Quantum Error Correction

And

Statistical Physics Model

Based on M. Rispler, D. Vodola, SK, M. Mueller, quantum 6 (2022) 618,

Seyong Kim

- Supercomputer and Quantum Computer?
- Computational speed?
- Qubit?
- Quantum Error correction

  and Threshold probability
- Statistical Physics model and Threshold probability

High Performance Computer

Columbia 64-node machine

주간조선 [2558호] 2019.05.20

[과학 연구의 최전선] 이론물리학자 세종대 김세용 교수

‘격자QCD’ 연구의 세계적 권위자 수퍼컴퓨터를 만들다

최준석 선임기자 jschoi@chosun.com

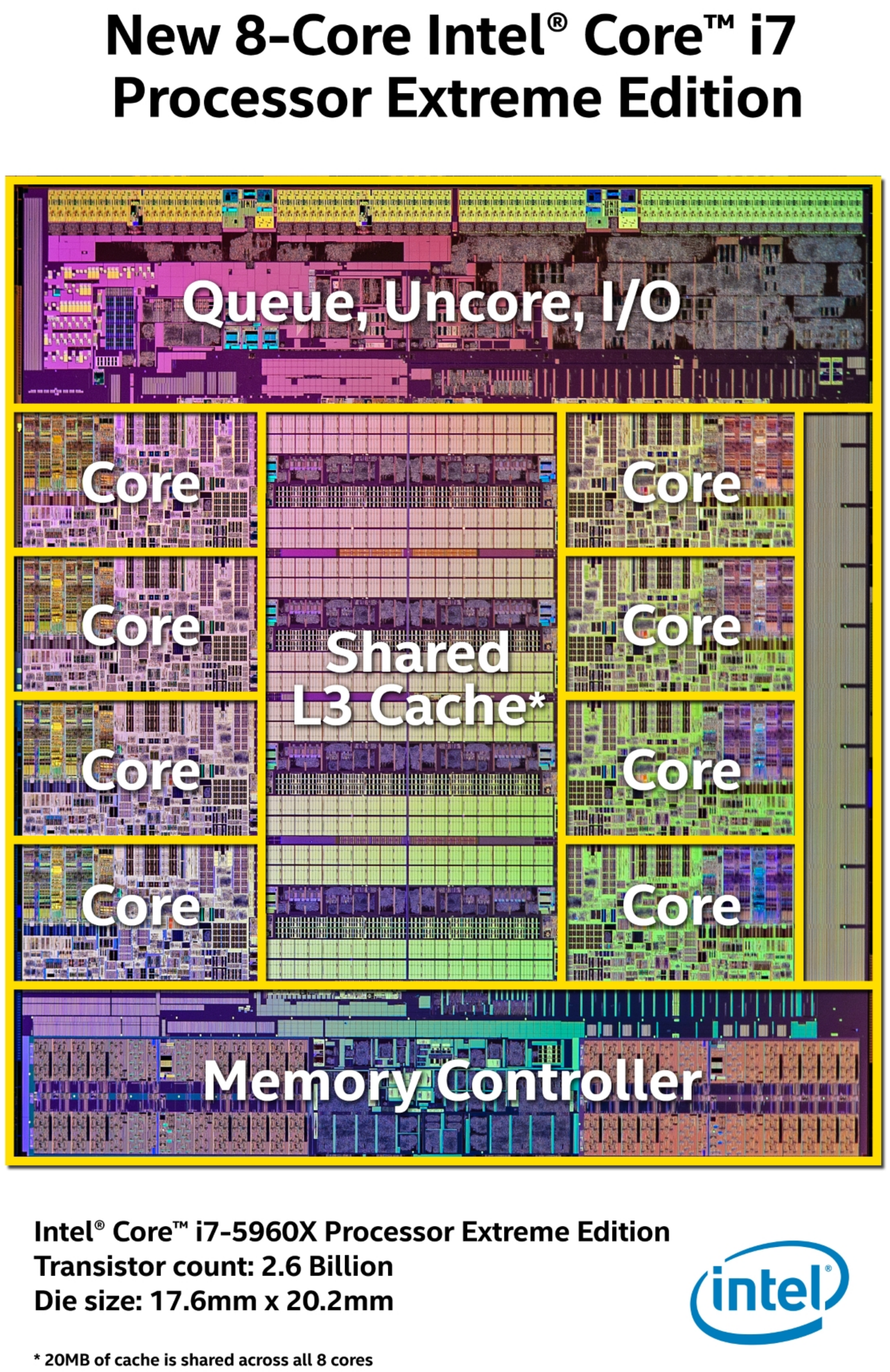
▲photo 한준호 영상미디어 기자

세종대 물리학과의 김세용 교수는 지난 3월부터 스위스 베른에 머물고 있다. 알베르트 아인슈타인 센터 부설 이론물리연구소에서 연구년을 보내는 중이다. 김 교수는 ‘격자(lattice)QCD(양자색역학)’라는 일반에게는 낯선 분야에서 국제적 지명도를 갖고 있다. 베른에서 서울에 잠시 온 그를 지난 4월 23일 세종대 연구실에서 만났다.

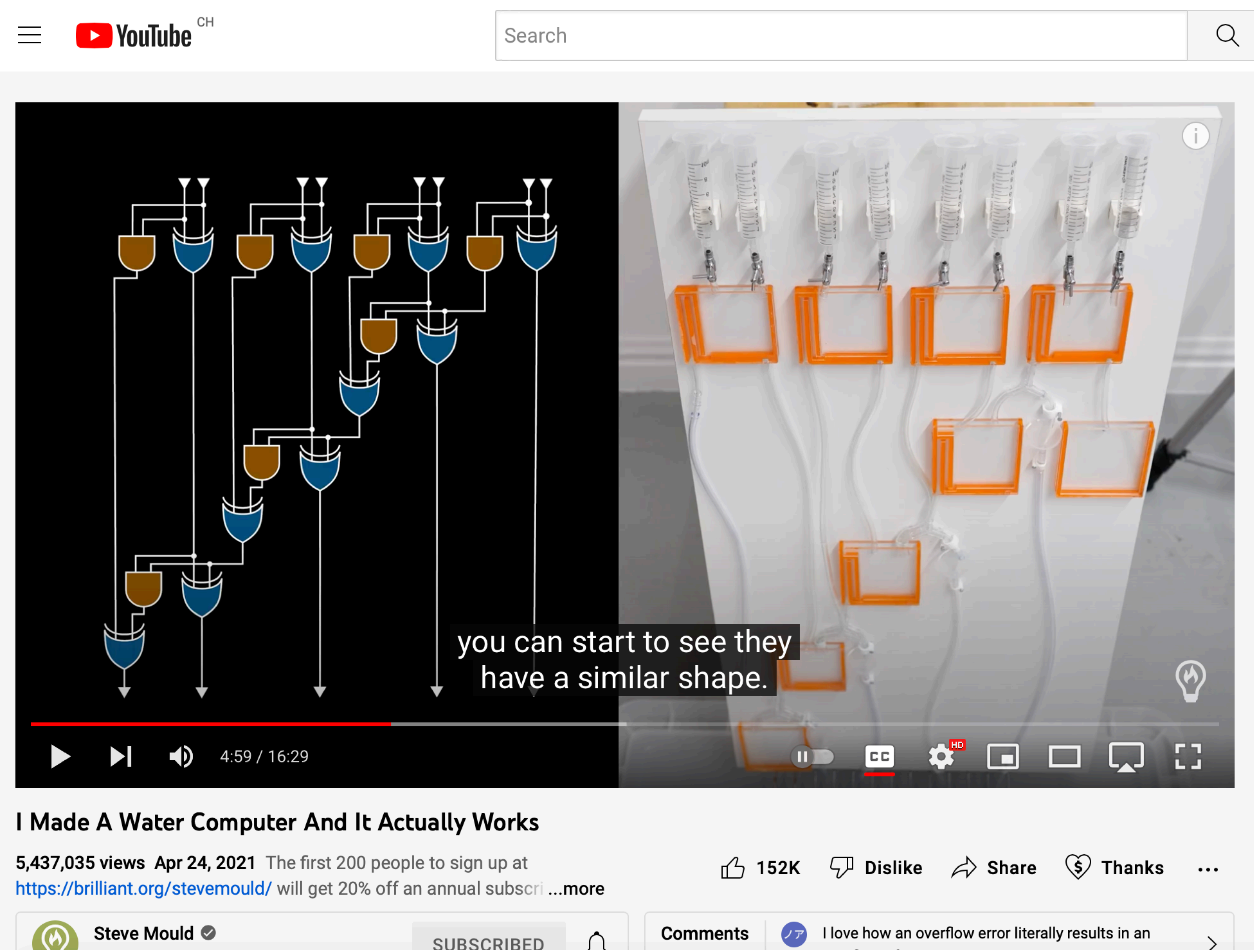
“High Performance Computing”

- “Big data problem?”
- “Large computational problem?”

Typical digital computer?



https://newsroom.intel.com/news-releases/intel-unleashes-its-first-8-core-desktop-processor/#gs.bsvihg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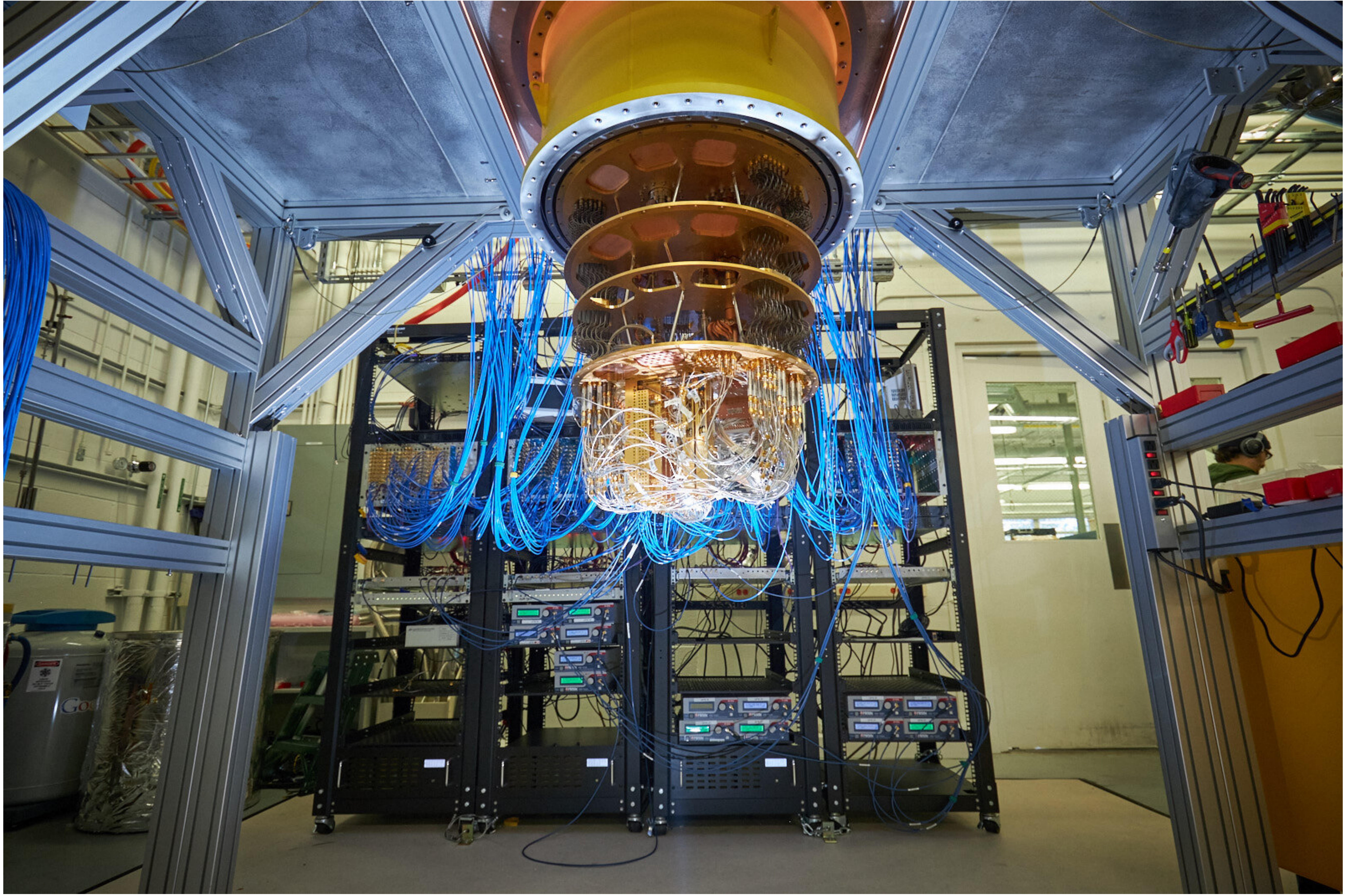
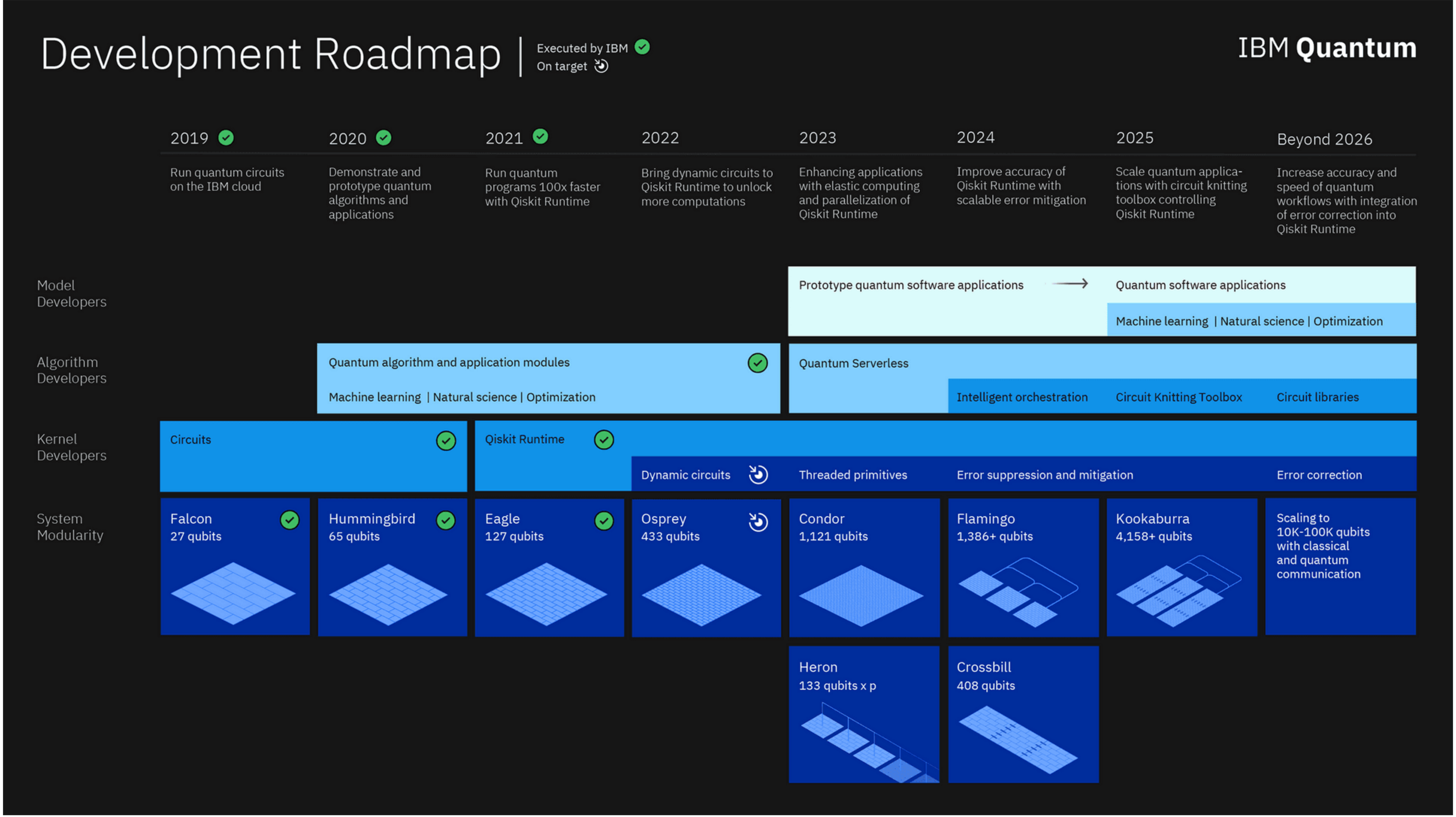
I love how an overflow error literally results in an

Quantum Computer,
Experimental Computing



Google's Sycamore processor mounted in a cryostat, recently used to demonstrate quantum supremacy and the largest quantum chemistry simulation on a quantum computer. Credit: Rocco Ceselin



https://research.ibm.com/blog/ibm-quantum-roadmap-2025

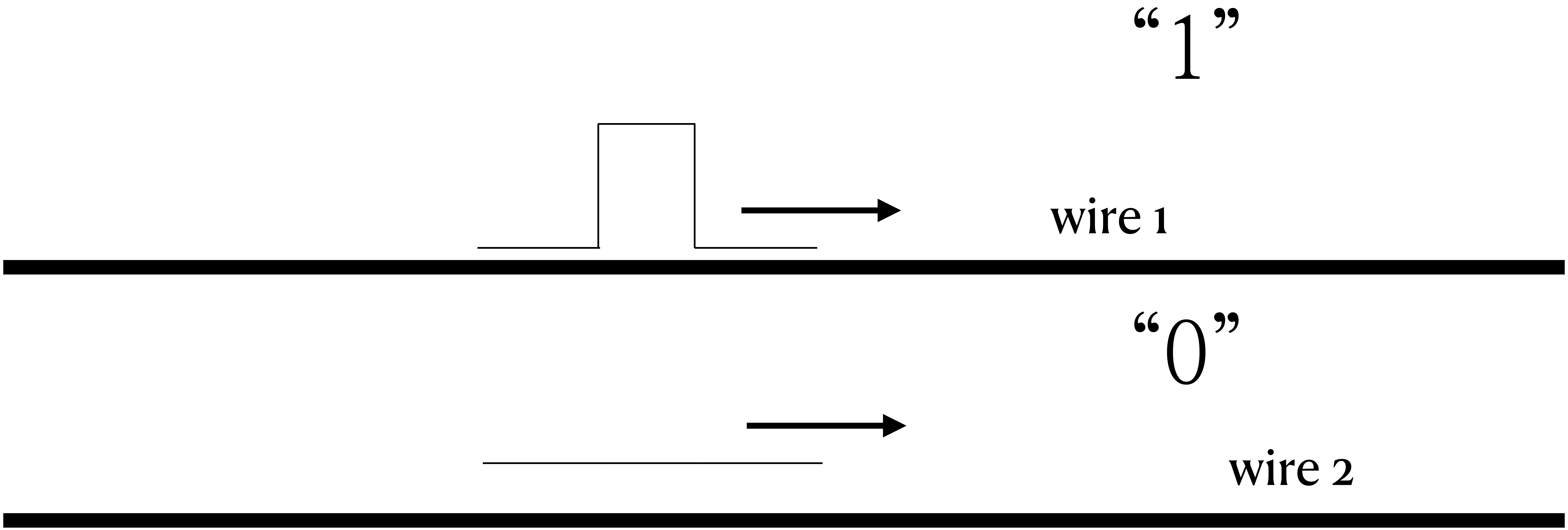
“High Performance Computing”

- “~~Big data~~ problem?”
- “Large computational problem?”

“Large computational problem”

- Need to choose algorithm —> necessary number of floating point operations (N = FLOP)
- Hardware/software performance (P = FLOPS)
- How long does it take? (S = N/P)

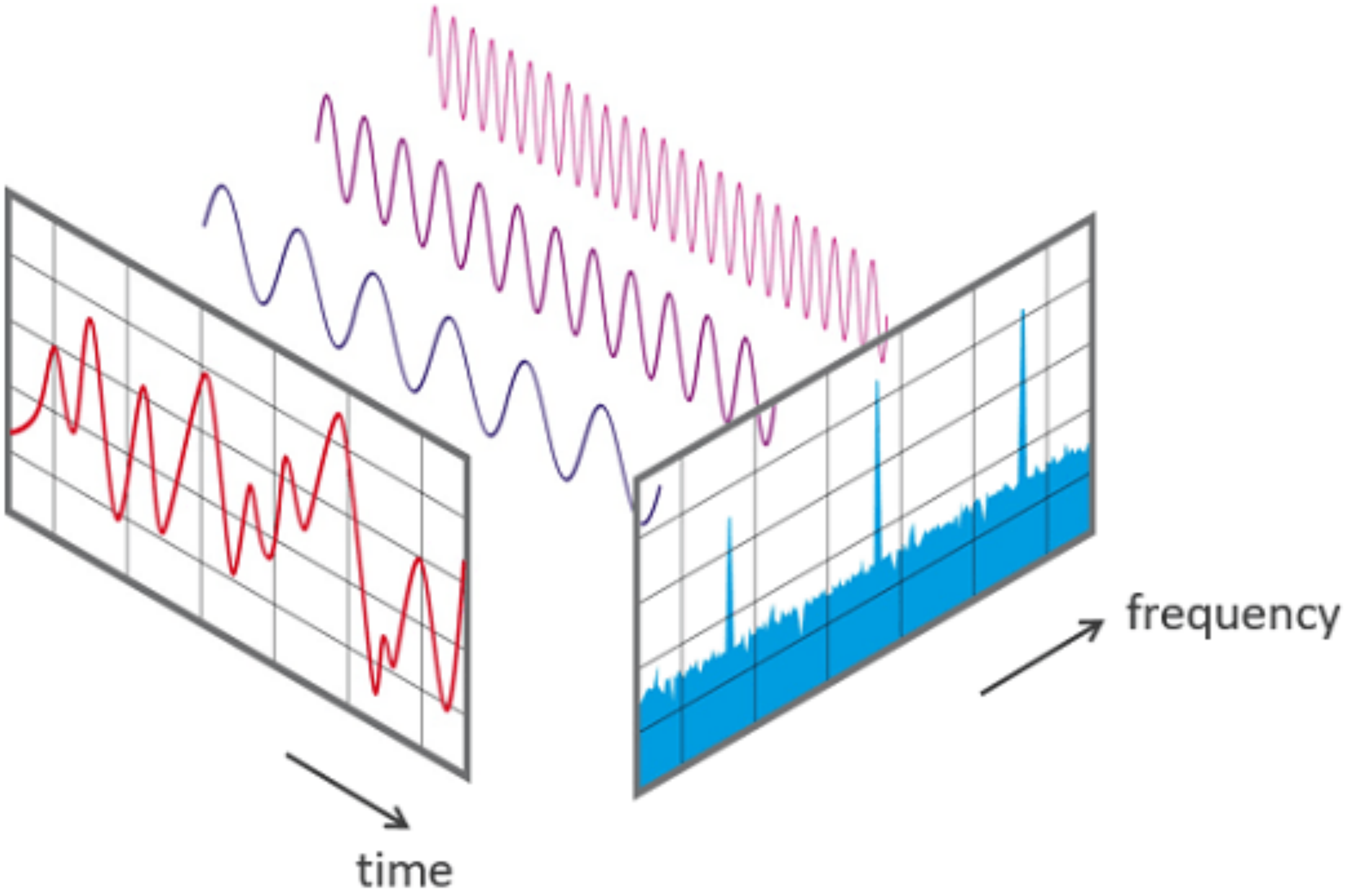
computational speed of digital computer



Shor's Algorithm

- RSA algorithm —> integer factoring problem
- change integer factoring problem into order-finding problem using a digital computer
- solve order-finding problem using a Quantum Computer
- P. Shor, proceedings of 35th annual symposium on the foundations of computer science, 1994, p124-134

Fourier transform



https://dev.to/trekhleb/playing-with-discrete-fourier-transform-algorithm-in-javascript-53n5

For digital computer,

the best Fourier transform algorithm is Cooley–Tukey algorithm and the number of operations is

$$\sim N \times \log_2 N, \quad N = 2^n$$

Cooley–Tukey for the Discrete Fourier Transform: sorting algorithm

$$F(k) = \frac{1}{2\pi} \int dx\, f(x)\, e^{-ikx}$$

$$\rightarrow \quad F_k = \sum_{m=0}^{N-1} f_m\, e^{-\frac{2\pi i}{N} mk} \quad , \; N = 2^n$$

Computational speed of Quantum Computer

Probability for 0 is 50%, and for 1 is 50%

$$\frac{1}{\sqrt{2}}(|0> + |1>)$$

wire 1

Computational speed of Quantum Computer

Bell state: Probability for 00 is 50%, and for 11 is 50 %

$$\frac{1}{\sqrt{2}}(|00> + |11>)$$

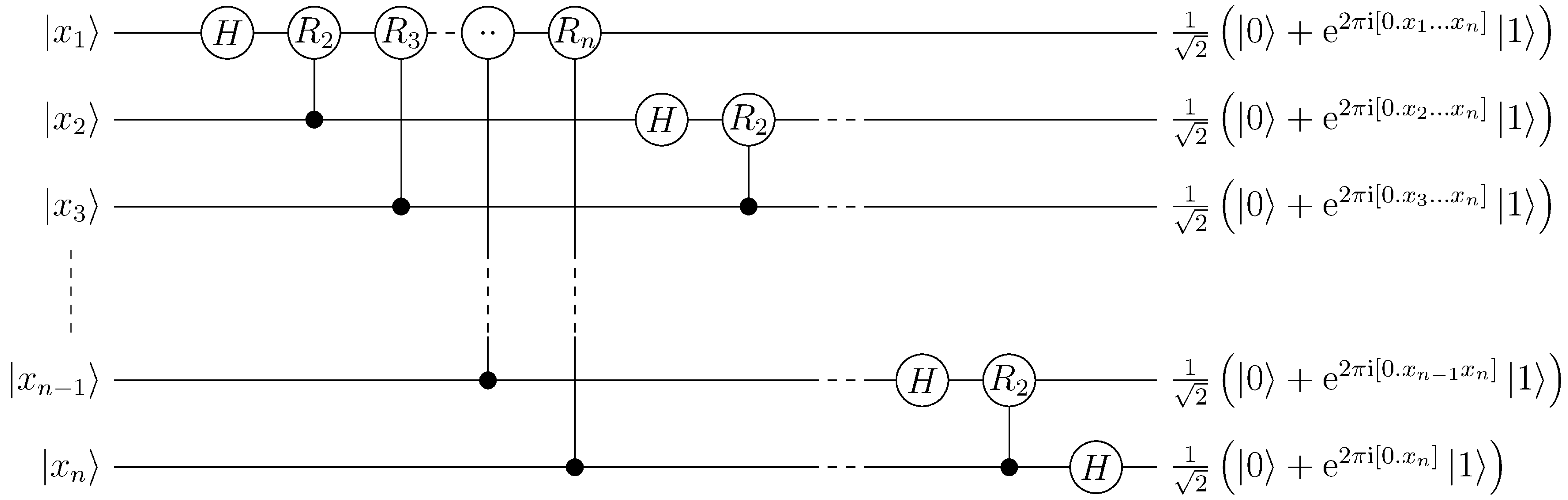
wire 1

wire 2

For Quantum computer,

the number of operations for Fourier transform is

$$\sim n^2$$



https://en.wikipedia.org/wiki/Quantum_Fourier_transform

Fourier transform of the same signal

requires

$\sim 2^n \times \log_2 2^n$ number of operations

for digital computer

$\sim n^2$ number of operations

for quantum computer

Qubits?

quantum error, decoherence, ⋯

Quantum Computing in "noisy environment"

- Fighting quantum decoherence with entanglement
- Quantum Error Correction (QEC)

cf. B.M Terhal, Rev. Mod. Phys. 87 (2015) 307

Digital computing in "noisy environment"

- Low density parity check code
- Hamming code
- "duplicate data–bits" so that the correct data can be reconstructed

Repetition code

- “11111111” = 1_L ,

  “00000000” = 0_L

  Is “10001111” 1_L or 0_L ?

Simple parity check

- Parity of “00” = 0, “11” = 0, “01” = 1, “10” = 1
- Form 3−bit data, “pdd”

Before discussing my work,

M. Rispler, D. Vodola, SK, M. Muller, "Fundamental Thresholds of Realistic Quantum Error Correction Circuits from Classical Spin Models", Quantum 6 (2022), 618 cf. J. Kelly et al, “State preservation by repetitive error detection in a superconducting quantum circuit”, Nature 519 (2015) 66

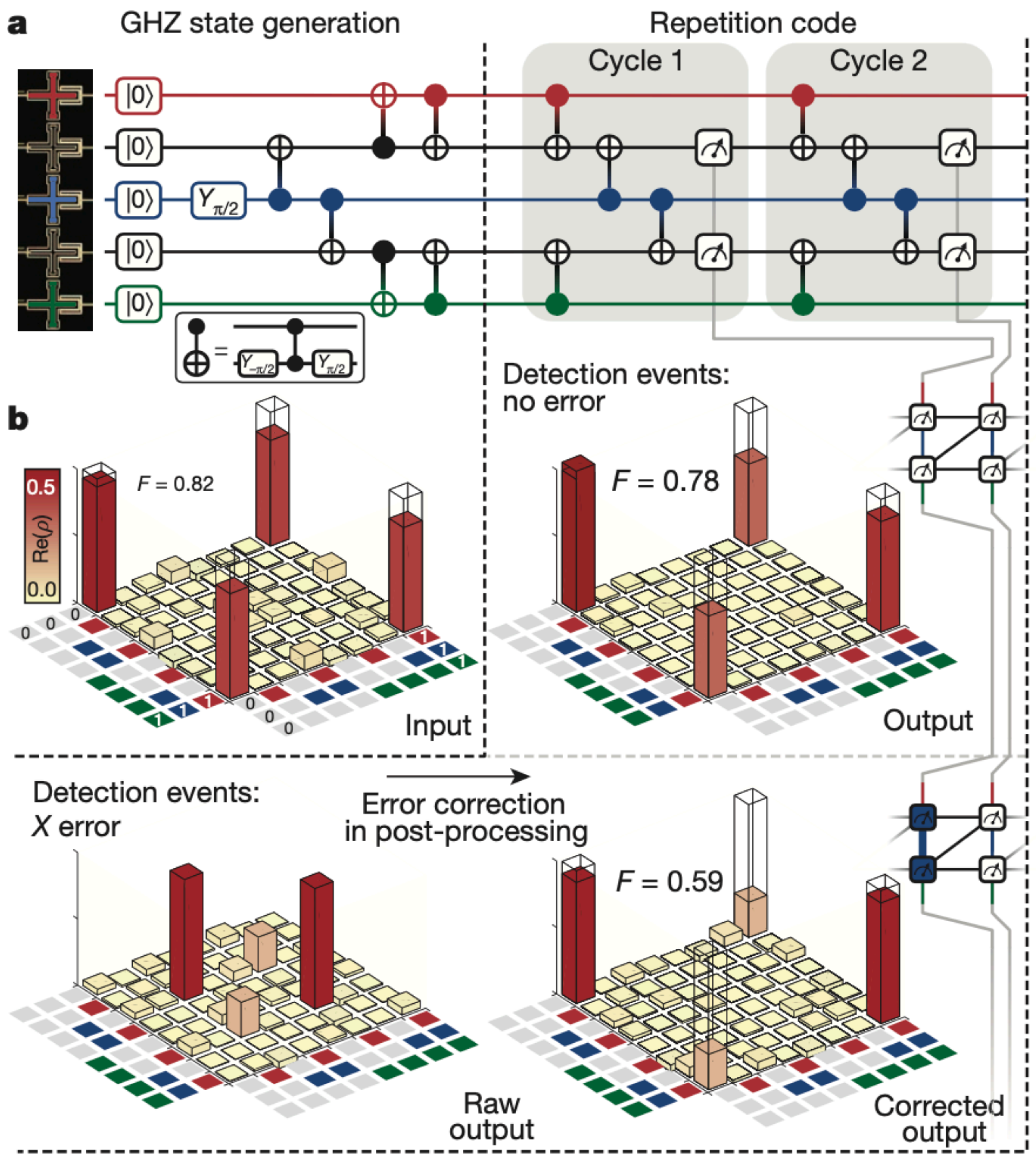


Figure 3 | Protecting the GHZ state from bit-flip errors. **a,** Quantum circuit for generating the GHZ state and two cycles of the repetition code. CNOT gates are physically implemented with controlled-phase (CZ) and single qubit gates. **b,** Quantum state tomography on the input (top left ‘Input’, left of black dashed line), and after the repetition code conditional on the detection events (between black dashed lines): we input a GHZ state with a fidelity (*F*) of 82%, and find, for the case of no detection events (top right ‘Output’, above grey dashed line), a 78% fidelity GHZ state. For the detection event connecting both measurement qubits (bottom left ‘Raw output’, below grey dashed line), indicating a likely bit-flip error on the central data qubit, we find that through

Quantum error and statistical model

- Specific quantum code
- Modeling quantum error pattern
- Mapping Q−error pattern to statistical model
- cf. simplest case: Dennis et al, J. Math. Phys. 43 (2002) 4452

Dennis et al, "Topological Quantum Memory"
J. Math. Phys. 43 (2002) 4452

- Toric Code
- Random 1 qubit error / measurement error
- 2-d quenched Ising Model or 3-d quenched lattice Z(2) gauge theory

Dennis et al,
Toric code

- Information on the data qubits is determined only after "measurement"
- After measurement, entanglement is lost
- Data qubits, Ancilla qubits, and Logical qubits on 2d lattice
- Stabilizer formalism and syndrome measurement

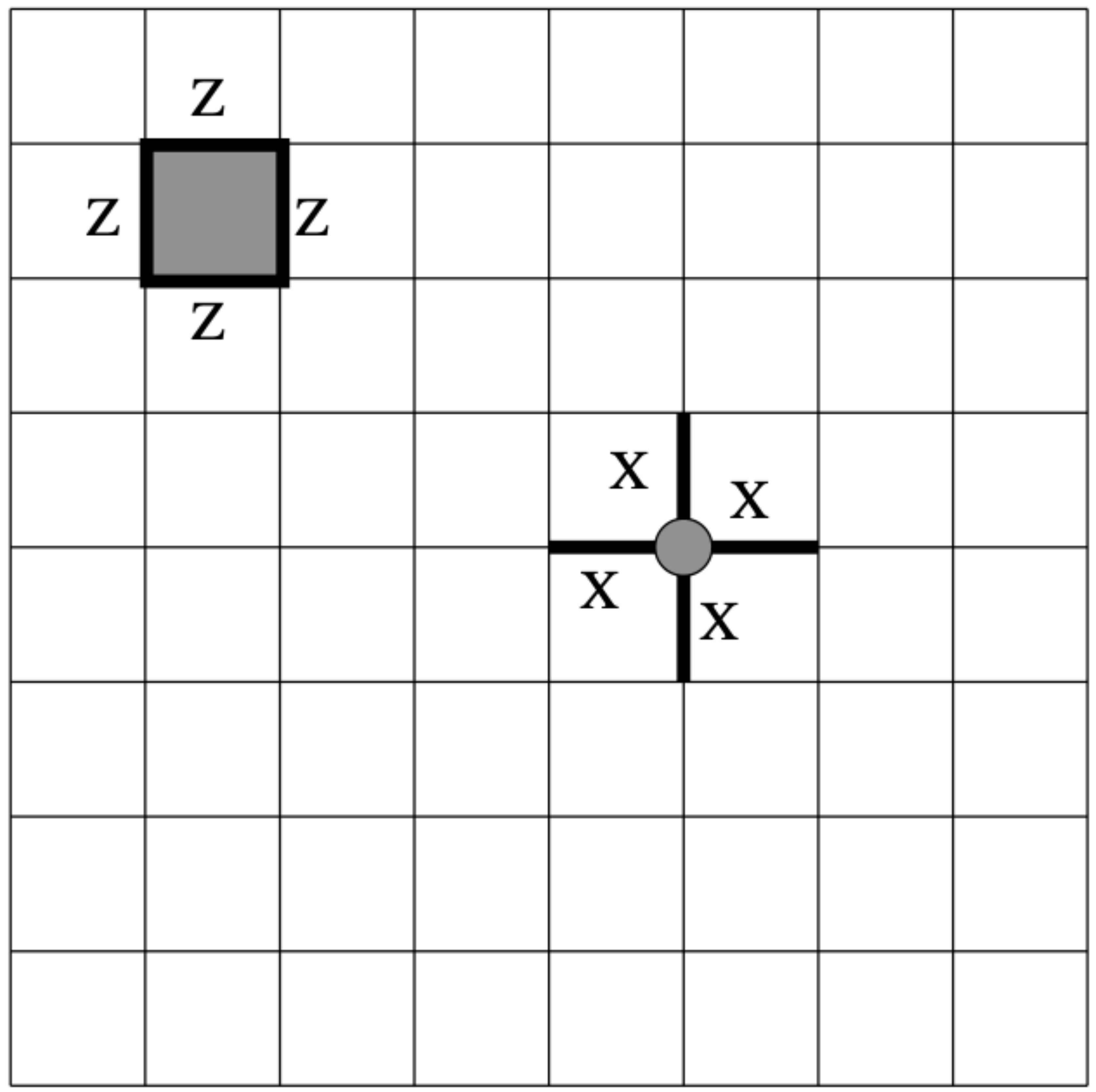


FIG. 1. Check operators of the toric code. Each plaquette operator is a tensor product of Z's acting on the four links contained in the plaquette. Each site operator is a tensor product of X's acting on the four links that meet at the site.

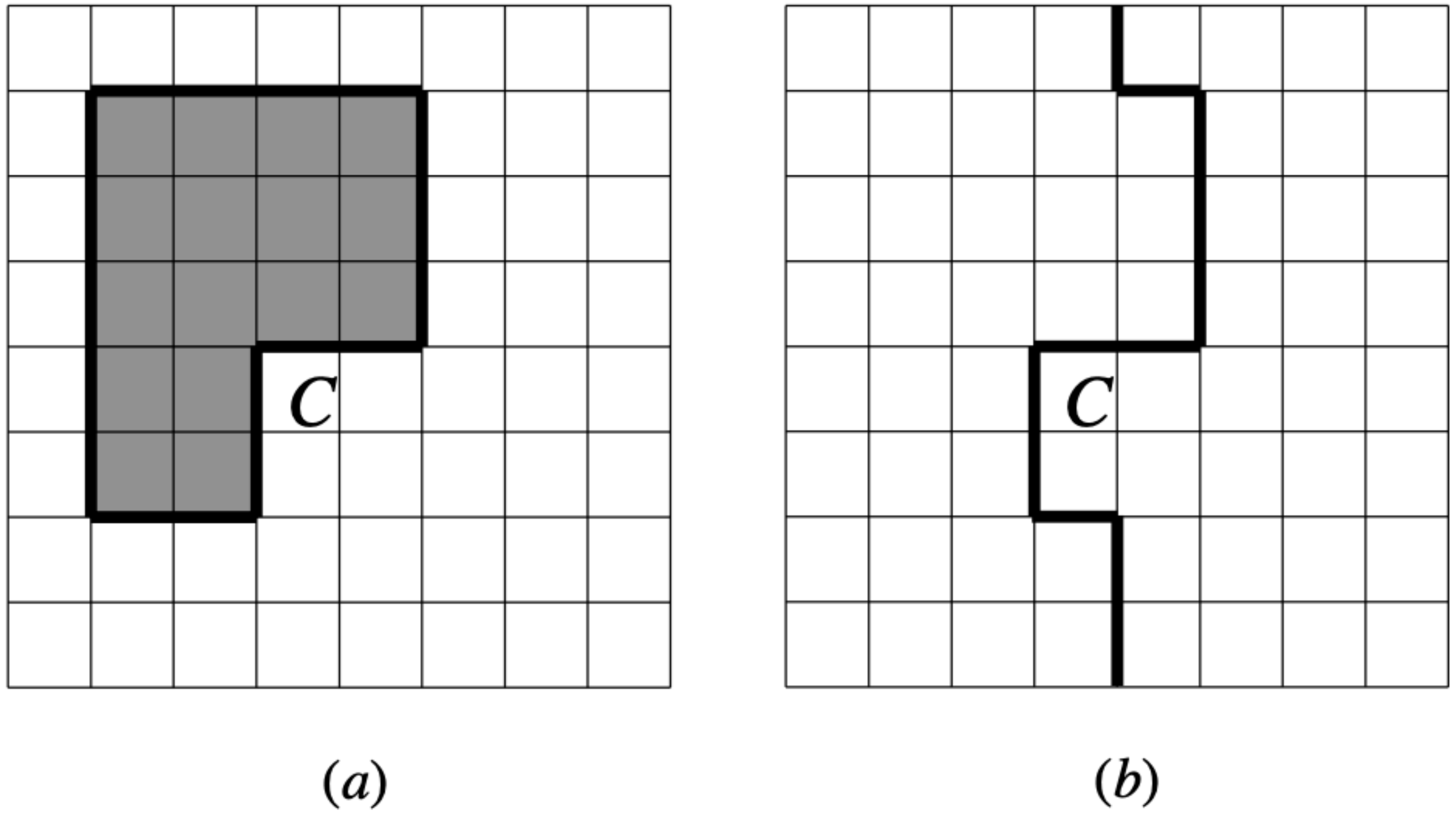


FIG. 2. Cycles on the lattice. (a) A homologically trivial cycle bounds a region that can be tiled by plaquettes. The corresponding tensor product of Z's lies in the stabilizer of the toric code. (b) A homologically nontrivial cycle is not a boundary. The corresponding tensor product of Z's commutes with the stabilizer but is not contained in it. It is a logical operation that acts nontrivially in the code subspace.

Dennis et al,
Quantum Error Model

- Random single data qubit error
- With/without a random measurement error
- Syndrome from cross-type stabilizers and plaquette-type stabilizers

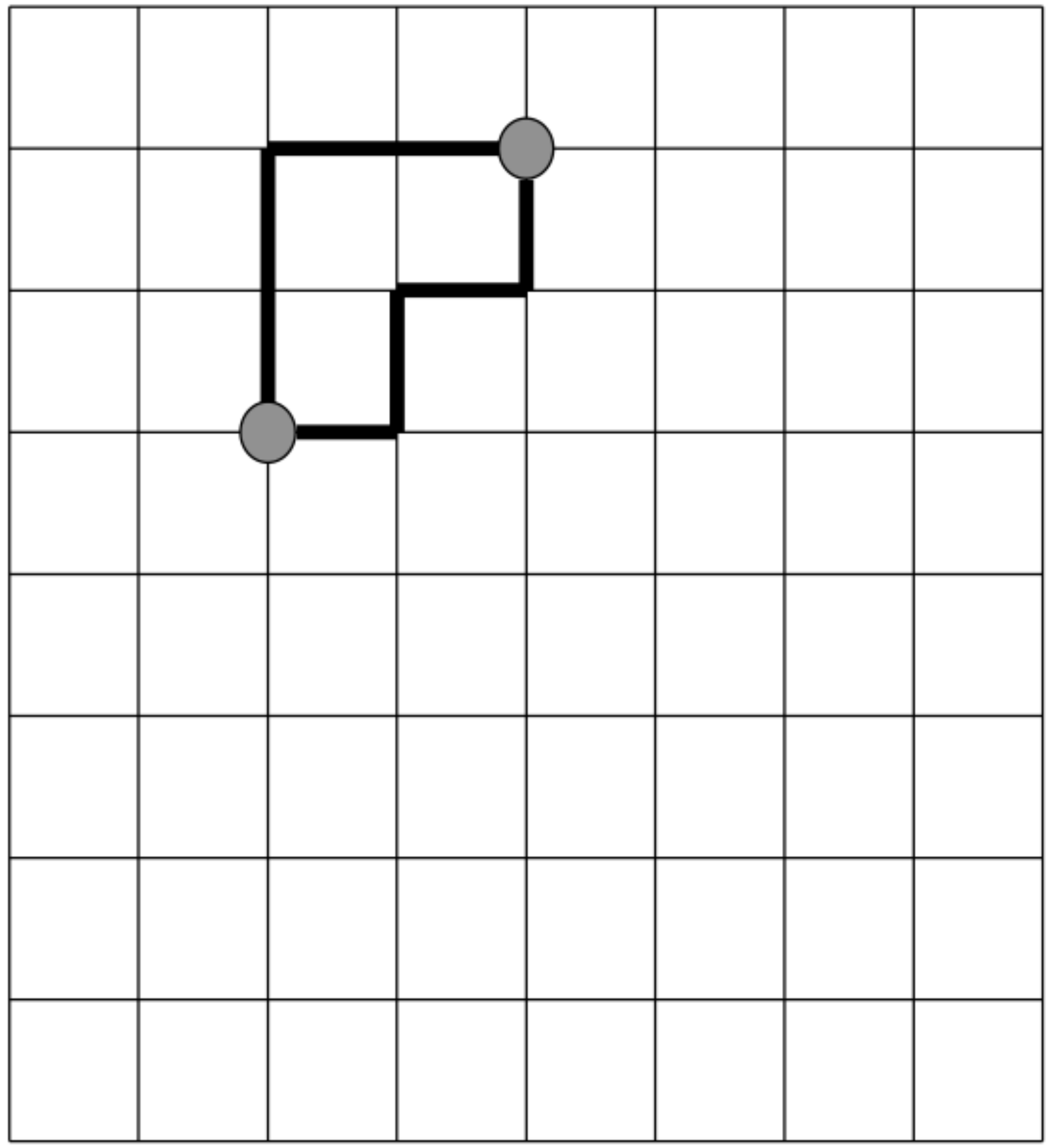


FIG. 4. The highly ambiguous syndrome of the toric code. The two site defects shown could arise from errors on either one of the two chains shown. In general, error chains with the same boundary generate the same syndrome, and error chains that are homologically equivalent act on the code space in the same way.

Dennis et al,
Statistical Model I

- Random data qubit error gives syndrome
- Given syndrome, deducing real error pattern should be done probabilistically
- Random bond Ising Model on 2-d lattice (without measurement error) or quenched Z(2) plaquette model on 3-d lattice (with measurement error)
- Quantum error is associated with anti-ferromagnetic coupling between Ising spin

Dennis et al,
Statistical Model II

- Ferromagnetic phase (ordered phase) —〉 Quantum Error Correction (QEC) is possible because quantum error pattern can be deduced from syndrome
- Disordered phase —〉 QEC is not possible

Dennis et al,
Nishimori Condition

- Data qubit error

  —> anti−ferromagnetic coupling
- Random distribution of anti−ferromagnetic coupling

  —> quenching
- "Thermal transition" and "Spin glass transition"
- Nishimori condition (H. Nishimori, PTP 66 (1981) 1169) is the line,

  "Thermal transition = Spin glass transition"

Dennis et al,
Statistical Model III

- There exists "threshold probability" above which "magnetization" disappears
- This probability is crucial for quantum computing with QEC because QC should have error rate lower than this

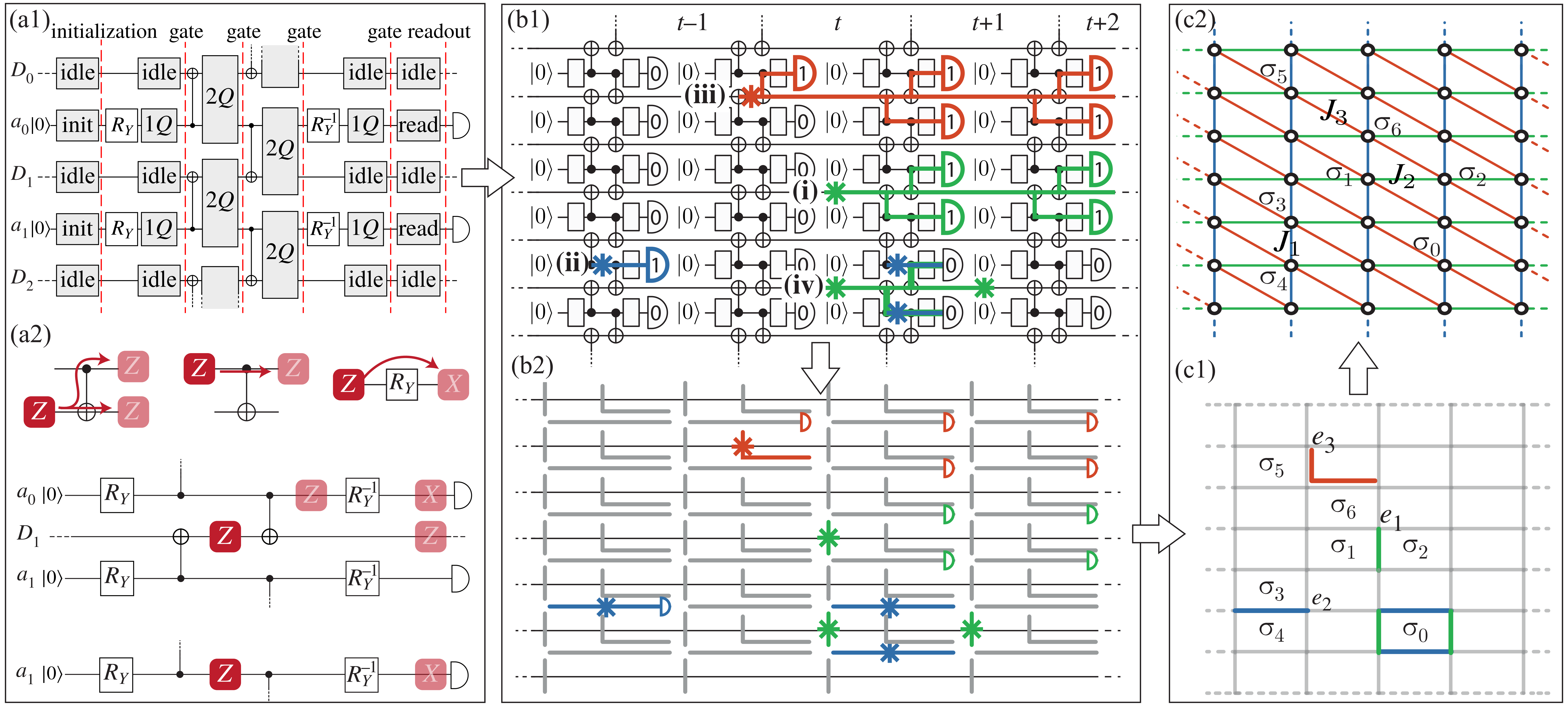
M. Rispler, D. Vodola, SK, M. Muller, “Fundamental Thresholds of Realistic Quantum Error Correction Circuits from Classical Spin Models”, Quantum 6 (2022), 618

1-d repetition code and correlated error

- Protecting against phase flip error (Z-error)
- Realistic quantum circuit which implements the algorithm
- Analysis of the correlated quantum error from 1-qubit error, 1-qubit gate error, 2-qubit gate error and etc
- Mapping into 2-d random bond Ising model on triangular lattice

Scheme

- realistic quantum circuit diagram for 1-D repetition code with phase flip error and mapping to a statistical model (quenched 2-D Ising model on a triangular lattice)



1-d repetition code
Quantum Code

- Protecting against phase flip error (Pauli Z-error)
- 1-d array of data qubit and ancilla qubit

1-d repetition code

Error Pattern

- Realistic quantum circuits which implements the algorithm
- 1-qubit error：data initialization error, phase-flip error, idling error, measurement error, 1-qubit gate error
- 2-quibit error：CNOT gate error
- Correlated error

1-d repetition code

Statistical Model

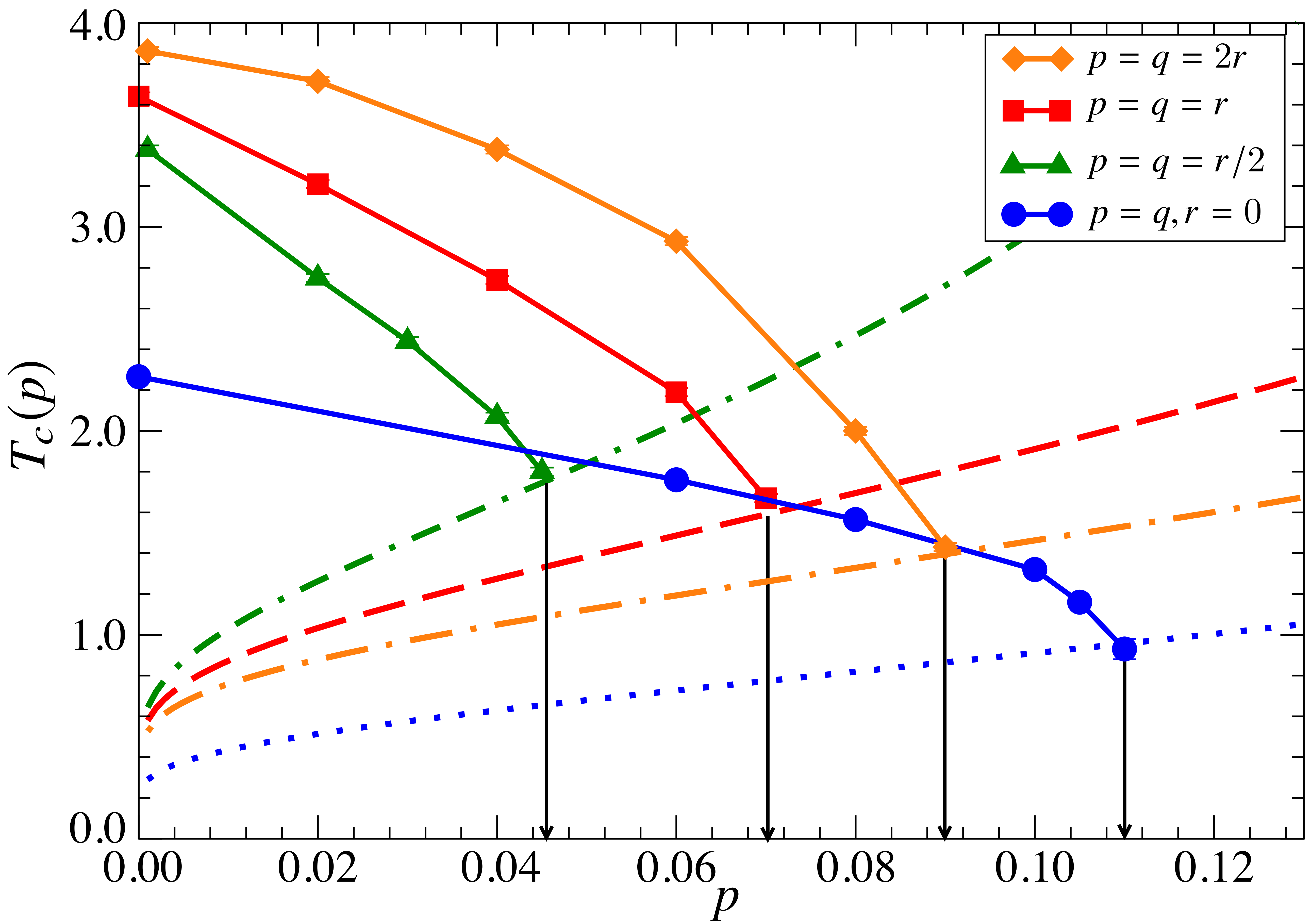
- Analysis of the correlated quantum error from various 1-qubit error, 1-qubit gate error, 2-qubit gate error
- Mapped into 2-d random bond Ising model on triangular lattice

Monte Carlo simulation of statistical physics model

- Standard Metropolis algorithm for Monte Carlo
- Parallel tempering
- Divergent correlation length near the critical point and finite size scaling of the correlation length

Monte Carlo result

- phase diagram of quenched 2-D Ising model corresponding to effective quantum error model

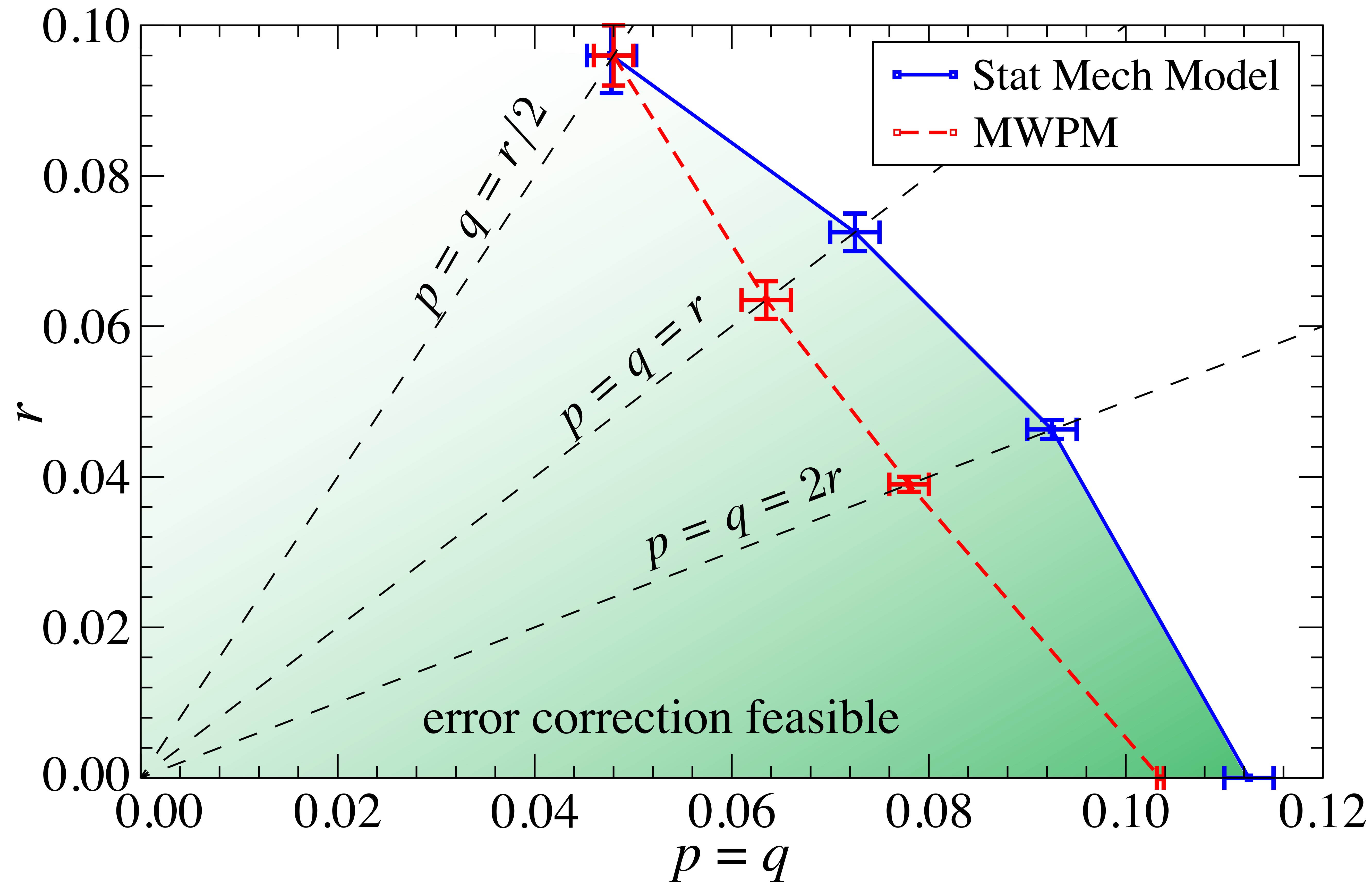


Minimum−Weight Perfect Matching

- Given syndrome, find the shortest distance between the syndrome sites
- QEC algorithm example

Comparison between a QEC algorithm (MWPM) result and the Monte Carlo result

- threshold probability from Monte Carlo study and that from MWPM



Outlook

- similar technique can be applied to surface code, color code, and concatenated code, and etc
- for more complex quantum circuits, there may be more complicated types of correlated effective noise processes and more sophisticated statistical mechanics model may be needed
- non-Clifford error dynamics
- ultimately, a target threshold probability for which a real quantum computer can aim may be suggested