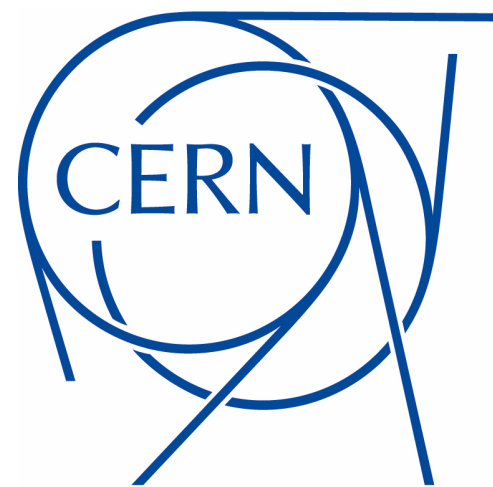
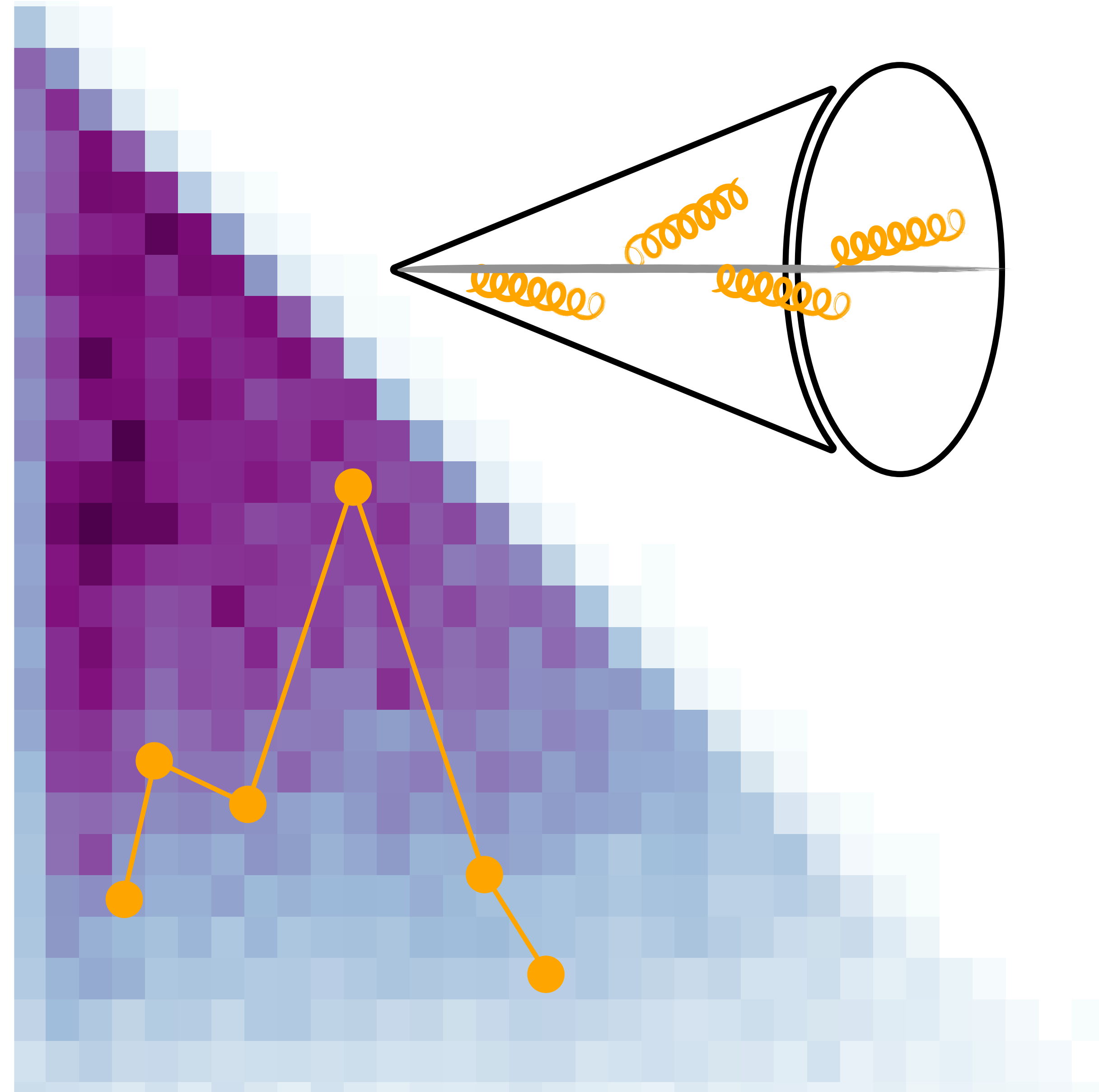


Logarithmic resummation of Lund-plane based observables



Alba Soto-Ontoso
Parton Showers and Resummation
Milano, 6th June, 2023



The Lund plane: core ingredient of PSR

Parton showers

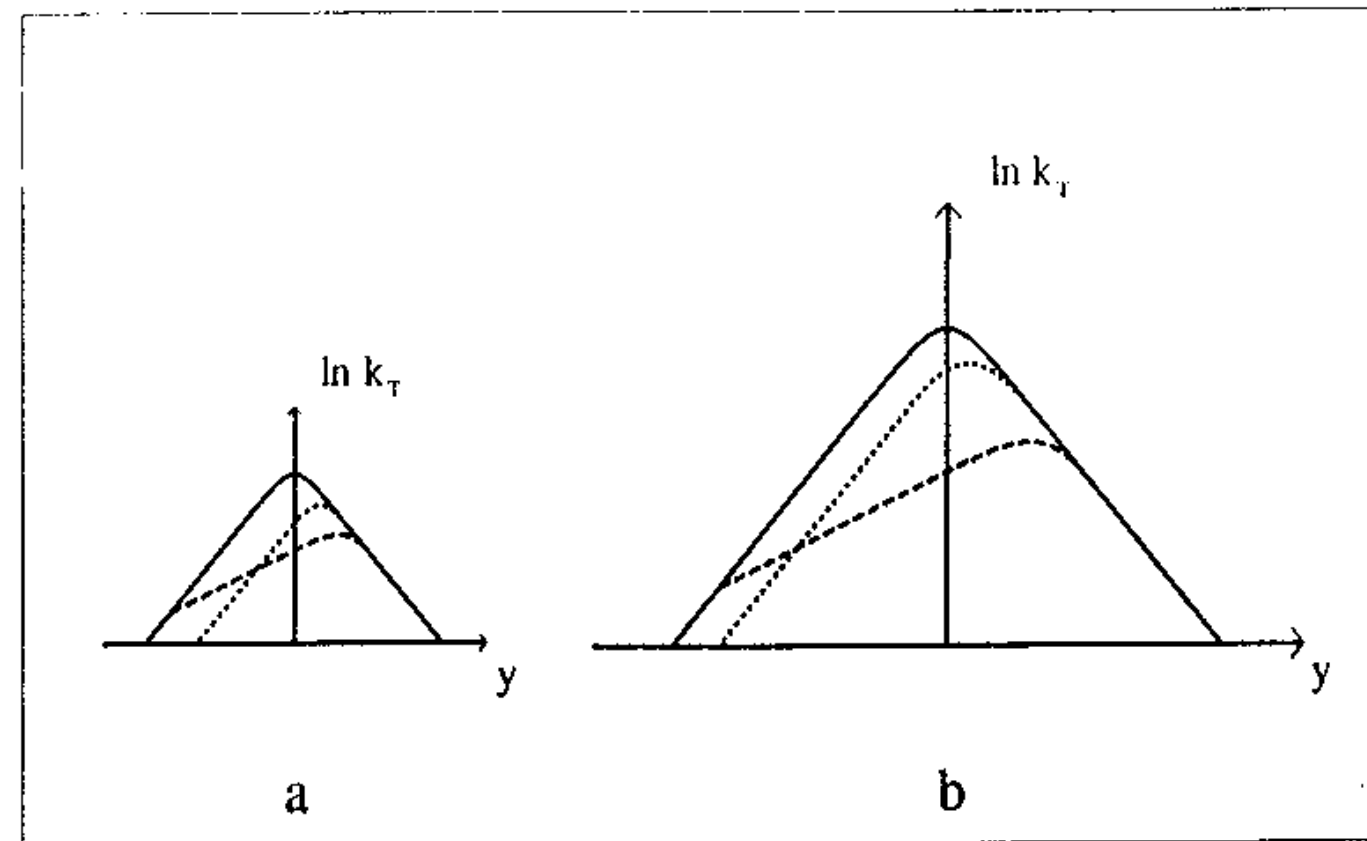
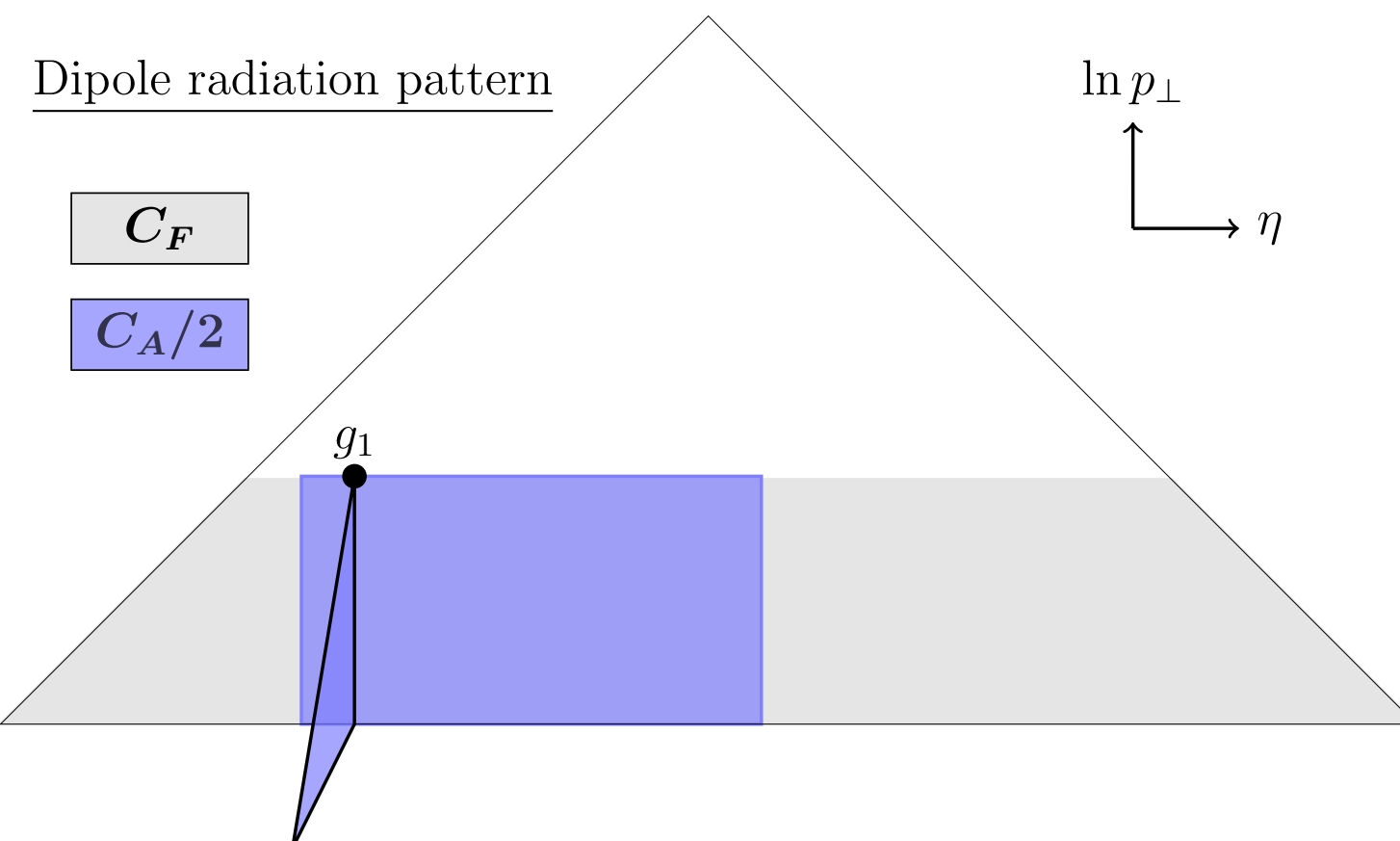
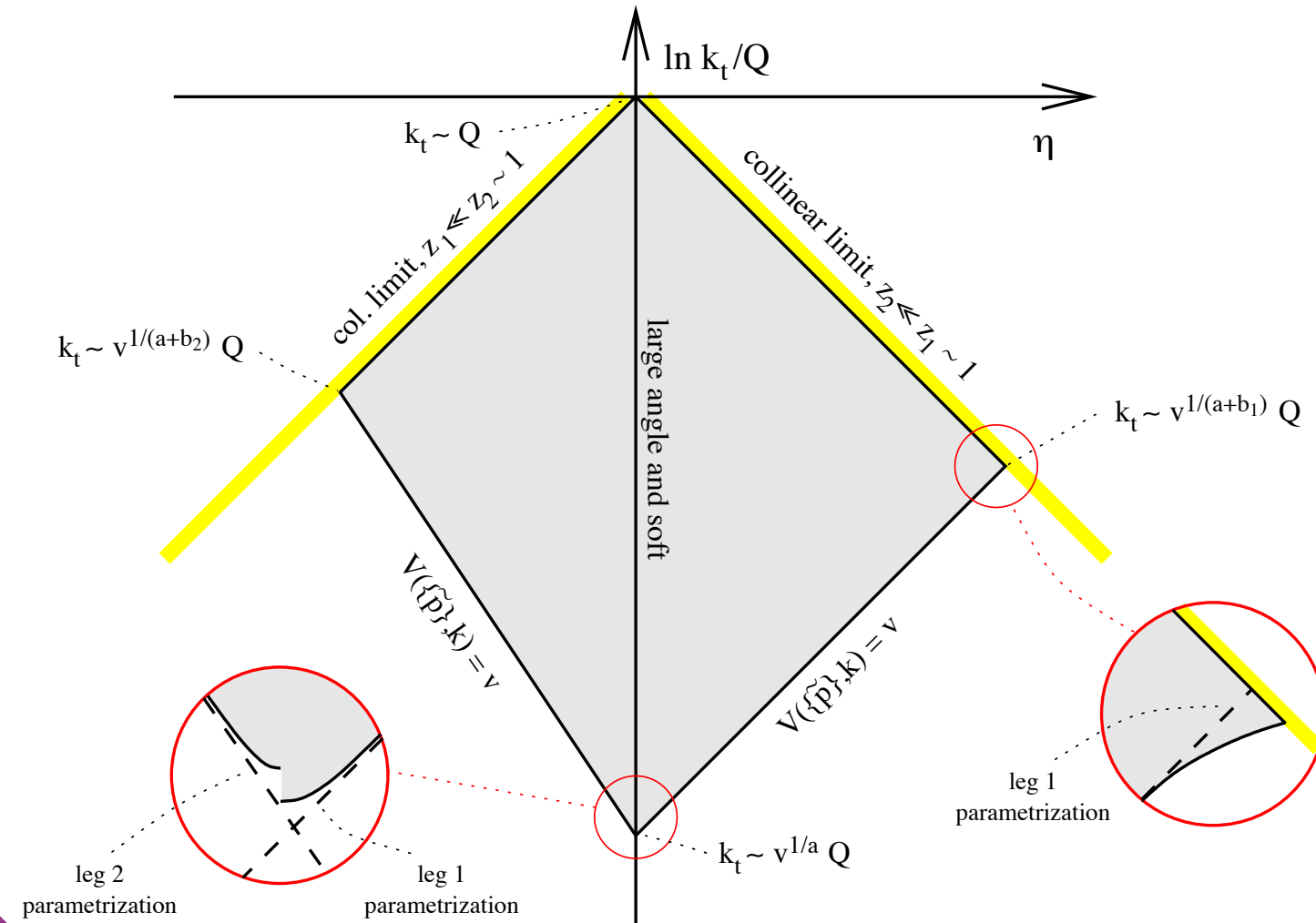


fig. 5
[Andersson et al. Z.Phys.C 43 (1989) 625]

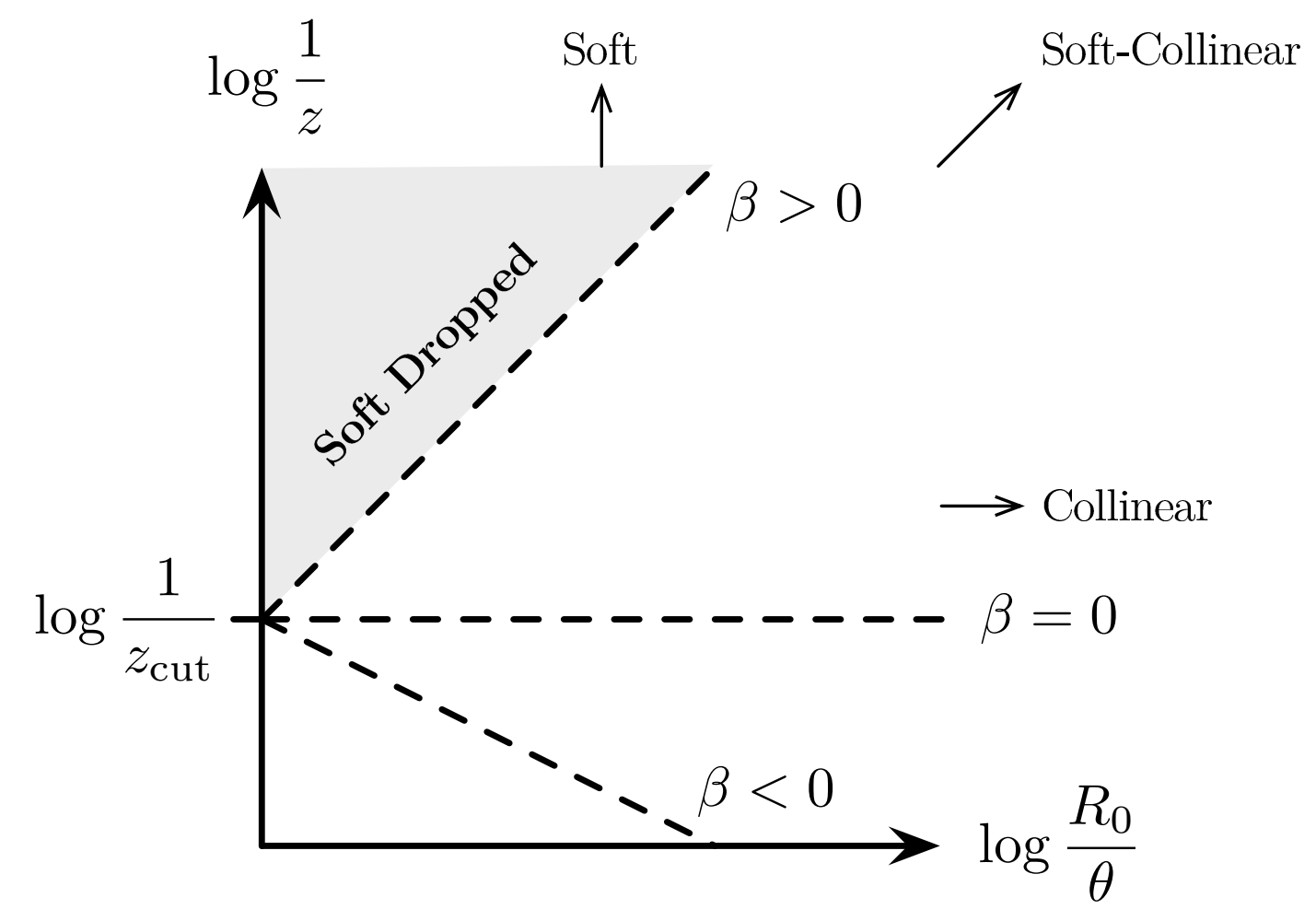


[Dasgupta et al. JHEP 09 (2018) 033]

Resummation

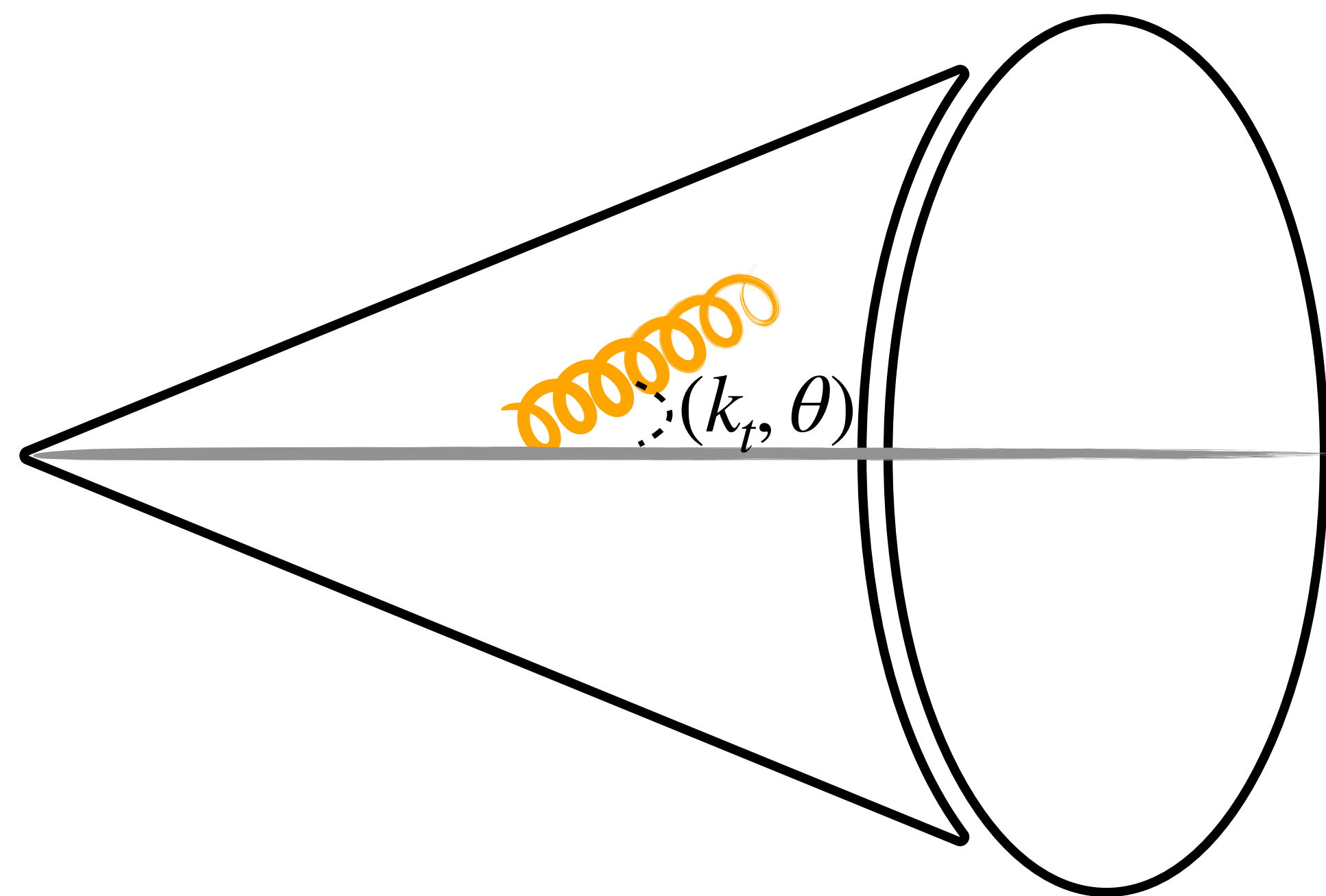


[Banfi, Salam, Zanderighi, JHEP 03 (2005) 073]

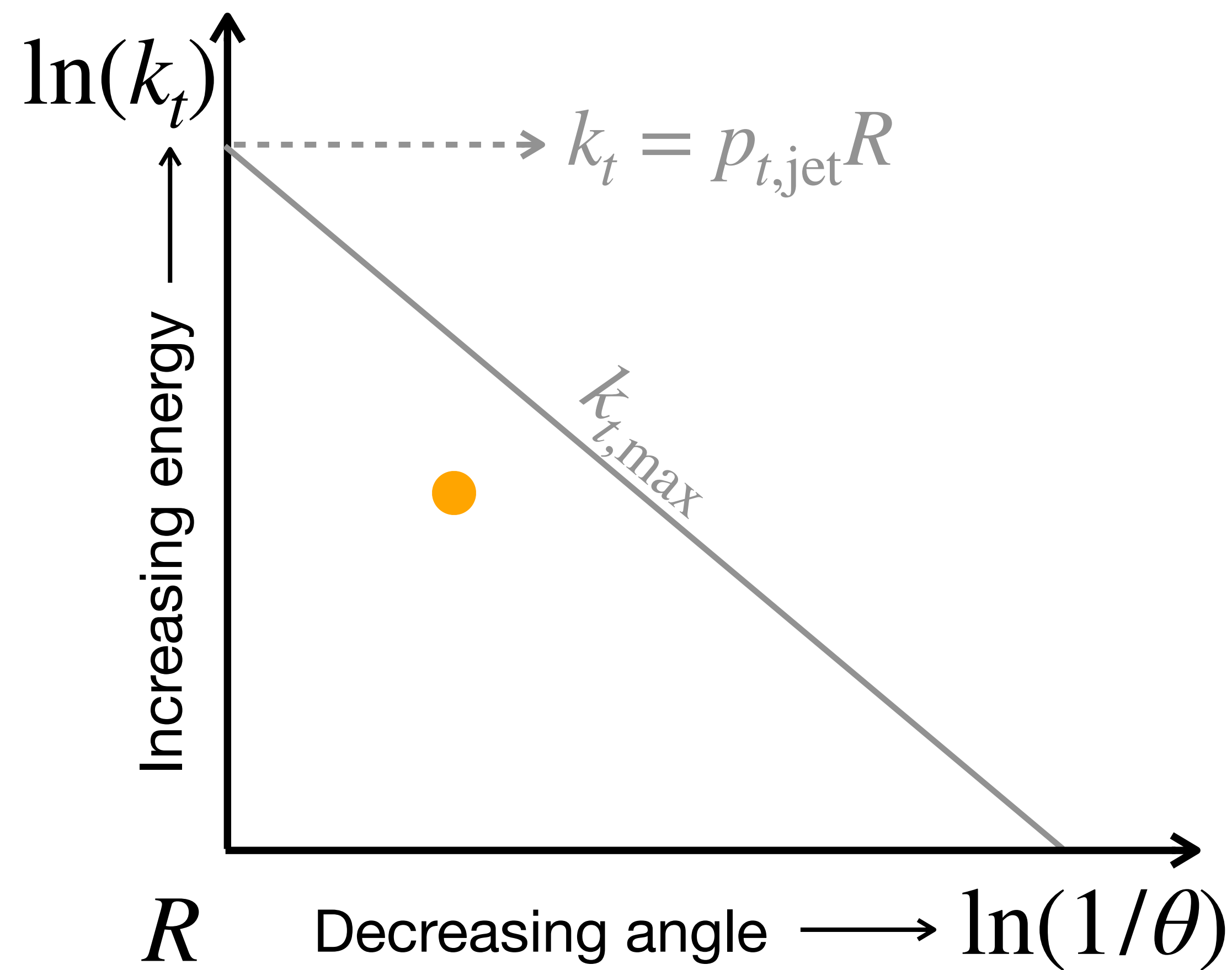


[Larkowski et al., JHEP 05 (2014) 146]

The Lund jet plane [Dreyer, Salam, Soyez JHEP 12 (2018) 064]

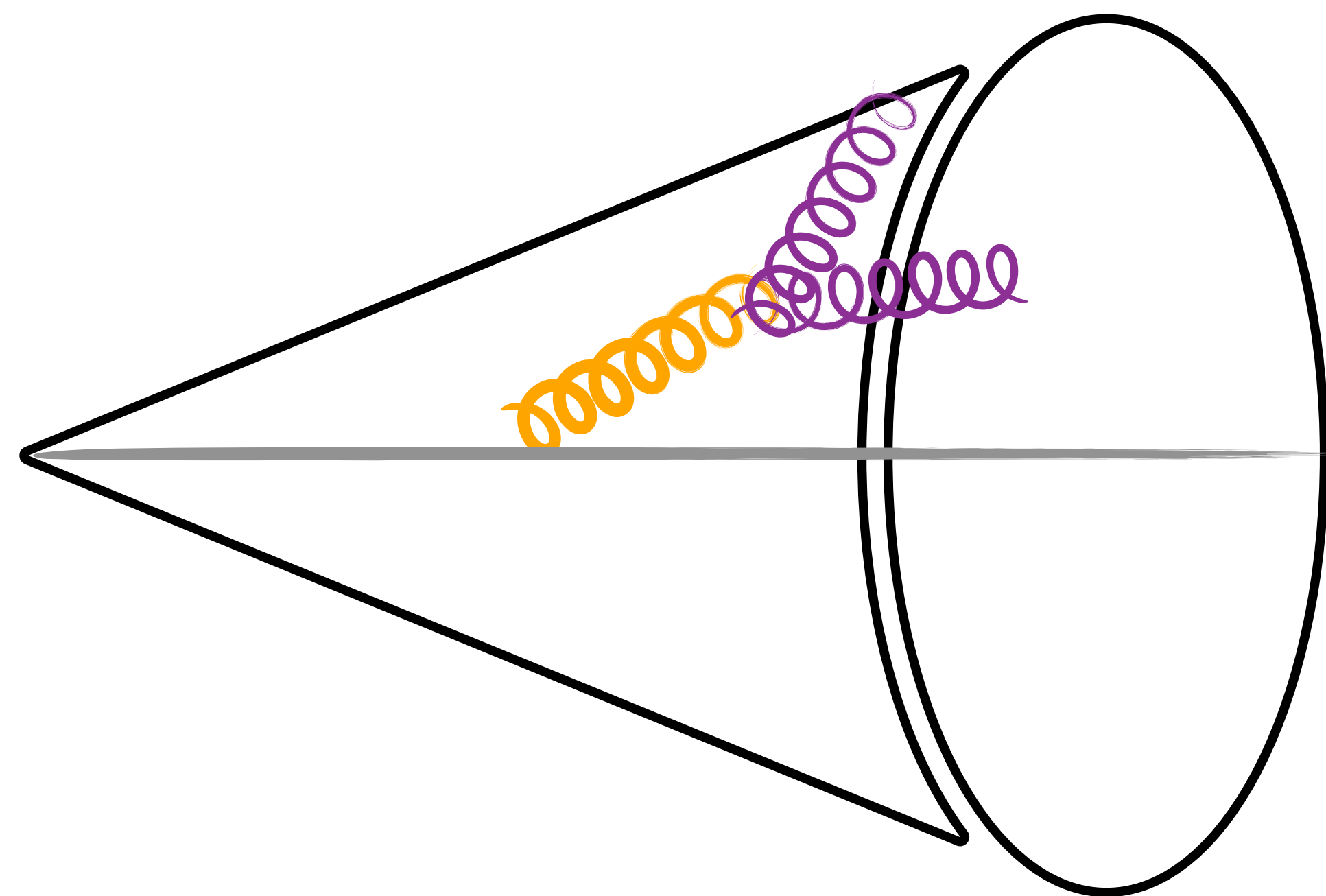


C/A reclustered jet

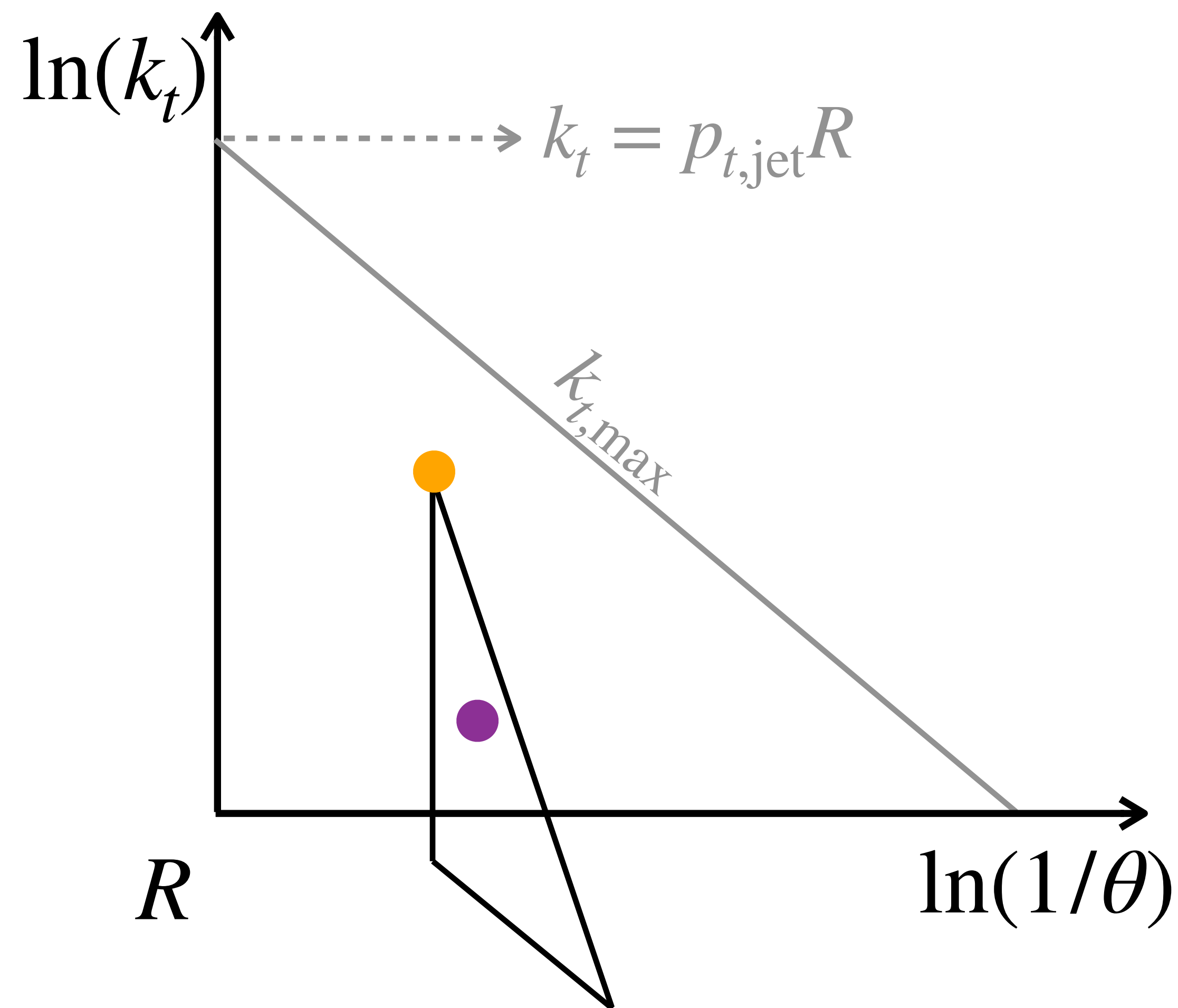


$$k_t = z p_{t,\text{jet}} \theta$$

The Lund jet plane [Dreyer, Salam, Soyez JHEP 12 (2018) 064]



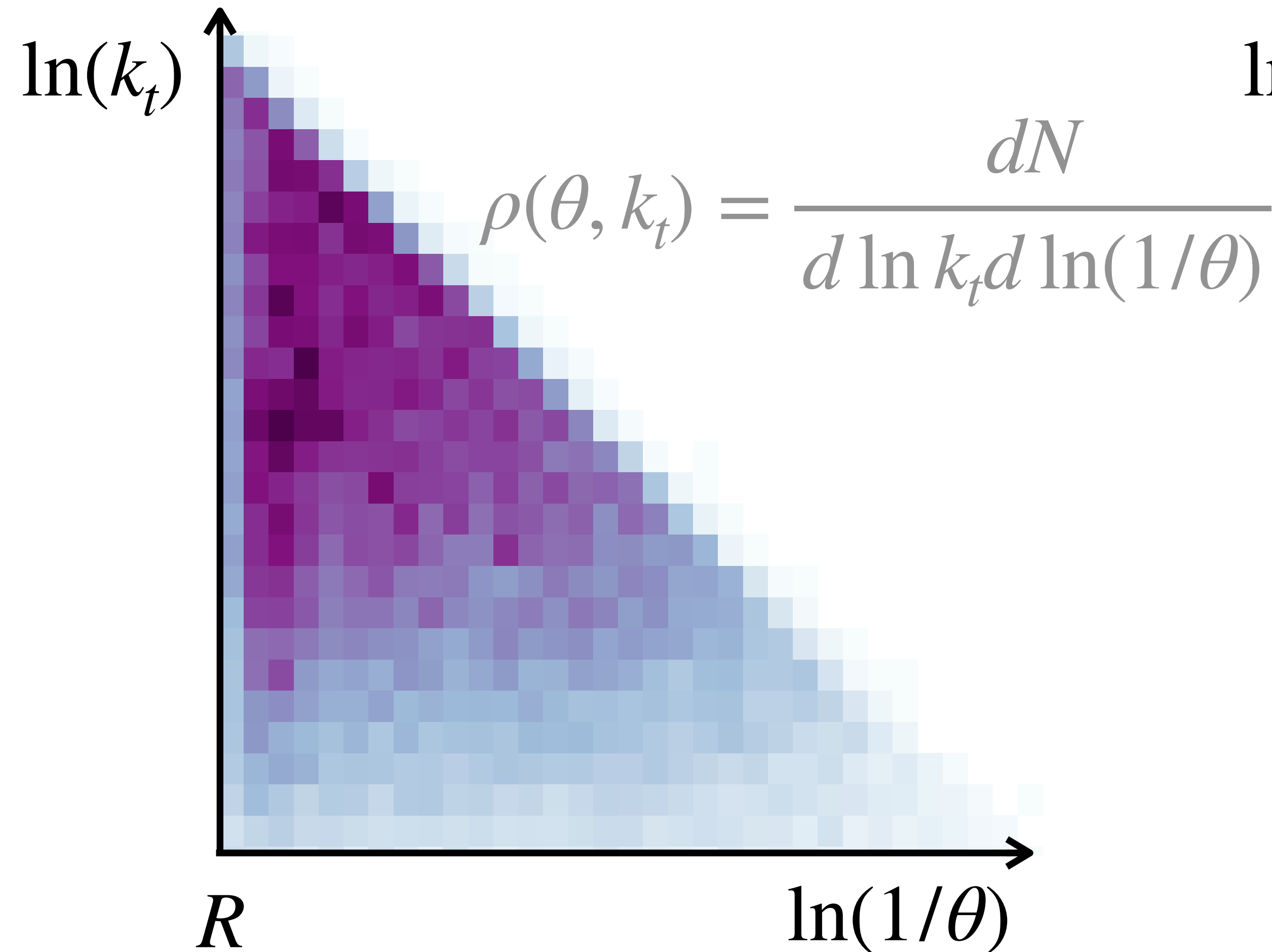
C/A reclustered jet



$$k_t = z p_{t,jet} \theta$$

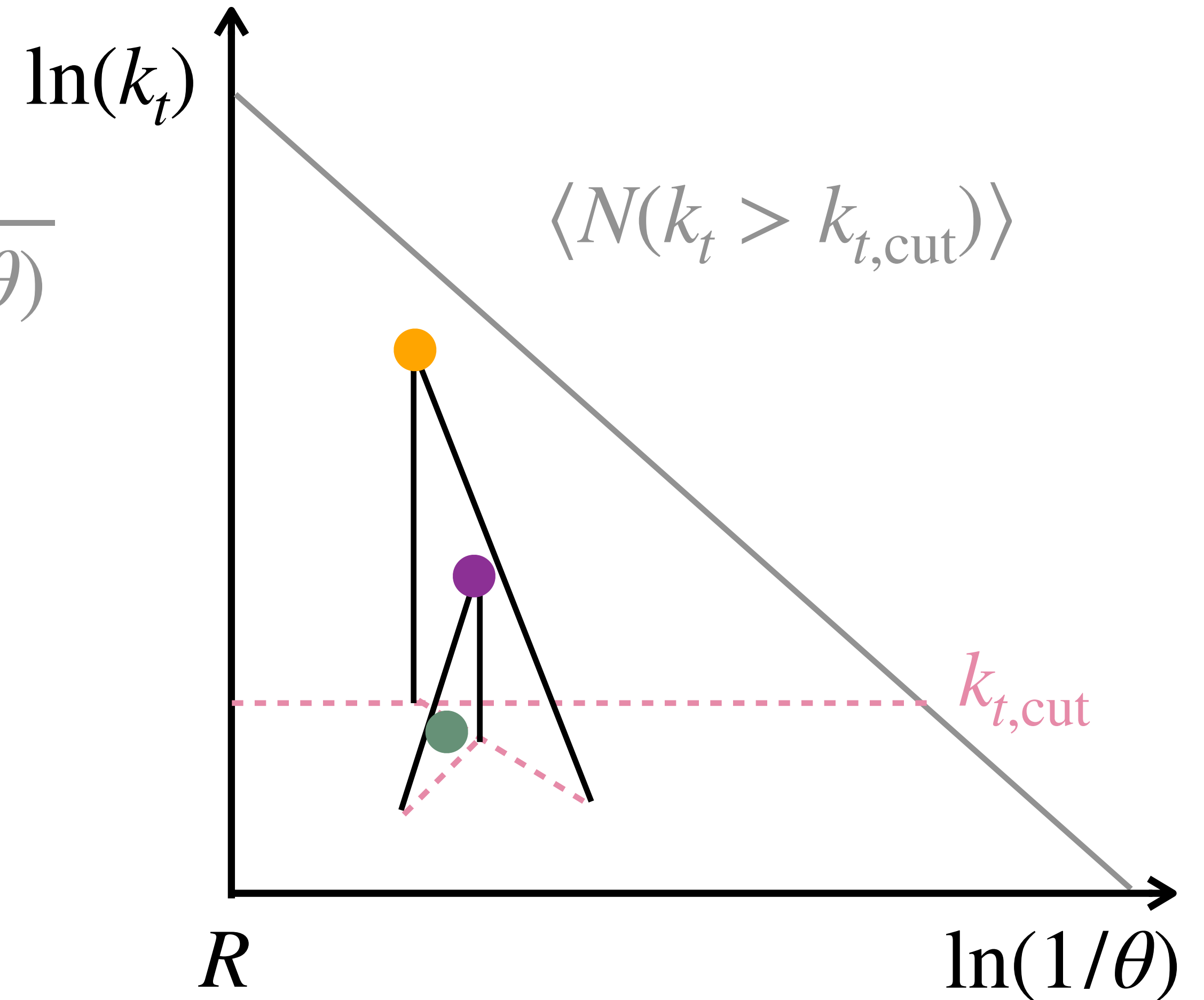
This talk: two spinoffs of the Lund jet plane

Primary Lund-plane density



[Lifson, Salam, Soyez JHEP 10 (2020) 170]

Lund multiplicity



[Medves, ASO, Soyez, JHEP 10 (2022) 156,
JHEP 04 (2023) 104]

The primary Lund-plane density: resummation structure

In the soft-and-collinear limit, the Lund plane density is simply given by

$$\rho_{\text{LO}}(\theta, k_t) = \frac{2\alpha_s C_i}{\pi}$$

Beyond LO/LL, two sources of logarithmic enhancements appear

$$\alpha_s^{n+1} \ln^m \theta \ln^{n-m} \frac{P_t}{k_t} \quad \text{with} \quad 0 \leq m \leq n$$

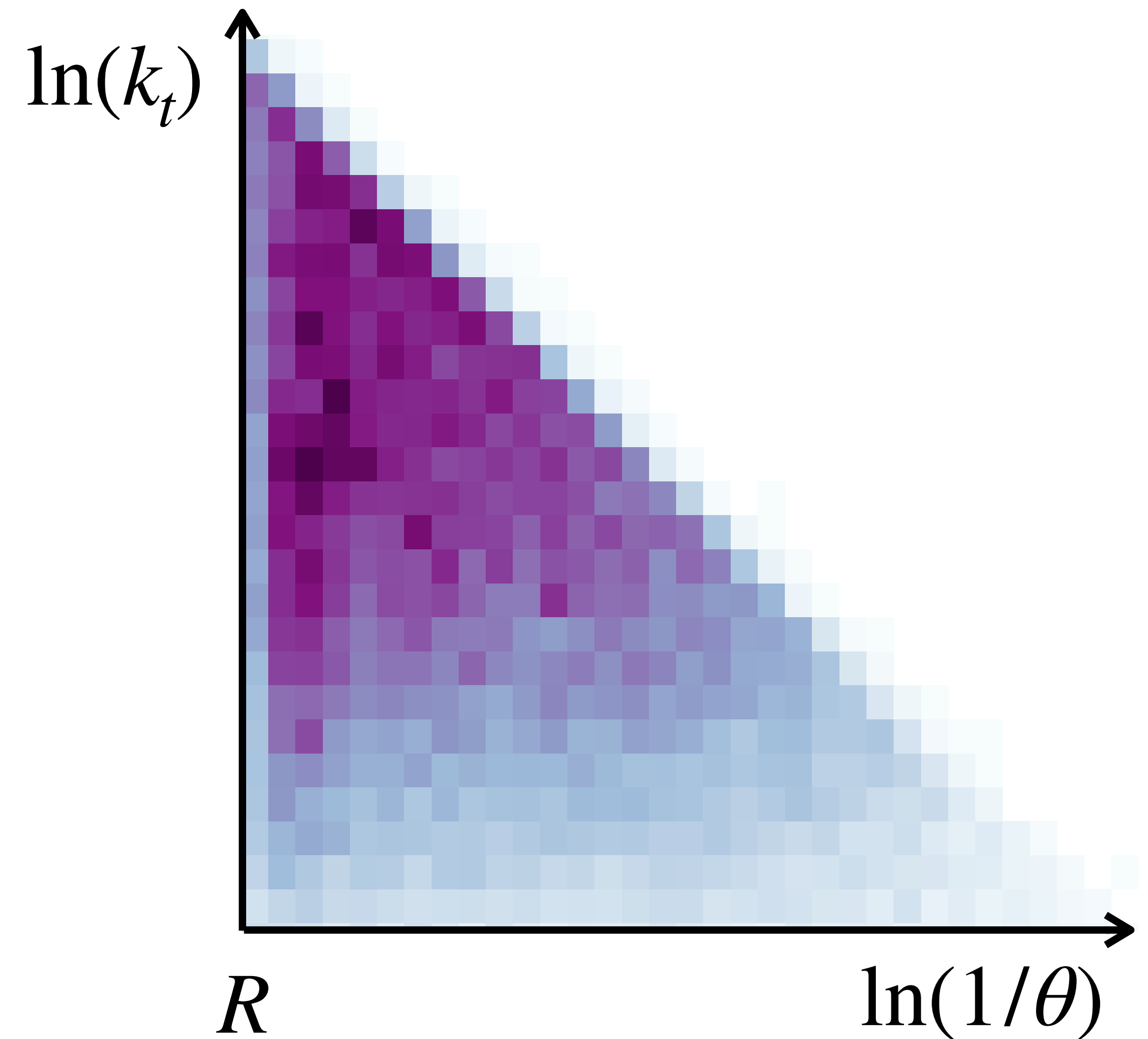
So far, the full set of single-logarithmic corrections has been computed

[Lifson, Salam, Soyez JHEP 10 (2020) 170]

The primary Lund-plane density: NLL resummation

- 1 Running coupling corrections

$$\rho_{\text{rc}}(\theta, k_t) = \frac{2\alpha_s^{1\ell}(k_t)C_i}{\pi}$$



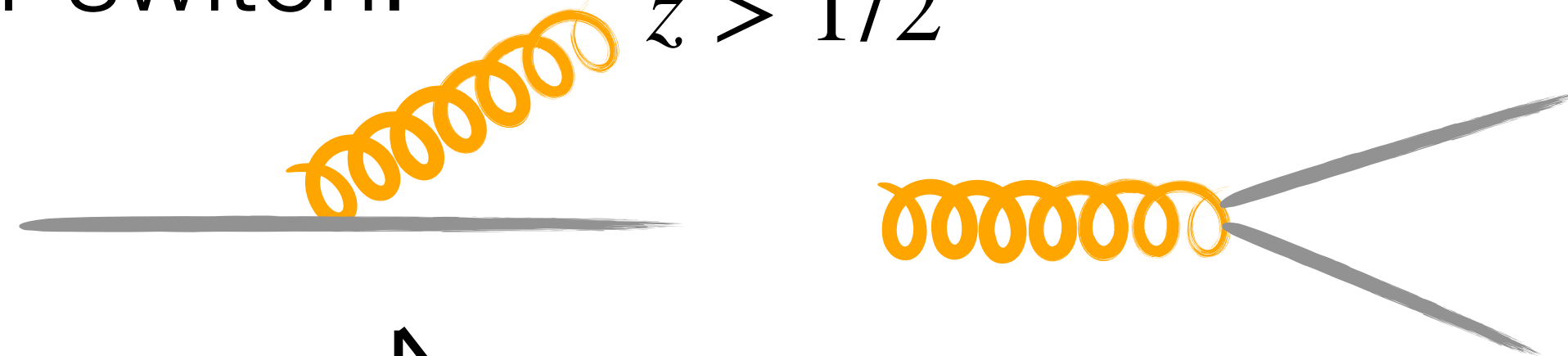
The primary Lund-plane density: NLL resummation

- 1 Running coupling corrections

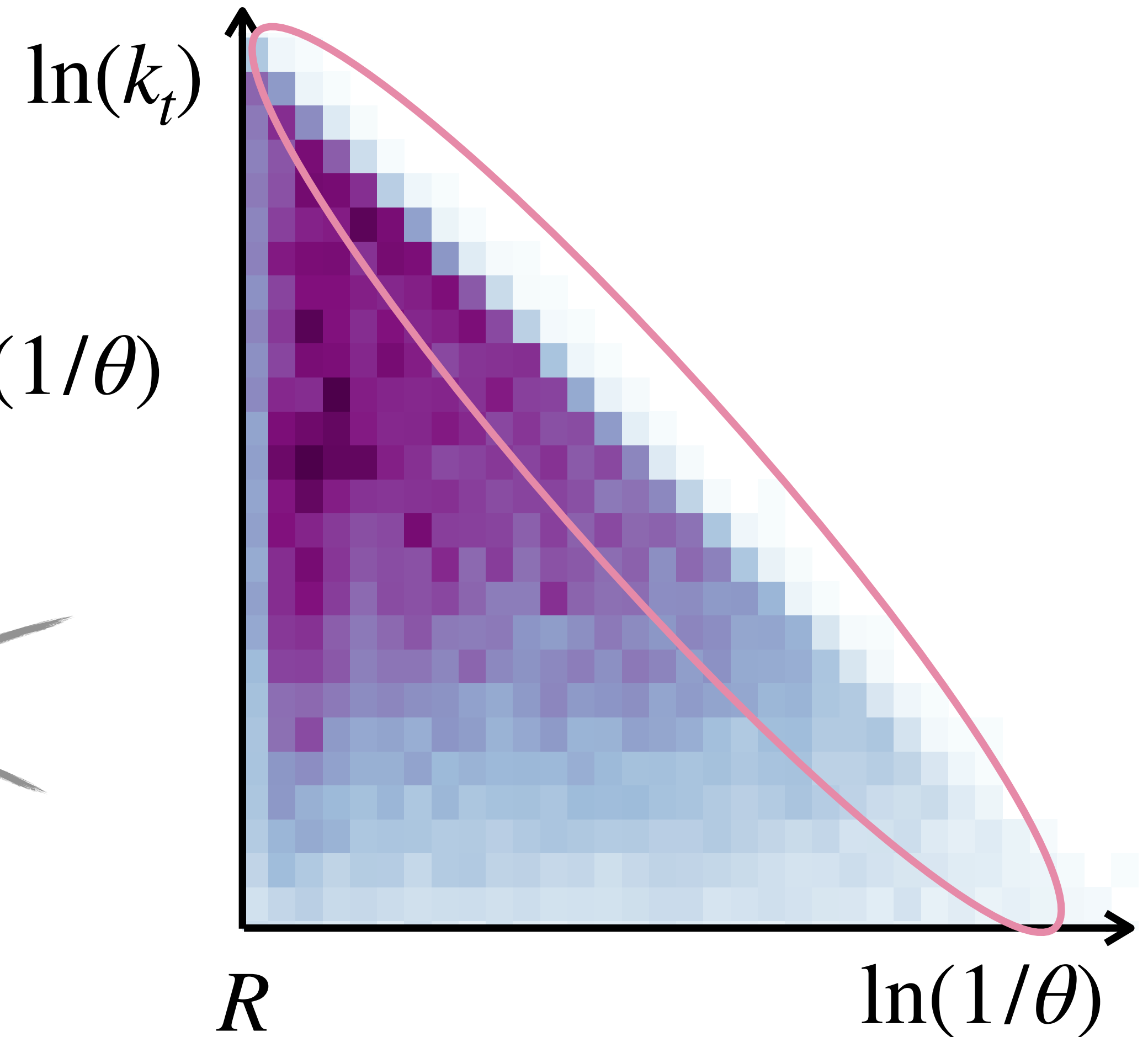
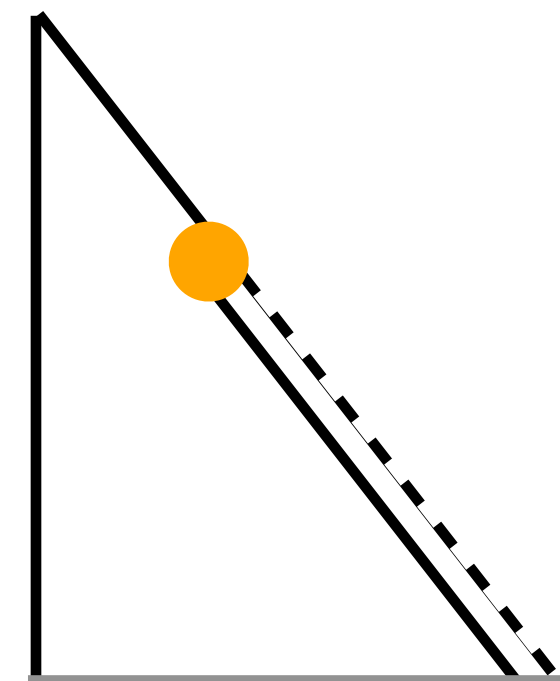
$$\rho_{\text{rc}}(\theta, k_t) = \frac{2\alpha_s^{1\ell}(k_t)C_i}{\pi}$$

- 2 Hard-collinear corrections $\alpha_s^{n+1} \ln^n(1/\theta)$

a) Flavour switch: $z > 1/2$



b) Energy loss:



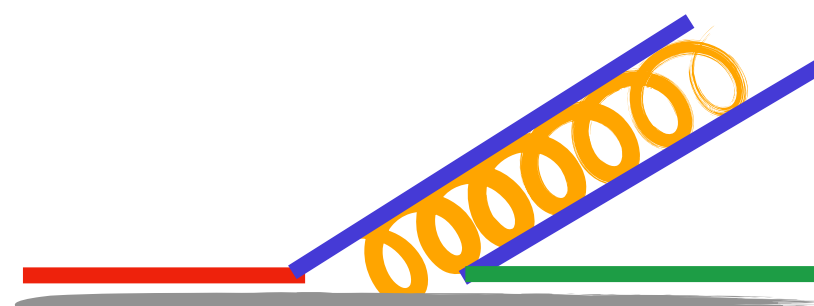
The primary Lund-plane density: NLL resummation

- 1 Running coupling corrections

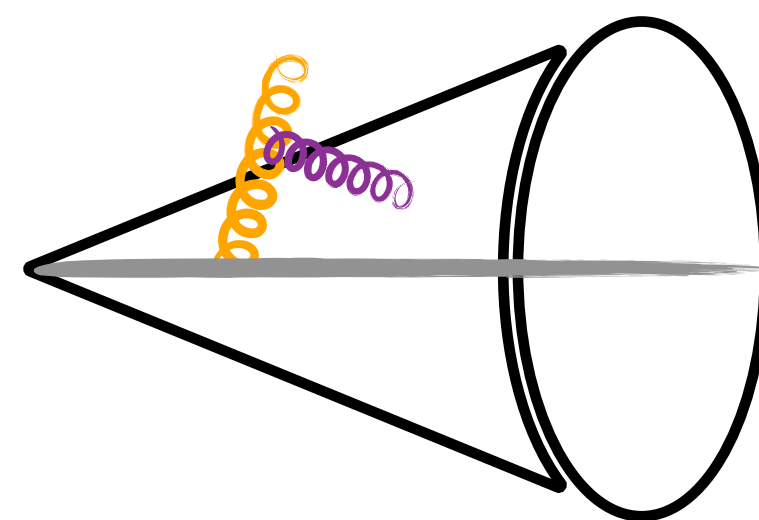
$$\rho_{\text{rc}}(\theta, k_t) = \frac{2\alpha_s^{1\ell}(k_t)C_i}{\pi}$$

- 2 Hard-collinear corrections $\alpha_s^{n+1} \ln^n \theta$

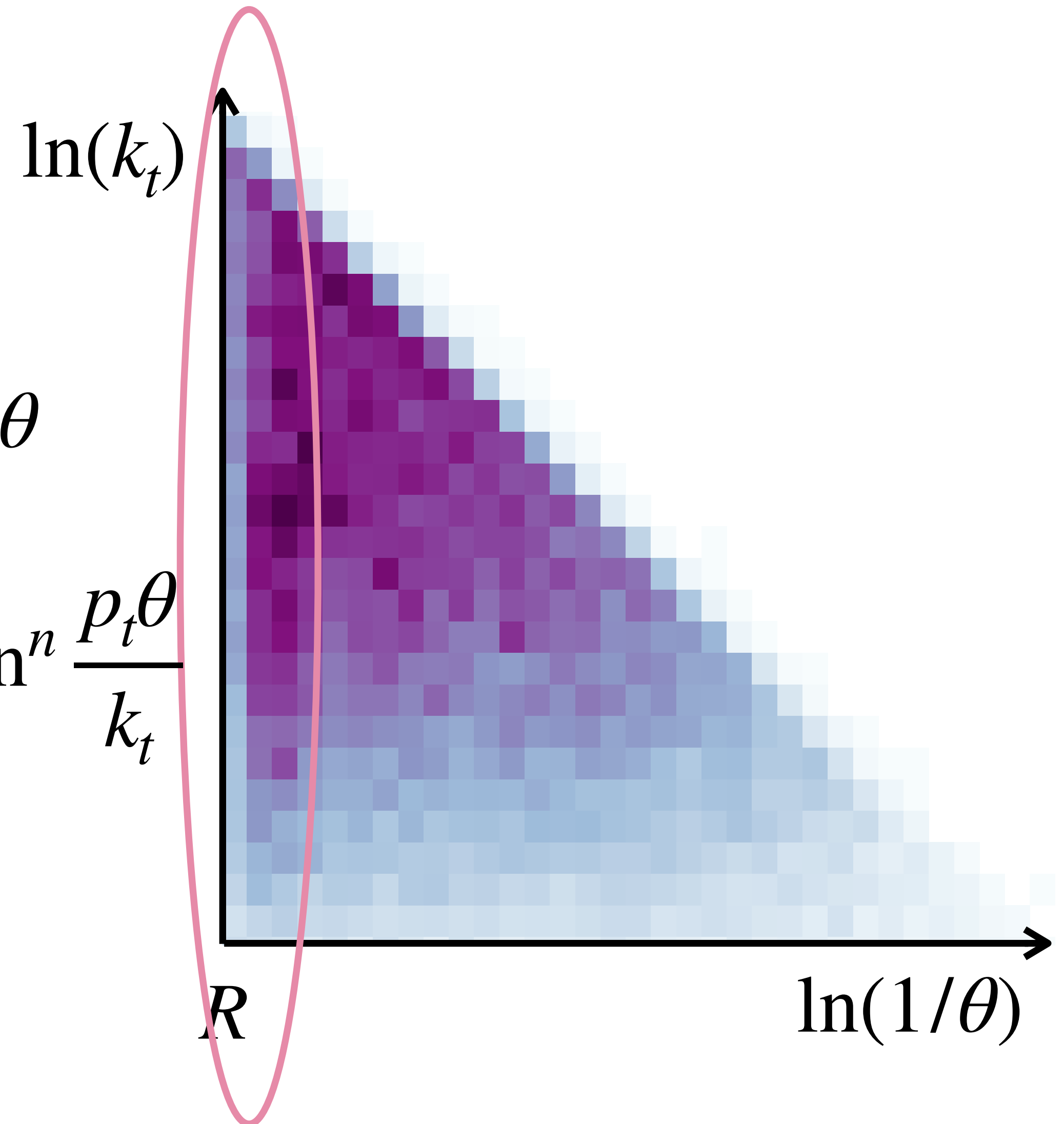
- 3 Soft, large-angle corrections $\alpha_s^{n+1} \ln^n \frac{p_t \theta}{k_t}$



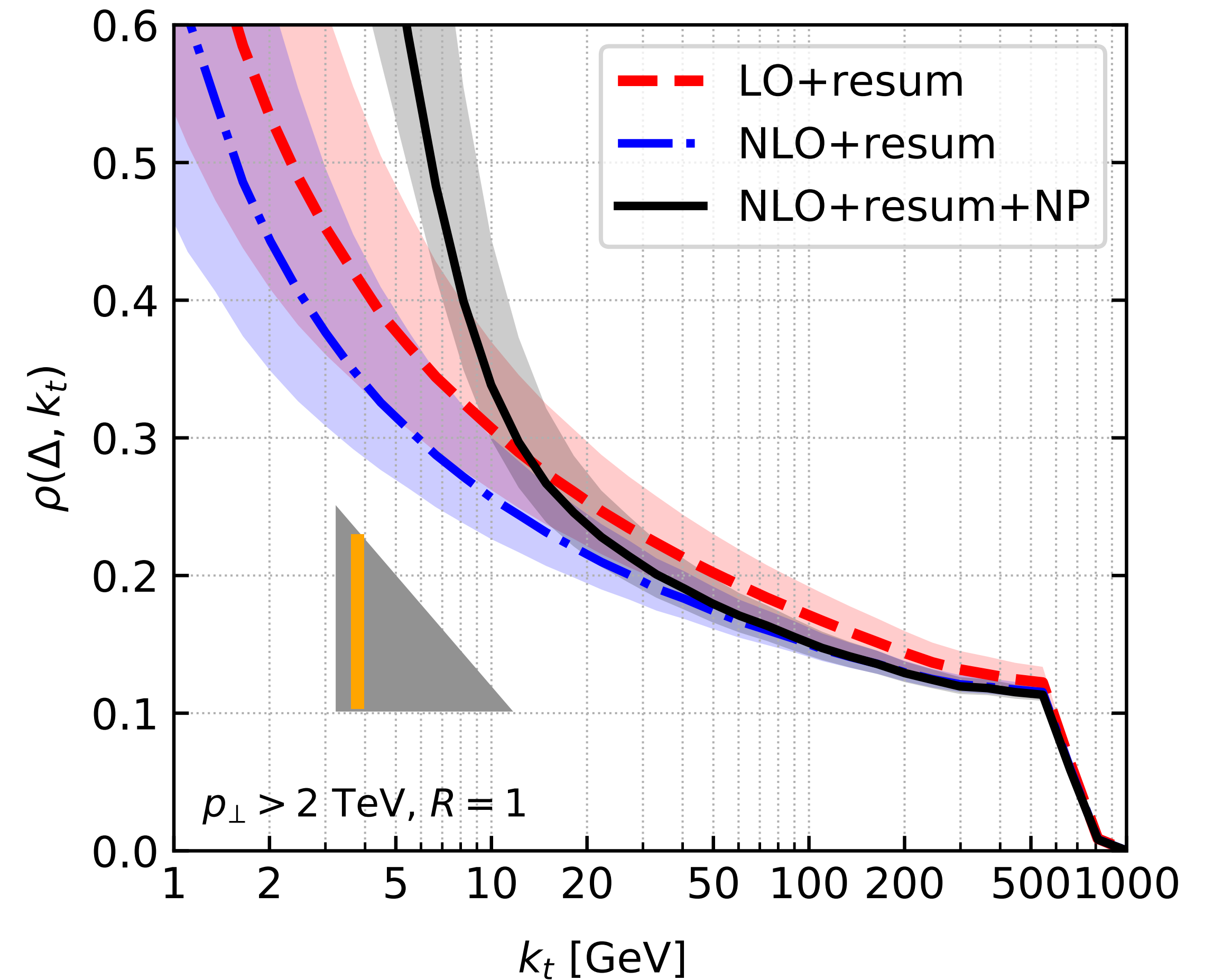
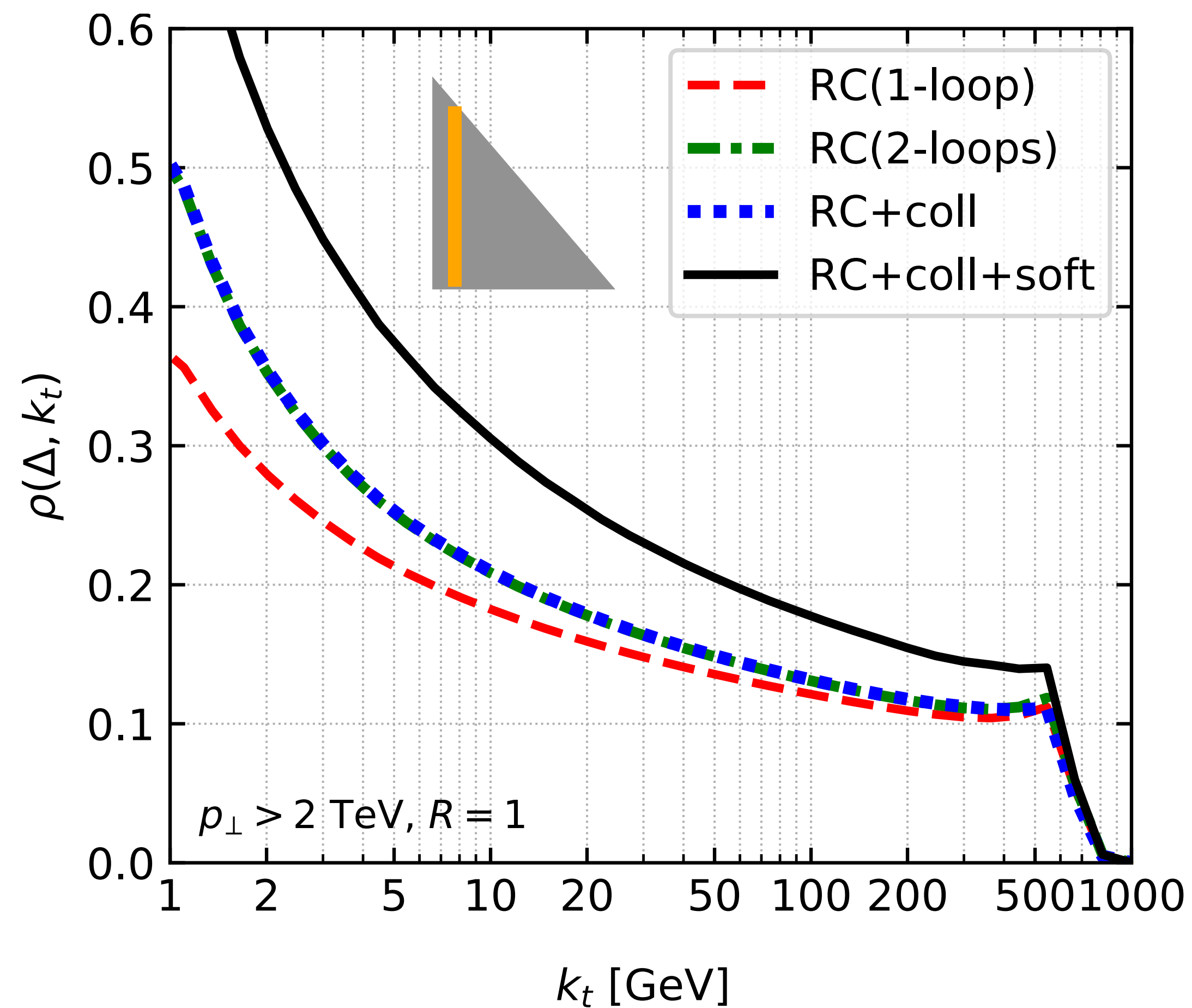
[Ellis, Marchesini, Webber.
Nucl.Phys.B 286 (1987) 643]



$\ln(R/(R - \theta))$

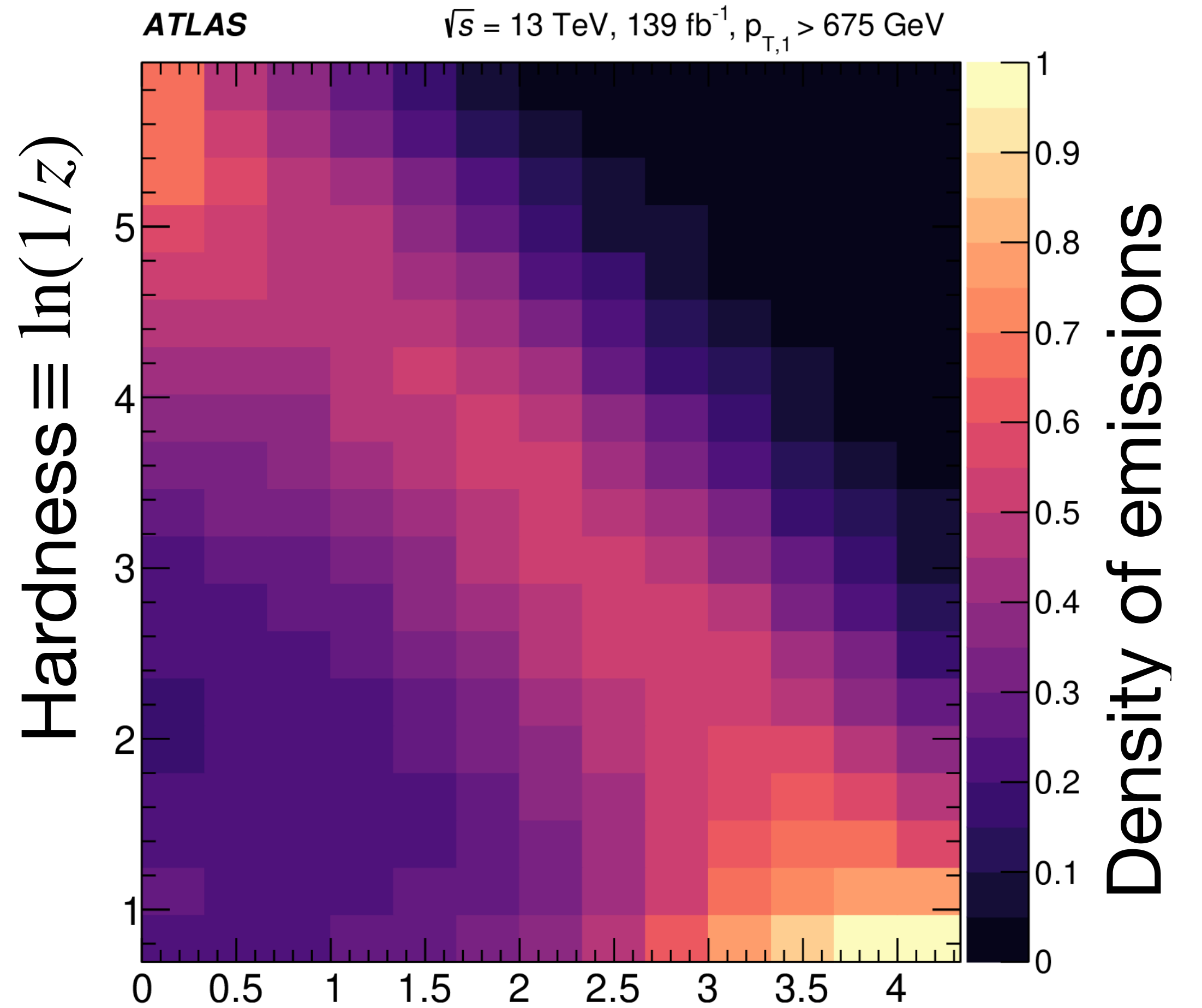


The primary Lund-plane density: NLL+NLO+NP result

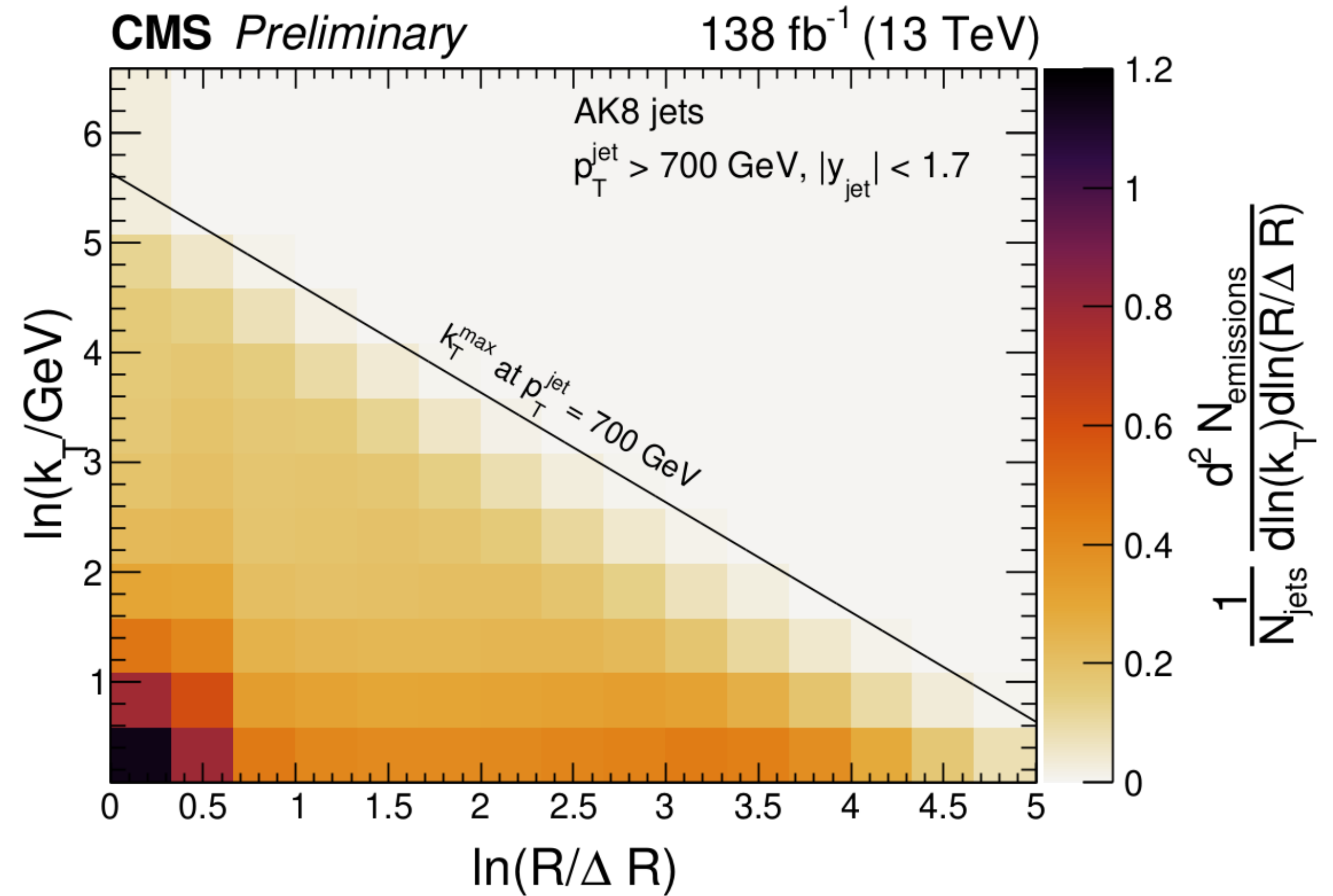


The primary Lund-plane density: measurements

[ATLAS PRL 124 (2020) 22, 222002]

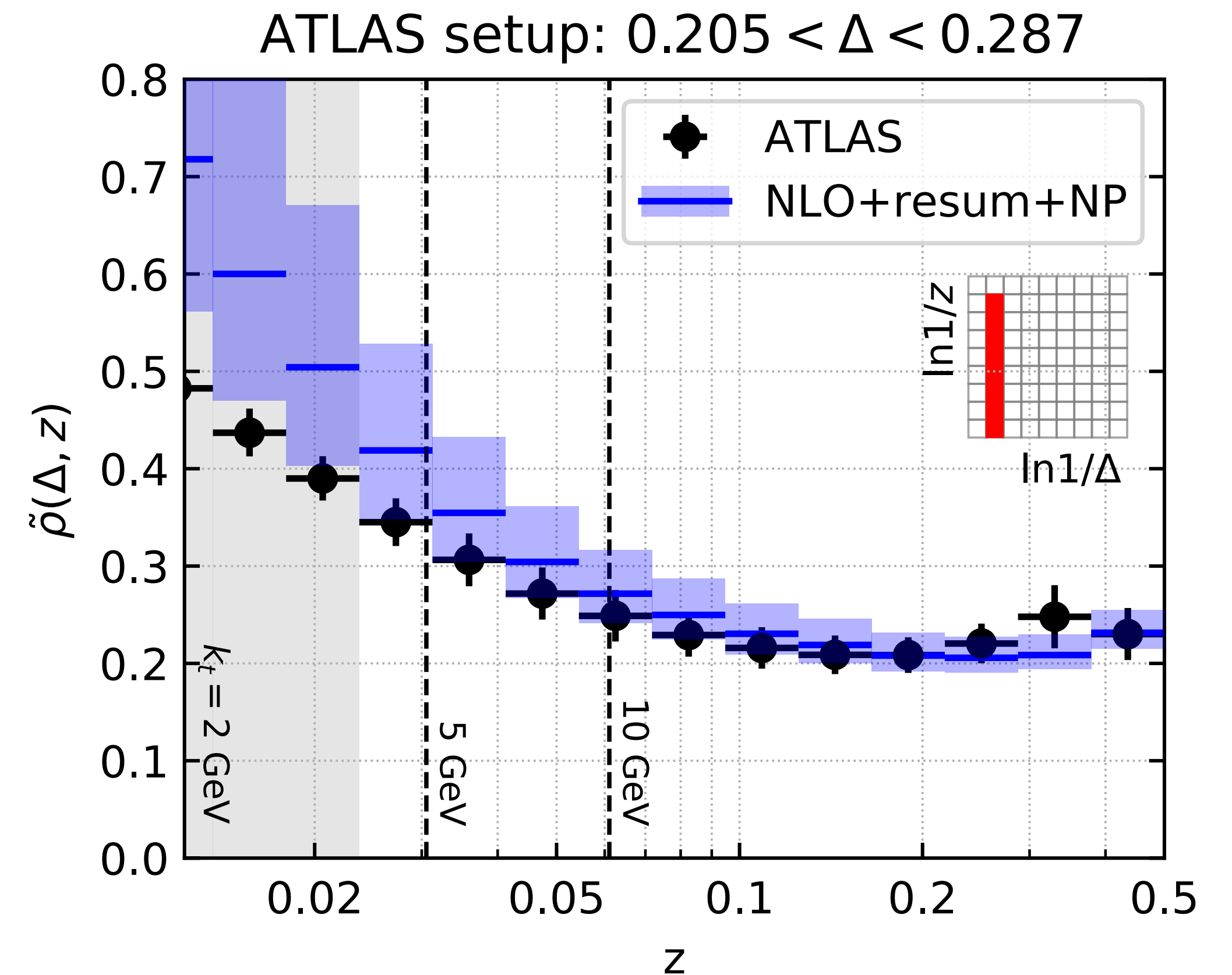
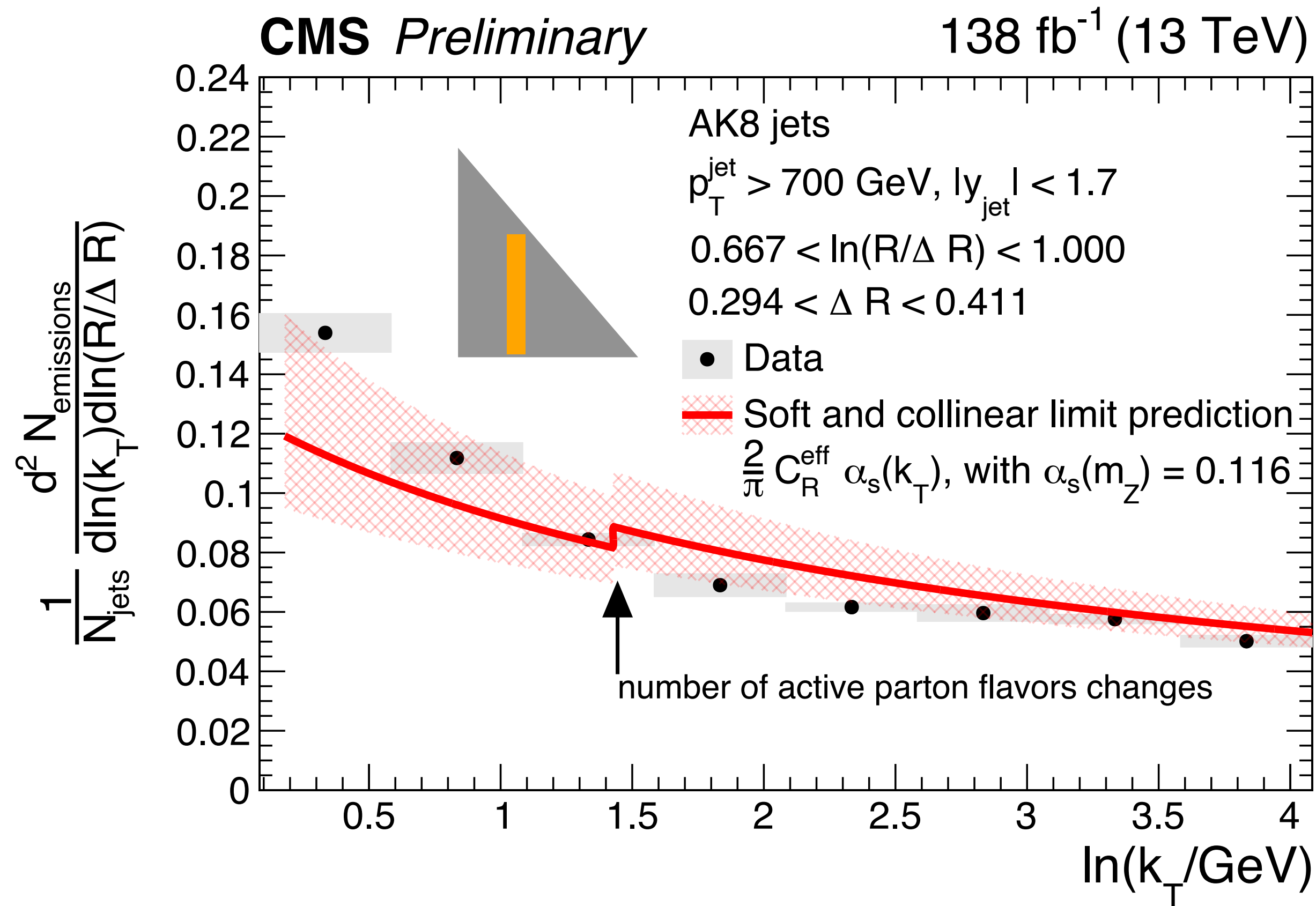


R Angular separation $\equiv \ln(R/\Delta R)$



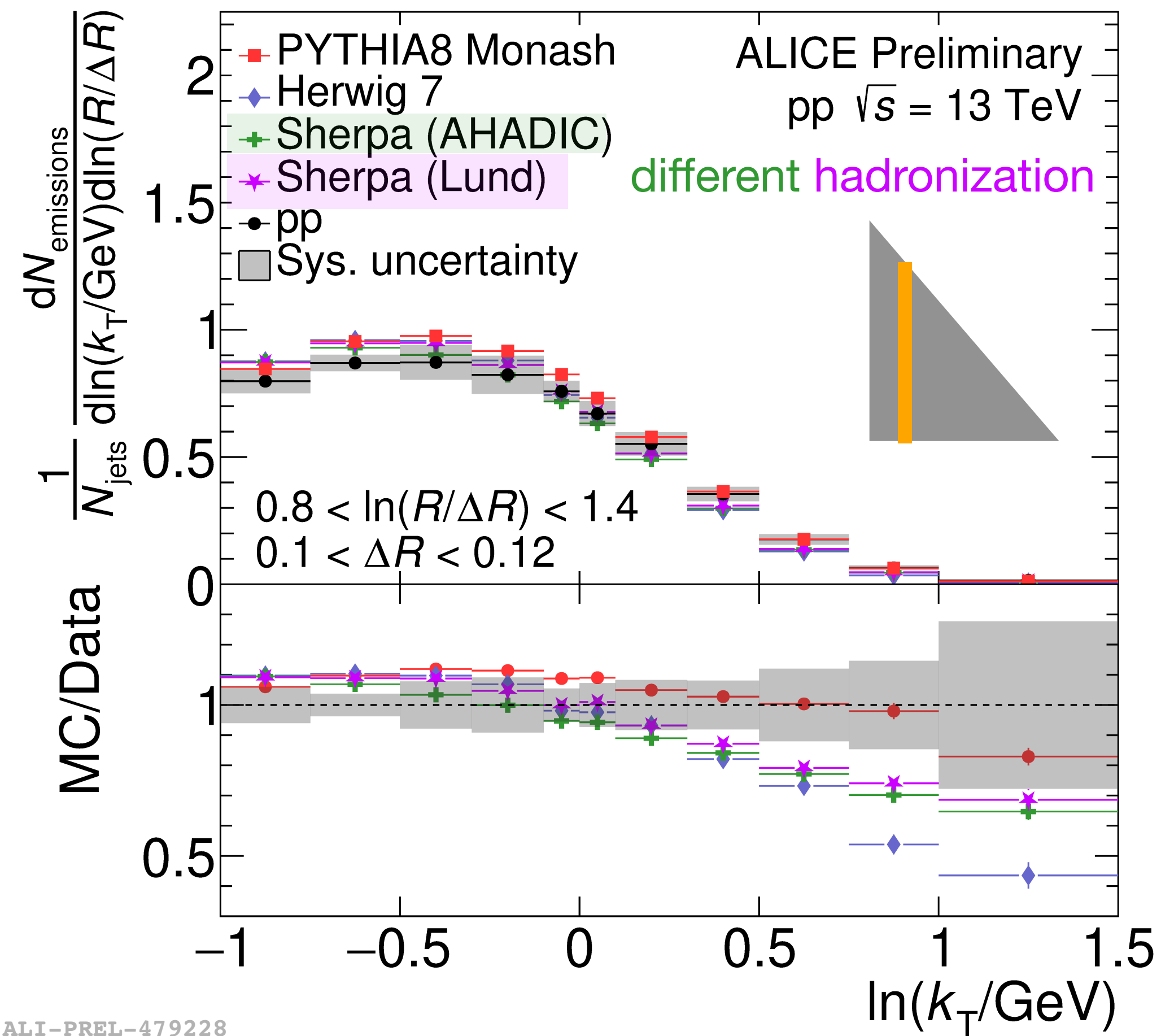
[CMS-PAS-SMP-22-007]

The primary Lund-plane density: theory-to-data

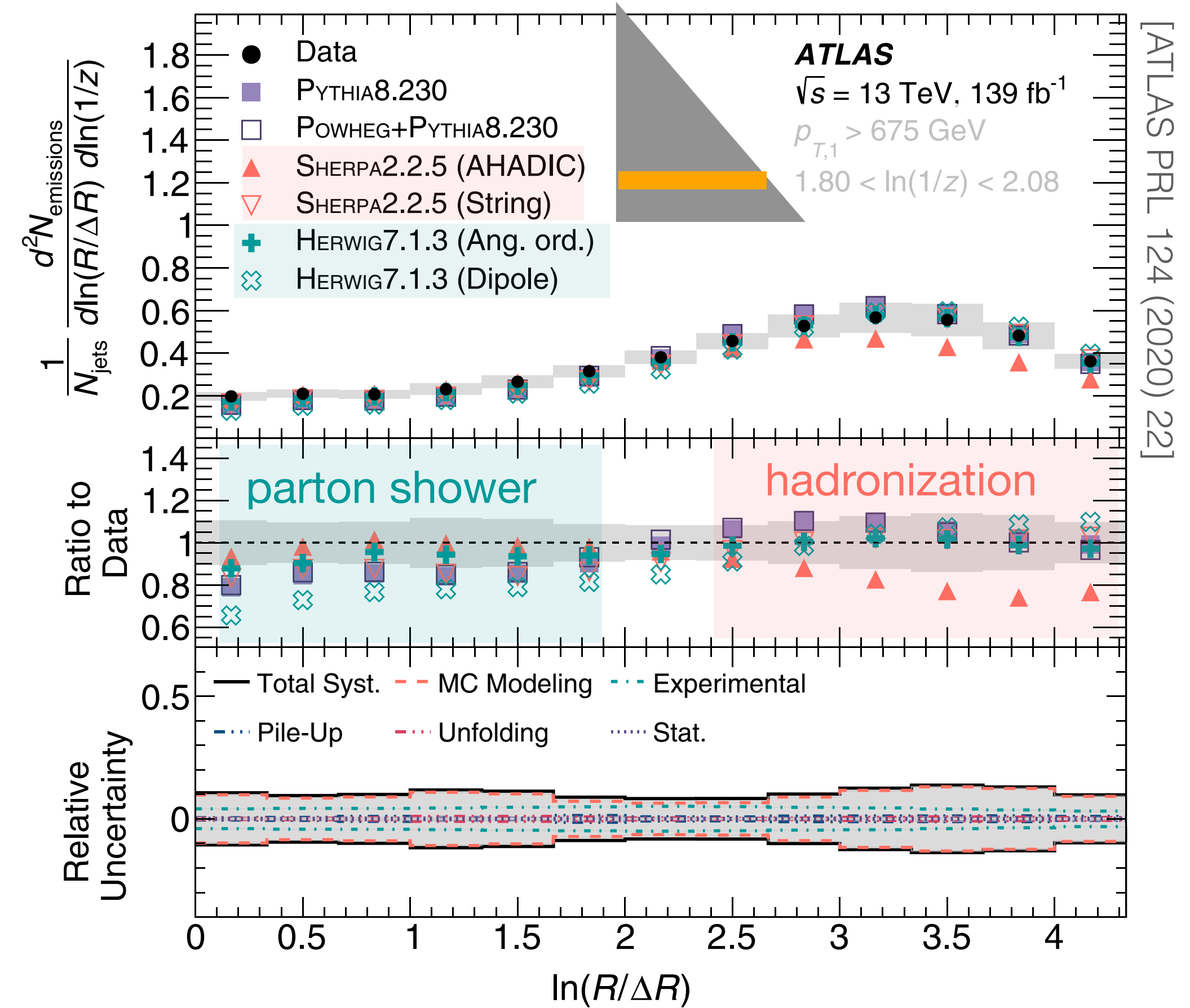


$\rho_{\text{LO}}(\theta, k_t)$ agrees with data in bulk of LP. N^kLL terms required elsewhere

The primary Lund-plane density: MC-to-data



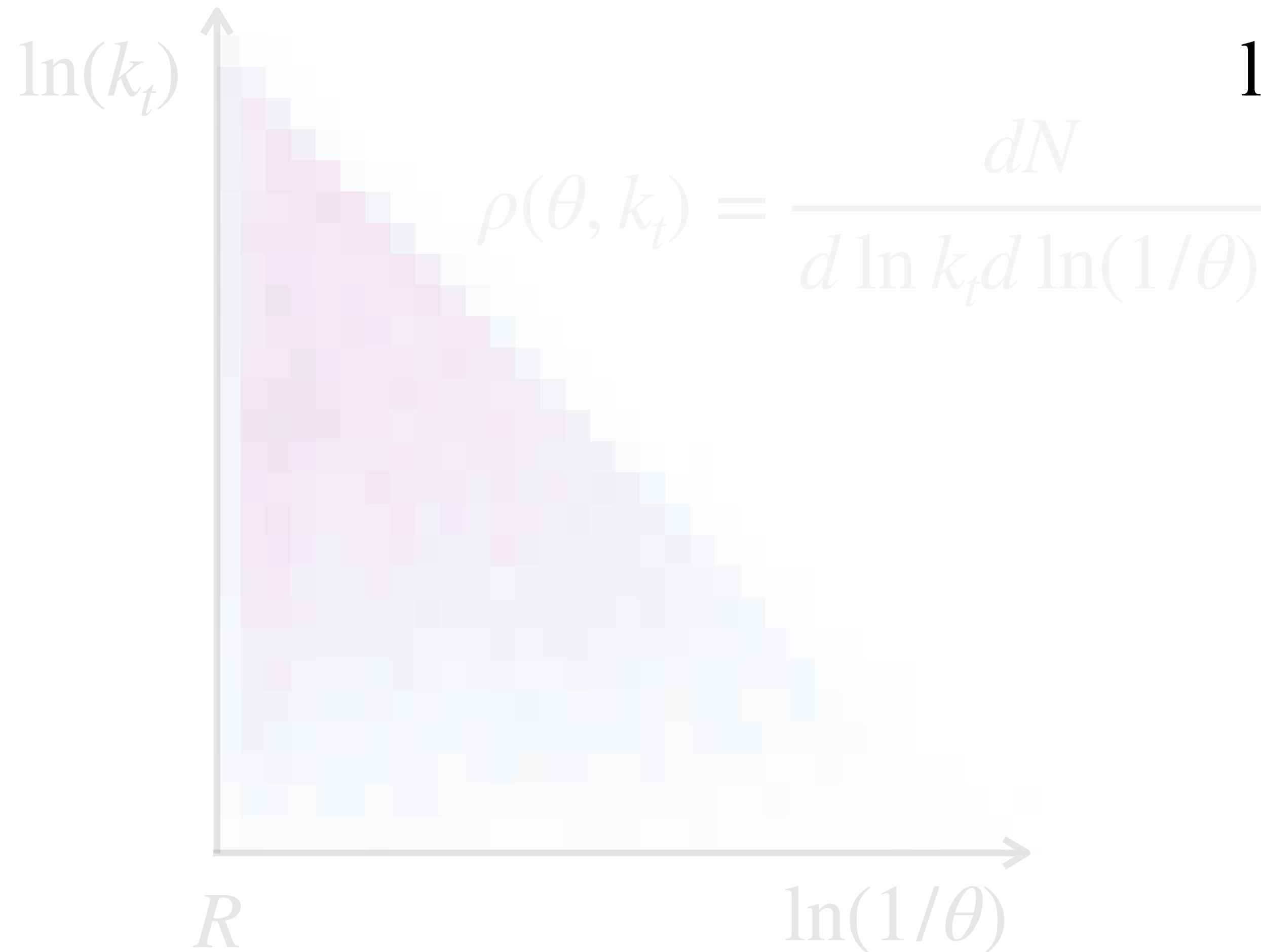
ALI-PREL-479228



Powerful tool to disentangle between different MC ingredients

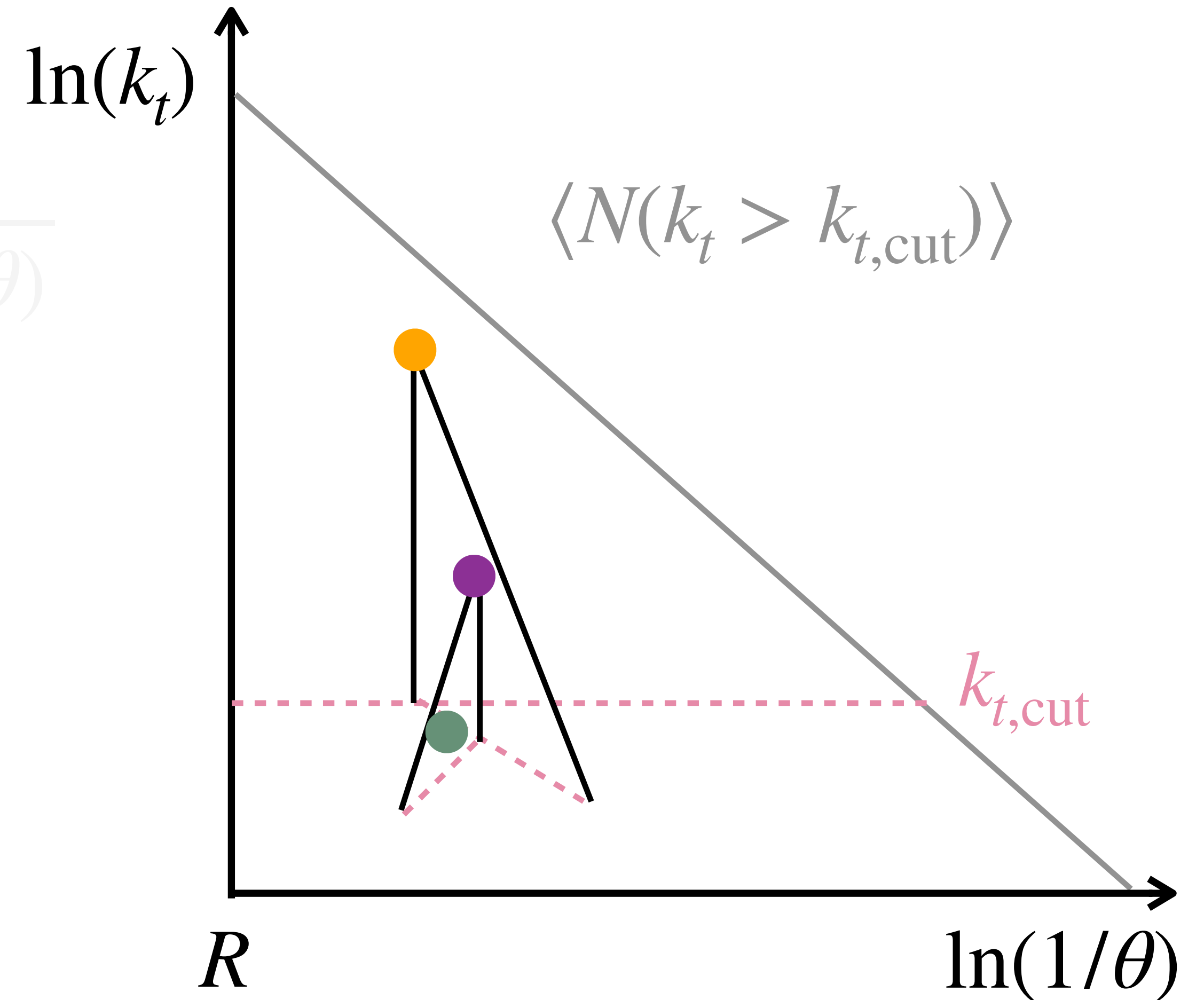
This talk: two spinoffs of the Lund jet plane

Primary Lund-plane density



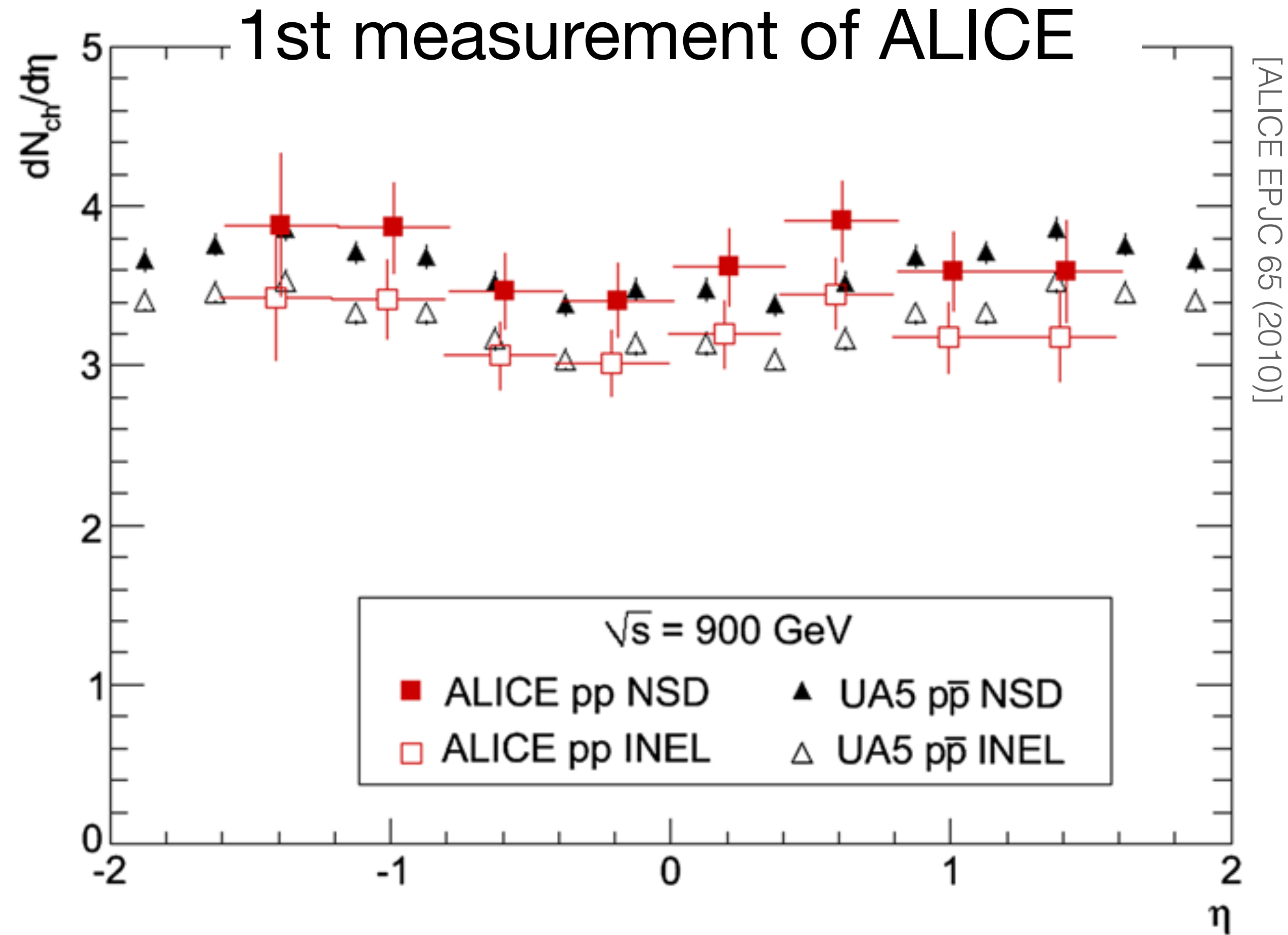
[Lifson, Salam, Soyez JHEP 10 (2020) 170]

Lund multiplicity



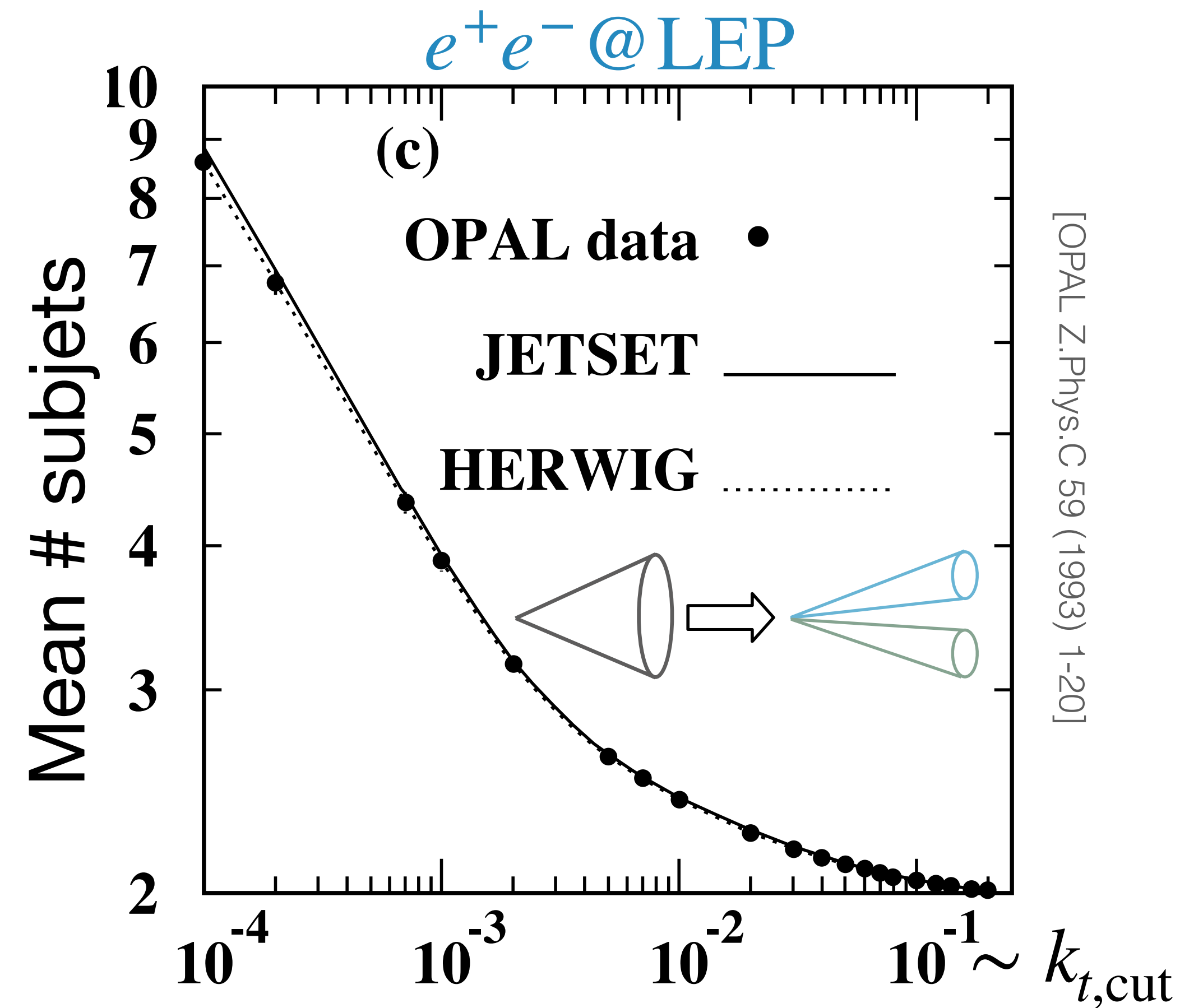
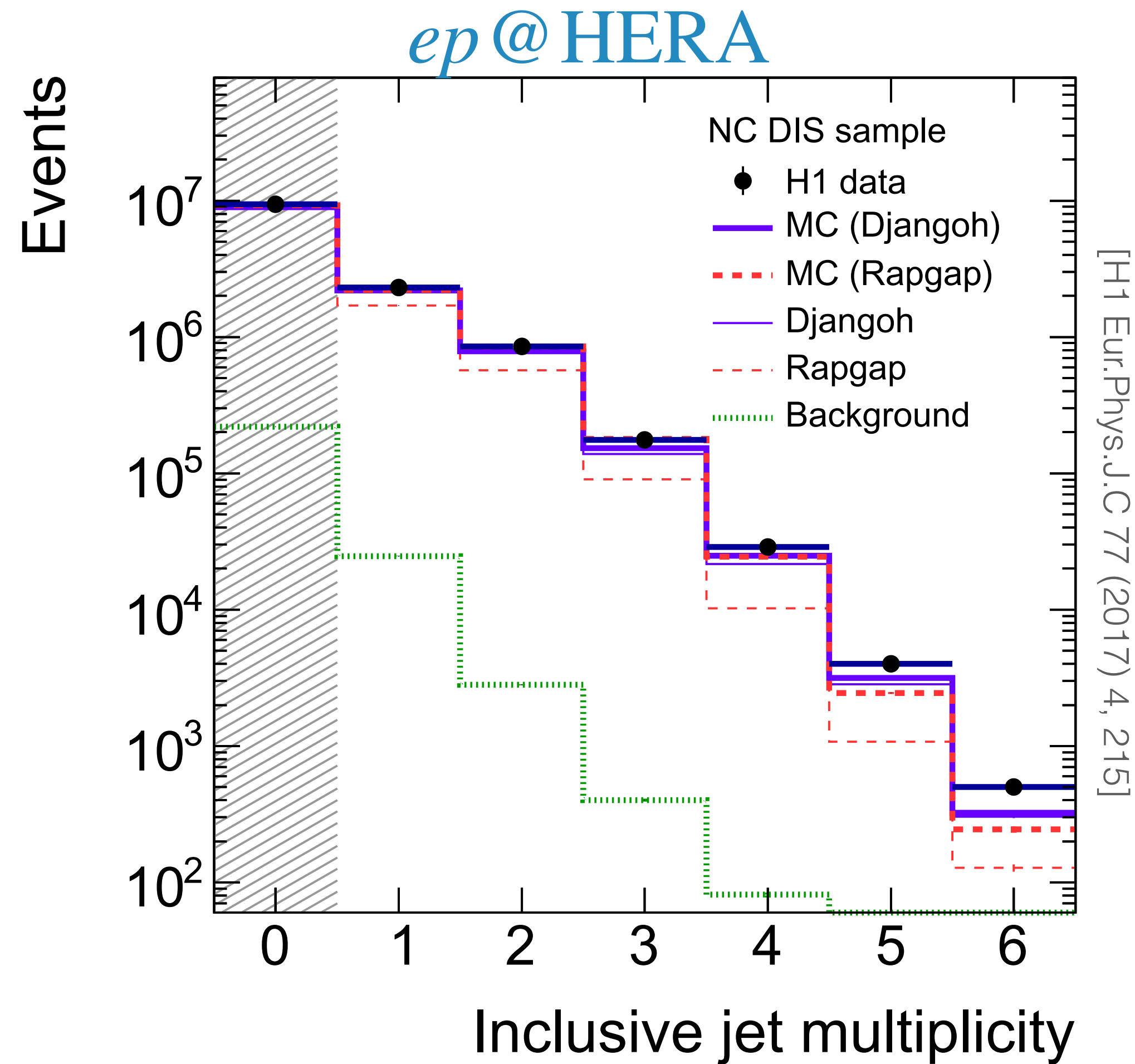
[Medves, ASO, Soyez, JHEP 10 (2022) 156,
JHEP 04 (2023) 104]

Why? One of the most studied observables in colliders



Particle multiplicities are non-perturbative objects

Why? One of the most studied observables in colliders



(Sub)jet multiplicities are calculable in pQCD $N(\alpha_s, L \equiv \ln(Q/k_{t,cut}))$

Analytic structure of the average subjet multiplicity

The perturbative expansion of the average subjet multiplicity reads

$$\langle N(\alpha_s; L) \rangle = \left[\underbrace{h_1(\alpha_s L^2)}_{\text{DL}} + \underbrace{\sqrt{\alpha_s} h_2(\alpha_s L^2)}_{\text{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\text{NNDL} \sim 10\%} + \dots \right]$$

$\alpha_s \ll 1$
 $\alpha_s L^2 \sim 1$

where N^k DL accuracy implies control over $\alpha_s^n L^{2n-k}$ terms with $0 < n < \infty$

Goal: NNDL calculation of average subjet multiplicity

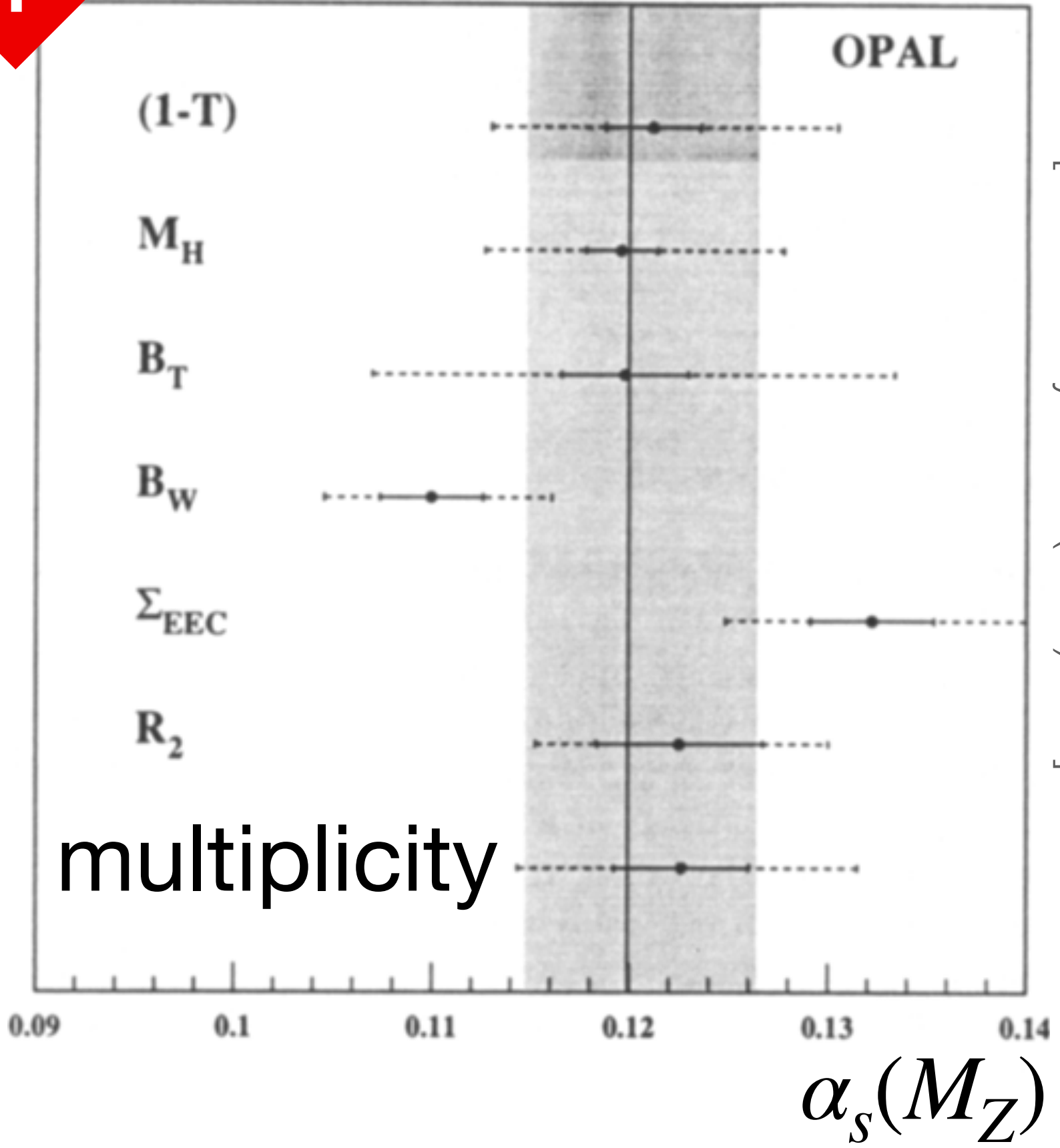
[Medves, ASO, Soyez, JHEP 10 (2022) 156, JHEP 04 (2023) 104]

[NDL: Catani, Dokshitzer, Fiorani, Webber NPB 377 (1992) 445-460]

Motivations to push for higher accuracy

1

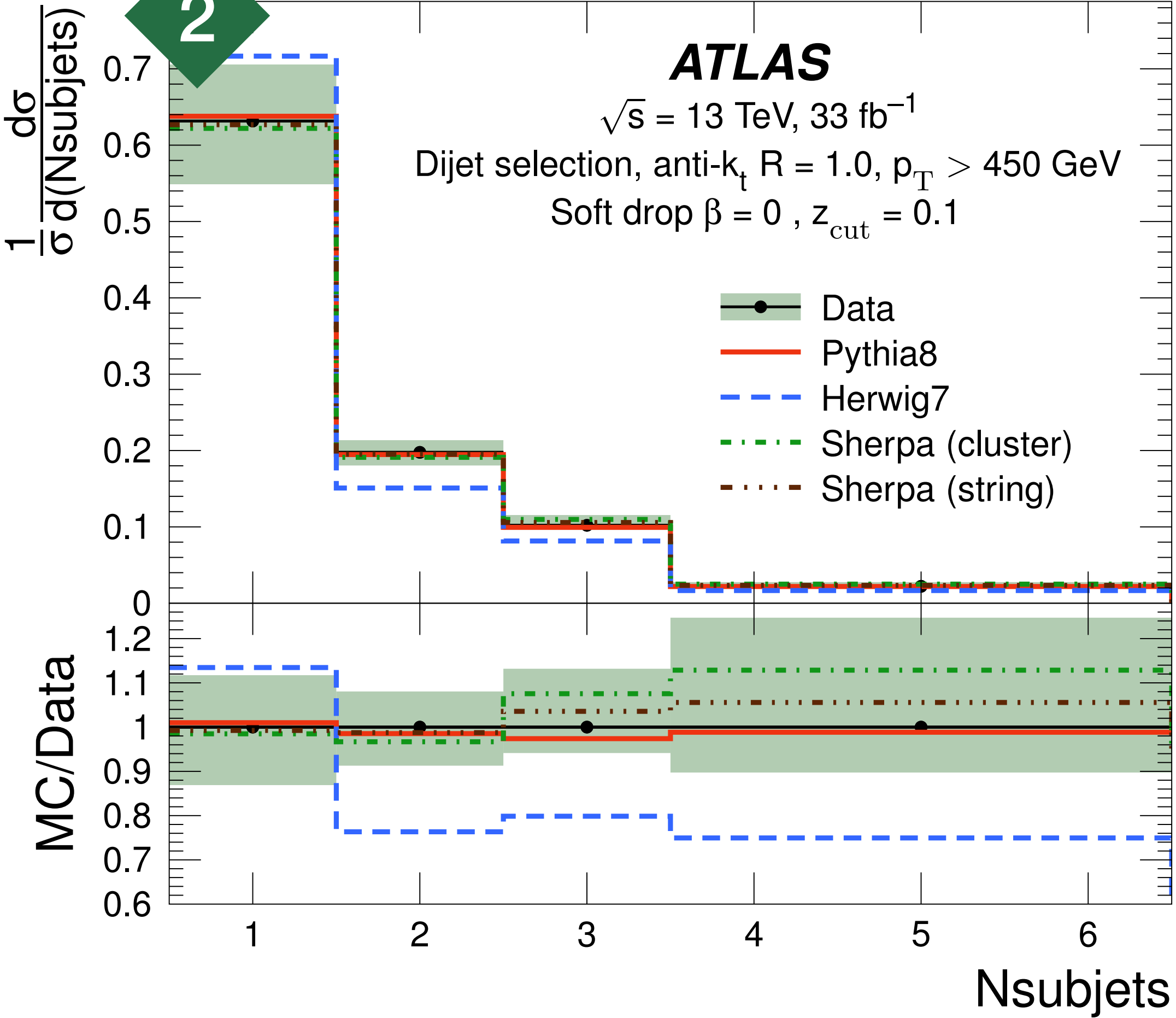
e^+e^- @ LEP



[Catani, Dokshitzer, Fiorani, Webber NPB 377 (1992) 445-460]

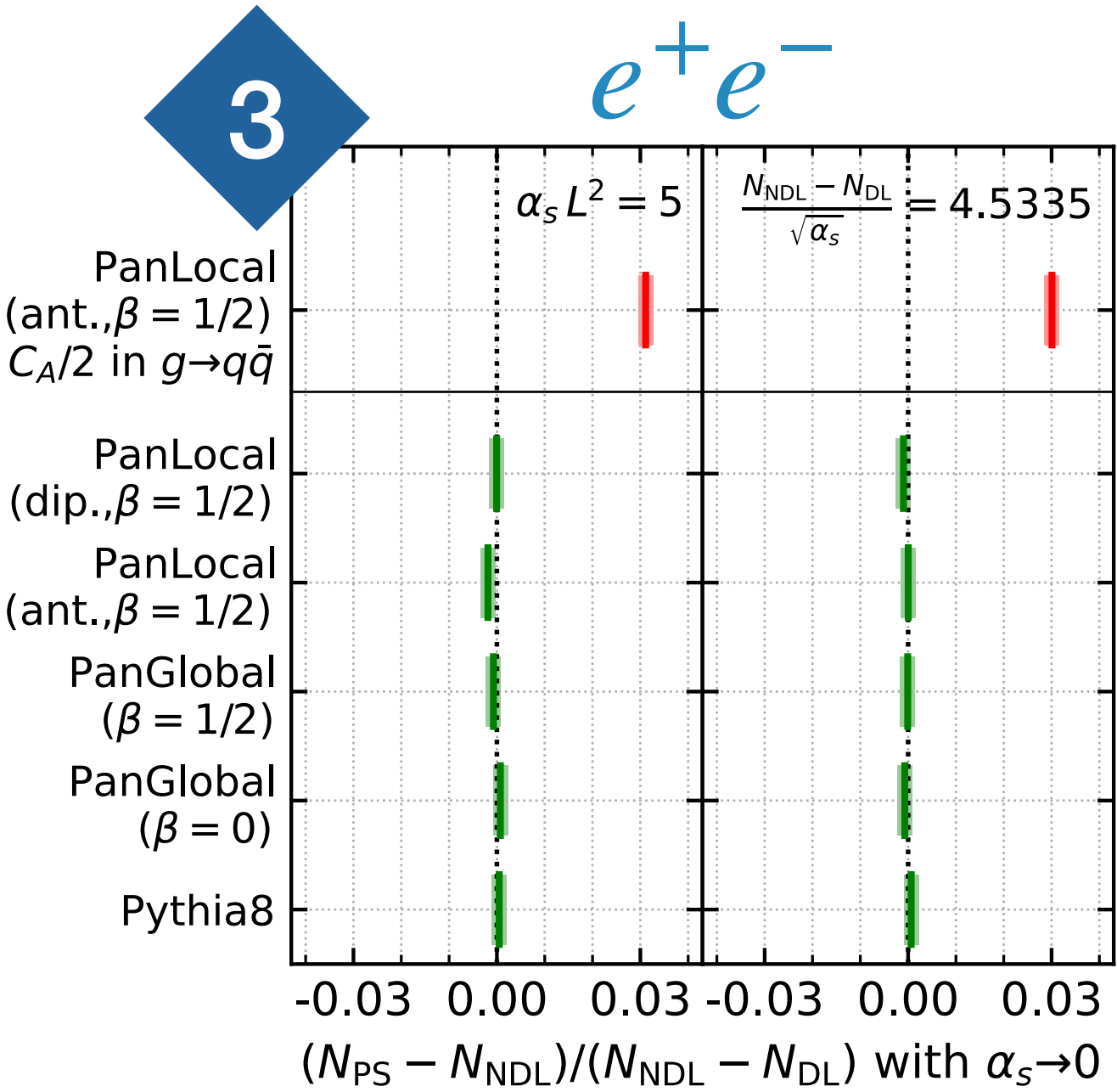
$\alpha_s(M_Z)$ extractions at e^+e^- colliders

2

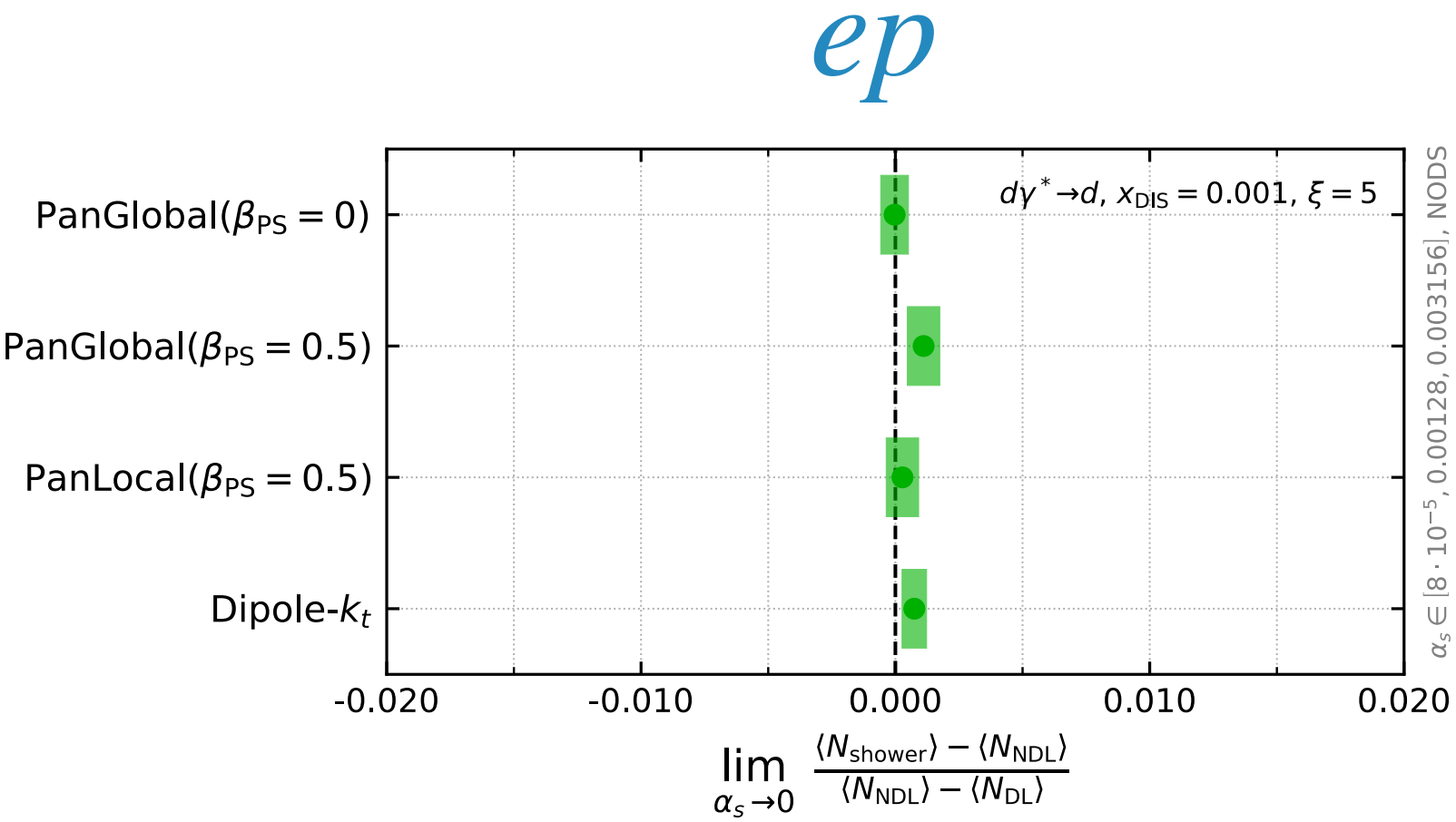


Calibrate MC generators

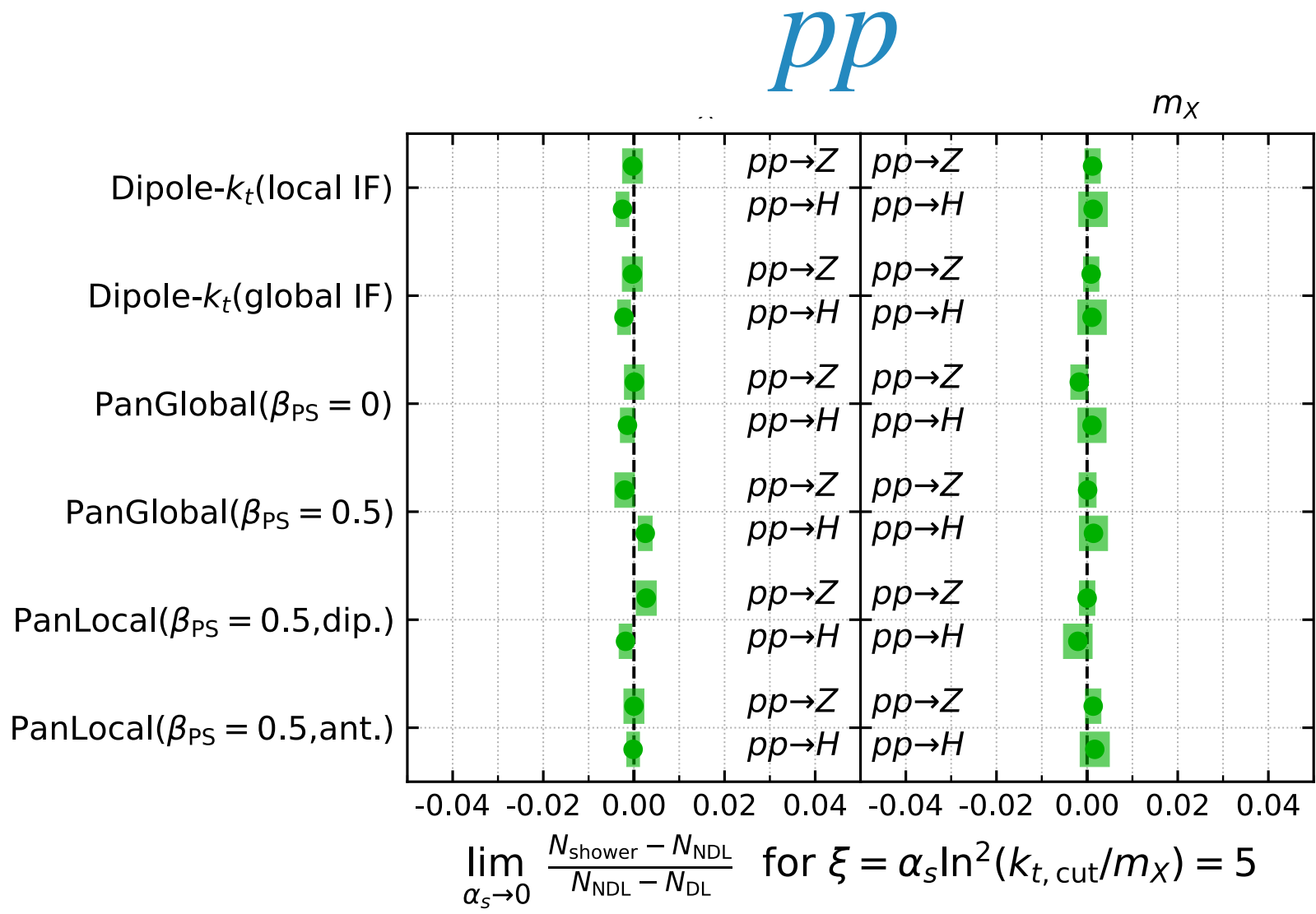
Motivations to push for higher accuracy



[Hamilton et al. JHEP 03 (2021) 041]



[van Beekveld, Ferrario Ravasio arXiv: 2305.08645]



[van Beekveld, ASO et al. JHEP 11 (2022) 020]

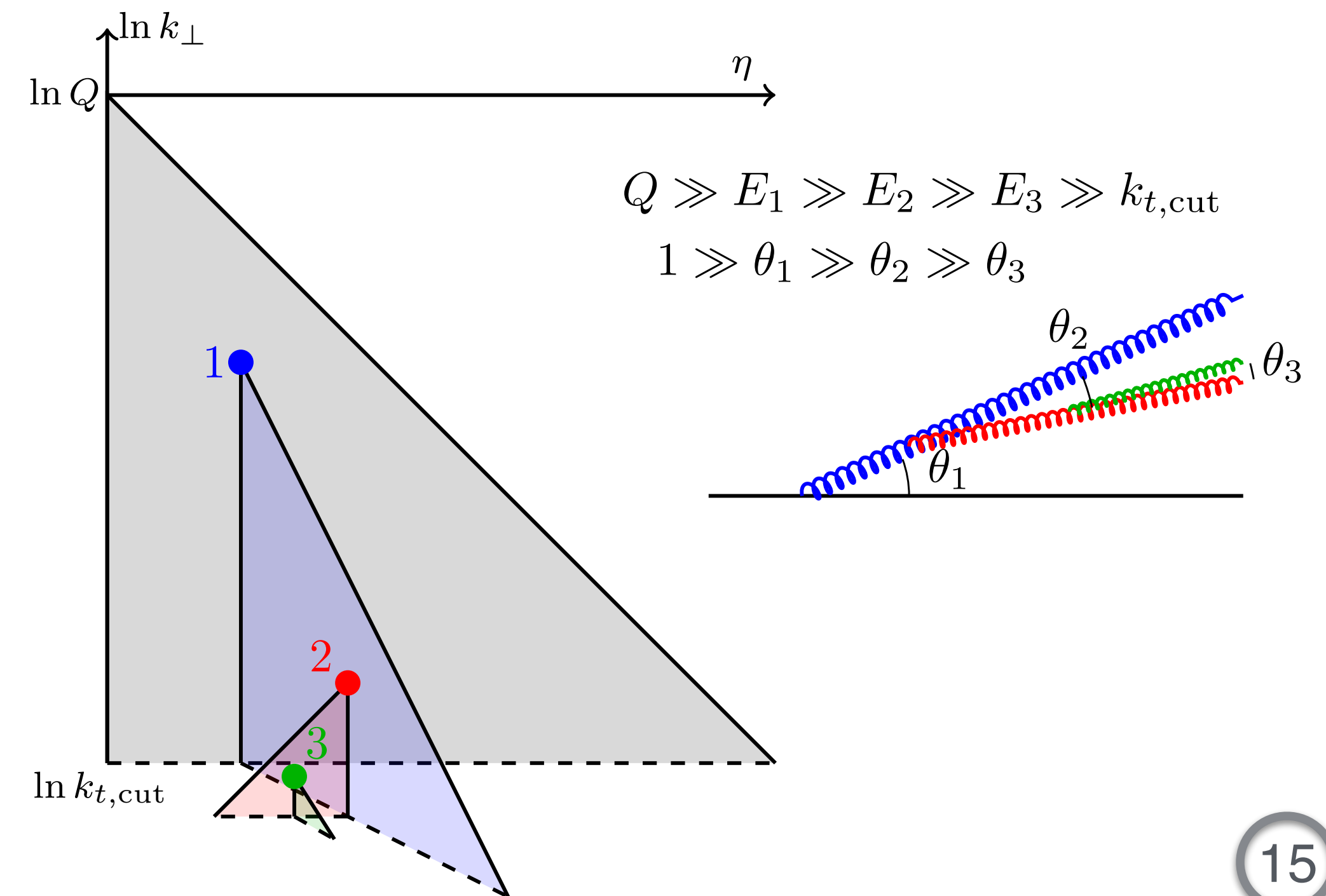
Test parton showers with improved logarithmic accuracy

Lund multiplicity: DL resummation $(\alpha_s L^2)^n$

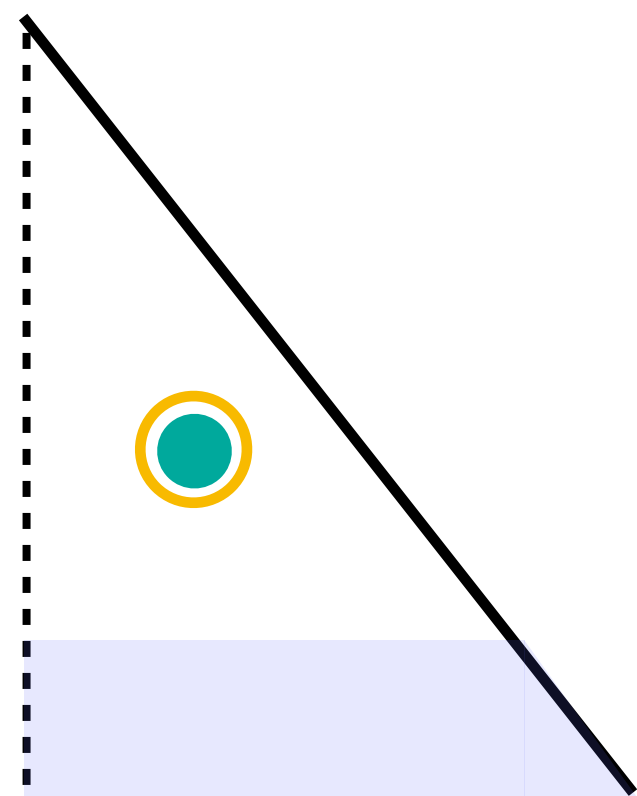
$$\langle N \rangle_{\text{DL}} = 1 + \frac{C_i}{C_A} \sum_{n=1}^{\infty} \bar{\alpha}^n \underbrace{\int_0^{\infty} d\eta_1 \int_{\eta_1}^{\infty} d\eta_2 \dots \int_{\eta_{n-1}}^{\infty} d\eta_n}_{\text{angular-ordering}} \underbrace{\int_0^1 \frac{dx_1}{x_1} \int_0^{x_1} \frac{dx_2}{x_2} \dots \int_0^{x_{n-1}} \frac{dx_n}{x_n}}_{\text{energy-ordering}} \underbrace{\Theta(x_n e^{-\eta_n} > e^{-L})}_{k_t > k_{t,\text{cut}}}$$

$$\langle N \rangle_{\text{DL}} = 1 + \frac{C_i}{C_A} \left[\cosh \nu - 1 \right]$$

$$\nu = \sqrt{2\alpha_s C_A L^2 / \pi}$$



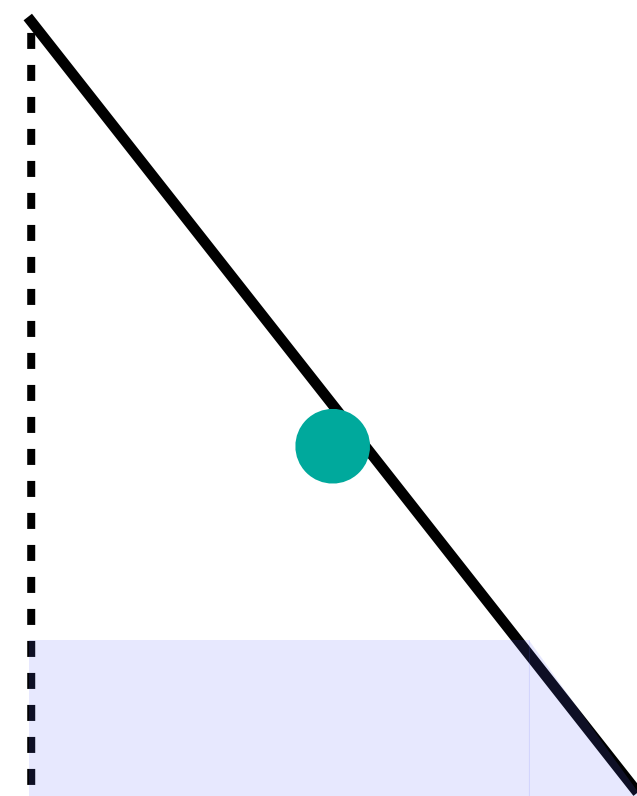
Lund multiplicity: NDL resummation $\alpha_s L (\alpha_s L^2)^n$



Running coupling

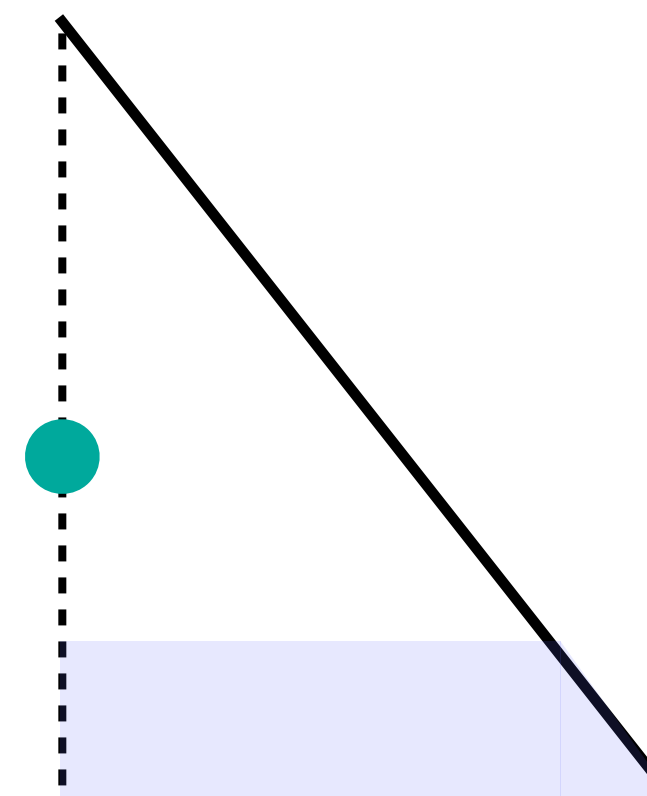
$$\alpha_s \rightarrow \alpha_s - 2\alpha_s^2 \beta_0 \ell + \mathcal{O}(\alpha_s^3)$$

with $\ell \equiv \ln(k_t/Q)$



Hard-collinear

$$\frac{1}{z} \rightarrow C_F \left(\frac{1-z}{z} + \frac{z}{2} \right)$$

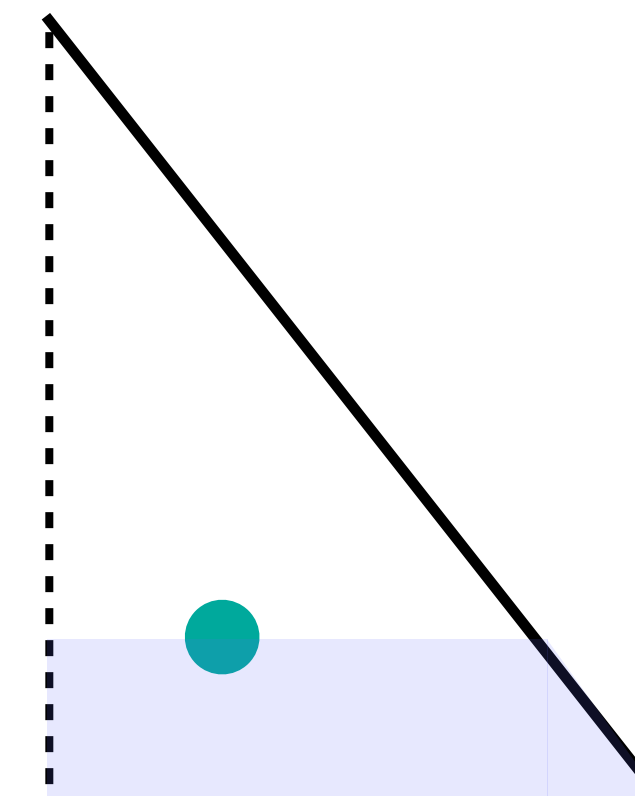


Large-angle

$$\frac{dz}{z} d\eta$$

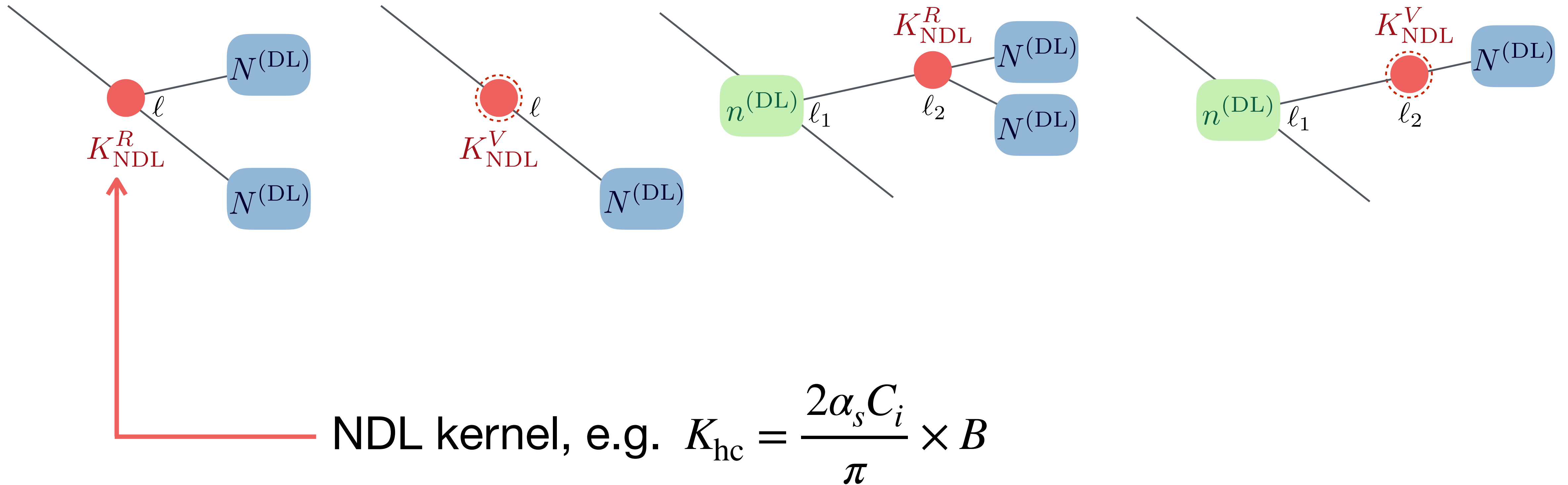


$$\frac{p_i \cdot p_j}{p_i \cdot p_k p_j \cdot p_k} \frac{d\phi}{2\pi} d \cos \theta$$



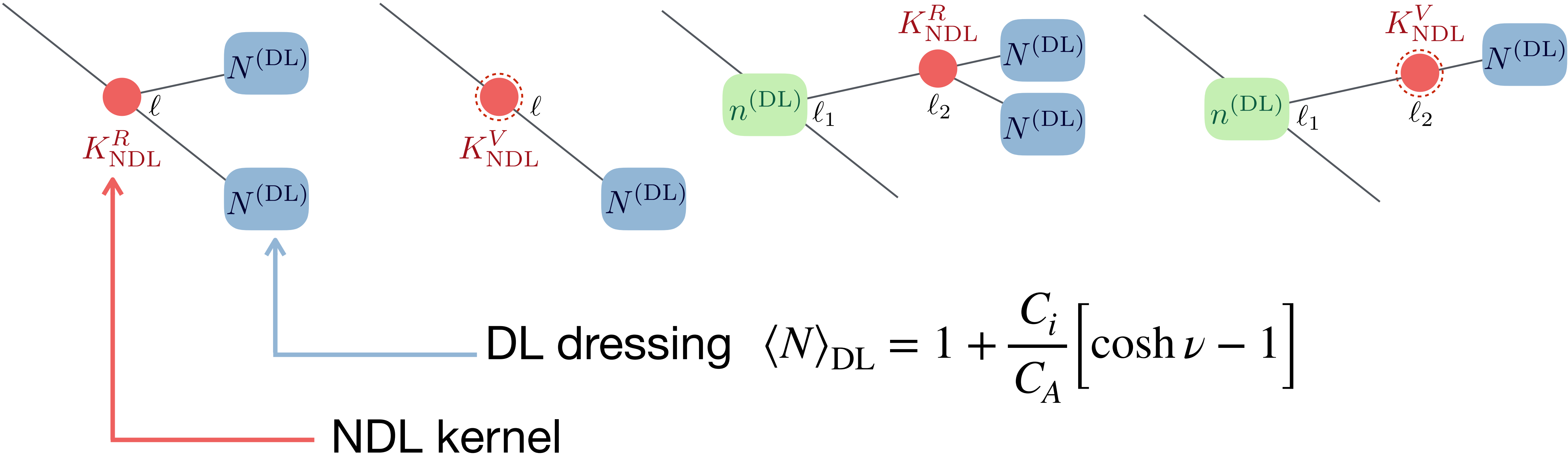
$k_t \sim k_{t,\text{cut}}$

Lund multiplicity: NDL resummation strategy

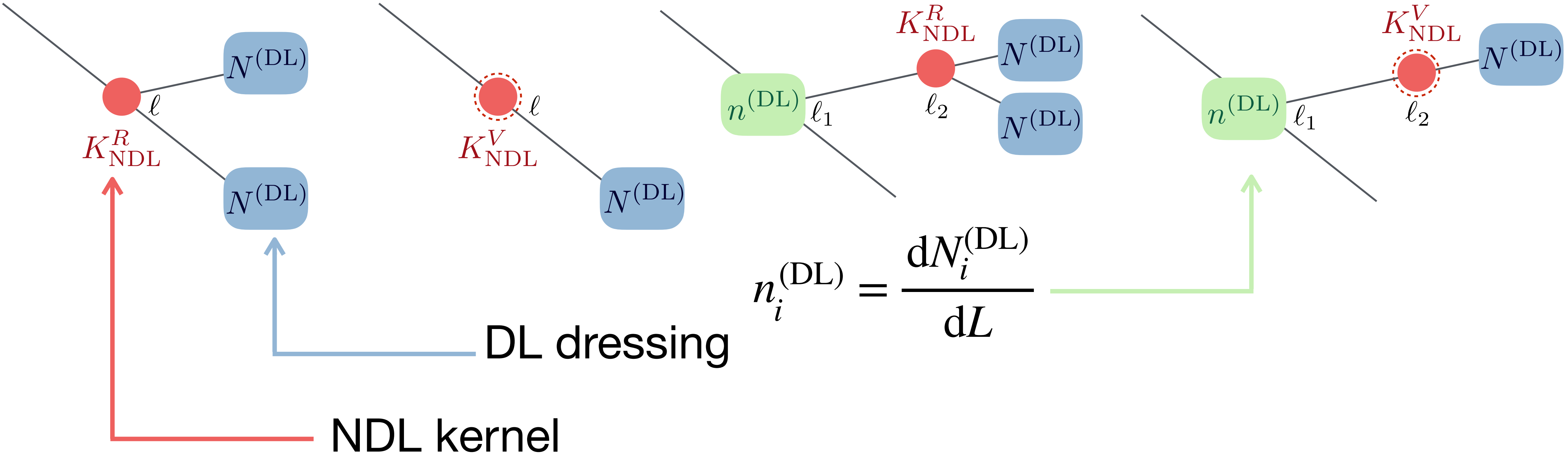


$$B = \int_0^1 \left(P(z) - \frac{1}{z} \right) dz$$

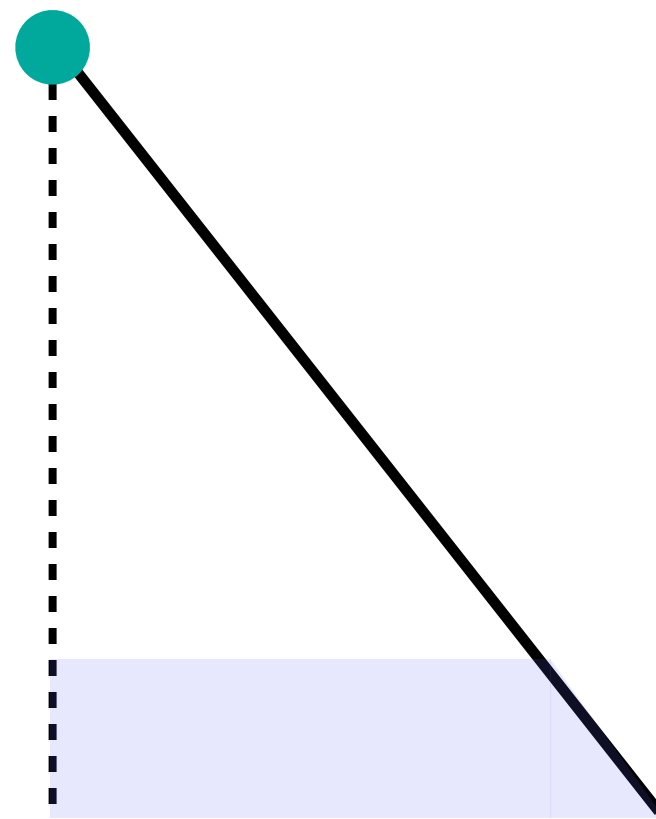
Lund multiplicity: NDL resummation strategy



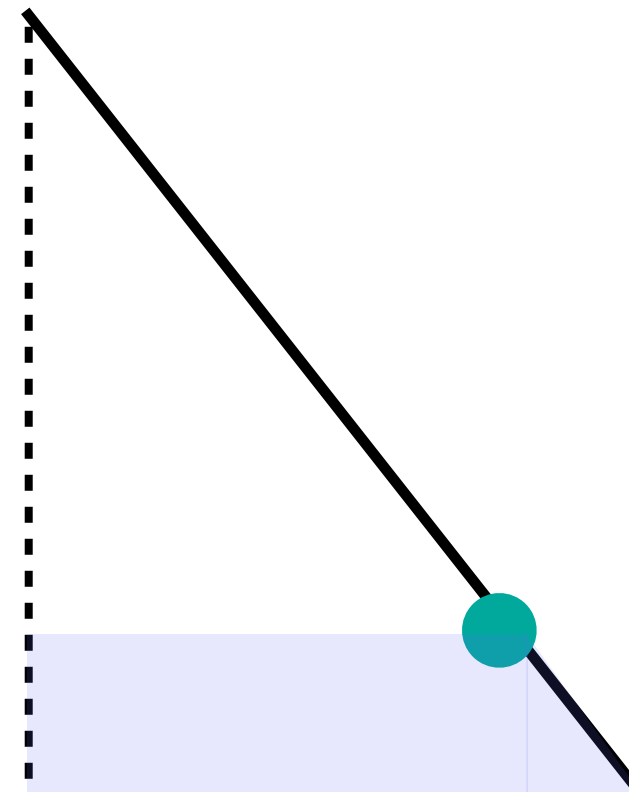
Lund multiplicity: NDL resummation strategy



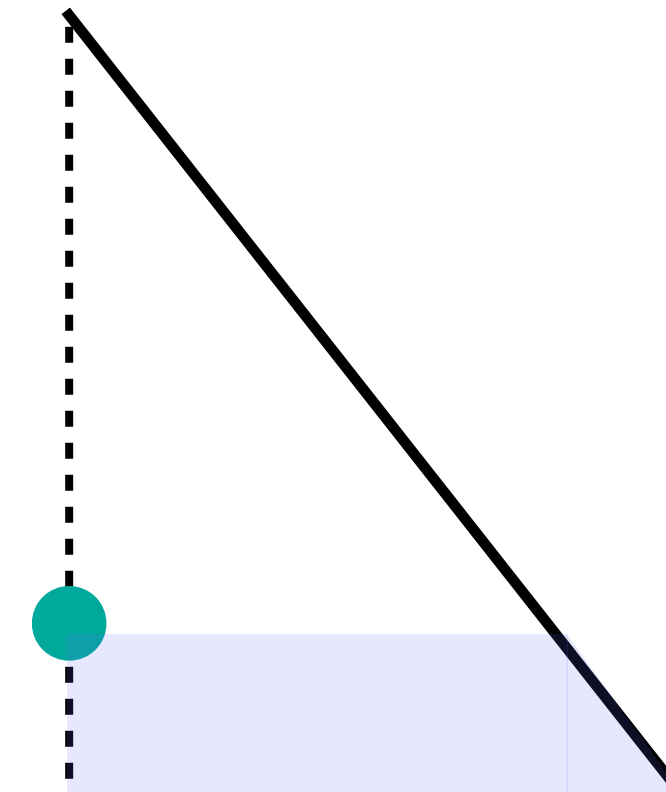
Lund multiplicity: NN DL resummation $\alpha_s(\alpha_s L^2)^n$



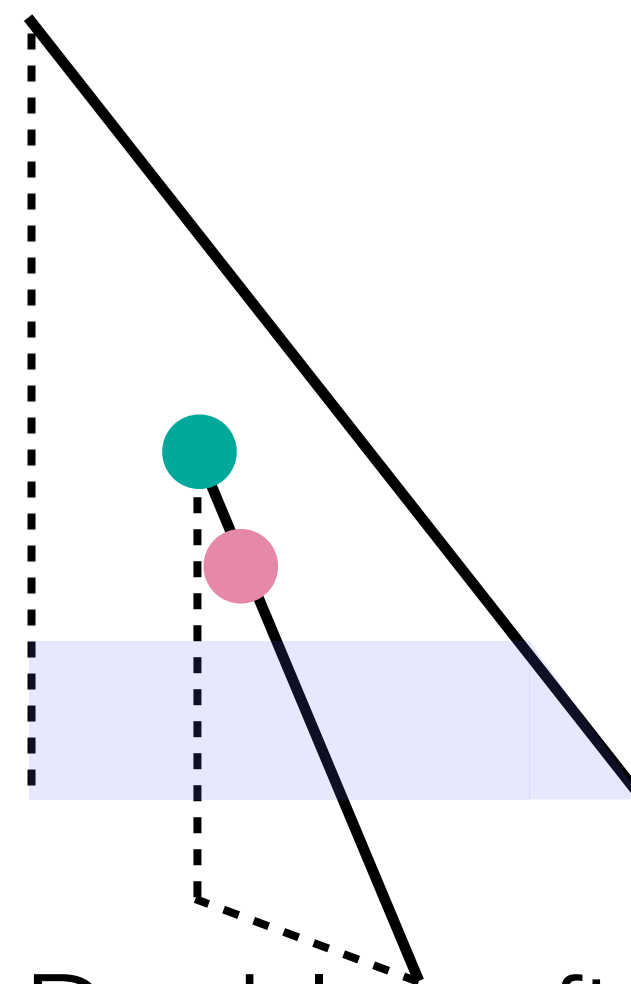
Hard ME



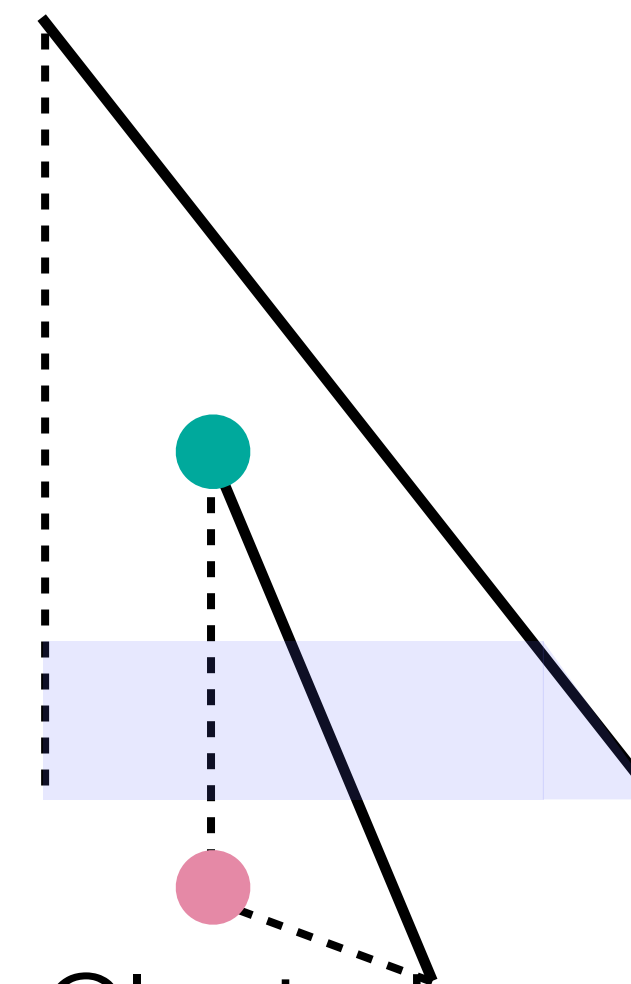
Collinear
end-point



Large angle
+ $k_t \sim k_{t,\text{cut}}$

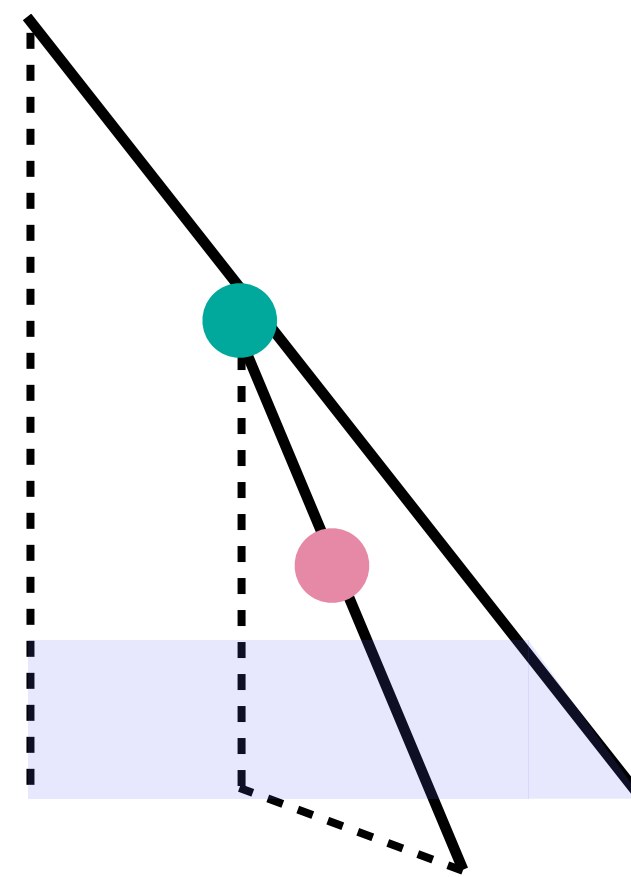


Double soft

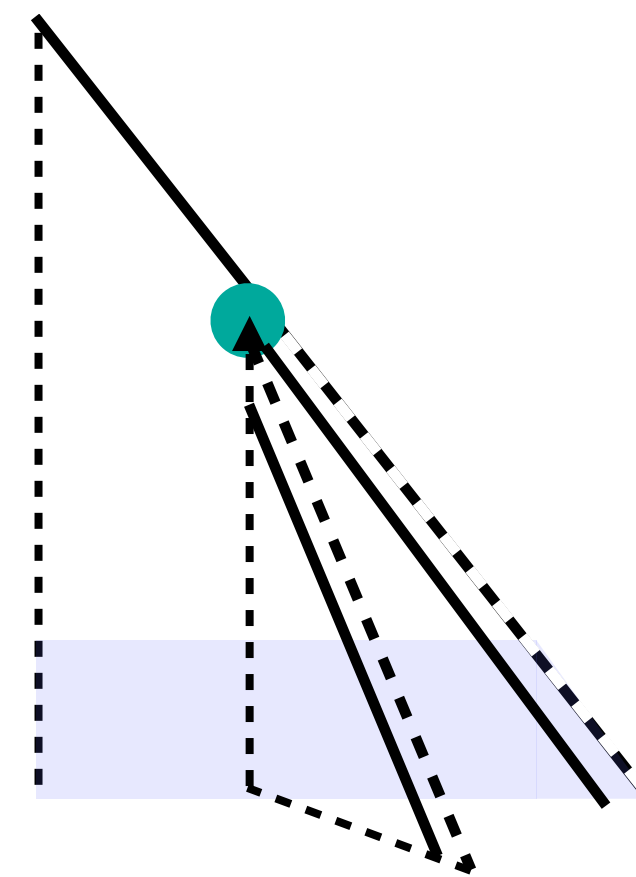


Clustering

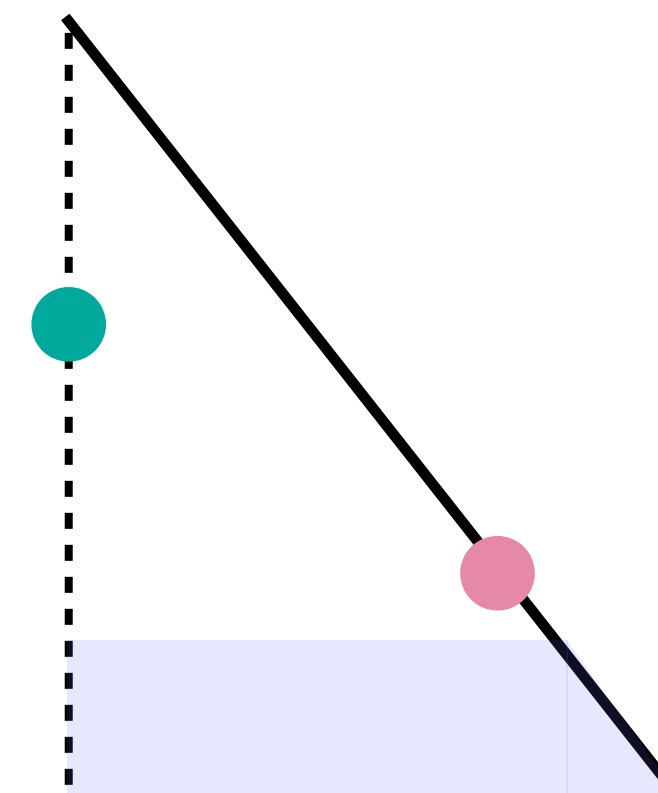
Lund multiplicity: NN DL resummation $(\alpha_s L)(\alpha_s L)(\alpha_s L^2)^n$



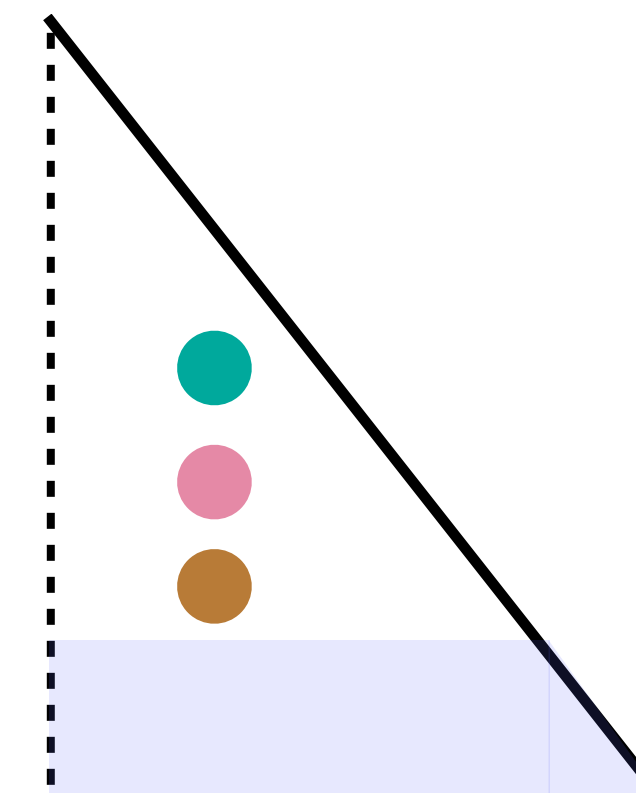
2 hard collinear



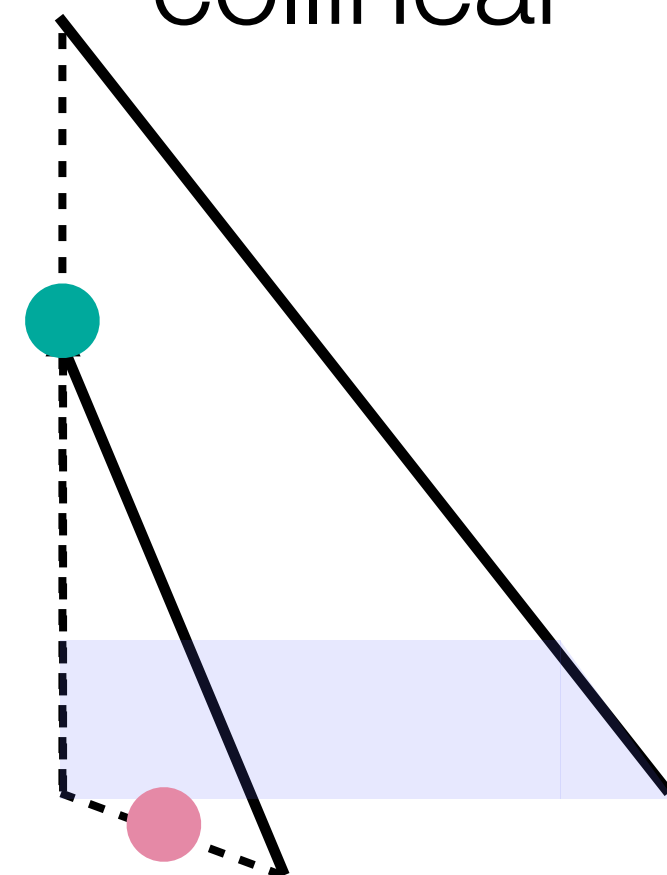
Energy loss



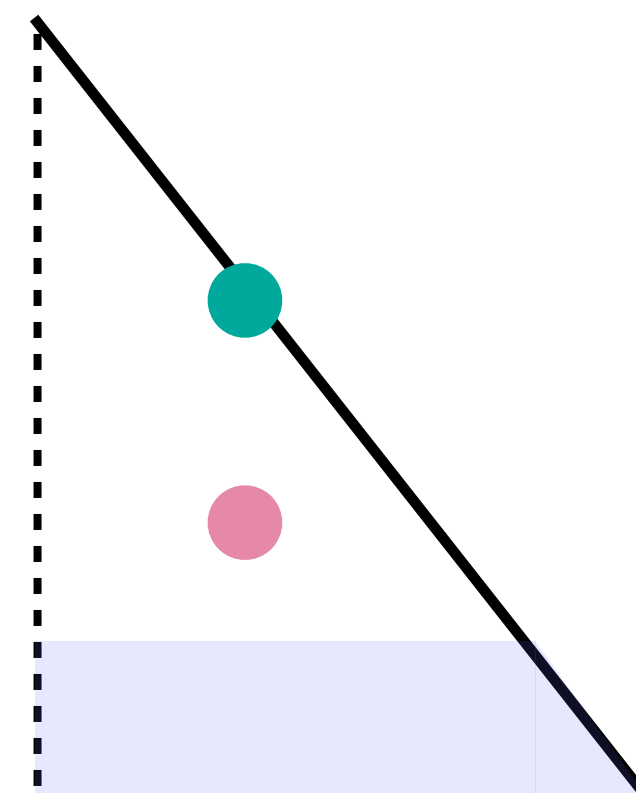
Large angle + hard-collinear



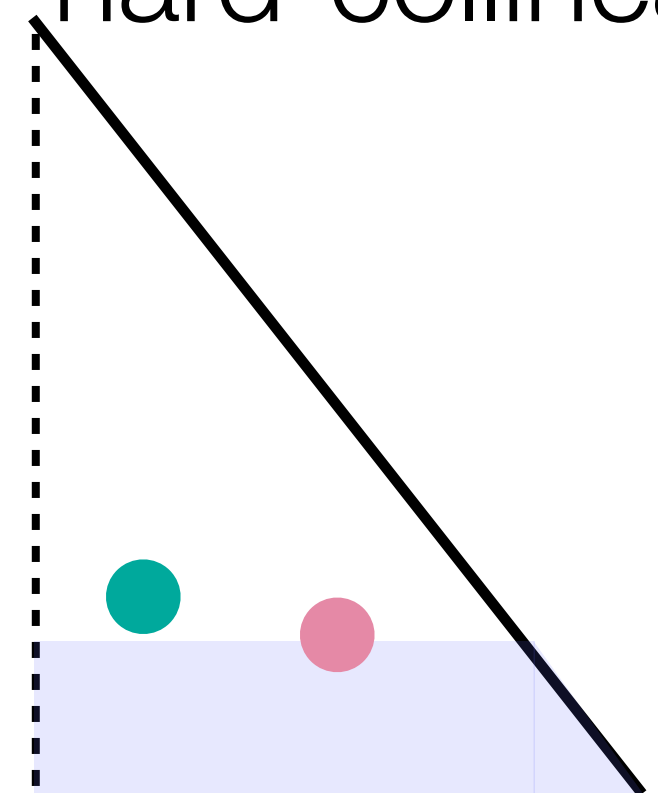
3 commensurate angles



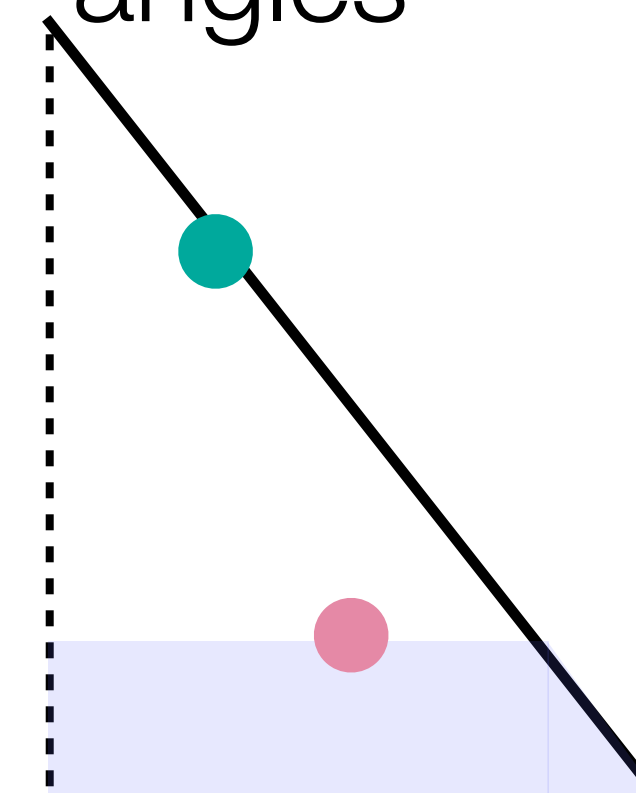
Large angle + $k_t \sim k_{t,cut}$



Hard coll. + commensurate η

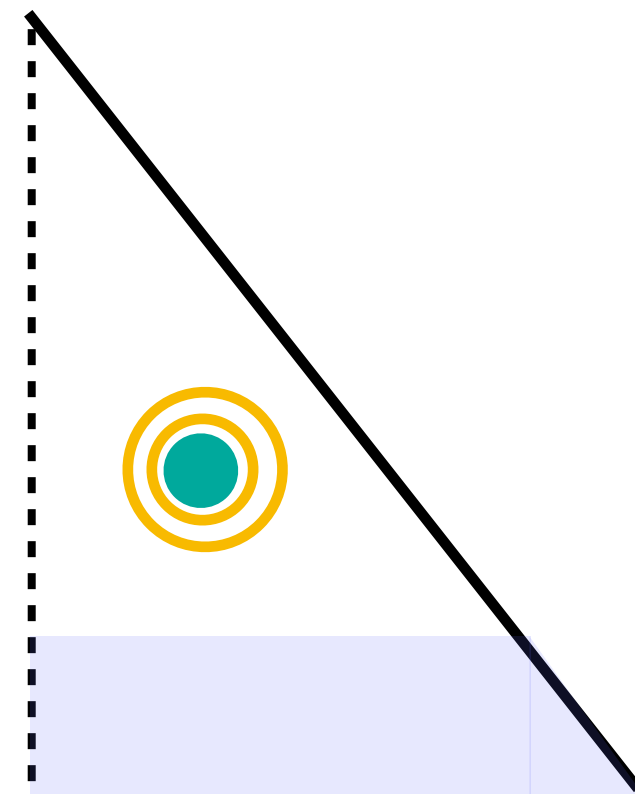


2 $k_t \sim k_{t,cut}$

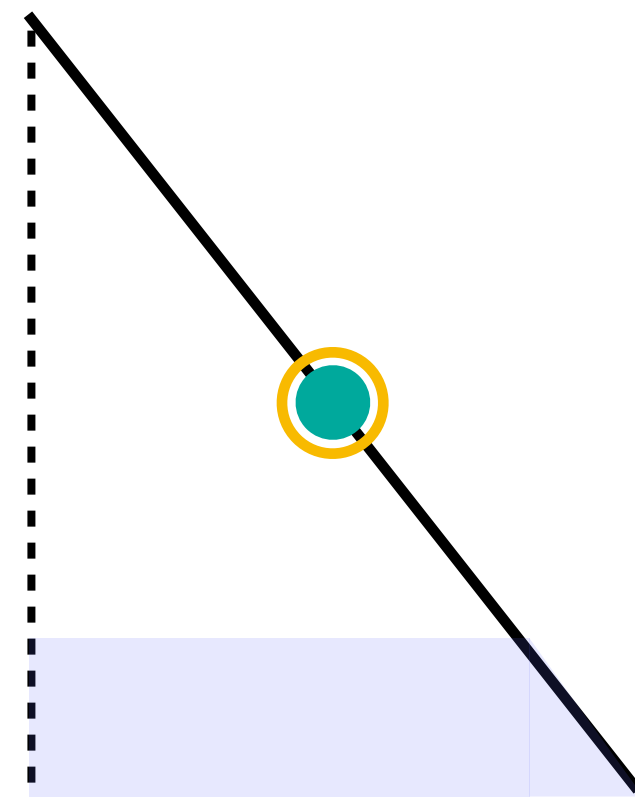


Hard coll. + $k_t \sim k_{t,cut}$

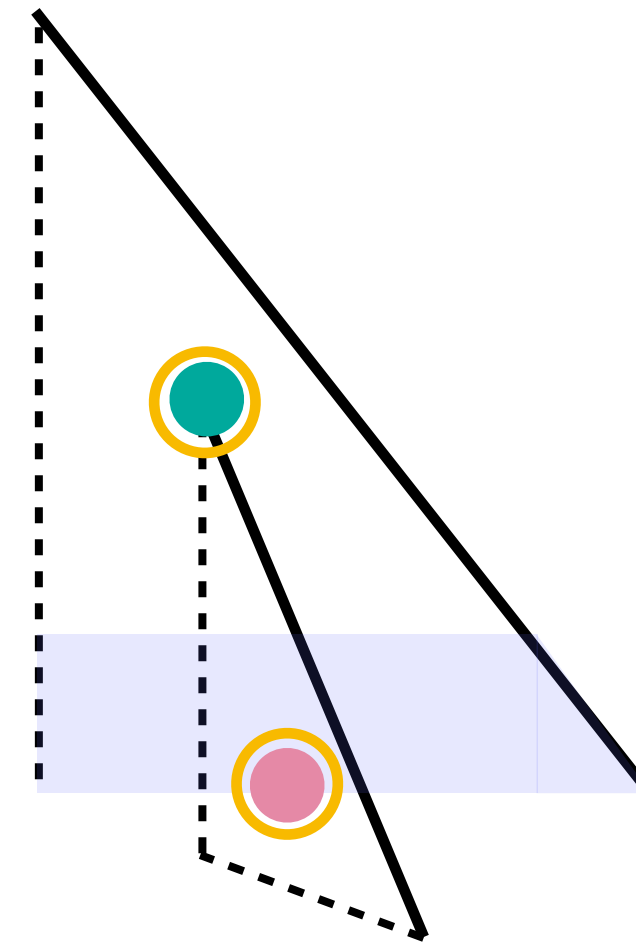
Lund multiplicity: NNDL resummation (running coupling)



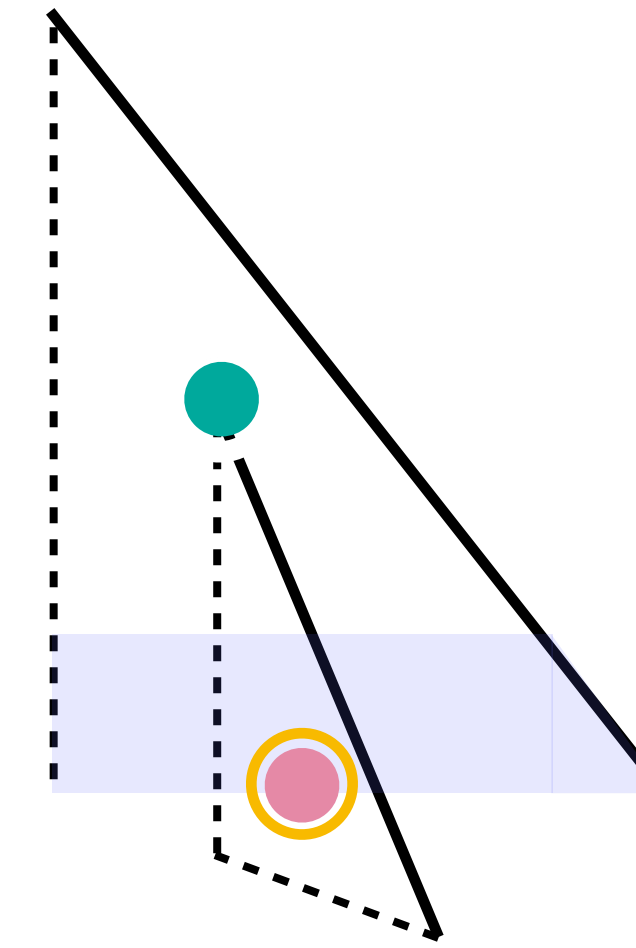
β_0^2



$\beta_0 + \text{hard-collinear}$



$\beta_0 \times \beta_0$

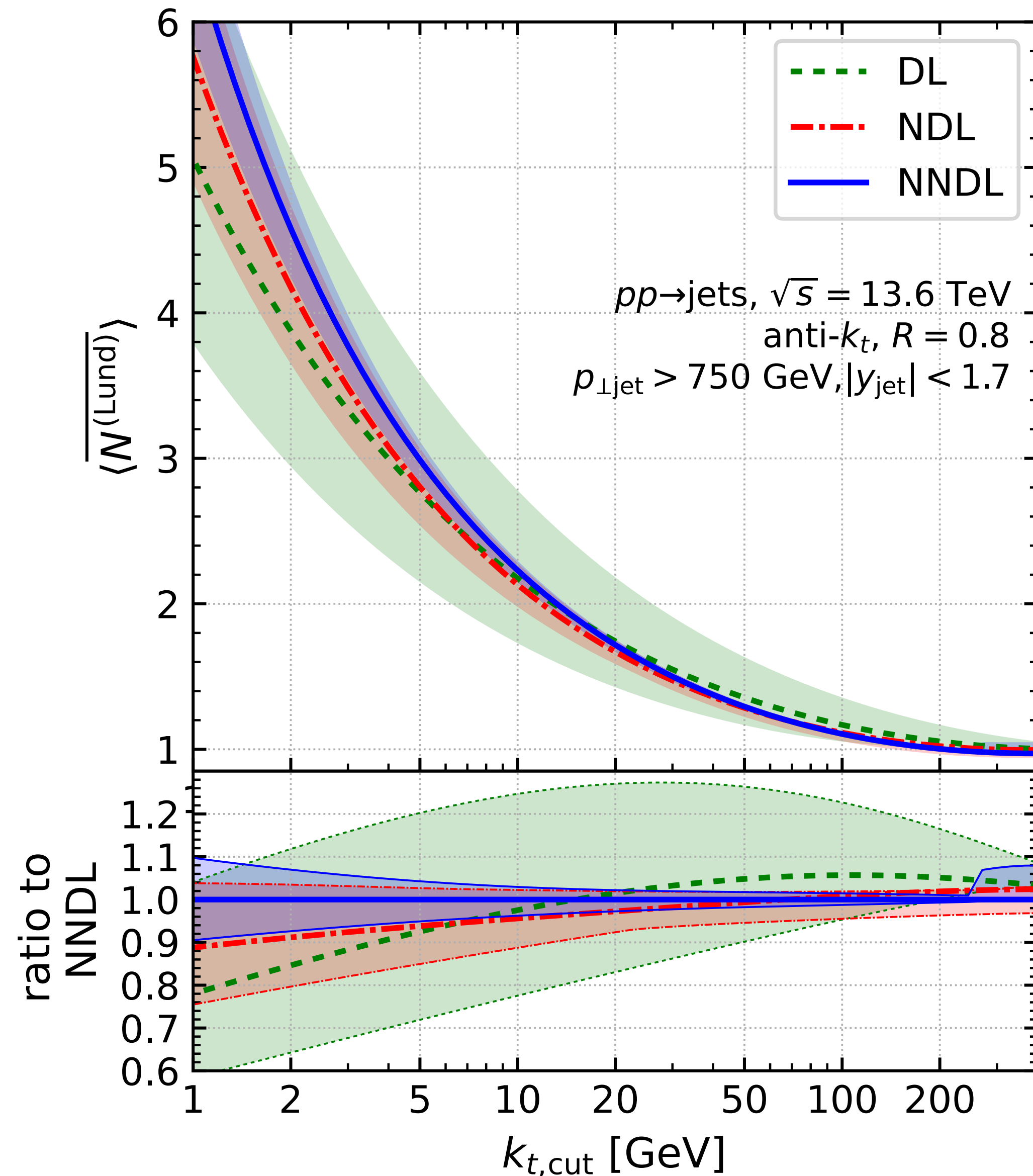


$\beta_0 \times \text{hard-collinear}$

Lund multiplicity: NNDL result

$$\begin{aligned}
 \text{NNDL: } 2\pi h_3^{(q)} = & D_{\text{end}}^{q \rightarrow qg} + \left(D_{\text{end}}^{g \rightarrow gg} + D_{\text{end}}^{g \rightarrow q\bar{q}} \right) \frac{C_F}{C_A} (\cosh \nu - 1) + D_{\text{hme}}^{qqg} \cosh \nu \\
 & + \frac{C_F}{C_A} \left[(1 - c_\delta) D_{\text{pair}}^{q\bar{q}} (\cosh \nu - 1) + \left(K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar{q}} \right) \frac{\nu}{2} \sinh \nu \right] \\
 & + C_F \left[\left(\cosh \nu - 1 - \frac{1 - c_\delta}{4} \nu^2 \right) D_{\text{clust}}^{(\text{prim})} + (\cosh \nu - 1) D_{\text{clust}}^{(\text{sec})} \right] \\
 & + \frac{C_F}{C_A} \left[D_{\text{e-loss}}^g \frac{\nu}{2} \sinh \nu + (D_{\text{e-loss}}^q - D_{\text{e-loss}}^g) (\cosh \nu - 1) \right] \\
 & + \frac{C_F}{2} \left\{ (B_{gg} + c_\delta B_{gq})^2 \nu^2 \cosh \nu + 8 \left[2c_\delta B_{gg} - 2c_\delta B_q - (1 - 3c_\delta^2) B_{gq} \right] B_{gq} \cosh \nu \right. \\
 & \quad + [4B_q (B_{gg} + (2c_\delta + 1) B_{gq}) - (B_{gg} + c_\delta B_{gq}) (B_{gg} + 9c_\delta B_{gq})] \nu \sinh \nu \\
 & \quad \left. + 4(1 - c_\delta^2) B_{gq}^2 \nu^2 + 8 \left[2c_\delta B_q - 2c_\delta B_{gg} + (1 - 3c_\delta^2) B_{gq} \right] B_{gq} \right\} \\
 & + \frac{C_F}{C_A} \frac{\pi \beta_0}{2} \left\{ (B_{gg} + c_\delta B_{gq}) \nu^3 \sinh \nu + [2B_q - 2B_{gg} + (6 - 8c_\delta) B_{gq}] \nu \sinh \nu \right. \\
 & \quad \left. + 2(B_q + B_{gg} + B_{gq}) \nu^2 \cosh \nu - 4(1 - c_\delta) B_{gq} (2 \cosh \nu - 2 + \nu^2) \right\} \\
 & + \frac{C_F}{C_A} \frac{\pi^2 \beta_0^2}{8C_A} [3\nu(2\nu^2 - 1) \sinh \nu + (\nu^4 + 3\nu^2) \cosh \nu]
 \end{aligned}$$

The importance of higher logarithmic accuracy

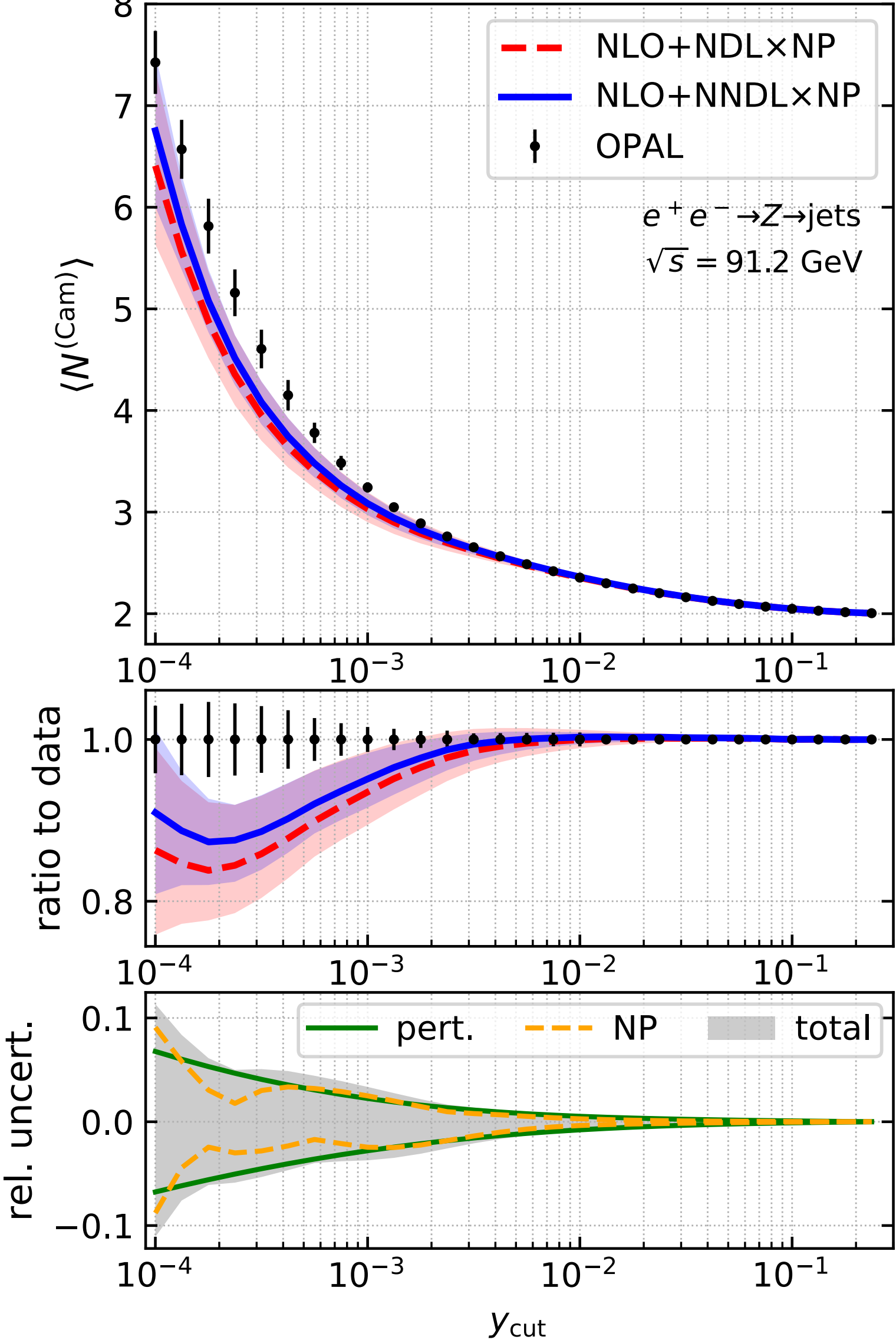


The uncertainty of the theoretical prediction at $k_{t,\text{cut}} = 5 \text{ GeV}$ is

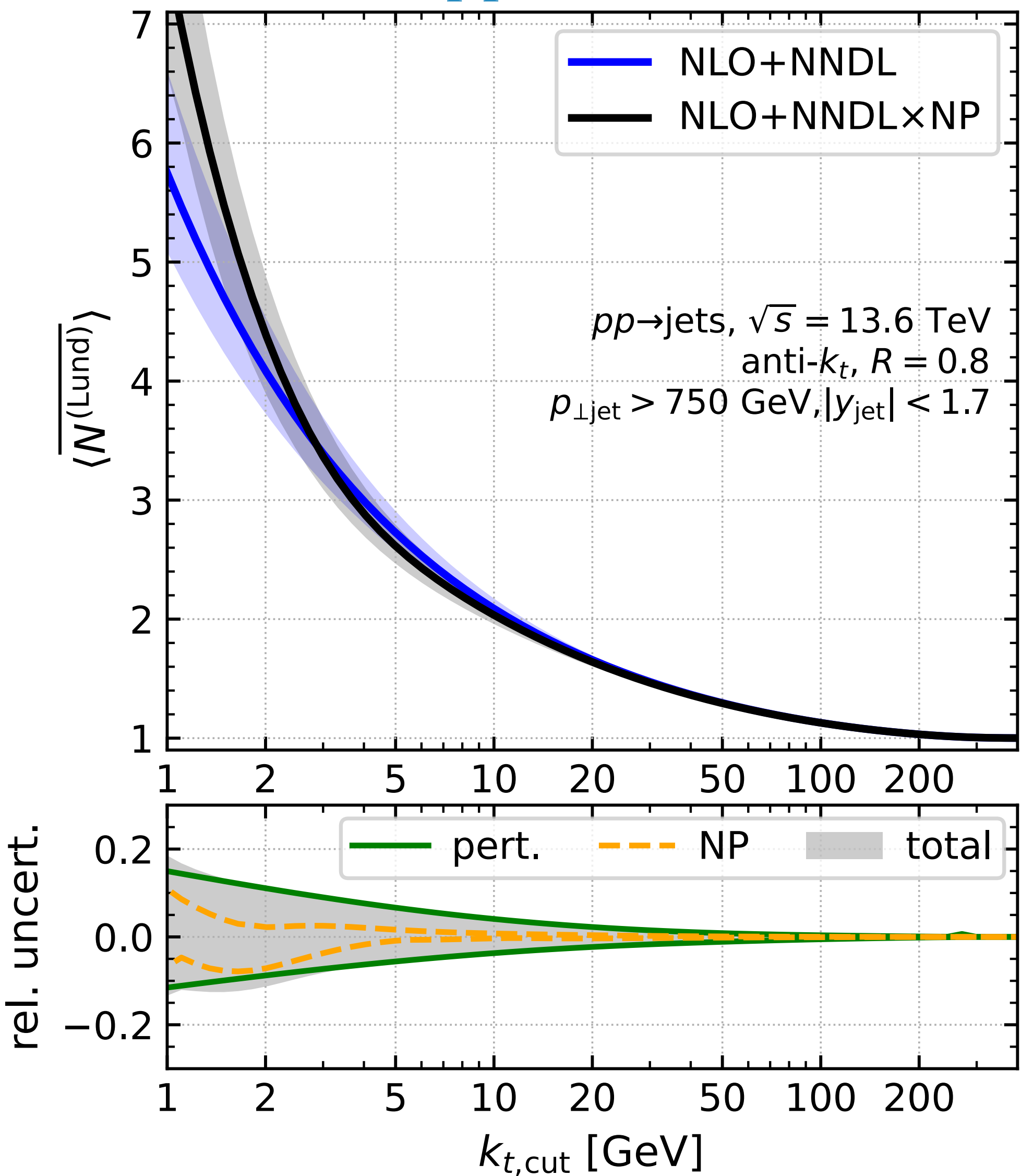
DL: 28 % , **NDL:** 10 % , **NNDL:** 5 %

Phenomenological studies

e^+e^- @ LEP



pp @ LHC



Wrap up

- The Lund plane plays a key role in parton showers and resummation R&D
- The Lund jet plane bridges the gap between theory and experiment
- The resummation toolkit for Lund plane observables is rapidly developing
- Predictions status: $<10\%$ uncertainty in a wide region of phase-space

Wrap up

- The Lund plane plays a key role in parton showers and resummation R&D
- The Lund jet plane bridges the gap between theory and experiment
- The resummation toolkit for Lund plane observables is rapidly developing
- Predictions status: $<10\%$ uncertainty in a wide region of phase-space

First Lund Jet Plane Institute

3–7 Jul 2023
CERN
Europe/Zurich timezone

<https://indico.cern.ch/event/1268020/>