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Amplitude evolution

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Parton Shower Resummation Workshop
Milano | 8 June 2023

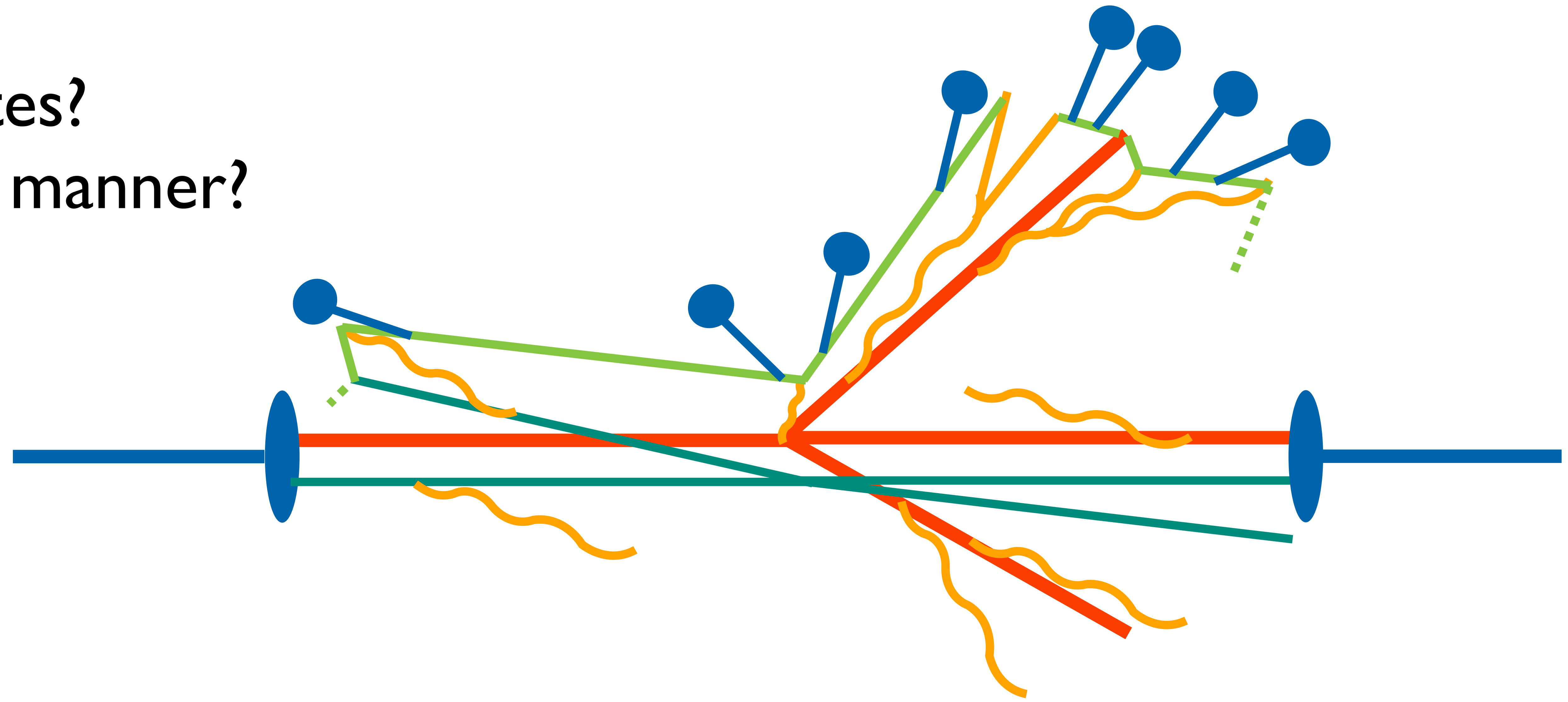
Current challenges

How do we accurately describe details of final states?
How do we quantify precision in a comprehensive manner?

Matching beyond NLO QCD?
Solve shower bottlenecks first?

How to benchmark precision of QCD algorithms?
How to accurately include EW and QED?

How to constrain hadronization models?
What is their response to perturbative variations?



$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Can we understand this better?

Perturbative precision is far from the last word:

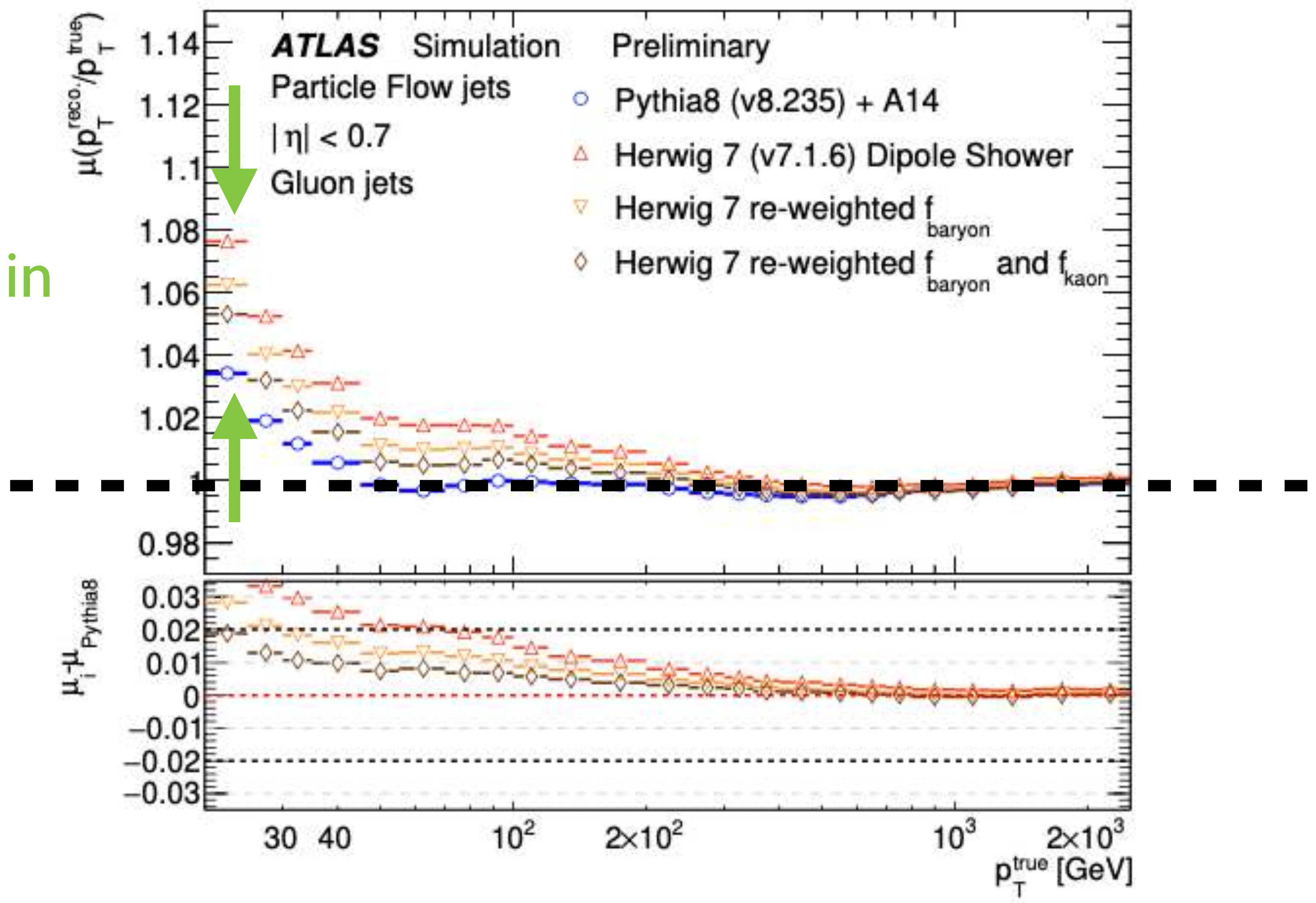
E.g. lack of understanding of baryon production is limiting the power of q/g discrimination.

Personal selection of some recent topics:
Parton showers, hadronization and their interface.
And new algorithms.

deviation of reconstructed pt

O(1) variation in the correction

1



simulated pt

[ATLAS-PUB-2022-021]

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Subleading colour corrections

Colour reconnection and hadronization is about subleading-N.
So are shower accuracy and interference terms.

Colour factor algorithms	Colour ME corrections	Full amplitude evolution
Coherent, NLL-accurate dipole showers	Colour-exact real emissions as far as possible	Colour-exact real and virtual corrections
[Gustafson] [PanScales '21] [Forshaw, Holguin, Plätzer '21]	[Plätzer, Sjödahl '12, '18] [Höche, Reichelt '20]	[Forshaw, Plätzer, Sjödahl, Holguin + … '13 …] [Nagy, Soper '12 …]

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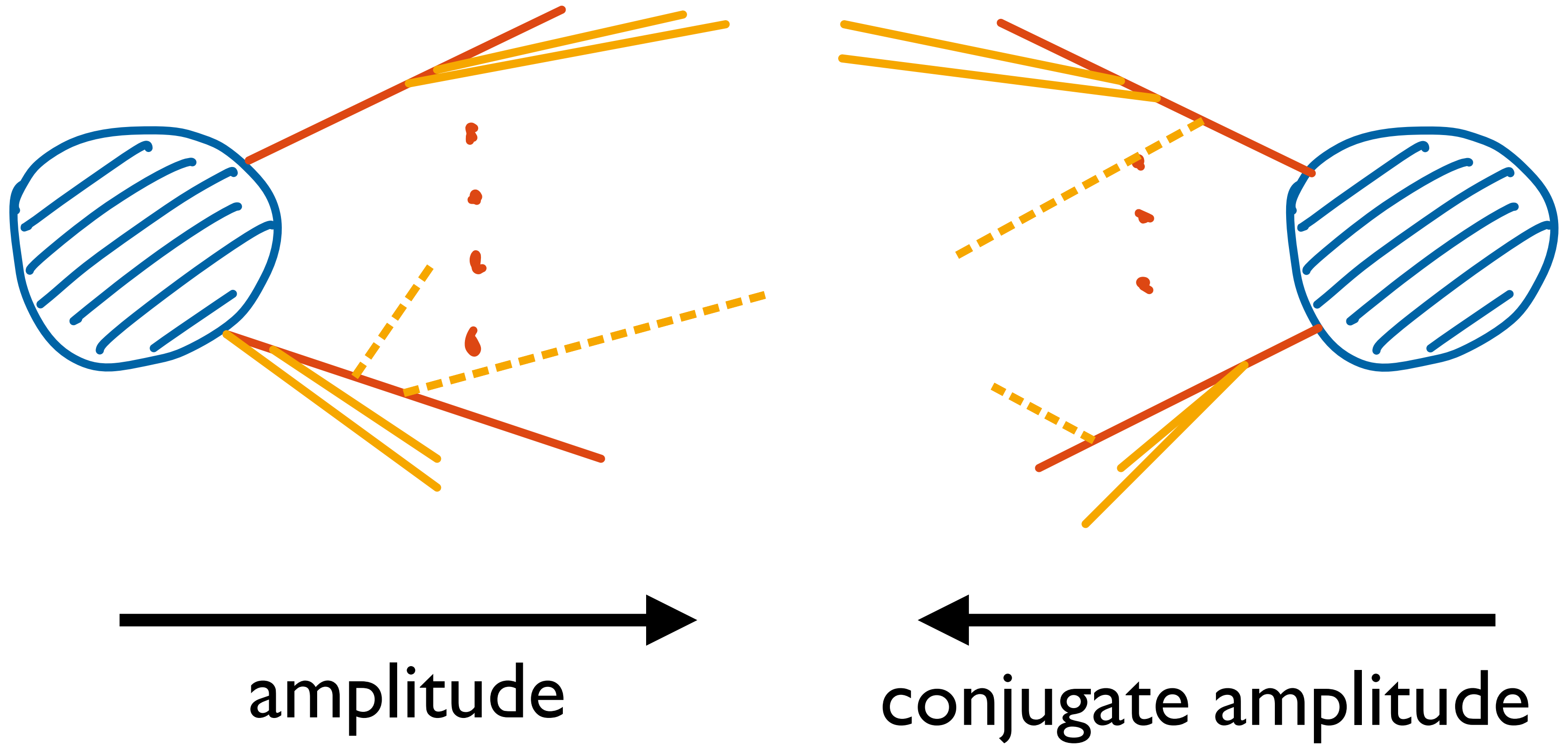
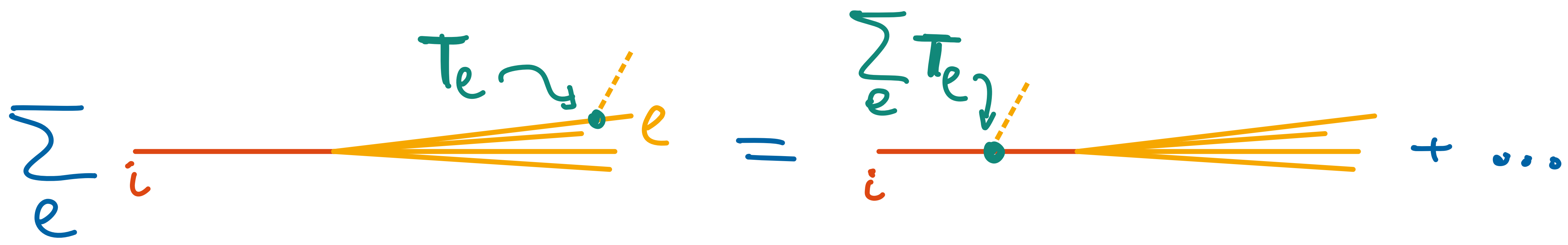
Coherent branching parton showers

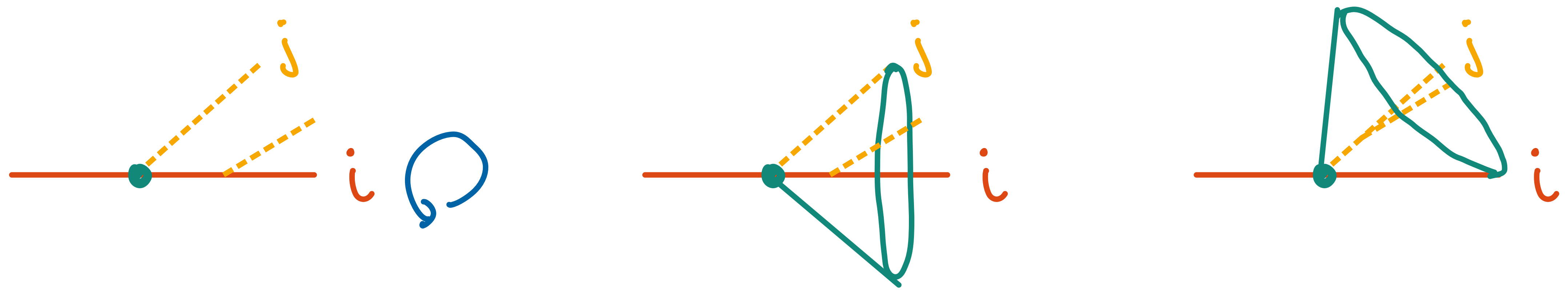
$$d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \to \mu) \times MPI \times Had(\mu \to \Lambda) \times \dots$$



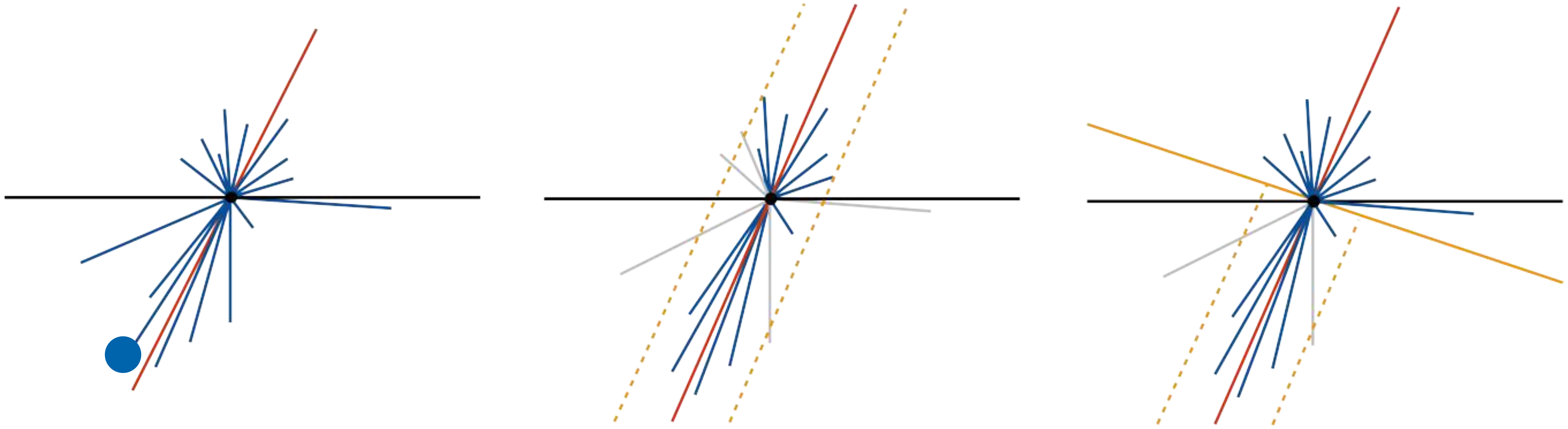


Move soft colour charges towards hard process and use angular ordering for azimuthal average around jet axes:

$$T_j T_l T_i \cdot T_i T_m T_j = C_i T_j T_l \cdot T_m T_j$$



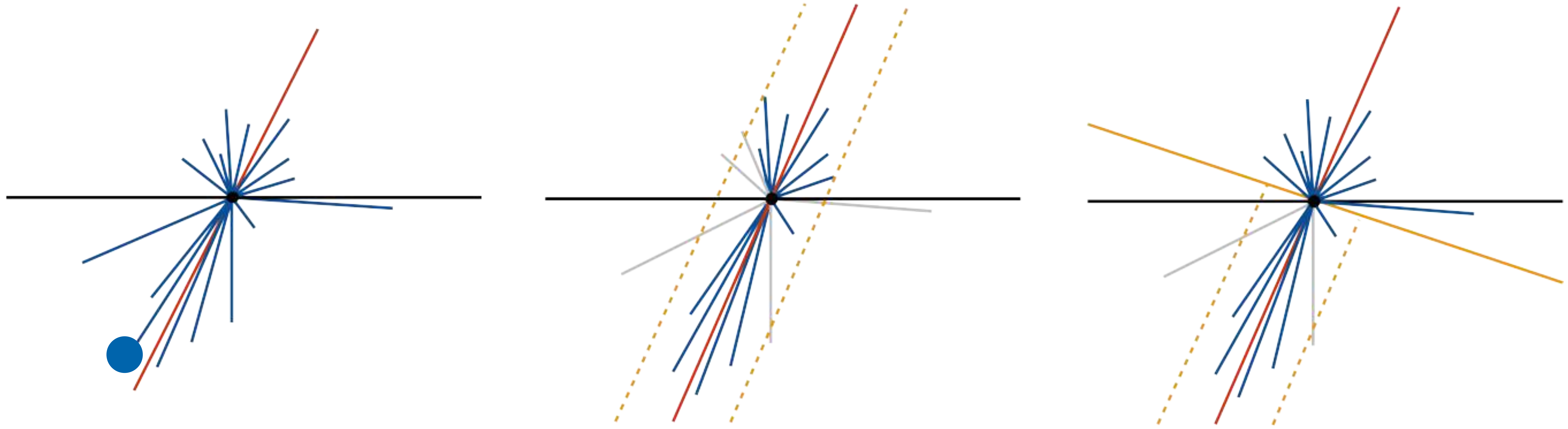
Accuracy of Parton Showers



Fragmentation is fine if we get collinear physics right.

Accuracy of Parton Showers

[Catani, Trentadue, Webber, Marchesini ….]



Fragmentation is fine if we get collinear physics right.

Global event shapes from coherent branching — for two jets.

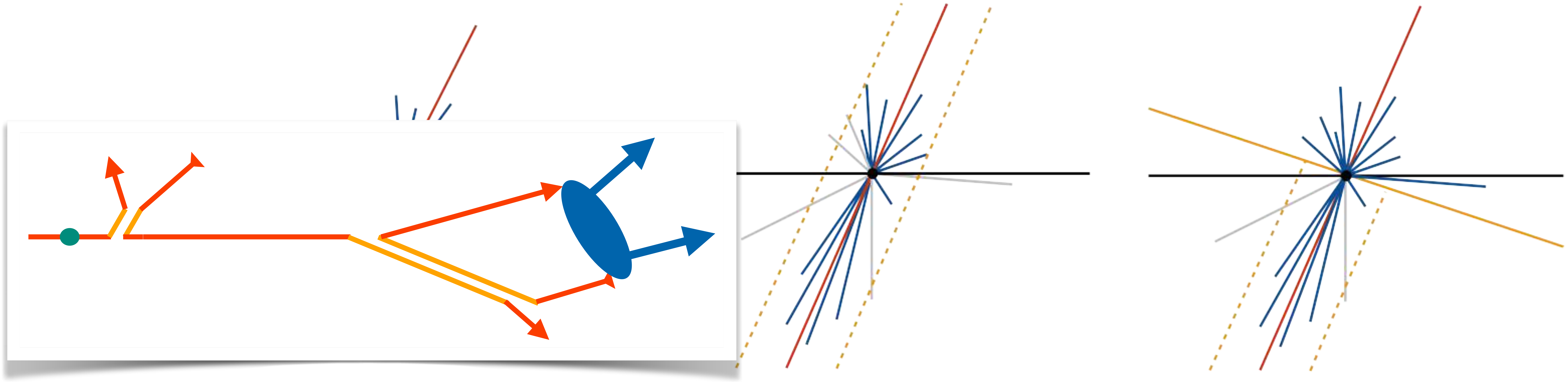
$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

LL — qualitative NLL — quantitative NNLL — precision

$$\alpha_s L \sim 1$$

Accuracy of Parton Showers

[Catani, Trentadue, Webber, Marchesini ….]



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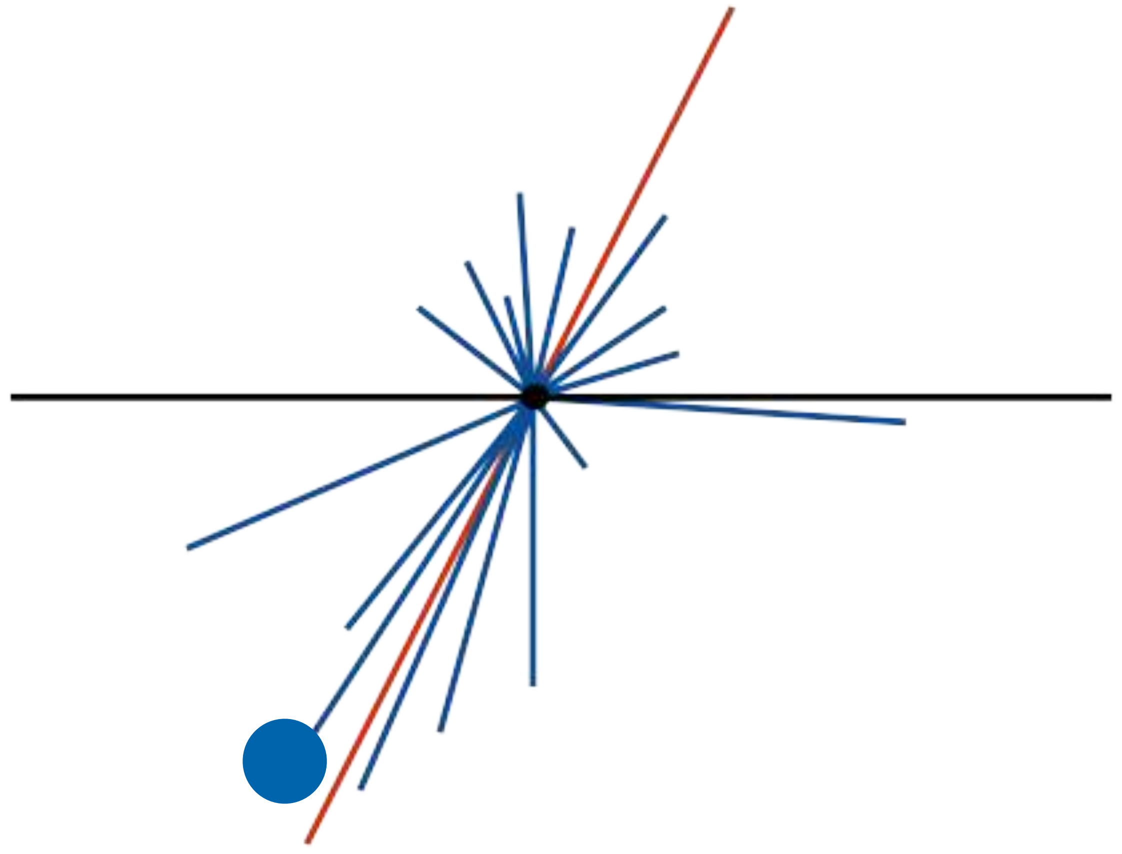

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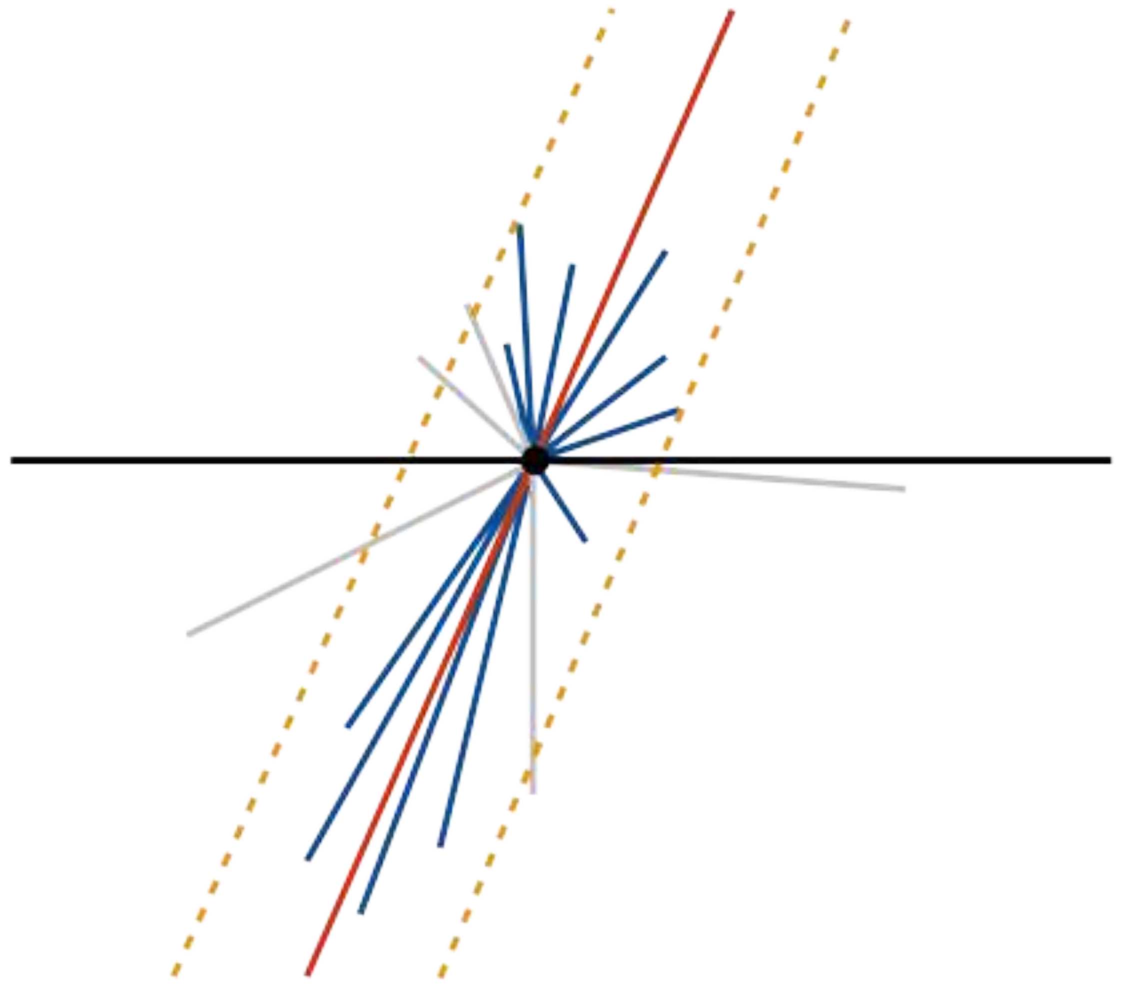

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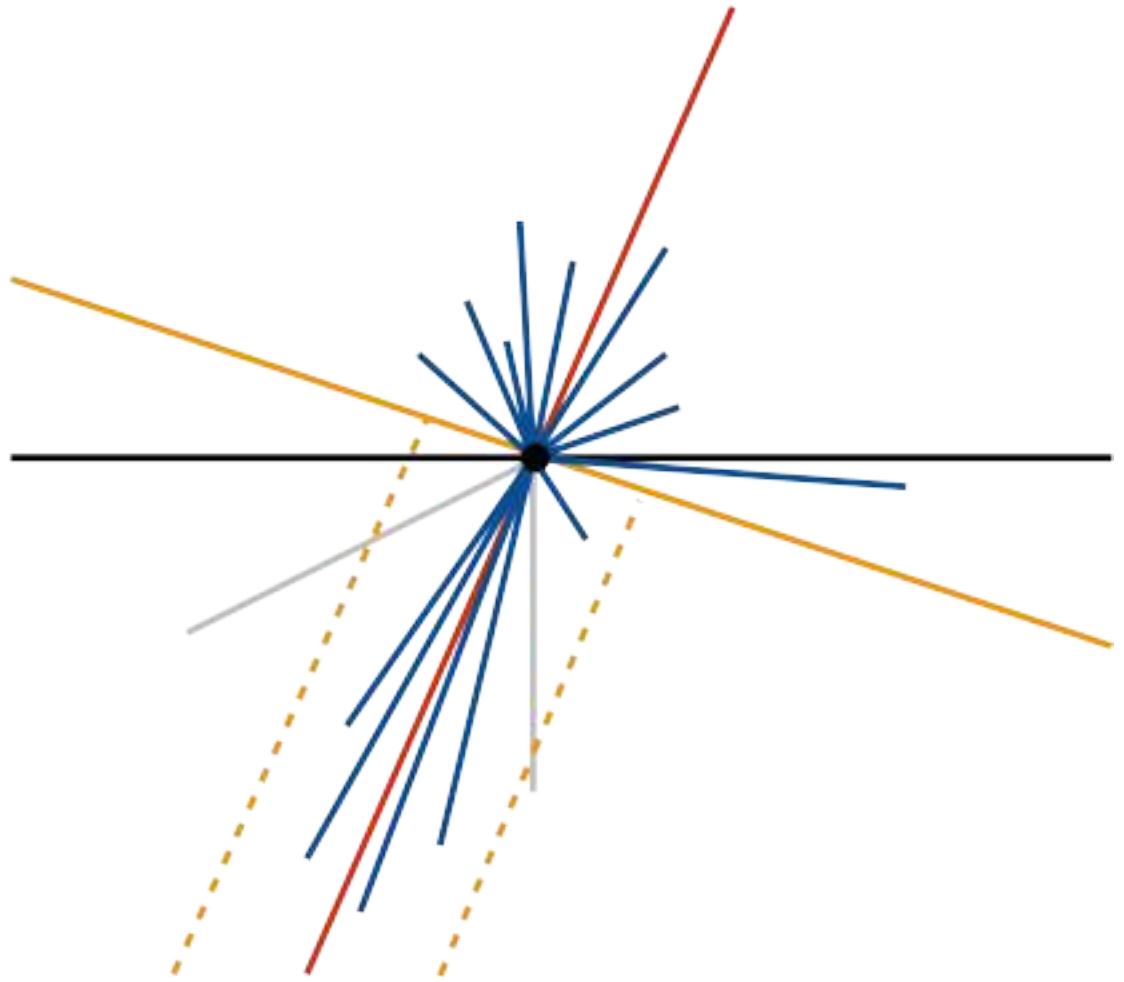


Coherence breaks down for non-global observables.

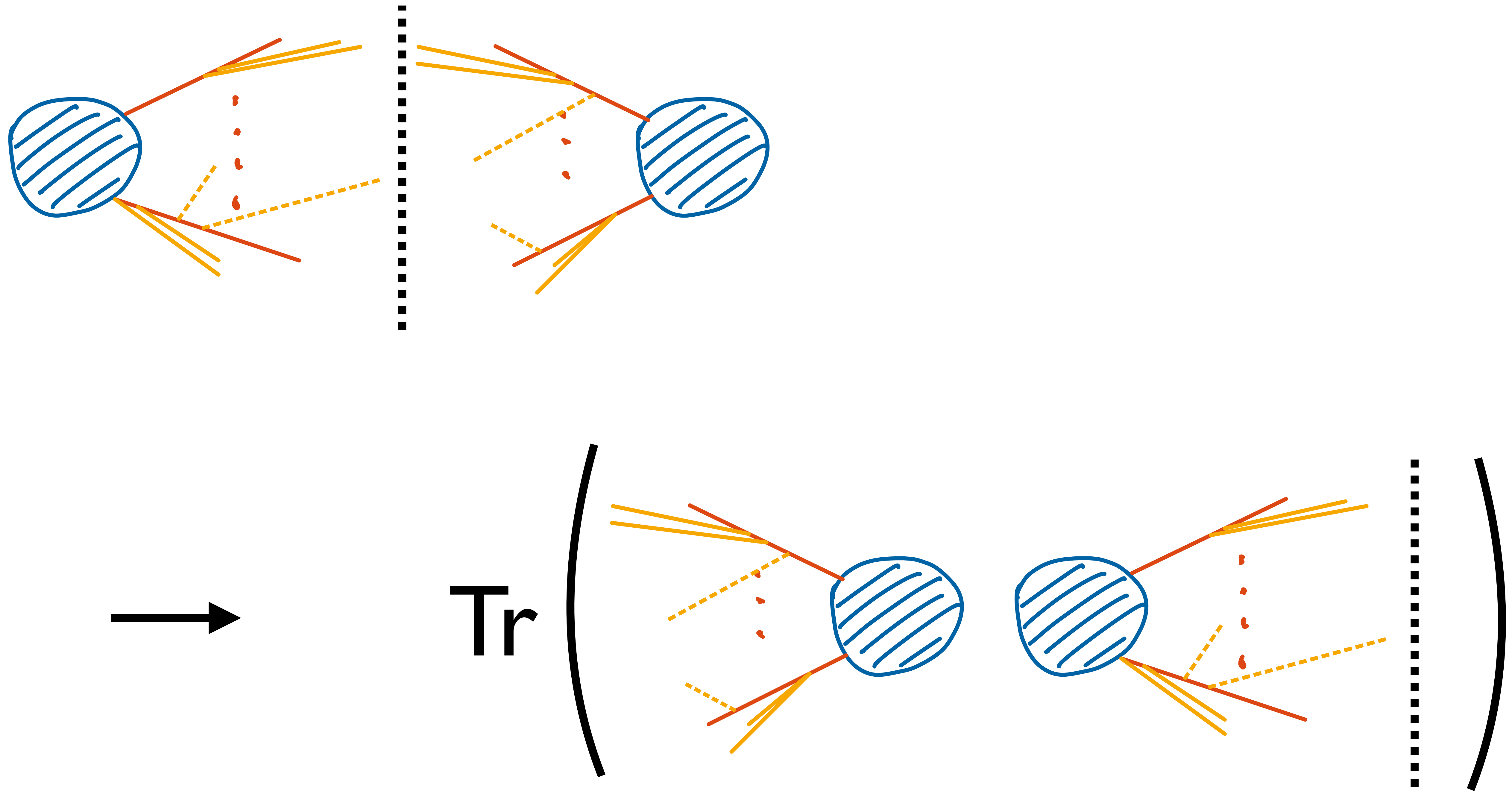
$T_k T_l T_i \cdot T_j T_m T_n$

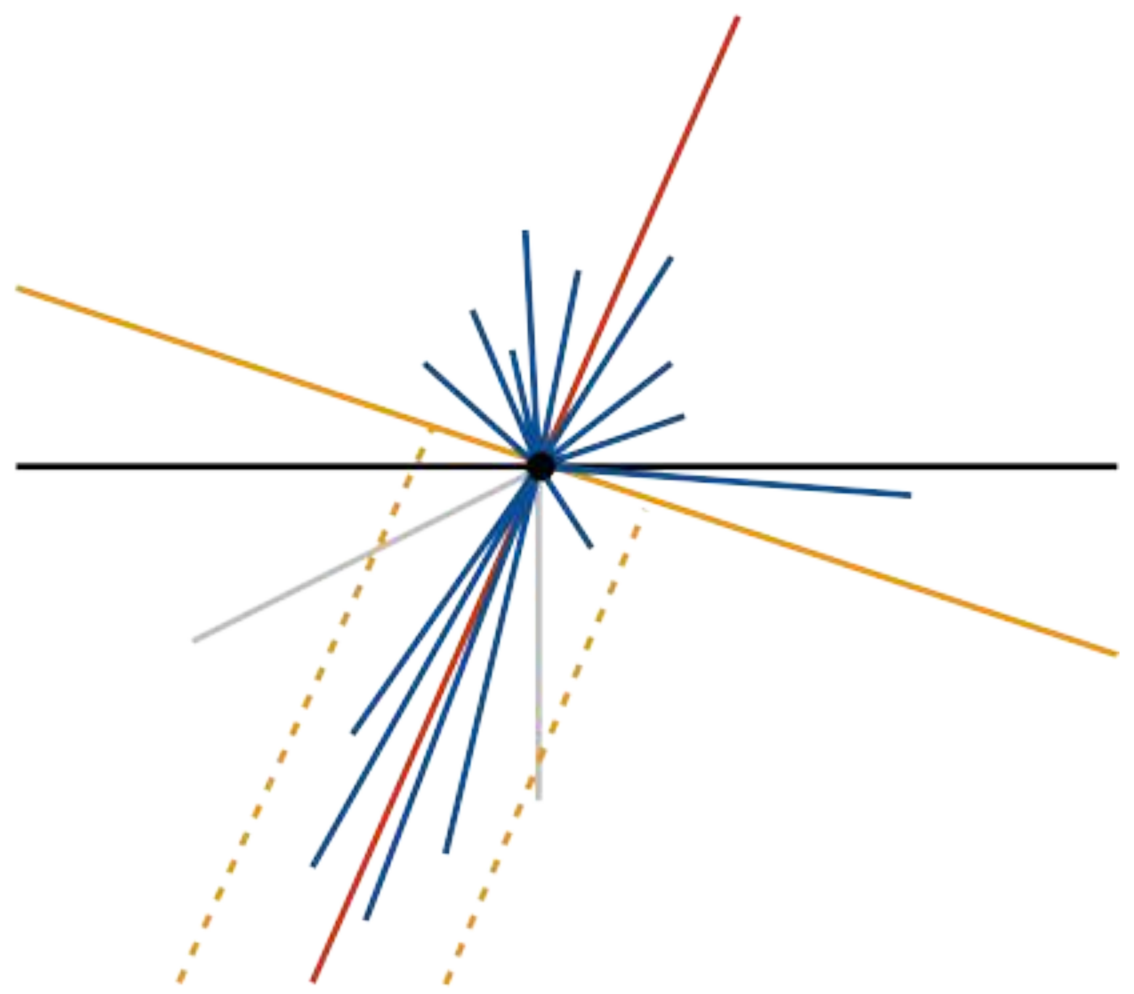
large-N limit ↓

$$\frac{\partial G_{ab}(t)}{\partial t} = -\int_{\text{in}} \frac{\mathrm{d}\Omega_k}{4\pi} \omega_{ab}(k) G_{ab}(t) + \int_{\text{out}} \frac{\mathrm{d}\Omega_k}{4\pi} \omega_{ab}(k) \left[G_{ak}(t) G_{kb}(t) - G_{ab}(t) \right]$$

[Banfi, Marchesini, Smye '02]

Amplitude evolution





Non-global observables set the level of complexity we need to address.
We cannot tell how subleading finite N is until we have the tools to test.

Amplitude evolution

Colour reconnection and hadronization is about subleading-N.
So are shower accuracy and interference terms.

Colour factor algorithms	Colour ME corrections	Full amplitude evolution
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[Gustafson] [PanScales '21] [Forshaw, Holguin, Plätzer '21]	[Plätzer, Sjödahl '12, '18] [Höche, Reichelt '20]	[Forshaw, Plätzer, Sjödahl, Holguin + … '13 …] [Nagy, Soper '12 …]

$$d\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q \to \mu)\mathrm{d}\mathbf{H}(Q)\mathbf{PS}^\dagger(Q \to \mu)\mathbf{Had}(\mu \to \Lambda)\right]$$

Colour matrix element corrections

[Plätzer, Sjödahl '12]
[Plätzer, Sjödahl, Thoren '18]

Colour matrix element corrections:
Real emissions only amplitude evolution —
first implementation in a shower algorithm.

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle$$

$$\mathcal{M}_n = (c_{n,1}, \ldots, c_{n,d_n})^T$$

$$|\mathcal{M}_n|^2 = \mathcal{M}_n^\dagger S_n \mathcal{M}_n = \mathrm{Tr}\left(S_n \times \mathcal{M}_n \mathcal{M}_n^\dagger\right)$$

$$\langle \mathcal{M}_n|\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}}|\mathcal{M}_n\rangle = \mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n}\mathcal{M}_n\mathcal{M}_n^\dagger T_{\tilde{ij},n}^\dagger\right)$$

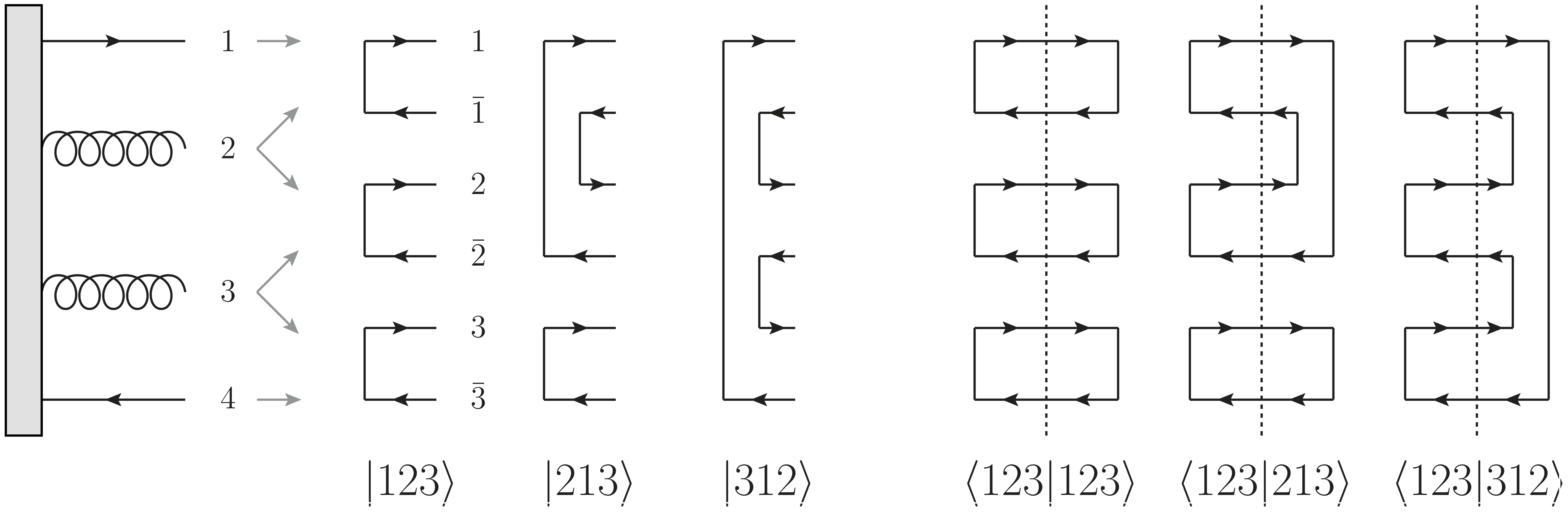
approximation: $V_{ij,k}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) \times$ correction factor: $\frac{-1}{\mathbf{T}_{\tilde{ij}}^2}\frac{\langle \mathcal{M}_n|\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_k|\mathcal{M}_n\rangle}{|\mathcal{M}_n|^2}$

$$M_{n+1} = -\sum_{i\neq j}\sum_{k\neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j}\frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{\tilde{ij}}^2} T_{\tilde{k},n} M_n T_{\tilde{ij},n}^\dagger$$

Tracking colour

Decompose amplitudes in flow of colour charge.

$$\mathrm{Tr}\left[\mathbf{A}_n\right] = \sum_{\sigma,\tau} A_{\tau\sigma} \langle \sigma | \tau \rangle$$

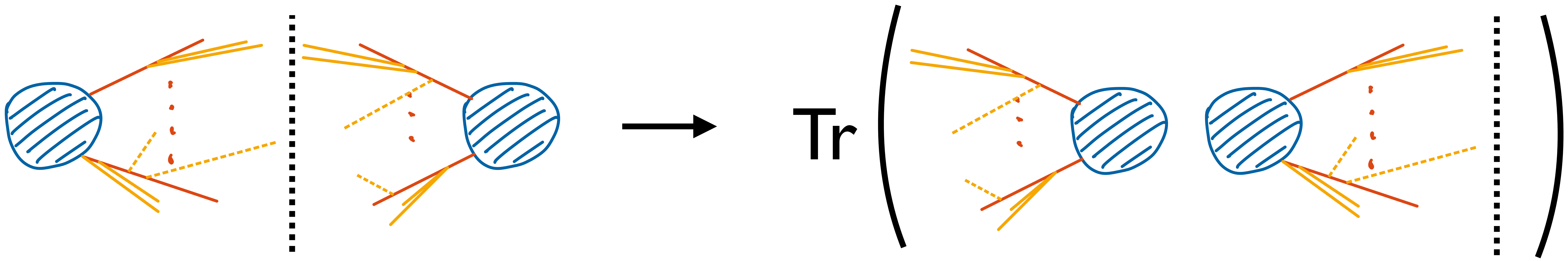


N^3 N^2 N

$$(t^a)^i{}_k (t^a)^j{}_l = T_R \left(\delta^i_l \delta^j_k - \frac{1}{N} \delta^i_k \delta^j_l \right)$$

[Plätzer '13]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour '18]

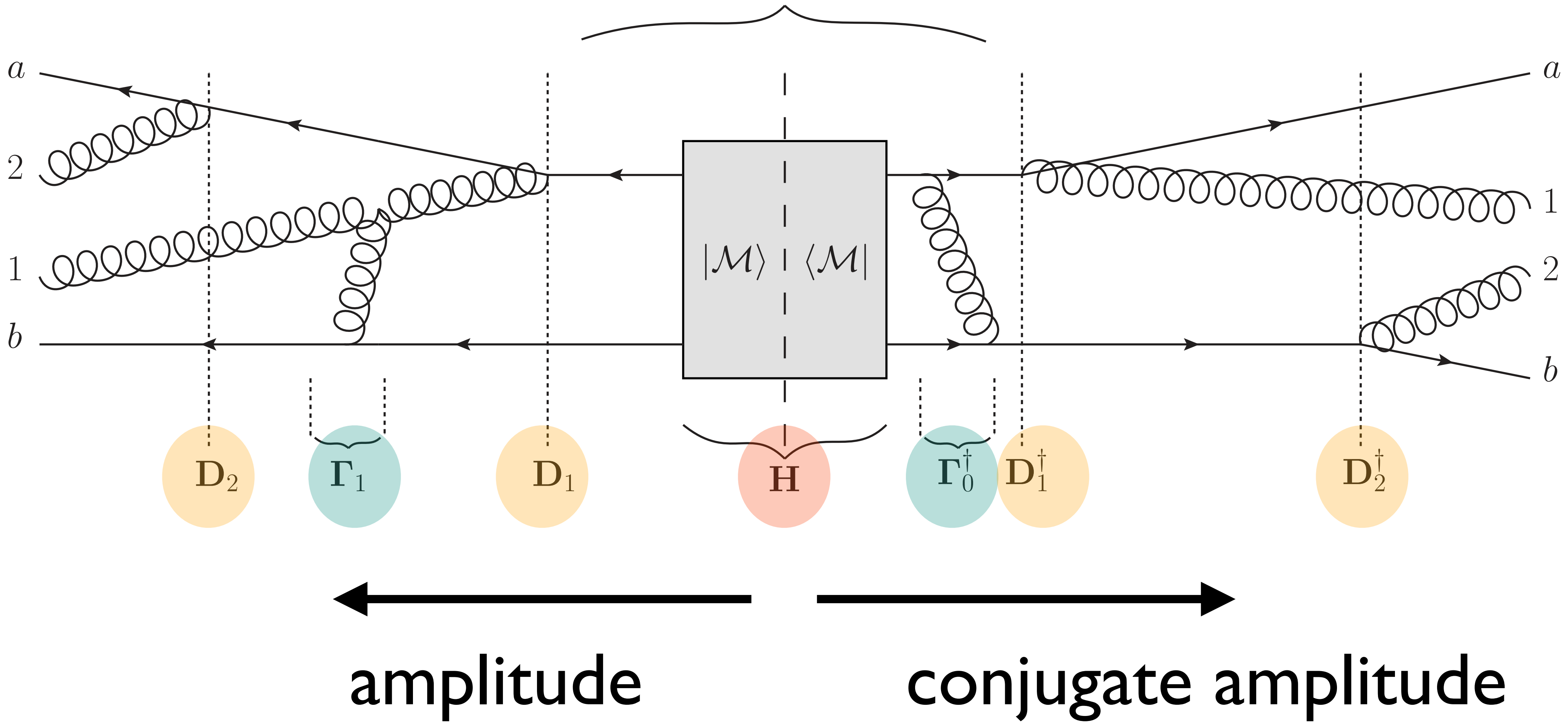
Amplitude evolution: CVolver



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

Different histories in amplitude and conjugate amplitude needed to include interference.



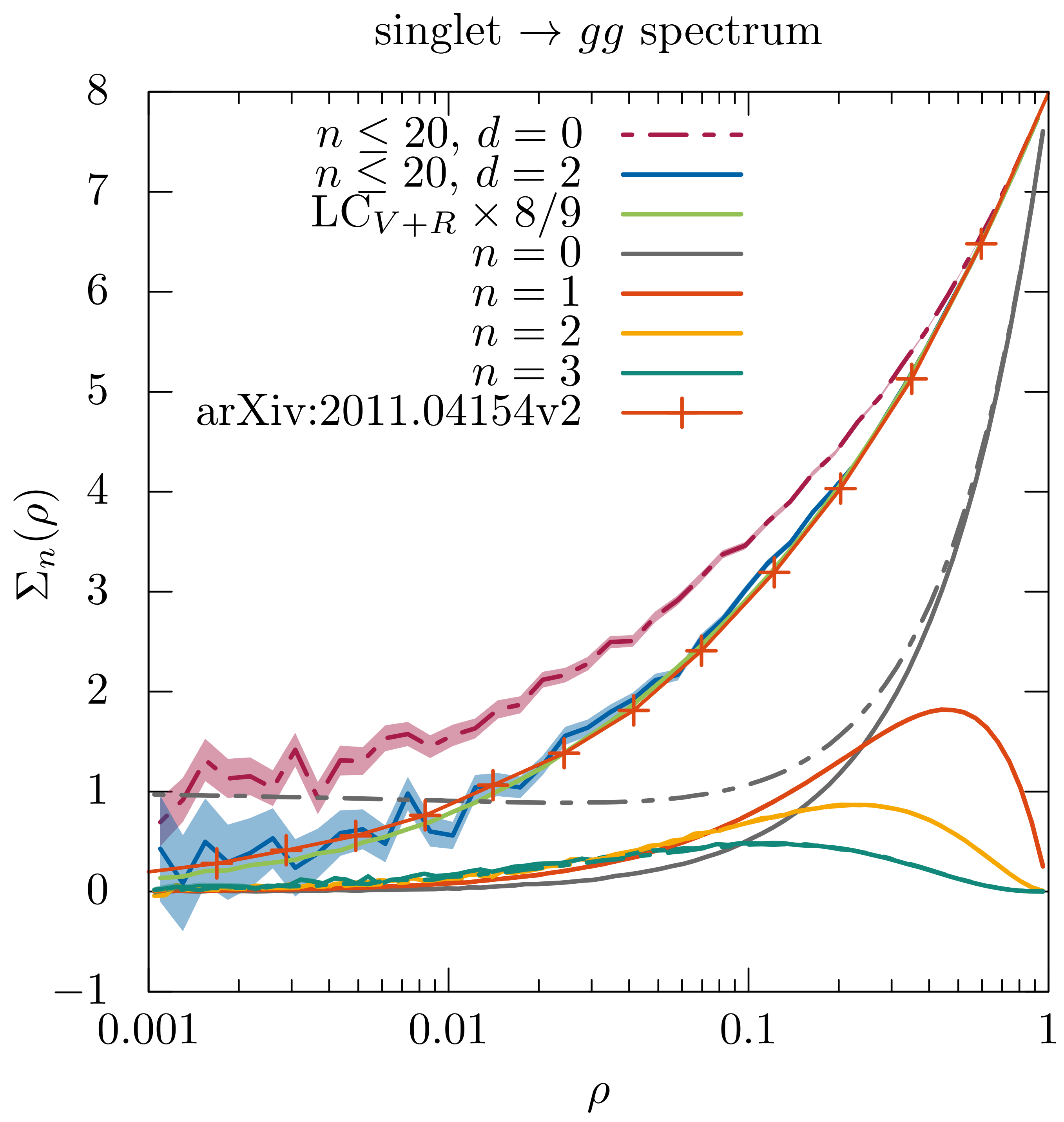
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – '18]
[Forshaw, Holguin, Plätzer – '19]

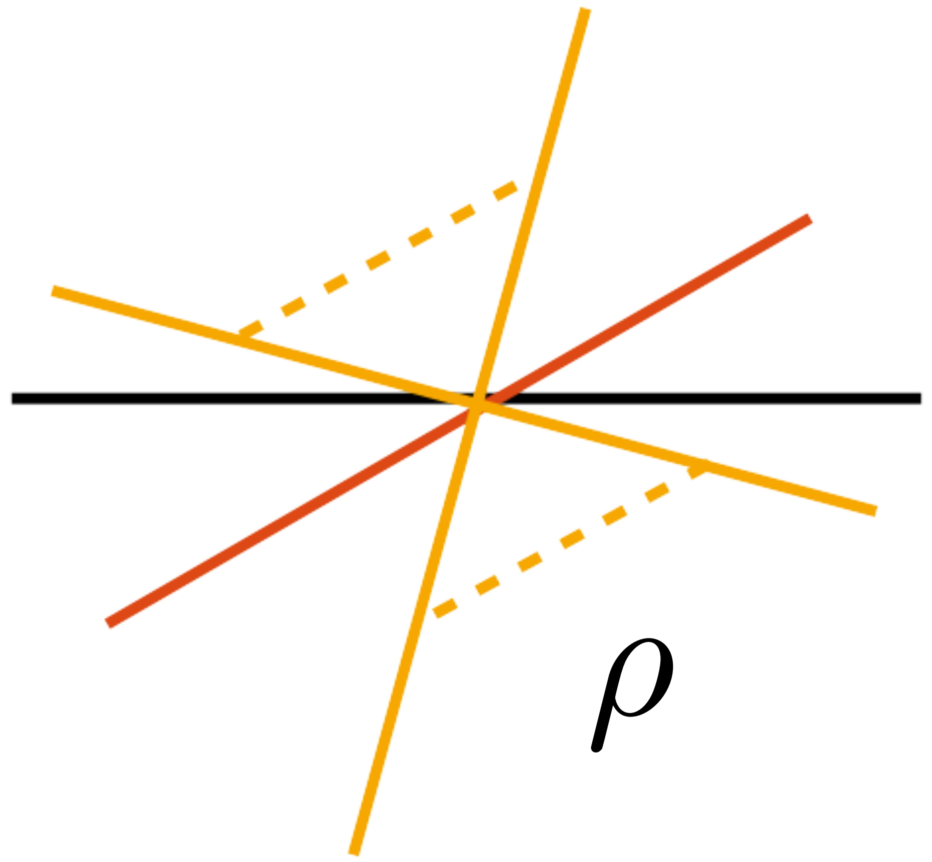
Amplitude evolution: CVolver

CVolver solves evolution equations in colour flow space

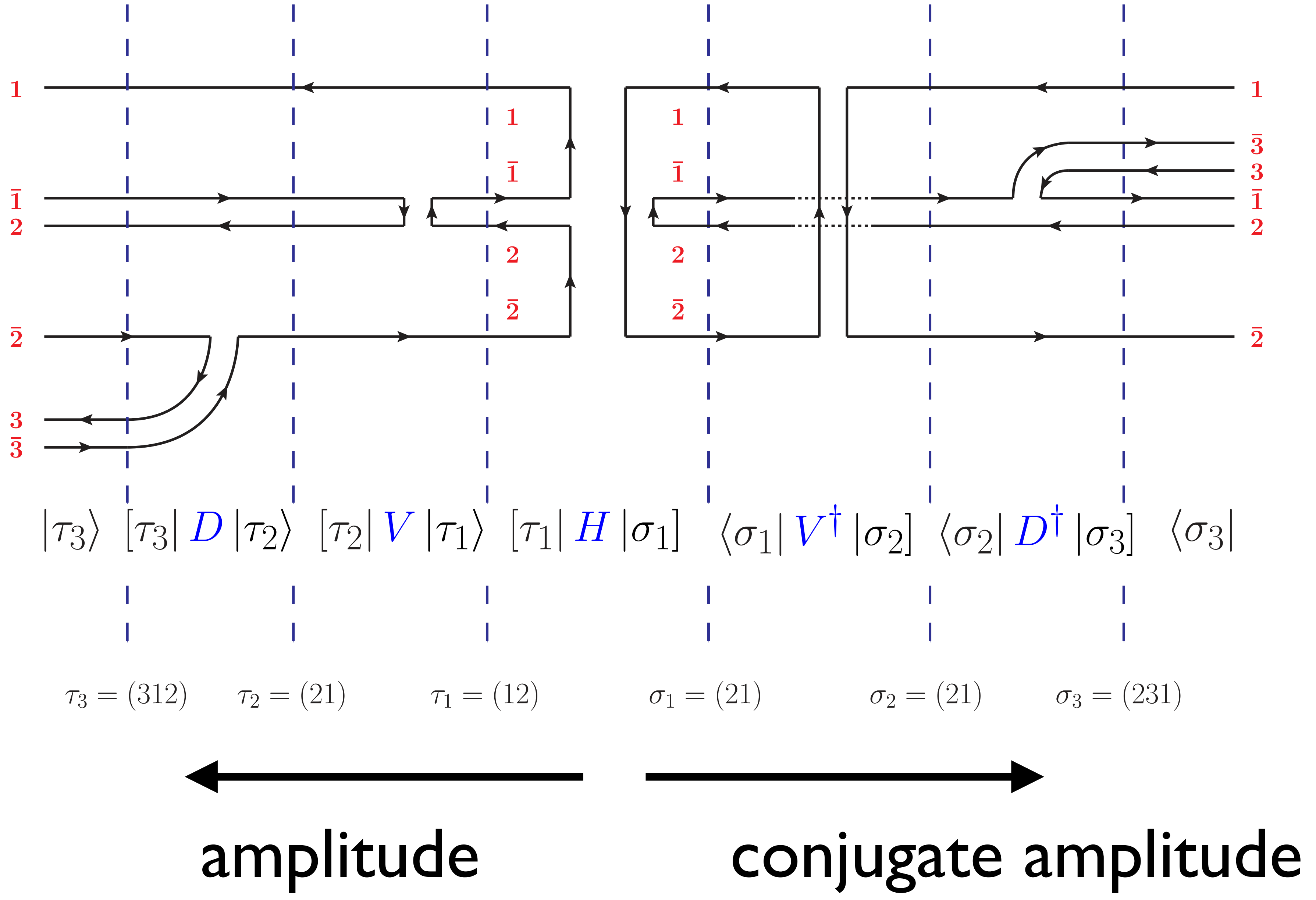
[De Angelis, Forshaw, Plätzer ‘21]
[Plätzer ’13]

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}$$



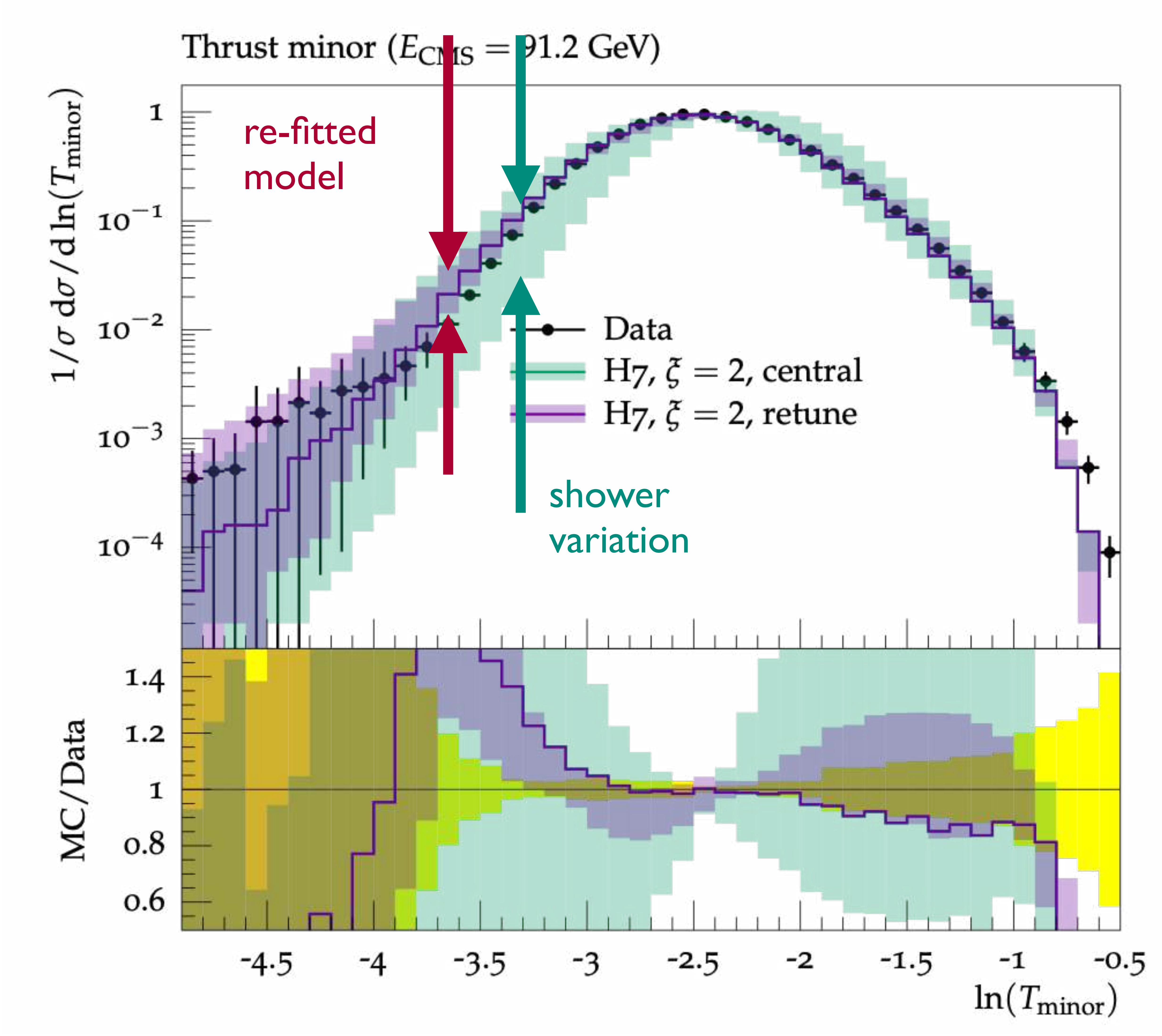


$$\Sigma(\rho) = \sum_n \int \mathrm{d}\sigma(\{p_i\}) \prod_i \theta_{\mathrm{in}}(\rho - E_i)$$



The interface to hadronization

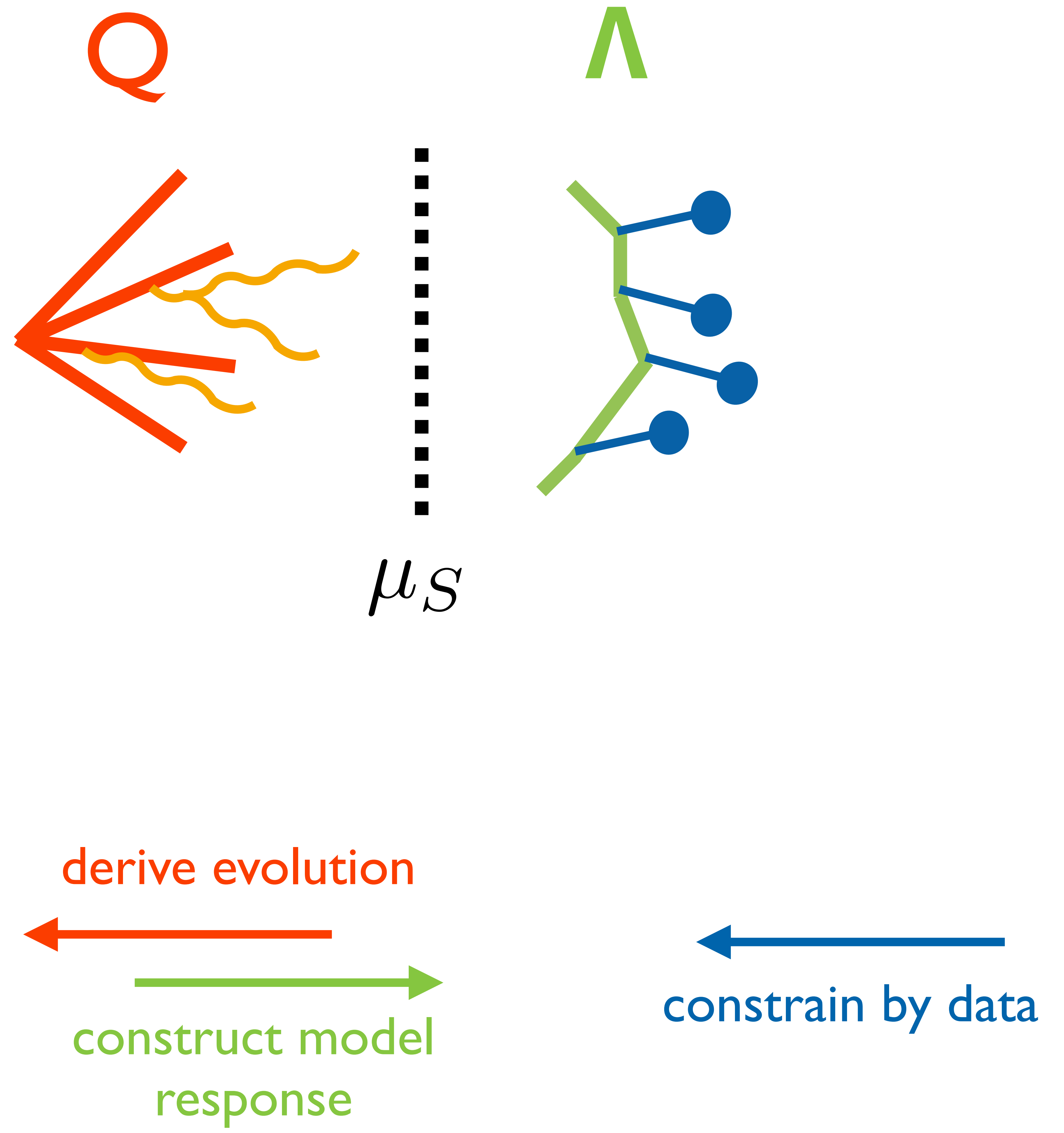
[Bellm, Lönnblad, Prestel, Plätzer, Samitz, Siodmok, Hoang — for Les Houches 2017]



Constructing evolution algorithms

IR cutoff of shower is UV cutoff of hadronization. Cross section is invariant under varying unphysical scales.

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



[Plätzer – '22]

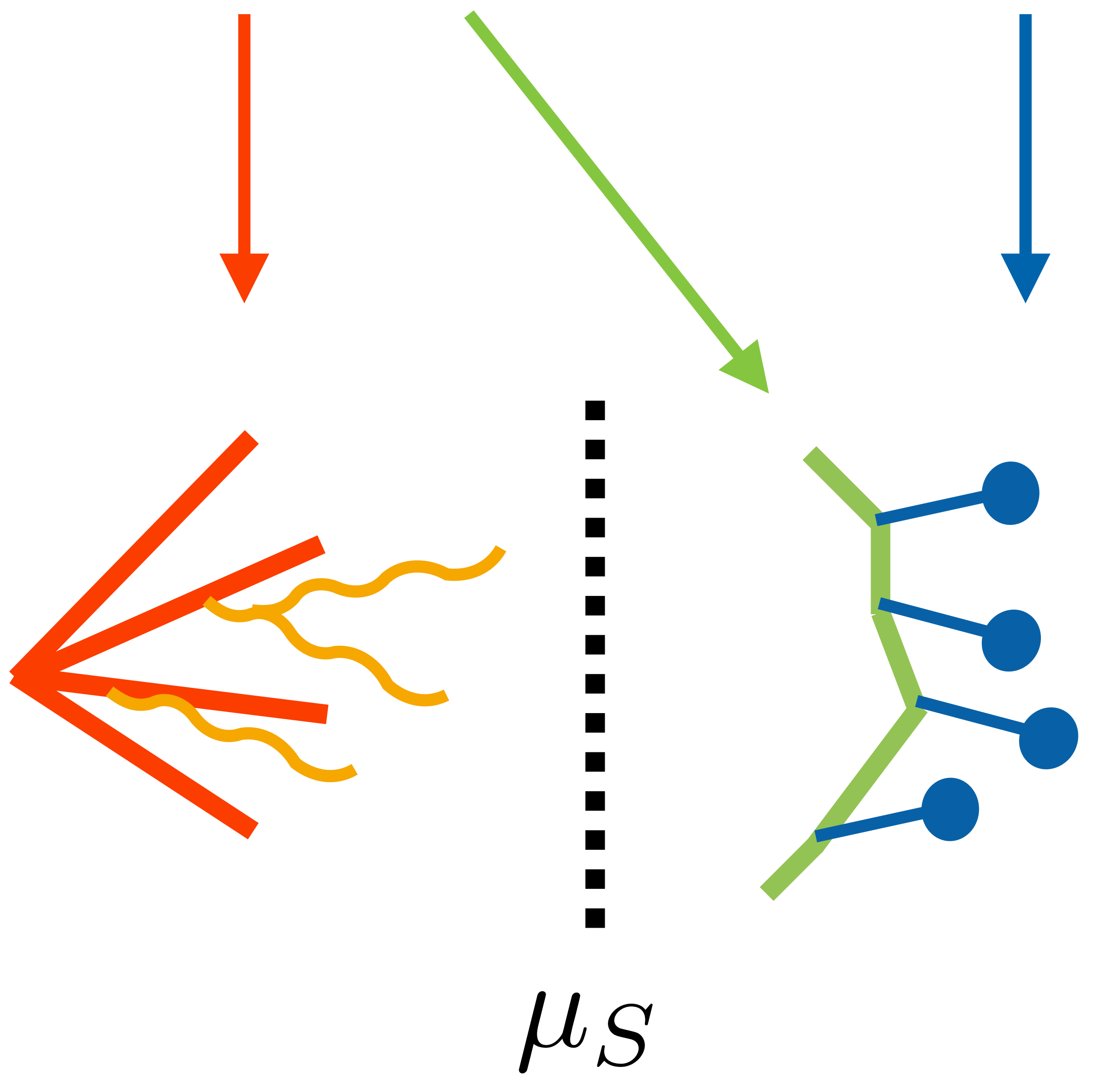
Factorisation and evolution

How do we consistently hadronize in light of (improved) shower algorithms?

How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



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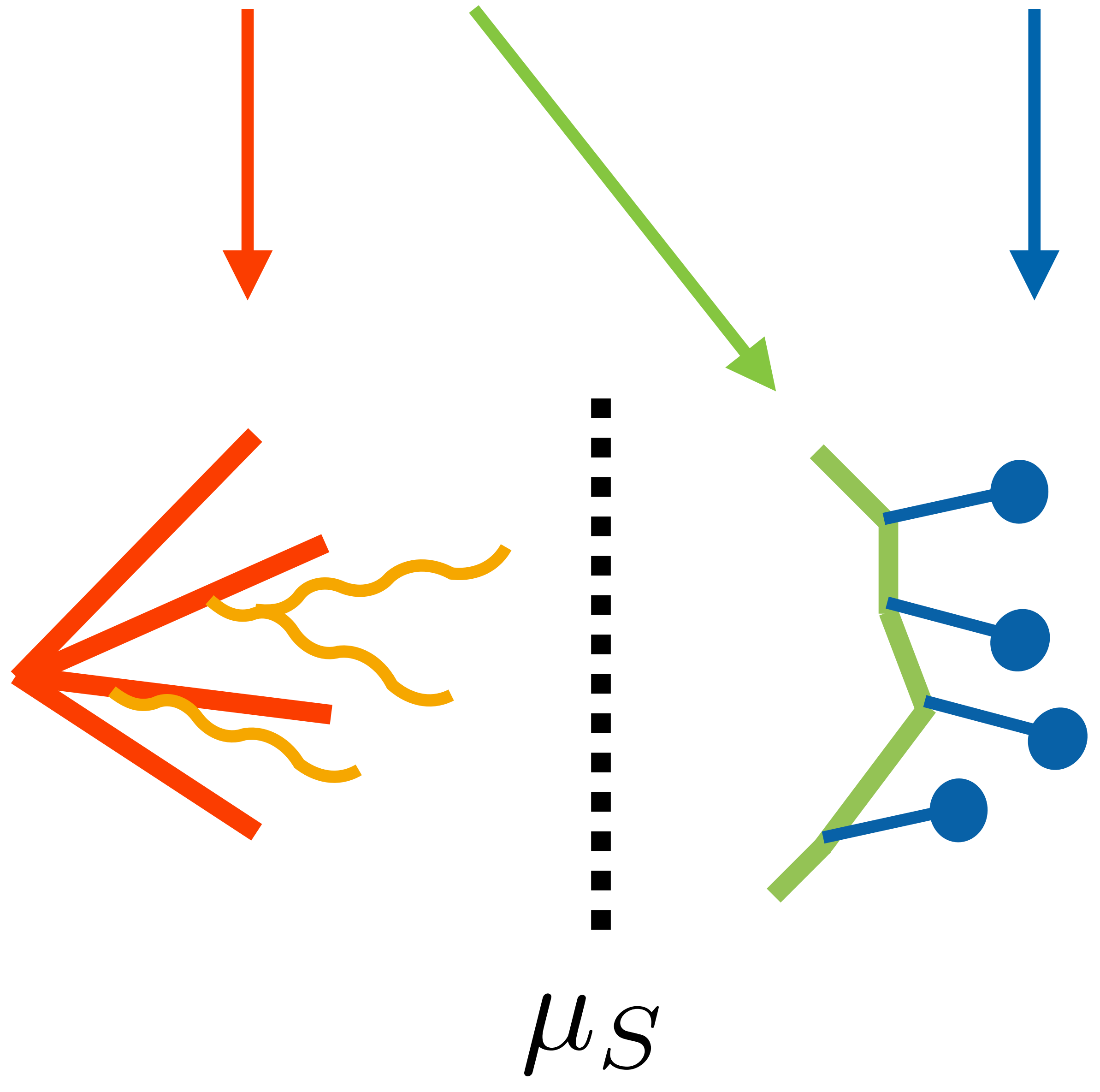
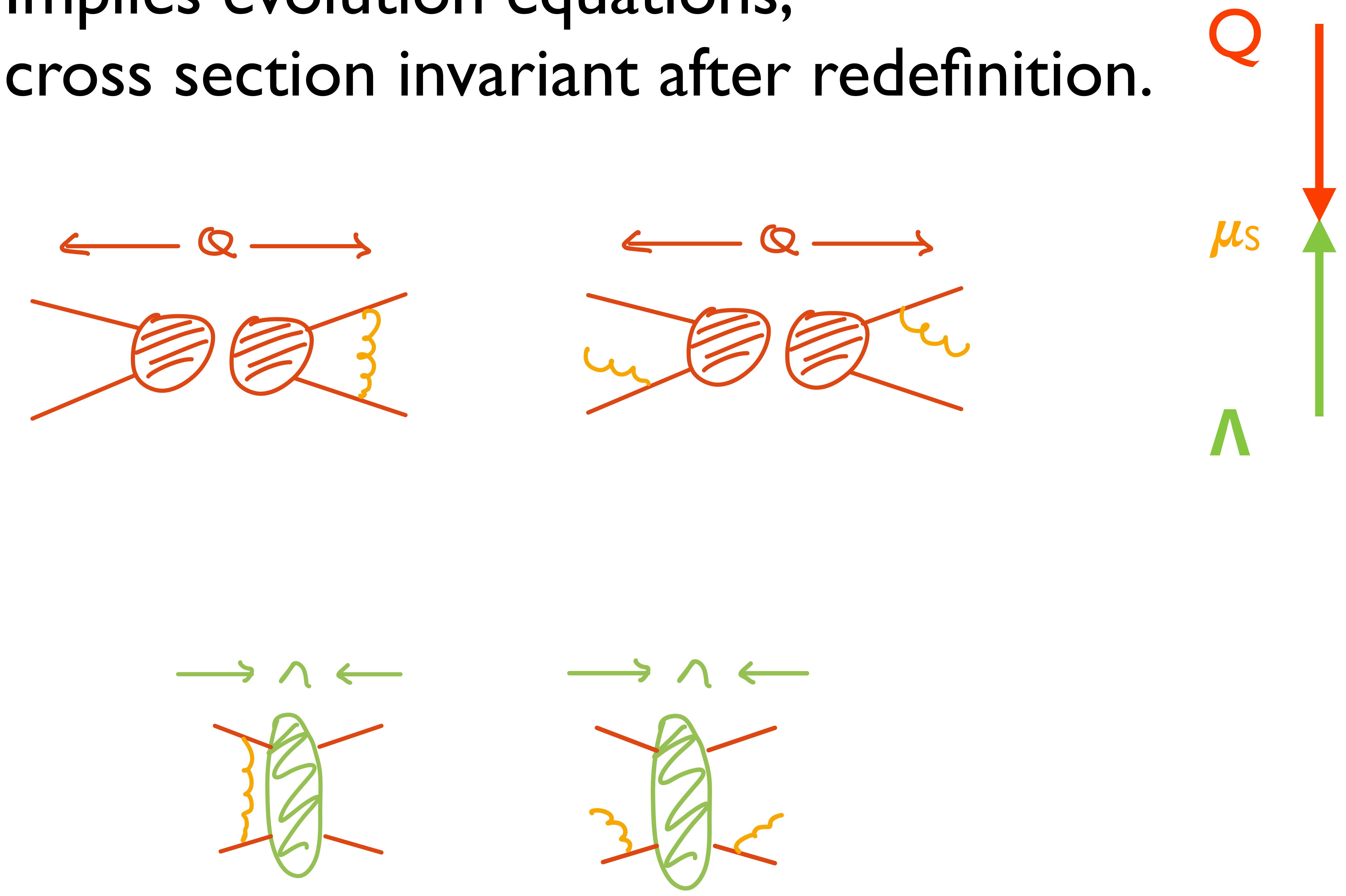


Implies evolution equations,
cross section invariant after redefinition.



Redefinitions of "bare" operators

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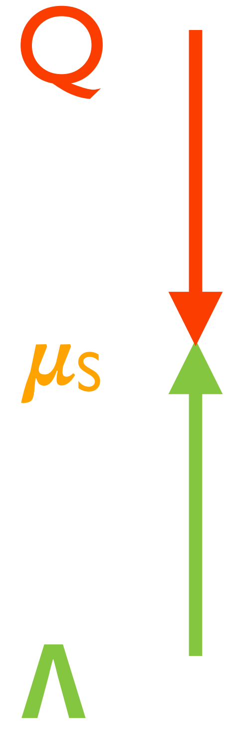
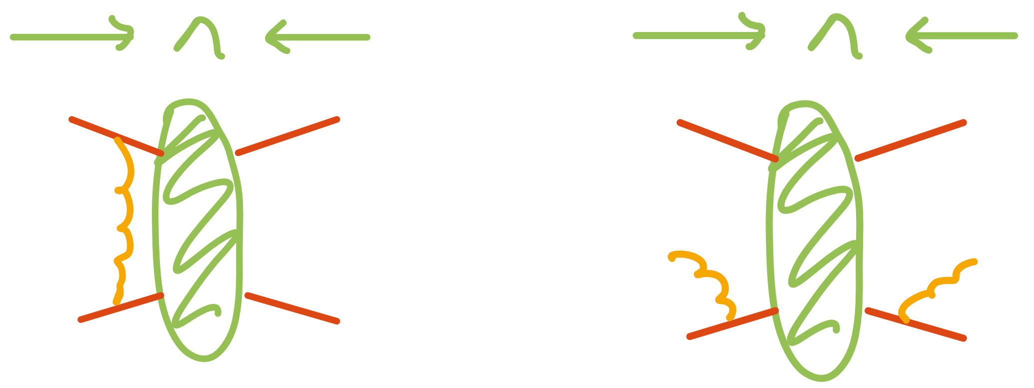


Remove UV divergencies

$$\alpha_0 \left(4\pi\mu^2\right)^{\epsilon} = \alpha_S(\mu_R)\mu_R^{2\epsilon} Z_g$$

Subtract IR divergencies in unresolved regions

$$\mathbf{U}_n = \mathbf{X}_n^{\dagger}\mathbf{S}_n\mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger}\mathbf{S}_{n+s}\mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$



Re-arrange to resum IR enhancements

$$\mathbf{M}_n Z_g^n = \mathbf{Z}_n\mathbf{A}_n\mathbf{Z}_n^{\dagger} + \sum_{s=1}^{n} \alpha_S^s \mathbf{E}_n^{(s)}\mathbf{A}_{n-s}\mathbf{E}_n^{(s)\dagger}$$

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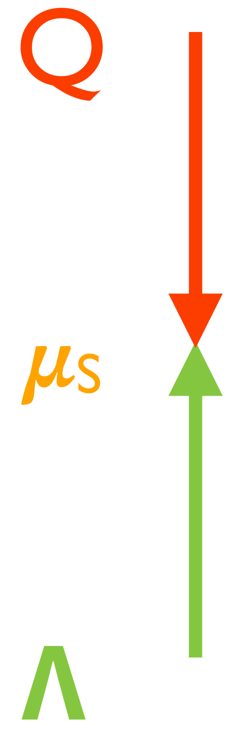
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Redefinitions of hard and soft factor **inverse** to each other:



$$\mathbf{Z}_n = \mathbf{X}_n^{-1}$$

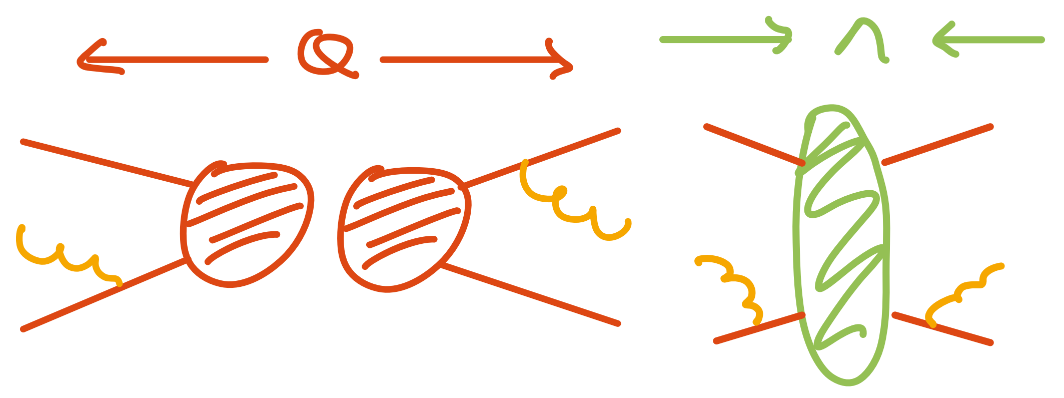
$$\mathbf{X}_n \mathbf{E}_n^{(s)} \circ \mathbf{E}_n^{(s)\dagger} \mathbf{X}_n^\dagger - \mathbf{F}_n^{(s)} \mathbf{Z}_{n-s} \circ \mathbf{Z}_{n-s}^\dagger \mathbf{F}_n^{(s)\dagger} - \sum_{t=1}^{s-1} \mathbf{F}_n^{(t)} \mathbf{E}_{n-t}^{(s-t)} \circ \mathbf{E}_{n-t}^{(s-t)\dagger} \mathbf{F}_n^{(t)\dagger} = 0$$

dressing of hard process ~ parton shower

soft evolution ~ hadronization model

$$\sum_n \int \alpha_S^n \; \mathrm{Tr}\left[(\mathbf{A}_n + \mathbf{\Delta}_n)\, \mathbf{S}_n\right] \mathrm{d}\phi(Q) \prod_{i=1}^{n} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

α_s corrections to tower of logarithms in A —
truncation error of relation of Z factors

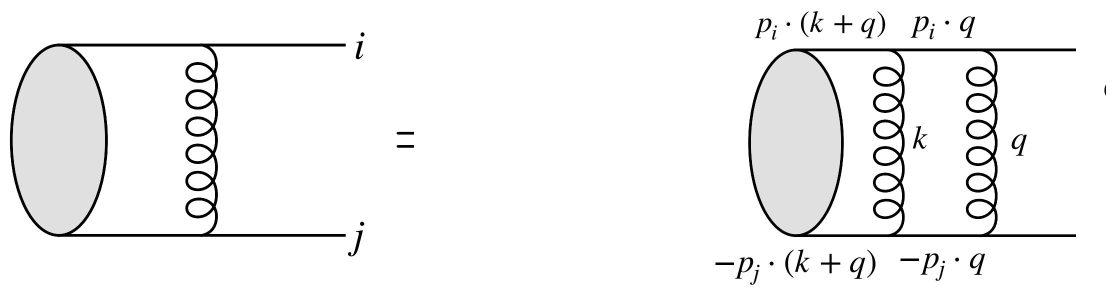


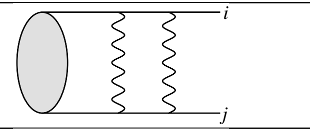
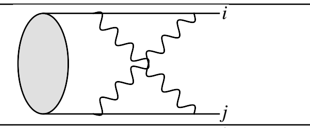
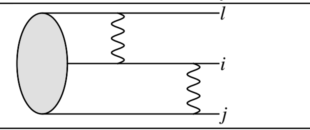
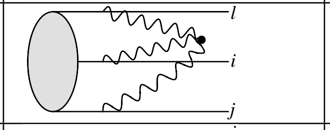
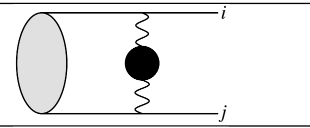
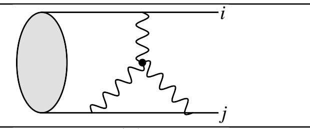
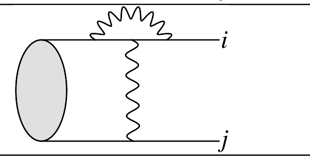
(Soft) factorisation of amplitudes

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

Factorisation of virtual contributions

$$\mathbf{M}_n^{(l)} = \mathbf{V}^{(1)} \mathbf{M}_n^{(l-1)} + \mathbf{M}_n^{(l-1)} \mathbf{V}^{(1)\dagger} + \mathbf{V}^{(1)} \mathbf{M}_n^{(l-2)} \mathbf{V}^{(1)\dagger} + \mathbf{V}^{(2)} \mathbf{M}_n^{(l-2)} + \mathbf{M}_n^{(l-2)} \mathbf{V}^{(2)\dagger} + \ldots$$

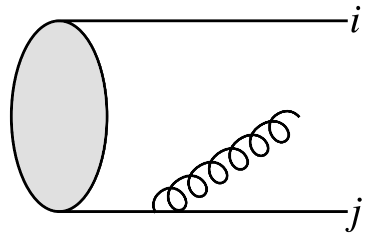


Coefficient	Diagram	Colour-factor
$\Omega_{ij}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\tilde{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^b \mathbf{T}_j^a$
$\Omega_{ijl}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_l)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ijl}^{(2)}$		$i f^{abc} \mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_l^c$
$\Omega_{ij,\mathrm{self-en.}}^{(2)}$		$T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\Omega_{ij,\mathrm{vertex-corr.}}^{(2)}$		$T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^b \mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^a$

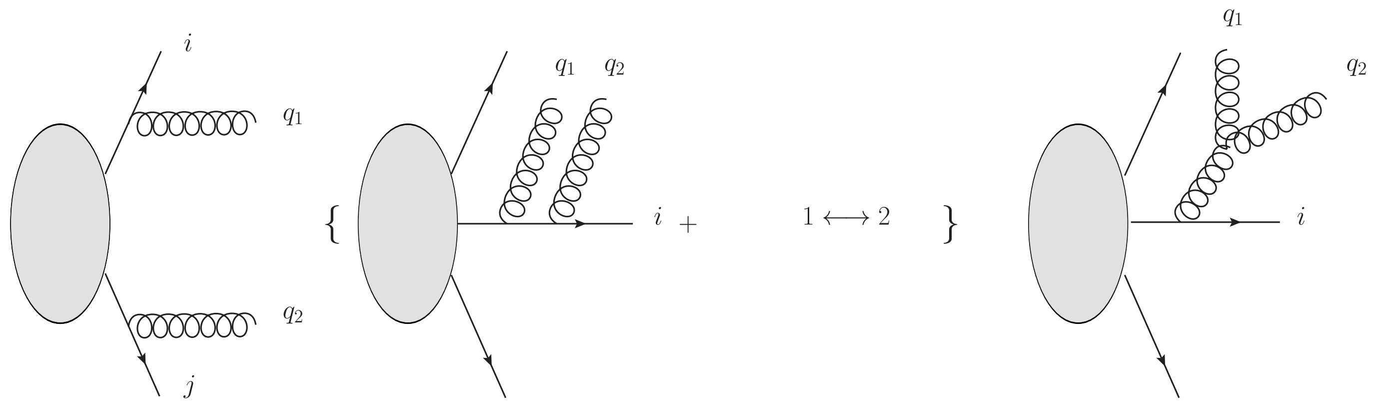
Handle virtual as phase-space type integrals to remove divergencies with subtractions.

[Plätzer, Ruffa — JHEP 06 (2021) 007]

Factorisation of real contributions



$$\mathbf{M}_n^{(l)} = \mathbf{D}_n^{(1,0)} \mathbf{M}_{n-1}^{(l)} \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,1)} \mathbf{M}_{n-1}^{(l-1)} \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)} \mathbf{M}_{n-1}^{(l-1)} \mathbf{D}_n^{(1,1)\dagger} + \mathbf{D}_n^{(2,0)} \mathbf{M}_{n-2}^{(l)} \mathbf{D}_n^{(2,0)\dagger} + \ldots$$

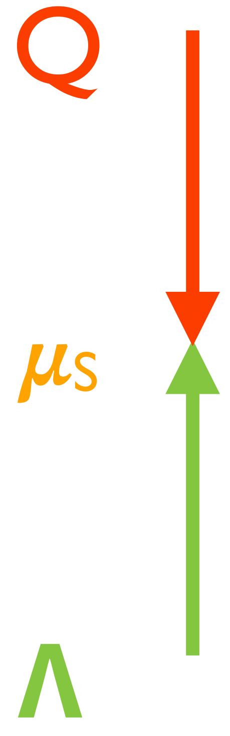


$$\sum_{(a,b),(c,d)} \sum_{i,j,k,l=1}^{n} \omega_{ijkl}^{abcd} \, \mathrm{T}_i^{(a)} \mathrm{T}_j^{(b)} \circ \mathrm{T}_k^{(c),\dagger} \mathrm{T}_l^{(d),\dagger}$$

[Majcen — M.Sc. thesis 2022]
based on Catani & Grazzini

Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$



Subtractions necessitate a resolution:
what is it we call 'unresolved'?
Encompass all singular regions!

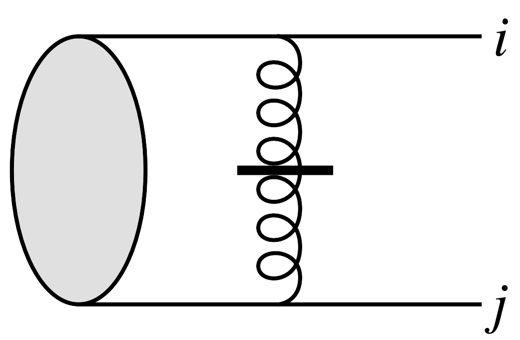
$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

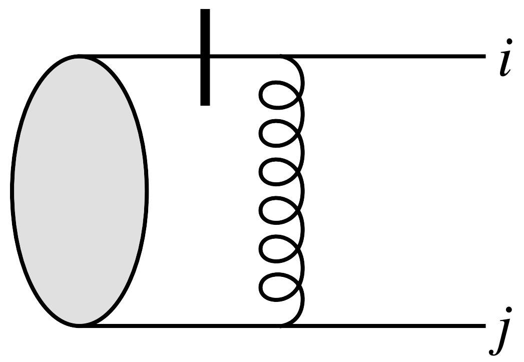
resolution function for (cut) loop momenta

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]$$

$$\hat{\mathbf{V}}_n^{(l)}[\Xi_{n,l}] = \sum_{\alpha} \int \mathcal{I}_{n,\alpha}^{(l)}(p_1, \ldots, p_n; k_1, \ldots, k_l)\; \Xi_{n,l}^{(\alpha)} \prod_{i=1}^{l} \mu_R^{2\epsilon}[\mathrm{d}k_i]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

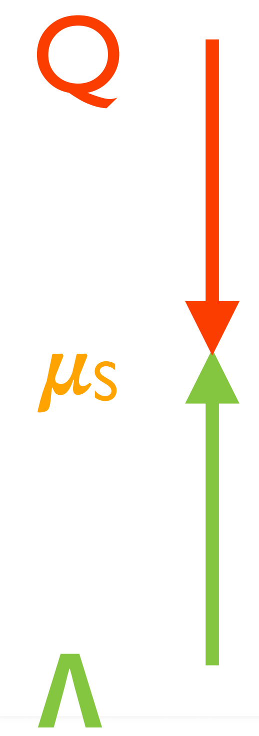




resolution function for real emission

Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$



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resolution function for (cut) loop mome

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission

Continues to higher orders …

$$\mathbf{X}_n^{(2)} = \hat{\mathbf{V}}_n^{(2)}[\Xi_{n,2}] - \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}] \hat{\mathbf{V}}_n^{(1)}$$

$$\begin{aligned} \mathbf{F}_n^{(1,1)} \circ \mathbf{F}_n^{(1,0)\dagger} = {} & \mathbf{D}_n^{(1,1)}\left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} \\ & + \mathbf{D}_n^{(1,1)}\left[1 - \Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} + \mathbf{D}_n^{(1,1)}\left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger}(1 - \Theta_{n,1}) \\ & - \hat{\mathbf{V}}_n^{(1)}\left[\Xi_{n-1,1}\right] \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)} \hat{\mathbf{V}}_{n-1}^{(1)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} \end{aligned}$$

$$\mathbf{F}_n^{(2,0)} \circ \mathbf{F}_n^{(2,0)\dagger} = \mathbf{D}_n^{(2,0)} \circ \mathbf{D}_n^{(2,0)\dagger} \Theta_{n,2} - \mathbf{D}_n^{(1,0)} \mathbf{D}_{n-1}^{(1,0)} \circ \mathbf{D}_{n-1}^{(1,0)\dagger} \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

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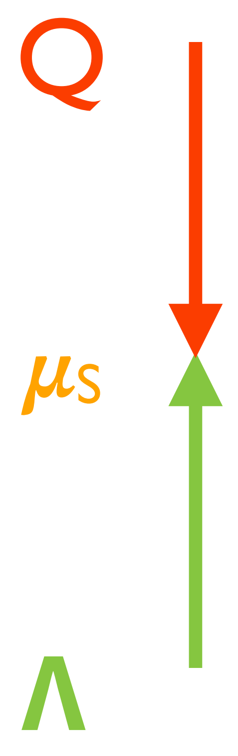

resolution function for (cut) loop momenta

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resolution function for real emission

Resolution functions introduce cutoff dependence, e.g. energy ordering:

$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1} \theta(E_n - \mu_S)$$

"soft or collinear"

$$\Theta_{n,2} = 1 - \hat{\Theta}_{n,2} \theta(E_{n-1} - \mu_S) \theta(E_n - \mu_S)$$

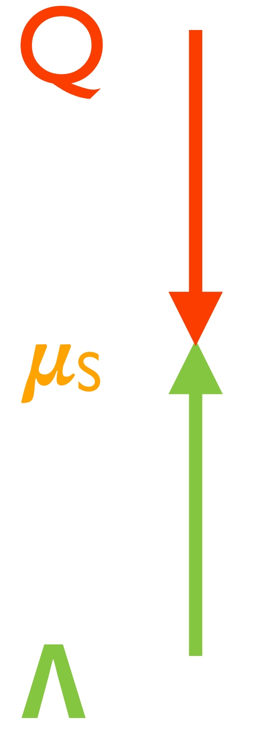
Infrared subtractions

How do we consistently hadronize in light of (improved) shower algorithms?

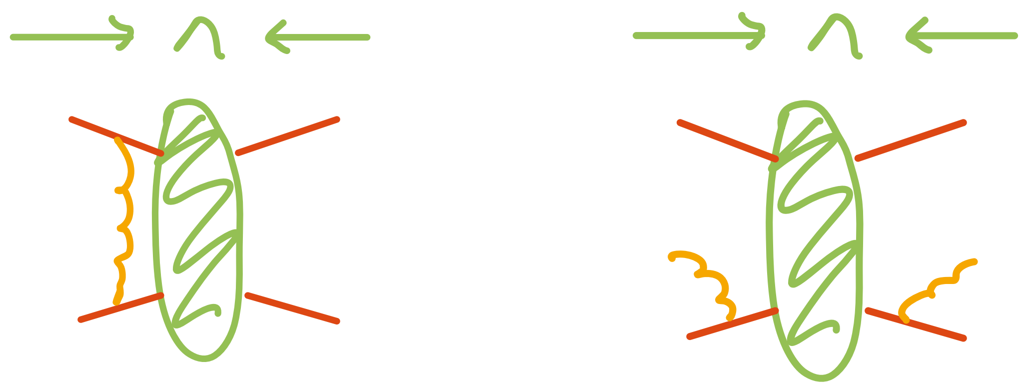
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.



$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^{\dagger}_{S,n} \mathbf{S}_n - \mathbf{S}_n \tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s \geq 1} \alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s} \mathbf{S}_{n+s} \tilde{\mathbf{R}}^{(s)}_{S,n+s} \prod_{i=n+1}^{n+s} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$



$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S} \mathbf{A}_n + \mathbf{A}_n \mathbf{\Gamma}^{\dagger}_{n,S} - \sum_{s \geq 1} \alpha_S^s \mathbf{R}^{(s)}_{S,n} \mathbf{A}_{n-s} \mathbf{R}^{(s)\dagger}_{S,n}$$

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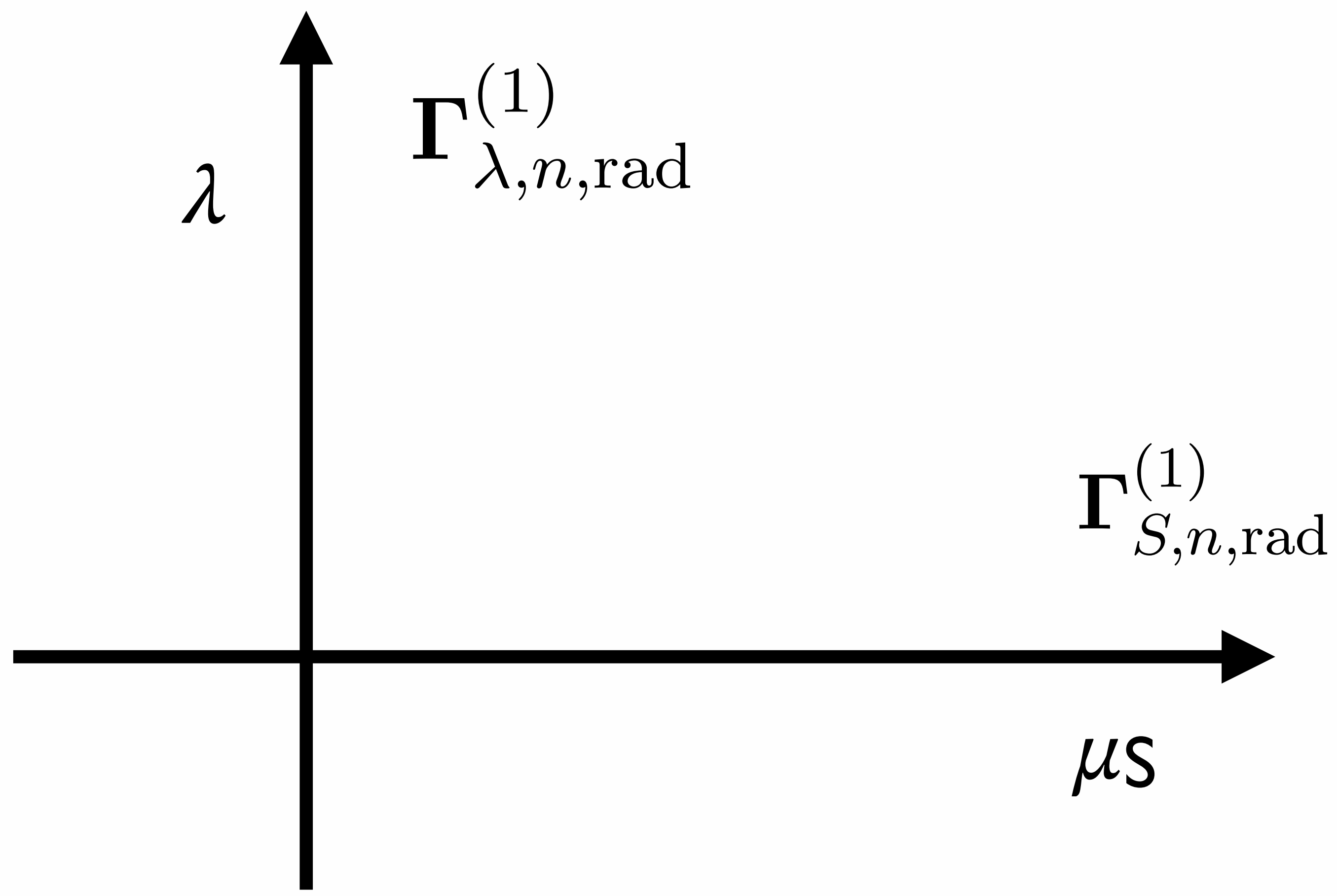
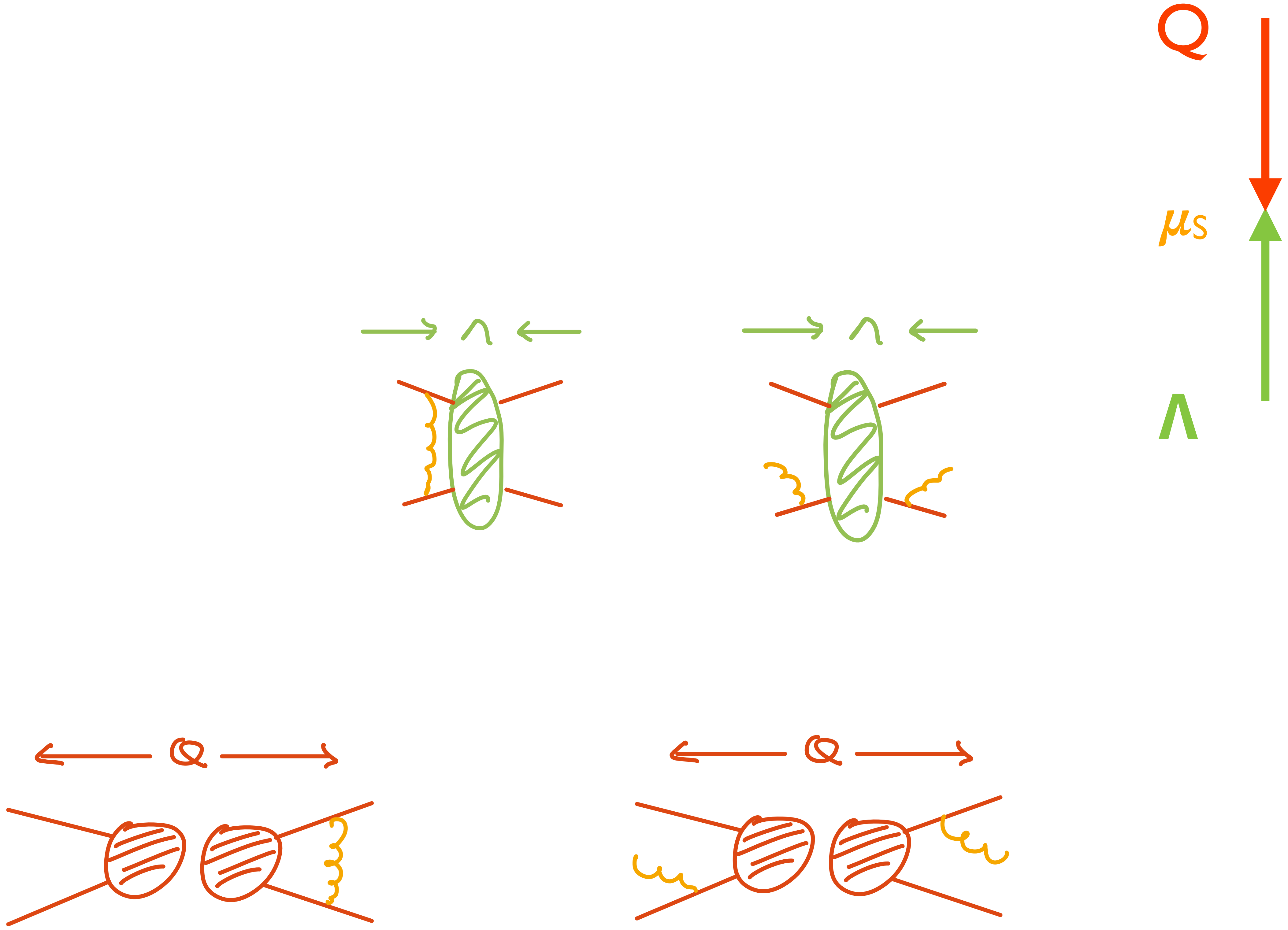
$\partial_S \mathbf{S}_{,}$ ... (p_i)

$\partial_S \mathbf{A}$



$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1}\theta(E_n - \mu_S)$$



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Q

μ_S

Λ

Enters the re-definition of observables, e.g. demanding a jet cross section

use unitarity and pick equal resolutions $\int \mathbf{D}_{n+1}^{(1,0)\dagger} \mathbf{D}_{n+1}^{(1,0)} \Theta_{n,1} \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) = -\frac{1}{2} \hat{\mathbf{V}}_n^{(1)} \left[\Theta_{n,1} \right]$

$$\mathbf{U}_n = \mathbf{1}_n u(p_1, ..., p_n)$$
$$- \alpha_s \int \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) \hat{\mathbf{D}}_{n+1}^{(1,0)\dagger} \hat{\mathbf{D}}_{n+1}^{(1,0)} \Theta_{n,1} \left[u(p_1, ..., p_n, p_{n+1}) - u(p_1, ..., p_n) \right] + \mathcal{O}(\alpha_s^2)$$

Proof this to vanish or to generate a power correction.

Constructing evolution algorithms

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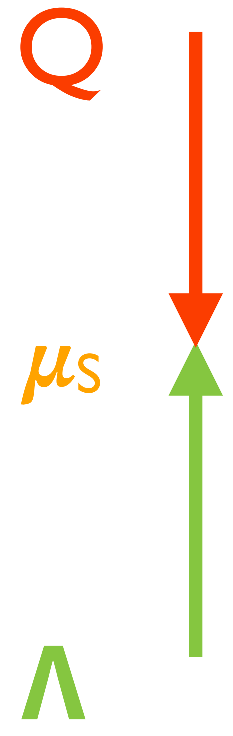
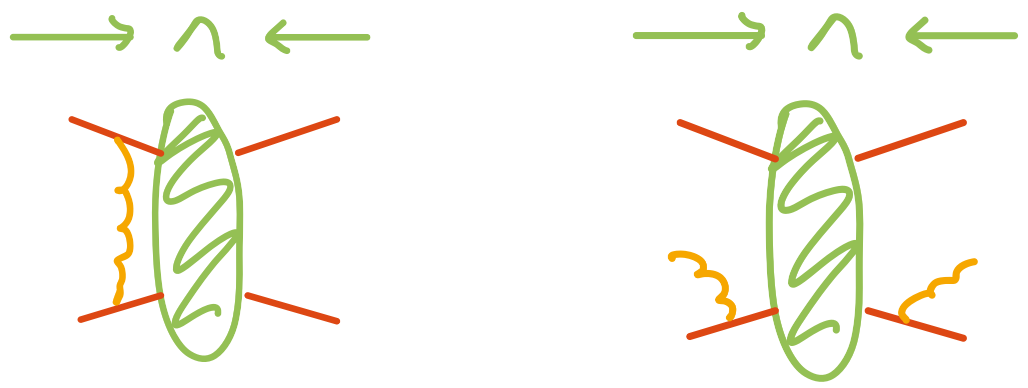


Subtract iterated contribution in ordered phase space.

$$\mathbf{R}_n^{(2,0)} \circ \mathbf{R}_n^{(2,0)\dagger} = \left(\hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger} \hat{\Theta}_{n,2} - \hat{\mathbf{D}}_n^{(0,1)} \hat{\mathbf{D}}_{n-1}^{(0,1)} \circ \hat{\mathbf{D}}_{n-1}^{(0,1)\dagger} \hat{\mathbf{D}}_n^{(0,1)\dagger} \hat{\Theta}_{n-1,1} \hat{\Theta}_{n,1}\right) \times \theta(E_{n-1} - \mu_S)\delta(E_n - \mu_s) + \hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger} \hat{\Theta}_{n,2} \theta(E_n - \mu_S)\delta(E_{n-1} - \mu_S)$$



Use full double gluon matrix element outside.

Similar consequences for virtual corrections.

Constructing evolution algorithms

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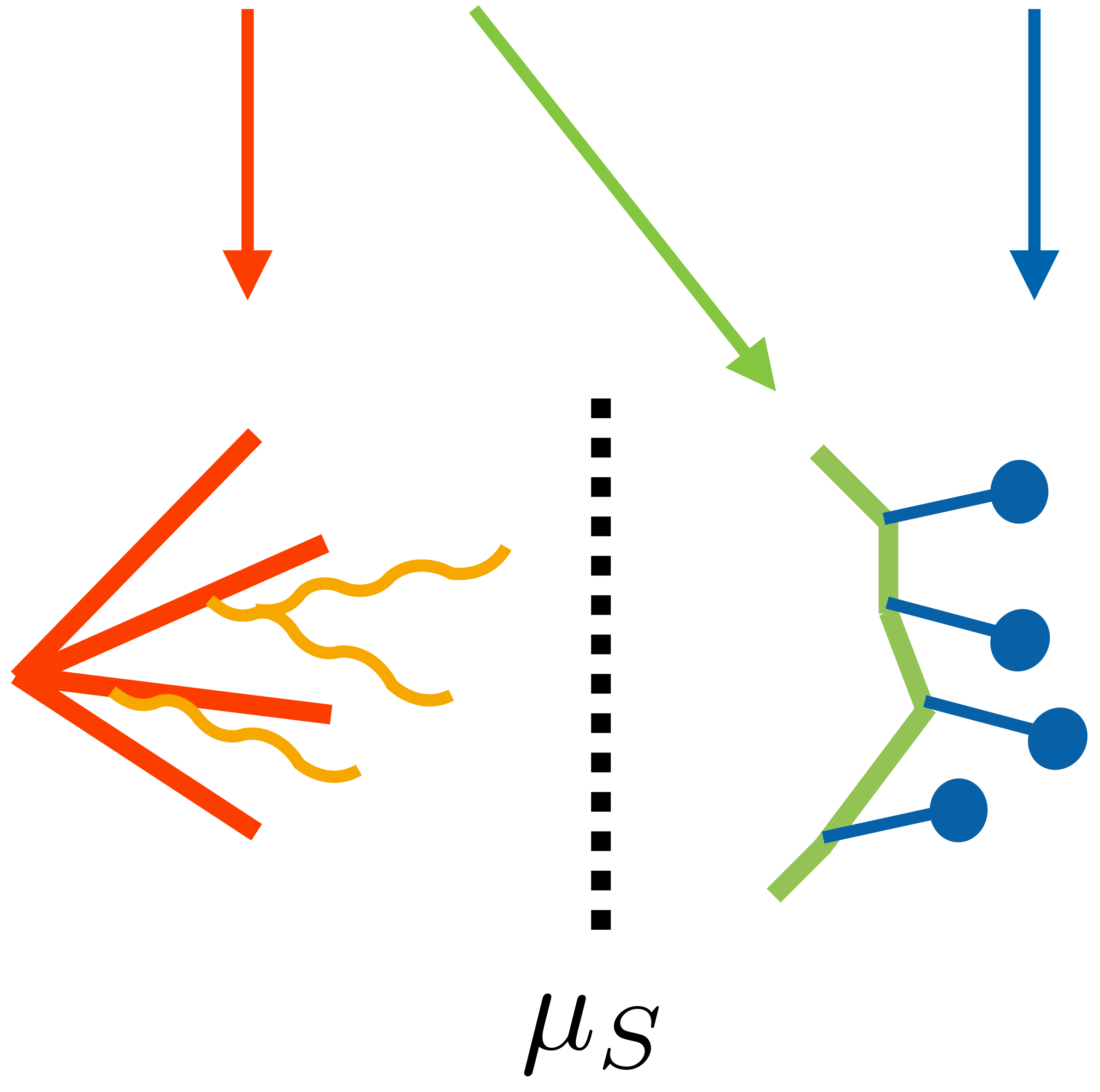
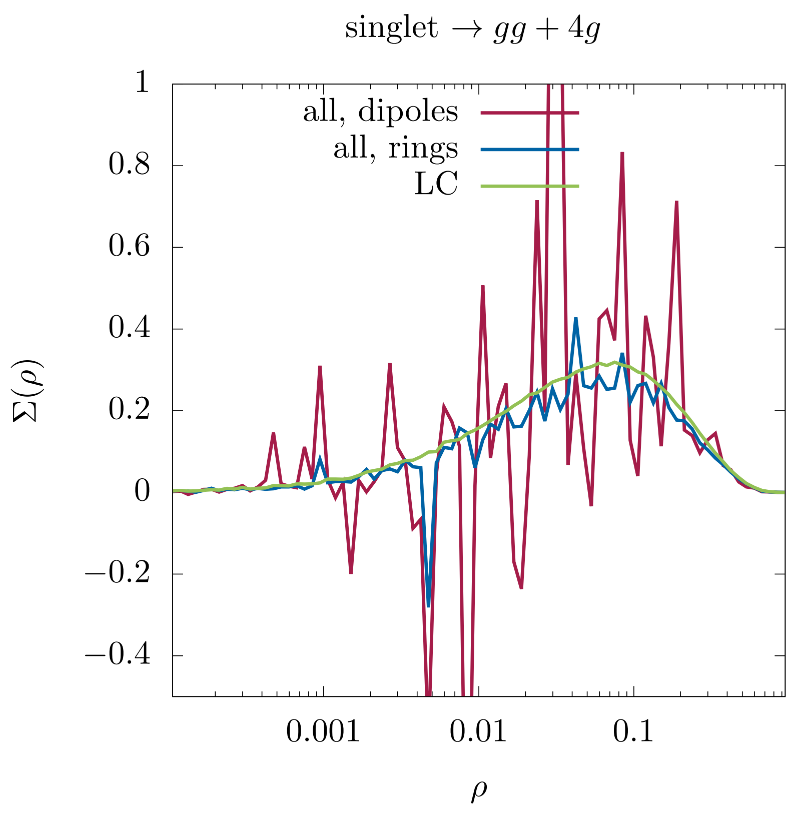
$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



Understand basis functions beyond large-N.



$$\omega_{ij}$$

$$\omega_{ij} + \omega_{ik} - \omega_{jk}$$

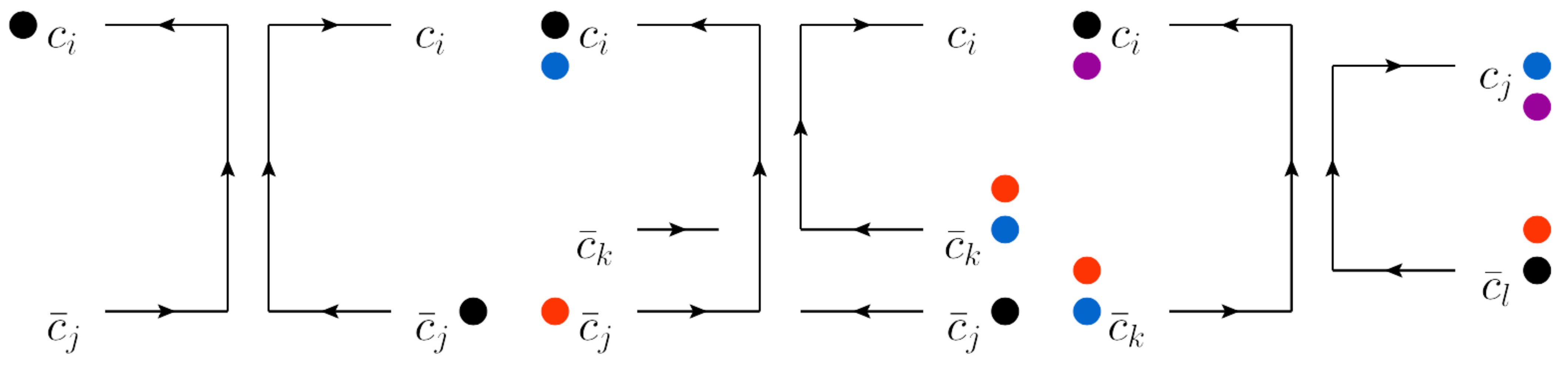
$$\omega_{il} + \omega_{kj} - \omega_{kl} - \omega_{ij}$$

[Holguin, Forshaw, Plätzer — '21]
[Plätzer, Majcen — in preparation]

Constructing evolution algorithms

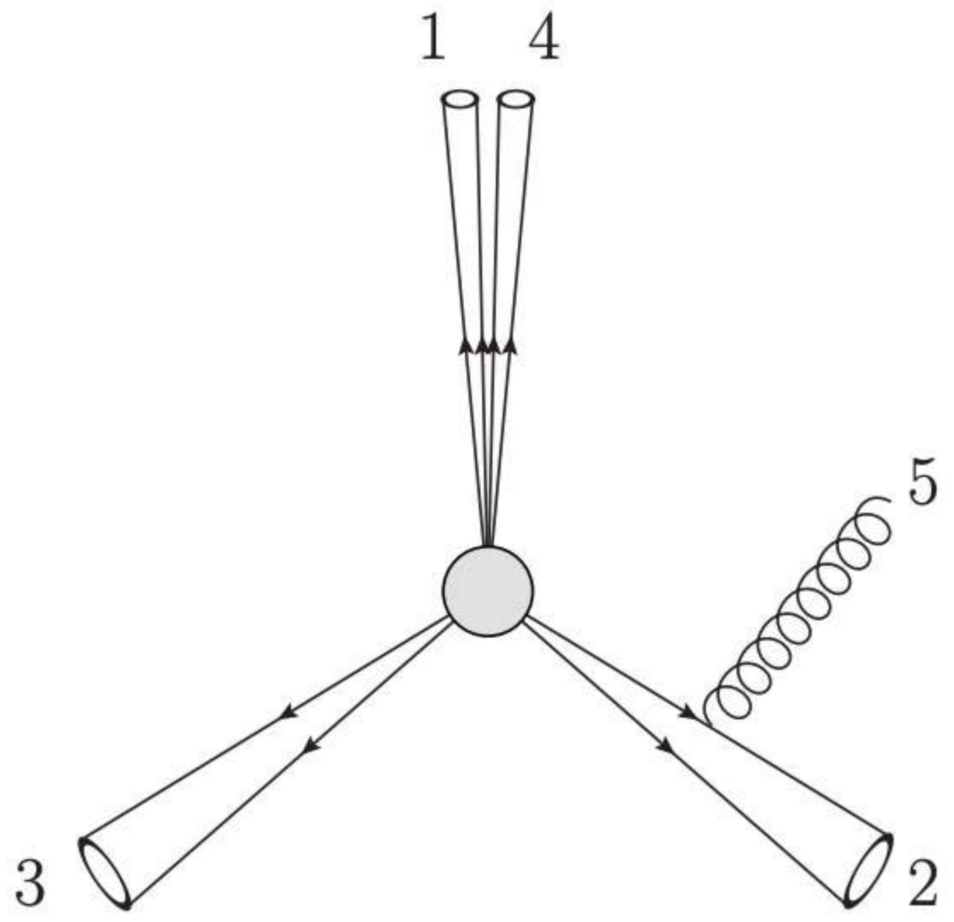
How do we consistently hadronize in light of (improved) shower algorithms?

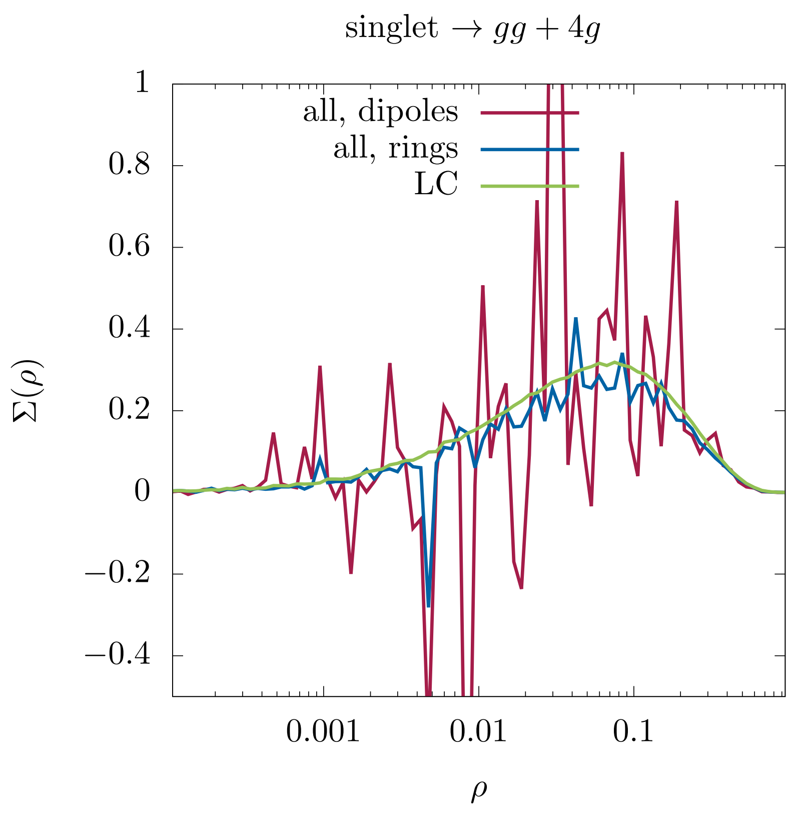
How to do this at subleading N and higher order shower evolution?



Understand basis functions beyond large-N.

Can extend coherence to three jets where colours are still trivial.





$$\omega_{ij}$$

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[Holguin, Forshaw, Plätzer — '20]

[Holguin, Forshaw, Plätzer — '21]
[Plätzer, Majcen — in preparation]

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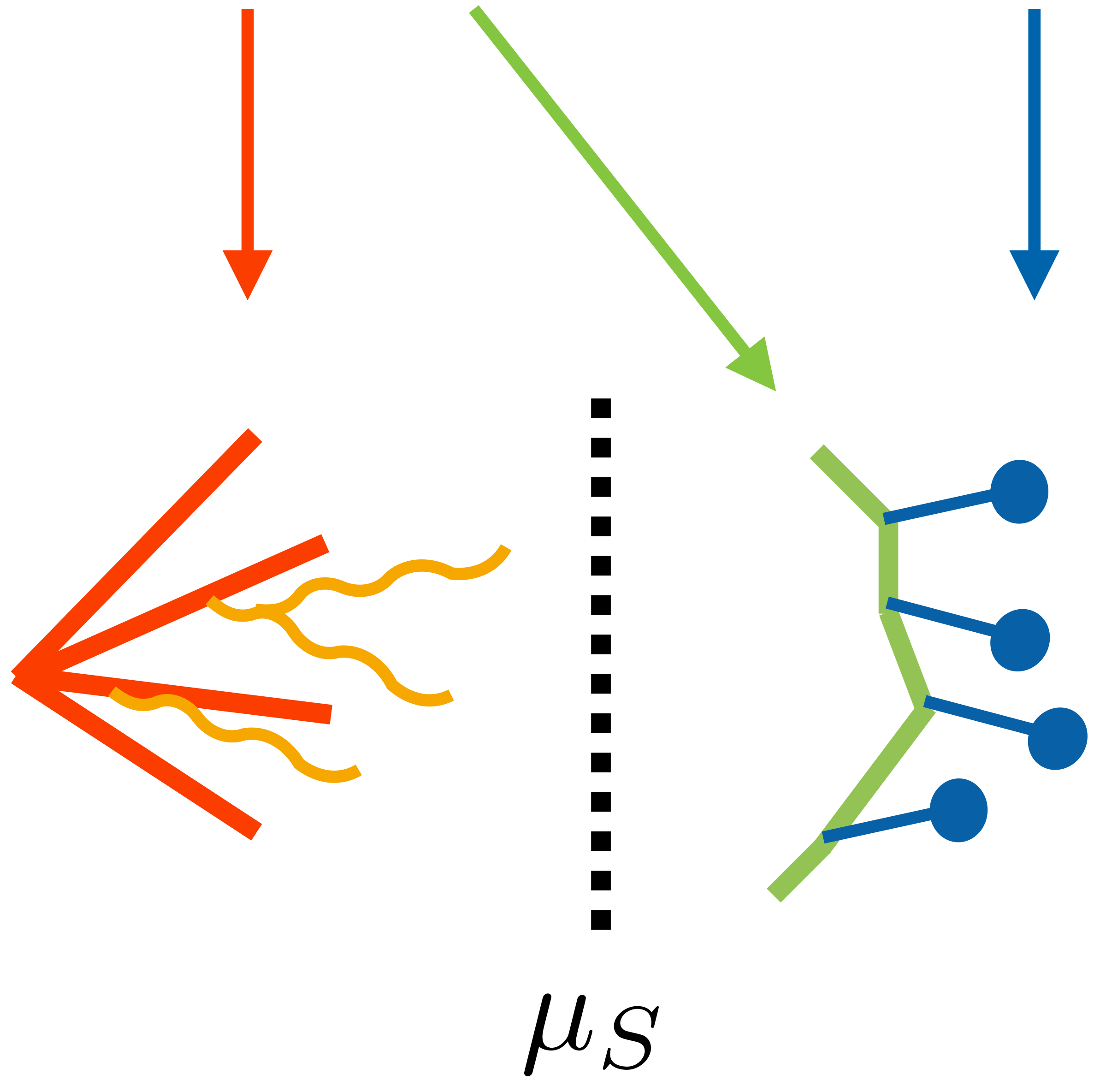
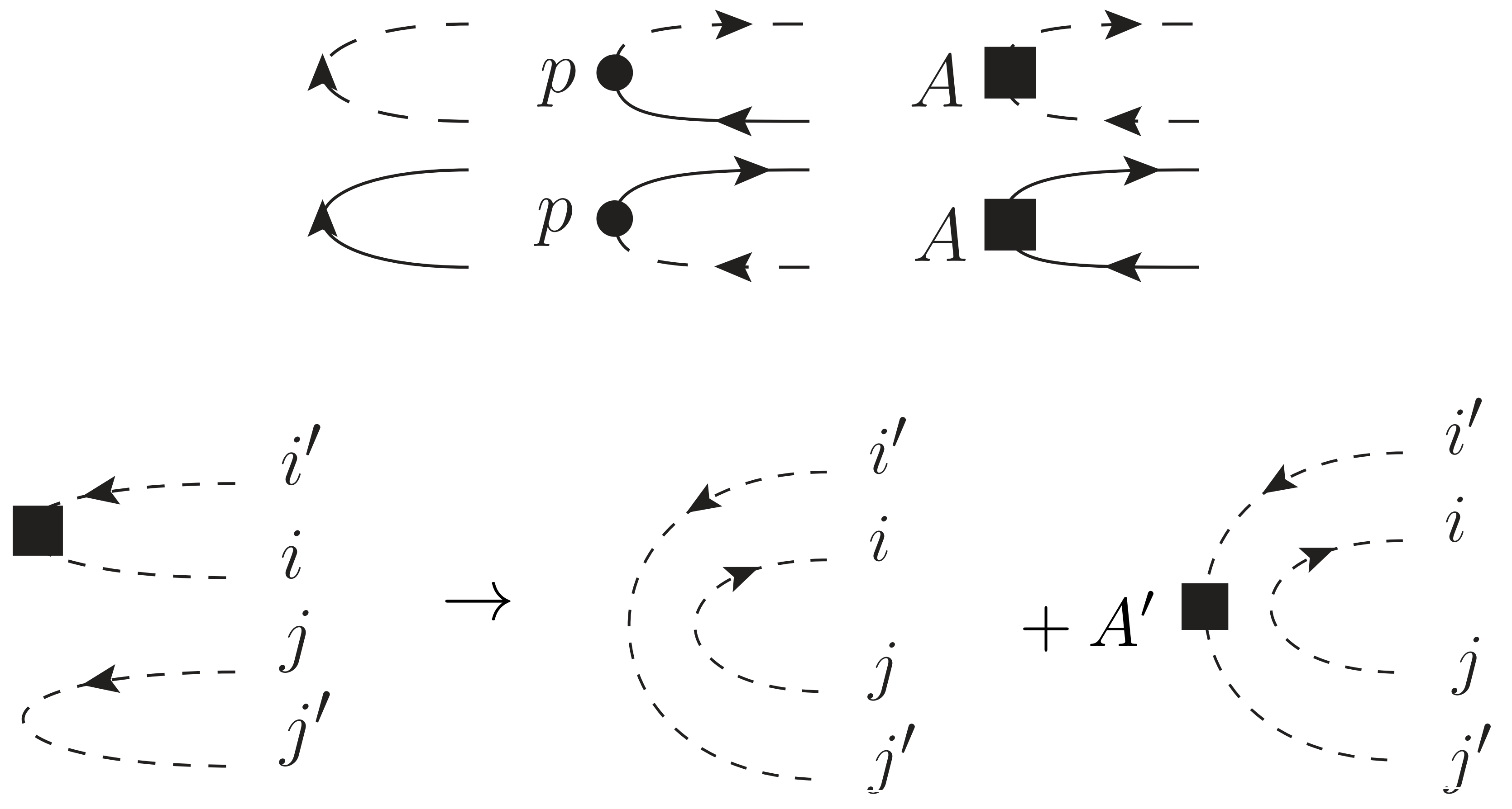


Construct electroweak evolution.
Measurement projection is ubiquitous.



Basis and mixing of chirality structures.

[Plätzer, Sjödahl — '21]

Constructing evolution algorithms

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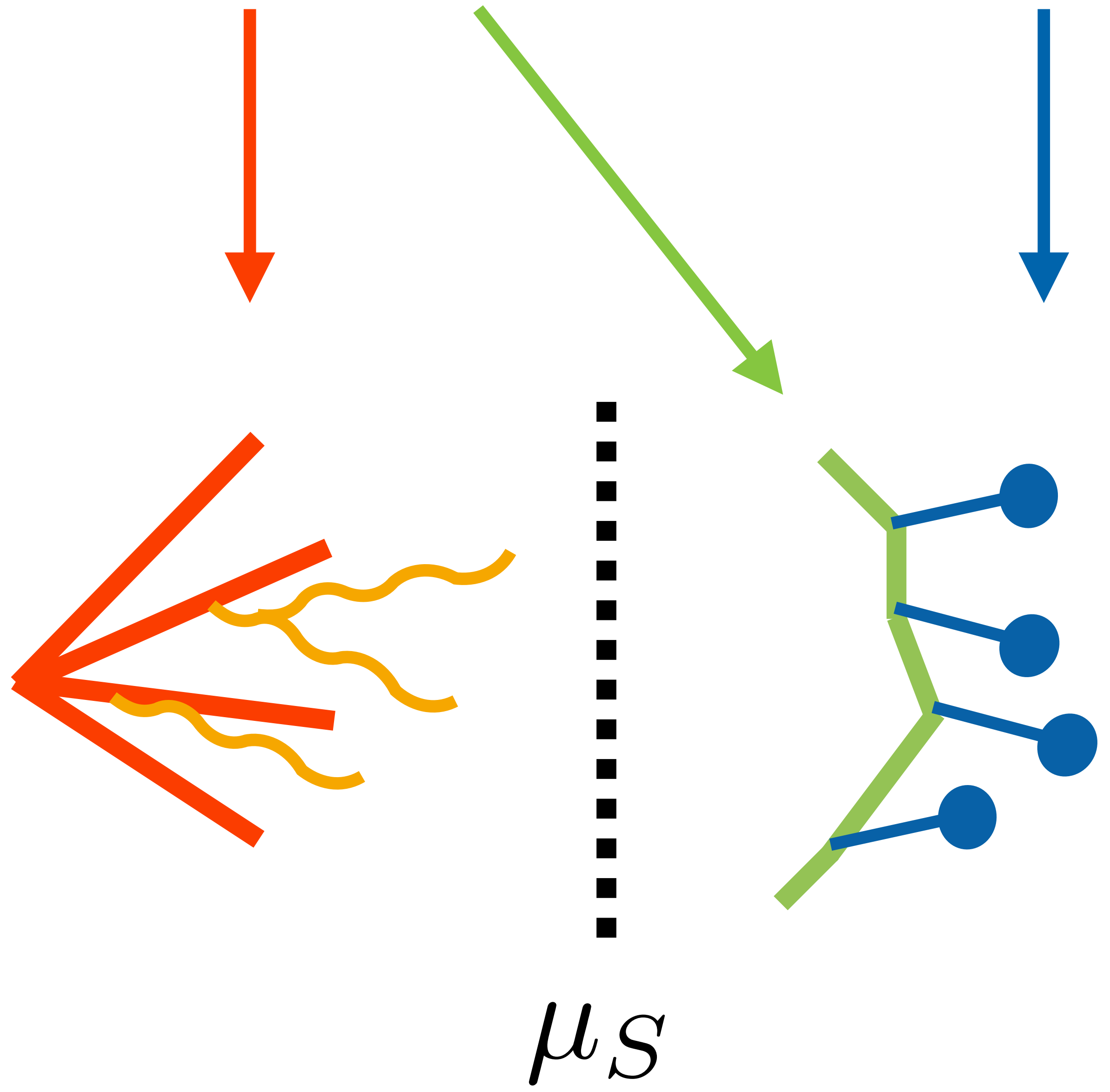

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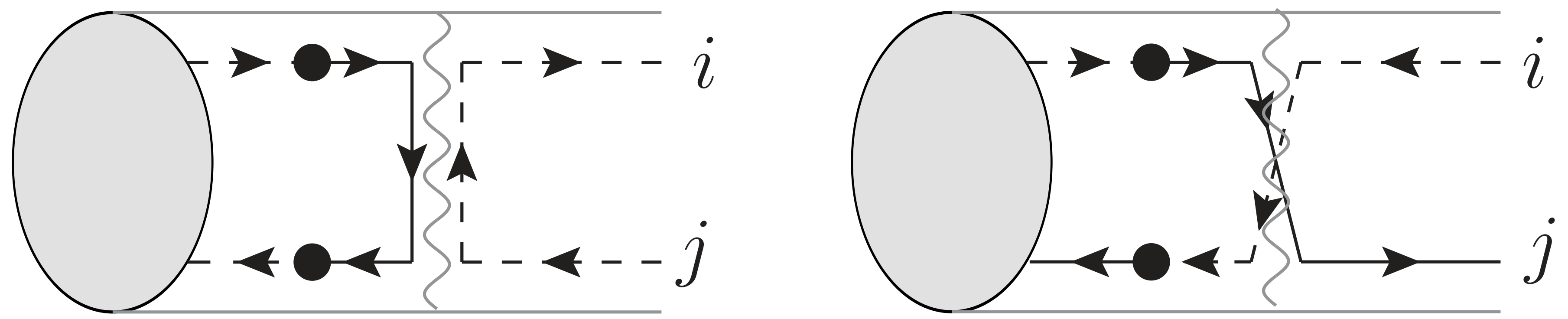

Factorisation and kinematics.

[Plätzer, Sjödahl — '21]

Constructing evolution algorithms

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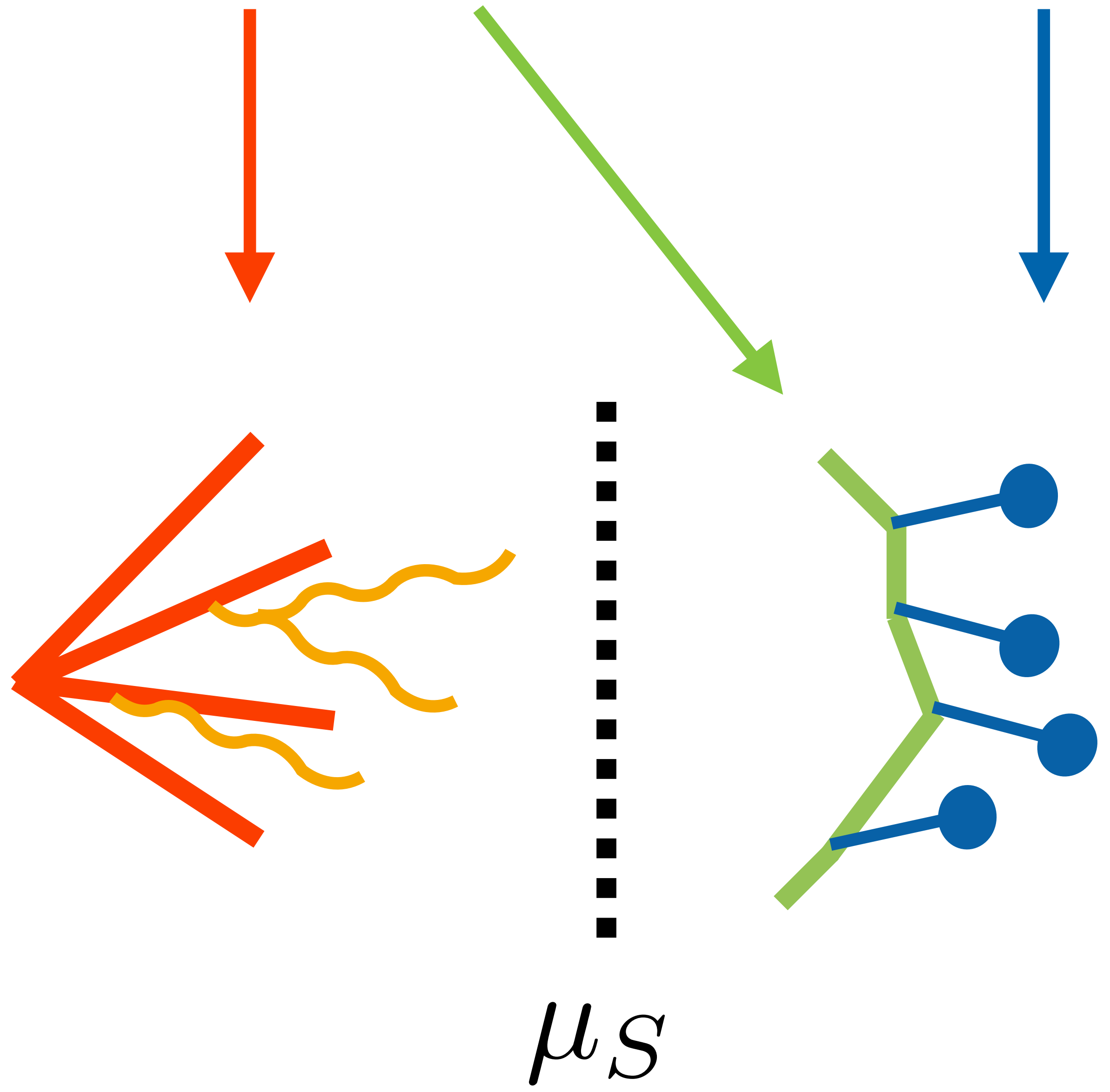


Construct electroweak evolution.

Cutting indicates that subtraction terms refer to different final states — unitarity?

$$\frac{1}{k^2 - m^2 - im\Gamma\ \mathrm{sign}(T\cdot k)} =$$

$$\frac{1}{k^2 - m^2 + im\Gamma} + 2i \frac{m\Gamma}{(k^2-m^2)^2 + m^2\Gamma^2}\theta(T\cdot k)$$

Factorisation and kinematics.

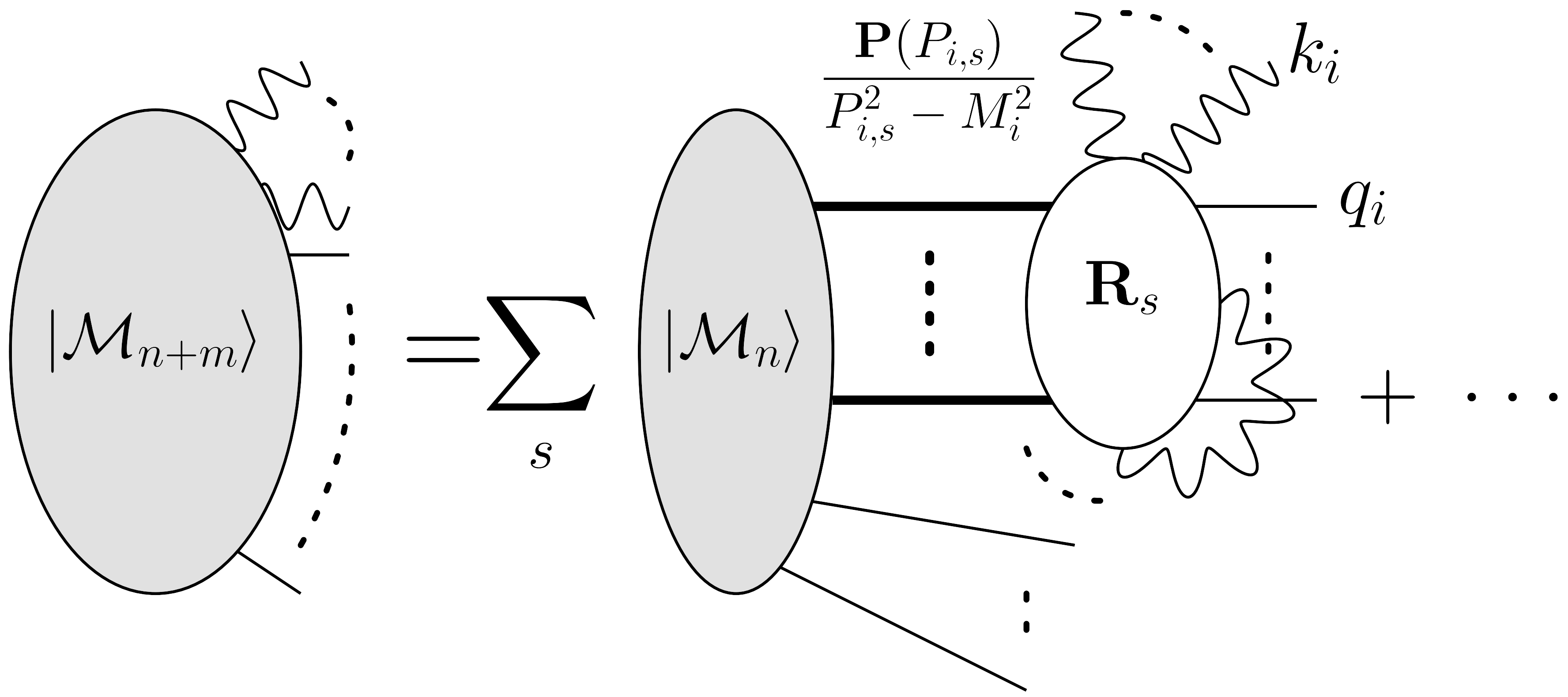
[Plätzer, Sjödahl — '21]

Electroweak evolution

Momentum remapping closely tie in with how factorisation is performed.
Eikonal approximation needs to separate true soft degrees of freedom.

$$(q_i + K_{i,s})^2 - M_i^2 = 2p_i \cdot Q_{i,s}$$

$$p_i \cdot Q_{i,s} \ll p_i \cdot n_{i,s} \equiv S_{i,s}$$



$$K_{i,s}^\mu = \Lambda^\mu{}_\nu \left(Q_{i,s}^\nu + \delta_{i,s}\ n_{i,s}^\nu\right)$$

$$q_i^\mu = \Lambda^\mu{}_\nu \left(\alpha p_i^\nu + \frac{(1-\alpha^2)M_i^2 + p_i \cdot Q_{i,s}}{2\alpha\ n_{i,s} \cdot p_i} n_{i,s}^\nu\right) - K_{i,s}^\mu$$

$$\sum_{n=0}^{\infty} \left(\frac{\mathbf{P}(q_i + K_{i,s}, M_i)}{(q_i + K_{i,s})^2 - \tilde{M}_{R,i}^2} \mathbf{\Sigma}(q_i + K_{i,s}) \right)^n \frac{\mathbf{P}(q_i + K_{i,s}, M_i)}{(q_i + K_{i,s})^2 - \tilde{M}_{R,i}^2} = \frac{1}{2p_i \cdot Q_{i,s}} \frac{\Psi(\Lambda p_i, M_i)\bar{\Psi}(\Lambda p_i, M_i)}{1 - \Sigma'(M_i^2)} + \mathcal{O}(\lambda) \ ;$$

[Plätzer, Sjödahl — '21]

Constructing evolution algorithms

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How to do this at subleading N and higher order shower evolution?

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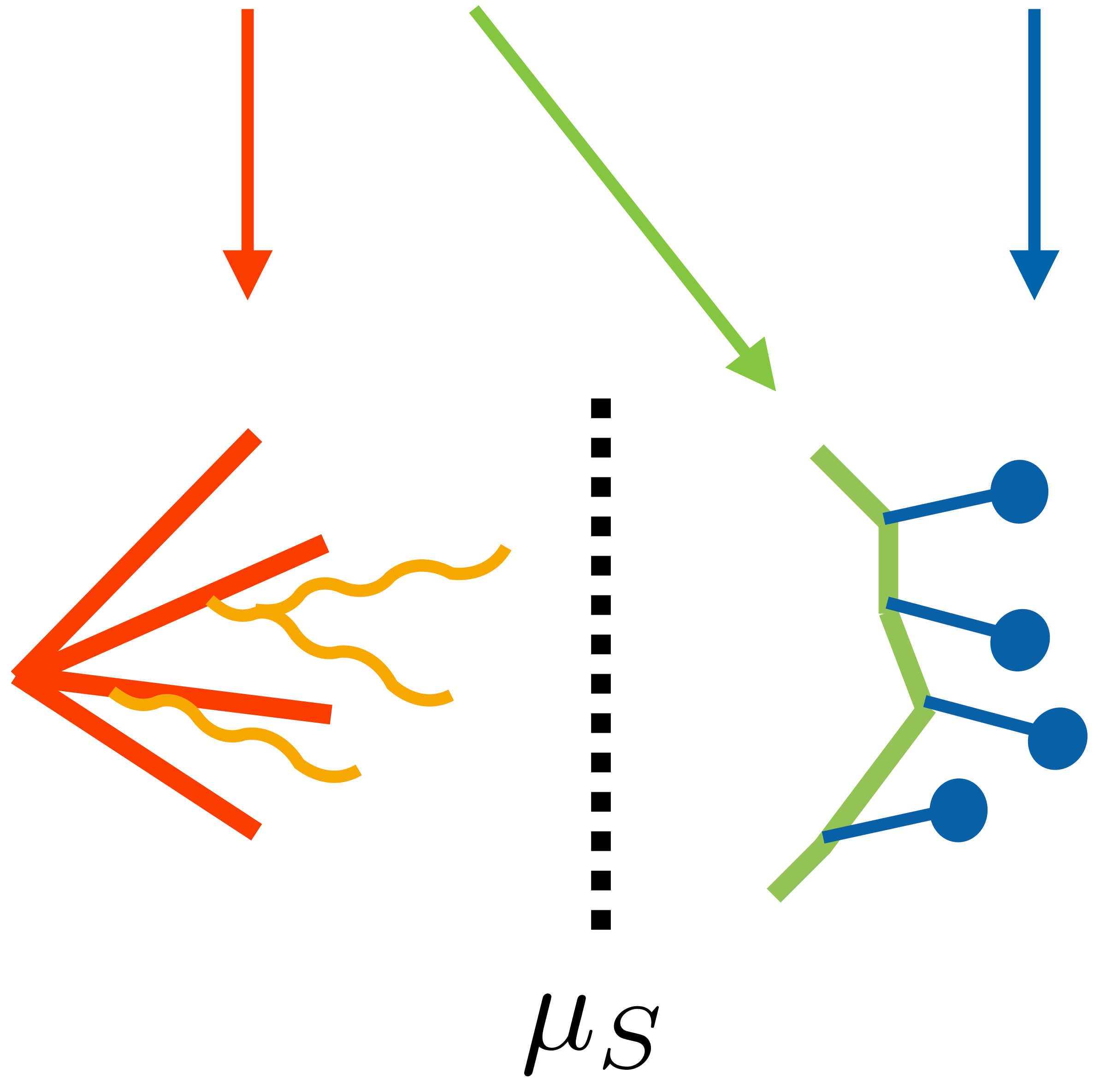
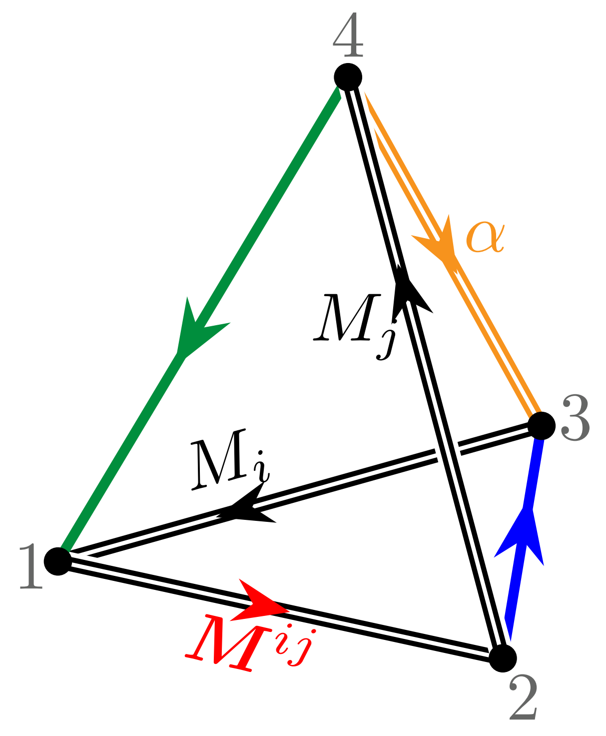
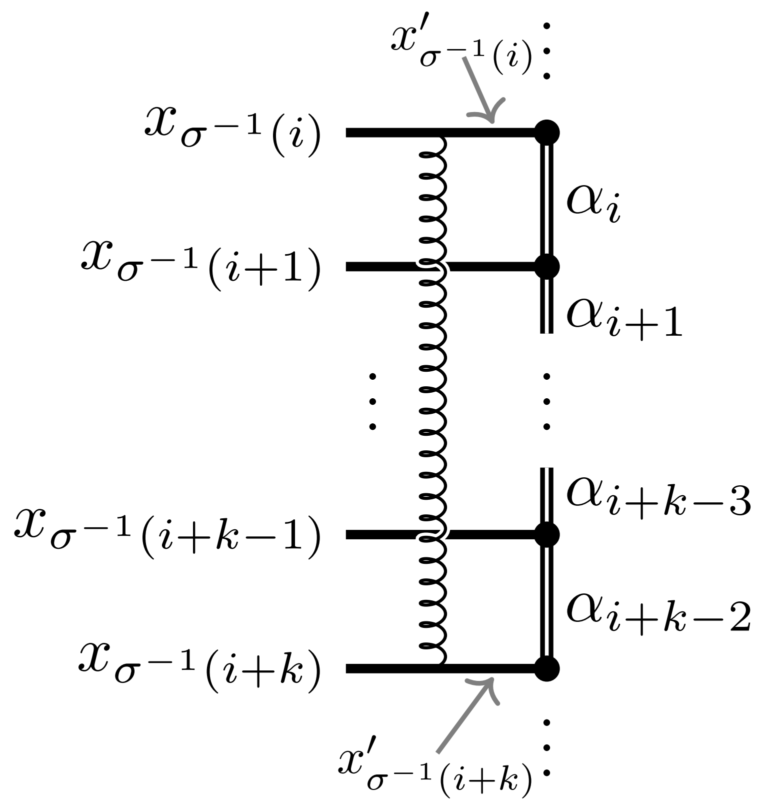


Understand colour multiplets for many legs.





[Alcock-Zeilinger, Keppeler, Plätzer, Sjödahl – '22 & in progress]

Constructing evolution algorithms

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[Plätzer – '22]

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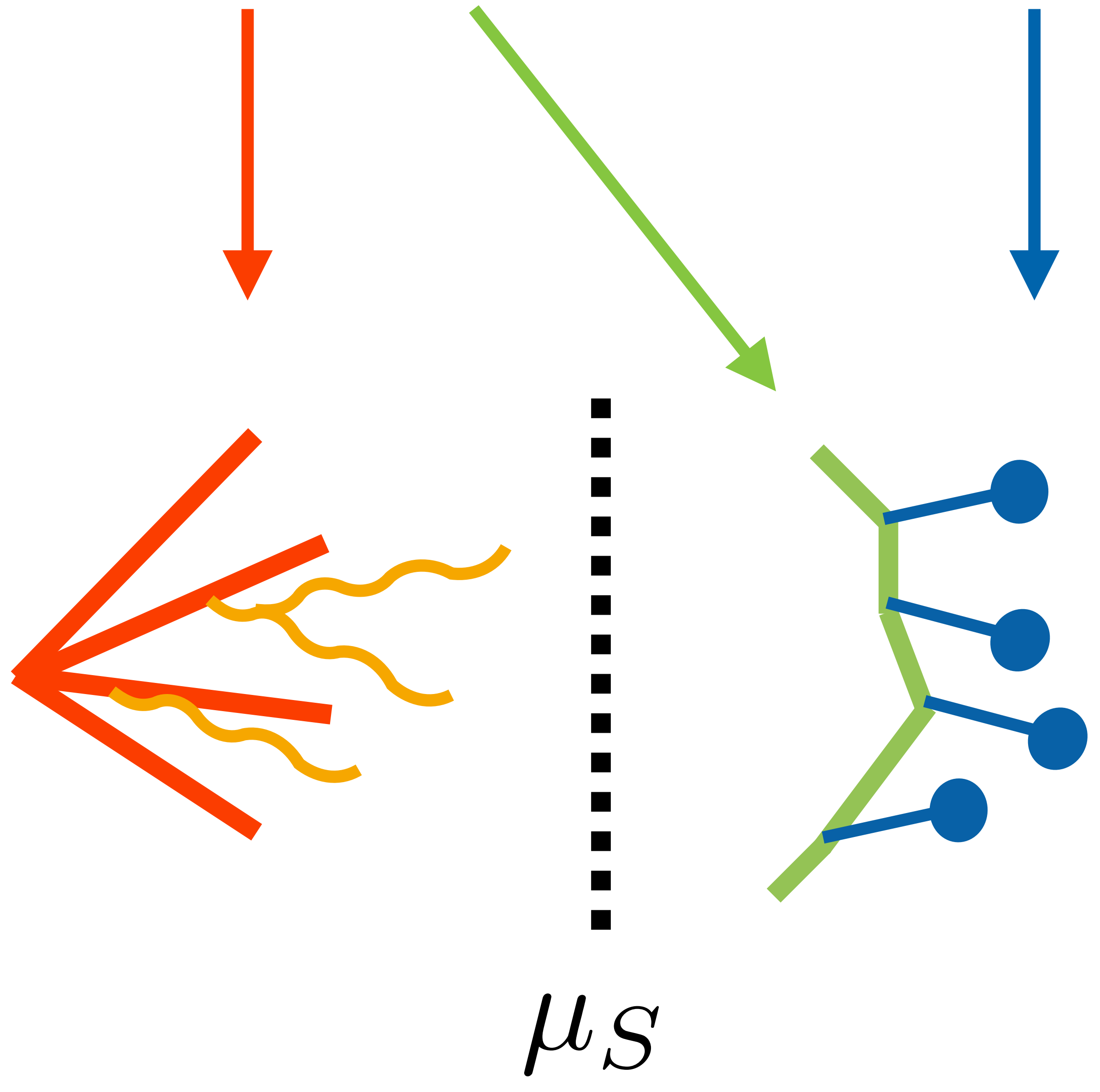
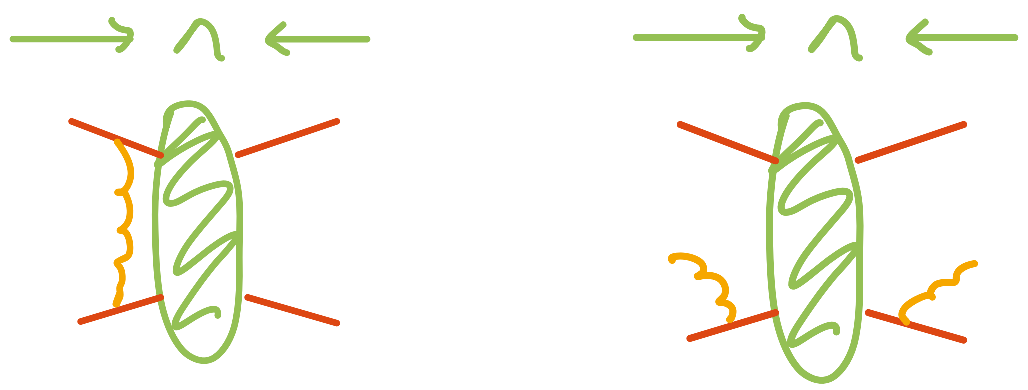
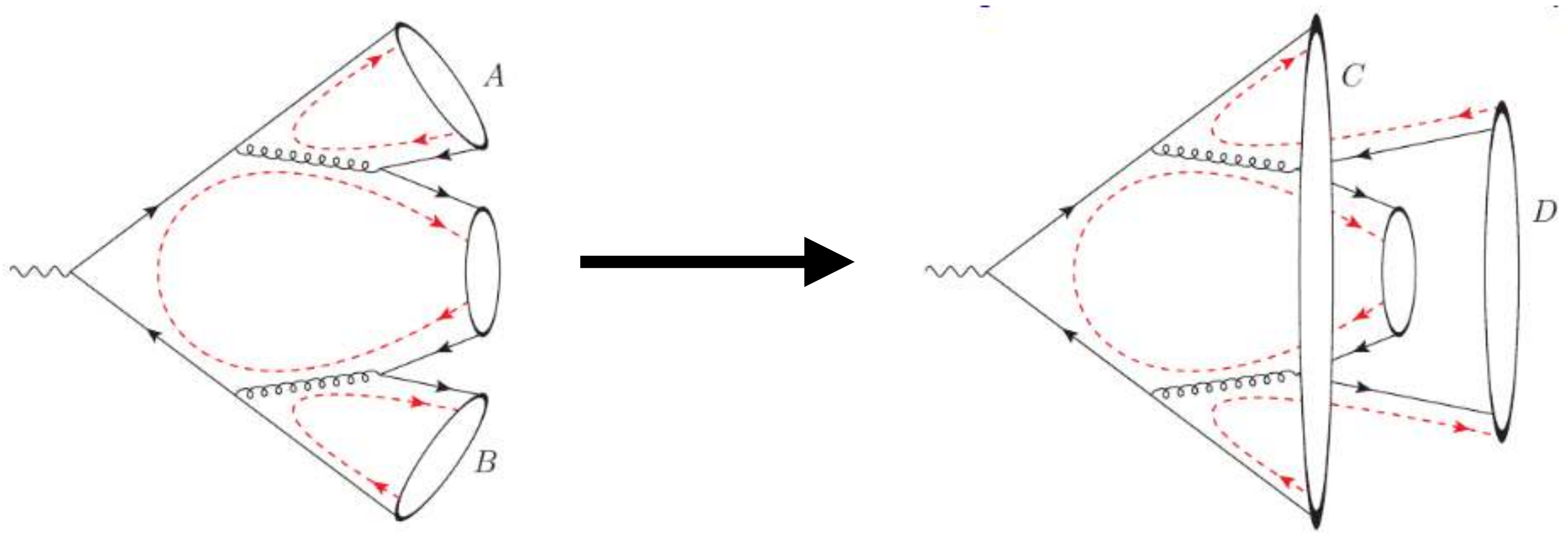


Construct perturbative end of hadronization.





e.g. colour reconnection *implied* just as observed in [Gieseke, Kirchgaesser, Plätzer – '18 …]

Summary

Amplitude evolution is much more than just studying subleading-N effects.
We use to as a theoretical tool and algorithm in its own right.

We can address the structure of evolution algorithms, at leading and higher orders.
Systematic break down in large-N allows us to solidify structure of new algorithms.

[in progress for second order]

Infrared cutoff is the factorisation scale to hadronization models and allows us to construct their high-energy end from perturbative considerations, including colour reconnection.

explored for non-globals and in Herwig, see Andrzej's talk
[Hoang, Plätzer, Samitz — in progress]
[Gieseke, Kiebacher, Plätzer, Priedigkeit — in progress]

If we want to thoroughly understand electroweak evolution beyond the quasi-collinear limit nothing allows us to bypass this framework.

[crucial input for ongoing activities in Herwig — see Andrzej's talk]

Thank you.