Sudakov Shoulder Resummation in Thrust and Heavy Jet Mass

Matthew Schwartz

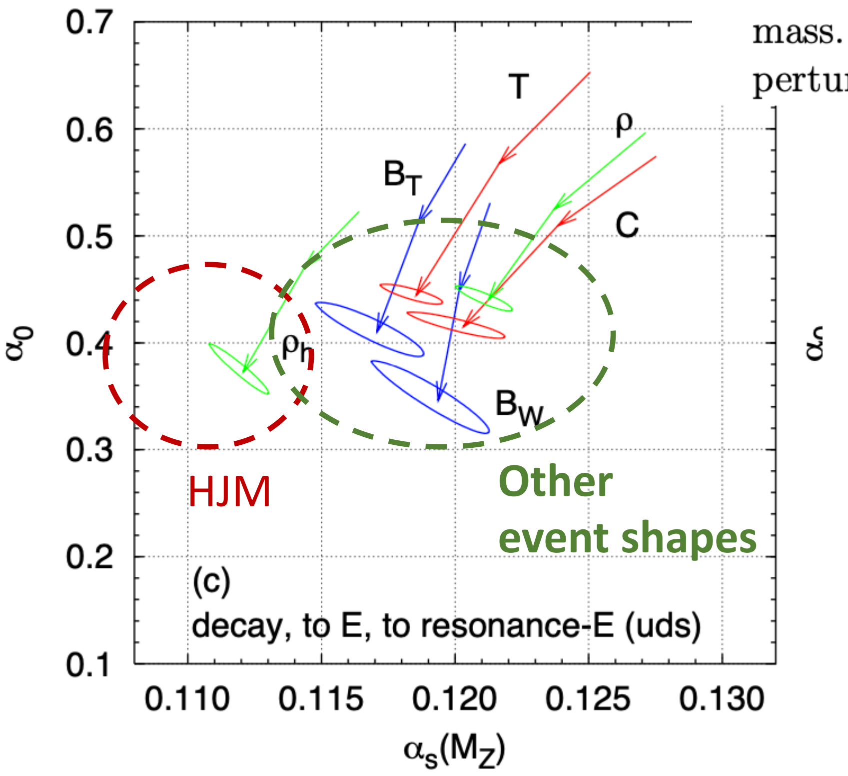
Harvard University

Parton Showers and Resummation Workshop
Milan, Italy
June 6, 2023

Based on
- arXiv:2205.05702 (PRD106.074011)
  - with Arindam Bhattacharya (Harvard) and Xiaoyuan Zhang (Harvard)
- arXiv:2306.xxxxx
  - with Arindam Bhattacharya (Harvard), Xiaoyuan Zhang (Harvard), Iain Stewart (MIT) and Johannes Michel (MIT)

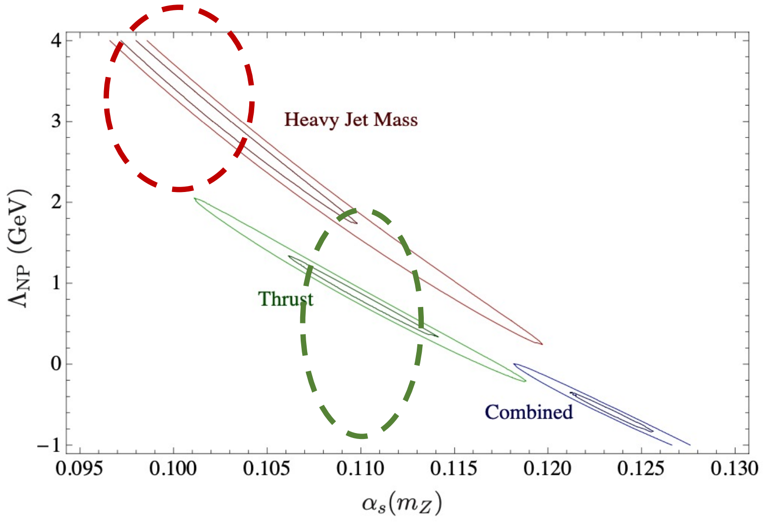
Motivation: HJM is an outlier

Salam and Wicke 2001 (hep-ph/0102343)



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo

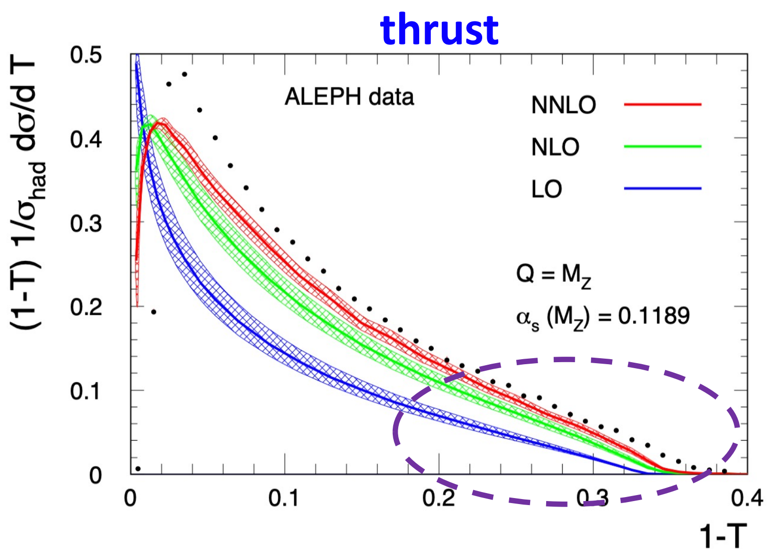
Chien and Schwartz 2010 (arXiv:1005.1644)
NNNLL resummation with NNLO matching

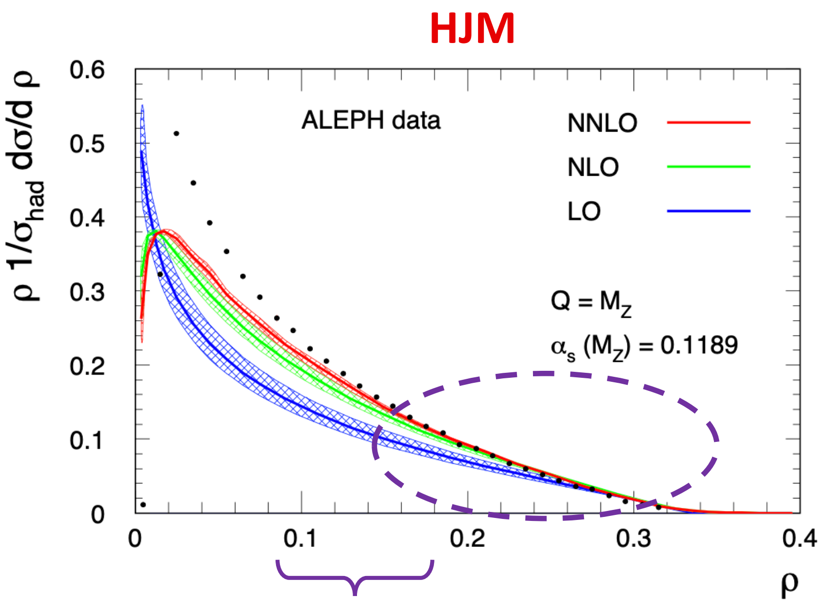


Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

Motivation: fixed order pertubation theory

- data for thrust seems matches shape of NNLO theory better than HJM in the far tail



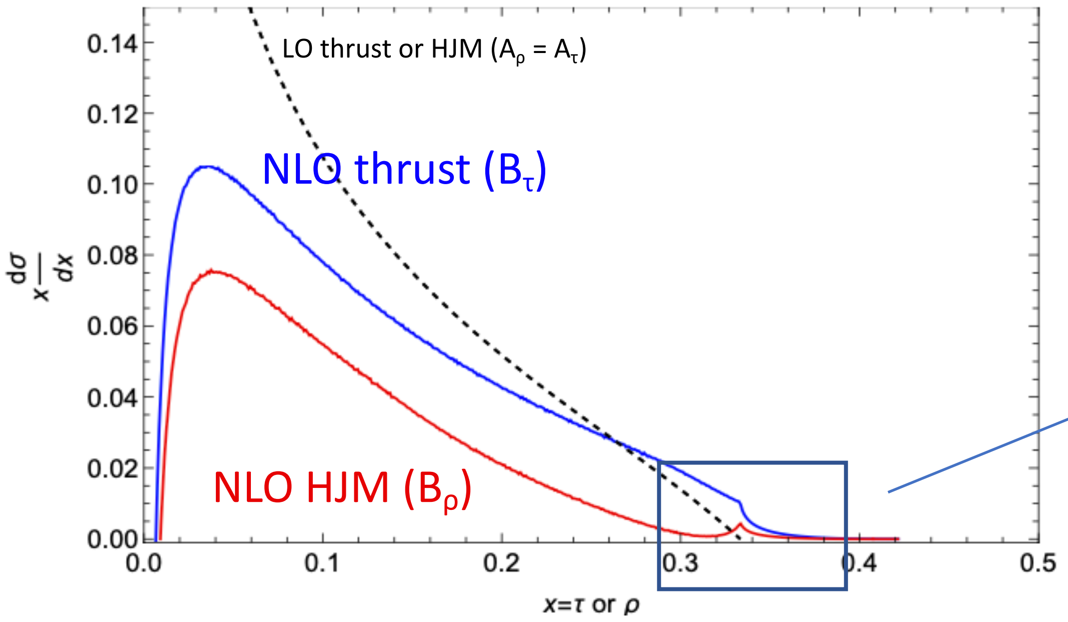


Fits used range $0.8 < \rho < 0.18$ [Dissertori et al 0712.0327]

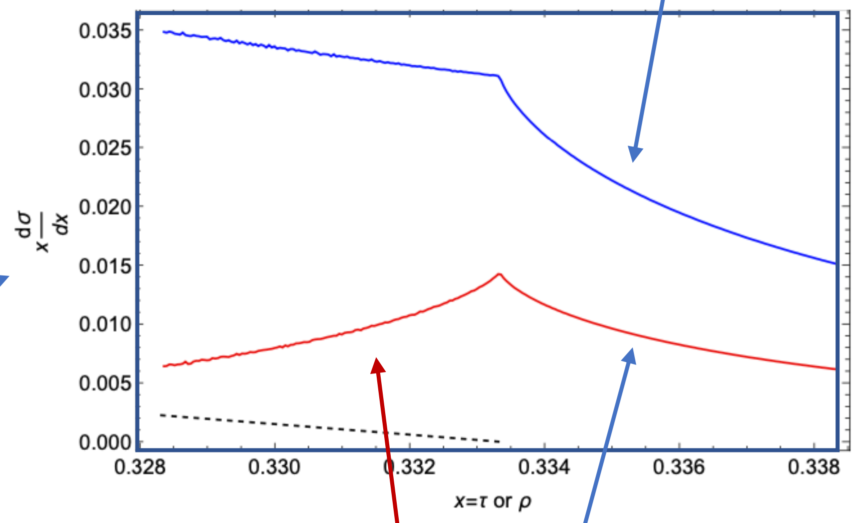
- Only uses 25% of data bins!

What is different about thrust and HJM?

1. Different perturbative behavior: Sudakov Shoulders (this talk)



- Thrust has **right** shoulder only



- HJM has **left** and **right** shoulders

Could resummation of the Sudakov shoulder improve the theory prediction?

What is different about thrust and HJM?

2. Different power corrections (not this talk)

Thrust

$$T \equiv \max_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

Thurst only involves 3 vectors

- Insensitive to mass scheme

$$E = |\vec{p}| \text{ or } E = \sqrt{\vec{p}^2 + m_\pi^2}$$

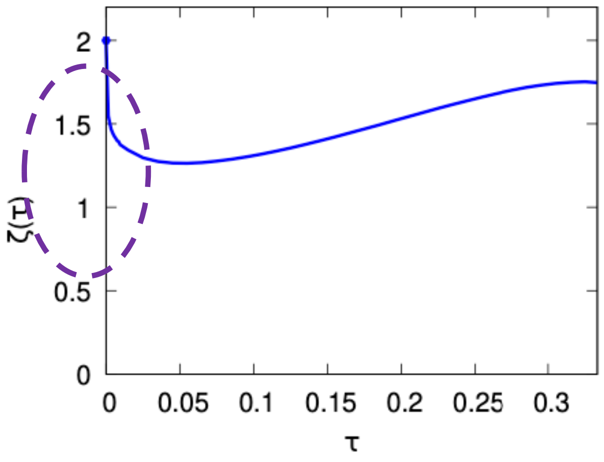
HJM

$$\rho = \frac{1}{Q^2}\max(M_L^2, M_R^2)$$

Heavy jet mass involves 4-vectors

- Sensitive to mass scheme

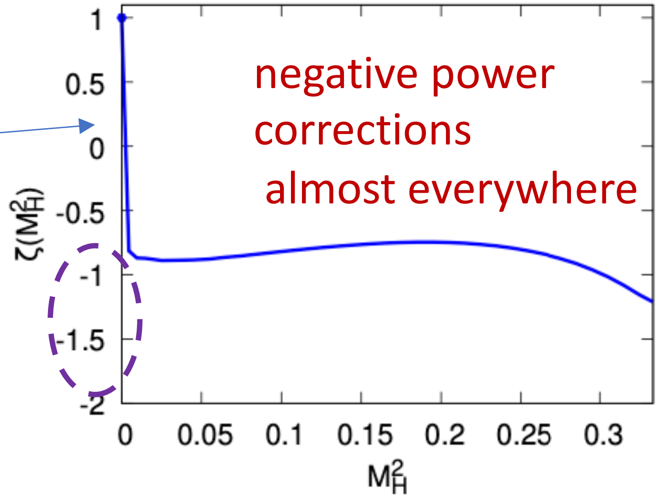
Nason & Zanderighi (2301.03607)

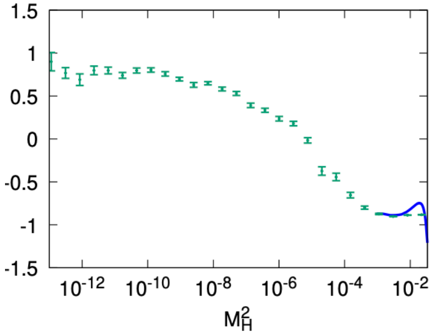


positive power corrections everywhere

positive power corrections near ρ = 0

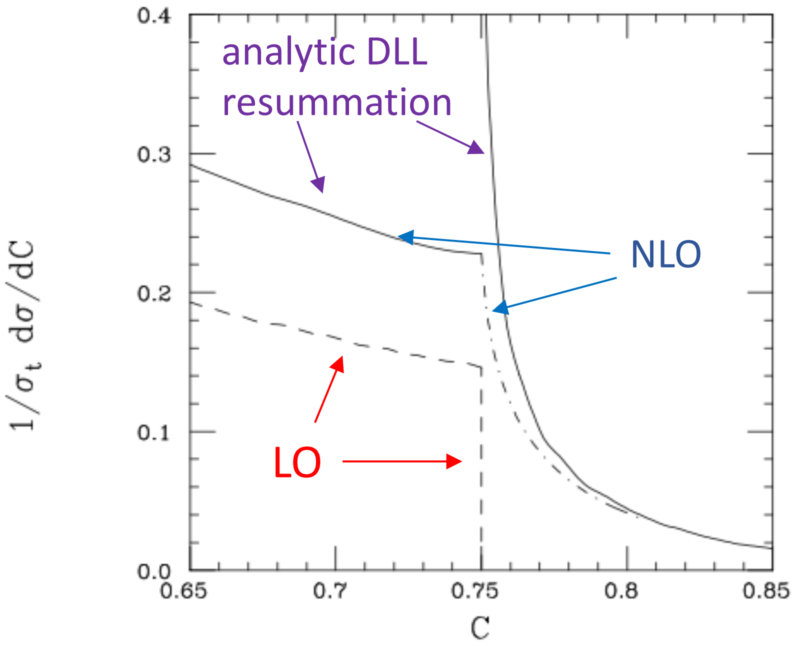
Nason & Zanderighi (2301.03607)



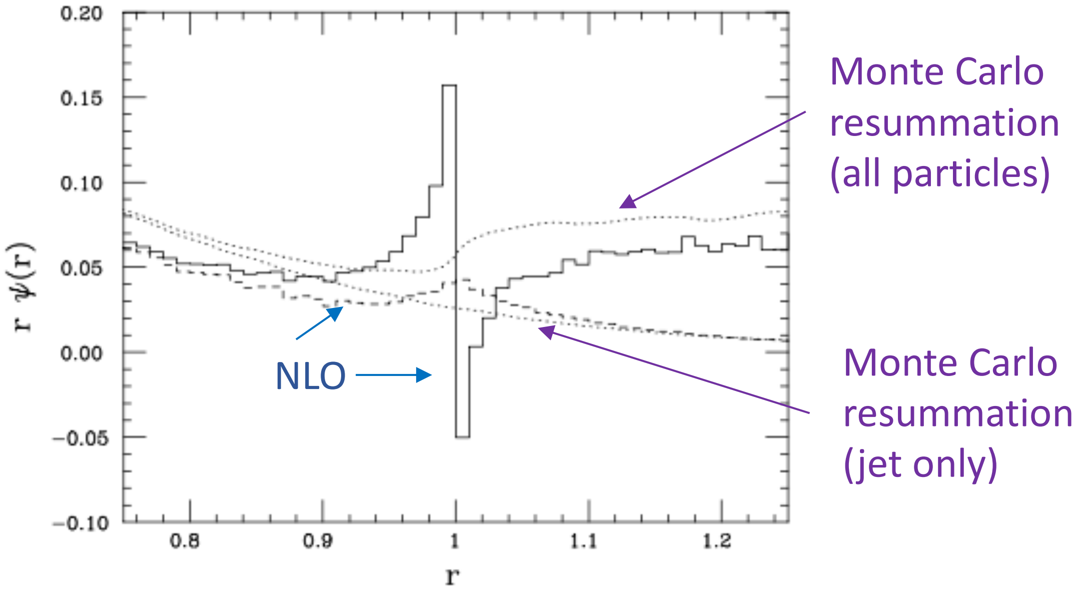


Sudakov Shoulders

- Catani and Webber (hep-ph/9710333)
  - C parameter has a right-shoulder
  - discontinuity at C=0.75



- Seymour (hep-ph/9707338)
  - Jet shape has a discontinuity at r=1

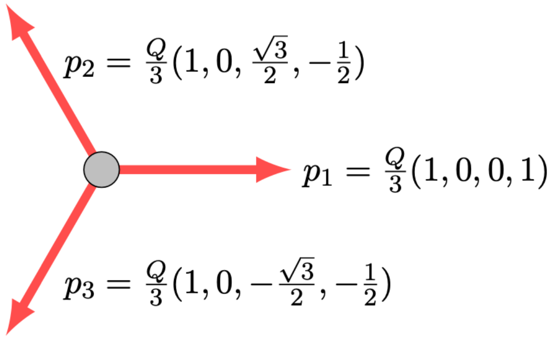


Sudakov Shoulders

Shoulders are a **generic feature** when range of observable changes order by order

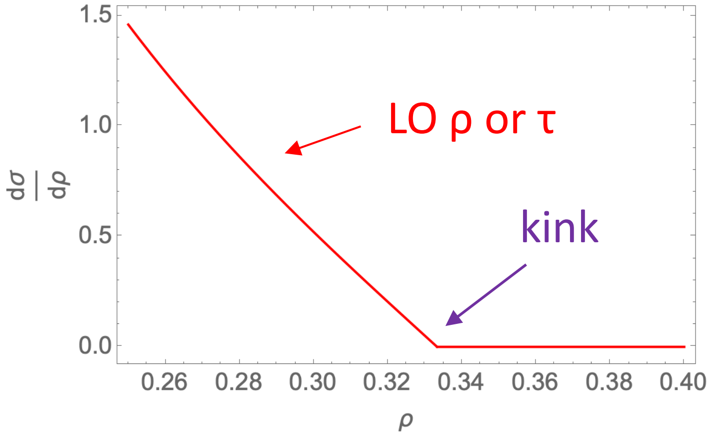
LO: C < ¾, ρ,τ <⅓

- Maximum value at unique trijet configuration



- Matrix element is finte at this endpoint
  - |M|~ constnat for ρ ~ ⅓

- Phase space zero volume at endpoint
  - Phase space volume vanishes linearly as ρ →⅓
- Cross section vanishes linearly as endpoint approached
  - Leads to kink (discontinuity in first derivative)

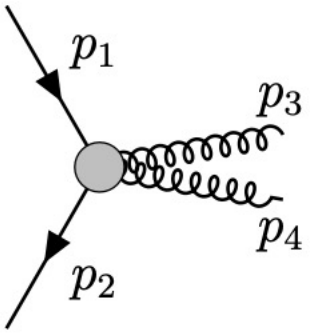


- at NLO get $\frac{d\sigma}{d\rho} \sim \alpha_s r \ln^2 r$ behavior

$$r \equiv \frac{1}{3} - \rho$$

Where do logs come from?

NLO: 4 partons

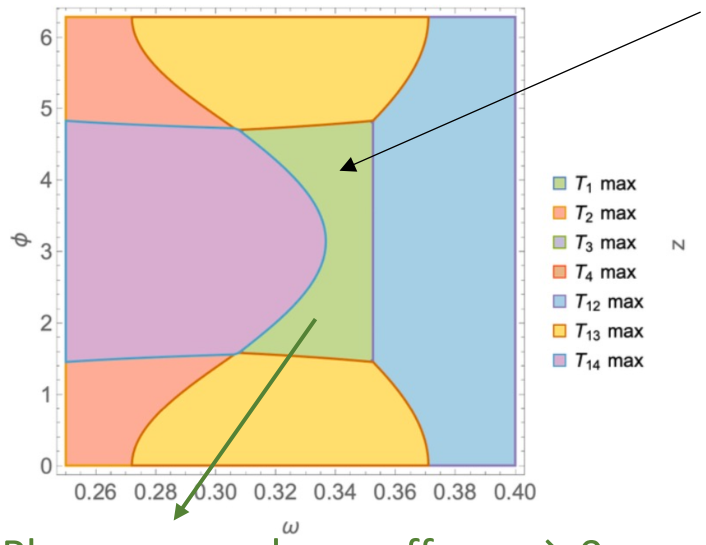


- 5 dimensional phse space
    - Choose 5 energies, angles, invariants

- Compute cross section in small r region

Thrust axis $T \equiv \max_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$

- 7 possible options for thrust axis



only region that contributes for

$r = \frac{1}{3} - \rho = 0.01$

Phase space closes off as r → 0

cross section

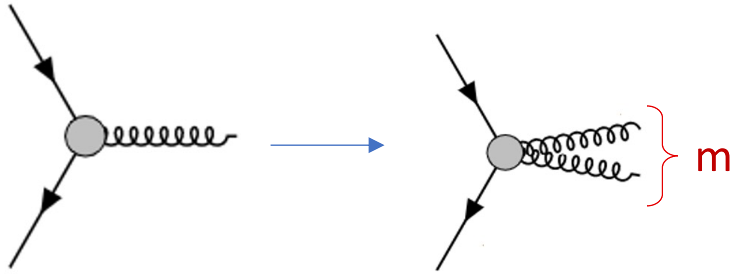
$$I \sim |\mathcal{M}_0|^2 \frac{\alpha_s}{4\pi} C_F^2 \int_0^r \frac{ds_{34}}{s_{34}} \int_{\frac{9}{4}s_{34}}^1 \frac{dz}{z} \int_{\frac{1}{3}-r}^{\frac{1}{3}+2r} ds_{234}$$

$$\cong |\mathcal{M}_0|^2 \frac{\alpha_s}{4\pi} C_F^2 r \ln^2 r$$

Large shoulder logs

Factorization

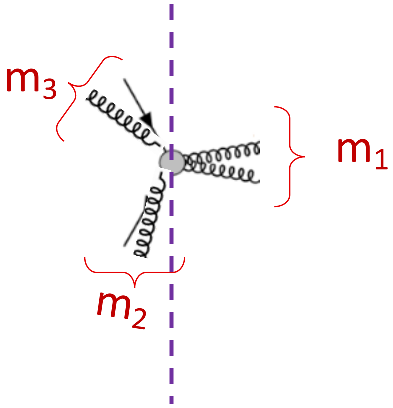
1. Soft and/or collinear emissions turn LO partons into jets



2. Derive constraint relating kinematics to ρ or τ

Consider the case of three massive partons (collinear radiation only)

- Phase space is 2d: described by s and t or $s_{12}=(p_1+p_2)^2$ and $r = ⅓-\rho$



Suppose thrust axis points in 1 direction (other cases by permuation)

- Two inequalities ($T_1 > T_2$ and $T_1 > T_3$), combined together into

$$T \equiv \max_{\vec{n}} \frac{\sum_j |\vec{p}_j \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

$$\frac{1}{3} - r + 2m_1^2 - 2m_3^2 < s_{12} < \frac{1}{3} + 2r - m_1^2 + 3m_2^2 + m_3^2$$

- For $m_1 = m_2 = m_3 = 0$ get $⅓-r < s_{12} < ⅓+ 2r$ so phase space cuts off as $r \to 0$
- With masses, phsae space is zero unless

$$m_1^2 < r + m_2^2 + m_3^2$$

3. Constraint turns double logs for jet mass into shoulder logs

Factorization

1. Soft and/or collinear emissions turn LO partons into jets

2. Derive constraint relating kinematics to ρ or τ

- Leading power constraint for HJM is

$$m_1^2 < r + m_2^2 + m_3^2$$

  - Solutions for r > 0 and r < 0
  - Large logs in both left and right shoulder

- Leading power constraint for thrust is

$$t < m_1^2 + m_2^2 + m_3^2$$

$$t \equiv \tau - \frac{1}{3}$$

  - Only solutions for t > 0
  - t < 0 no phase space limits
    - not sensitive to emissions
  - Only right shoulder

3. Constraint turns double logs for jet mass into shoulder logs

e.g. One collinear emission (one nonzero mass) $m_1^2 < r$

$$\int_{\frac{1}{3}-r}^{\frac{1}{3}+2r} ds_{12} \int_0^r dm_1^2 \frac{d\sigma}{dm^2} \approx \int_{\frac{1}{3}-r}^{\frac{1}{3}+2r} ds_{12} \int_0^r dm_1^2 \frac{\ln m_1^2}{m_1^2} = r \ln^2 r$$

Phase space closes off as r → 0
- generates r prefactor

cross section
- generates sudakov logs

Soft radiation

Including soft and collienar radiation constraint is

HJM

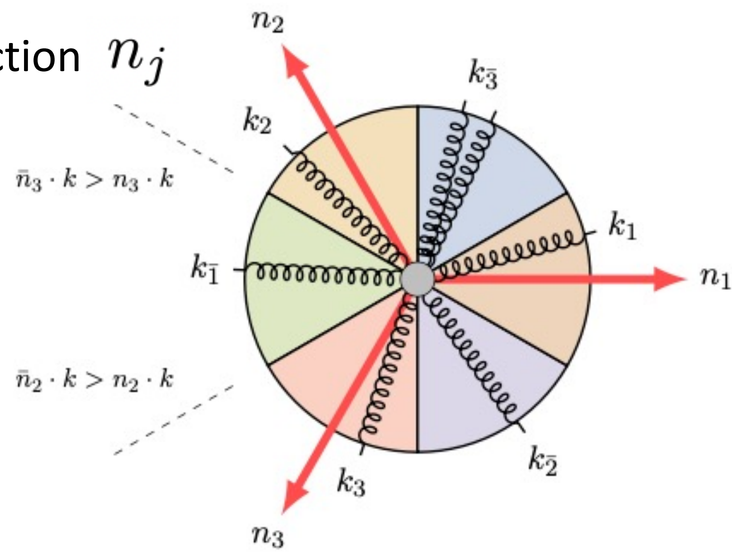
$$m_1^2 + 2p_1k_1 + 2v_{\bar{2}}k_{\bar{2}} + 2v_{\bar{3}}k_{\bar{3}} < r + m_2^2 + 2p_2k_2 + m_3^2 + 2p_3k_3 + 2v_{\bar{1}}k_{\bar{1}}$$

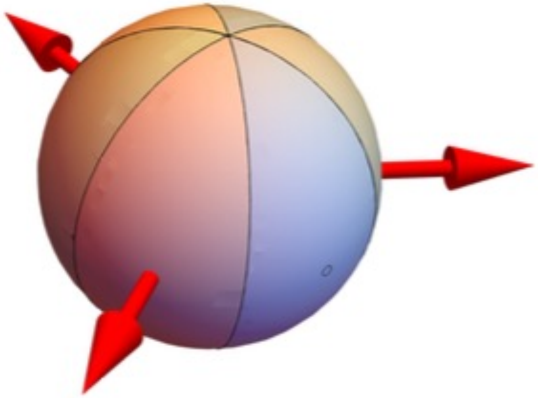
$$t < m_1^2 + m_2^2 + m_3^2 + 2p_1k_1 + 2p_2k_2 + 2p_3k_3 + 2v_{\bar{1}}k_{\bar{1}} + 2v'_{\bar{2}}k_{\bar{2}} + 2v'_{\bar{3}}k_{\bar{3}}$$

thrust

k_j = soft radiation in the sextant aligned with direction n_j

$k_{\bar{j}}$ = soft radiation in the sextant opposite n_j





Compute 6-argument soft function

$$S(k_1, k_2, k_3, k_{\bar{1}}, k_{\bar{2}}, k_{\bar{3}})$$

Final factorization formula

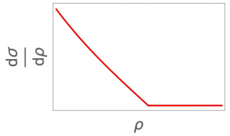
Soft

Measurement function

$$\frac{1}{\sigma_1}\frac{d\sigma}{dr} = H(Q)\int d^3m^2 d^6q\, J(m_1^2)J(m_2^2)J(m_3^2)S_6(q_i)W(m_j,q_i,r)\theta[W(m_j,q_i,r)]$$

Hard Jet Jet Jet

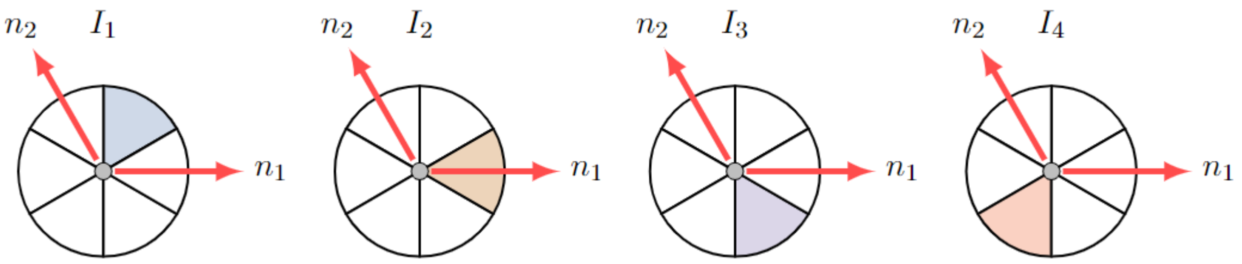
0[th] order → $\frac{1}{\sigma_0}\frac{d\sigma}{dr} = r\theta(r)$



$$W(m_j, q_i, r) = r - m_1^2 + m_2^2 + m_3^2 + \frac{2Q}{3}(q_2 + q_3 + q_{\bar{1}} - q_1 - q_{\bar{3}} - q_{\bar{2}})$$

1-loop soft function

Need to compute 4 integrals



$$I_1 = \mathcal{N}\left[4\kappa - \frac{4}{3}\pi \ln 2 + \epsilon c_3\right], \quad I_2 = \mathcal{N}\left[-2\kappa + 2\pi \ln 2 + \epsilon c_4\right]$$

$$I_3 = \mathcal{N}\left[4\kappa + \frac{4}{3}\pi \ln 2 + \epsilon c_5\right], \quad I_4 = \mathcal{N}\left[-2\kappa + 2\pi \ln 2 + \epsilon c_6\right]$$

- Gieseking's constant (transcendality 2 number)

$$\kappa = \mathrm{Im}\,\mathrm{Li}_2\left(e^{\frac{i\pi}{3}}\right) = 1.014\ldots$$

- Like Catalan $C = \mathrm{Im}\,\mathrm{Li}_2\left(e^{\frac{i\pi}{2}}\right) = 0.915\ldots$
- Or $-\dfrac{\pi^2}{12} = \mathrm{Li}_2(e^{i\pi})$

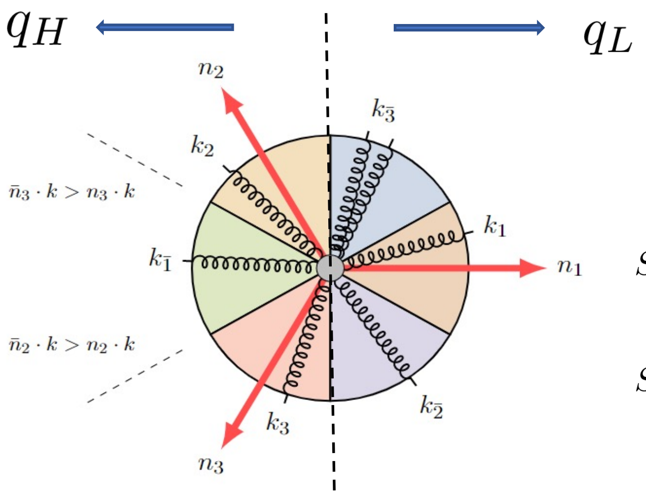
For NNLL resummation we need the constants.
Some of these we were only above to get numerically

$$\kappa = \Im\left[\mathrm{Li}_2\left(e^{\frac{i\pi}{3}}\right)\right], \quad c_1 = \Im\left[\mathrm{Li}_3\left(\frac{i}{\sqrt{3}}\right)\right], \quad c_2 = \Im\left[\mathrm{Li}_3\left(1 + i\sqrt{3}\right)\right], \quad c_3 = -1.89958$$

$$c_4 = -3.83452, \quad c_5 = -12.3488, \quad c_6 = 5.56704, \quad c_7 = 15.9482, \quad c_8 = 3.52263$$

1-loop soft function

Combine integrals together to produces trijet hemisphere soft function



$$S(q_L, q_H) = \int d^6 q_i \; S_6(q_i)\delta(q_L - q_1 - q_{\bar{2}} - q_{\bar{3}})\delta(q_H - q_{\bar{1}} - q_2 - q_3)$$

$$S_g^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-4C_F\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqq}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_A\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sg}}{k_L}\right]_\star + \cdots$$

$$S_q^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-2(C_F + C_A)\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqg}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_F\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sq}}{k_L}\right]_\star$$

$\gamma_{sqq} = -4C_F \ln 6,$ $\gamma_{sg} = -2C_A \ln 3 + 4C_F \ln 2$

$\gamma_{sqg} = -2(C_A + C_F)\ln 6,$ $\gamma_{sq} = -2C_F \ln\frac{3}{2} + 2C_A \ln 2$

- RGE Consistency check - $-\gamma_h = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqq} + \gamma_{sg} = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqg} + \gamma_{sq}$ ✓

Previously computed in literature

Resummed distribution

HJM, ρ< ⅓

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dr} = \Pi_g(\partial_{\eta_\ell},\partial_{\eta_h}) r \left(\frac{rQ}{\mu_s}\right)^{\eta_\ell}\left(\frac{rQ}{\mu_s}\right)^{\eta_h}\frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}\frac{\sin(\pi\eta_\ell)}{\sin(\pi(\eta_\ell+\eta_h))}$$

HJM, ρ > ⅓

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{ds} = \Pi_g(\partial_{\eta_\ell},\partial_{\eta_h}) s \left(\frac{sQ}{\mu_s}\right)^{\eta_\ell}\left(\frac{sQ}{\mu_s}\right)^{\eta_h}\frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}\frac{\sin(\pi\eta_h)}{\sin(\pi(\eta_\ell+\eta_h))}$$

Thrust, τ > ⅓

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dt} = \Pi_g(\partial_{\eta_\ell},\partial_{\eta_h}) t \left(\frac{tQ}{\mu_s}\right)^{\eta_\ell}\left(\frac{tQ}{\mu_s}\right)^{\eta_h}\frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}$$

$$\eta_\ell = 2C_A\, A_\Gamma(\mu_j,\mu_s) \quad -2C_A\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

$$\eta_h = 4C_F\, A_\Gamma(\mu_j,\mu_s) \quad \sim -4C_F\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

$$\begin{aligned}\Pi_g(\partial_{\eta_\ell},\partial_{\eta_h}) &= \exp\left[4C_F S(\mu_h,\mu_j) + 4C_F S(\mu_s,\mu_j) + 2C_A S(\mu_h,\mu_j) + 2C_A S(\mu_s,\mu_j)\right]\\ &\times \exp\left[2A_{\gamma sg}(\mu_s,\mu_h) + 2A_{\gamma sqq}(\mu_s,\mu_h) + 2A_{\gamma jg}(\mu_j,\mu_h) + 4A_{\gamma jq}(\mu_j,\mu_h)\right]\\ &\times H(Q,\mu_h)\widetilde{j}_q\left(\partial_{\eta_h} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\widetilde{j}_{\bar q}\left(\partial_{\eta_h} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\widetilde{j}_g\left(\partial_{\eta_\ell} + \ln\frac{Q\mu_s}{\mu_j^2}\right)\widetilde{s}_{qq}(\partial_{\eta_h})\widetilde{s}_g(\partial_{\eta_\ell})\end{aligned}$$

Check fixed order expansion

$$\begin{aligned}\frac{1}{\sigma_1}\frac{d\sigma^{\rm sub}}{dr} &= \frac{\alpha_s}{4\pi}\left\{-\frac{1}{2}(2C_F + C_A)\Gamma_0 r\ln^2 r + \left[(C_A + 2C_F)\Gamma_0 + \gamma_{jg} + 2\gamma_{jq} + 2\gamma_{sg} + 4\gamma_{sq}\right] r\ln r\right\}\\ &= \frac{\alpha_s}{4\pi}\left\{-2(2C_F + C_A) r\ln^2 r + \left[2C_F\left(1 + 4\ln\frac{4}{3}\right) + C_A\left(\frac{1}{3} + 4\ln\frac{4}{3}\right) + \frac{4}{3}n_f T_F\right] r\ln r\right\} + c_0(\alpha_s) + c_1(\alpha_s) r\end{aligned}$$

Check fixed order expansion

Expanding to order α_s for the HJM left shoulder

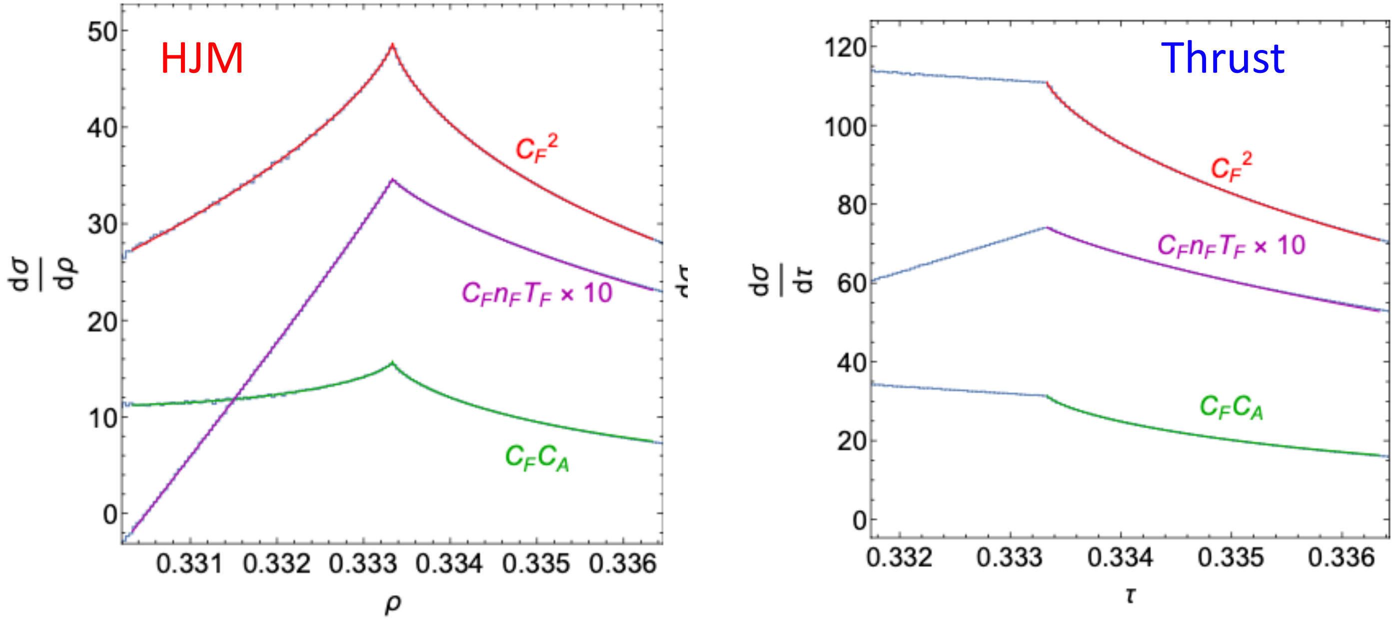
$$\frac{1}{\sigma_1}\frac{d\sigma^{\rm sub}}{dr} = \frac{\alpha_s}{4\pi}\left\{-\frac{1}{2}(2C_F + C_A)\Gamma_0 r\ln^2 r + \left[(C_A + 2C_F)\Gamma_0 + \gamma_{jg} + 2\gamma_{jq} + 2\gamma_{sg} + 4\gamma_{sq}\right] r\ln r\right\}$$

$$= \frac{\alpha_s}{4\pi}\left\{-2(2C_F + C_A)r\ln^2 r + \left[2C_F\left(1 + 4\ln\frac{4}{3}\right) + C_A\left(\frac{1}{3} + 4\ln\frac{4}{3}\right) + \frac{4}{3}n_f T_F\right] r\ln r\right\} + c_0(\alpha_s) + c_1(\alpha_s) r$$

And similarly (with differnet constants) for the HJM right shoulder and thrust

constant and linear term not predicted (more on this soon)

Excellent agreement with event 2 (NLO fixed order)



- used cutoff = 10^{-12}, 12 trillion events for event2

Resummed distribution

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dr} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) r \left(\frac{rQ}{\mu_s}\right)^{\eta_\ell} \left(\frac{rQ}{\mu_s}\right)^{\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)} \frac{\sin(\pi\eta_\ell)}{\sin(\pi(\eta_\ell+\eta_h))}$$

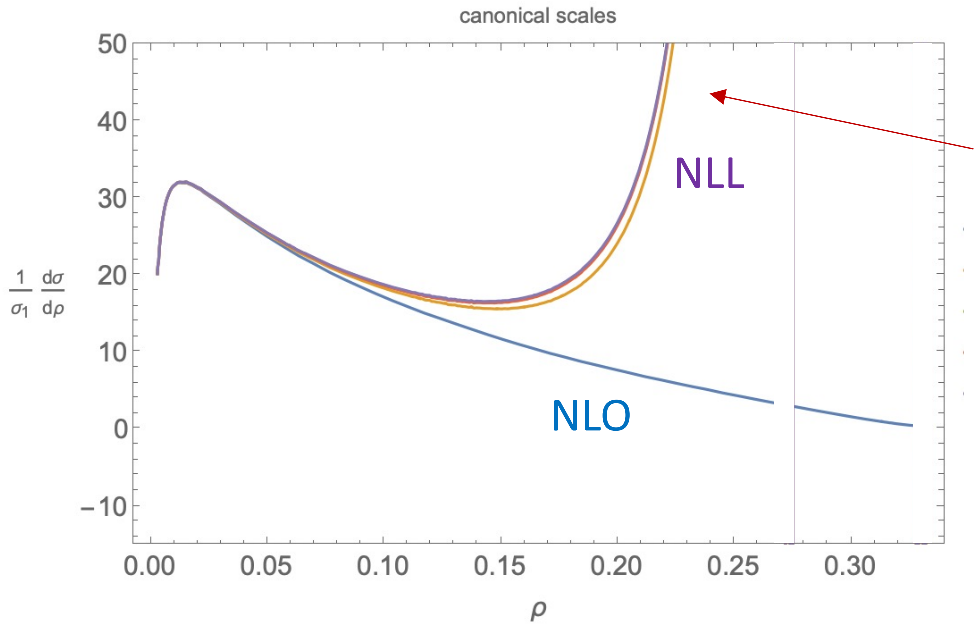
$$\eta_\ell = 2C_A A_\Gamma(\mu_j, \mu_s) \qquad \eta_h = 4C_F A_\Gamma(\mu_j, \mu_s)$$

canonical scales

$$\mu_h = Q$$
$$\mu_j = \sqrt{r}Q$$
$$\mu_s = rQ$$

$$\eta_\ell = -C_A \frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r \qquad \eta_h = -2C_F \frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r$$

Plot it



Blows up at ρ∼0.25

- Arises when

$$\sin(\pi(\eta_\ell+\eta_h)) = 0 \quad \Leftrightarrow \quad \eta_\ell+\eta_h = (C_A+2C_F)\frac{\Gamma_0}{2}\frac{\alpha_s}{4\pi}\ln r = 1, 2, 3, \cdots$$

- Comes from running associated with cusp anomalous dimension
- We call this a "**Sudakov Landau pole**"
  - Similar effects seen in q_T resummation for Drell-Yan
  - Present even if β=0

UV divergences

Factorization formula is of the form

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma_g}{d\rho} = \Pi_g \int_0^\infty dm_\ell^2 \int_0^\infty dm_h^2\ \widetilde{sj}_g \cdot \left(\frac{Qm_l^2}{\mu_{s\ell}}\right)^a \left(\frac{Qm_h^2}{\mu_{sh}}\right)^b \frac{1}{m_l^2 m_h^2}\frac{e^{-\gamma_E(a+b)}}{\Gamma(a)\Gamma(b)}\Bigg|_{\substack{a=\eta_\ell \\ b=\eta_h}} \times (r+m_h^2-m_\ell^2)\theta(r+m_h^2-m_\ell^2)$$

- can have $r \ll 1, \quad m_h^2 \gg 1, \quad m_\ell^2 \gg 1$
- integral is linearly UV divergent in this region for $0 < a,b < 1$

Simplest to **take two more derivatives** and consider

$$f(r) \equiv \frac{1}{\Gamma(a)}\frac{1}{\Gamma(b)}\int_0^\infty dx \int_0^\infty dy\ x^{a-1}y^{b-1}\delta(r+y-x)$$
$$= \frac{1}{\Gamma(a+b)}\left[r^{a+b-1}\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) + (-r)^{a+b-1}\frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

- Now UV finite
  - Will need to integrate twice and set integration constants $c_0 + c_1 r$
- Formula is valid for $\rho < ⅓$ ($r>0$) as well as $\rho > ⅓$ ($r<0$)
- Still has Sudakov Landau pole at *a+b=1,2,3,…*

Position vs momentum space

In momentum space, distribution is complicated and non-analytic

$$f(r) \equiv \frac{1}{\Gamma(a)}\frac{1}{\Gamma(b)}\int_0^\infty dx \int_0^\infty dy\; x^{a-1}y^{b-1}\delta(r+y-x)$$
$$= \frac{1}{\Gamma(a+b)}\left[r^{a+b-1}\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) + (-r)^{a+b-1}\frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

In **position space**, distribution is remarkably simple

$$\tilde{f}(z) = \int_{-\infty}^{\infty} dr\, f(r) e^{izr} = (-iz)^a (iz)^b$$

difficult to compute

most carefully track analytic continuation

No longer has Sudakov landau poles at a+b = 1,2,3...

- In **Laplace space** distribution is **not simple**
    - For 1-sided Laplace transform, need to flip sign in exponent to make integral convergent

$$\mathcal{L}[f](\nu) = \int_0^\infty dr\; e^{-\nu r} f(r) + \int_{-\infty}^0 dr\; e^{\nu r} f(r) = \nu^{-a-b}\frac{\sin(\pi a)+\sin(\pi b)}{\sin(\pi(a+b))}\,.$$

    - Still has Sudakov Landau pole

Γ_0 approximation

To undestand better what is happening, consider the "Γ_0 approximation"

- set all γ_j=0 and β=0
- keep only leading cusp anomalous dimension Γ_0

Momentum space

$$\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma_g}{d\rho^3} = e^{-2\hat\alpha C_F \ln^2\frac{\mu_h}{\mu_{jh}} - 2\hat\alpha C_F \ln^2\frac{\mu_{sh}}{\mu_{jh}} - \hat\alpha C_A \ln^2\frac{\mu_h}{\mu_{j\ell}} - \hat\alpha C_A \ln^2\frac{\mu_{s\ell}}{\mu_{j\ell}}} \times \frac{e^{-\gamma_E(a+b)}}{\Gamma(a+b)}$$

$$\times\left[\frac{1}{r}\left(\frac{rQ}{\mu_{s\ell}}\right)^a\left(\frac{rQ}{\mu_{sh}}\right)^b\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) + \frac{1}{(-r)}\left(\frac{-rQ}{\mu_{s\ell}}\right)^a\left(\frac{-rQ}{\mu_{sh}}\right)^b\frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

$$\mu_{s\ell} = \mu_{sh} = |r|Q$$
$$\mu_{j\ell}^2 = \mu_{jh}^2 = Q\mu_{s\ell} = Q\mu_{sh}$$

canonical scales

$$\left(\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma}{d\rho^3}\right) = \frac{1}{r}e^{-\Gamma_0\frac{1}{2}C_A\hat\alpha\ln^2|r| - \Gamma_0 C_F\ln^2|r|}\frac{e^{-\gamma_E(a+b)}}{\Gamma(a+b)}\left[\frac{\sin(\pi a)}{\sin(\pi(a+b))}\theta(r) - \frac{\sin(\pi b)}{\sin(\pi(a+b))}\theta(-r)\right]$$

$$a = -C_A\hat\alpha\Gamma_0\ln|r|$$
$$b = -2C_F\hat\alpha\Gamma_0\ln|r|$$

- Sudakov Landau poles are there
- Not of the double-logarithmic form exp[L g_1(α L)]
  - Includes problematic subleading terms

Position space

$$\tilde\sigma(z) = \int_{-\infty}^{\infty} dr\, e^{izr}\,\frac{1}{\sigma_{\rm LO}}\frac{d^3\sigma}{d\rho^3}$$

$$= e^{-2\hat\alpha C_F \ln^2\frac{\mu_h}{\mu_{jh}} - 2\hat\alpha C_F \ln^2\frac{\mu_{sh}}{\mu_{jh}} - \hat\alpha C_A \ln^2\frac{\mu_h}{\mu_{j\ell}} - \hat\alpha C_A \ln^2\frac{\mu_{s\ell}}{\mu_{j\ell}}}\left(-iz\frac{\mu_{s\ell}e^{\gamma_E}}{Q}\right)^{-a}\left(iz\frac{\mu_{sh}e^{\gamma_E}}{Q}\right)^{-b}$$

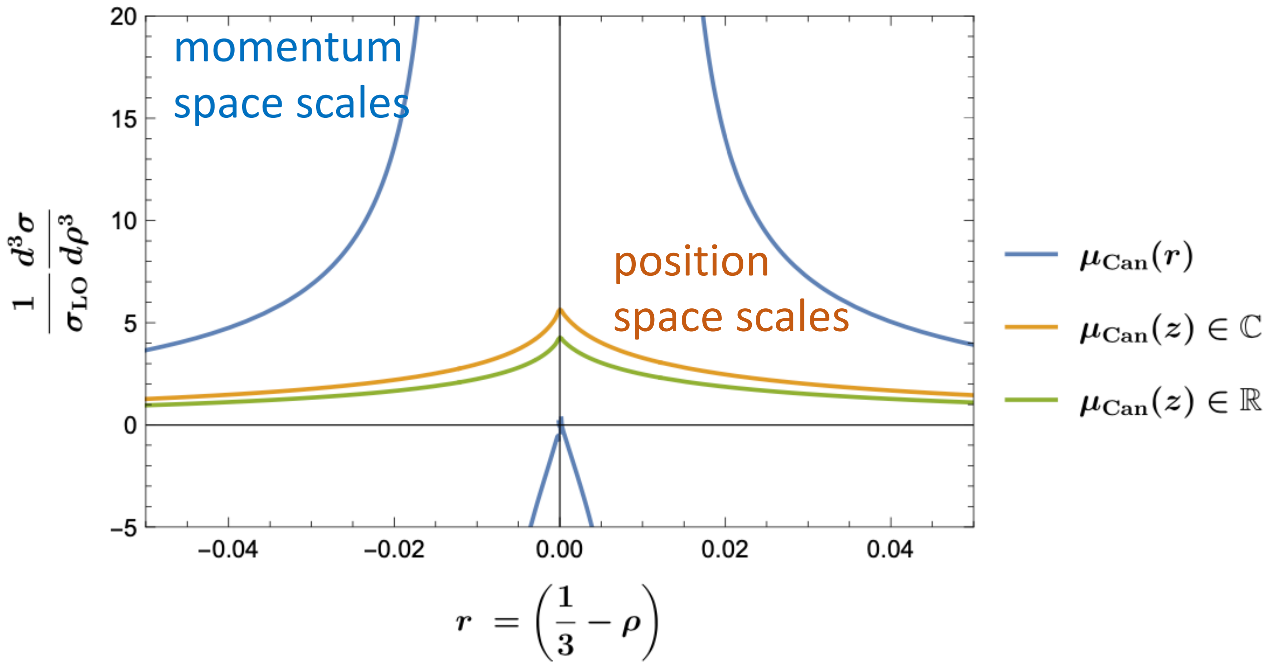
canonical complex scales

$$\mu_{s\ell} = i\frac{Qe^{-\gamma_E}}{z}, \quad \mu_{sh} = -i\frac{Qe^{-\gamma_E}}{z}$$

$$\tilde\sigma(z) = \exp\left[-\frac{1}{2}\hat\alpha C_A\Gamma_0\ln^2(-ize^{\gamma_E}) - \hat\alpha C_F\Gamma_0\ln^2(ize^{\gamma_E})\right]$$

- Sudakov Landau poles are gone
- Exactly of the double-logarithmic form exp[L g_1(α L)]
  - Problematic subleading terms are absent

Γ_0 approximation



With position space scale setting

- Sudakov Landau poles are gone
- Distribution is continuous across r=0
- With running coupling wil be smooth across r=0

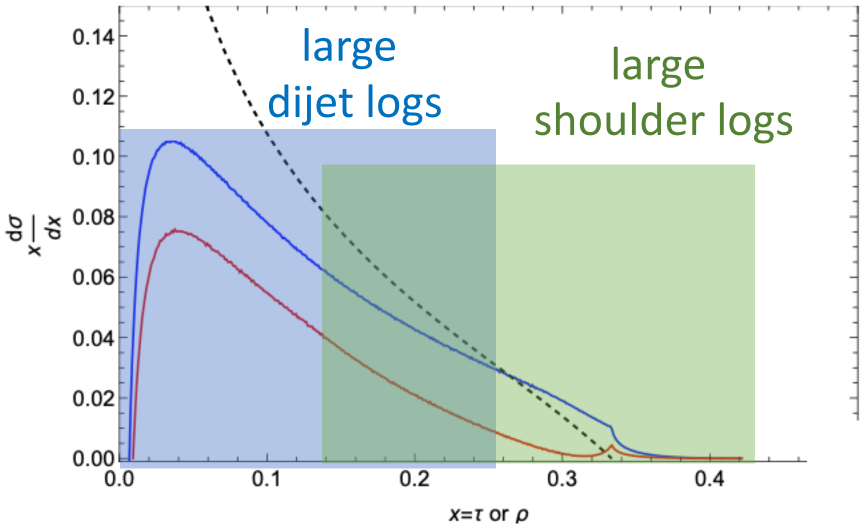
Can show that

$$\left(\frac{1}{\sigma_{\text{LO}}}\frac{d^3\sigma}{d\rho^3}\right)_{\text{pos}} = e^{-\hat{\alpha}\Gamma_0 \frac{1}{2} C_A \partial_a^2 - \hat{\alpha}\Gamma_0 C_F \partial_b^2} \left(\frac{1}{\sigma_{\text{LO}}}\frac{d^3\sigma}{d\rho^3}\right)_{\text{mom}}$$

- derivatives blow up at Sudakov Landau pole
- exp(-∂^2) supresses the divergence

Matching to the dijet region

We need to match bewteen the shoulder region and the dijet region



Look at fixed order

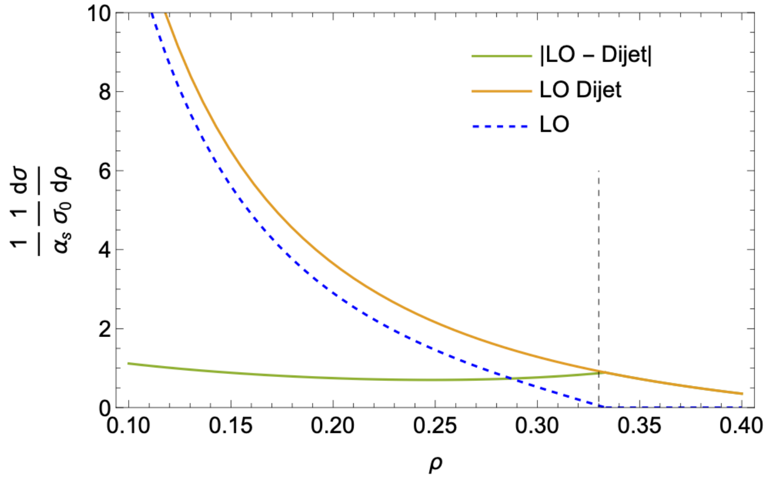
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\rho} = C_F\frac{\alpha_s}{2\pi}\left\{\frac{3(1+\rho)(3\rho-1)}{\rho} + \frac{[4+6\rho(\rho-1)]\ln\frac{1-2\rho}{\rho}}{\rho(1-\rho}\right]\theta\left(\frac{1}{3}-\rho\right)$$

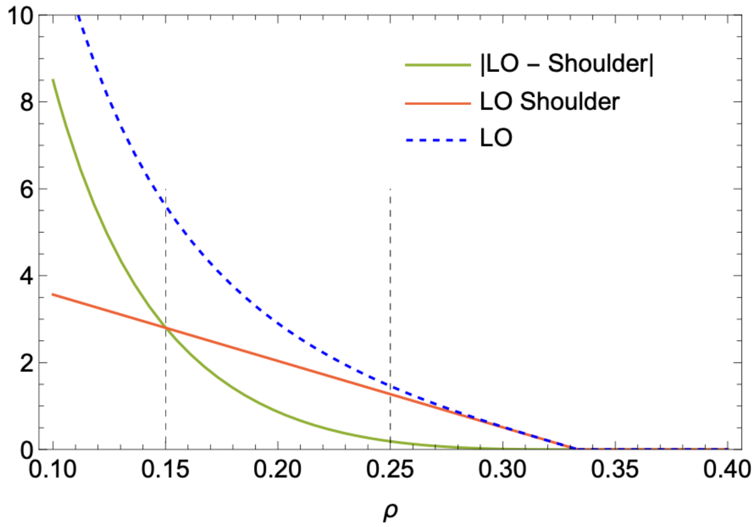
expand near ρ=0

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{dij}}}{d\rho} = C_F\frac{\alpha_s}{2\pi}\left(-\frac{3}{\rho}-\frac{4\ln\rho}{\rho}\right)$$

expand near ρ=⅓

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{shoulder}}}{d\rho} = C_F\frac{\alpha_s}{2\pi}72\left(\frac{1}{3}-\rho\right)\theta\left(\frac{1}{3}-\rho\right)$$



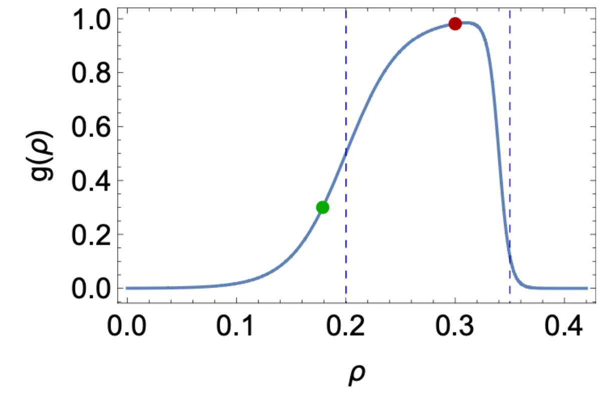


- want pure shoulder for $\rho > 0.25$
- fade to dijet by $\rho < 0.15$

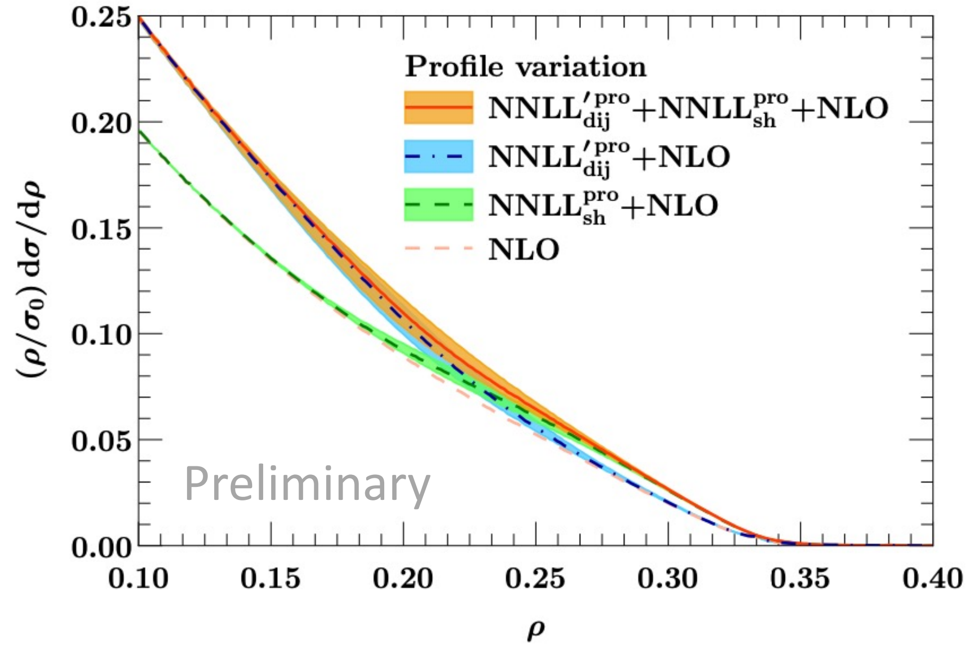
Matching to the dijet region

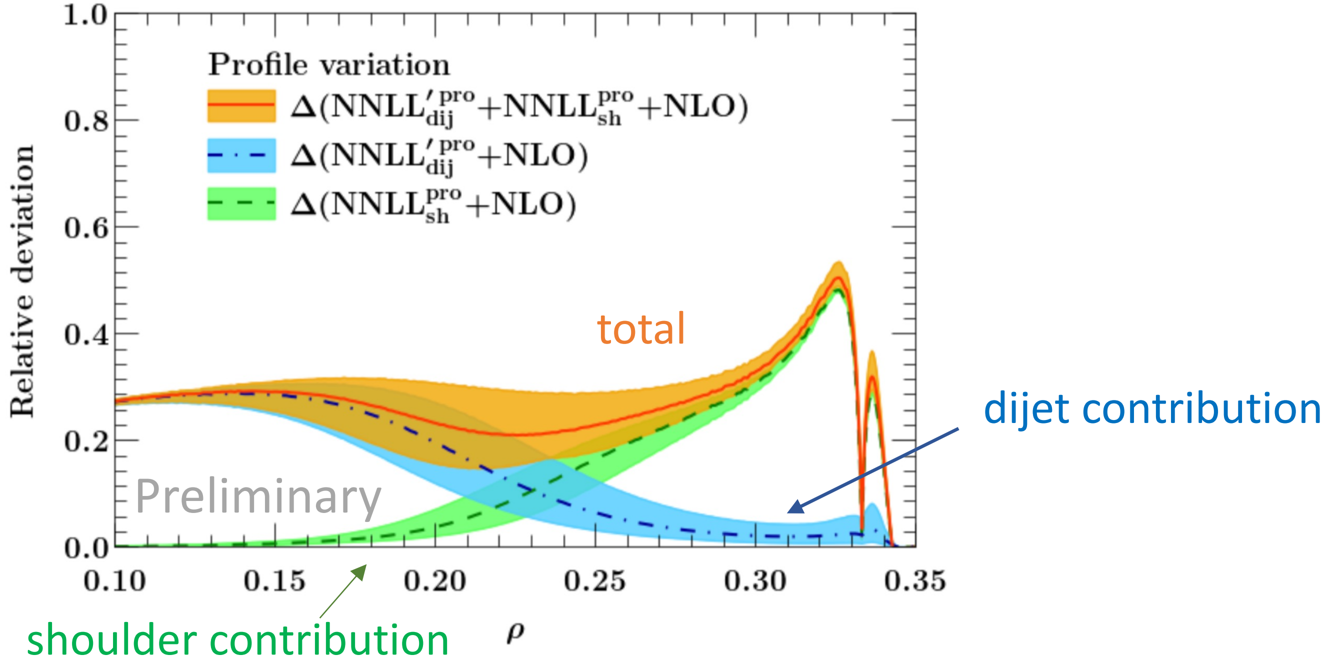
Turn off resummation in shoulder and dijet region by using ρ dependent profile functions for soft and jet scales:

$$\mu_{j,s}^{\mathrm{pro}}(z,\rho) = \mu_h^{1-g(\rho)}\left[\mu_{j,s}^{\mathrm{can}}(z)\right]^{g(\rho)}$$

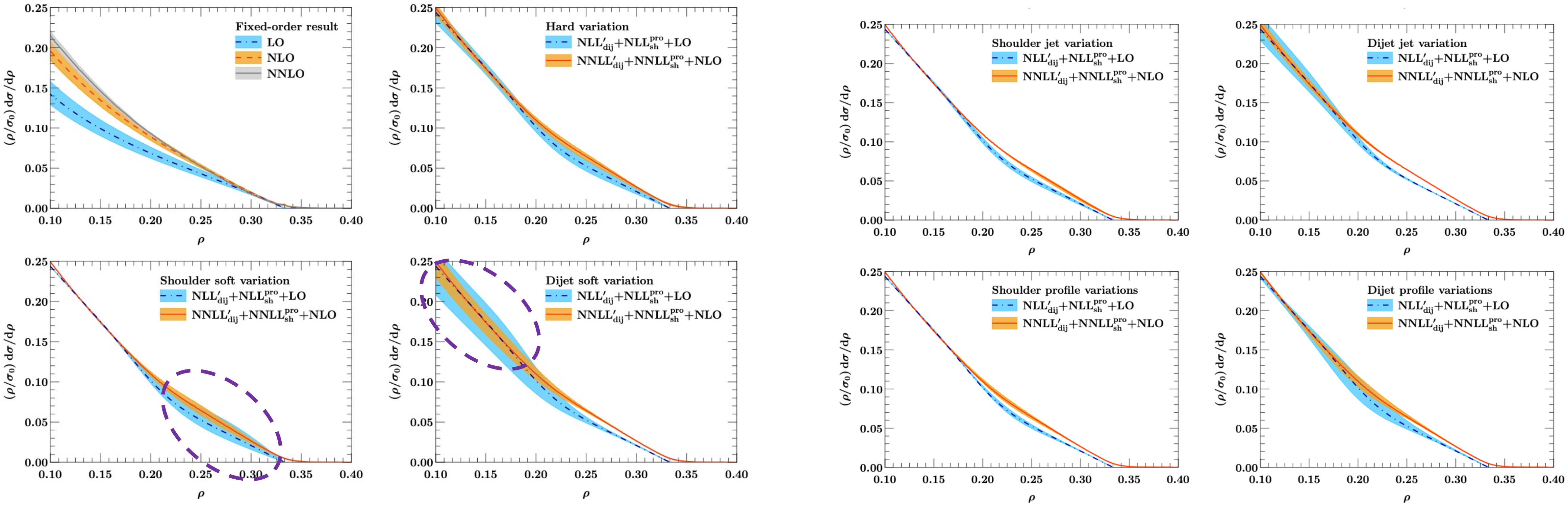


We use sigmoid functions to
- fade between shoulder and dijet
- fade out resummation in the right shoulder.





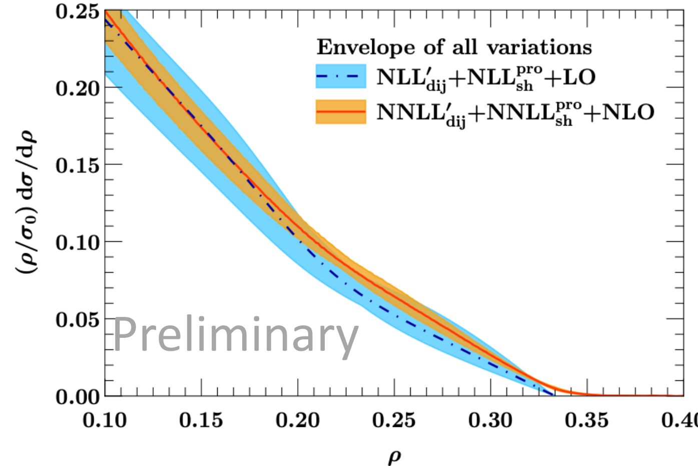
Scale variations



$$\mu_h = 2^{v_h} Q$$

$$\mu_{s,\mathrm{sh}}(z) = \sqrt{\left(2^{v_h} 2^{v_s} \frac{Q e^{-\gamma_E}}{|z|}\right)^2 + \left(\mu_s^{\min}\right)^2}$$

$$\mu_{j,\mathrm{sh}}(z) = \left(\mu_h \mu_{s,\mathrm{sh}}(z)\right)^{v_j} Q^{1-2v_j}$$

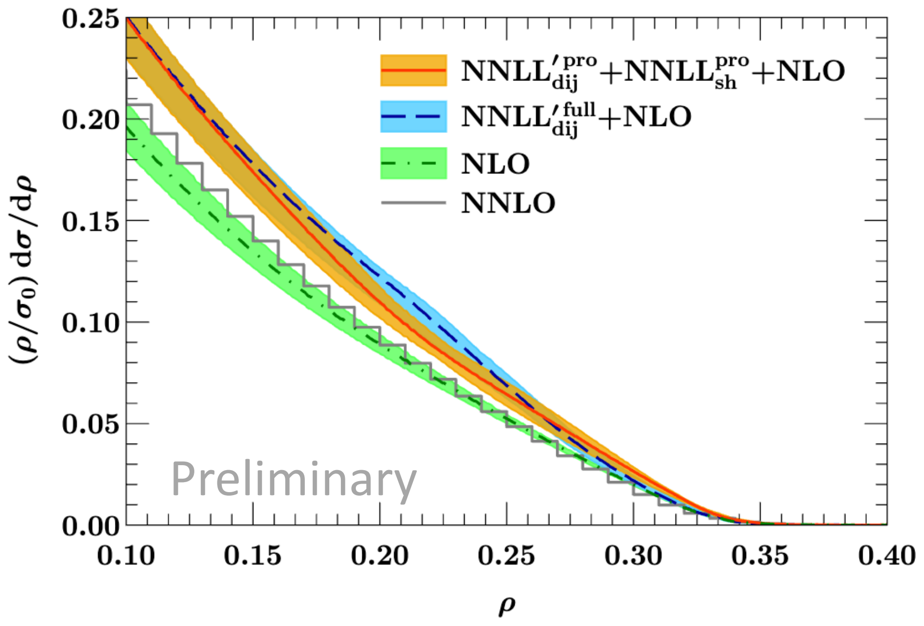


Envelope of all variations

- dominated by soft variation
- will go down a bit with N^3LL dijet resummation

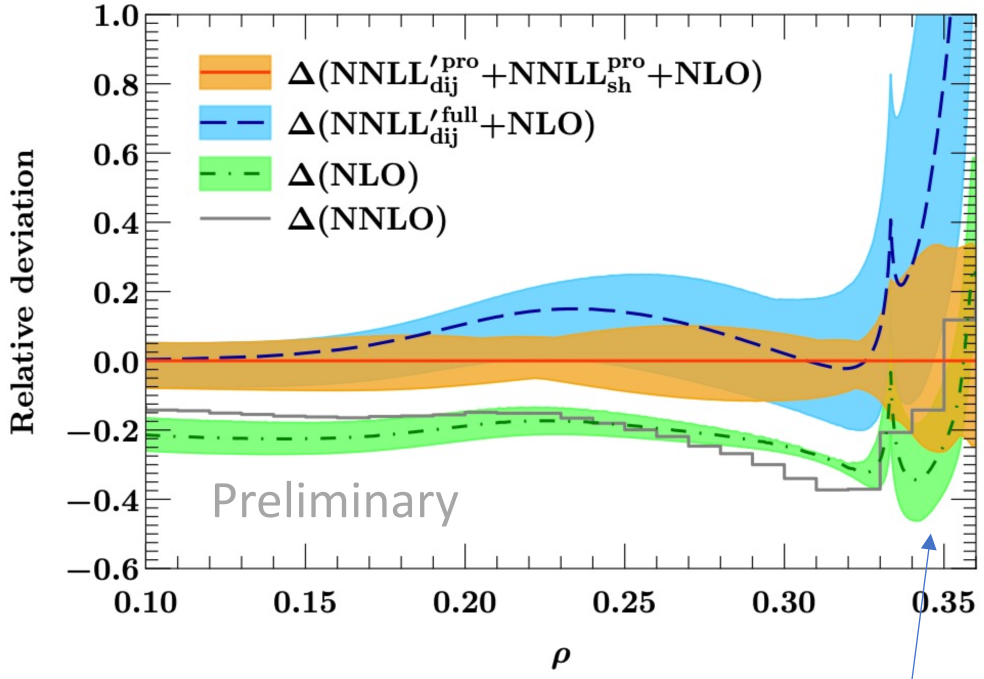
Final result

Compare to NLO and just dijet resummation



ratio to best

$$\Delta(d\sigma) = \frac{d\sigma - d\sigma_{\mathrm{sh+dij+NLO}}}{d\sigma_{\mathrm{sh+dij+NLO}}}$$

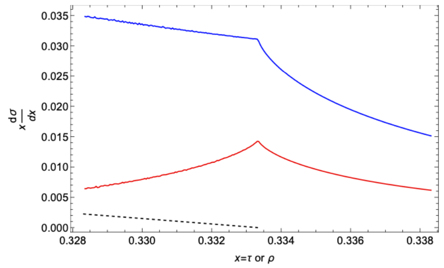


- Matched result is
  - 20% higher than NLO
  - More important than NNLO correction
- Pure dijet + NLO is 5% to 20% larger for $0.15 < \rho < 0.3$

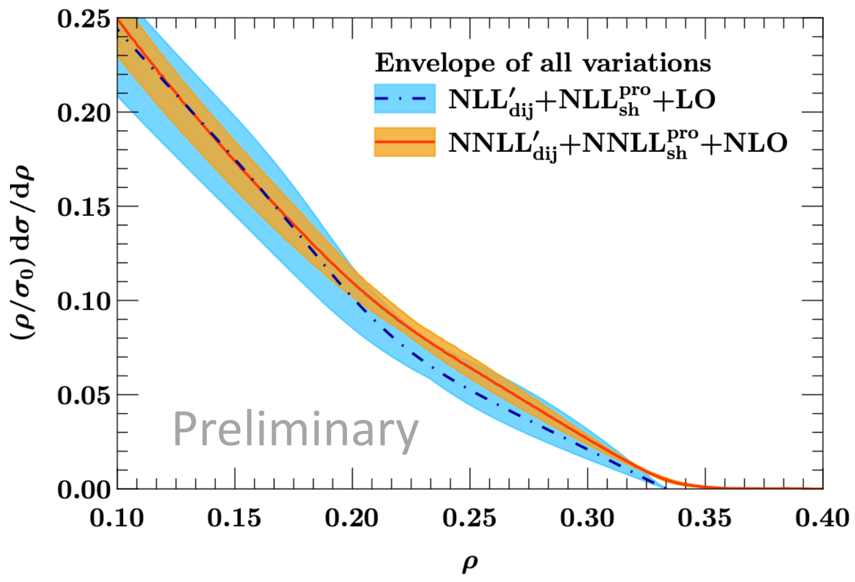
things go nuts because denominator is zero

Conclusions

- Sudakov shoulders are large logarithms associated with perturbative phase space boundaries
  - Heavy jet mass has a left Sudakov shoulder ($\rho < ⅓$) and a right sudakov shoulder ($\rho > ⅓$)
    - large logs extend down to $\rho = 0.15$ where α_s fits are done
  - Thrust only has a right shoulder ($\tau > ⅓$)
    - shoulder resummation is not critical for α_s fits for thrust



- Factorization theorem for the Sudakov shoulder allows for systematic resummation
  - Scales must be set in **position space** before Fourier transforming to avoid spurious Sudakov Landau poles
  - We resummed the HJM shoulder to NNLL, matching to NNLL dijet resummation and NLO fixed order



- **Effect is signifcant**
  - 5% to 100% larger than dijet+NLO for $0.15 < \rho < 0.3$

Next steps

- Extend matching to NNLO and N^3LL dijet
- Study **power corrections**
  - Factorization formula valid in the trijet region gives operator definition
  - How to interpolate power corrections from dijet to shoulder region?
  - How many non-perturbative parameters are needed?
- Extract α_s from e^+e^- data