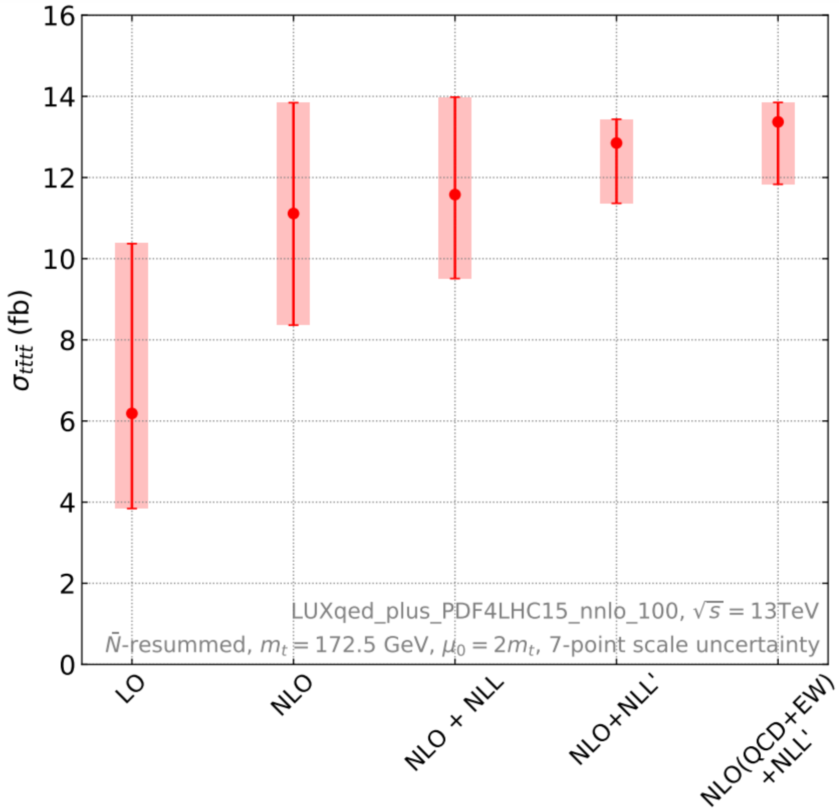
Soft gluon resummation for the production of four top quarks at the LHC



Laura Moreno Valero

in collaboration with Melissa van Beekveld and Anna Kulesza

Institute for Theoretical Physics, University of Münster

(based on the paper **arXiv:2212.03259**)

WWU MÜNSTER

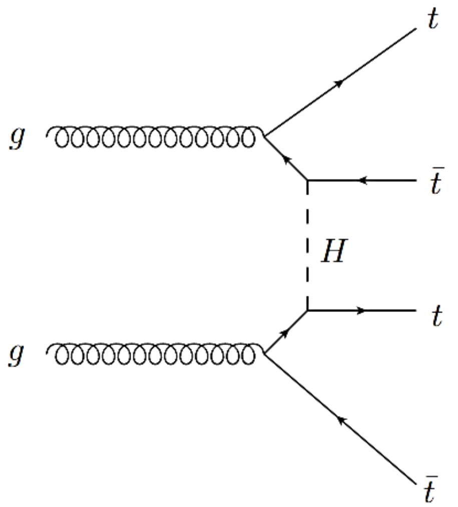
Graduiertenkolleg 2149 Research Training Group

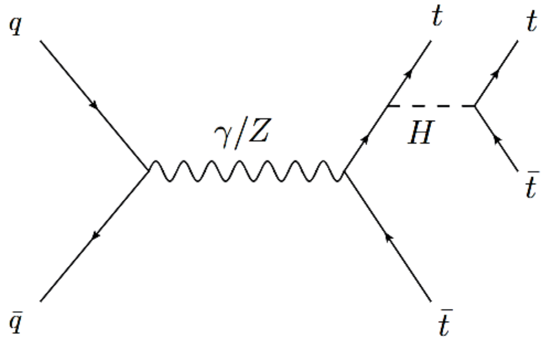
PSR 2023 Milan 6-8 June 2023

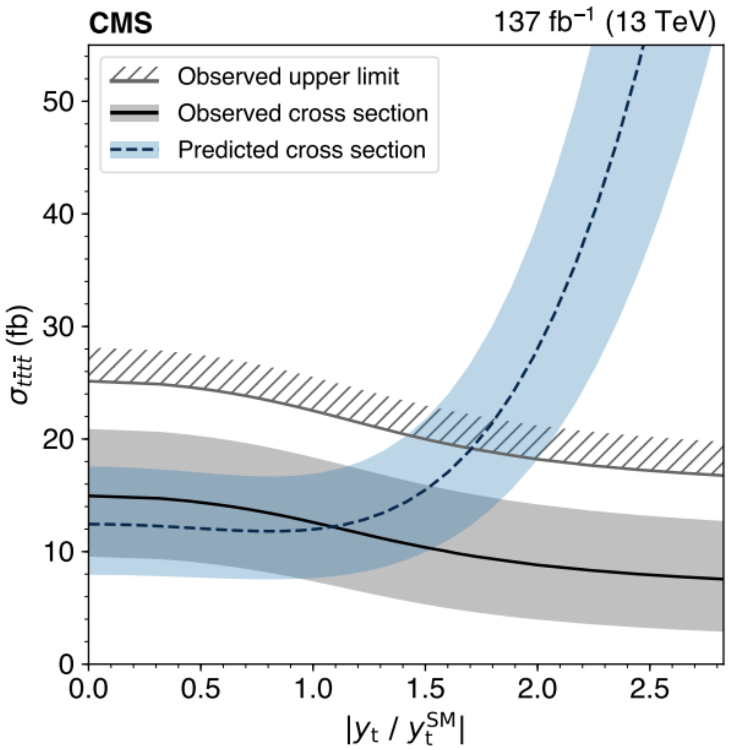
- Four coloured massive particles in the final state

WHY 4 TOP?

- Four coloured massive particles in the final state
- Sensitive to the Yukawa coupling







[Eur. Phys. J. C (2020) 80, 75]

WHY 4 TOP?

- Four coloured massive particles in the final state
- Sensitive to the Yukawa coupling
- Sensitive to new physics (gluinos, scalar gluons, heavy scalar bosons, ...)

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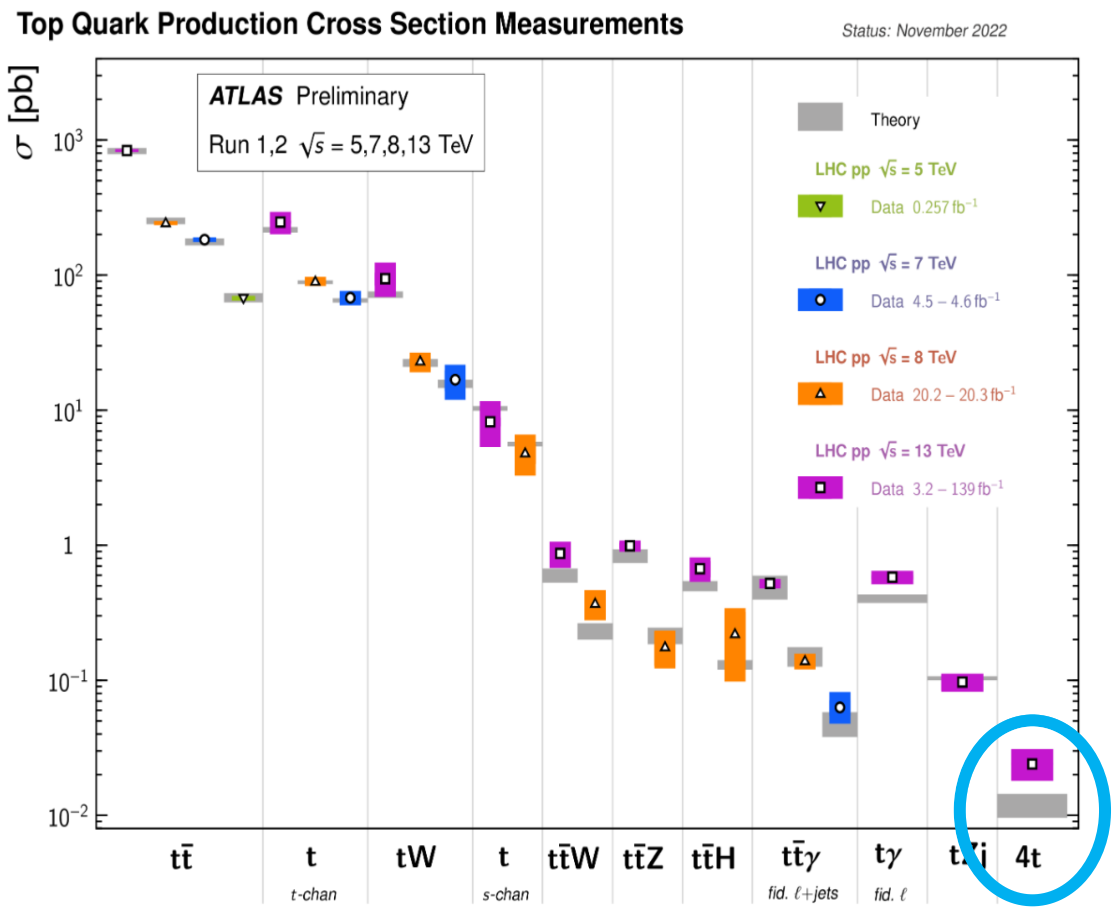
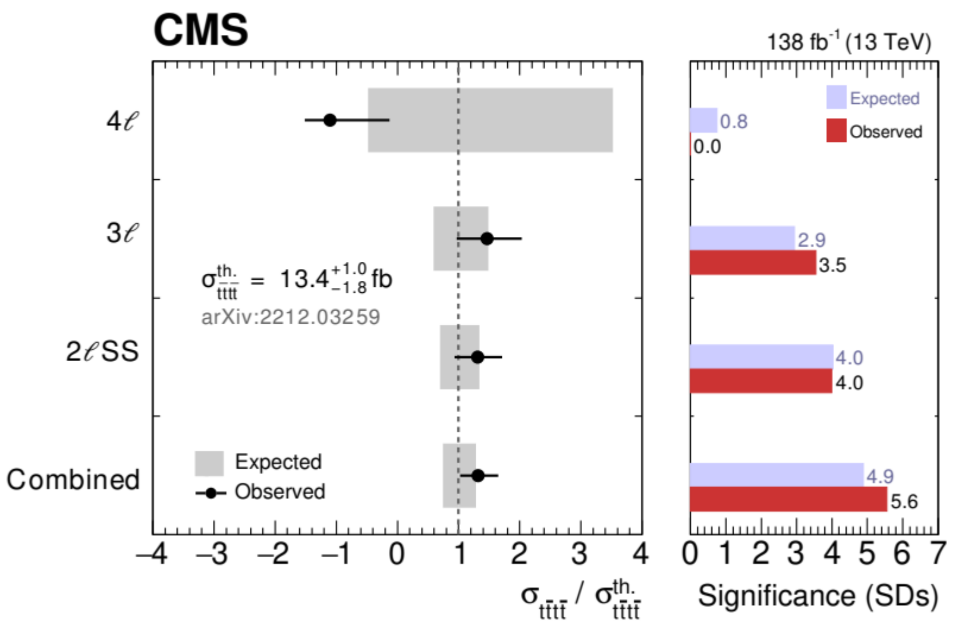
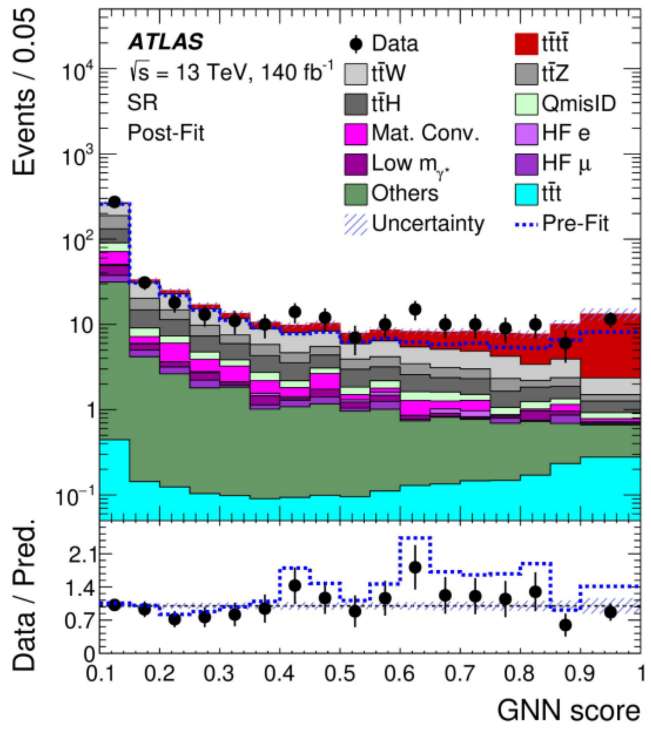
- Four coloured massive particles in the final state
- Sensitive to the Yukawa coupling
- Sensitive to new physics (gluinos, scalar gluons, heavy scalar bosons, ...)
- Constrains SMEFT coefficients: e.g. four-fermion operator

- Measured at the LHC
  - ATLAS [Eur. Phys. J. C (2020) 80, 1085; JHEP 11 (2021), 118; Phys. Rev. D 99, 052009 (2019), arXiv:2303.15061]
  - CMS [Eur. Phys. J. C (2020) 80, 75; JHEP 11 (2019), 082, arXiv:2303.03864, CMS-PAS-TOP-22-013, arXiv:2305.13439]





- Significance > 5σ obtained in multilepton channels (2lSS,3l,4l)



5.6σ

$$\sigma^{\text{SM, NLO (QCD+EW)}}_{t\bar{t}t\bar{t}} = 12.0 \pm 2.4 \text{ fb}$$

[Frederix, Pagani, Zaro (2017)]

6.1σ

$$\sigma^{\text{CMS}}_{t\bar{t}t\bar{t}} = 17.7\,^{+3.7}_{-3.5}(\text{stat.})\,^{+2.3}_{-1.9}(\text{syst.}) \text{ fb}$$

$$\sigma^{\text{ATLAS}}_{t\bar{t}t\bar{t}} = 22.5\,^{+4.7}_{-4.3}(\text{stat.})\,^{+4.6}_{-3.4}(\text{syst.}) \text{ fb}$$

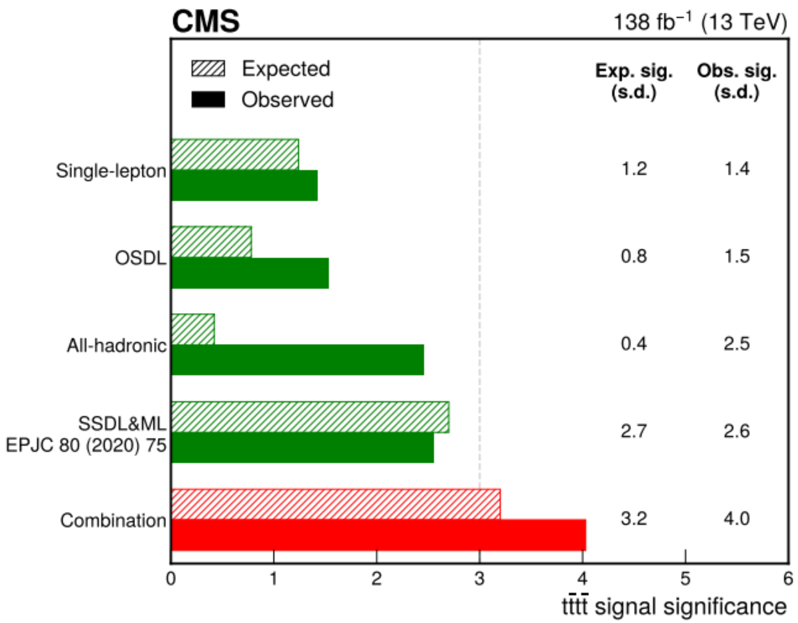
[arXiv:2305.13439]

[arXiv:2303.15061]

- Evidence in other channels (1l, 2lOS)

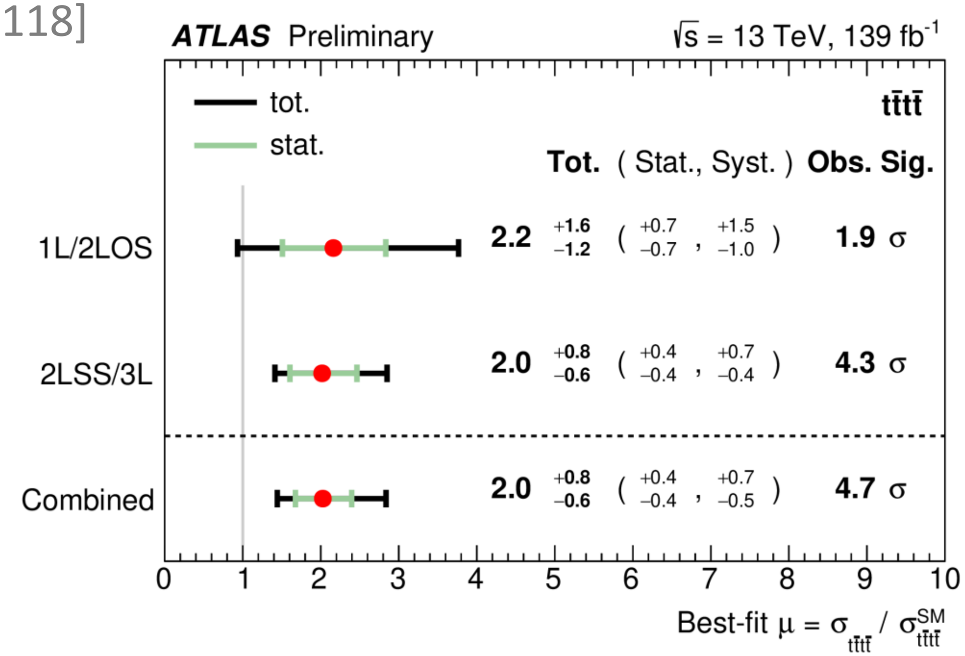
CMS

[arXiv:2303.03864]



ATLAS

[JHEP 11 (2021), 118]



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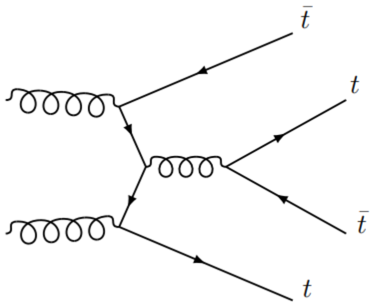
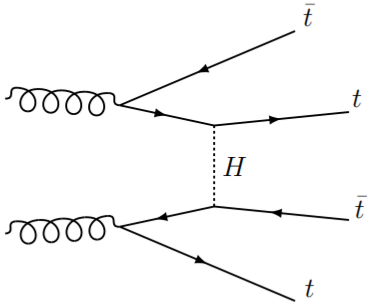
- Theoretical predictions for total rate: NLO (QCD + EW) available

[Bevilacqua, Worek (2012)], [Frederix, Pagani, Zaro (2017)],[Ježo, Kraus (2021)]

... at LO e.g. pure QCD



and



QCD + EW

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GOAL: extend the precision of theoretical predictions beyond NLO for $pp \to t\bar{t}t\bar{t}$ by means of **resummation** techniques

- Perturbative expansion cross-section

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1\alpha_s + c_2\alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n} d_{nk} L^k \quad , \quad L^k = \ln^k(1 - M^2/s)$$

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- Large logs *L* can spoil predictive power, $L^k \to \infty$ for $M^2 \simeq s$ with $M = 4m_t$

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- Large logs L can spoil predictive power, $L^k \to \infty$ for $M^2 \simeq s$ with $M = 4m_t$
- **Soft Gluon Resummation**
  - Systematic treatment to all orders: resummation
  - Relies on ME and Phase space factorization → Mellin space $L := \ln(1 - M^2/s) \quad \to \quad \tilde{L} := \ln N$

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$$\hat{\sigma}^{\text{res}}(N) \sim \mathcal{F}(\alpha_s) \exp\left[\tilde{L}\, g_1(\alpha_s \tilde{L}) + g_2(\alpha_s \tilde{L}) + \ldots\right]$$

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LL

$\alpha_s^n \ln^{2n} N$

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LL: $\alpha_s^n \ln^{2n} N$

NLL: $\alpha_s^n \ln^{2n-1} N$

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Finite contributions — $\mathcal{F}(\alpha_s)$

LL — $\alpha_s^n \ln^{2n} N$

NLL — $\alpha_s^n \ln^{2n-1} N$

- Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

$$\hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}} = \mathrm{Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}} \;\; \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\, \mathbf{S}_{ij\to t\bar{t}t\bar{t}}\, \mathbf{U}_{ij\to t\bar{t}t\bar{t}} \,\right]\; \Delta_i\, \Delta_j$$

- Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

Incoming jet funcions

(soft-)collinear enhancements

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}} = \text{Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}} \;\; \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}} \, \mathbf{S}_{ij\to t\bar{t}t\bar{t}} \, \mathbf{U}_{ij\to t\bar{t}t\bar{t}} \,\right] \boxed{\Delta_i \, \Delta_j}$$

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Hard function

constant contributions as $N \to \infty$

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Hard function

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Soft function

soft wide-angle enhancements

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

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Soft anomalous dimension (SAD) matrix

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s\right) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

- $q\bar{q} \to t\bar{t}t\bar{t}: \; \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

6-dimensional colour space

- $gg \to t\bar{t}t\bar{t}: \; \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

14-dimensional colour space

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right] \longrightarrow$$

Diagonal SAD matrix

$$\bar{\mathbf{U}}_R\,\mathbf{S}_R\,\mathbf{U}_R = \mathbf{S}_R\,\exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}(\alpha_s) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

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$$
\begin{aligned}
c_1^{q\bar{q}} &= \frac{1}{\sqrt{N_c^3}}\delta_{c_1c_3}\delta_{c_2c_4}\delta_{c_6c_8}\\
c_2^{q\bar{q}} &= \frac{1}{T_R\sqrt{N_c(N_c^2-1)}}\delta_{c_1c_3}t^{a_1}_{c_2c_4}t^{a_1}_{c_6c_8}\\
c_3^{q\bar{q}} &= \frac{1}{T_R\sqrt{N_c(N_c^2-1)}}t^{a_1}_{c_1c_3}\delta_{c_2c_4}t^{a_1}_{c_6c_8}\\
c_4^{q\bar{q}} &= \frac{1}{T_R\sqrt{N_c(N_c^2-1)}}t^{a_1}_{c_1c_3}t^{a_1}_{c_2c_4}\delta_{c_6c_8}\\
c_5^{q\bar{q}} &= \frac{\sqrt{N_c}}{T_R^2\sqrt{2(N_c^4-5N_c^2+4)}}t^{a_1}_{c_1c_3}d^{a_1a_2b_3}t^{a_2}_{c_2c_4}t^{b_3}_{c_6c_8}\\
c_6^{q\bar{q}} &= \frac{1}{T_R^2\sqrt{2N_c(N_c^2-1)}}t^{a_1}_{c_1c_3}if^{a_1a_2b_3}t^{a_2}_{c_2c_4}t^{b_3}_{c_6c_8}\,,
\end{aligned}
$$

[Keppeler, Sjodahl (2012)]

6x6 SAD matrix

$$
\begin{aligned}
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},11} &= -C_F\left(L_{\beta_{34}}+L_{\beta_{56}}+2\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},12} &= \frac{\sqrt{N_c^2-1}}{2N_c}\left(L_{\beta_{35}}+L_{\beta_{46}}-L_{\beta_{36}}-L_{\beta_{45}}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},13} &= \frac{\sqrt{N_c^2-1}}{2N_c}\left(T_{15}-T_{16}-T_{25}+T_{26}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},14} &= \frac{\sqrt{N_c^2-1}}{2N_c}\left(T_{13}-T_{14}-T_{23}+T_{24}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},15} &= 0\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},16} &= 0\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},22} &= -2C_F+\frac{1}{2N_c}\left(L_{\beta_{34}}+L_{\beta_{56}}\right)-\frac{1}{N_c}\left(L_{\beta_{35}}+L_{\beta_{46}}\right)-\frac{N_c^2-2}{2N_c}\left(L_{\beta_{36}}+L_{\beta_{45}}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},23} &= \frac{1}{2N_c}\left(T_{13}-T_{14}-T_{23}+T_{24}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},24} &= \frac{1}{2N_c}\left(T_{15}-T_{16}-T_{25}+T_{26}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},25} &= \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}\left(T_{13}-T_{14}+T_{15}-T_{16}-T_{23}+T_{24}-T_{25}+T_{26}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},26} &= \frac{1}{2\sqrt{2}}\left(-T_{13}-T_{14}+T_{15}+T_{16}+T_{23}+T_{24}-T_{25}-T_{26}\right)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},33} &= -\frac{N_c^2-2}{2N_c}-C_FL_{\beta_{34}}+\frac{1}{2N_c}L_{\beta_{56}}+\frac{N_c^2-2}{2N_c}T_{15}+\frac{1}{N_c}T_{16}+\frac{N_c^2-2}{2N_c}T_{26}+\frac{1}{N_c}T_{25}\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},34} &= \frac{1}{2N_c}\left(L_{\beta_{35}}+L_{\beta_{46}}-L_{\beta_{36}}-L_{\beta_{45}}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},35} &= \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}\left(L_{\beta_{35}}-L_{\beta_{36}}-L_{\beta_{45}}+L_{\beta_{46}}+T_{13}-T_{14}-T_{23}+T_{24}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},36} &= \frac{1}{2\sqrt{2}}\left(-L_{\beta_{35}}-L_{\beta_{36}}+L_{\beta_{45}}+L_{\beta_{46}}-T_{13}+T_{14}-T_{23}+T_{24}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},44} &= -C_F+\frac{1}{2N_c}-C_FL_{\beta_{56}}+\frac{1}{2N_c}L_{\beta_{34}}+\frac{N_c^2-2}{2N_c}\left(T_{13}+T_{24}\right)+\frac{1}{N_c}\left(T_{14}+T_{23}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},45} &= \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}\left(L_{\beta_{35}}-L_{\beta_{36}}-L_{\beta_{45}}+L_{\beta_{46}}+T_{15}-T_{16}-T_{25}+T_{26}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},46} &= \frac{1}{2\sqrt{2}}\left(L_{\beta_{35}}-L_{\beta_{36}}+L_{\beta_{45}}-L_{\beta_{46}}+T_{15}-T_{16}+T_{25}-T_{26}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},55} &= -C_F+\frac{1}{2N_c}+\frac{1}{2N_c}\left(L_{\beta_{34}}-3L_{\beta_{35}}-3L_{\beta_{46}}+L_{\beta_{56}}-\frac{N_c^2-6}{2}L_{\beta_{36}}-\frac{N_c^2-6}{2}L_{\beta_{45}}\right)\\
&\quad+\frac{3}{2N_c}\left(T_{14}+T_{16}+T_{23}+T_{25}\right)+\frac{N_c^2-6}{4N_c}\left(T_{13}+T_{15}+T_{24}+T_{26}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},56} &= \frac{\sqrt{N_c^2-4}}{4}\left(-L_{\beta_{36}}+L_{\beta_{45}}-T_{13}+T_{15}+T_{24}-T_{26}\right)\\
\bar{\Gamma}^{(1)}_{q\bar{q}\to4\text{top},66} &= -C_F+\frac{1}{2N_c}-\frac{N_c^2-2}{4N_c}\left(L_{\beta_{36}}+L_{\beta_{45}}\right)+\frac{1}{2N_c}\left(L_{\beta_{34}}-L_{\beta_{35}}-L_{\beta_{46}}+L_{\beta_{56}}\right)\\
&\quad+\frac{N_c^2-2}{4N_c}\left(T_{13}+T_{15}+T_{24}+T_{26}\right)+\frac{1}{2N_c}\left(T_{14}+T_{16}+T_{23}+T_{25}\right)
\end{aligned}
$$

SAD matrix diagonal at threshold

COLOUR STRUCTURE

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \boxed{\mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}}$

$$c_1^{gg} = \frac{1}{T_R}\frac{1}{N_c^2-1} t^{a_1}_{c_2c_4} t^{a_2}_{c_6c_8}\,, \qquad c_2^{gg} = \frac{1}{Nc\sqrt{N_c^2-1}}\delta_{a_1a_2}\delta_{c_2c_4}\delta_{c_6c_8}\,,$$

$$c_3^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}}\delta_{c_2c_4} d_{a_1a_2b_1} t^{b_1}_{c_6c_8}\,, \qquad c_4^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}}\delta_{c_2c_4} i f_{a_1a_2b_1} t^{b_1}_{c_6c_8}\,,$$

$$c_5^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} t^{b_1}_{c_2c_4} d_{b_1a_1a_2}\delta_{c_6c_8}\,, \qquad c_6^{gg} = \frac{1}{T_R^2}\frac{N_c}{2(N_c^2-4)\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_7^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_8^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} t^{b_1}_{c_2c_4} i f_{b_1a_1a_2}\delta_{c_6c_8}\,,$$

$$c_9^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_{10}^{gg} = \frac{1}{T_R^2}\frac{1}{2N_c\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{11}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{10}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{12}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{\overline{10}}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{13}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2+2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{27}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{14}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2-2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{0}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,.$$

⇒ SAD matrix not diagonal at threshold

[Keppeler, Sjodahl (2012)]

⇒ 14x14 SAD matrix

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,1} = \frac{1}{N_c} + \frac{1}{2N_c}\left(L_{\beta_{34}} + L_{\beta_{34}}\right) + \frac{N_c}{2}\left(T_{13}+T_{14}+T_{25}+T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,2} = \frac{1}{2N_c\sqrt{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,3} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,4} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,5} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,6} = \frac{N_c^2-4}{4N_c\sqrt{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,7} = \frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,8} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(-L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}} - 2T_{15} + 2T_{16}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,9} = -\frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} - L_{\beta_{36}} + L_{\beta_{45}} - L_{\beta_{46}} + 2T_{15} - 2T_{16}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,10} = -\frac{N_c}{4\sqrt{N_c^2-1}}\Big(4 + L_{\beta_{35}} + L_{\beta_{36}} + L_{\beta_{45}} + L_{\beta_{46}} + 2T_{15} + 2T_{16} + 2T_{23} + 2T_{24}\Big)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,11} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,12} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,13} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,14} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,2} = -2C_F - C_F\left(L_{\beta_{34}} + L_{\beta_{56}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,3} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,4} = \frac{1}{\sqrt{2}}\left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,5} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,6} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,7} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,8} = \frac{1}{\sqrt{2}}\left(T_{13} - T_{14} - T_{23} + T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,9} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,10} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,11} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,12} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,13} = \frac{1}{4}\sqrt{\frac{N_c+3}{N_c+1}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,14} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,3} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15}+T_{16}+T_{25}+T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,4} = \frac{\sqrt{N_c^2-4}}{4}\left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,5} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,6} = \frac{N_c^2-12}{4N_c\sqrt{2(N_c^2-4)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,7} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,8} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,9} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 4T_{13} - 4T_{14} - 2T_{23} + 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,10} = \frac{\sqrt{2(N_c^2-4)}}{8N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,11} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{35}} - L_{\beta_{45}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{36}} - L_{\beta_{46}}\right) + \frac{1}{2\sqrt{2}}\left(T_{23} - T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,12} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{46}} - L_{\beta_{36}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{45}} - L_{\beta_{35}}\right) + \frac{1}{2\sqrt{2}}\left(T_{24} - T_{23}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,13} = \frac{1}{4\sqrt{2}}\sqrt{\frac{(N_c-2)(N_c+3)}{(N_c+1)(N_c+2)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,14} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},4\,4} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15}+T_{16}+T_{25}+T_{26}\right)$$

+ 65 more components!

- $gg \to t\bar{t}t\bar{t}$: $\mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \boxed{\mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}}$

$$c_1^{gg} = \frac{1}{T_R}\frac{1}{N_c^2-1} t^{a_1}_{c_2c_4} t^{a_2}_{c_6c_8}\,,$$

$$c_3^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} \delta_{c_2c_4} d_{a_1a_2\,b_1} t^{b_1}_{c_6c_8}\,,$$

$$c_5^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} t^{b_1}_{c_2c_4} d_{b_1a_1a_2} \delta_{c_6c_8}\,,$$

$$c_7^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} if_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_9^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} if_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{11}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{10}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{13}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2+2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{27}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_2^{gg} = \frac{1}{Nc\sqrt{N_c^2-1}} \delta_{a_1a_2}\delta_{c_2c_4}\delta_{c_6c_8}\,,$$

$$c_4^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} \delta_{c_2c_4} if_{a_1a_2\,b_1} t^{b_1}_{c_6c_8}\,,$$

$$c_6^{gg} = \frac{1}{T_R^2}\frac{N_c}{2(N_c^2-4)\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_8^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} t^{b_1}_{c_2c_4} if_{b_1a_1a_2} \delta_{c_6c_8}\,,$$

$$c_{10}^{gg} = \frac{1}{T_R^2}\frac{1}{2N_c\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} if_{b_1a_1b_2} if_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{12}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{\overline{10}}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{14}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2-2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{0}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,.$$

[Keppeler, Sjodahl (2012)]

⇒ SAD matrix not diagonal at threshold

⇒

$$\bar{c}_1^{gg} = \frac{3\sqrt{3}}{8}c_1^{gg} + \frac{3}{10}\sqrt{\frac{3}{2}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{3}{2}}c_{10}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{11}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{12}^{gg} + \frac{7}{40}c_{13}^{gg}\,,$$

$$\bar{c}_2^{gg} = -\frac{\sqrt{5}}{4}c_1^{gg} + \sqrt{\frac{2}{5}}c_6^{gg} - \frac{1}{2\sqrt{2}}c_{11}^{gg} - \frac{1}{2\sqrt{2}}c_{12}^{gg} + \frac{1}{4}\sqrt{\frac{3}{5}}c_{13}^{gg}\,,$$

$$\bar{c}_3^{gg} = -\frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}\,,$$

$$\bar{c}_5^{gg} = -\frac{1}{2\sqrt{2}}c_1^{gg} - \frac{1}{2}c_6^{gg} - \frac{1}{2}c_{10}^{gg} + \frac{1}{2}\sqrt{\frac{3}{2}}c_{13}^{gg}\,,$$

$$\bar{c}_6^{gg} = -\frac{1}{2}\sqrt{\frac{5}{14}}c_1^{gg} + \frac{3}{2\sqrt{35}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{5}{7}}c_{10}^{gg} + \frac{2}{\sqrt{7}}c_{12}^{gg} - \frac{3}{2}\sqrt{\frac{3}{70}}c_{13}^{gg}\,,$$

$$\bar{c}_7^{gg} = -\frac{1}{2\sqrt{7}}c_1^{gg} + \frac{3}{5\sqrt{14}}c_6^{gg} - \frac{1}{\sqrt{14}}c_{10}^{gg} + \sqrt{\frac{7}{10}}c_{11}^{gg} - \frac{3}{\sqrt{70}}c_{12}^{gg} - \frac{3}{10}\sqrt{\frac{3}{7}}c_{13}^{gg}\,,$$

$$\bar{c}_8^{gg} = \frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}\,,$$

$$\bar{c}_{13}^{gg} = \frac{1}{8}c_1^{gg} + \frac{1}{2\sqrt{2}}c_6^{gg} + \frac{1}{2\sqrt{2}}c_{10}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{11}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{12}^{gg} + \frac{3\sqrt{3}}{8}c_{13}^{gg}\,,$$

$$\bar{c}_4^{gg} = c_{14}^{gg}\,,\quad \bar{c}_9^{gg} = c_8^{gg}\,,\quad \bar{c}_{10}^{gg} = c_5^{gg}\,,\quad \bar{c}_{11}^{gg} = c_4^{gg}\,,\quad \bar{c}_{12}^{gg} = c_3^{gg}\,,\quad \bar{c}_{14}^{gg} = c_2^{gg}\,.$$

⇒ SAD matrix diagonal at threshold

SAD MATRIX

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}(\alpha_s) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

Diagonal SAD matrix

$$\bar{\mathbf{U}}_R\,\mathbf{S}_R\,\mathbf{U}_R = \mathbf{S}_R \exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

- $q\bar{q} \to t\bar{t}t\bar{t}$: $\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \to \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

- $gg \to t\bar{t}t\bar{t}$: $\mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \to \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}(\alpha_s) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

Diagonal SAD matrix

$$\bar{\mathbf{U}}_R\,\mathbf{S}_R\,\mathbf{U}_R = \mathbf{S}_R \exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

- $q\bar{q} \to t\bar{t}t\bar{t}$: $\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

$$2\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\left(0, 0, -3, -3, -3, -3\right)$$

- $gg \to t\bar{t}t\bar{t}$: $\mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

$$2\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\left(-8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, -3, 0, 0\right)$$

- Quadratic Casimir Invariants $N_c = 3$

$$\begin{aligned} C_2(\mathbf{1}) &= 0 \\ C_2(\mathbf{8}_{(S/A)}) &= 3 \\ C_2(\mathbf{10},\overline{\mathbf{10}}) &= 6 \\ C_2(\mathbf{27}) &= 8 \\ C_2(\mathbf{0}) &= 4 \end{aligned}$$

- $q\bar{q} \to t\bar{t}t\bar{t}: \ \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{1} \oplus \mathbf{8} = \mathbf{0} \oplus \boxed{(2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$

$$2\mathrm{Re}\left[\boldsymbol{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(0, 0, -3, -3, -3, -3\Big)$$

- $gg \to t\bar{t}t\bar{t}: \ \mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \boxed{\mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}}$

$$2\mathrm{Re}\left[\boldsymbol{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(-8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, -3, 0, 0\Big)$$

RESUMMATION: NLL and NLL'

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)} + \ldots$$

$$\mathbf{S} = \mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{S}^{(1)} + \ldots$$

- **NLL** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{HS}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\mathbf{S}^{(0)}\right]$$

- **NLL'** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{HS}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)}\mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(0)}\mathbf{S}^{(1)}\right]$$

RESUMMATION: NLL and NLL'

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)} + \ldots$$

$$\mathbf{S} = \mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{S}^{(1)} + \ldots$$

with

$$\mathbf{H}^{(1)} = \mathbf{V}^{(1)} + \mathbf{C}^{(1)}$$

$\mathbf{V}^{(1)}$: One-loop virtual corrections (extracted from aMC@NLO)

$\mathbf{C}^{(1)}$: Constant terms from collinear contribution

- **NLL** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{HS}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\mathbf{S}^{(0)}\right]$$

- **NLL'** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{HS}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)}\mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(0)}\mathbf{S}^{(1)}\right]$$

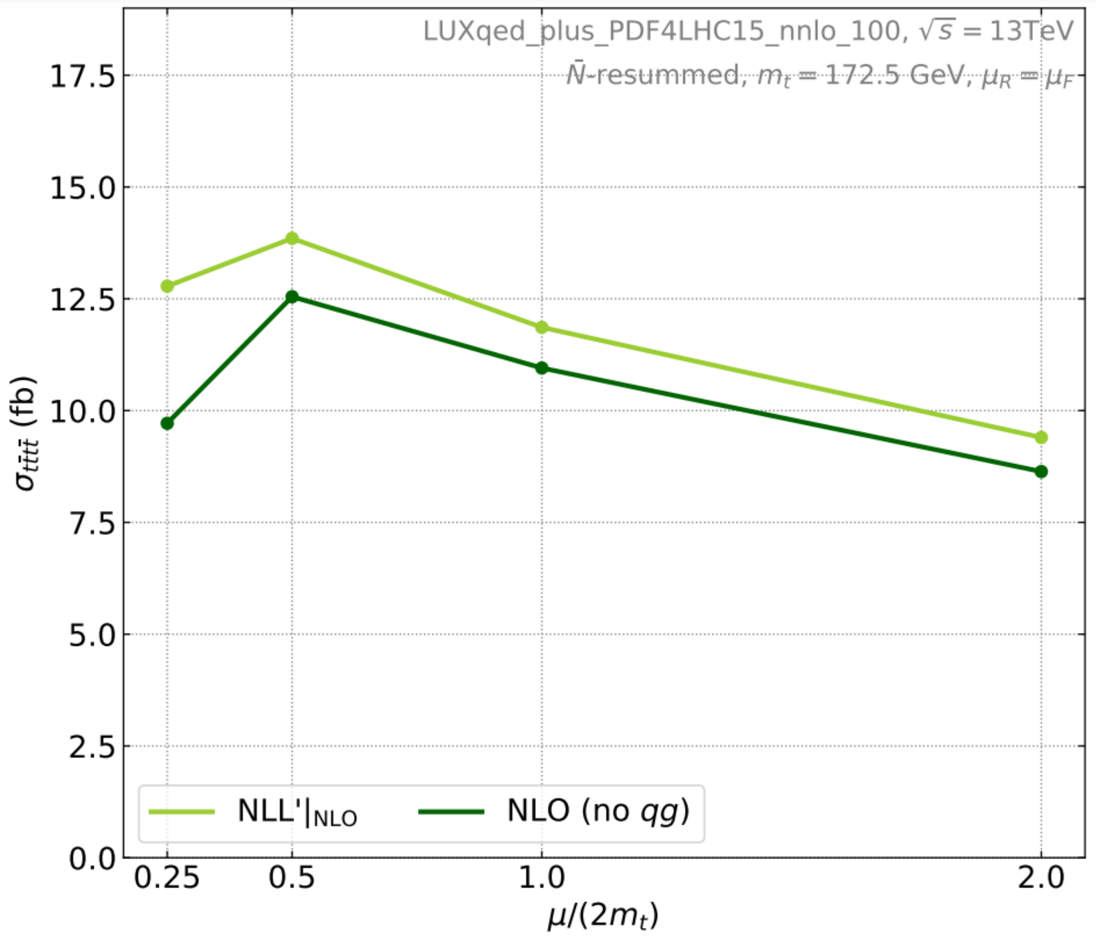
$$\sigma^{\mathrm{NLL}(')}_{t\bar{t}t\bar{t}}(\tau) = \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N}\, f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2)\, \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)$$

- Combination fixed-order + resummation → **Matching**

$$\sigma^{\mathrm{NLO+NLL}(')}_{t\bar{t}t\bar{t}}(\tau) = \sigma^{\mathrm{NLO}}_{t\bar{t}t\bar{t}}(\tau) + \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N}\, f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2) \times \underbrace{\left[\hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N) - \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)\big|_{\mathrm{NLO}}\right]}_{\text{avoid double counting with NLO!}}$$

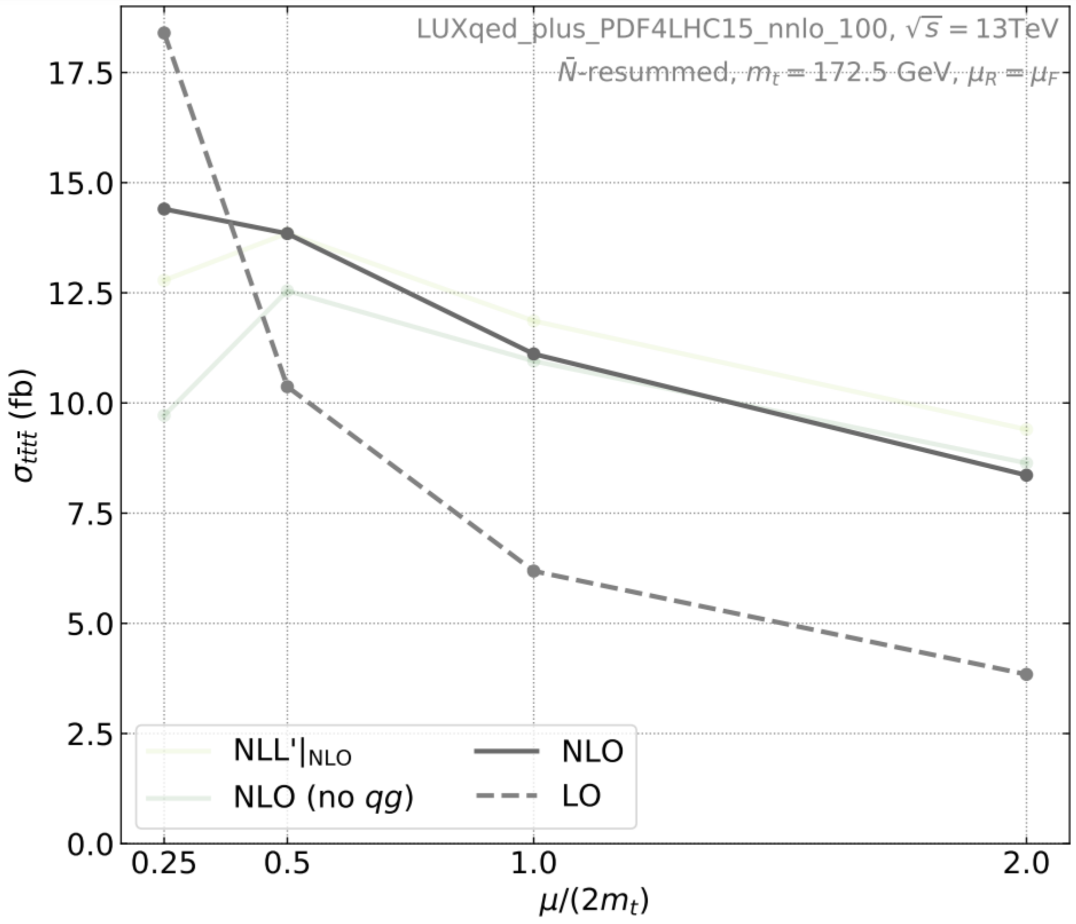
QCD-only NLO and QCD + EW NLO

- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Comparison expanded resummed cross section (NLL'|NLO) against NLO (no qg)
  - Dominant part of NLO corrections captured by soft-gluon radiation



Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD



NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

Fixed-order results obtained with
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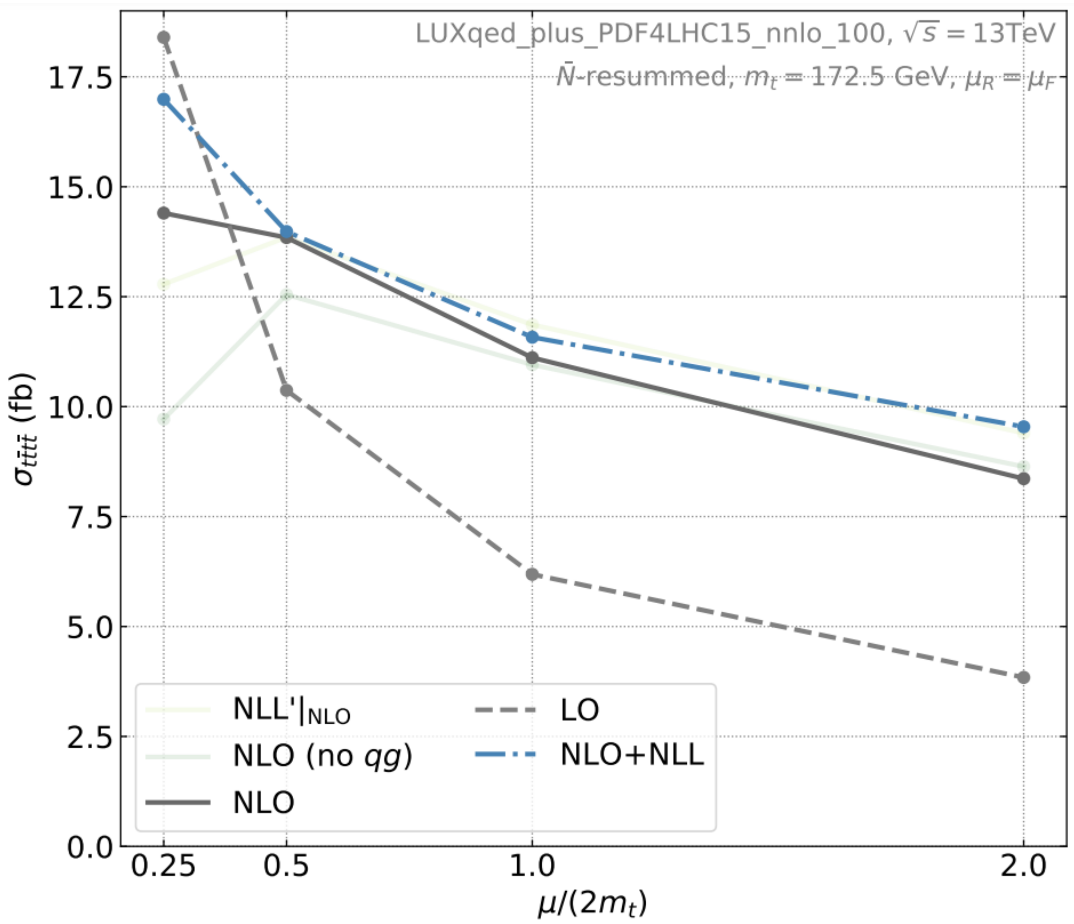
- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD
- Exponentials at NLL

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$



Fixed-order results obtained with
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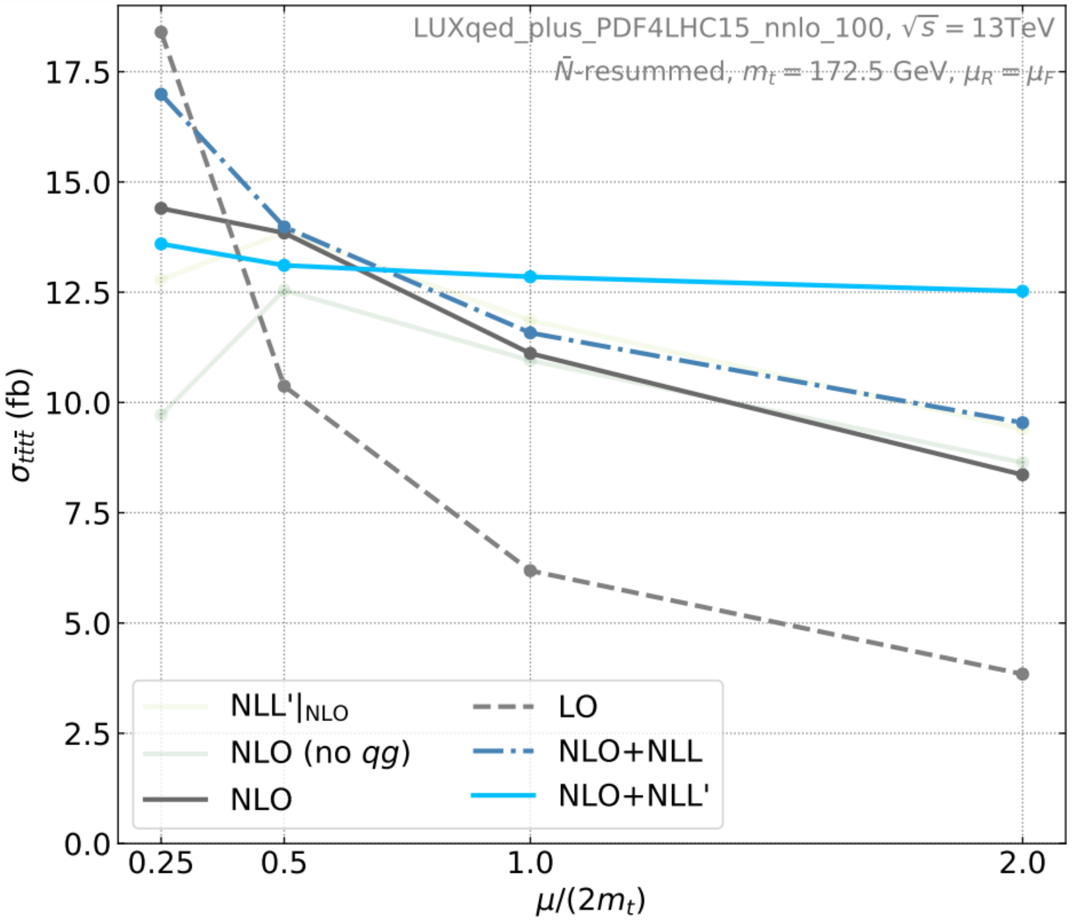
- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD
- Exponentials at NLL
- Upgrade to NLL’

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$



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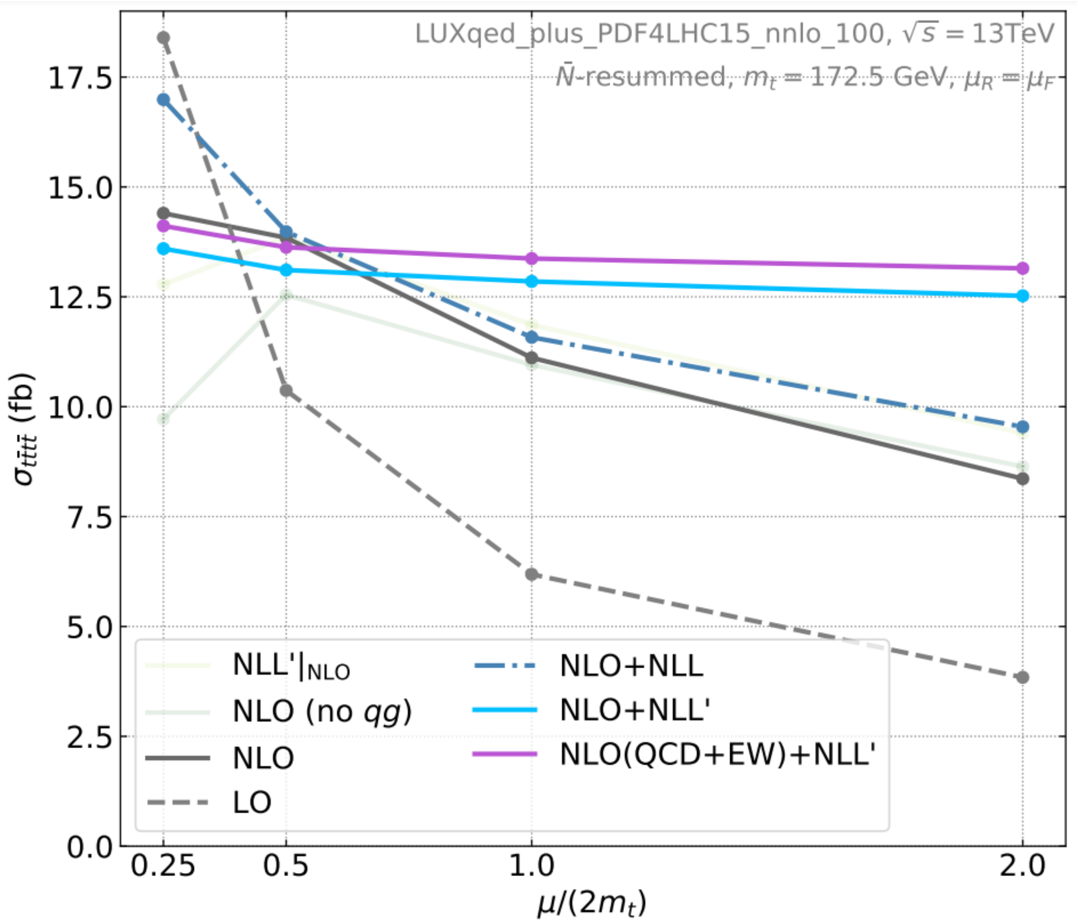
- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD
- Exponentials at NLL
- Upgrade to NLL'
- Match to NLO (QCD+EW)

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$



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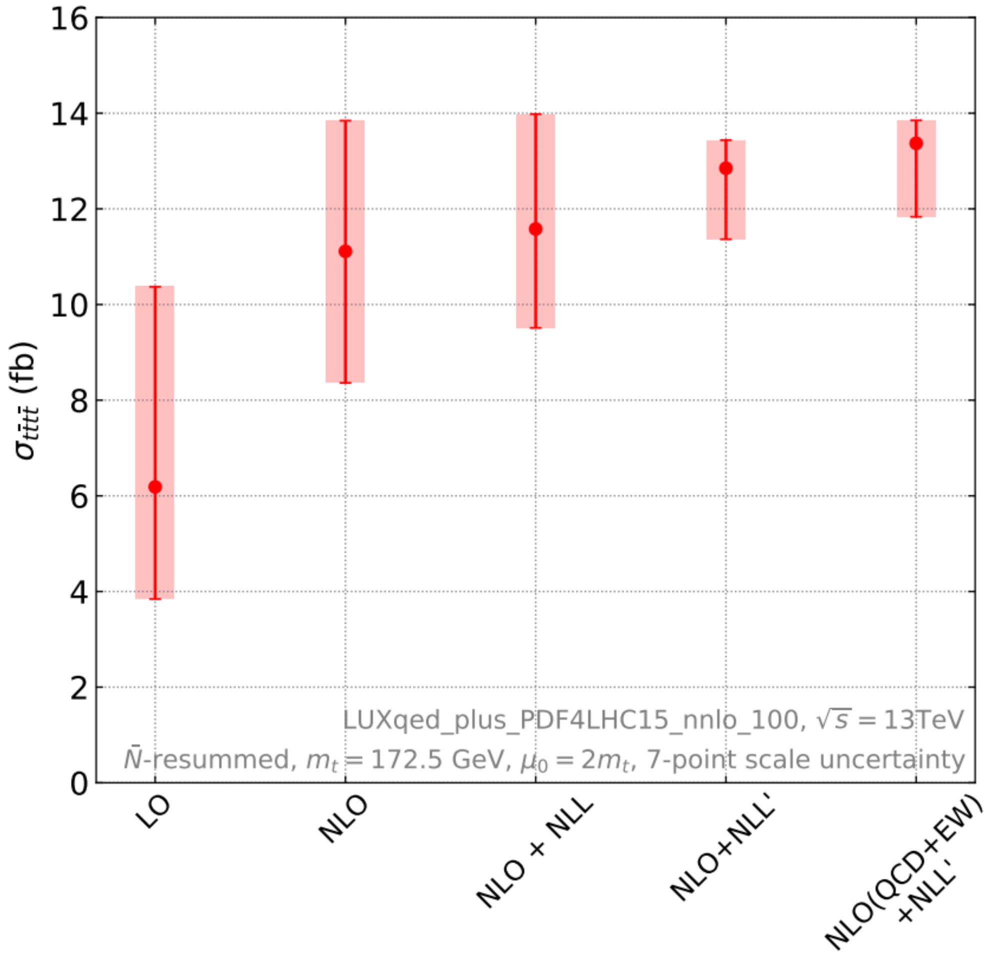
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

- 7-point scale variation

	$\sigma_{t\bar{t}t\bar{t}}$ [fb]	K-factor
NLO	$11.00(2)^{+25.2\%}_{-24.5\%}$	
NLO+NLL	$11.46(2)^{+21.3\%}_{-17.7\%}$	1.04
NLO+NLL′	$12.73(2)^{+4.1\%}_{-11.8\%}$	1.16
NLO (QCD+EW)	$11.64(2)^{+23.2\%}_{-22.8\%}$	
NLO (QCD+EW)+NLL′	$13.37(2)^{+3.6\%}_{-11.4\%}$	1.15

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

PDF error: $\pm$ 6.9%



Fixed-order results obtained with
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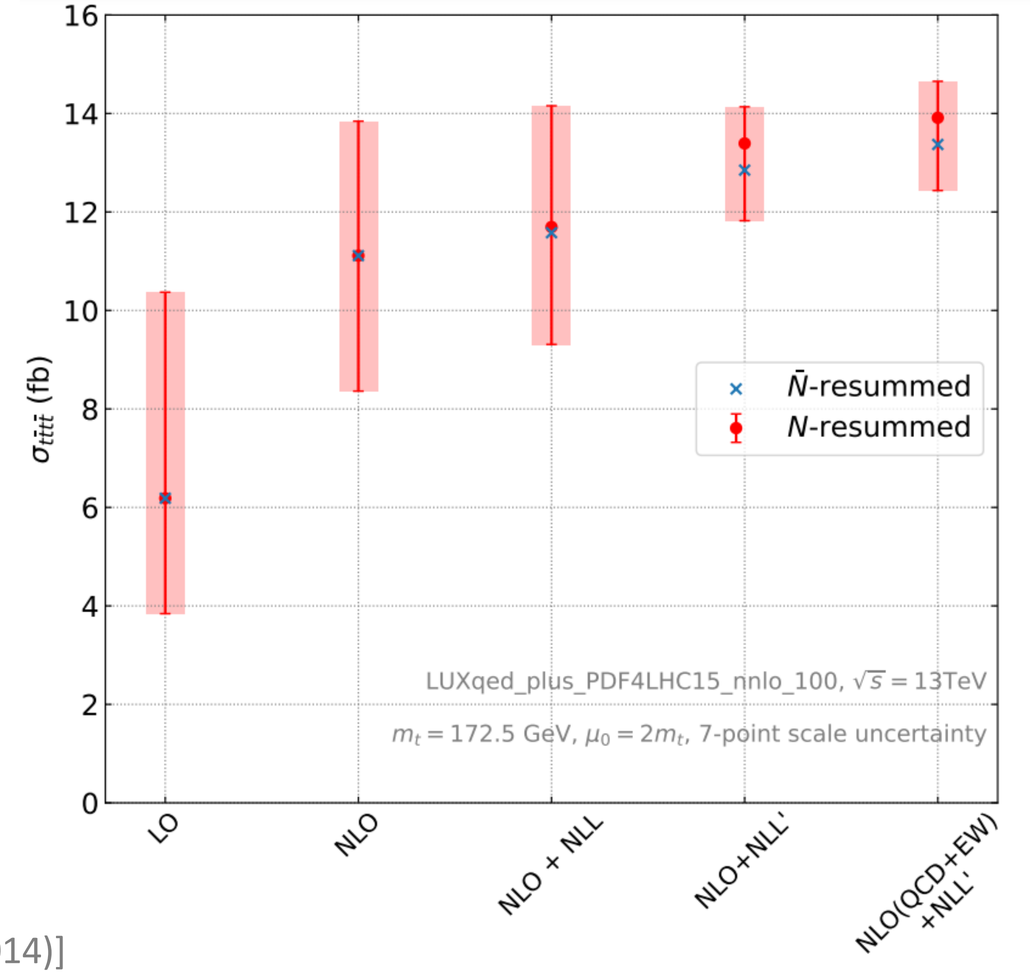
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

- N versus $\bar{N} = e^{\gamma_E} N$

e.g. up to NLO

$$\hat{\sigma}_{ij}^{\mathrm{NLL}'}\big|_{\mathrm{NLO}}(\bar{N}) - \hat{\sigma}_{ij}^{\mathrm{NLL}'}\big|_{\mathrm{NLO}}(N) = \frac{\alpha_s}{\pi}\,\mathrm{Tr}\Bigg[\mathbf{H}_R^{(0)}\,\mathbf{S}_R^{(0)}\Bigg\{\sum_{n=\{i,j\}} C_n\Bigg(\log^2(\bar{N}+1) - \log^2\left(\mathrm{e}^{\gamma_E}(N+1)\right) - \log\frac{s}{\mu_F^2}\log\left(\frac{\bar{N}+1}{(N+1)\mathrm{e}^{\gamma_E}}\right)\Bigg) - 2\mathrm{Re}\left[\mathbf{\Gamma}_R^{(1)}\right]\log\left(\frac{\bar{N}+1}{(N+1)\mathrm{e}^{\gamma_E}}\right)\Bigg\}\Bigg]$$



Fixed-order results obtained with
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[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

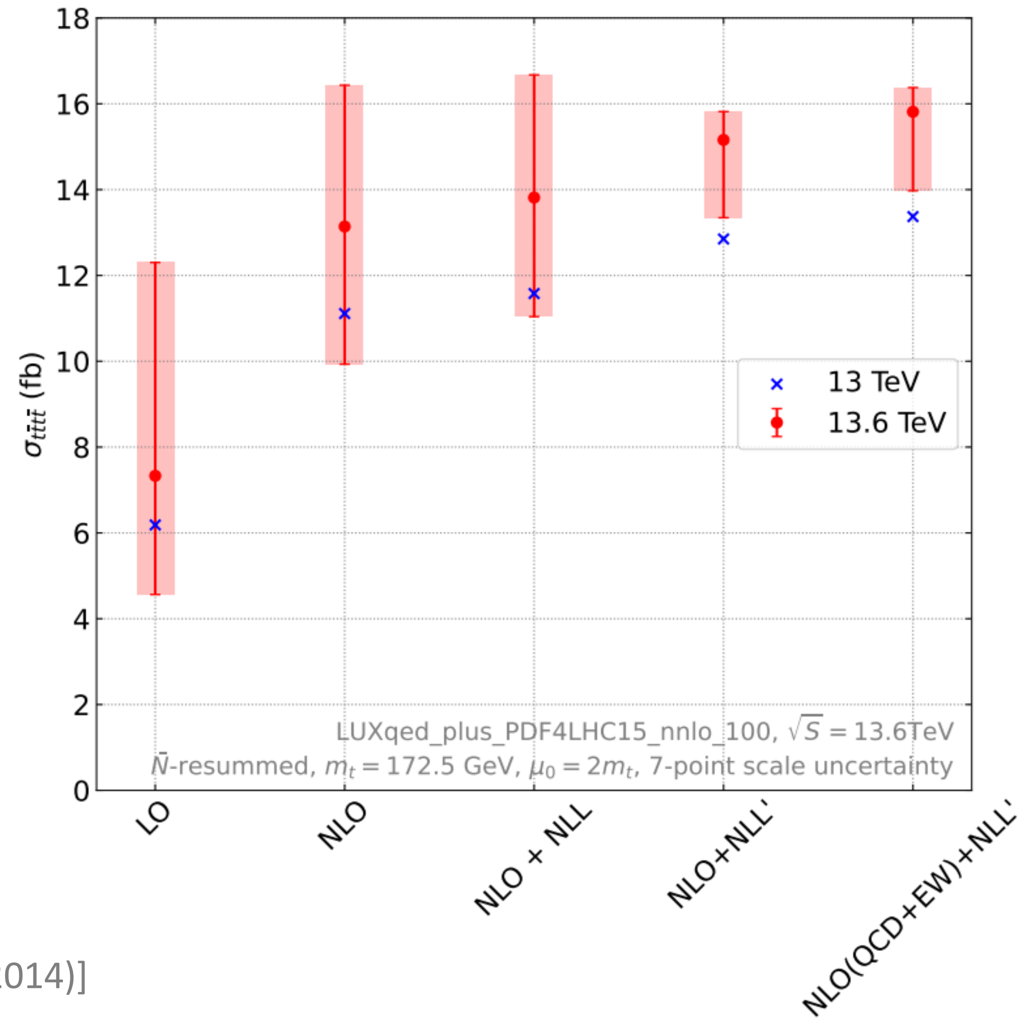
RESULTS for $\sqrt{S} = 13.6$ TeV

- 7-point scale variation

	$\sigma_{t\bar{t}t\bar{t}}$ [fb]	K-factor
NLO	$13.14(2)^{+25.1\%}_{-24.4\%}$	
NLO+NLL	$13.81(2)^{+20.7\%}_{-20.1\%}$	1.05
NLO+NLL′	$15.16(2)^{+4.3\%}_{-11.9\%}$	1.15
NLO (QCD+EW)	$13.80(2)^{+22.9\%}_{-22.6\%}$	
NLO (QCD+EW)+NLL′	$15.81(2)^{+3.6\%}_{-11.6\%}$	1.14

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

PDF error: $\pm$ 6.7%



Fixed-order results obtained with
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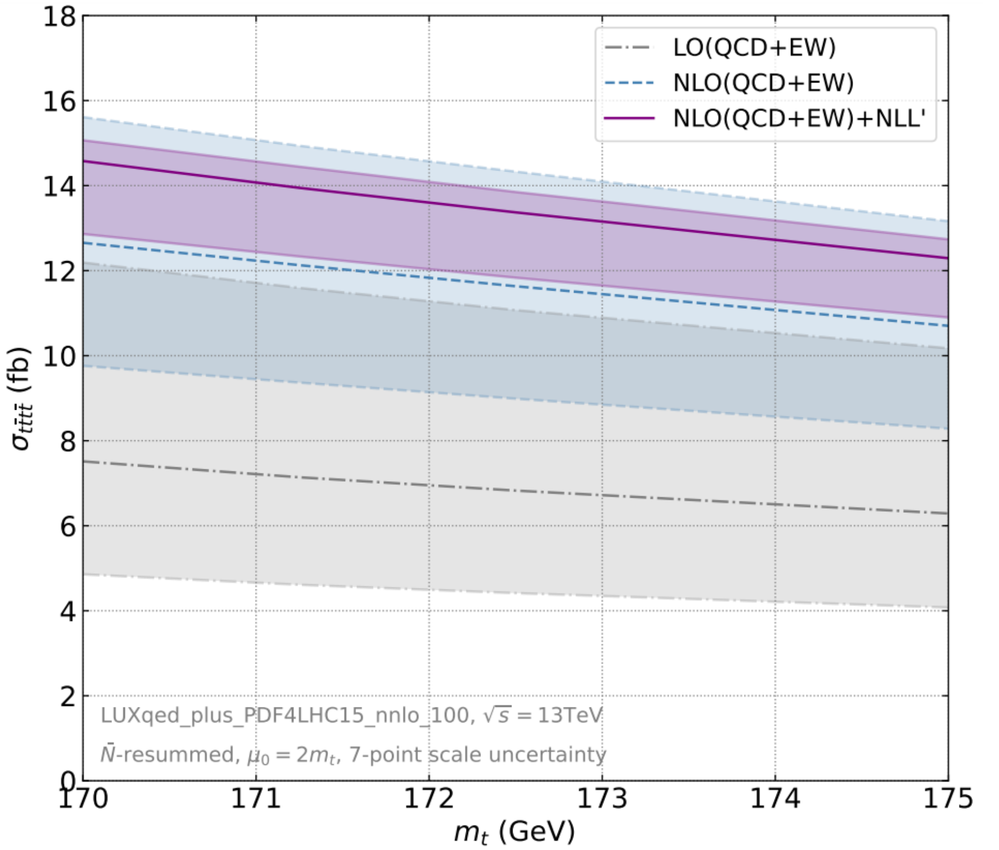
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

TOP MASS DEPENDENCE

- Top mass $\in$ [170,175] GeV



Error bands: 7-point scale uncertainty

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

Fixed-order results obtained with
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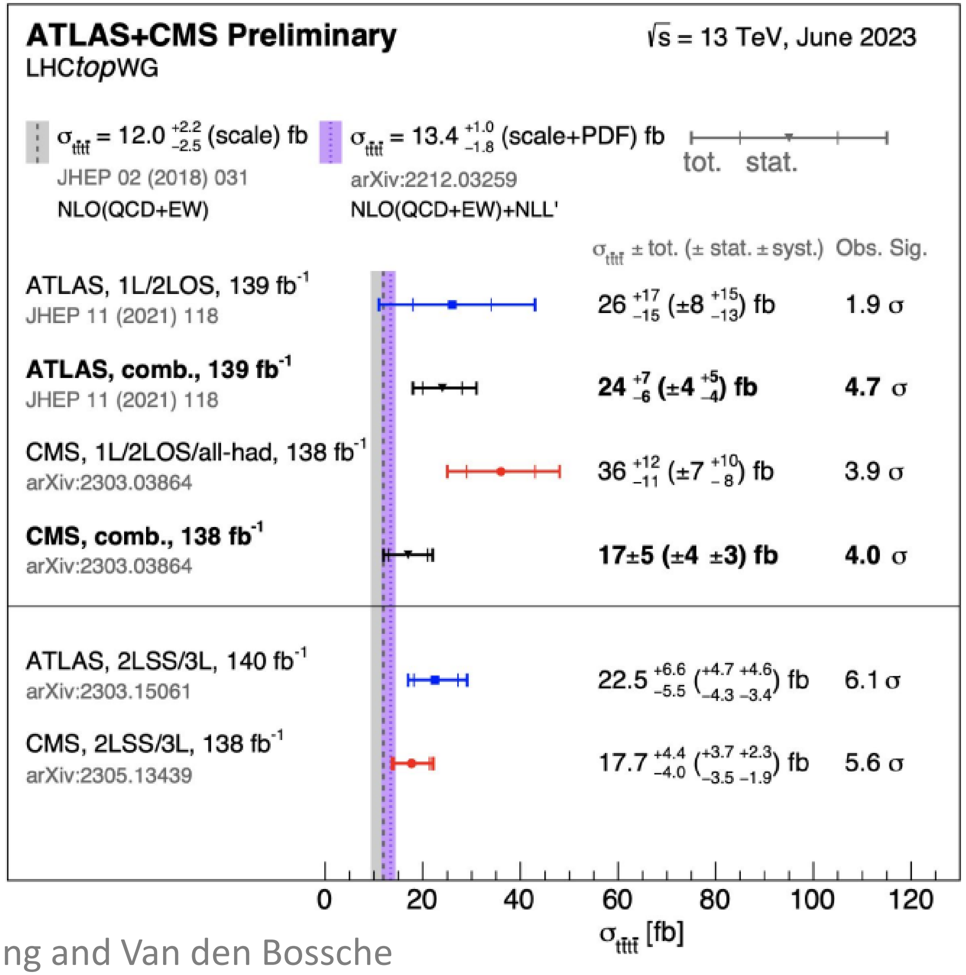
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]



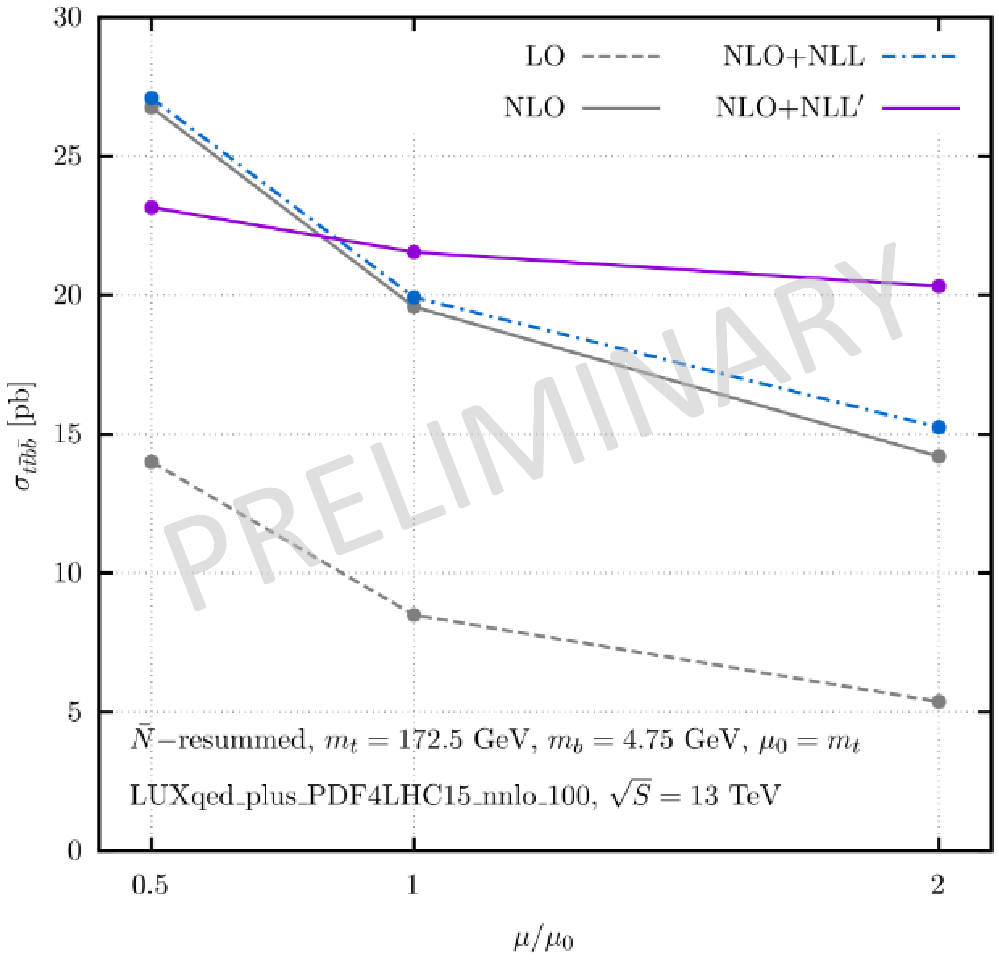
Talk by Zheng and Van den Bossche (23rd LHC TOP WG meeting)

- Soft gluon resummation at NLL’ accuracy
- Significant reduction of the total scale uncertainty

NLO (QCD + EW) + NLL’ at 13 TeV

$$13.37\,(2)\,{}^{+3.6\%}_{-11.4\%}\;{}^{+6.9\%}_{-6.9\%}\ \text{fb}$$

CONCLUSIONS



- Soft gluon resummation at NLL’ accuracy
- Significant reduction of the total scale uncertainty

NLO (QCD + EW) + NLL’ at 13 TeV

$$13.37\,(2)\,{}^{+3.6\%}_{-11.4\%}\,{}^{+6.9\%}_{-6.9\%}\ \text{fb}$$

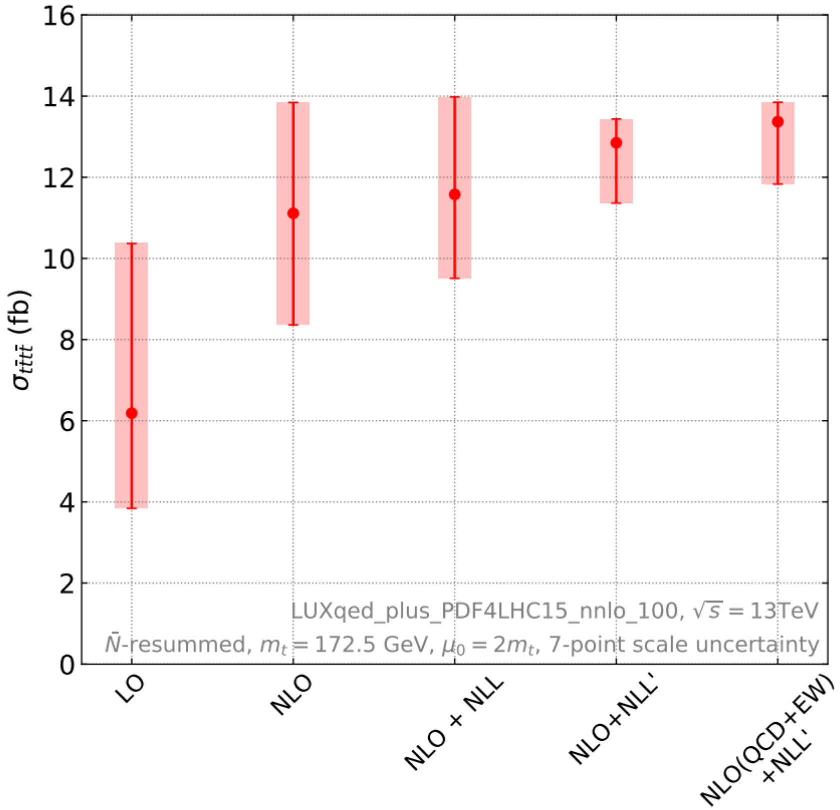
- Next: $t\bar{t}b\bar{b}$ production, invariant mass threshold resummation, ...

Soft gluon resummation for the production of four top quarks at the LHC

Laura Moreno Valero

in collaboration with Melissa van Beekveld and Anna Kulesza

Institute for Theoretical Physics, University of Münster



(based on the paper **arXiv:2212.03259**)

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