A Logarithmically Accurate Resummation In C++

Parton Showers and Resummation 2023, 7 June 2023

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A Logarithmically Accurate Resummation In C++

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - Underlying event
  - Hadronisation
  - QED radiation
  - Hadron Decays

See also talks by Basem El-Menoufi, Christian Preuss

See also talks by Silvia Ferrario Ravasio, Alexander Karlberg

This Talk:

Why?

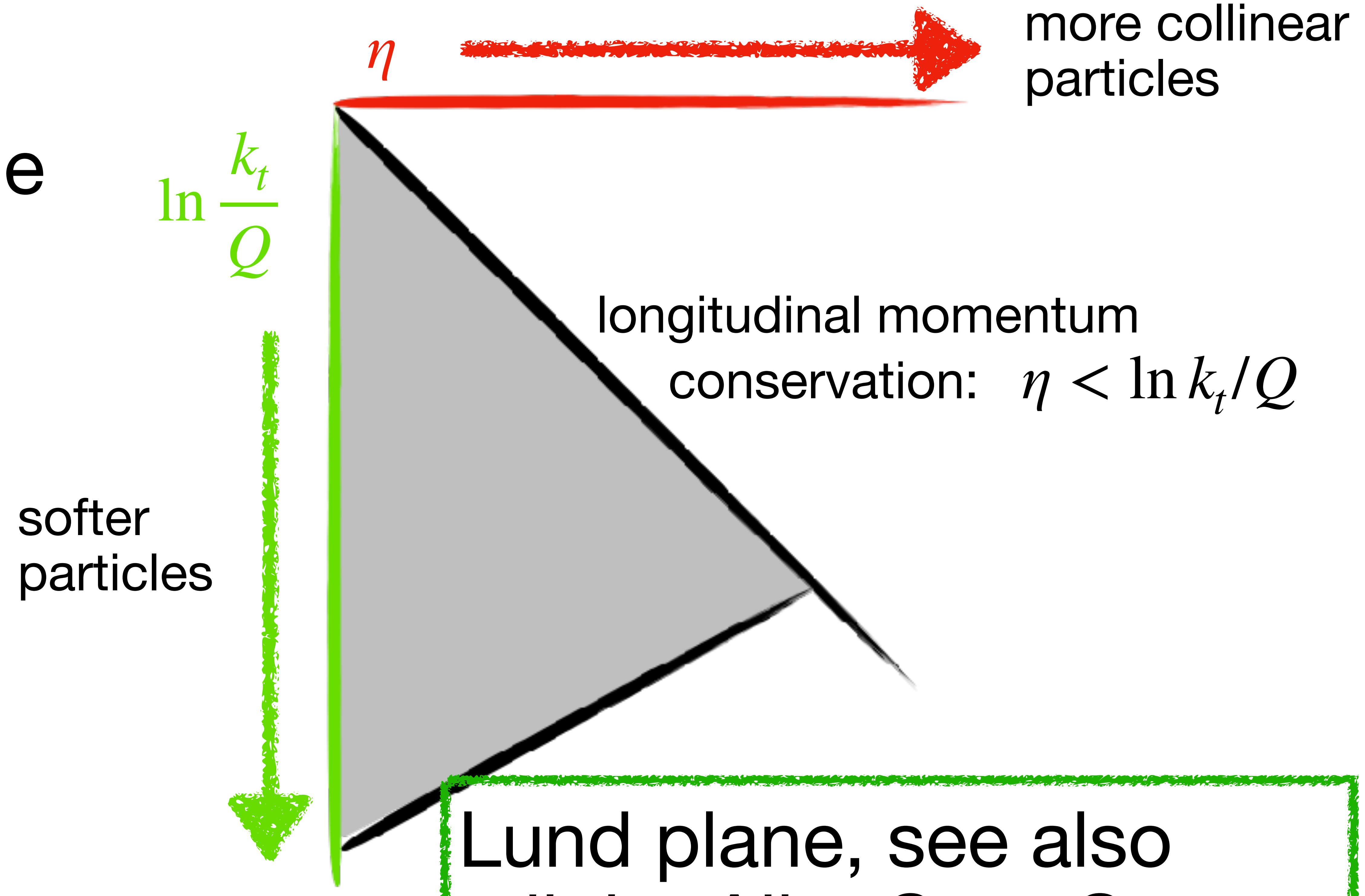
- parton showers resum large logs ~ NLL, but open questions on actual accuracy
- starting work towards NNLL/NLO evolution → probably better resolve this first
- recent formal discussion → current dipole showers need reworking [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]

parton showers - Cliff notes version

- no-emission probability (sudakov factor)

$$\sim \exp\left[-\int_{t_0}^{t_1} \frac{dk_t}{k_t} dz \frac{\alpha_S}{2\pi} P(z)\right]$$

- splitting kernels $P(z)$ captures soft and collinear limits of matrix elements
- fill phase space ordered in evolution variable $(k_t, \theta, q^2, \ldots) \Rightarrow$ here k_t ordered shower
- generate new final state after emission according to recoil scheme



Lund plane, see also talk by Alba Soto Ontoso

splitting of Eikonal

Starting point: eikonal

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2} \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} \equiv \frac{W_{ik,j}}{E_j^2}$$

naive implementation leads to soft double counting need to split into ij and kj collinear terms [Marchesini, Webber ‘88]

Option 1:

$$W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}\ , \qquad \text{where} \qquad \tilde{W}^i_{ik,j} = \frac{1}{2}\left(\frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} + \frac{1}{1 - \cos\theta_{ij}} - \frac{1}{1 - \cos\theta_{jk}}\right)$$

- e.g. Angular ordered shower, downside: problems with NGLs

Option 2: follow [Catani, Seymour ’97]

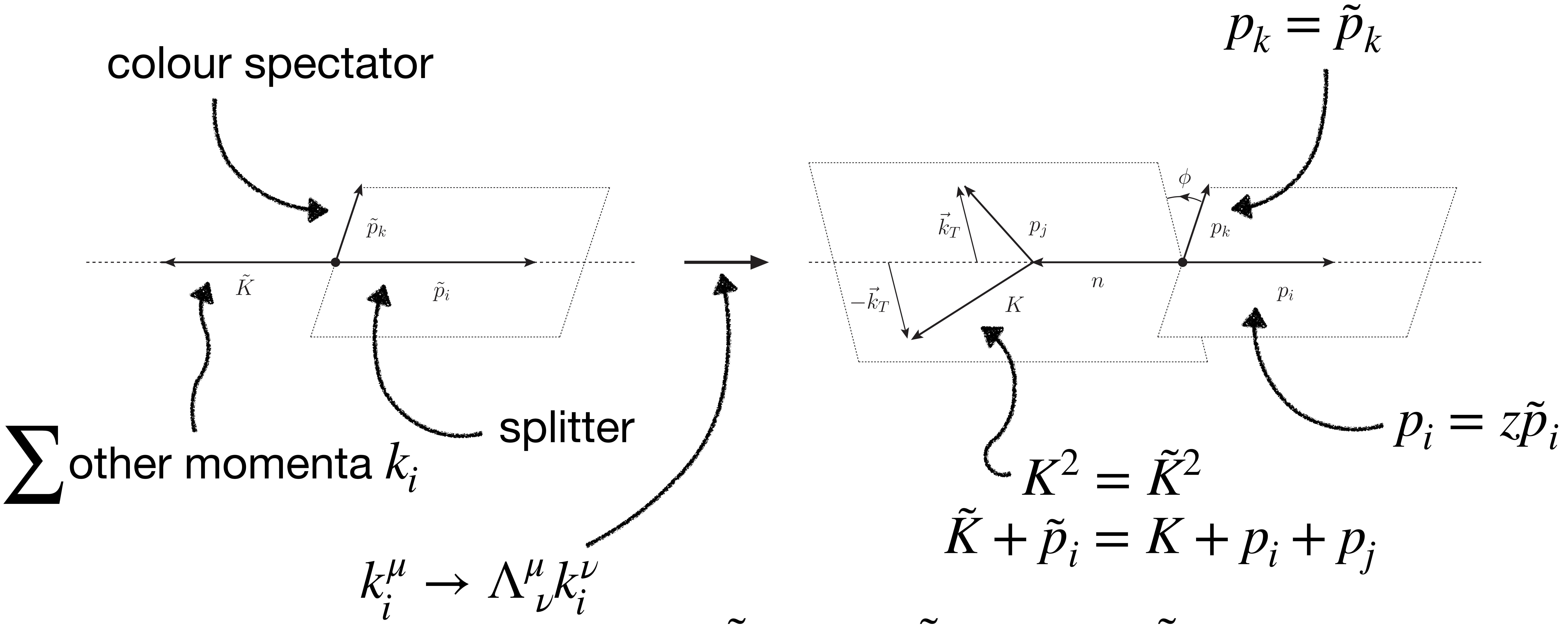
$$W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}\ , \qquad \text{where} \qquad \bar{W}^i_{ik,j} = \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(2 - \cos\theta_{ij} - \cos\theta_{jk})}$$

- full phase space coverage, splitting functions remain positive definite

Note related ideas in [Forshaw, Holguin, Plätzer ’20]

kinematics - global recoil scheme

- Before splitting:
- After splitting:



[Catani, Seymour '97]

$$\Lambda^\mu{}_\nu = g^\mu{}_\nu - \frac{(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{K\cdot\tilde{K} + \tilde{K}^2} + 2\frac{K^\mu \tilde{K}_\nu}{\tilde{K}^2} \quad\to\quad \Lambda^\mu{}_\nu \tilde{K}^\nu = K^\mu$$

effect of recoil on accuracy - multiple emissions

- QCD coherence → factorised emissions
- observables dependece correlated → how to extract NLL without additional information?
- method from [Banfi, Salam, Zanderighi ’05]: need explicit soft-collinear limit*:

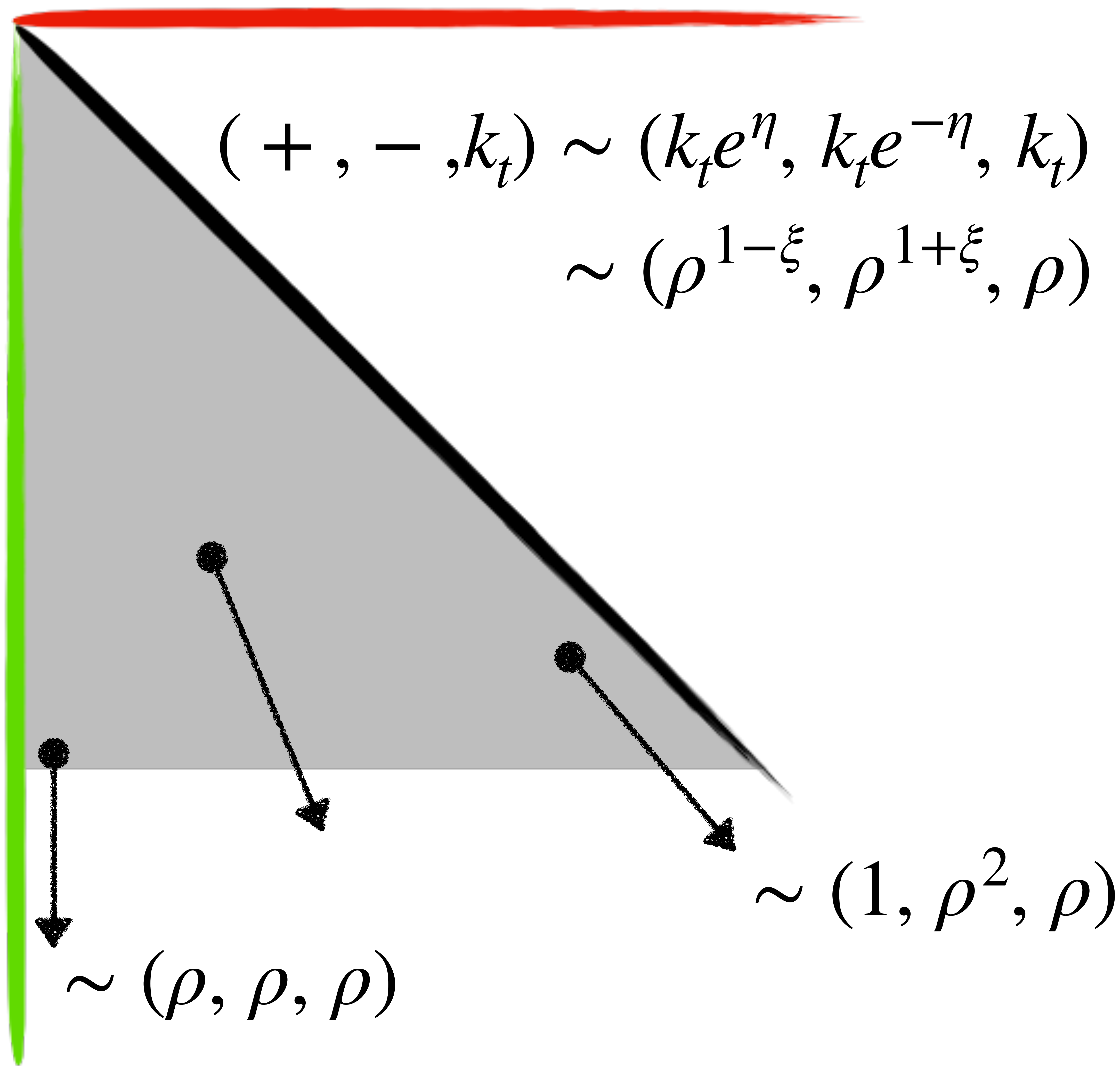
$$k_t^\rho = k_t \rho \qquad \xi = \frac{\eta}{\eta_{\max}}$$

$$\eta^\rho = \eta - \xi \ln \rho$$

and assume

$$V(k_i^\rho) = \rho V(k_i)$$

→ numerically evaluate integrals in this limit



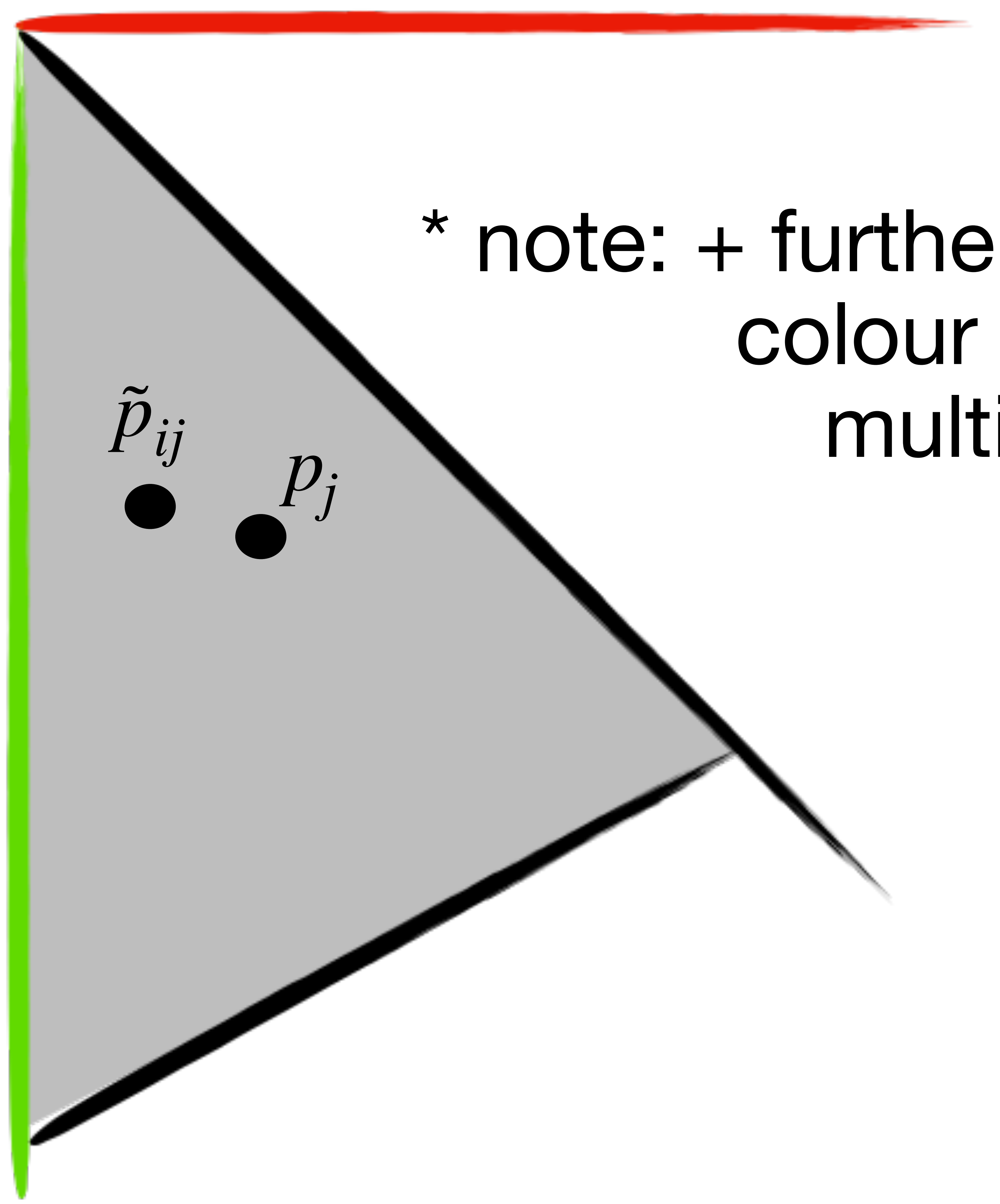
* again assume $V(k_t, \eta) \sim k_t/Q$ for brevity

effect of recoil on accuracy

- question: do recoil effects indeed vanish in soft limit (i.e. $\rho \to 0$)?*
  [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]
- consider situation where we first emit $\tilde{p}_{ij}$ from p_a, p_b, then emit p_j, $\tilde{p}_{ij} \to p_i, p_j$
- transverse momentum of p_i will be $\sim k_t^{ij} + k_t^j$

$$\Rightarrow \frac{\Delta k_t^{ij}}{k_t^{ij}} \to \frac{\rho k_t^j}{\rho k_t^{ij}} = \mathcal{O}(1)$$

$$p_i = z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\perp$$
$$p_j = (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\perp$$
$$p_k = (1-y)\tilde{p}_k \ .$$



* note: + further problems for colour assignment in multiple emissions

analytic proof of accuracy

$$\Lambda^{\mu}{}_{\nu}(K, \tilde{K}) = g^{\mu}_{\nu} + \tilde{K}^{\mu} A_{\nu} + X^{\mu} B_{\nu}$$

vanishes in soft limit

work out $\rho \to 0$ limit: $A^{\nu} \xrightarrow{\rho \to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^{\nu}}{\tilde{K}^2} - \frac{X^{\nu}}{\tilde{K}^2}\,,$ and $B^{\nu} \xrightarrow{\rho \to 0} \frac{\tilde{K}^{\nu}}{\tilde{K}^2}$

apply to soft momentum p_l:

$$\frac{\Delta p_l^{0,3}}{p_l^{0,3}} \sim \rho^{1-\max(\xi_i,\xi_j)}$$

$$\frac{\Delta p_l^{1,2}}{p_l^{1,2}} \sim \rho^{(1-\xi_l)(\max(\xi_i,\xi_j)-\xi_l)}$$

compare to $\frac{\Delta k_t}{k_t} \sim \mathcal{O}(1)$ from local dipole scheme

numerical validation I

- Limit $\alpha_s \to 0$ with $\lambda = \alpha_s L =$ const. of

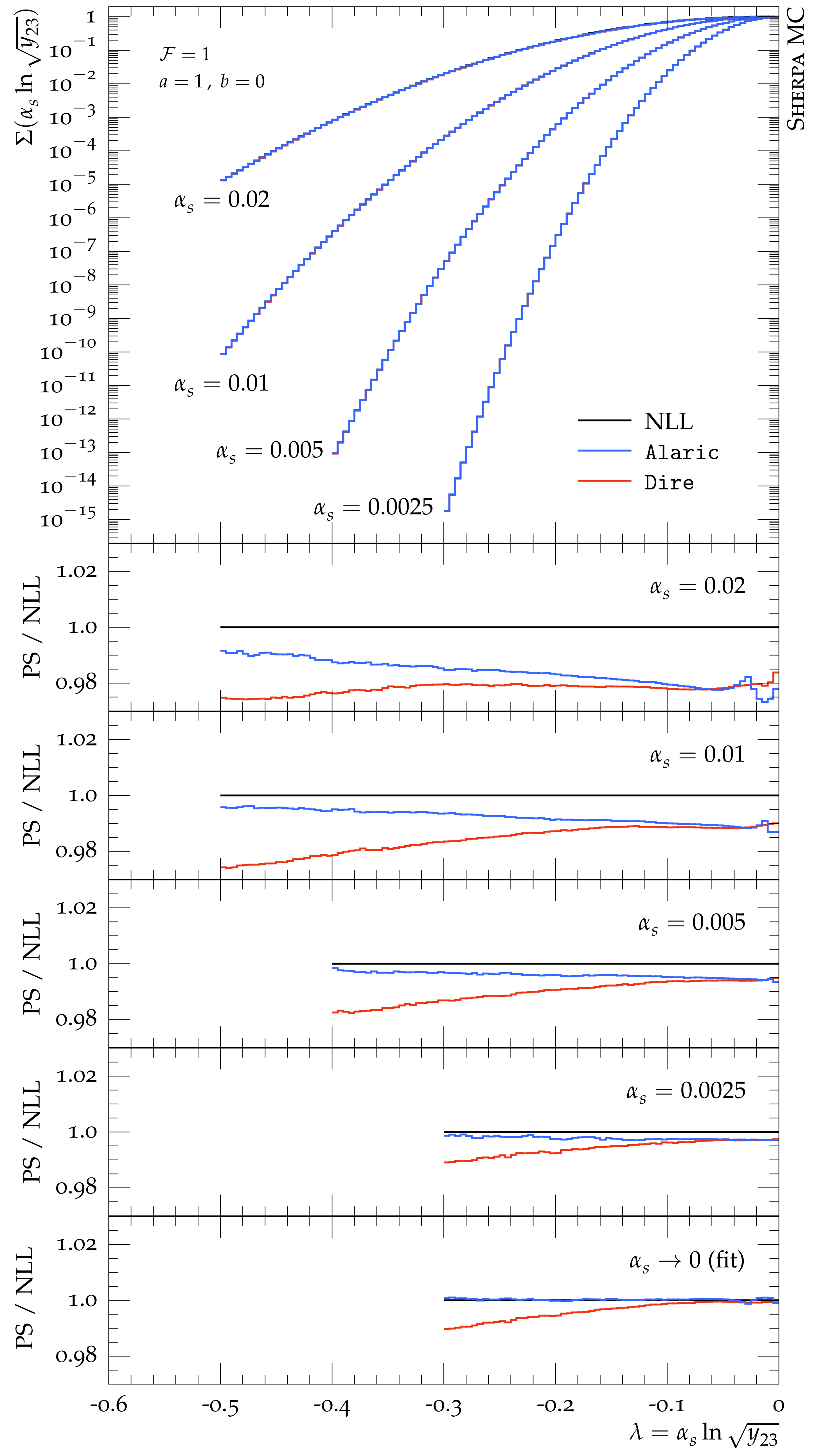
$$\frac{\Sigma^{\text{Shower}}}{\Sigma^{\text{NLL}}} \sim \exp\left(f^{LL}_{\text{Shower}} - L g_1(\alpha_s^n L^n)\right)$$

$$\times \exp\left(f^{NLL}_{\text{Shower}} - g_2(\alpha_s^n L^n)\right)$$

$$\times \exp\left(\mathcal{O}(\alpha_s^{n+1} L^n)\right)$$

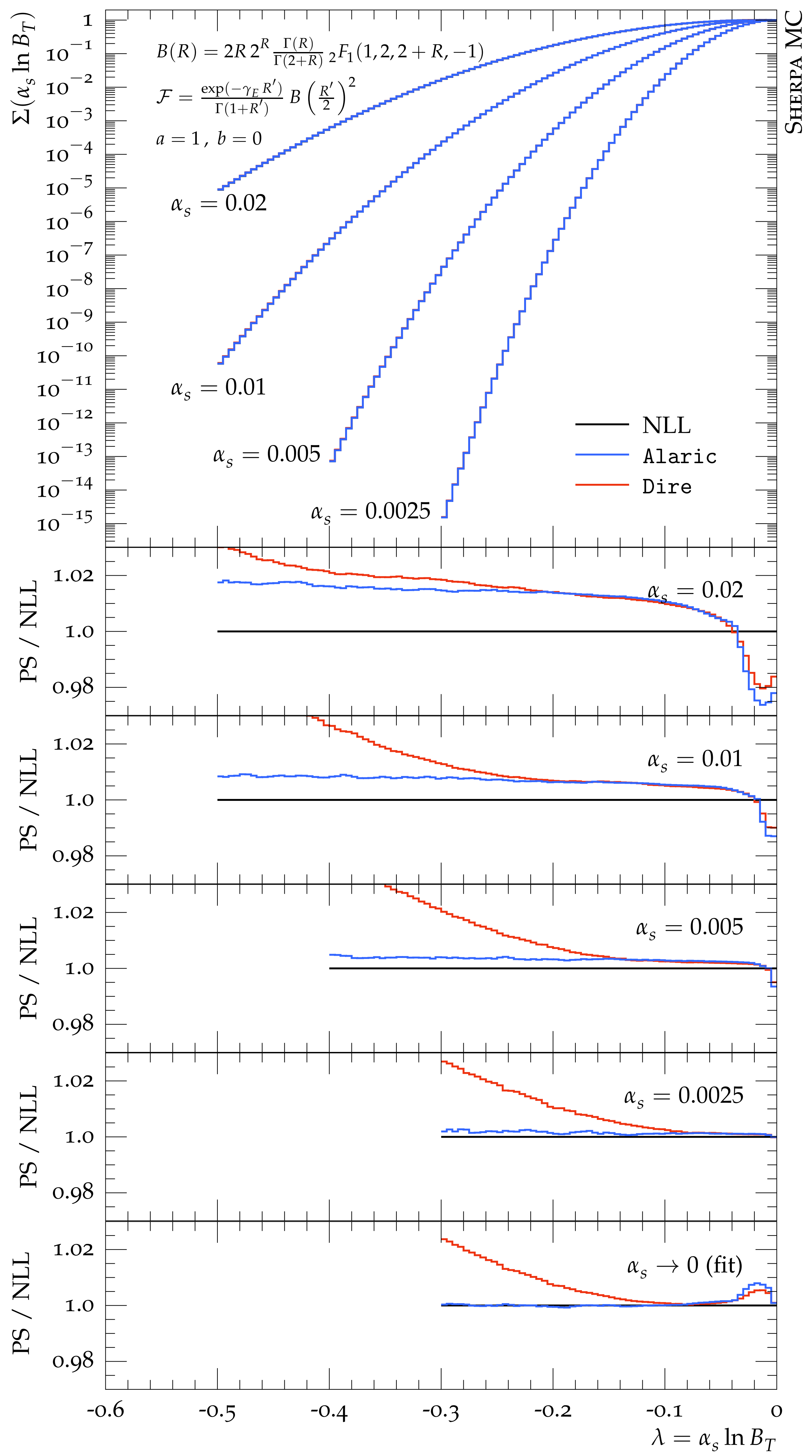
$\to 1$ if shower reproduces LL, NLL logs

- Observable: jet resolution y_{23} in Cambridge jet measure, $\mathcal{F} = 1 \to$ only largest emission matters, check that additional shower emissions vanish



numerical validation II

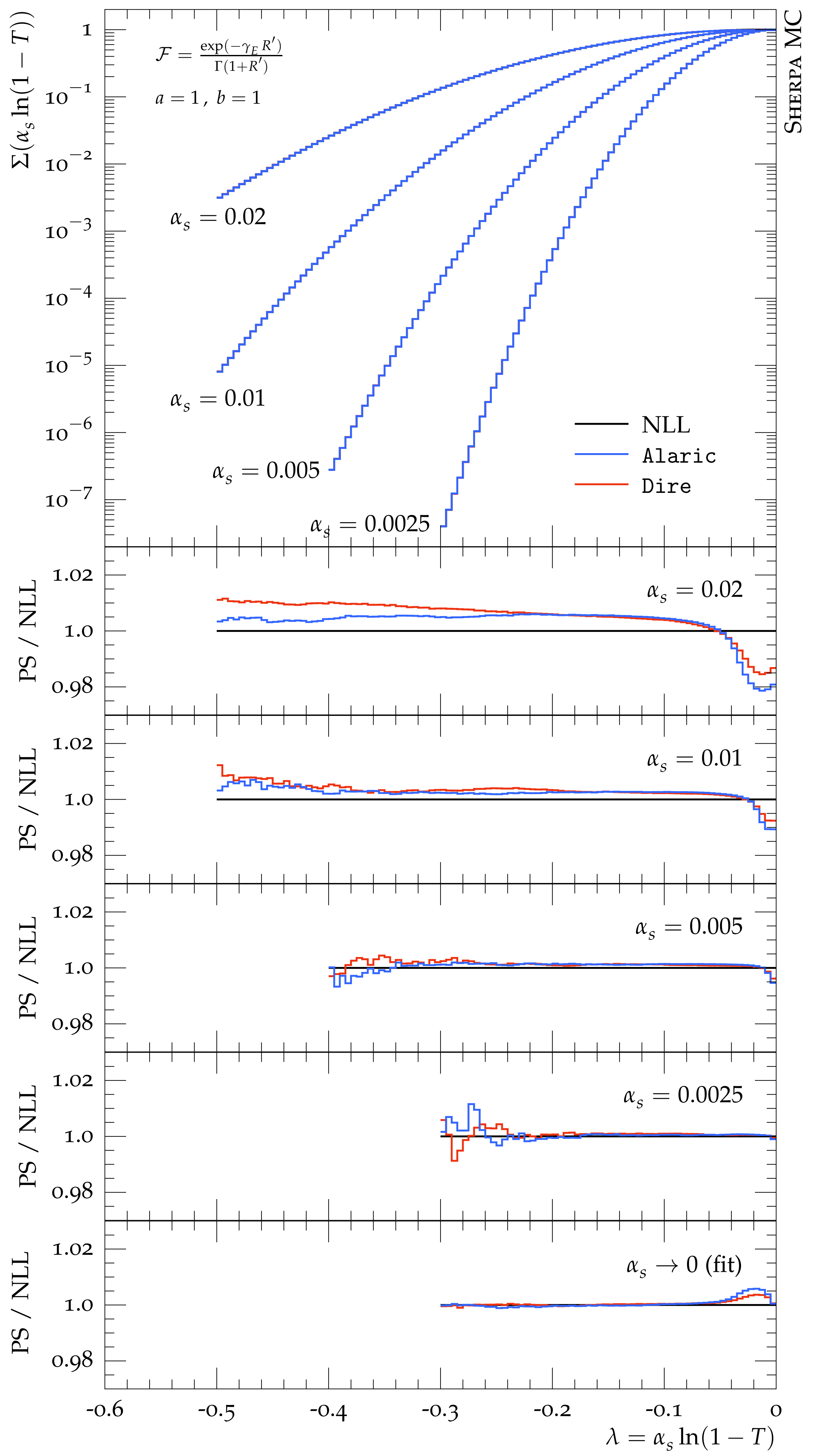
- total broadening $B_T = B_L + B_R$
- scaling k_t like, similar to y_{23}
- but non-trivial $\mathcal{F}$ function



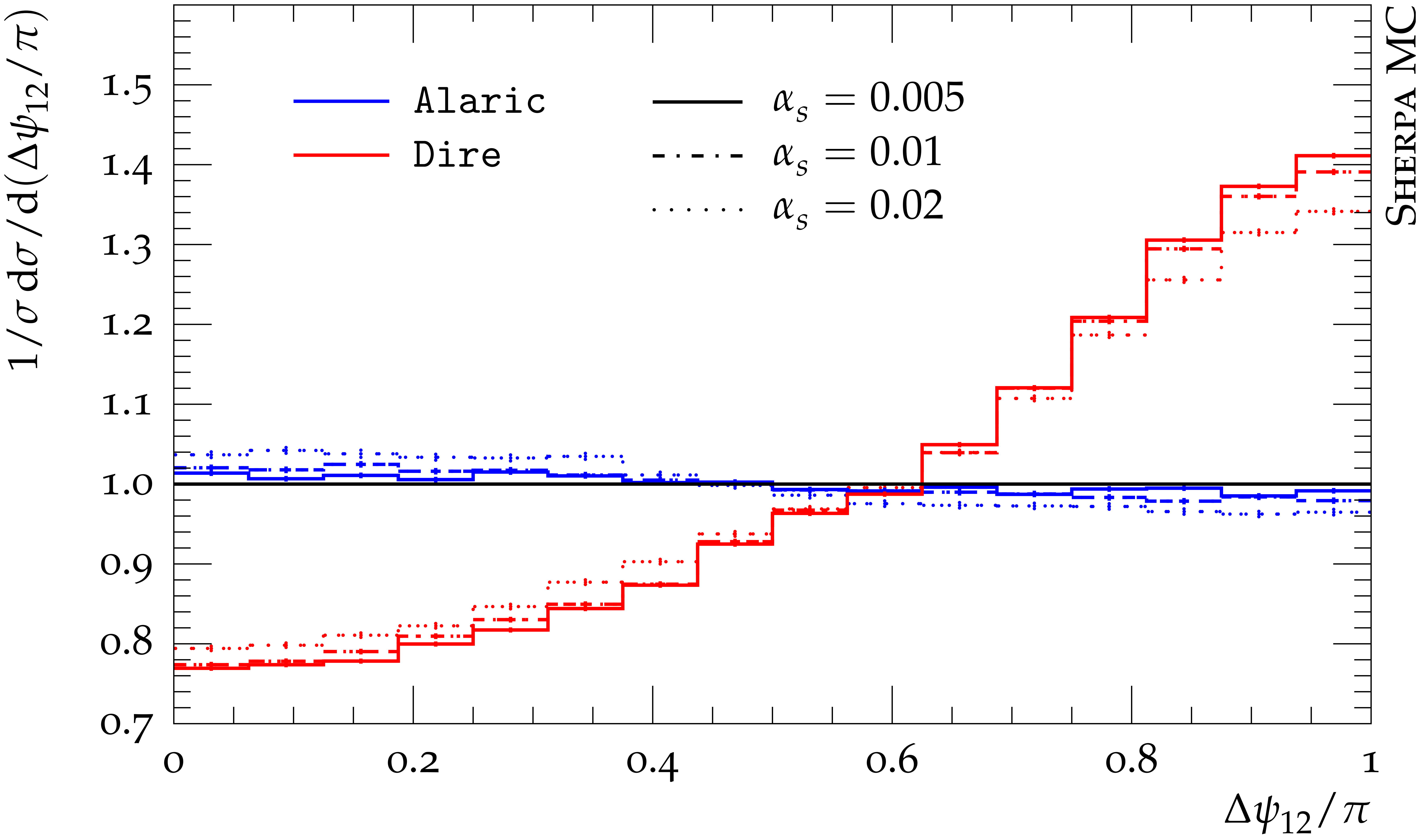
- thrust $\tau = 1 - t$
- scaling like virtuality $k_t e^{-\eta}$
- standard function

$$\mathcal{F} = \frac{\exp(-\gamma_E R)}{\Gamma(1+R')}$$

- no evidence for NLL violation even for standard showers



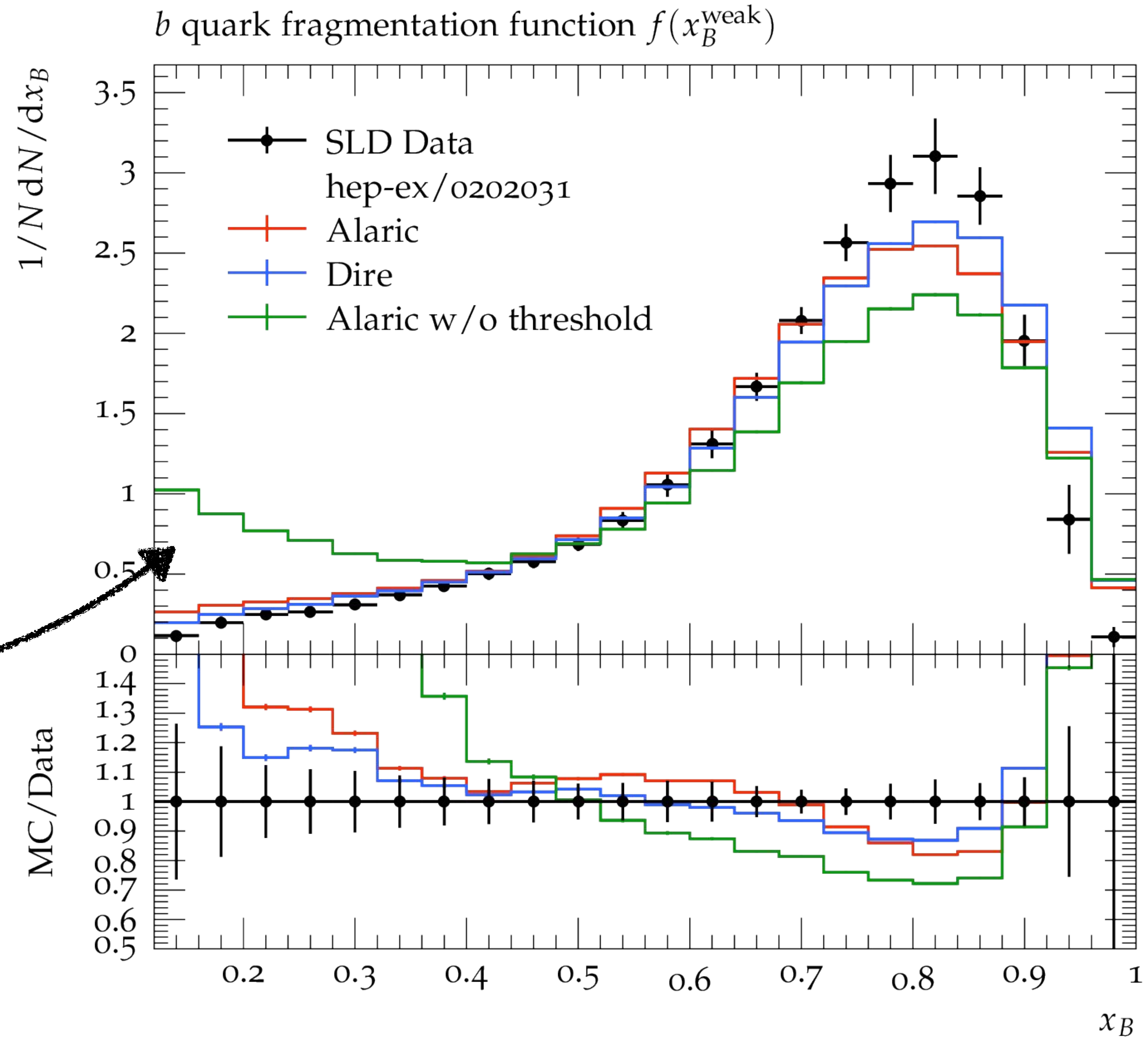
numerical validation III



See also talks by Gregory Soyez

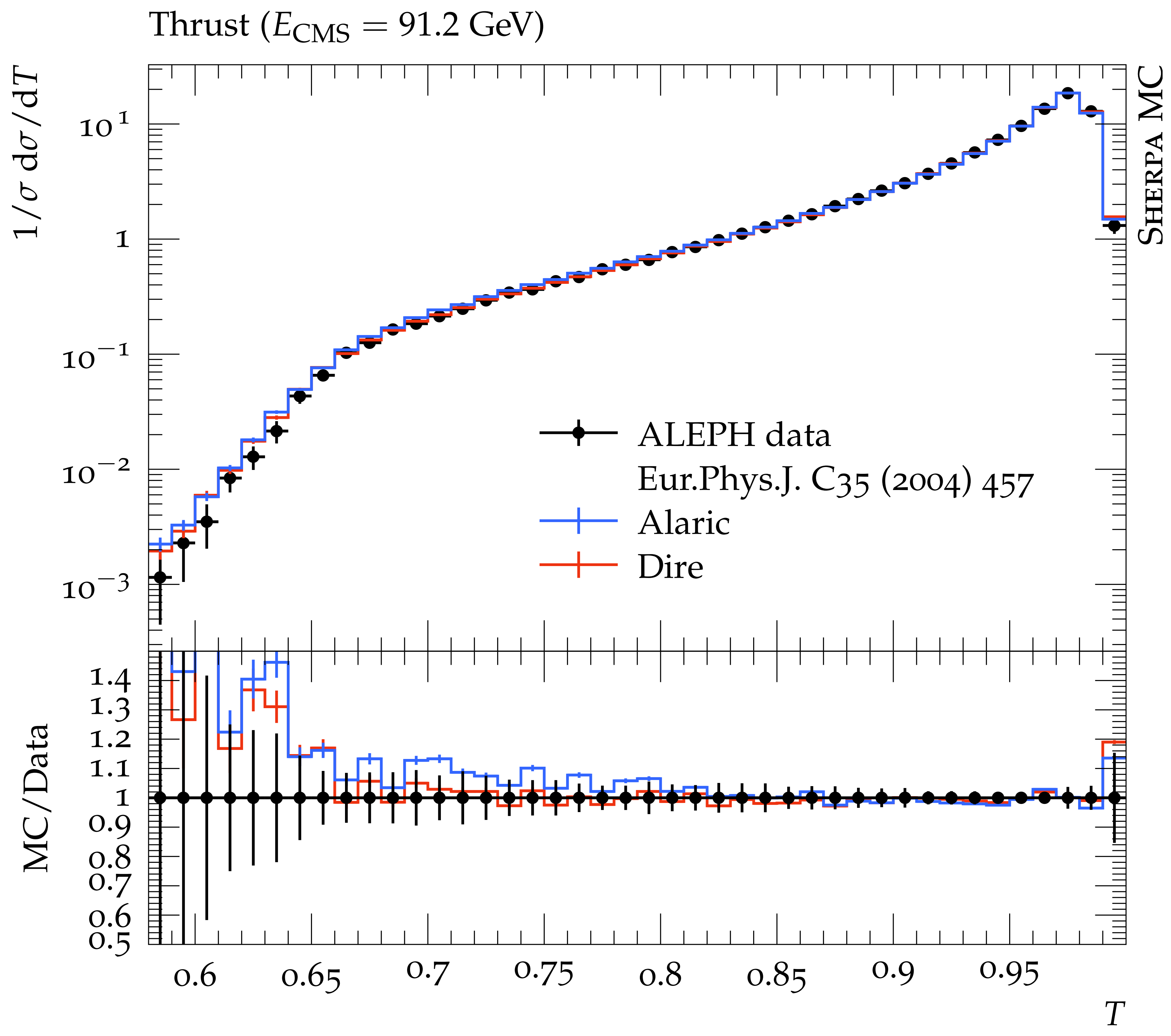
pheno, details and b fragmentation

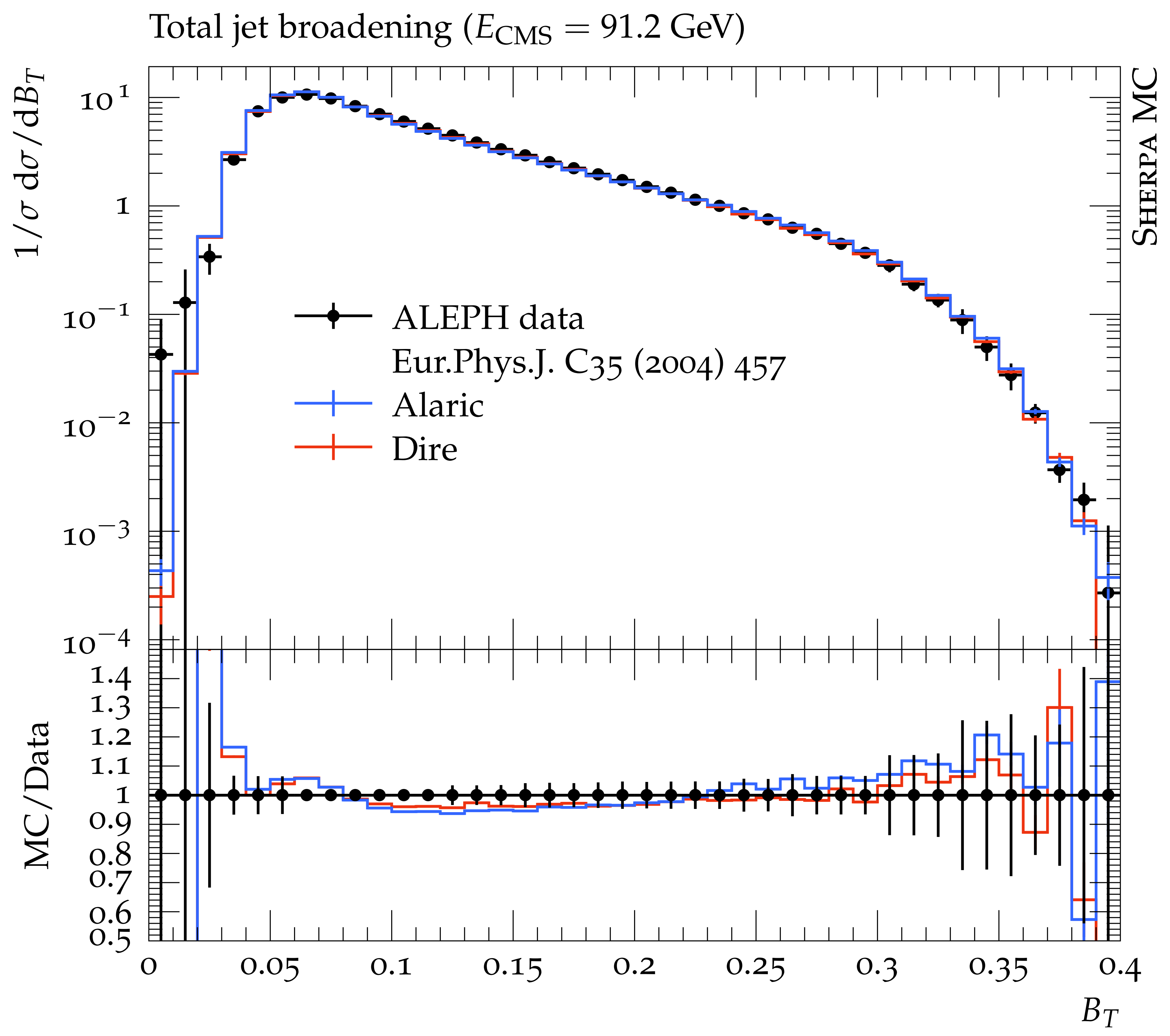
- first caveat: no quark masses implemented yet
- problem for cluster hadronisation → use Lund model via Pythia
- + need flavour threshold for $g \to b\bar{b}/g \to c\bar{c}$ splittings
- Dire parton shower as implemented in Sherpa as reference, Lund model tuned for Alaric $\sigma = 0.3$ GeV, $a = 0.4$, $b = 0.36$ GeV^{-2}
  and for Dire $\sigma = 0.3$ GeV, $a = 0.4$, $b = 0.46$ GeV^{-2}



$$x_B \sim \frac{E_{B\text{-Hadron}}}{E_{\text{tot.}}/2}$$

pheno, LEP observables





Thrust:

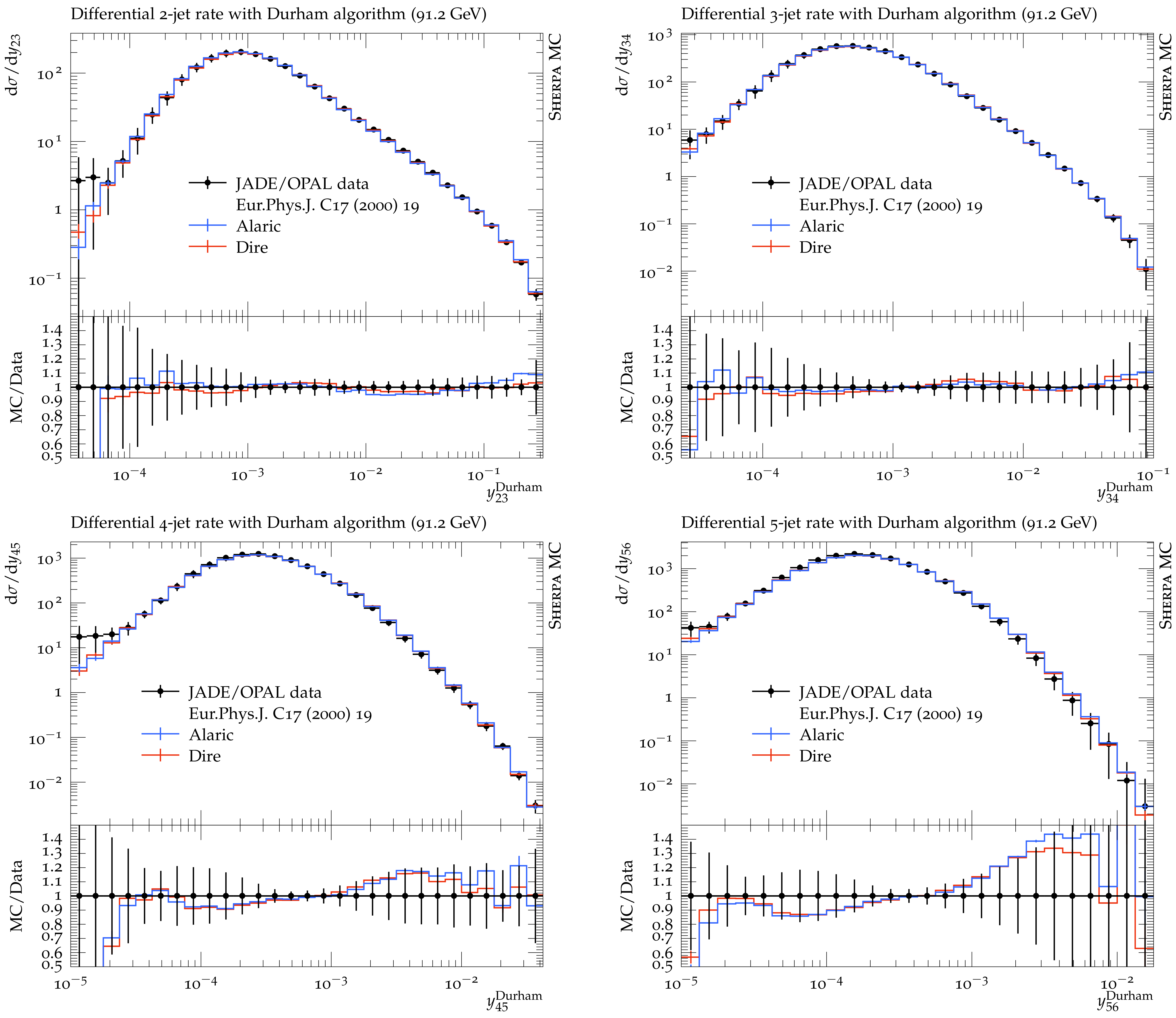
- Note this is T, not 1-T: soft physics is to the right
- Note there is no matching, relevant for small T

Total Broadening:

- soft physics is left hand side
- some deviations from data, but similar to Dire

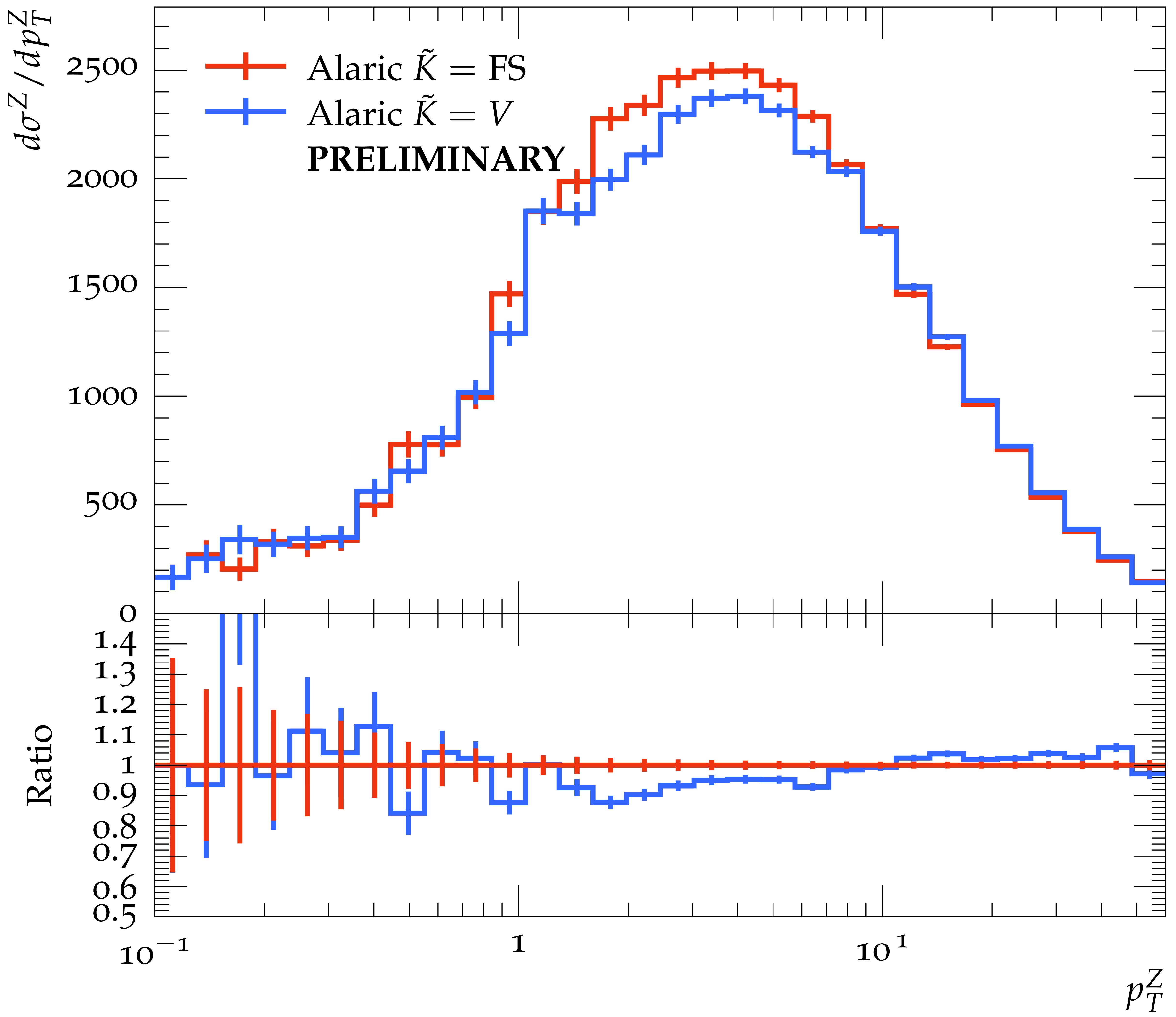
pheno, LEP observables

- Durham resolution scales $y_{n,n+1} \sim k_t^2/Q^2$
- higher Born multiplicities → sensitivity to multiple emissions increased
- again, note no matching/merging involved



Alaric initial state shower (outlook)

- Formalism presented in [Herren, Höche, Krauss, DR, Schönherr '22] general and applicable to initial state evolution
- practical considerations:
  - precise definition of evolution variable
  - PDFs, clear in principle, but more choices to make
  - distribution of recoil (i.e. definition of $\tilde{K}$)
    - e.g. Drell-Yan process, could be EW boson, or full final state (or …?)



transverse momentum of Z bosons at the LHC at 13 TeV

A Logarithmically Accurate Resummation In C++

- NLL resummation in CAESAR formalism as definition and validation of parton shower accuracy
- New parton shower Alaric
  - partial fractioning of eikonal → positive definite splitting function with full phase space coverage
  - global kinematics scheme enables analytic proof of NLL accuracy + numerical validation
  - included in Sherpa framework and first pheno results