Collinear fragmentation of gluon jets at NNLL

Dissecting the collinear structure of quark splitting at NNLL

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To appear soon

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A bird's eye view

- (Semi)-analytic resummation has achieved an impressive accuracy (NNLL and N^{3}LL) over previous decades.

$1-T$	0803.0342,1006.3080,1105.4560
ρ_H	1005.1644
B_T, B_W	1210.0580
C-parameter	1411.6633
EEC	hep-ph/0407241,1708.04093,1801.02627
Angularities	1806.10622,1807.11487
D-parameter	1912.09341

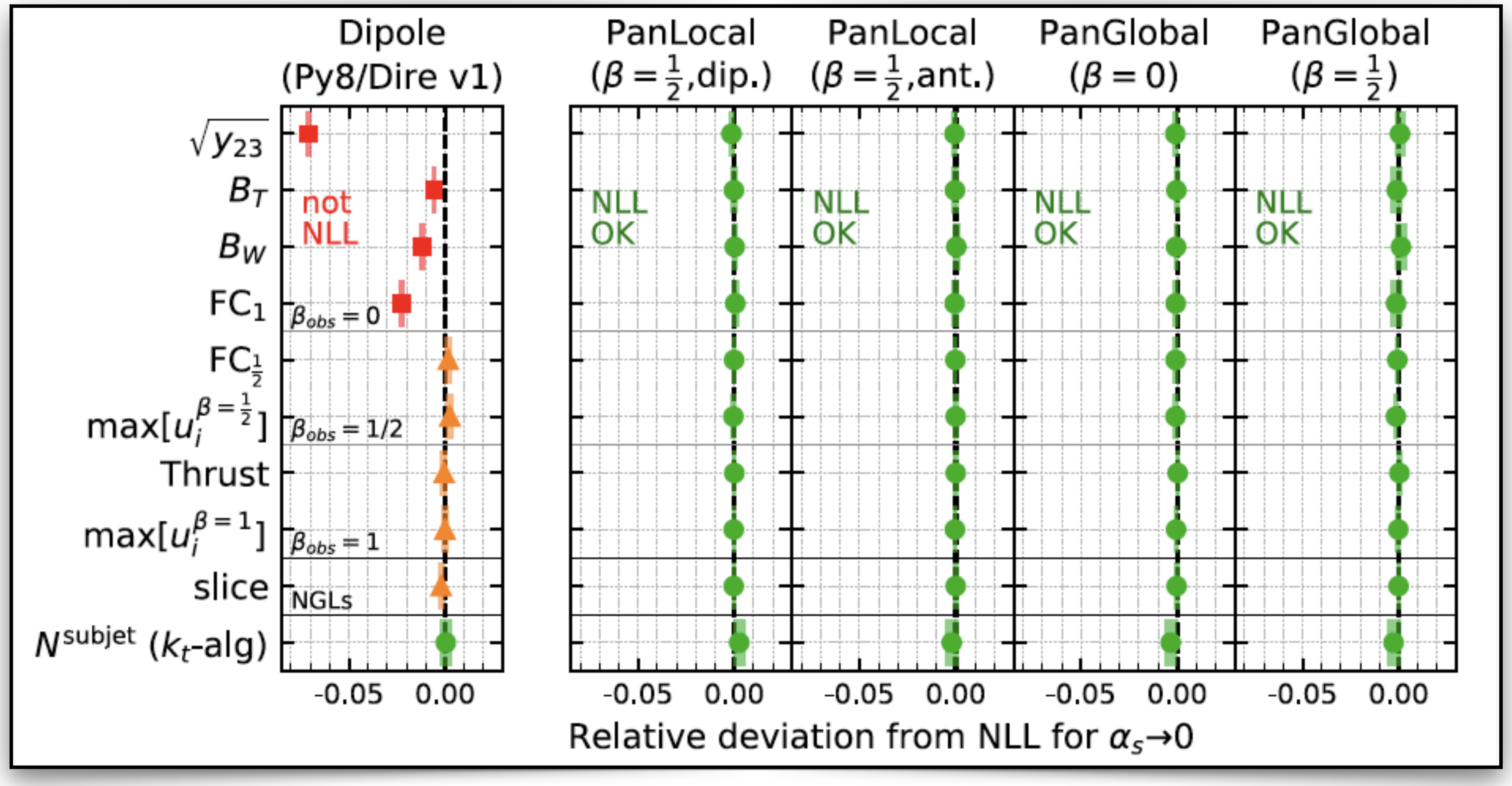
- Parton showers (PS) have not kept up with such progress.

- PS are essential due to their versatility: It is much more efficient to simulate QCD dynamics than to resum a specific observable.

Motivation: Recent progress in NLL accurate PS

- The PanScales family of PS has been able to achieve NLL accuracy for any recursive IRC safe observable:

Dasgupta et. al. (2002.11114), colour and spin (2011.10054,2103.16526,2111.01161), G. Salam "The power and limits of parton showers" https://gsalam.web.cern.ch/gsalam/talks/repo/202109-SLAC-seminar\\-SLAC-panscales-seminar.pdf



- The crux of this development is to design recoil maps that preserve the correct physical limits required for NLL.

- More in Silvia's, Alexander's & Alba's talks.

Outline: what do we need to achieve NNLL?

- The goal is to define and compute a differential *anomalous dimension* which encodes collinear dynamics at NNLL.
- Outline:
    1. Review the results for quark jets as these form the conceptual basis of the physics.
    2. Compute the relevant *anomalous dimension* for gluon jets.
        - Tool kit: triple-collinear splitting functions (double-real) and 1L correction to 1→2 splitting (real-virtual).
    3. Derive new resummed results for groomed jet observables.

Look back at NLL

- Over 30 years ago Catani, Marchesini & Webber introduced the notion of a soft physical coupling:

$$\mathrm{d}\mathcal{P}_{\mathrm{sc}} = C_i \frac{\alpha_s^{\mathrm{phys}}}{\pi} \frac{\mathrm{d}k_t^2}{k_t^2} \frac{\mathrm{d}z}{1-z}, \qquad \alpha_s^{\mathrm{phys}} = \alpha_s(k_t^2)\left(1 + K_{\mathrm{CMW}} \frac{\alpha_s(k_t^2)}{2\pi}\right)$$

- The CMW coupling represents the intensity of soft gluon radiation.

$$K_{\mathrm{CMW}} = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_F$$

- For showers that intertwine real and virtual corrections through unitarity, specifying the scheme and scale of the coupling is the sole NLO ingredient to achieve NLL accuracy.

Questions for NNLL PS: key concepts

- How to include NLO virtual corrections in a shower algorithm?

- The CMW lesson: at NNLL can we properly define an inclusive emission probability, i.e.

$$d\mathcal{P}_q \stackrel{!}{=} \frac{C_F}{2\pi}\frac{d\theta^2}{\theta^2}dz\, p_{qq}(z)\, \alpha_{\text{eff.}}\left(z, \theta^2\right)$$

- The inclusive limit of the double-soft function defines the CMW coupling ↔ furnish a commensurate understanding of the triple-collinear splitting functions.

- The coefficient B_2 → define a suitable differential version thereof?

Introduction into B_2

- So what exactly is $B_2^{q/g}$?
- Let us take an example from the transverse momentum distribution in hadronic collisions:

$$\frac{d\sigma_{ab\to F}}{dp_t^2} = \frac{1}{2}\int b\, db J_0(bp_t)\, W_{ab}^F(s, Q, b)$$

de Florian & Grazzini hep-ph/0108273 (see also the references therein)

- The interesting piece is the function W_{ab}^F, which includes the quark/gluon form factor:

$$S_{q/g}(Q, b) = \exp\left(-\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A_{q/g}(\alpha_s)\ln\frac{Q^2}{q^2} + B_{q/g}(\alpha_s)\right]\right)$$

Introduction into B_2

- Each function has a perturbative expansion. The A function has a soft origin, while the B function has a hard-collinear origin.

$$A_{q/g} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n A_{(n)}^{q/g}, \quad B_{q/g} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n B_{(n)}^{q/g}$$

- Let us focus on the B series. Going back to direct space, one finds a hard-collinear logarithm:

$$\left(\frac{\alpha_s}{2\pi}\right) B_1^{q/g} \quad || \quad \left(\frac{\alpha_s}{2\pi}\right)^2 B_2^{q/g}$$

- This talk is about a suitably defined differential $\mathscr{B}_2(z)$, which controls the effective coupling.

- To give an example, at leading order we have:

$$B_1^q = -\frac{3C_F}{2} \rightarrow \mathscr{B}_1^q(z) = -C_F(1+z)$$

$$B_1^g = -b_0 \rightarrow \mathscr{B}_1^g(z) = C_A(-2+z(1-z)) + T_R n_f \left(z^2 + (1-z)^2\right)$$

Introduction into B_2

- What do we know about the structure of $B_2^{q/g}$?

- For final-state observables, we have:

Banfi, BKE & Monni 1807.11487, Banfi et. al. 1412.2126

See also de Florian & Grazzini hep-ph/0407241, Davies & Stirling Nucl. Phys. B 244 (1984)

$$B_2^q = -\gamma_q^{(2)} + b_0\, X_v^q, \quad B_2^g = -\gamma_g^{(2)} + b_0\, X_v^g$$

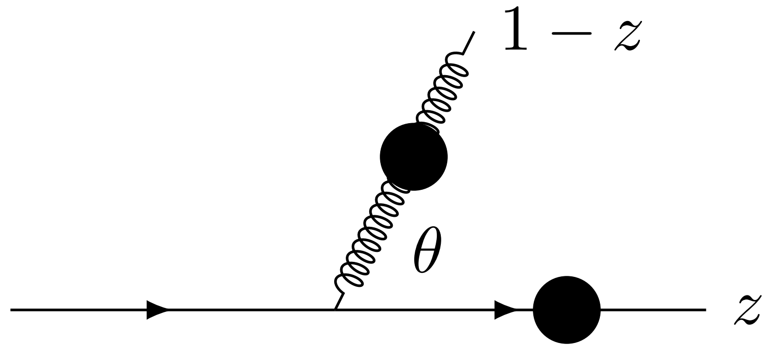
- We have two pieces. An observable dependent constant, X_v, that comes multiplied by b_0. The other pieces, $\gamma_{q/g}^{(2)}$, is universal and represents the endpoint contribution, i.e. $\delta(1-x)$, to the NLO DGLAP kernel obtained from sum rules.

$$\gamma_q^{(2)} = C_F^2\left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3)\right) + C_F C_A\left(\frac{17}{24} + \frac{11\pi^2}{18} - 3\zeta(3)\right) - C_F T_R n_f\left(\frac{1}{6} + \frac{2\pi^2}{9}\right)$$

$$\gamma_g^{(2)} = C_A^2\left(\frac{8}{3} + 3\zeta(3)\right) - C_F T_R n_f - \frac{4}{3} C_A T_R n_f$$

Quark jets

- Let us recap the results of **arXiv:2109.07496**



$$d\mathscr{P}^q_{\rm NNLL} = \frac{d\theta^2}{\theta^2} dz \left\{ \left(\frac{\alpha_s}{2\pi}\right) \left(C_F\, p_{qq}(z) - \frac{\alpha_s}{2\pi}\frac{2\,C_F}{1-z} b_0 \ln\frac{\theta^2}{4} - \frac{\alpha_s}{2\pi}\mathscr{B}^q_1(z)\, b_0 \ln\frac{\theta^2}{4} - \frac{\alpha_s}{2\pi}\frac{2\,C_F}{1-z} b_0 \ln(1-z)^2 + \frac{\alpha_s}{2\pi}\frac{2\,C_F}{1-z} K^{(1)} \right) + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathscr{B}^q_2(z) \right\}$$

- Here we see the following features:
  - The emergence of the scale of the coupling $k_t^2 = (1-z)^2\,\theta^2$
  - The CMW piece.
  - The LO collinear anomalous dimension.
  - The NLO collinear anomalous dimension.

Quark jets (contd.)

- $\mathscr{B}_2^q(z)$ is decomposed by colour factors:

$$\mathscr{B}_2^q(z) = C_F^2\,\mathscr{B}_2^{q,(\mathrm{ab.})}(z) + C_F C_A\,\mathscr{B}_2^{q,(\mathrm{nab.})}(z) + C_F T_R n_f\,\mathscr{B}_2^{q,n_f}(z) + C_F\left(C_F - \frac{C_A}{2}\right)\mathscr{B}_2^{q,(\mathrm{id.})}(z)$$

- The individual expressions are given in **arXiv:2109.07496**

- This function is, by construction, free from any soft physics and is integrable in $z \in [0,1]$

$$B_2^q\left(\theta^2\right) = \int_0^1 dz\,\mathscr{B}_2^q(z;\theta^2) = -\gamma_q^{(2)} + b_0\,X_{\theta^2}^q \rightarrow X_{\theta^2}^q = C_F\left(\frac{2\pi^2}{3} - \frac{13}{2}\right)$$

- The take-home message is that we can determine X_v^q for any observable using $d\mathscr{P}_{\mathrm{NNLL}}^q$ and the one-gluon form of the observable.

Gluon jets (to appear soon arXiv:2306.xxxx)

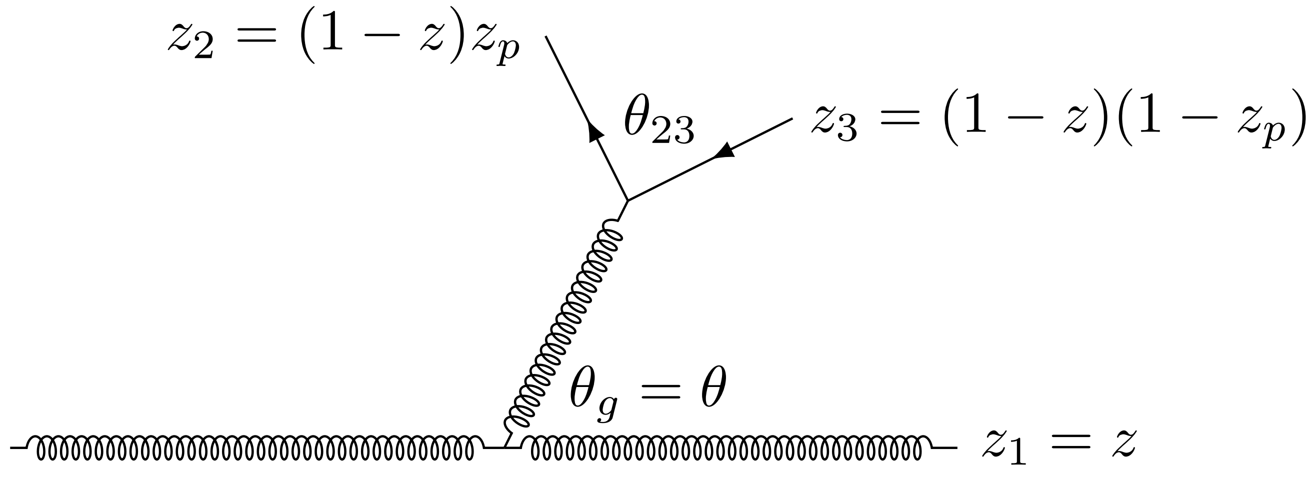
- $\mathscr{B}_2^g(z)$ is decomposed by colour factors:

$$d\mathscr{P}^g_{\rm NNLL} = \frac{d\theta^2}{\theta^2} dz \left\{ \left(\frac{\alpha_s}{2\pi}\right) \left(C_A\, p_{gg}(z) + T_R n_f\, p_{qg}(z) - \frac{\alpha_s C_A}{2\pi} \frac{b_0}{z(1-z)} \ln\frac{\theta^2}{4} - \frac{\alpha_s}{2\pi} \mathscr{B}_1^g(z)\, b_0 \ln\frac{\theta^2}{4} - \frac{\alpha_s C_A}{2\pi} \frac{b_0}{z(1-z)} \ln(1-z)^2 + \frac{\alpha_s}{2\pi} \frac{C_A}{z(1-z)} K^{(1)} \right) + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathscr{B}_2^g(z) \right\}$$

- We observe immediately the Casimir replacement, $C_F \to C_A$, for all soft-enhanced structures.
- The LO collinear anomalous dimension sits in the right place.
- The computation carries through identically to the quark case, let us dive right in.

Triple-collinear splitting functions: Catani & Grazzini hep-ph/9810389, Campbell & Glover hep-ph/9710255

- This channel has the feature that there is a single collinear singularity as $\theta_{q\bar{q}} \to 0$



    - The kinematics variables fixed are both θ_g and z

- The computation is conveniently done using the “web variables”:

$$d\Phi^{\text{web}}_{1\to 3} = \frac{(4\pi)^{2\epsilon}}{256\pi^4} \frac{2z^{1-2\epsilon}dz}{1-z} \frac{1}{\Gamma(1-\epsilon)} \frac{d^{2-2\epsilon}k_\perp}{\Omega_{2-2\epsilon}} \frac{d\cancel{s_{12}}\ s_{23}}{(\cancel{s_{12}})^\epsilon} \frac{dz_p}{(z_p(1-z_p))^\epsilon} \frac{1}{\Gamma(1-\epsilon)} \frac{d\Omega_{2-2\epsilon}}{\Omega_{2-2\epsilon}}$$

$C_A T_R$

- We obtain the following analytic result:

$$\left(\frac{\theta^2}{\sigma_0}\frac{d^2\sigma^{(2)}_{\mathscr{R}}}{d\theta^2\,dz}\right)^{C_A T_R} = \frac{1}{2!}C_A T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 z^{-3\epsilon}(1-z)^{-3\epsilon}\,\theta_g^{-4\epsilon}\left[p_{gg}(z)\left(-\frac{8}{3\epsilon}-\frac{40}{9}\right)-\frac{4}{3}(1+z)\,\ln z-\frac{4}{3}(2-z)\,\ln(1-z)+\frac{26}{9}\left(z^2+(1-z)^2-\frac{1}{z(1-z)}\right)+\frac{10}{3}\right]$$

- To extract the anomalous dimension we need two more inputs
  - Virtual corrections (Sborlini, de Florian & Rodrigo **arXiv:1310.6841**)
  - Subtract the iteration of the NLL result, i.e.

$$\left(\frac{\theta^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta^2 dz}\right)_{\text{itr.}1\to 2} = \left(\frac{\alpha_s}{2\pi}\right)^2 z^{-2\epsilon}(1-z)^{-2\epsilon}\theta^{-4\epsilon}\left(C_A\,p_{gg}(z)+T_R n_f\,p_{qg}(z,\epsilon)\right)\frac{1}{-\epsilon}\int dz_p z_p^{-2\epsilon}(1-z_p)^{-2\epsilon}\left(C_A\,p_{gg}(z_p)+T_R n_f\,p_{qg}(z_p,\epsilon)\right)$$

$C_A T_R$

- After adding virtual corrections and subtracting NLL structures, we get:

$$\mathscr{B}_2^{g,C_AT_R}(z) = -\,p_{qg}\left(\ln^2 z + \ln^2(1-z)\right) + \frac{1}{9}(28 - 41z + 41z^2) + \ln z\left(\frac{4}{3(1-z)} - \frac{26}{3}z^2 + 8z - 7\right) + \ln(1-z)\left(\frac{4}{3z} - \frac{26}{3}(1-z)^2 + 8(1-z) - 7\right)$$

- The $T_R^2 n_f^2$ colour channel has no double-real correction, it comes purely from virtual corrections:

$$\mathscr{B}_2^{g,T_R^2n_f^2}(z) = \; p_{qg}(z)\left(\frac{1}{3} + \frac{4}{3}\ln(z(1-z))\right)$$

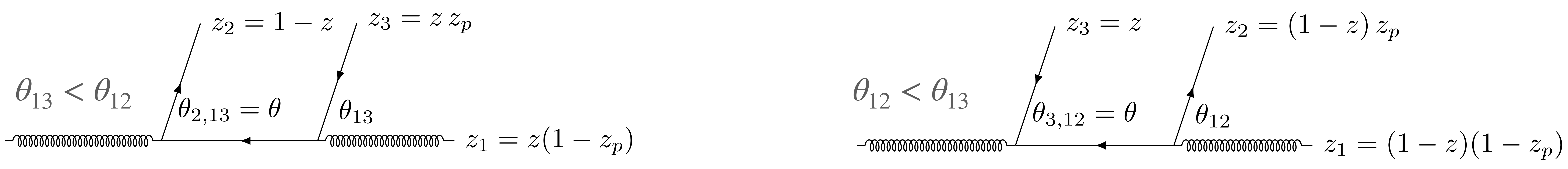
- Integrating both functions:

$$X_{\theta^2}^{g} = -\,C_A\left(\frac{67}{9} - \frac{2\pi^2}{3}\right) + \frac{23}{9}T_R n_f$$

- The observable dependent constant can be computed for any observable using the inclusive emission probability integrated against the observable in the limit of a single splitting.

$C_F T_R$

- This channel has two collinear singularities $\theta_{qg} \to 0$ and $\theta_{\bar{q}g} \to 0$
- The situation if identical to the abelian C_F^2 contribution for quark jets.
- We divide the phase space into two regions, where at most a single collinear singularity appears in each:



- Only starts at NNLL:

$$\mathscr{B}_2^{g,C_F T_R}(z) = p_{qg}(z)\left(\frac{1}{2}\ln^2\left(\frac{z}{1-z}\right) + \frac{1}{2}\ln^2\left(\frac{1-z}{z}\right) - \frac{\pi^3}{3} + 5\right) + H_{\text{fin.}}(z) \longrightarrow \int_0^1 dz\, \mathscr{B}_2^{g,C_F T_R}(z) = 1$$

C_A^2

- This channel has a collinear singularity as *any* $\theta_{g_i g_j} \to 0$

- Unlike the situation for quark jets, correlated and independent emission terms are mixed together.

- 'Divide and conquer' by partitioning the phase space into 3 identical regions, e.g. $\min\{\theta_{ij}\} = \theta_{12}$, and fix $z = z_3$.

- One can instead leverage the universality of the double-soft limit and *locally* subtract off the double-soft function.

Divide and conquer ⟶

$$\langle \hat{P}^{\text{sub.}}_{g_1 g_2 g_3} \rangle \equiv \frac{1}{s_{123}^2} \langle \hat{P}_{g_1 g_2 g_3} \rangle - \langle \hat{P}_{g_1 g_2 ; g_3} \rangle - \langle \hat{P}_{g_1 g_3 ; g_2} \rangle - \langle \hat{P}_{g_2 g_3 ; g_1} \rangle$$

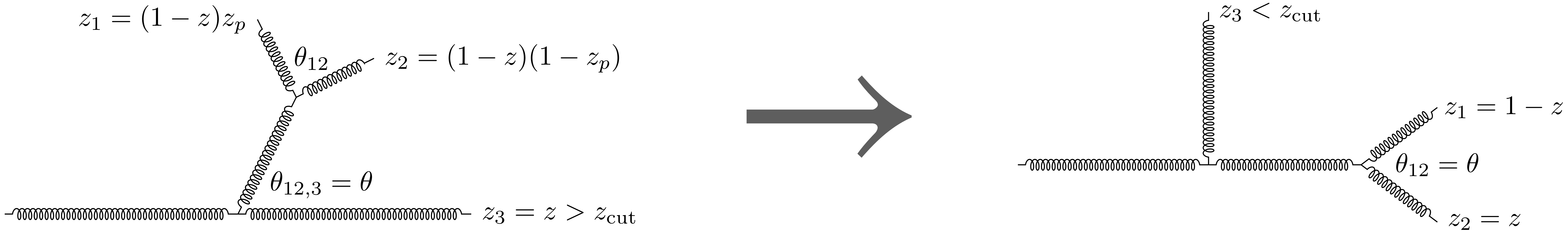
Full splitting function

DS limit: $z_i, z_j \to 0$

- When the DS functions are *added* back, they are to be treated identically to the quark case. Here the separation between correlated and independent is dynamical.

C_A^2 (contd.)

- One complication when dealing with $\langle \hat{P}^{\text{sub.}}_{g_1 g_2 g_3} \rangle$ manifests as $z \to 0$. As the gluon becomes soft, it is allowed to fly off at wide angle and the angle is rendered ill-defined.



- Putting everything in, subtracting the NLL pieces, we find:

$$\mathscr{B}_2^{g,C_A^2}(z) = G^{\text{sub.}}_{z_3 > z_{\text{cut}}}(z) + \mathscr{B}_2^{g,\,C_A^2,\,\text{analytic}}(z)$$

$$\int_0^1 dz\, G^{\text{sub.}}_{z_3 > z_{\text{cut}}}(z) \simeq 6.976(1) \qquad \int_0^1 dz\, \mathscr{B}_2^{g,C_A^2,\text{analytic}}(z) = -14.832.... \longrightarrow \left(-\gamma_g^{(2)} + b_0 X^g_{\theta^2}\right)_{C_A^2}$$

Resummation of groomed observables

- As a practical application of these results, we can obtain new resummed results for a host of groomed jet observables.

- Following the ARES formalism, Ref. **arXiv:2211:03820** showed that knowing the anomalous dimension B_2 of the ungroomed variant of observables allows for a straightforward NNLL resummation of mMDT groomed observables.

- Ref. **arXiv:2211:03820** initiated such a formalism for quark jets, and focused on WTA angularities in e^+e^-:

$$\lambda_x = \frac{1}{E}\sum_i E_i \,|\sin\theta_i|^x (1-|\cos\theta_i|)^{1-x}$$

- The resummed cross section takes a very simple form

$$\Sigma(v) = \left(1 + \frac{\alpha_s(E^2)}{2\pi} C_v^{(1)}(z_{\text{cut}})\right) e^{-R_v^q(v, z_{\text{cut}})} \left(1 + \mathscr{F}_{\text{clust}}(v)\right)$$

- The collinear logarithm in the Sudakov has B_2^q as its coefficient.

Resummation of groomed observables

- It requires no work to generalise the resummed formula to capture gluon jets as well:

$$C_v^{q(1)}(z_{\text{cut}}) = H^{q(1)} + X_v^q + C_F\left(4\ln 2\ln z_{\text{cut}} - \frac{\pi^2}{6}\right) \longrightarrow C_v^{g(1)}(z_{\text{cut}}) = H^{g(1)} + X_v^g + C_A\left(4\ln 2\ln z_{\text{cut}} - \frac{\pi^2}{6}\right)$$

- The whole problem reduces to the determination of X_v for any observable, which we can obtain quite easily using our inclusive emission probability.

$$X_{\lambda_x}^q = C_F\frac{(9-\pi^2+9\ln 2)}{3(2-x)} + X_{\theta^2}^q \qquad X_{\lambda_x}^g = \frac{1}{2-x}\left(C_A\left(\frac{137}{36}-\frac{\pi^2}{3}+\frac{44\ln 2}{12}\right) - T_R n_f\left(\frac{29}{18}+\frac{4\ln 2}{3}\right)\right) + X_{\theta^2}^g$$

- We can do much more with very simple computation!

$$FC_x = \frac{1}{E^2}\sum_{i\neq j} E_i E_j |\sin\theta_{ij}|^x (1-|\cos\theta_{ij}|)^{1-x}$$

Summary & Outlook

- Understanding collinear dynamics at NNLL is one core ingredient of next generation parton showers.
- We have been able to define and compute a NLO differential anomalous dimension, which will control the no-emission probability in a self-similar branching process.
- This anomalous dimension ties in very well with what we know from analytic resummations.
- Phenomenological study of multiple jet observables is one future avenue.

→ **The inclusion of the differential anomalous dimension in a shower Monte Carlo.**

Thanks PSR23!