Combining QCD and QED transverse-momentum resummation for Z and W production

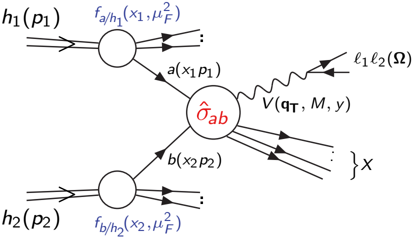
Giancarlo Ferrera

Milan University & INFN, Milan

Parton Shower and Resummation

Milan – 8/6/2023

Drell–Yan q_T distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\rightarrow\; V + X \;\rightarrow\; \ell_1 + \ell_2 + X}$$

where $\quad V = Z^0/\gamma^*, W^\pm$

QCD factorization formula:

$$\frac{d\sigma}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\; f_{a/h_1}(x_1, \mu_F^2)\; f_{b/h_2}(x_2, \mu_F^2)\; \frac{d\hat{\sigma}_{ab}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\mathbf{\Omega}} (\alpha_S(\mu_R^2), \mu_R^2, \mu_F^2).$$

Fixed-order perturbative expansion reliable

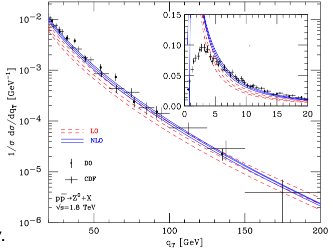
only for $q_T \sim M$. When $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2\, \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \sim 1 + \alpha_S \left[c_{12} L_{q_T}^2 + c_{11} L_{q_T} + \cdots \right]$$

$$+ \alpha_S^2 \left[c_{24} L_{q_T}^4 + \cdots + c_{21} L_{q_T} + \cdots \right] + \mathcal{O}(\alpha_S^3)$$

with $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m(M^2/q_T^2) \gg 1$.

Resummation of logarithmic corrections mandatory.



Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.

$$dw_n(q_1,\ldots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform)[Parisi,Petronzio('79)]

$$\int d^2\mathbf{q_T}\ \exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta^{(2)}\left(\mathbf{q_T}-\sum_{j=1}^{n}\mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$.

$\mathbf{q_T}$ resummation in QCD [Catani,de Florian,Grazzini('01)] [Bozzi,Catani,de Florian,Grazzini('03,'06)]

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \, \mathcal{W}(b,M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)} \, \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\}$$

with $\quad L \equiv \log(M^2 b^2)$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; $\cdots$ N^{k}LL ($\sim \alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, $\mathcal{H}^{(k)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD

[Catani, de Florian, Grazzini('01)]
[Bozzi, Catani, de Florian, Grazzini('03,'06)]

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)}\, \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\}$$

with $L \equiv \log(M^2 b^2)$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim\alpha_S^n L^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim\alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; $\cdots$ N^{k}LL ($\sim\alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, $\mathcal{H}^{(k)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD [Catani,de Florian,Grazzini('01)] [Bozzi,Catani,de Florian,Grazzini('03,'06)]

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)} \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\}$$

with $L \equiv \log(M^2 b^2)$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim\alpha_S^n L^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim\alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; $\cdots$ N^{k}LL ($\sim\alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, $\mathcal{H}^{(k)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD [Catani, de Florian, Grazzini ('01)] [Bozzi, Catani, de Florian, Grazzini ('03, '06)]

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \, \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)} \, \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\}$$

with $L \equiv \log(M^2 b^2)$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, $(\hat{\sigma}^{(0)})$; NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; $\cdots$ N^{k}LL ($\sim \alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, $\mathcal{H}^{(k)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$ via all-order formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln\left(M^2 b^2\right) \;\to\; \ln\left(Q^2 b^2\right) + \ln\left(M^2/Q^2\right)$$

- Perturbative unitarity constraint: recover *exactly* the total cross-section (upon integration on q_T)

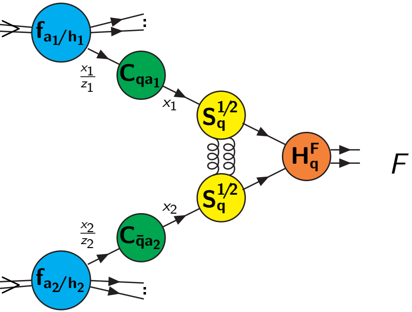
$$\ln\left(Q^2 b^2\right) \;\to\; \widetilde{L} \equiv \ln\left(Q^2 b^2 + 1\right) \quad\Rightarrow\quad \exp\left\{\alpha_S^n \widetilde{L}^k\right\}\Big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right) = \hat{\sigma}^{(tot)};$$

- General procedure to treat the q_T recoil [Catani,de Florian,G.F.,Grazzini('15)]:

$$\frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} = \hat{\sigma}^{(0)}(M^2)\; F(\mathbf{q_T}; M^2, \mathbf{\Omega}) \;\; with \;\; F(\mathbf{q_T}; M^2, \mathbf{\Omega}) = F(\mathbf{0}; M^2, \mathbf{\Omega}) + \mathcal{O}(\mathbf{q_T^2}/M^2)$$

Connection with TMD formalism

[Collins,Soper,Sterman('05)]



$$M \gg \Lambda_{QCD} \;, \quad b \gg 1/M \;, \quad b \ll 1/\Lambda_{QCD}$$

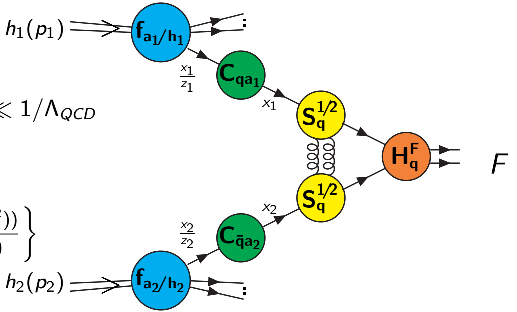
$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times \;\exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\; \beta(\alpha_S(q^2)) \;\frac{d\ln C(\alpha_S(q^2))}{d\ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{dq_T^2} = \frac{M^2}{s}\,\sigma^{(0)}_{q\bar{q},F}\, H_q^F(\alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{2\pi}\; e^{i\mathbf{b}\cdot\mathbf{q_T}}\, S_q(M,b)$$
$$\times \int_{x_1}^{1} \frac{dz_1}{z_1}\, C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^{1} \frac{dz_2}{z_2}\, C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\, \sqrt{S_q(M,b)}\, C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\; f_{a/h}(x/z, b_0^2/b^2)$$

Connection with TMD formalism

[Collins,Soper,Sterman('05)]



$$M \gg \Lambda_{QCD} \;, \quad b \gg 1/M \;, \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times \;\exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\; \beta(\alpha_S(q^2))\; \frac{d\ln C(\alpha_S(q^2))}{d\ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{dq_T^2} = \frac{M^2}{s}\, \sigma_{q\bar{q},F}^{(0)}\, H_q^F(\alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{2\pi}\; e^{i\mathbf{b}\cdot\mathbf{q_T}}\, S_q(M,b)$$

$$\times \int_{x_1}^{1} \frac{dz_1}{z_1}\, C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\, f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^{1} \frac{dz_2}{z_2}\, C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\, \sqrt{S_q(M,b)}\, C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\; f_{a/h}(x/z, b_0^2/b^2)$$

q_T resummation up N^4LL+N^4LOa

- We have implemented the calculation in the publicly available code:

  DYTurbo: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda,Boonekamp,Bozzi,Catani,Cieri,Cuth,G.F.,de Florian,Glazov, Grazzini,Vincter,Schott('20)]

  https://dyturbo.hepforge.org.

- q_T resummation performed for Drell–Yan process up to N^4LL+N^4LOa We have included
  - N^4LL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-3})$);
  - Approximated N^4LO corrections (i.e. up to $\mathcal{O}(\alpha_S^4)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;
- Matching with NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at large q_T from results in [Boughezal et al.('16)], [Gehrmann-De Ridder et al.('16)], [MCFM ('23)];
- Results up to N^3LO (i.e. up to $\mathcal{O}(\alpha_S^3)$) recovered for the total cross section (from unitarity).

q_T resummation up N^{4}LL+N^{4}LOa

- We have implemented the calculation in the publicly available code:

  DYTurbo: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott('20)]

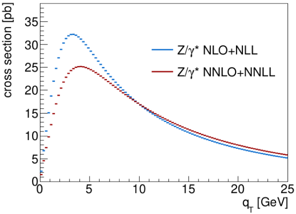
  https://dyturbo.hepforge.org.

- q_T resummation performed for Drell–Yan process up to N^{4}LL+N^{4}LOa We have included
  - N^{4}LL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-3})$);
  - Approximated N^{4}LO corrections (i.e. up to $\mathcal{O}(\alpha_S^4)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;
- Matching with NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at large q_T from results in [Boughezal et al.('16)], [Gehrmann-De Ridder et al.('16)], [MCFM ('23)];
- Results up to N^{3}LO (i.e. up to $\mathcal{O}(\alpha_S^3)$) recovered for the total cross section (from unitarity).

Fast predictions for Drell-Yan processes: DYTurbo

[Camarda,Boonekamp,Bozzi,Catani,Cieri,Cuth,G.F.,de Florian,Glazov, Grazzini,Vincter,Schott ('20)]

Example calculation



- Example calculation for Z p_T spectrum at 13 TeV
  - No cuts on the leptons
  - Full rapidity range
  - 100 p_T bins
  - 20 parallel threads

Time required	RES	CT	V+jet
NLO+NLL	6 s	0.2 s	4 min
NNLO+NNLL	10 s	0.7 s	3.4 h

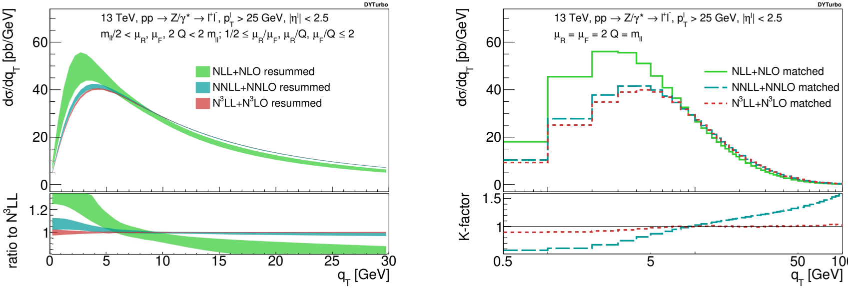
- The most demanding calculation is V+jet

  → can use APPLgrid/FASTnlo for this term

Stefano Camarda 11

Z/γ^* production at N^{3}LL+N^{3}LO

[Camarda,Cieri,G.F.('21)]

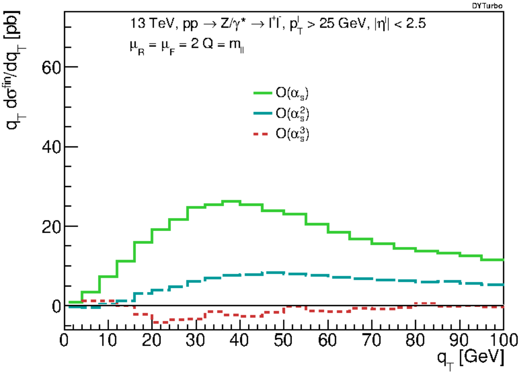


DYTurbo results. Resummed (left) and matched (right) NLL, NNLL and N^{3}LL bands for Z/γ^* q_T spectrum.

Lower panel: ratio with respect to the N^{3}LL central value.

Z/γ^* production at N^{3}LL+N^{3}LO

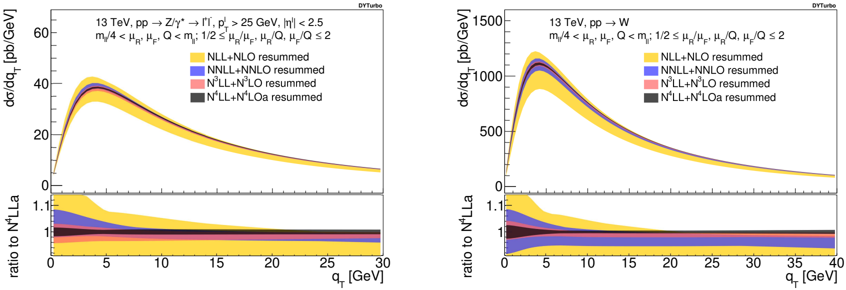
[Camarda,Cieri,G.F.('21)]



Finite part at $\mathcal{O}(\alpha_S)$, $\mathcal{O}(\alpha_S^2)$ and $\mathcal{O}(\alpha_S^3)$.

Z/γ^* and W production at N^{4}LL+N^{4}LOa

[Camarda,Cieri,G.F.('23)]

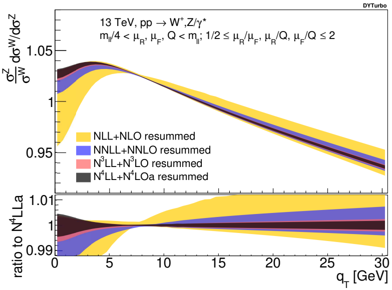


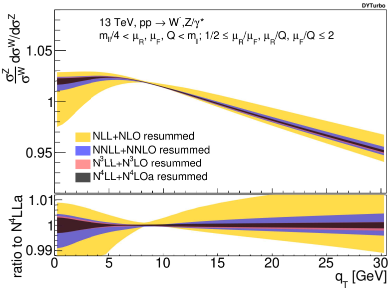
DYTurbo results. Resummed NLL, NNLL, N^{3}LL and N^{4}LLa bands for Z/γ^* (left) and W (right) q_T spectrum.

Lower panel: ratio with respect to the N^{4}LLa central value.

Z/γ^* and W production at N^{4}LL+N^{4}LOa

[Camarda,Cieri,G.F.('23)]



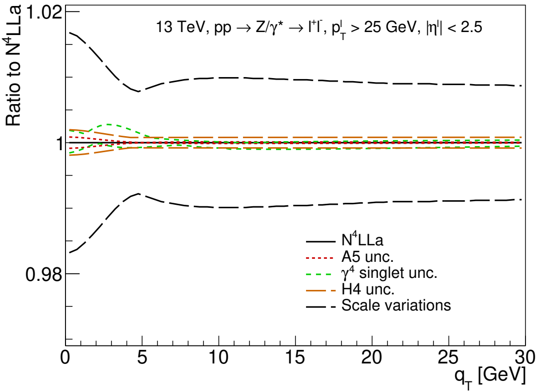


DYTurbo results. Resummed NLL, NNLL, N^{3}LL and N^{4}LLa bands for q_T spectrum of W^+ (left) and W^- (right) over Z/γ^* ratio.

Lower panel: ratio with respect to the N^{4}LLa central value.

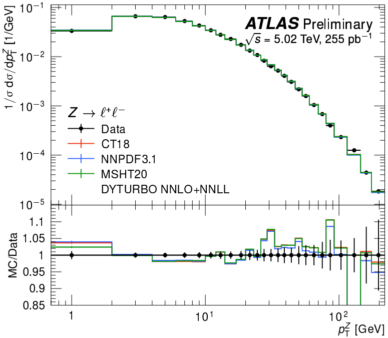
Z/γ^* at N^{4}LL+N^{4}LOa

[Camarda,Cieri,G.F.('23)]

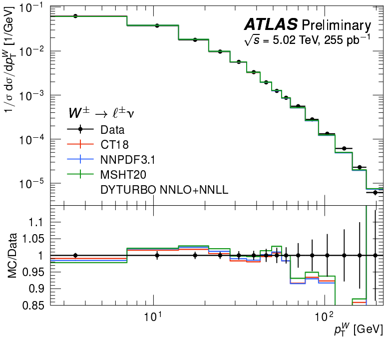


Uncertainties from approximations of the perturbative coefficients at N4LL+N4LOa compared to scale variations.

DYTurbo vs LHC data comparison



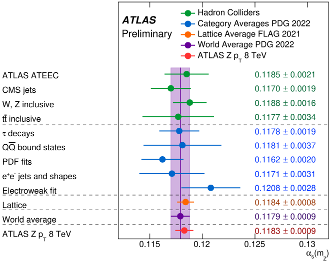
NNLL+NNLO DYTURBO predictions for Z qt spectrum with different PDF sets compared with data [ATLAS Coll.('23)]



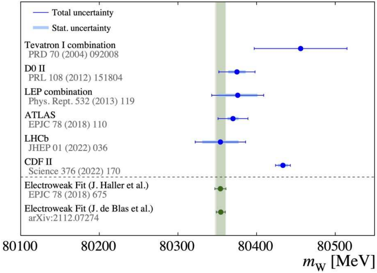
NNLL+NNLO DYTURBO predictions for W qt spectrum with different PDF sets compared with data [ATLAS Coll.('23)]

Modelling W and Z production for α_S and M_W determination

Theoretical predictions from DYTurbo used for determination of SM parameters.



Comparison of determinations of $\alpha_S(m_Z)$.

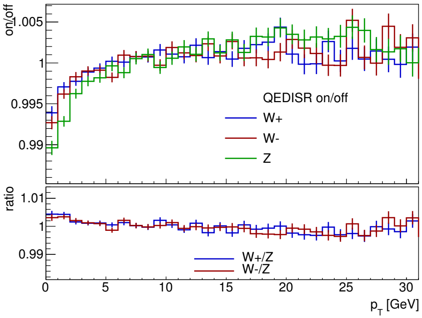


Measured values of M_W compared with the prediction of from the global electroweak fit

Combining QED and QCD q_T resummation

W and Z q_T distributions sensitive to QED effects.

Pythia 8 QED ISR



October 2, 2017 Stefano Camarda 6

Combining QED and QCD q_T resummation

[Cieri,G.F.,Sborlini('18)]

We start considering QED contributions to the q_T spectrum in the case of colourless and **neutral** high mass systems, e.g. on-shell Z boson production

$$h_1 + h_2 \ \rightarrow\ Z^0 + X$$

In the impact parameter and Mellin spaces resummed partonic cross section reads:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)}\, \mathcal{H}'_N(\alpha_S,\alpha) \times \exp\left\{\mathcal{G}'_N(\alpha_S,\alpha,L)\right\}$$

$$\mathcal{G}'(\alpha_S,\alpha,L) = \mathcal{G}(\alpha_S,L) + L\, g'^{(1)}(\alpha L) + g'^{(2)}(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}(\alpha L)$$

$$+\ g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L,\alpha L)$$

$$\mathcal{H}'(\alpha_S,\alpha) \quad = \quad \mathcal{H}(\alpha_S) + \frac{\alpha}{\pi}\mathcal{H}'^{(1)} + \sum_{n=2}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n} \mathcal{H}_N'^{(n)} + \sum_{n,m=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n}\left(\frac{\alpha}{\pi}\right)^{m} \mathcal{H}_N'^{F\,(n,m)}$$

LL QED ($\sim\alpha^n L^{n+1}$): $g'^{(1)}$; NLL QED ($\sim\alpha^n L^n$): $g'^{(2)}$, $\mathcal{H}'^{(1)}$;
LL mixed QCD-QED ($\sim\alpha_S^n\alpha^n L^{2n}$): $g'^{(1,1)}$;

The LL and NLL QED functions $g'^{(1)}$ and $g'^{(2)}$ has the same *functional* form of the QCD ones:

$$g'^{(1)}(\alpha L) = \frac{A_q'^{(1)}}{\beta_0'} \frac{\lambda' + \ln(1-\lambda')}{\lambda'} \quad ,$$

$$g_N'^{(2)}(\alpha L) = \frac{\widetilde{B}_{q,N}'^{(1)}}{\beta_0'} \ln(1-\lambda') - \frac{A_q'^{(2)}}{\beta_0'^2}\left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda')\right)$$
$$+ \frac{A_q'^{(1)}\beta_1'}{\beta_0'^3}\left(\frac{1}{2}\ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'}\right) \quad ,$$

the *novel* LL mixed QCD-QED function reads:

$$g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)}\beta_{0,1}}{\beta_0^2\beta_0'} \, h(\lambda,\lambda') + \frac{A_q'^{(1)}\beta_{0,1}'}{\beta_0'^2\beta_0} \, h(\lambda',\lambda) \quad ,$$

$$h(\lambda,\lambda') = -\frac{\lambda'}{\lambda-\lambda'}\ln(1-\lambda) + \ln(1-\lambda')\left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln\left(\frac{-\lambda'(1-\lambda)}{\lambda-\lambda'}\right)\right]$$
$$- \mathrm{Li}_2\left(\frac{\lambda}{\lambda-\lambda'}\right) + \mathrm{Li}_2\left(\frac{\lambda(1-\lambda')}{\lambda-\lambda'}\right),$$

where $\lambda = \frac{1}{\pi}\beta_0\,\alpha_S\,L$, $\lambda' = \frac{1}{\pi}\beta_0'\,\alpha\,L$, and β_0, β_0', β_1', $\beta_{0,1}$, $\beta_{0,1}'$ are the coefficients of the QCD and QED β functions.

Abelianization procedure

$$\frac{d\ln\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S(\mu^2),\alpha(\mu^2)) = -\sum_{n=0}^{\infty}\beta_n\left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta_{n,m}\left(\frac{\alpha_S}{\pi}\right)^{n+1}\left(\frac{\alpha}{\pi}\right)^{m},$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2),\alpha_S(\mu^2)) = -\sum_{n=0}^{\infty}\beta'_n\left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta'_{n,m}\left(\frac{\alpha}{\pi}\right)^{n+1}\left(\frac{\alpha_S}{\pi}\right)^{m}.$$

Novel QED coefficients obtained through an Abelianization algorithm

$$A_q'^{(1)} = e_q^2\,, \qquad A_q'^{(2)} = -\frac{5}{9}\,e_q^2\,N^{(2)} \qquad \widetilde{B}_{q,N}'^{(1)} = B_q'^{(1)} + 2\gamma_{qq,N}'^{(1)}\,,$$

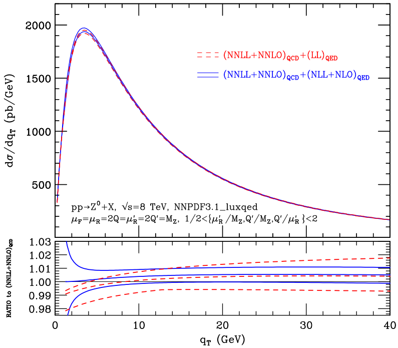
$$\text{with} \quad B_q'^{(1)} = -\frac{3}{2}\,e_q^2\,, \quad N^{(n)} = N_c\sum_{q=1}^{n_f} e_q^n + \sum_{l=1}^{n_l} e_l^n\,,$$

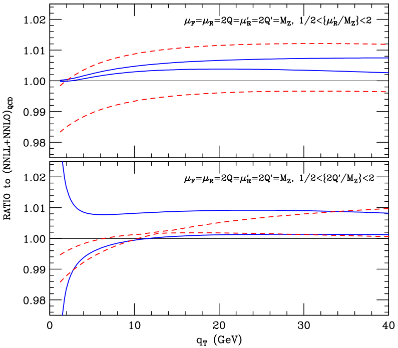
$$\gamma_{qq,N}'^{(1)} = e_q^2\left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1)\right), \quad \gamma_{q\gamma,N}'^{(1)} = \frac{3}{2}\,e_q^2\,\frac{N^2+N+2}{N(N+1)(N+2)}.$$

$$\mathcal{H}_{q\bar{q}\leftarrow q\bar{q},N}'^{F\,(1)} = \frac{e_q^2}{2}\left(\frac{2}{N(N+1)} - 8 + \pi^2\right), \quad \mathcal{H}_{q\bar{q}\leftarrow\gamma q,N}'^{F\,(1)} = \frac{3\,e_q^2}{(N+1)(N+2)},$$

Resummed result *matched* with corresponding finite $\mathcal{O}(\alpha)$ term.

Combined QED and QCD q_T resummation for Z production at the LHC [Cieri,G.F.,Sborlini ('18)]

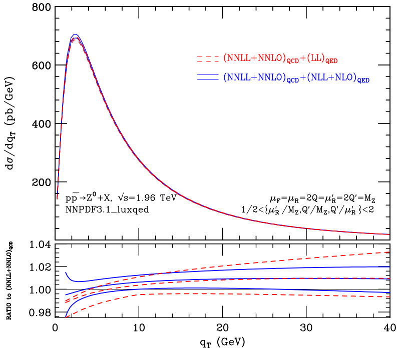


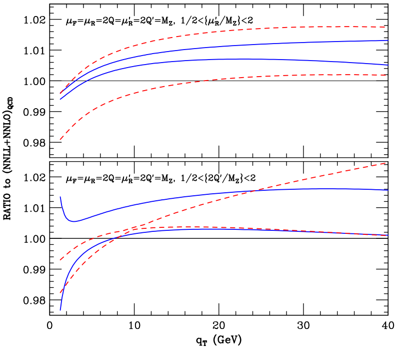


Z qT spectrum at the LHC. NNLL+NNLO QCD combined with the LL (red dashed) and NLL+NLO (blue solid) QED with the corresponding QED uncertainty bands.

Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Combined QED and QCD q_T resummation for Z production at the Tevatron [Cieri,G.F.,Sborlini ('18)]





Z qT spectrum at the LHC. NNLL+NNLO QCD combined with the LL (red dashed) and NLL+NLO (blue solid) QED with corresponding QED uncertainty bands.

Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Combining QED and QCD q_T resummation for W production

[Autieri,Cieri,G.F.,Sborlini ('23)]

We next consider QED contributions to the q_T spectrum in the case of colourless and **charged** high mass systems, e.g. on-shell $W^{\pm}$ boson production

$$h_1 + h_2 \;\to\; W^{\pm} + X$$

- Initial state QED emissions sensitive to different quark charges ($q\bar{q}' \to W^{\pm}$):

$$2e_q^2 \to e_q^2 + e_{\bar{q}'}^2$$

- Final state QED emissions: *abelianizion* of QCD resummation formula q_T resummation for $t\bar{t}$ production [Catani,Grazzini,Torre('14)]:

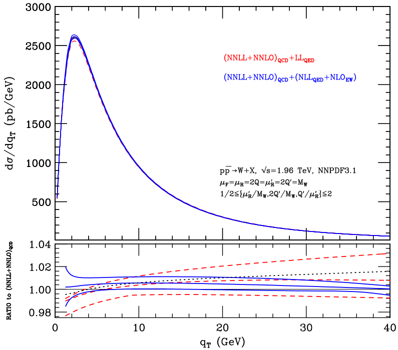
$$\Delta'(b, M) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} D'(\alpha(q^2)) \right\}$$

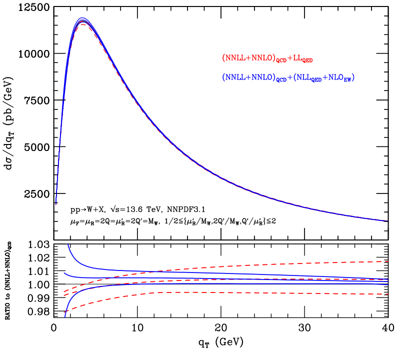
$$\text{with} \quad D'(\alpha) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n D'^{(n)}\,, \qquad \text{and} \qquad D'^{(1)} = -\frac{e^2}{2}\,.$$

- Factor $\Delta'(b, M)$ resums soft (non collinear) QED emissions from final state (and from initial-final interference). Effects from $D'(\alpha)$ start to contribute at NLL. Same functional dependence, in terms of $g'^{(i)}$ functions, as the $B'(\alpha)$ term.

Combined QED and QCD q_T resummation on W production

[Autieri,Cieri,G.F.,Sborlini ('23)]



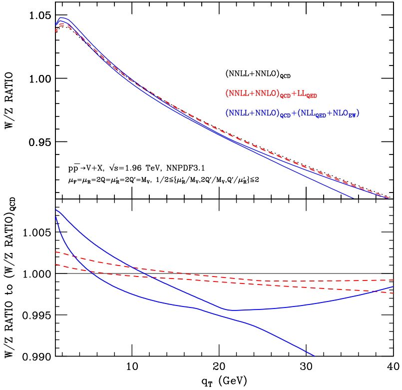


W qT spectrum at the Tevatron. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with corresponding QED uncertainty bands.

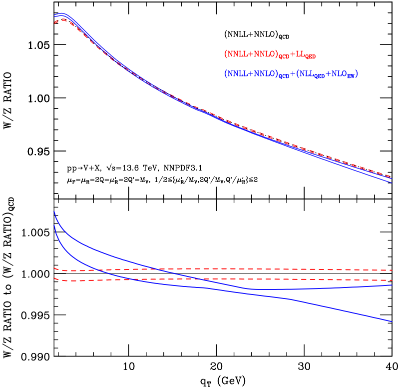
W qT spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.

Combined QED and QCD q_T resummation on W/Z spectrum

[Autieri,Cieri,G.F.,Sborlini ('23)]



W over *Z qT* spectrum at the Tevatron. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.



W over *Z qT* spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.

Conclusions

- Discussed formalism to perform q_T resummed predictions up to N^{4}LL+N^{4}LOa and presented results for Drell–Yan production at the Tevatron and the LHC.
- Discussed perturbative uncertainty performing μ_R, μ_F and Q scale variation and its reduction going to higher accuracy.
- Discussed formalism to perform combined QCD and QED q_T resummation for Z and W production at hadron colliders.
- Presented a fast and numerically precise publicly available code DYTurbo: https://dyturbo.hepforge.org