}_{-4.34}$	$42.35^{+4.55}_{-4.41}$	$42.33^{+4.54}_{-4.40}$



Time-like resummation. Different choice of hard scale



Comparison to Data



Zero-jettiness resummation for top-quark pair production at the LHC

Based on arXiv:2111.03632, S. Alioli, AB, M.A. Lim

0-jettiness resummation for $t\bar{t}$ production

- NNLO+PS for $t\bar{t}$ production available in MINNLOPS framework [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi `20, `21]. GENEVA will include higher-order resummation.
- To reach NNLO+PS accuracy in GENEVA: NLO calculations for $t\bar{t}$ and $t\bar{t}$+jet and resummed calculation at NNLL` in $\mathcal{T}_0$
- Definition of 0-jettiness has to be adapted with *top-quarks* in the final state, we choose to *treat them like EW particles* and exclude them from the sum over radiation. First develop resummation framework.

We derived a factorization formula (see 2111.03632 Appendix A) using SCET+HQET in the region $\mathcal{T}_0 \to 0$ when $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales (in case of boosted regime $M_{t\bar{t}} \gg m_t$ situation similar to [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21])

Hard functions (colour matrices) known to NLO [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]

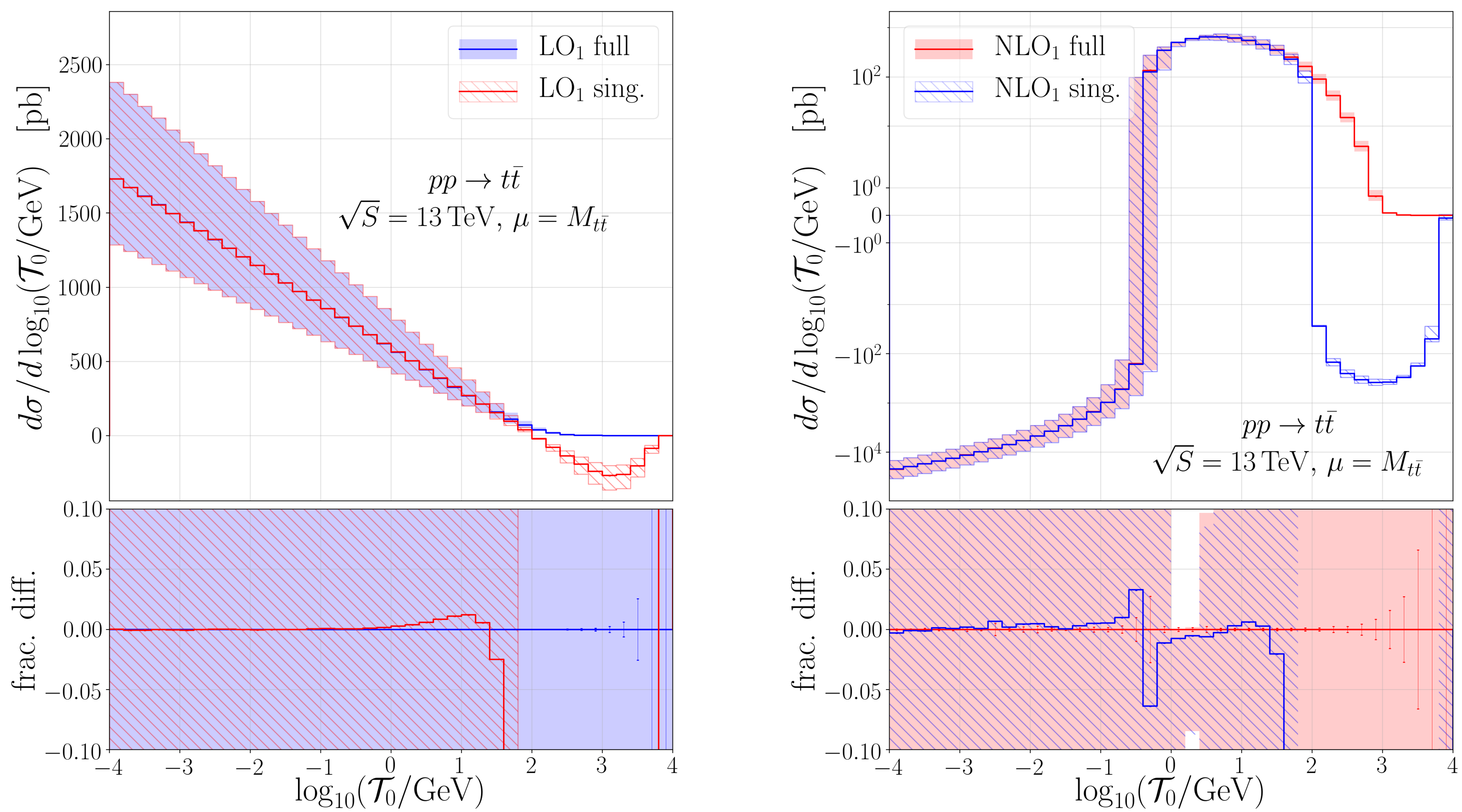
$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^3LO

Soft functions (colour matrices) computed to NLO

Singular vs Fixed order

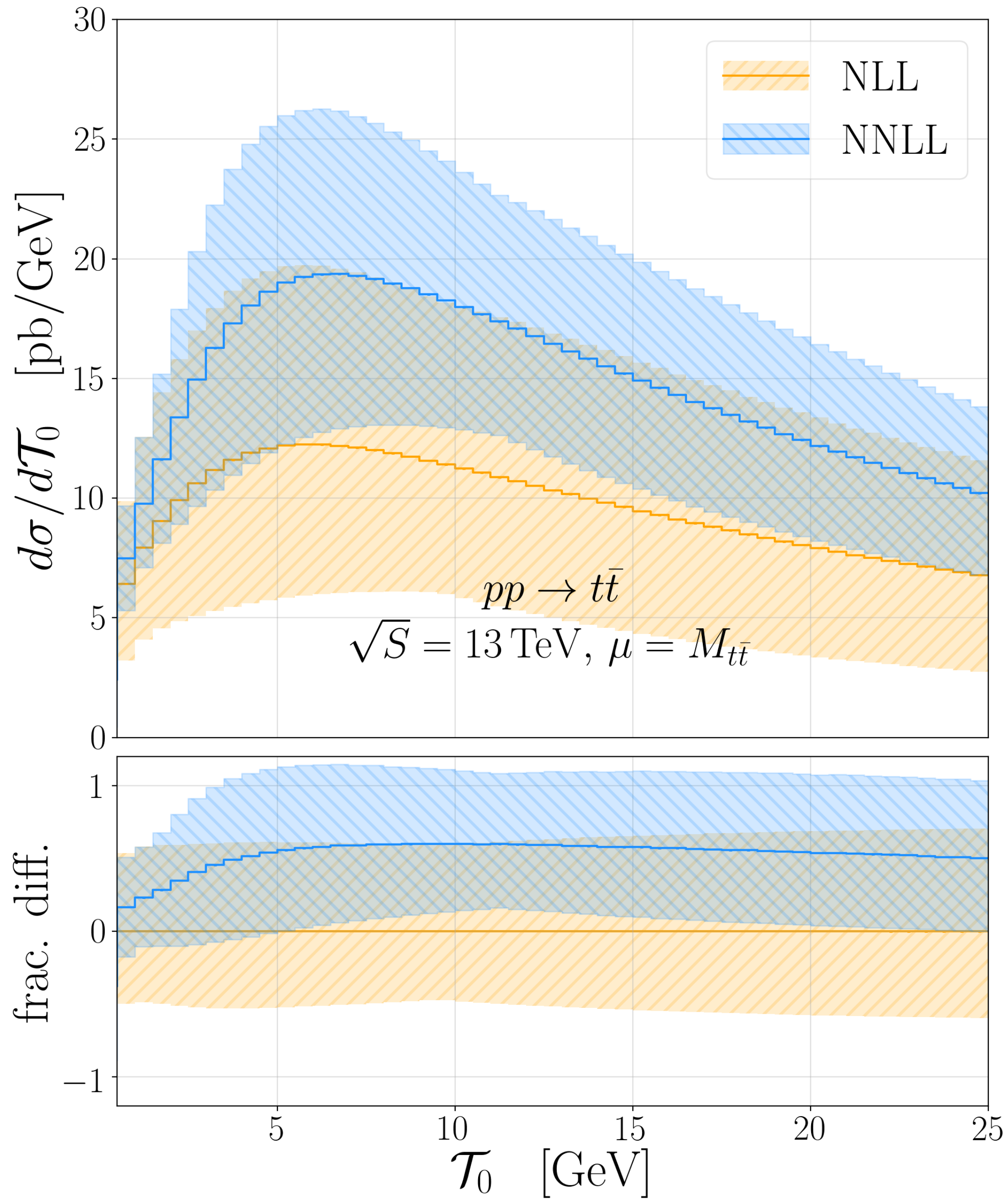
- We have: beam functions at NNLO (both for $q\bar{q}$ and gg channels), hard functions at NLO, soft functions at NLO, by knowing the two-loop soft anomalous dimensions we can solve the RG equations order by order and obtain all the NNLO logarithmic contributions, we miss $\delta(\mathcal{T}_0)$ terms at NNLO
- We can resum to NNLL. We are missing $\delta(\mathcal{T}_0)$ terms (NNLO hard functions and NNLO soft). If we include everything we know we obtain a NNLL'_a result
- We construct an approximate (N)NLO formula which reproduces the fixed-order behaviour of the spectrum (for $\mathcal{T}_0 > 0$)

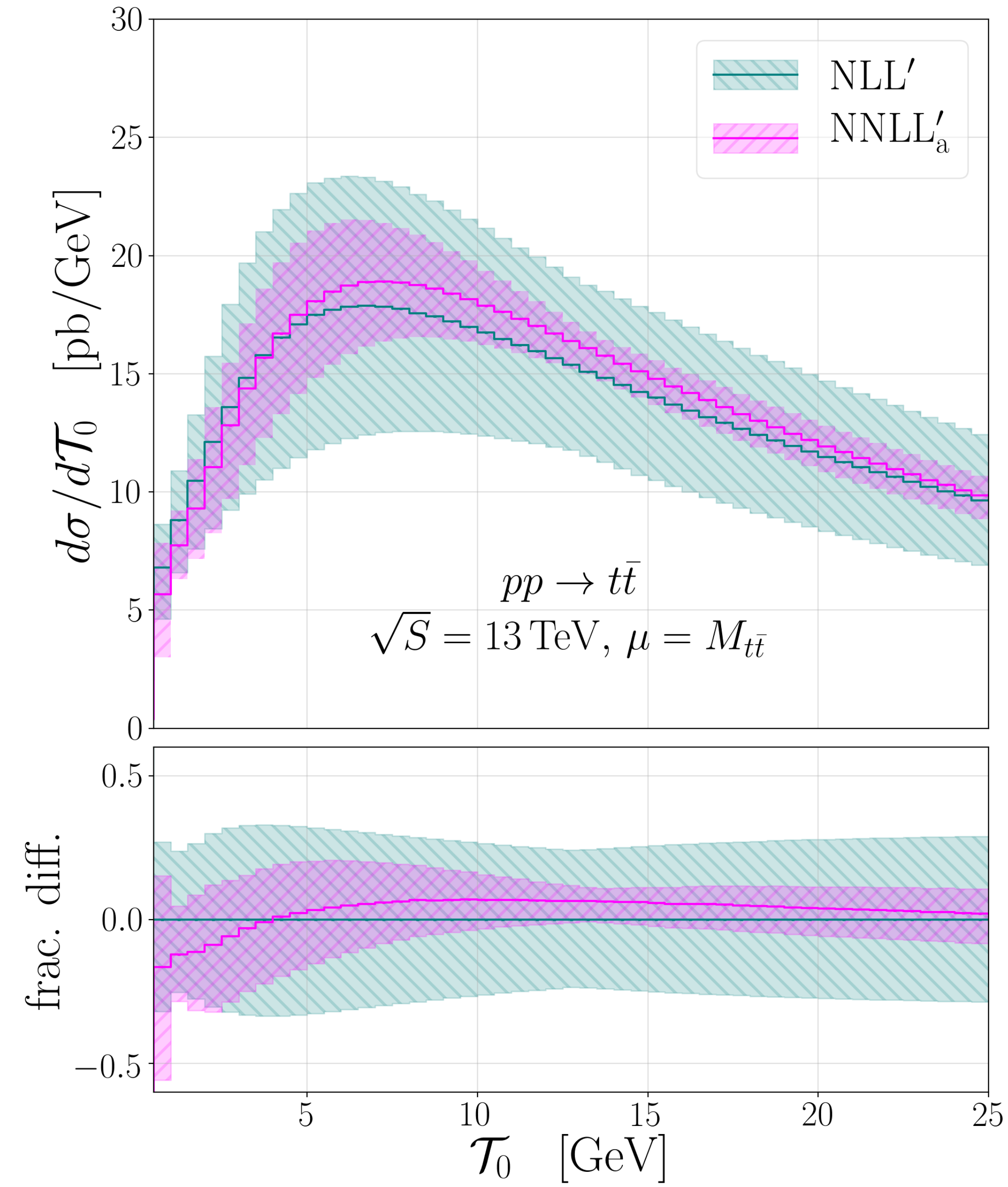


Fixed-order comparisons, approximate NLO and approximate NNLO vs LO_1 and NLO_1

Resummed results

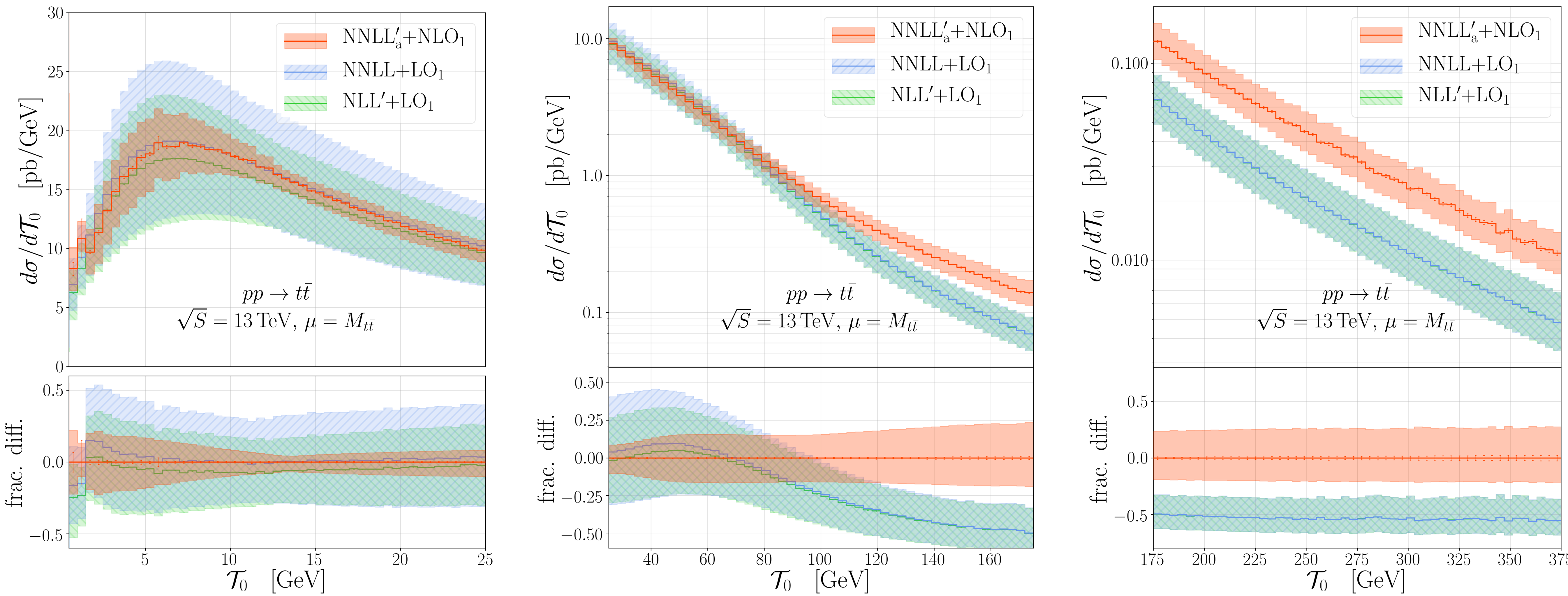
$NNLL'_a$ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales





Matched results to fixed-order

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$



One-jettiness resummation for Z+jet production at the LHC

work in progress…

1-jettiness

- Start from expression for 1-jettiness in the Born frame, where $\rho_i = 1$

$$\hat{\mathcal{T}}_1 = \sum_k \min\{\frac{\hat{q}_a \cdot \hat{p}_k}{\rho_a}, \frac{\hat{q}_b \cdot \hat{p}_k}{\rho_b}, \frac{\hat{q}_J \cdot \hat{p}_J}{\rho_J}\}$$

- 1-jettiness in the color singlet frame by making a different choice of the ρ_i's (similar way to go to the laboratory frame)

$$\rho_a = e^{\hat{Y}_V}\,,$$
$$\rho_b = e^{-\hat{Y}_V}\,,$$
$$\rho_J = \frac{e^{-\hat{Y}_V}(\hat{p}_J)_+ + e^{\hat{Y}_V}(\hat{p}_J)_-}{2\hat{E}_J}$$

- We also employ a Fully-Recursive (FR) version of one-jettiness which is used in the fixed order calculations. Closest particles in the one-jettiness metric are merged together.

- Factorization formula in the region $\mathcal{T}_1 \ll M_{ll} \sim \sqrt{s} \sim M_{T,ll}$ [Stewart, Tackmann,Waalewijn \`09,\`10]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_{\kappa=\{q\bar{q}g,qgq,ggg\}} H_\kappa(\Phi_1) \int \mathrm{d}t_a \mathrm{d}t_b \mathrm{d}s_J\, B_{\kappa_a}(t_a) B_{\kappa_b}(t_b) J_{\kappa_J}(s_J) \times S_\kappa\left(n_{a,b}\cdot n_J, \mathcal{T}_1 - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \frac{s_J}{Q_J}\right)$$

Dependence on the frame

Hard, Soft, Beam and Jet functions

- Hard functions: two-loop amplitudes for $q\bar{q} \to Zg$ known from [T. Gehrmann and L. Tancredi 1112.1531]. Recently available also the axial vector couplings [T. Gehrmann,T. Peraro,L. Tancredi 2211.13596] *but not-included yet*. IR-finite functions taken from [T. Becher, G. Bell, C. Lorentzen, S. Marti 1309.3245]. $\gamma^*/Z^* \to l^+l^-$ added, squared amplitude complete analytic result. At NNLL` accuracy included the 1loop squared $gg \to Zg$.

- Beam and quark Jet functions known up to N^{3}LO [M. Ebert, B. Mistlberger, G. Vita 2006.03056] and [R. Bruser, Z.L. Liu, M. Stahlhofen 1804.09722], only needed up to NNLO here Beams [J.R. Gaunt, M. Stahlhofen, F. Tackmann 1401.5478, 1405.1044] and Jets [T. Becher and M. Neubert 0603140], [T. Becher and G. Bell 1104.4108].
- Soft function boundary terms at NLO implemented as on-the-fly integrals using results in [T.T. Jouttenus, I.W. Stewart, F. Tackmann, W. Waalewijn 1302.0846], kept full dependence on $\mathcal{T}_1$ frame dependence.
- Frame dependent NNLO soft function boundary contribution is provided by using the SoftSERVE [G. Bell, R. Rahn, J. Talbert 1812.08690, 2004.08396] method (thanks to Bahman Dehnadi, Guido Bell, Rudi Rahn) in the form of an interpolation grid over the parameters $\{\cos\theta_J, 1/\rho_a, 1/\rho_J\}$
- Validation against NLO result in different frames, at NNLO validated in UB frame against the interpolation in MCFM [J. Campbell, K. Ellis, R. Mondini, C. Williams, 1711.09984]. In CS and Lab frames new results.

Resummation formula to NNLL$'$

Combine the solutions to the RG equations for the hard, soft, beam and jet functions to obtain

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_\kappa \exp\Big\{ & 4(C_{\kappa_a}+C_{\kappa_b})K_{\Gamma_{\rm cusp}}(\mu_B,\mu_H) + 4C_{\kappa_J}K_{\Gamma_{\rm cusp}}(\mu_J,\mu_H) \\
& - 2(C_{\kappa_a}+C_{\kappa_b}+C_{\kappa_J})K_{\Gamma_{\rm cusp}}(\mu_S,\mu_H) - 2C_{\kappa_J}L_J\, \eta_{\Gamma_{\rm cusp}}(\mu_J,\mu_H) \\
& - 2(C_{\kappa_a}L_B + C_{\kappa_b}L_B')\eta_{\Gamma_{\rm cusp}}(\mu_B,\mu_H) + \bigg[C_{\kappa_a}\ln\left(\frac{Q_a^2 u}{st}\right) + C_{\kappa_b}\ln\left(\frac{Q_b^2 t}{su}\right) \\
& \quad + C_{\kappa_j}\ln\left(\frac{Q_J^2 s}{tu}\right) + (C_{\kappa_a}+C_{\kappa_b}+C_{\kappa_J})L_S\bigg]\eta_{\Gamma_{\rm cusp}}(\mu_S,\mu_H) + K_{\gamma_{\rm tot}}\Big\} \\
& \times \tilde{B}_{\kappa_a}(\partial_{\eta_B}+L_B, x_a,\mu_B)\tilde{B}_{\kappa_b}(\partial_{\eta_B'}+L_B', x_b,\mu_B)\, \tilde{J}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \\
& \times H_\kappa(\Phi_1,\mu_H)\tilde{S}^\kappa_{\mathcal{T}_1}\left(\partial_{\eta_S}+L_S,\mu_S\right)\frac{Q^{-\eta_{\rm tot}}}{\mathcal{T}_1^{1-\eta_{\rm tot}}}\frac{\eta_{\rm tot}\, e^{-\gamma_E \eta_{\rm tot}}}{\Gamma(1+\eta_{\rm tot})} + \mathcal{O}\left(\frac{\mathcal{T}_1}{Q}\right)
\end{aligned}
$$

where we defined

$$
L_H = \ln\left(\frac{Q^2}{\mu_H^2}\right) \qquad L_B = \ln\left(\frac{Q_a Q}{\mu_B^2}\right), \qquad L_B' = \ln\left(\frac{Q_b Q}{\mu_B^2}\right)
$$

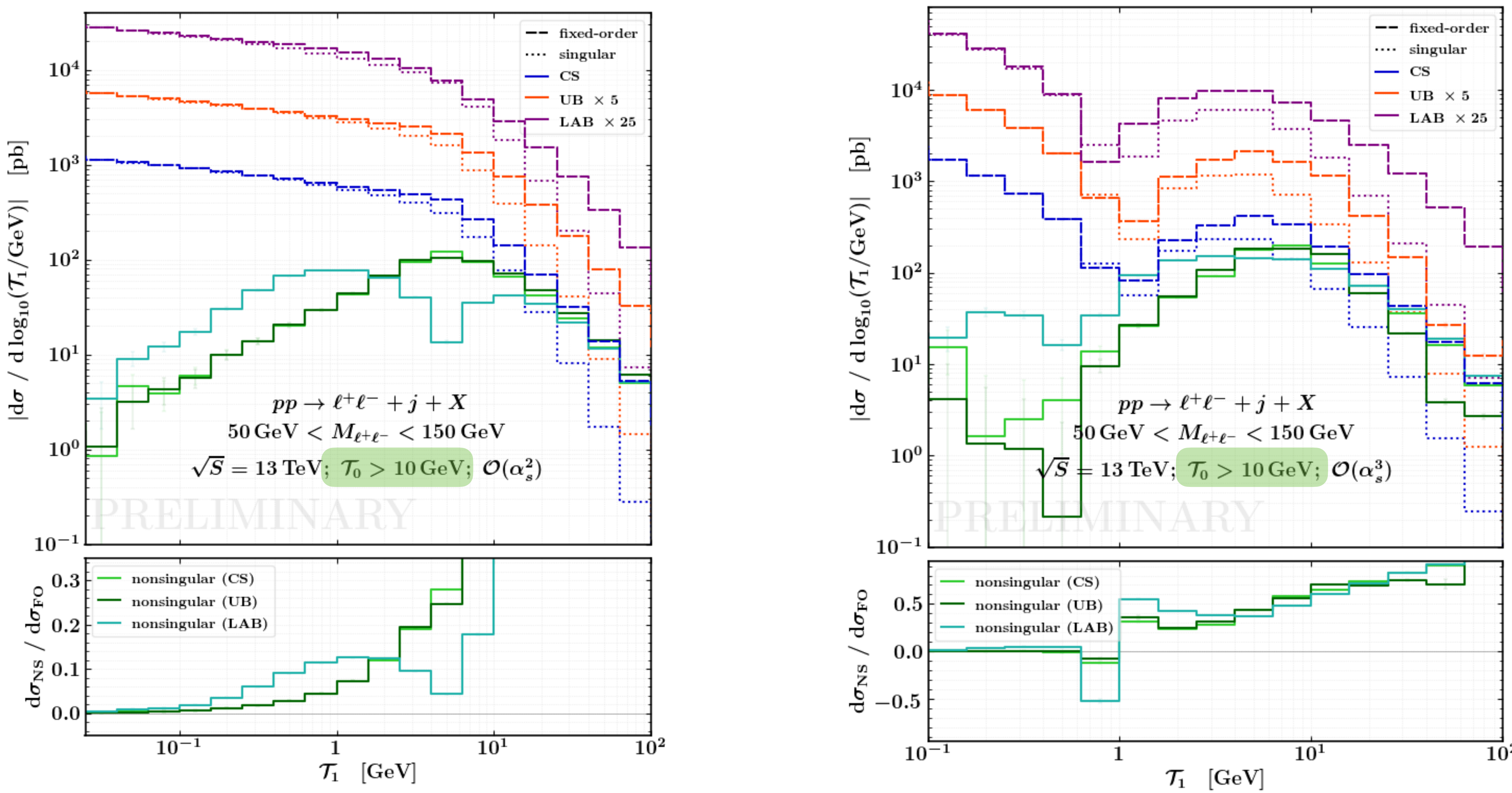
$$
L_J = \ln\left(\frac{Q_J Q}{\mu_J^2}\right) \qquad L_S = \ln\left(\frac{Q^2}{\mu_S^2}\right)
$$

$$
\begin{aligned}
K_{\gamma_{\rm tot}} = & -2n_g K_{\gamma_C^g}(\mu_S,\mu_H) + 2(n_g-3)K_{\gamma_C^q}(\mu_S,\mu_H) \\
& - (n_g - n_g^{\kappa_J})K_{\gamma_J^g}(\mu_J,\mu_B) - n_g K_{\gamma_J^g}(\mu_S,\mu_J) \\
& + (n_g - 2 - n_g^{\kappa_J})K_{\gamma_J^q}(\mu_J,\mu_B) + (n_g-3)K_{\gamma_J^q}(\mu_S,\mu_J)
\end{aligned}
$$

$$
\eta_{\rm tot} = -2(C_{\kappa_a}+C_{\kappa_b})\eta_{\Gamma_{\rm cusp}}(\mu_B,\mu_J) + 2(C_{\kappa_a}+C_{\kappa_b}+C_{\kappa_J})\eta_{\Gamma_{\rm cusp}}(\mu_S,\mu_J)
$$

Singular vs Nonsingular

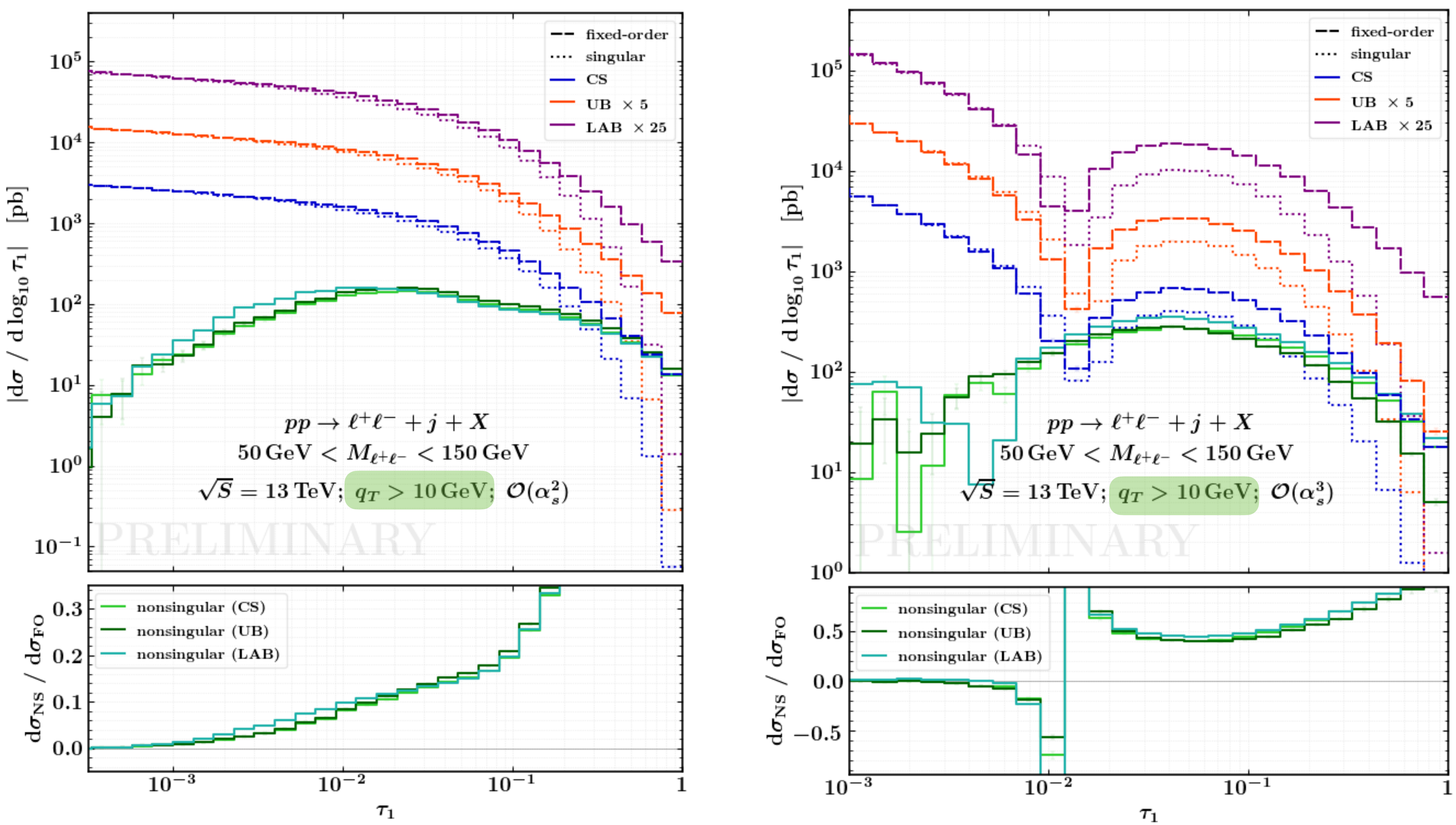
- Different frame choices for one-jettiness definition have different sizes of power corrections (fully-recursive results below, only fixed-order is different for $\mathscr{T}_1 > 0$)
- CS frame as good as UB frame for different cuts, Lab. frame is worse



Singular vs Nonsingular

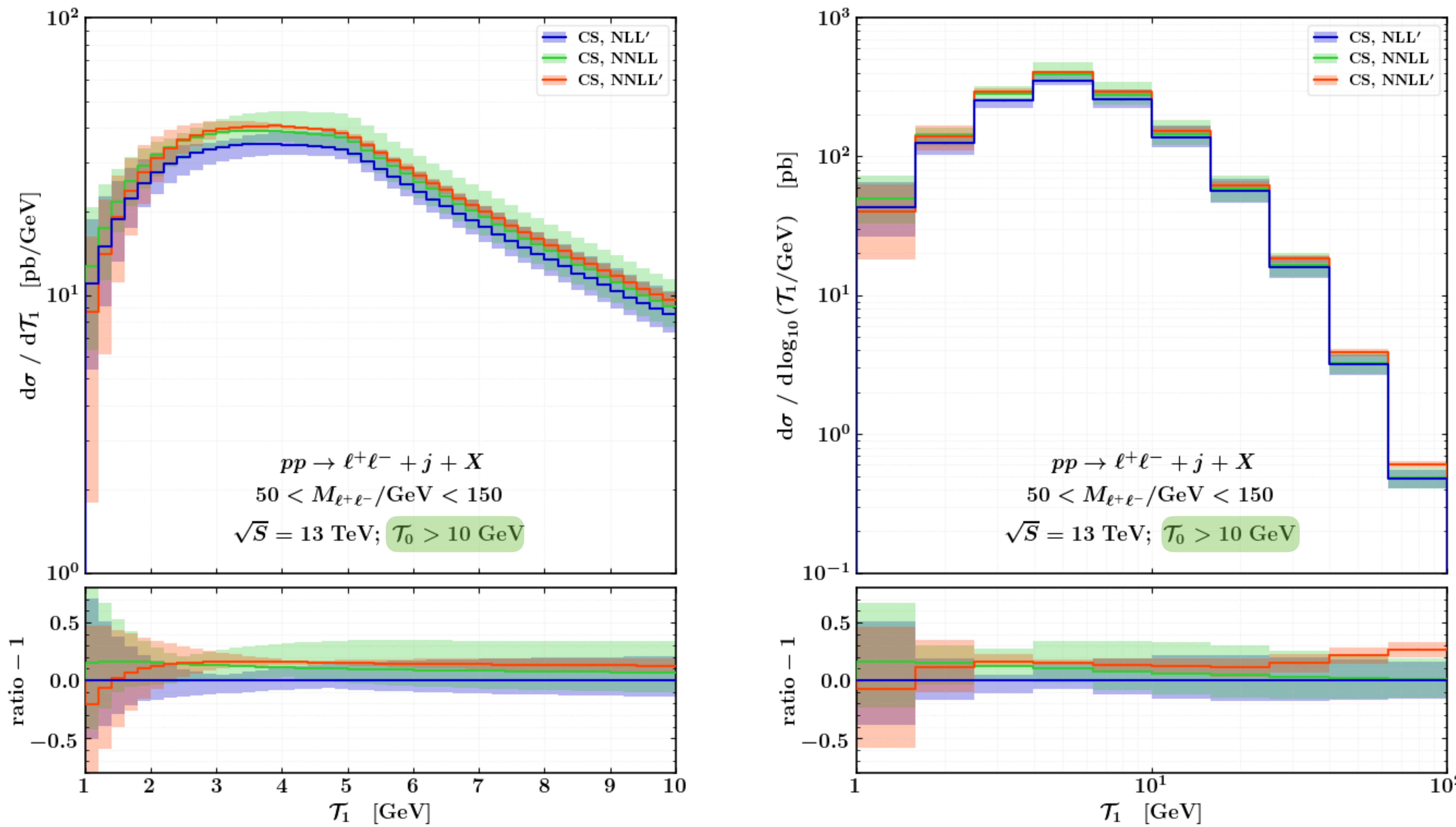
- Reduced definition $\tau_1 = 2\,\mathcal{T}_1/\sqrt{M^2_{l^+l^-} + q_T^2}$

- When we use as born defining cut the Z boson transverse momentum q_T, differences in power corrections among the different definitions are reduced



Resummed results up to NNLL'

- We use profile scales to switch off resummation at $\mu_H = \sqrt{M_{l^+l^-}^2 + q_T^2}$



N^{3}LL Resummation

For every channel ($q\bar{q}g$, qgq,ggg,…), hard anomalous dimension has the form [T. Becher and M. Neubert 1908.11379]

$$\Gamma(\{\underline{s}\},\mu) = \frac{\gamma_{\text{cusp}}(\alpha_s)}{2}\left[(C_{R_3} - C_{R_1} - C_{R_2})\ln\frac{\mu^2}{(-s_{12})} + \text{cyclic permutations}\right]$$ 4-loop

$$+\gamma^1(\alpha_s) + \gamma^2(\alpha_s) + \gamma^3(\alpha_s) + \frac{C_A^2}{8} f(\alpha_s)\,(C_{R_1} + C_{R_2} + C_{R_3})$$ 3-loop

$$+\sum_{(i,j)}\left[-f(\alpha_s)\,\mathcal{T}_{iijj} + \sum_R g^R(\alpha_s)\left(3\mathcal{D}^R_{iijj} + 4\mathcal{D}^R_{iiij}\right)\ln\frac{\mu^2}{-s_{ij}}\right]$$

we explicitly evaluated these contributions as functions of N_c using colour space formalism

$$\mathcal{D}^R_{ijkl} = d_R^{abcd}\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c\mathbf{T}_l^d \qquad \mathcal{T}_{ijkl} = f^{ade}f^{bce}(\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c\mathbf{T}_l^d)_+$$

$$d_R^{a_1\dots a_n} = \text{Tr}_R\big(\boldsymbol{T}^{a_1}\dots\boldsymbol{T}^{a_n}\big)_+ \equiv \frac{1}{n!}\sum_\pi \text{Tr}\big(\boldsymbol{T}_R^{a_{\pi(1)}}\dots\boldsymbol{T}_R^{a_{\pi(n)}}\big)$$

We found the following relations

$$\Gamma^{ij}_{\mathcal{G}} = -\sum_{R=F,A} g^R(\alpha_s)\frac{3\langle\mathcal{D}^R_{iijj}\rangle + 4\langle\mathcal{D}^R_{iiij}\rangle}{\langle\mathcal{M}|\mathcal{M}\rangle}$$

symmetrized in a, b →

$$\Gamma^{\{ab\}}_{\mathcal{G}} = \sum_{R=F,A} g^R(\alpha_s)\left[C_4(R_a,R) + C_4(R_b,R) - C_4(R_c,R)\right]$$

$$\Gamma^{\{ac\}}_{\mathcal{G}} = \sum_{R=F,A} g^R(\alpha_s)\left[C_4(R_a,R) + C_4(R_c,R) - C_4(R_b,R)\right]$$

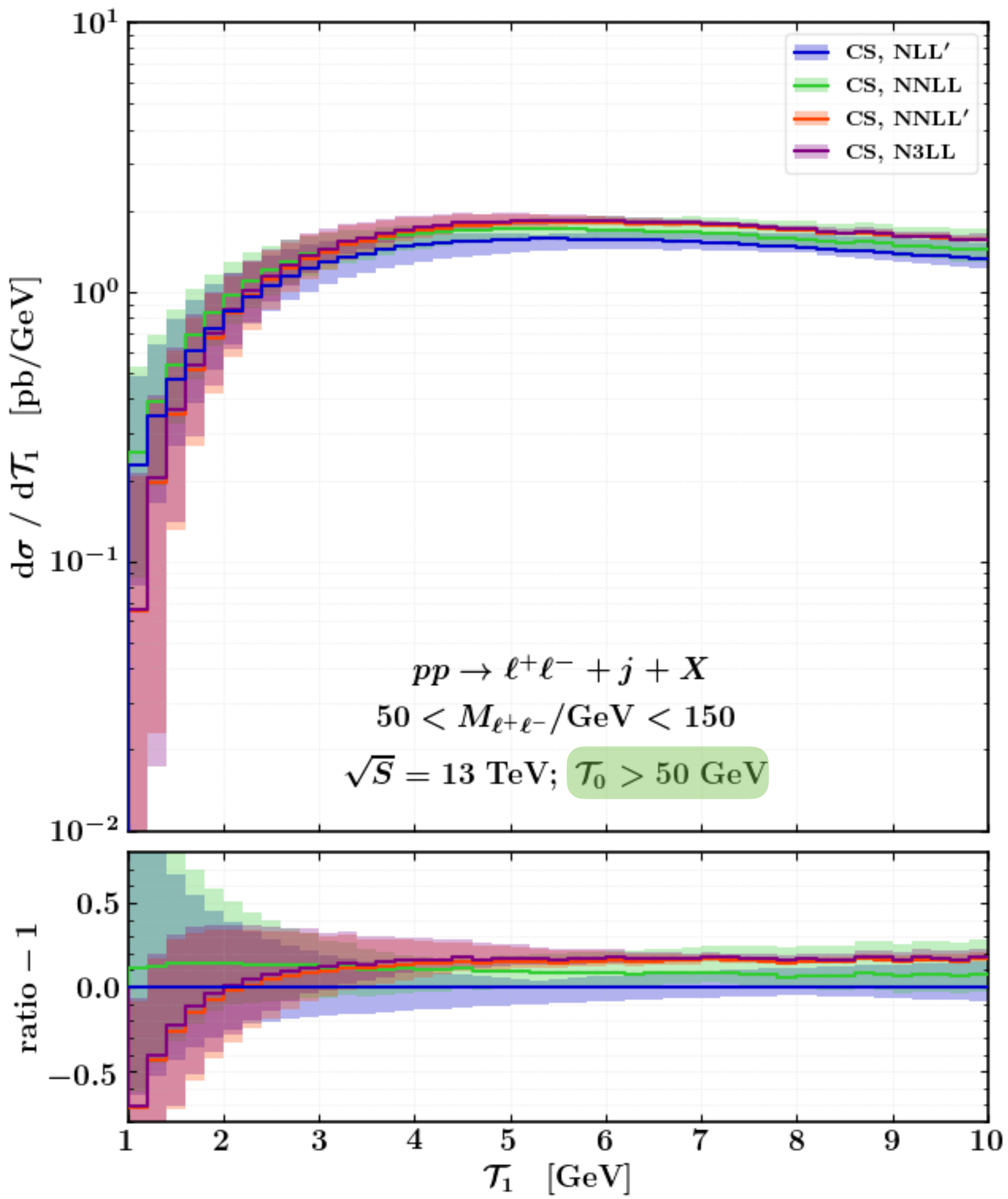
$$\Gamma^{\{bc\}}_{\mathcal{G}} = \sum_{R=F,A} g^R(\alpha_s)\left[C_4(R_b,R) + C_4(R_c,R) - C_4(R_a,R)\right]$$

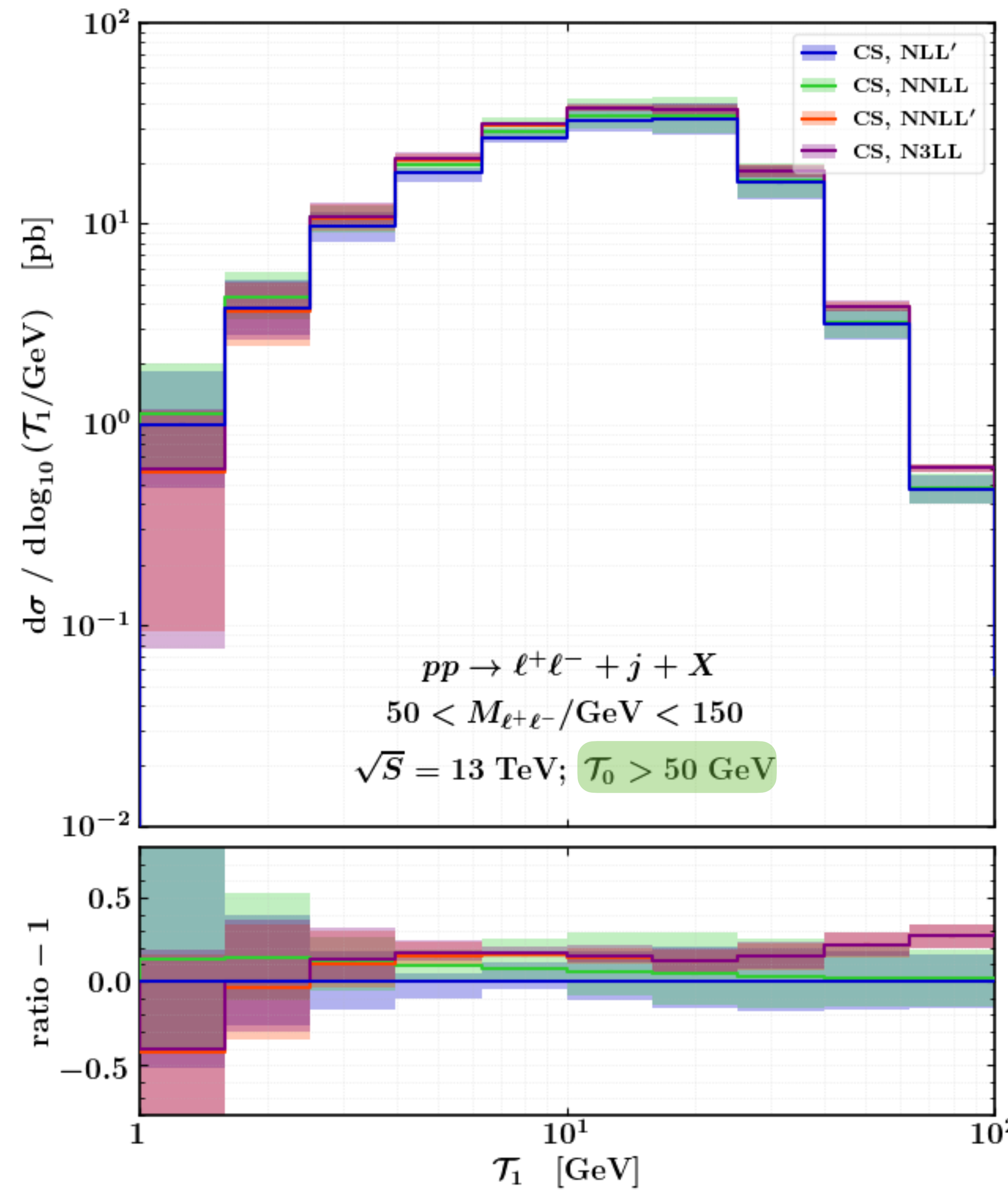
Quartic Casimirs

$$C_4(R_i,R) = \frac{d_{R_i}^{abcd}d_R^{abcd}}{N_{R_i}}$$

Resummed results up to N^{3}LL

- We use profile scales to switch off resummation at $\mu_H = \sqrt{M^2_{l^+l^-} + q_T^2}$

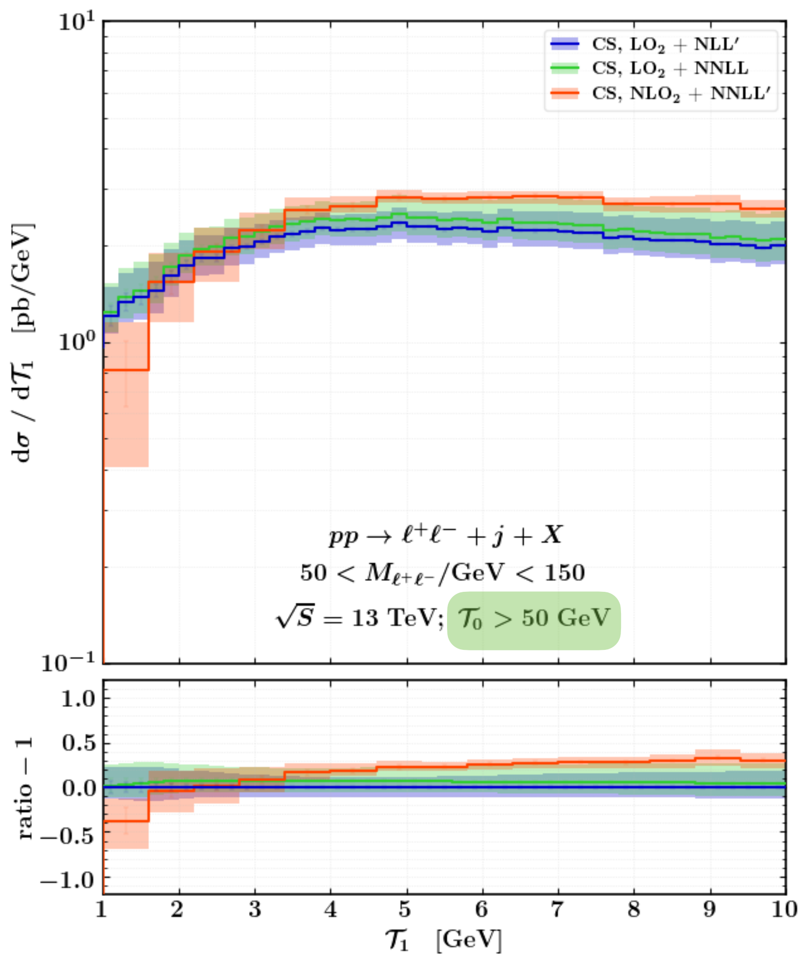


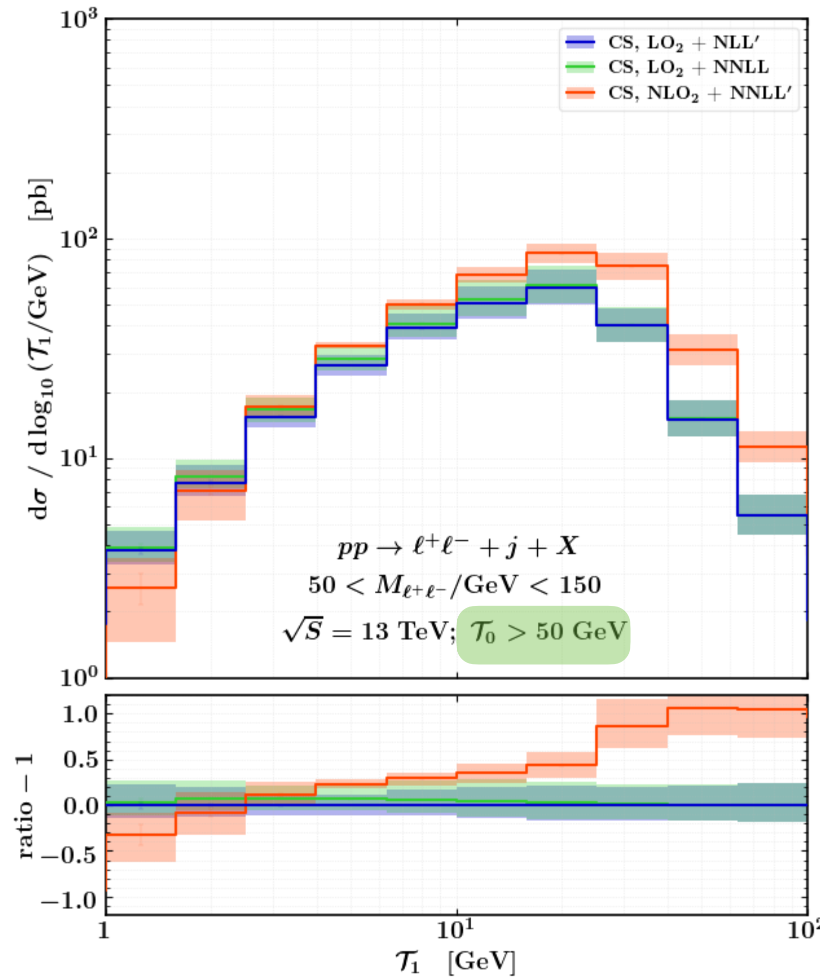


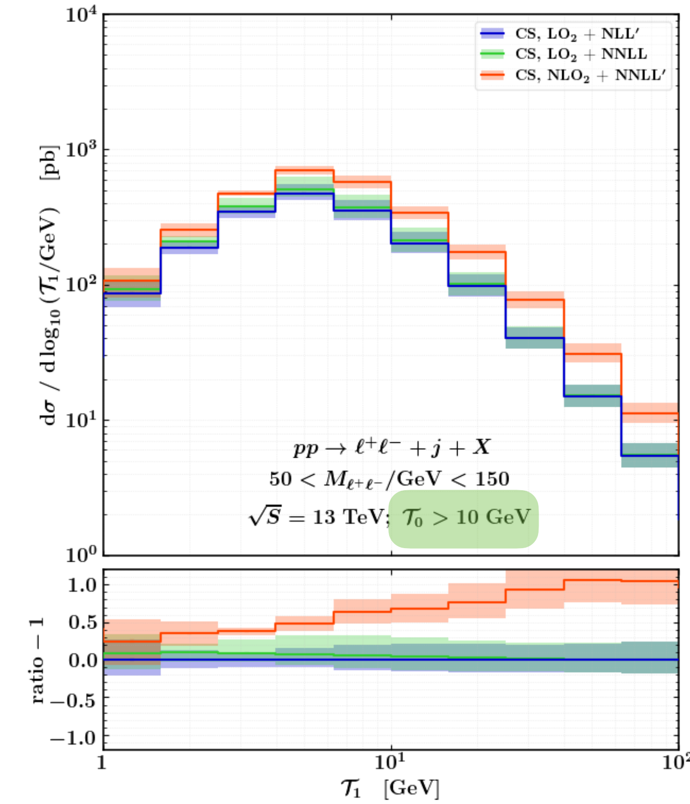
Very Preliminary!

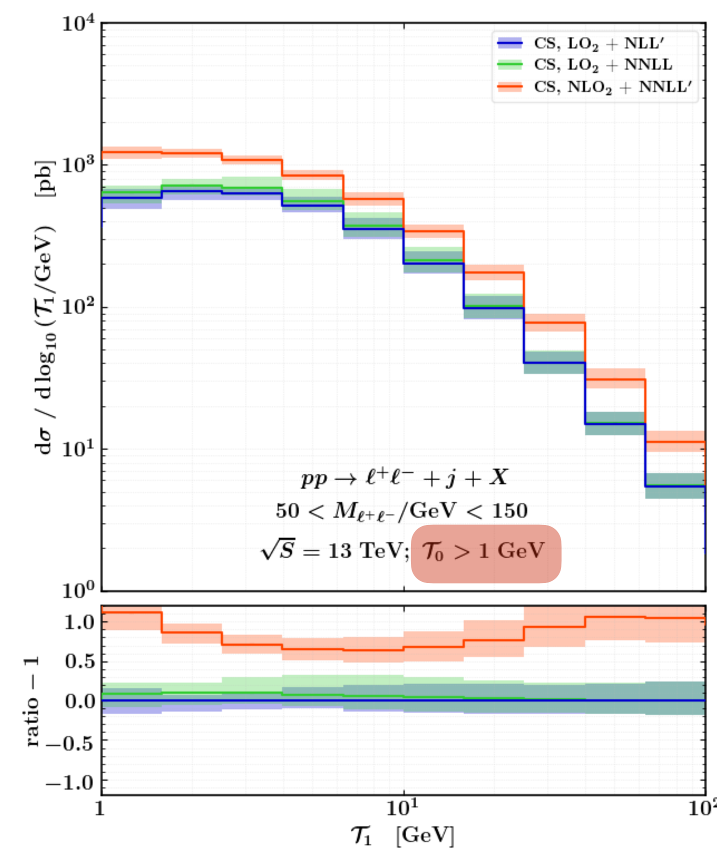
Matched results

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_1} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_1} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_1} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_1}\right]_{\mathrm{FO}}$$









- $\mathcal{O}(\alpha_s^3)$ large corrections especially for small values of $\mathcal{T}_0^{\mathrm{cut}}$
- We know that nonsingular in $\mathcal{T}_1$ is divergent for $\mathcal{T}_0 \to 0$
- We sum in quadrature profile scales variations and fixed-order scale variations

Outlook

- Calculate and extract all the missing ingredients to reach NNLL$'$ accuracy for the top-quark pair production process (hard and soft functions). Implement in GENEVA event generator

- Extend top-quark pair to study associated production of a top-pair and a heavy boson $t\bar{t}V$ $(V = H, W^{\pm}, Z)$ [AB,Ferroglia,Pecjak,Signer, Yang `15], [AB,Ferroglia,Pecjak,Ossola `16], [AB,Ferroglia,Pecjak,Yang `16],[AB,Ferroglia,Pecjak,Ossola,Sameshima `17],[AB,Ferroglia,Frederix, Pagani,Pecjak,Tsinikos `19]

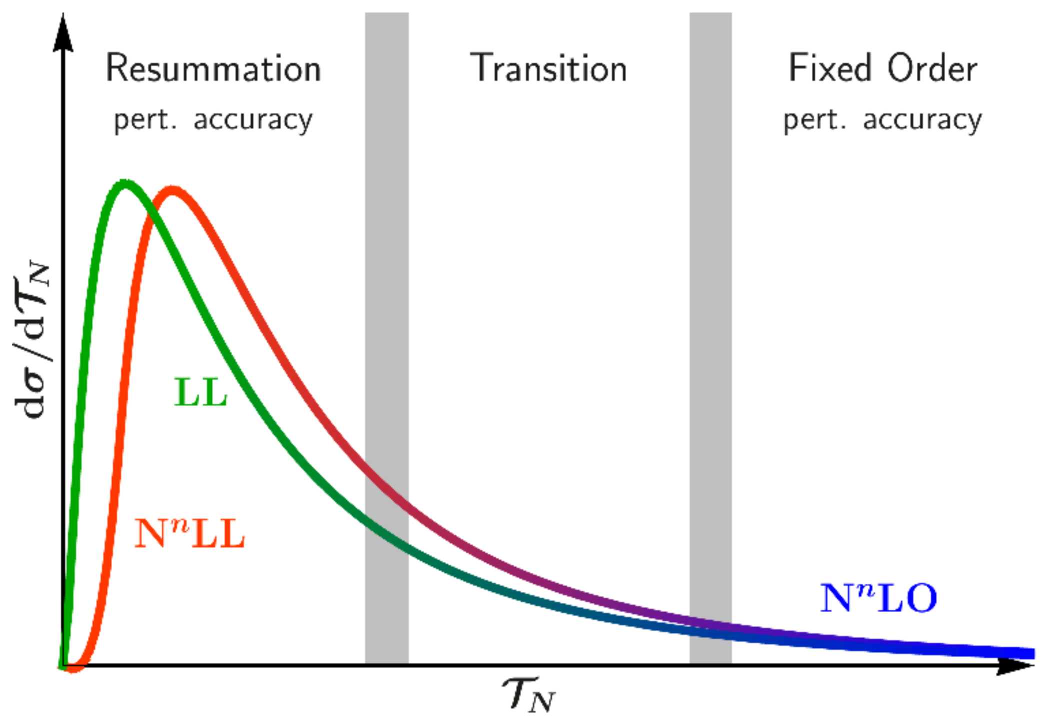
- Implementation of Monte Carlo event generator for Z+jet production.

Thank you!

Backup slides

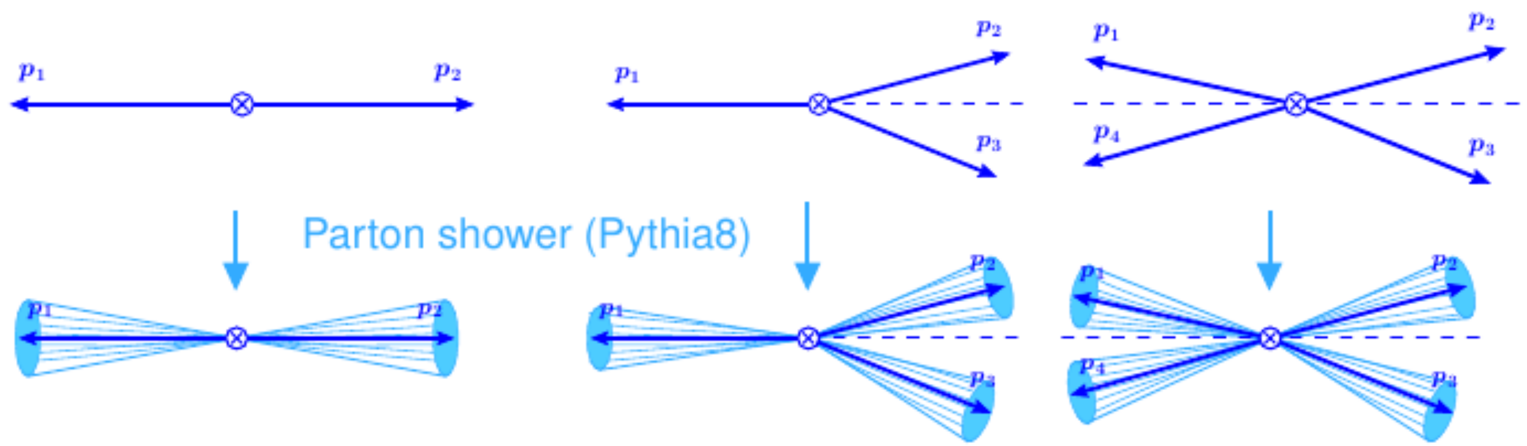
N-Jettiness and Resummation

- At NNLO one needs a 0-jet and a 1-jet (for Z+j also 2-jet) resolution parameters
- Emissions below $\mathcal{T}_N^{\rm cut}$ are unresolved (integrated over) and the kinematic considered is the one of the event before extra emissions
- Emissions above $\mathcal{T}_N^{\rm cut}$ are kept and the full kinematics is considered
- When we take $\mathcal{T}_N^{\rm cut} \to 0$, large logarithms of $\mathcal{T}_N^{\rm cut}$, $\mathcal{T}_N$ appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum



Matching to a parton shower

- Parton shower makes the calculation differential in higher multiplicities by filling the 0- and 1-jet exclusive bins with radiation and by adding more emissions to the inclusive 2-jet bin



- Not allowed to affect the accuracy of the cross sections reached at partonic level

- $\mathcal{T}_i^{\rm cut}$ constraints must be respected by the shower

- Φ_0 events have $\mathcal{T}_0 = 0$. The shower should restore the emissions which were integrated, but should respect the constraint $\mathcal{T}_0(\Phi_N) < \mathcal{T}_0^{\rm cut}$. The shape is completely given by PYTHIA

- Φ_1 events, the first shower emission should satisfy $\mathcal{T}_1(\Phi_2) < \mathcal{T}_1^{\rm cut}$ and $\mathcal{T}_0(\Phi_2) = \mathcal{T}_0(\Phi_1)$ (map) → First emission is done in GENEVA after that $\mathcal{T}_1(\Phi_N) < \mathcal{T}_1^{\rm cut}$

- Φ_2 events (>95% of total cross section) with nonzero values of $\mathcal{T}_0$ and $\mathcal{T}_1$: PYTHIA first emission affects the $\mathcal{T}_0$ distribution only beyond NNLL' [Alioli,Bauer,Berggren,Tackmann, Walsh `15]

Factorization

We derived a factorization formula (see 2111.03632 Appendix A) using SCET+HQET in the region $\mathcal{T}_0 \to 0$ when $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales (in case of boosted regime $M_{t\bar{t}} \gg m_t$ situation similar to [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21])

Hard functions (colour matrices) known to NLO [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^3LO

Soft functions (colour matrices) computed to NLO

Hard function anomalous dimension: split into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$\mathbf{\Gamma}_H(M, \beta_t, \theta, \mu) = \Gamma_{\text{cusp}}(\alpha_s)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + \boldsymbol{\gamma}^h(M, \beta_t, \theta, \alpha_s)$$

[Ferroglia, Neubert, Pecjak, Yang,`09]

One can average over the two hemisphere momenta, soft function satisfies the RG equation in Laplace space, we used the consistency relation among anomalous dimensions $\boldsymbol{\gamma}^s = \boldsymbol{\gamma}^h + \gamma^B\, \mathbf{1}$

$$\frac{d}{d\ln\mu}\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) = \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s\dagger}\right]\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) + \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu)\left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^s\right]$$

Beam functions

The beam functions are given by convolutions of perturbative kernels with the standard PDFs $f_i(x,\mu)$

$$B_i(t,z,\mu) = \sum_j \int_z^1 \frac{d\xi}{\xi} I_{ij}(t, z/\xi, \mu) f_j(\xi,\mu)$$

I_{ij} kernels are known up to N^{3}LO, process independent

RG equation in Laplace space is given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{B}_i(L_c,z,\mu) = \left[-2\Gamma_{\text{cusp}}(\alpha_s)L_c + \gamma_i^B(\alpha_s)\right]\tilde{B}_i(L_c,z,\mu)$$

$$C_{k_i}\gamma_{\text{cusp}} \to C_{k_i}\gamma_{\text{cusp}} + 2\sum_R C_4(R_{k_i},R)g^R(\alpha_s)$$ At N^{3}LL

with solution in momentum space

$$B(t,z,\mu) = \exp\left[-4S(\mu_B,\mu) - a_{\gamma^B}(\mu_B,\mu)\right]\tilde{B}(\partial_{\eta_B},z,\mu_B)\frac{1}{t}\left(\frac{t}{\mu_B^2}\right)^{\eta_B}\frac{e^{-\gamma_E\eta_B}}{\Gamma(\eta_B)}$$

where $\eta_B \equiv 2a_\Gamma(\mu_B,\mu)$ and the collinear log is given by $L_c = \ln\left(M\kappa/\mu^2\right)$

Hard functions

The hard functions arise from matching the full theory onto the EFT, they can be extracted from colour decomposed loop amplitudes. At NLO it was first computed in [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]. They satisfy the RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{\Gamma}_H(M,\beta_t,\theta,\mu)\,\mathbf{H}(M,\beta_t,\theta,\mu) + \mathbf{H}(M,\beta_t,\theta,\mu)\,\mathbf{\Gamma}_H^\dagger(M,\beta_t,\theta,\mu)$$

Solution:

$$\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{U}(M,\beta_t,\theta,\mu_h,\mu)\mathbf{H}(M,\beta_t,\theta,\mu_h)\mathbf{U}^\dagger(M,\beta_t,\theta,\mu_h,\mu)$$

$$\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)$$

We have split the anomalous dimension into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$\mathbf{\Gamma}_H(M,\beta_t,\theta,\mu) = \Gamma_{\mathrm{cusp}}(\alpha_s)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + \boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha_s)$$ [Ferroglia, Neubert, Pecjak, Yang,`09]

$$\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu) = \mathcal{P}\exp\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)}\frac{\mathrm{d}\alpha}{\beta(\alpha)}\boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha)$$

We evaluate the matrix exponential **u** as a series expansion in α_s [1003.5827], [Buchalla,Buras,Lautenbacher `96]

Soft functions

We computed the soft functions matrices at NLO which were unknown for this observable

$$\mathbf{S}^{(1)}_{\text{bare},\,ij}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = \sum_{\alpha,\,\beta} \boldsymbol{w}_{ij}^{\alpha\beta}\, \hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu)$$

$$\hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = -\frac{2(\mu^2 e^{\gamma_E})^\epsilon}{\pi^{1-\epsilon}} \int \mathrm{d}^d k \frac{v_\alpha \cdot v_\beta}{v_\alpha \cdot k\, v_\beta \cdot k}\, \delta(k^2)\Theta(k^0)$$
$$\times \left[\delta(k_a^+ - k\cdot n_a)\Theta(k\cdot n_b - k\cdot n_a)\,\delta(k_b^+) + \delta(k_b^+ - k\cdot n_b)\Theta(k\cdot n_a - k\cdot n_b)\,\delta(k_a^+)\right]$$

One can average over the two hemisphere momenta, the soft function satisfies the RG equation in Laplace space

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) = \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s\dagger}\right] \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) + \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s}\right]$$

Solution in momentum space, where we used the consistency relation among anomalous dimensions $\boldsymbol{\gamma}^s = \boldsymbol{\gamma}^h + \gamma^B\,\mathbf{1}$

$$\mathbf{S}_B(l^+, \beta_t, \theta, \mu) = \exp\left[4S(\mu_s, \mu) + 2a_{\gamma^B}(\mu_s, \mu)\right]$$
$$\times\ \mathbf{u}^\dagger(\beta_t, \theta, \mu, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s}, \beta_t, \theta, \mu_s)\, \mathbf{u}(\beta_t, \theta, \mu, \mu_s)\, \frac{1}{l^+}\left(\frac{l^+}{\mu_s}\right)^{2\eta_s} \frac{e^{-2\gamma_E \eta_s}}{\Gamma(2\eta_s)}$$

$$\eta_s \equiv -2a_\Gamma(\mu_s, \mu)$$

Resummed result for the cross section

We can combine the solutions for the hard, soft and beam functions to obtain

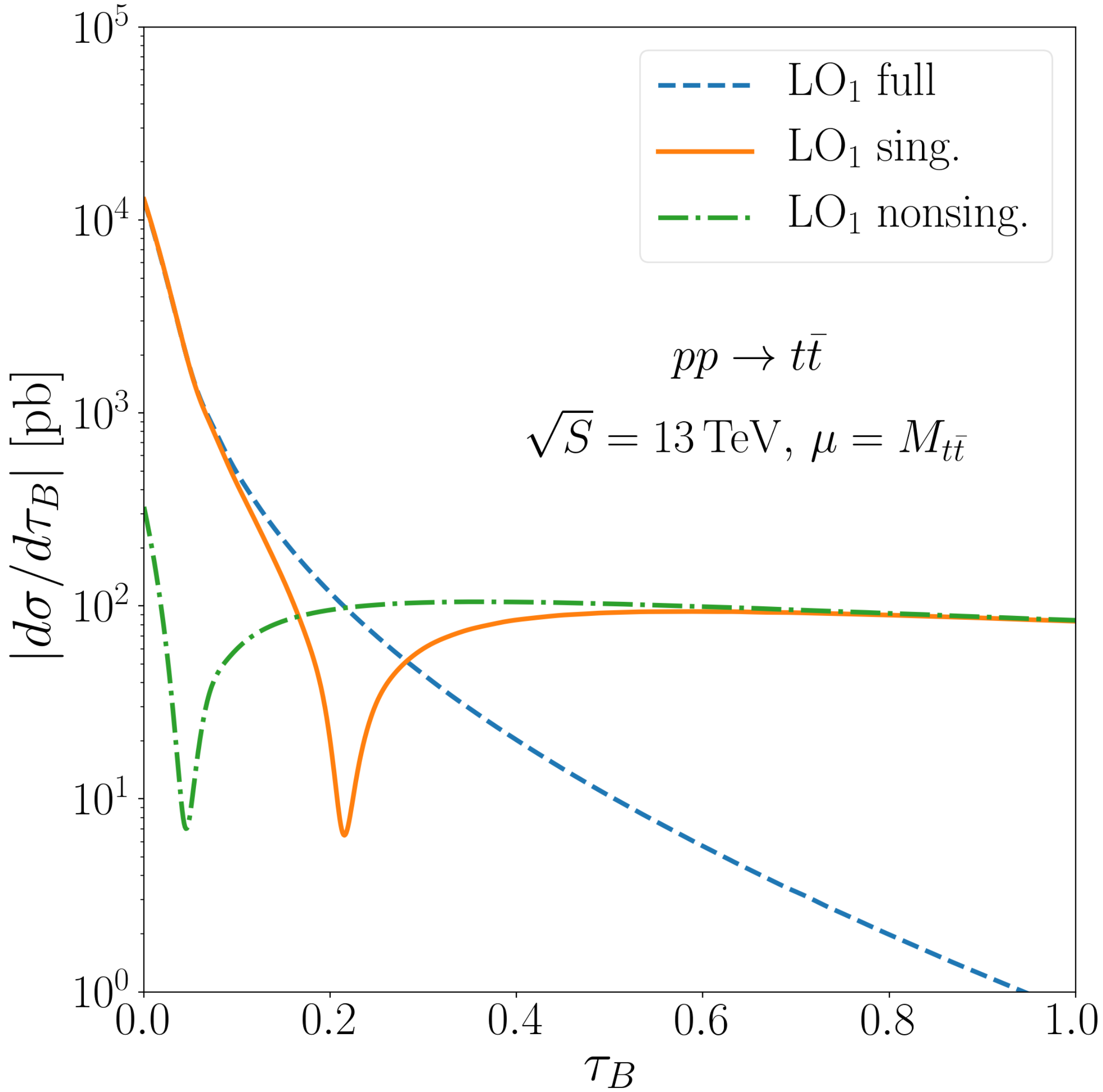
$$\frac{d\sigma}{d\Phi_0 d\tau_B} = U(\mu_h, \mu_B, \mu_s, L_h, L_s)$$
$$\times \mathrm{Tr}\left\{ \mathbf{u}(\beta_t, \theta, \mu_h, \mu_s) \mathbf{H}(M, \beta_t, \theta, \mu_h) \mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s) \tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \right\}$$
$$\times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\tau_B^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})}$$

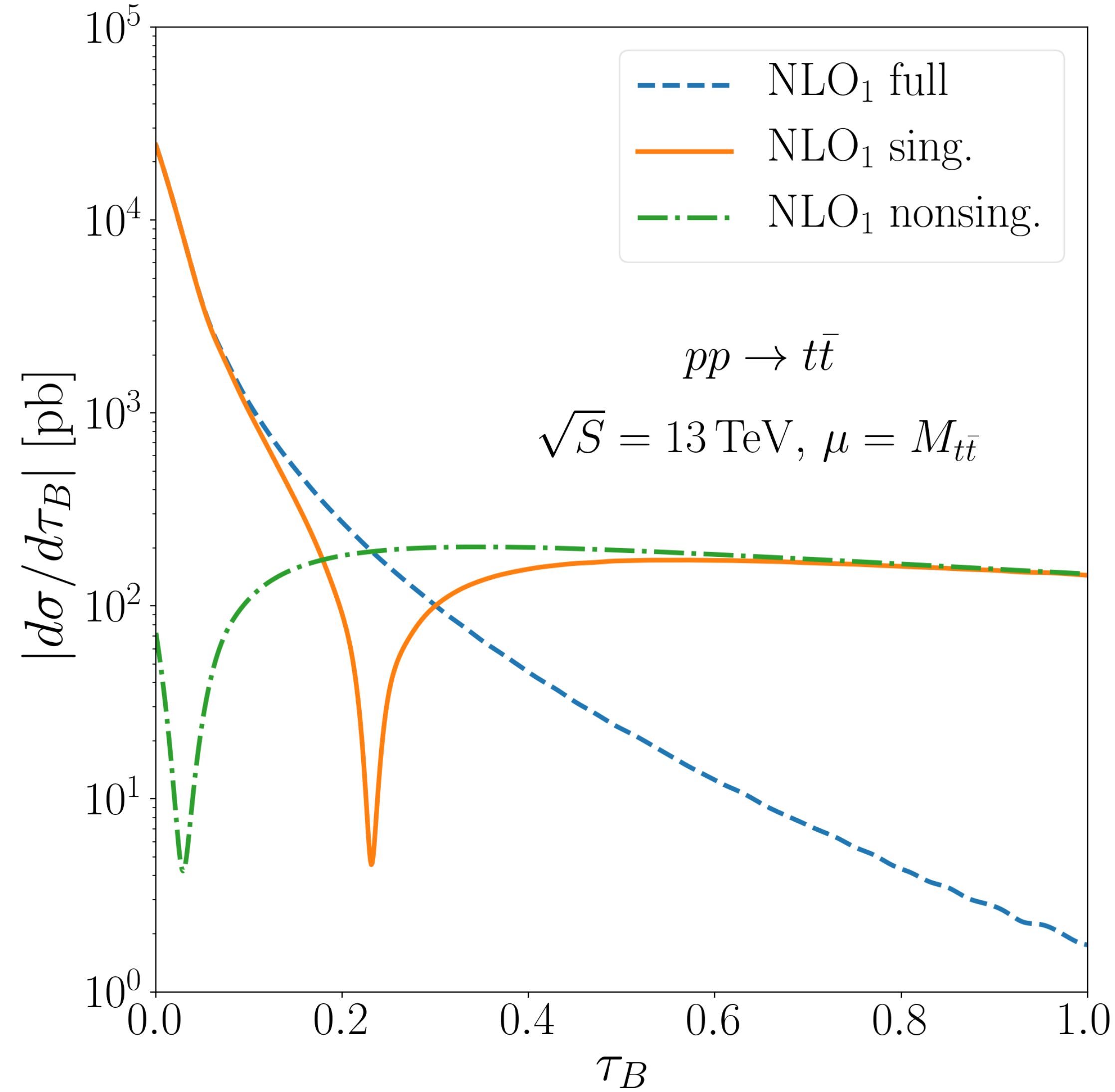
where

$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) =$$
$$\exp\left[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\, L_h - 2a_\Gamma(\mu_s, \mu_B)\, L_s \right]$$

and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\mathrm{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

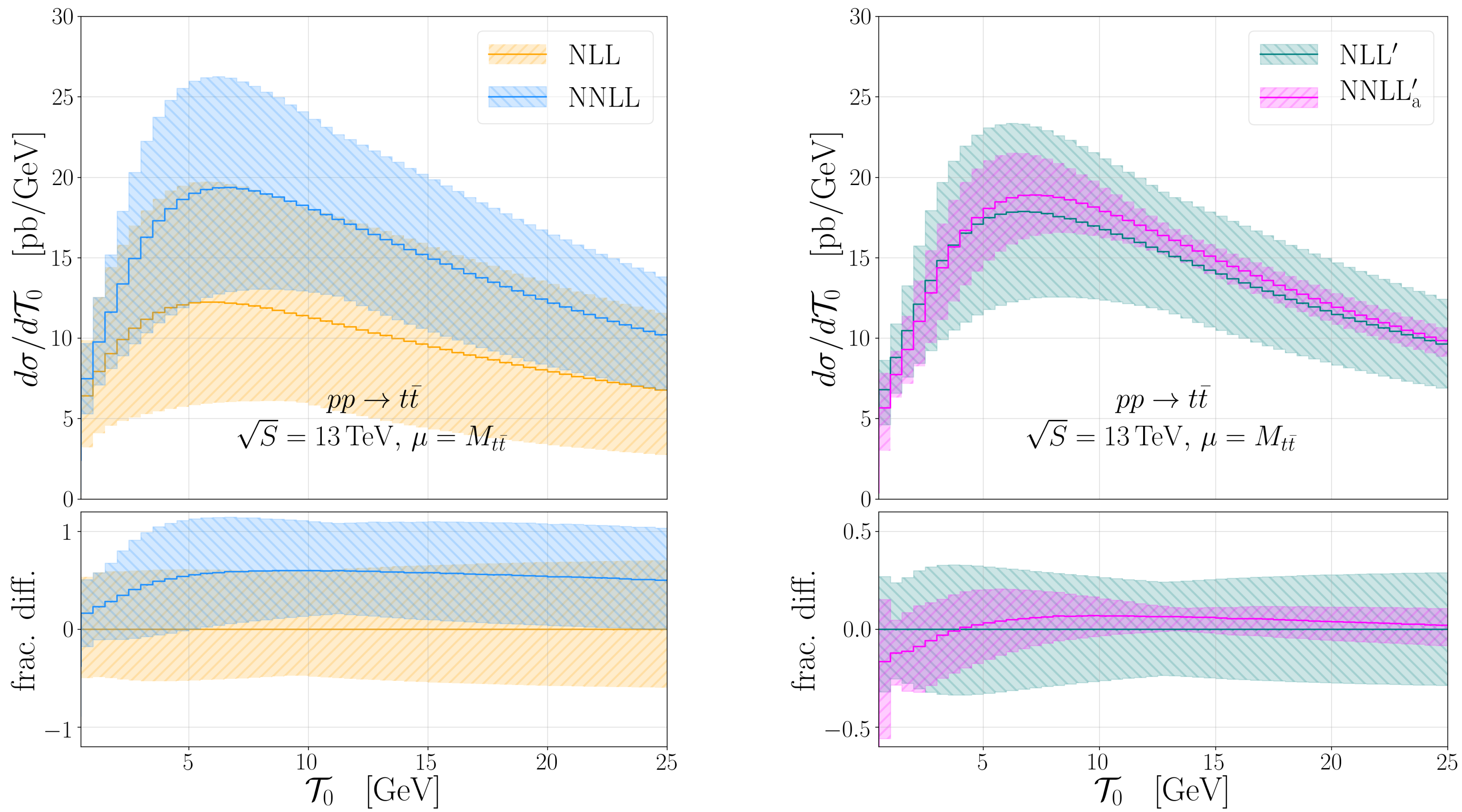
Singular vs Nonsingular contributions





Resummed results

NNLL′ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



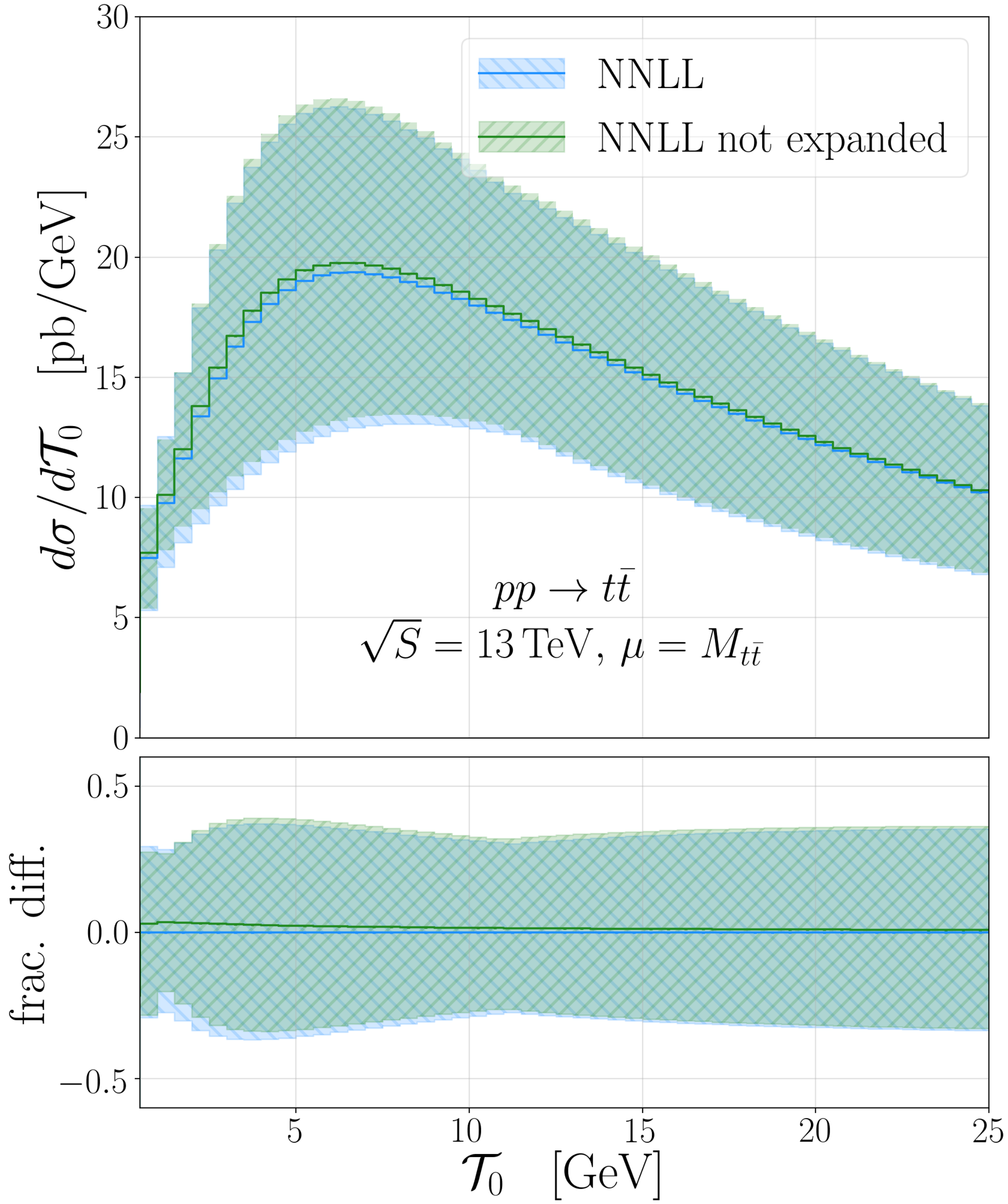
$$
\begin{aligned}
\mu_H &= \mu_{\rm NS}\,, \\
\mu_S(\mathcal{T}_0) &= \mu_{\rm NS}\ f_{\rm run}(\mathcal{T}_0/M)\,, \\
\mu_B(\mathcal{T}_0) &= \mu_{\rm NS}\ \sqrt{f_{\rm run}(\mathcal{T}_0/M)}
\end{aligned}
$$

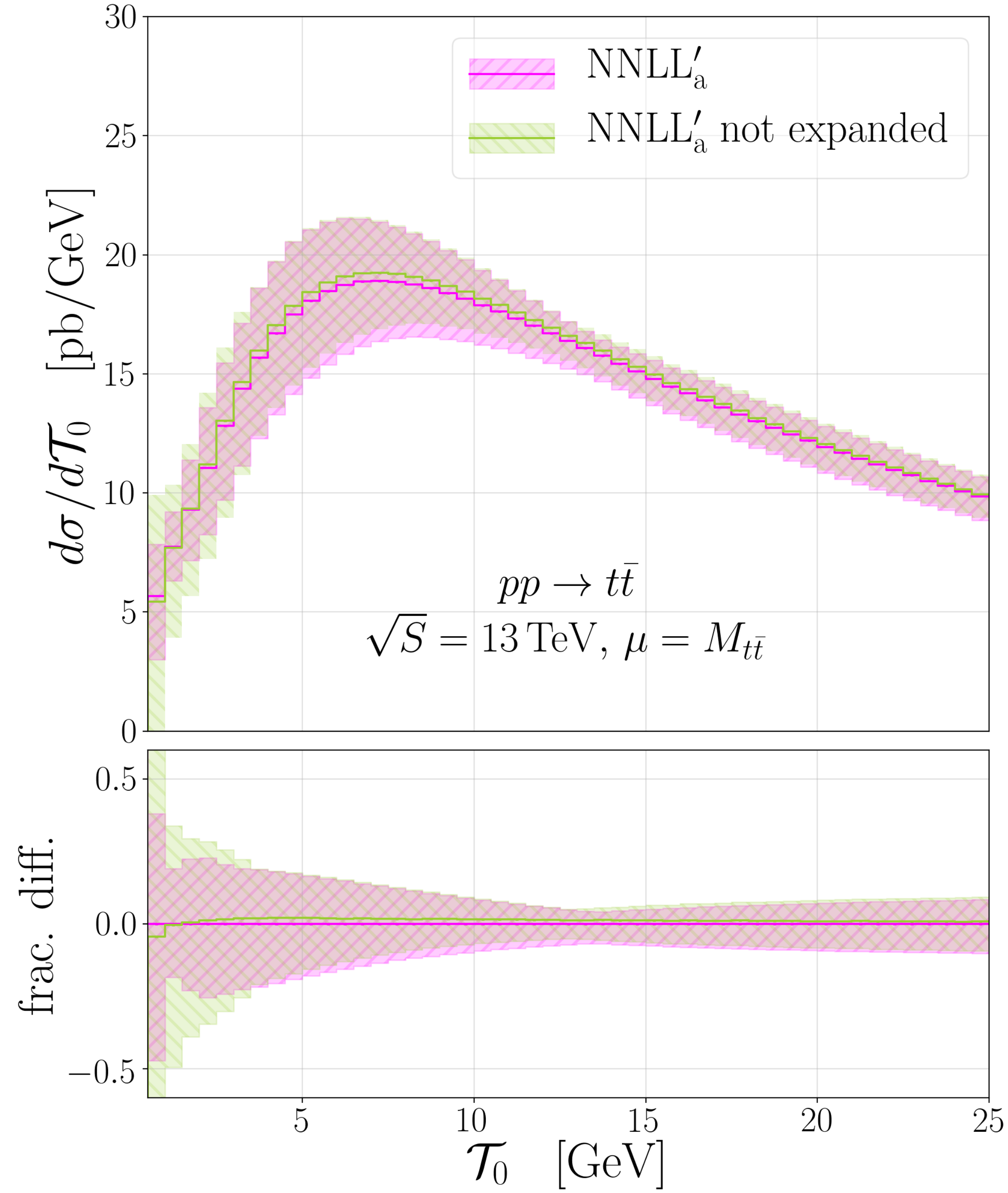
$$
f_{\rm run}(y) = \begin{cases} y_0\big[1+(y/y_0)^2/4\big] & y \le 2y_0\,, \\ y & 2y_0 \le y \le y_1\,, \\ y + \frac{(2-y_2-y_3)(y-y_1)^2}{2(y_2-y_1)(y_3-y_1)} & y_1 \le y \le y_2\,, \\ 1 - \frac{(2-y_1-y_2)(y-y_3)^2}{2(y_3-y_1)(y_3-y_2)} & y_2 \le y \le y_3\,, \\ 1 & y_3 \le y\,. \end{cases}
$$

$$
y_0 = 1.0\,\text{GeV}/M\,, \quad \{y_1, y_2, y_3\} = \{0.1, 0.175, 0.25\}
$$

Resummed results

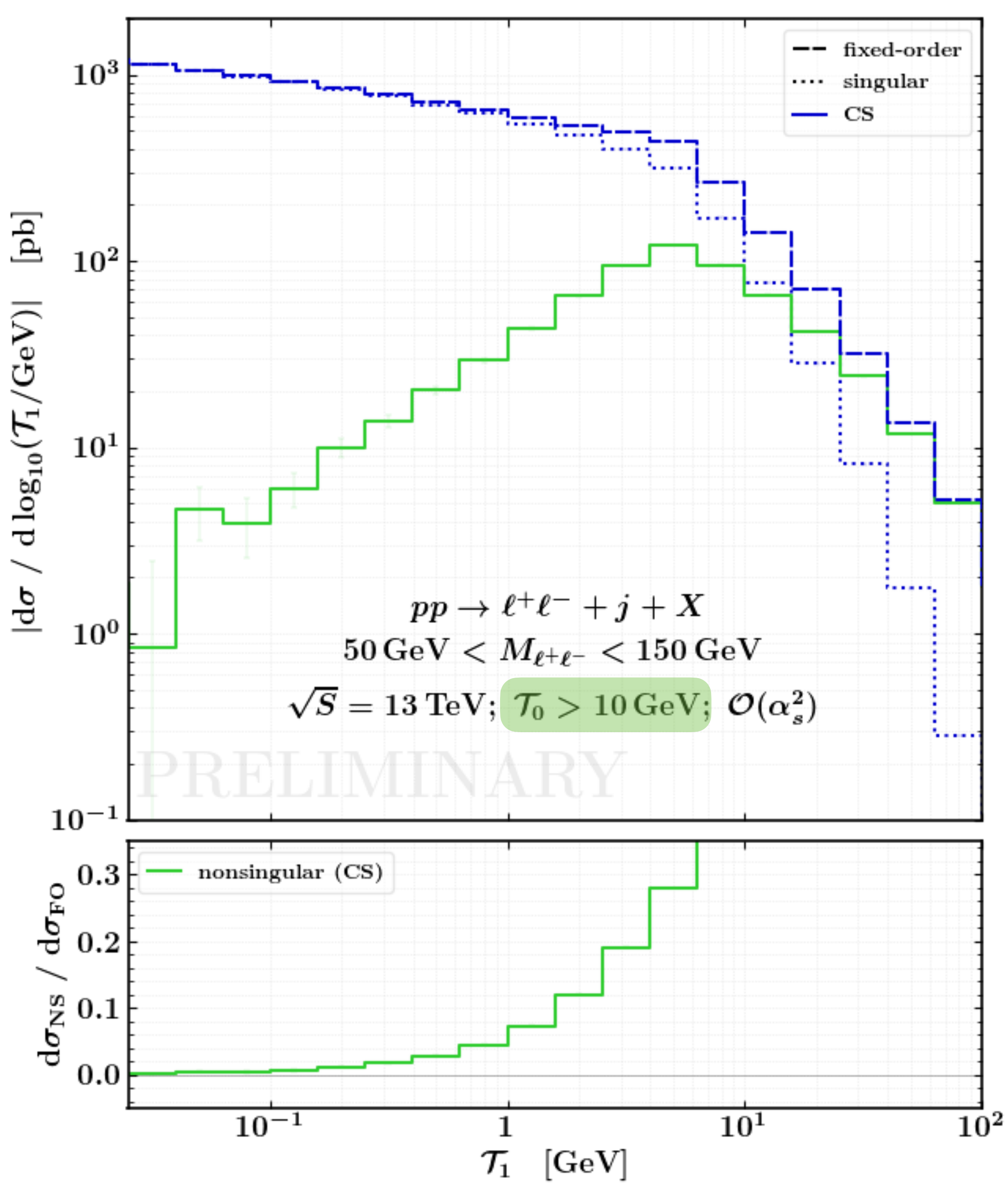
The evolution matrix **u** is evaluated in α_s expansion, we can choose to expand or not expand U, the difference is quite small

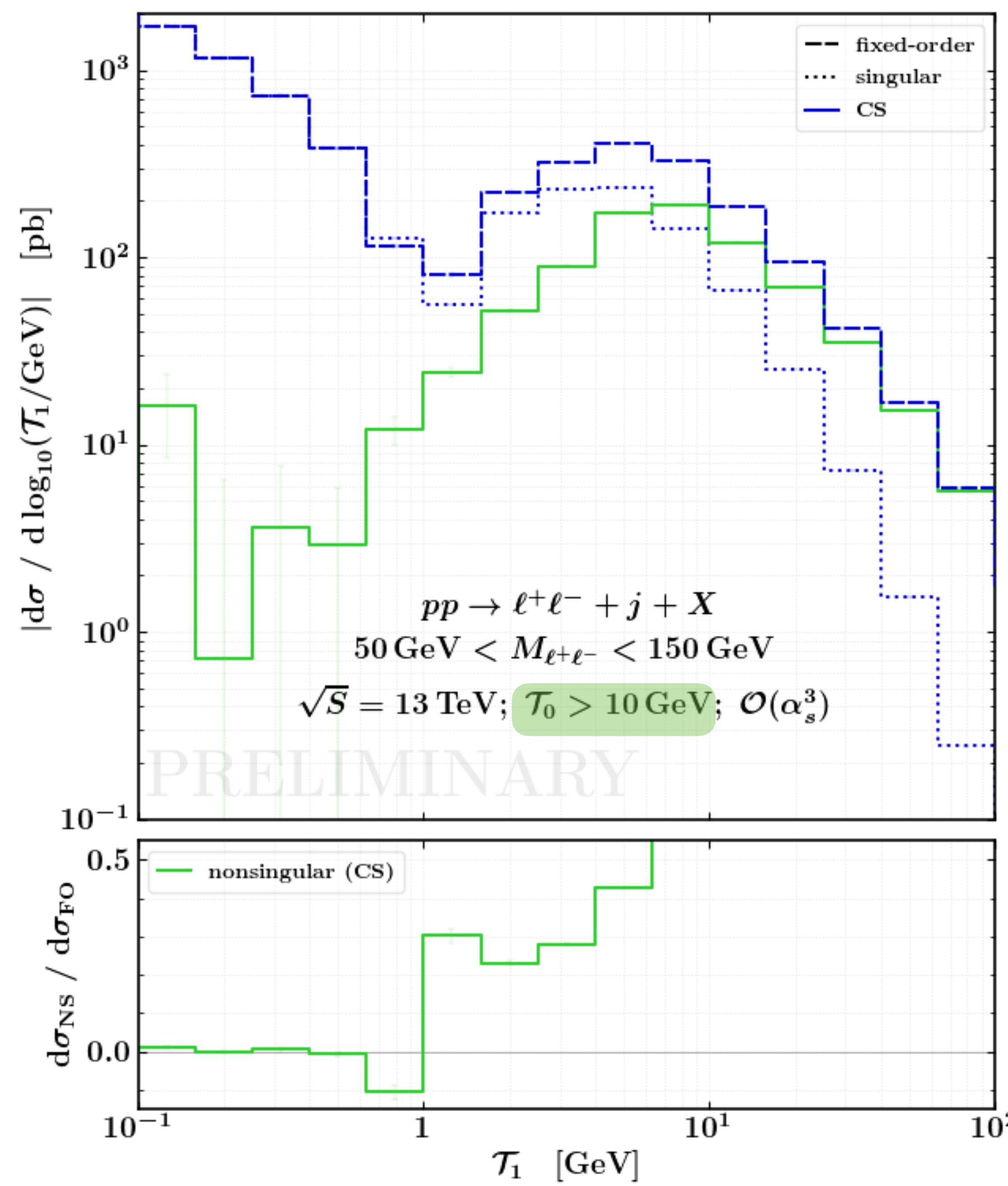




Singular vs Nonsingular

- Result for *exact* one-jettiness in CS frame, very similar results to FR

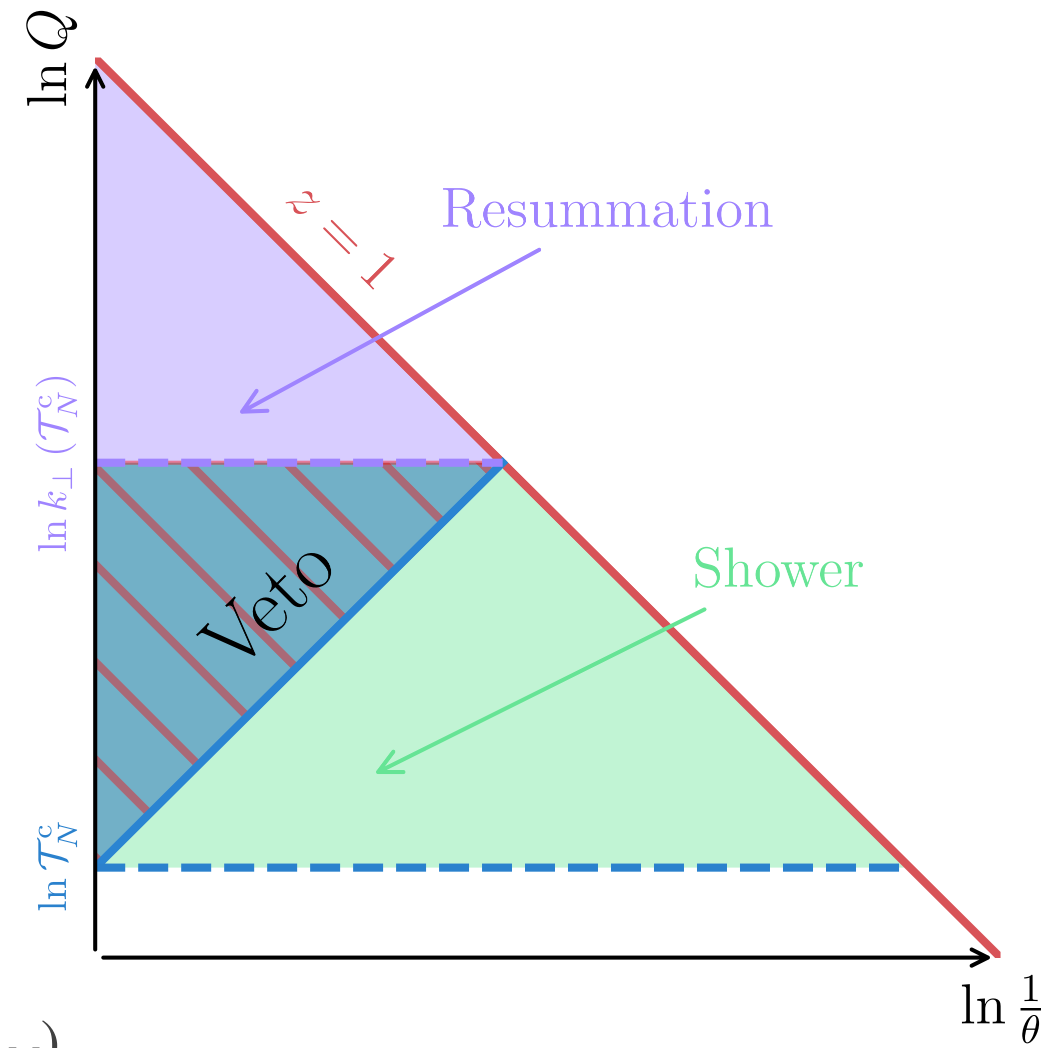




Interface to the parton shower

$\mathcal{T}_N(\Phi_{N+1})$ measures the hardness of the N+1-th emission

- If shower ordered in k_T, start from largest value allowed by N-jettiness
- Let the shower evolve unconstrained.
- At the end veto an event if after shower emissions $\mathcal{T}_N(\Phi_{N+M}) > \mathcal{T}_N(\Phi_N + 1)$ and **retry** the whole shower.



$$\mathcal{T}_{N+M-1}(\Phi_{N+M}) \leq \mathcal{T}_{N+M-2}(\Phi_{N+M}) \leq \ldots \leq \mathcal{T}_N(\Phi_{N+M})$$

Ensures the relevant phase space is correctly covered to avoid spoiling the resummation accuracy for $\mathcal{T}$ and the shower accuracy for other observables.

0-jet and 1-jet bins are treated differently: starting scale is resolution cutoff.

Method rather independent from shower used: PYTHIA8, DIRE & SHERPA.