Mesons on the light front

$e^- + Au \to J/\psi + X$,

$e^+ + e^- \to e^+ + e^- + \eta_c$

$\Downarrow$

mesons

Lattice QCD, DSE $\qquad$ LFH

QCD

Lagrangian	Hamiltonian
Euclidean	Minkowski

- Hamiltonian formalism

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iEt}|\psi(0)\rangle$$

$$\boxed{H|\psi(0)\rangle = E|\psi(0)\rangle}$$

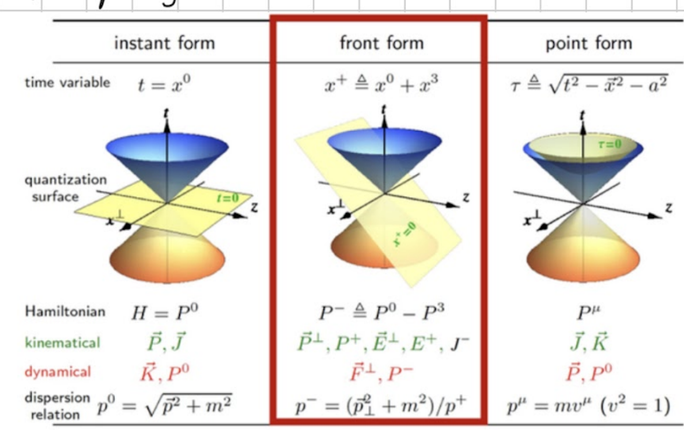
$\hookrightarrow |\psi\rangle, E$

$\langle O\rangle = \langle\psi_h|\hat{O}|\psi_h\rangle$

t. "time"

H. conjugate to t

- forms of dynamics



Dirac's forms of relativistic dynamics, Rev. Mod. Phys. 21, 392 1949

§ I. Canonical quantization of QCD Hamiltonian on the LF

$\mathcal{L}_{QCD} \to P^-_{QCD}$

conventions

$g^{-+} = g^{+-} = 2, \quad g_{+-} = g_{-+} = \frac{1}{2}, \quad g^{ii} = g_{ii} = -1$

$x^{\pm} = x^0 \pm x^3$

$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu} + \boxed{\bar{\psi}(i\gamma^\mu D_\mu - m)\psi}$

$\frac{1}{2}[\bar{\psi}(i\gamma^\mu D_\mu - m)\psi + \bar{\psi}(-i\gamma^\mu \overleftarrow{D}_\mu - m)\psi]$

$F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a - g f^{abc} A^\mu_b A^\nu_c$

$a = 1, 2, \cdots 8$

$D_\mu = \partial_\mu I_3 + i g A^\mu$

$A^\mu = A^\mu_a T^a$

ψ_c, $c = 1, 2, 3$

$\bar{\psi}_c [A^\mu_a T^a]_{cc'} \psi_c$

$\mathcal{L} = \mathcal{L}[\phi_r, \partial_\mu \phi_r]$

$A^\mu, \psi, \bar{\psi}$

$$\boxed{\begin{aligned} \pi^k_r &= \frac{\delta \mathcal{L}}{\delta(\partial_k \phi_r)} \\ \partial_k \pi^k_r &- \delta\mathcal{L}/\delta\phi_r = 0 \quad (\star) \end{aligned}}$$

① A^μ as ϕ_r

$\pi^\lambda_{A^s_k} = -F^{\lambda k}_s$

$\frac{\delta \mathcal{L}}{\delta A^s_k} = -\frac{1}{4} F^{k\mu}_a (-g f^{asc} A^a_\mu) \times 4 + \bar{\psi}(i\gamma^k(igT^s))\psi$

$= -g f^{sac} F^{k\mu}_a A^c_\mu - g\bar{\psi}\gamma^k T^s \psi$

E.O.M $(\star) \Rightarrow$ $\boxed{\partial_\lambda F^{\lambda k}_s = g J^k_s}$ (Eq. c-Maxwell)

$J^k_s \equiv f^{sac} F^{k\mu}_a A^c_\mu + \bar{\psi}\gamma^k T^s \psi$

LC gauge $A^+_a = 0$

$\nu = +$

$$g J^+_a = \partial_\lambda F^{\lambda +}_a = -\partial^+\partial_- \underset{\sim}{A^-_a} - \partial^+\partial_i A^i_a$$

no x^+

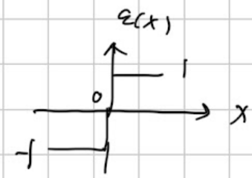
$$\frac{1}{2} A^-_a = -g \frac{1}{(\partial^+)^2} J^+_a - \frac{1}{\partial^+}\partial_i A^i_a$$

$\frac{1}{\partial^+}$

$$\frac{1}{\partial^+} f(x^-) = \frac{1}{4}\int_{-\infty}^{+\infty} \underset{\sim}{\varepsilon}(x^- - y^-) f(y^-)$$

$$\partial^+ = \frac{\partial}{\partial x_+} = \frac{\partial}{\frac{1}{2}\partial x^-} = 2\frac{\partial}{\partial x^-} = 2\partial_-$$

$$\frac{1}{i\partial^+} e^{-ikx} = \frac{1}{k^+} e^{-ikx}$$



$$\tilde{A}^\mu = \lim_{g\to 0} A^\mu \quad , \quad \text{"free" field}$$

$$\tilde{A}^\mu = (0, \tilde{A}^-_a, A^i_a)$$

$$\frac{1}{2}\tilde{A}^-_a = -\frac{1}{\partial^+}\partial_i A^i_a$$

② ψ_a as ϕ_r

$$\Pi^\lambda_\psi = \frac{i}{2}\bar{\psi}\gamma^\lambda$$

$$\frac{\delta \mathcal{L}}{\partial \psi} = -g\bar{\psi}\gamma^\mu A_\mu - m\bar{\psi} - \frac{i}{2}\bar{\psi}\gamma^\mu \overleftarrow{\partial}_\mu$$

E.O.M (★) ⇒ $$\boxed{\bar{\psi}\,[i\gamma^\mu(\overleftarrow{\partial}_\mu - igA_\mu) + m] = 0}$$ (Eq. c-Dirac†)

$\uparrow$

$\bar{\psi} = \psi^\dagger\gamma^0$

$\downarrow$ adjoint

$$[-i\gamma^{\mu\dagger}(\partial_\mu + igA_\mu) + m]\gamma^0\psi = 0 \qquad \gamma^0\gamma^\mu = \gamma^{\mu\dagger}\gamma^0$$

$$\Rightarrow \boxed{[i\gamma^\mu(\partial_\mu + igA_\mu) - m]\psi = 0}$$ (Eq. c-Dirac)

$\psi_\pm = \Lambda^\pm \psi$ $+ \to G$ ψ_G

$\underset{\sim}{\Lambda^\pm} \equiv \frac{1}{2}\gamma^0\gamma^\pm$ $- \to B$ ψ_B

$1 = \Lambda^+ + \Lambda^-$

$(\Lambda^\pm)^2 = \Lambda^\pm$ $\gamma^0 \times$

$\Lambda^+\Lambda^- = \Lambda^-\Lambda^+ = 0$

$\Lambda^+\Lambda^- = \Lambda^-\Lambda^+ = 0$

$$[i(\gamma^0\gamma^+ D_+ + \gamma^0\gamma^- D_- + \alpha^i D_i) - m\beta]\psi = 0 \qquad \beta \equiv \gamma^0, \quad \alpha^i \equiv \gamma^0\gamma^i$$

$$\Rightarrow [i(2\Lambda^+ D_+ + 2\Lambda^- D_- + \alpha^i D_i) - m\beta]\psi = 0$$

$$\Big\downarrow \Lambda^\pm \times$$

$$[i(2D_\pm \Lambda^\pm + \alpha^i D_i \Lambda^\mp) - m\beta\Lambda^\mp]\psi = 0$$

$$\begin{cases} 2i\partial_+\psi_+ = (-i\alpha^i D_i + m\beta)\psi_- + 2gA_+\psi_+ & \leftarrow \quad 1^\circ \\ 2i\partial_-\psi_- = (-i\alpha^i D_i + m\beta)\psi_+ + \underline{2gA_-\psi_-} & \quad 2^\circ \end{cases}$$

$A_- = \frac{1}{2}A^+ = 0$ LC gauge

$$2^\circ \Rightarrow \psi_- = \frac{1}{2i\partial_-}(m\beta - i\alpha^i D_i)\underset{\text{不}}{\psi_+} \qquad 3^\circ$$

3° in 1°

$\Downarrow$

$$2iD_+\psi_+ = (m\beta - i\alpha^i D_i)\frac{1}{2i\partial_-}(m\beta - i\alpha^i D_i)\psi_+$$

$\tilde\psi = \lim_{g\to 0}\psi$ "free" field

$\tilde\psi = \tilde\psi_+ + \tilde\psi_-$

$\tilde\psi_+ = \psi_+$

$$\tilde\psi_- = \frac{1}{2i\partial_-}(m\beta - i\alpha^i\partial_i)\psi_+$$

③ $\tilde\psi$ E.O.M. (☆) ⇒ (Eq. c-Dirac)

$$\underbrace{\frac{1}{2}P^-}_{\equiv\, P_+} x^+$$

$$\mathcal{P}_+ = (\partial_+ A^s_k)\Pi^+_{A^s_k} + (\partial_+\psi)\Pi^+_{\psi} + (\partial_+\bar\psi)\Pi^+_{\bar\psi} - \mathcal{L}$$

$$= \underbrace{-F^{+k}_s\partial_+ A^s_k + \frac{1}{2}[i\bar\psi\gamma^+\partial_+\psi + h.c.] + \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}}$$

$$P^- = 2\int dx_+ d^2x_\perp \mathcal{P}_+ = \int dx^- d^2x_\perp \mathcal{P}_+$$

$$P^- = \int dx_+ d^2x_\perp \mathcal{P}^-$$

$$-\partial_k(F^{k+}_s A^s_+)$$

$$P^- = \int dx^- d^2x_\perp\ -F^{+k}_s\partial_+ A^s_k + \frac{1}{2}[i\bar\psi\gamma^+\partial_+\psi + h.c.] \qquad (Eq.\ P^-)$$

$$+\frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu} - \partial_k(F^{k+}_s A^s_+)$$

① ②

$$\sim\ = -F^{k+}_s F^s_{k+} - g\bar\psi\gamma^+ T^s A^s_+\psi$$

$$\frac{1}{2}[i\bar\psi\gamma^+ D_+\psi + h.c.]$$

① $\frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu} - F^{\mu+}_a F^a_{\mu+}$

$$= \frac{1}{4}(F^{ij}_a F^a_{ij} + F^{\mu+}_a F^a_{\mu+} + F^{+\nu}_a F^a_{+\nu} + F^{\mu-}_a F^a_{\mu-} + F^{-\nu}_a F^a_{-\nu} - F^{+-}_a F^a_{+-} - F^{-+}_a F^a_{-+}) - F^{\mu+}_a F^a_{\mu+}$$

$$F^{\mu-}_a = g^{\mu\nu}F^a_{\nu+}g^{+-}$$

$$= \frac{1}{4}\underbrace{F^{ij}_a F^a_{ij}}_{(1.1)} - \frac{1}{2}\underbrace{F^{+-}_a F^a_{+-}}_{(1.2)}$$

$$(1.1) = 2A^i_a\nabla_\perp^2 A^a_j - 2(\partial_j A^j_a\partial^i A^a_i) - 4g f^{abc}\partial^i A^j_a A^b_i A^c_j + g^2 f^{abc}A^i_b A^j_c f^{aef}A^e_i A^f_j$$

$A^+=0$

$$(1.2) = F^{+-}_a F^a_{+-} = -(\partial^+ A^-_a)(\partial_- A^a_+)$$

$$= -\frac{1}{4}\partial^+ A^-_a\partial^+ A^-_a$$

$$= g^2 J^+_a\frac{1}{(\partial^+)^2}J^+_a - (\partial_i A^i_a)^2$$

$$-gJ^+_a\tilde{A}^-_a$$

② $i\bar{\psi}\gamma^+ D_+\psi = i\psi^\dagger\underbrace{\gamma^0\gamma^+}_{2\Lambda^+ \overset{\text{u}}{=} (\Lambda^+)^2} D_+\psi = 2i\underset{\sim}{\psi_+^\dagger} D_+ \underset{\sim}{\psi_+}$

$\Rightarrow 2i\psi_+^\dagger D_+\psi_+$

2,1 $= \boxed{\psi_+^\dagger(m\beta - i\alpha^i\partial_i)\frac{1}{2i\partial_-}(m\beta - i\alpha^i\partial_i)\psi_+}$

2,2 $+ \boxed{g^2\psi_+^\dagger\alpha^i A_i\frac{1}{2i\partial_-}\alpha^i A_i\psi_+} + \boxed{g\psi_+^\dagger\alpha^i A_i\tilde{\psi}_-}$

$+ \boxed{g\tilde{\psi}_-^\dagger\alpha^i A_i\psi_+}$ 2,3

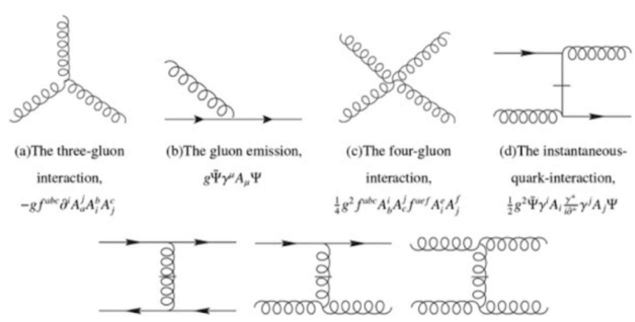
2.1 $\Rightarrow \frac{1}{2}\bar{\tilde{\psi}}\gamma^+\frac{m^2-\nabla_\perp^2}{2i\partial_-}\tilde{\psi}$

2,2 $\Rightarrow \frac{g^2}{2}\bar{\tilde{\psi}}\gamma^i A_i\frac{\gamma^+}{2i\partial_-}\gamma^j A_j\tilde{\psi}$

2,3 $\Rightarrow g\bar{\tilde{\psi}}\gamma^i A_i\tilde{\psi}$

putting all pieces back to (Eq. P^-)

$$\Rightarrow \quad P^-_{QCD} = \int \mathrm{d}x^-\,\mathrm{d}^2x_\perp\; -\frac{1}{2}\tilde{A}^j_a(i\nabla_\perp)^2\tilde{A}^a_j + \frac{1}{2}\bar{\tilde{\Psi}}\gamma^+\frac{m^2-\nabla_\perp^2}{i\partial^+}\tilde{\Psi}$$
$$- g f^{abc}\partial^i\tilde{A}^j_a\tilde{A}^b_i\tilde{A}^c_j + g\tilde{J}^+_a\tilde{A}^a_+ + g\bar{\tilde{\Psi}}\gamma^i\tilde{A}_i\tilde{\Psi}$$
$$-\frac{1}{2}g^2\tilde{J}^+_a\frac{1}{(\partial^+)^2}\tilde{J}^+_a + \frac{g^2}{4}f^{abc}\tilde{A}^i_b\tilde{A}^j_c f^{aef}\tilde{A}^e_i\tilde{A}^f_j$$
$$+\frac{g^2}{2}\bar{\tilde{\Psi}}\gamma^i\tilde{A}_i\frac{\gamma^+}{i\partial^+}\gamma^j\tilde{A}_j\tilde{\Psi}\,.$$



(a)The three-gluon interaction, $-gf^{abc}\partial^i A^j_a A^b_i A^c_j$

(b)The gluon emission, $g\bar{\Psi}\gamma^\mu A_\mu\Psi$

(c)The four-gluon interaction, $\frac{1}{4}g^2 f^{abc}A^i_b A^j_c f^{aef}A^e_i A^f_j$

(d)The instantaneous-quark-interaction, $\frac{1}{2}g^2\bar{\Psi}\gamma^i A_i\frac{\gamma^+}{i\partial^+}\gamma^j A_j\Psi$

(e)The instantaneous-gluon-interaction, $-\frac{1}{2}g^2 J^+_a\frac{1}{(i\partial^+)^2}J^+_a$

mode expansions of A^μ, ψ (see notes)

§2. Mesons as $\bar{P}_{QCD}$ eigenstates

$$|\psi_h\rangle = c_0|q\bar{q}\rangle + c_1|q\bar{q}g\rangle + \cdots$$

$$(P^+\hat{P}^- - \vec{P}_\perp^{\,2})|\psi_h\rangle = M_h^2|\psi_h\rangle$$

single particle coordinates in Eq(46)

$$(l_i^+, \vec{l}_{i\perp}) \qquad \Sigma l_i^+ = P^+$$
$$\Sigma \vec{l}_{i\perp} = \vec{P}_\perp$$

relative

$$x_i \equiv \frac{l_i^+}{P^+}, \quad \vec{k}_{i\perp} = \vec{l}_{i\perp} - x\vec{P}_\perp$$

under Lorentz boosts

- $v^+ \to v^+$
  $\vec{v}_\perp \to \vec{v}_\perp + v^+ \underset{\uparrow\ \text{c-number}}{\vec{\beta}_\perp}$

- $v^+ \to \overset{\text{c-number}}{C_\omega}\, v^+$
  $\vec{v}_\perp \to \vec{v}_\perp$

$x_i \to \underline{x_i}$

$$\left\{\begin{array}{l} \vec{k}_{i\perp} \to \vec{k}_{i\perp} + \underset{\approx\, x_i P^+}{k_i^+} \vec{\beta}_\perp \\ \vec{P}_\perp \to \vec{P}_\perp + P^+ \vec{\beta}_\perp \end{array}\right.$$

$\vec{k}_{i\perp} \to \underline{\vec{k}_{i\perp}}$

$x_i \to \underline{x_i}$

$\vec{k}_{i\perp} \to \underline{\vec{k}_{i\perp}}$

invariant

III. Two studies of meson light-front wavefunctions

1. Basis Light-Front Quantization (BLFQ)
2. Small-basis Light-Front Wavefunction (sLFWF) by design

1. BLFQ

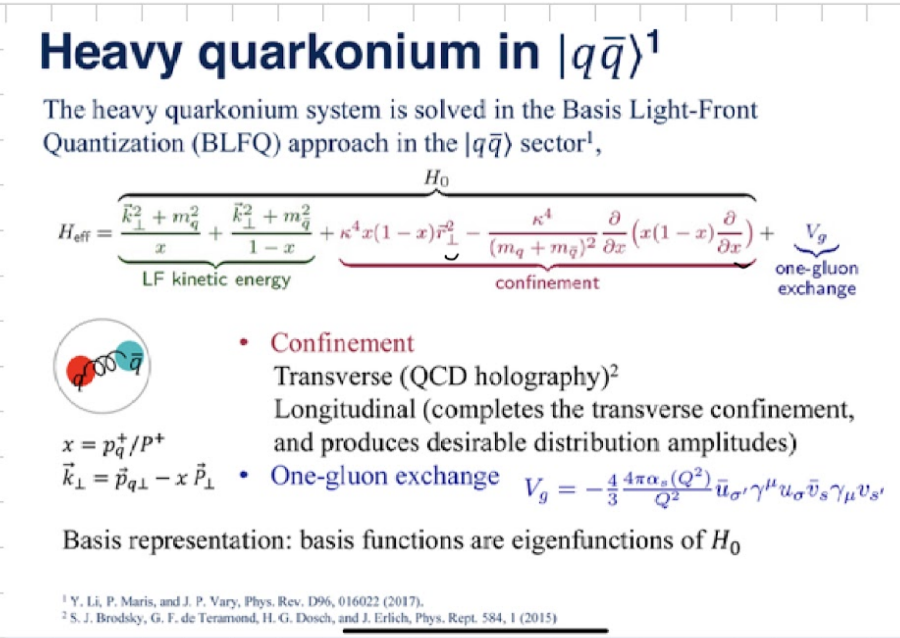
$$|\psi\rangle = \sum_i c_i \, |\beta_i\rangle$$

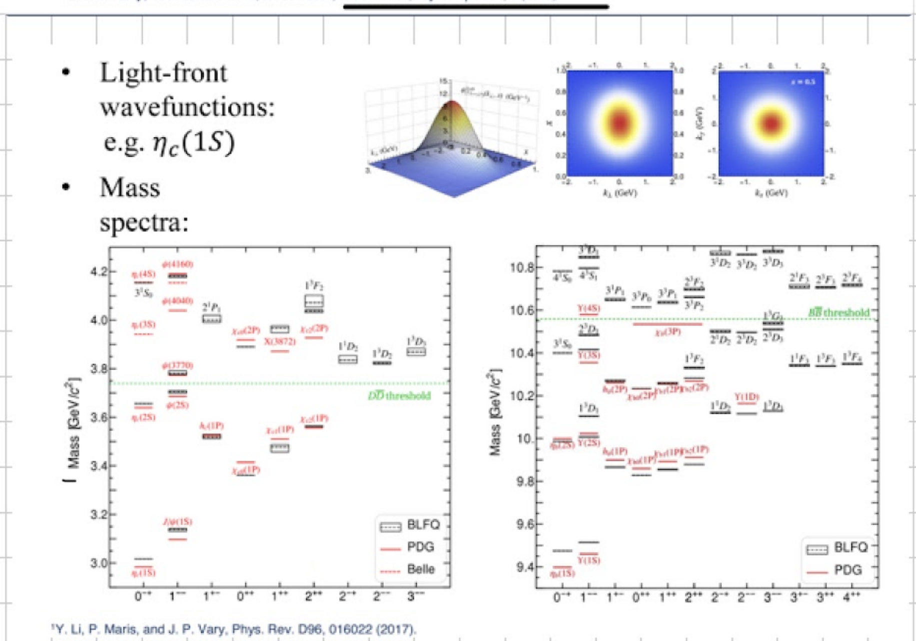
$$H|\psi\rangle = E|\psi\rangle$$

$$\begin{array}{cc} & |\beta_j\rangle \\ \langle\beta_i| & \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1n} \\ H_{21} & \ddots & & \\ \vdots & & \ddots & \\ H_{n1} & & & H_{nn} \end{bmatrix} \end{array} \xrightarrow{\text{diagonalization}} \begin{array}{cc} \lambda_1 & |\psi_1\rangle \\ \lambda_2 & |\psi_2\rangle \\ \vdots & \vdots \\ \lambda_n & |\psi_n\rangle \end{array}$$

$$H_{ij} \equiv \langle\beta_i|H|\beta_j\rangle$$

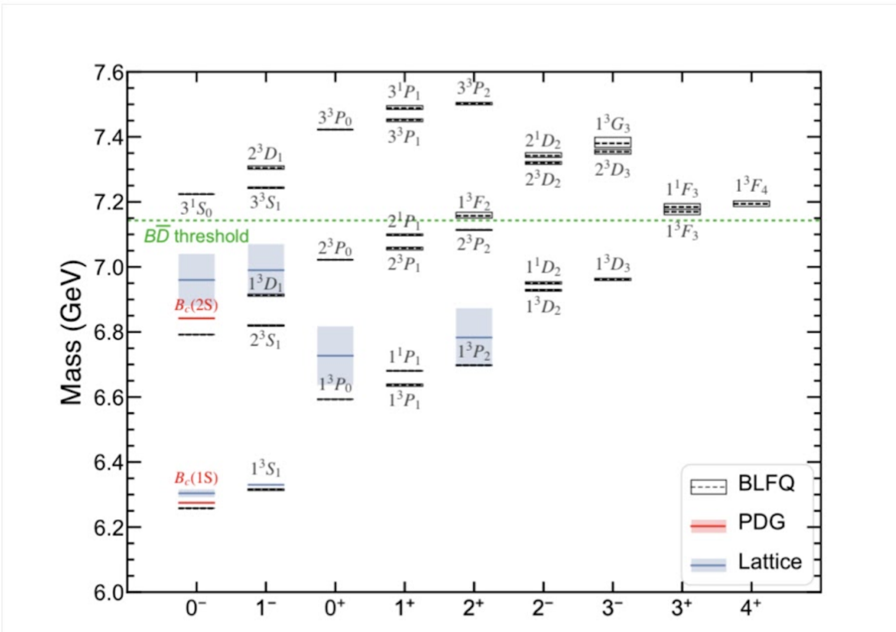
Heavy quarkonium, Y. Li, P. Maris, X. Zhao, and J. P. Vary, Phys. Lett. B758, 118 (2016); Y. Li, P. Maris, and J. P. Vary, Phys. Rev. D96, 016022 (2017).





B_c mesons,S. Tang, Y. Li, P. Maris, and J. P. Vary,
Phys.Rev.D 98 (2018) 11, 114038
Heavy-light mesons, S. Tang, Y. Li, P. Maris, and J. P. Vary, Eur. Phys. J. C80, 522 (2020)

D, D_s, B, B_s,...



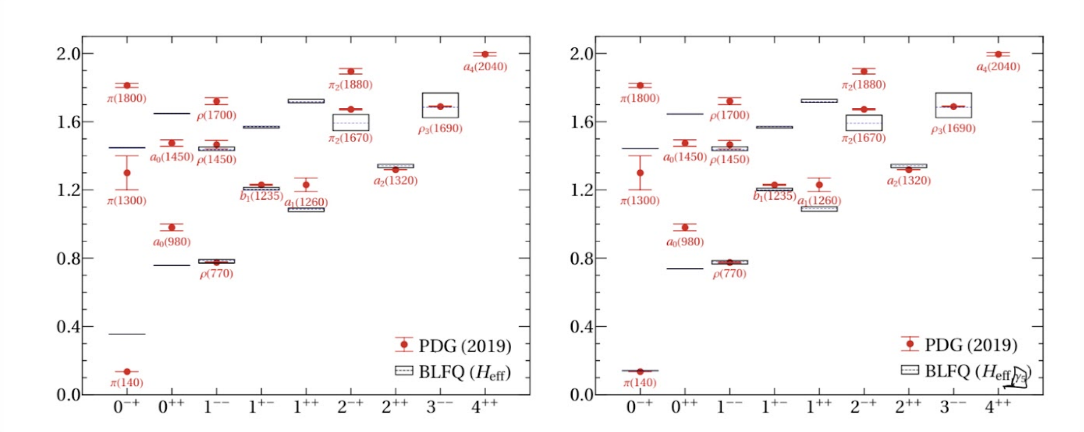
Light mesons, S. Jia, and J. P. Vary, *Phys.Rev.C* 99 (2019) 3, 035206; W. Qian, S. Jia, Y. Li, and J. P. Vary, Phys. Rev. C102, 055207 (2020)
(on quantum computers, W. Qian, R. Basili, S. Pal, G. Luecke, J. P. Vary, arXiv: 2112.01927)

pion, rho, ...

Light mesons, S. Jia, and J. P. Vary, *Phys.Rev.C* 99 (2019) 3, 035206; W. Qian, S. Jia, Y. Li, and J. P. Vary, Phys. Rev. C102, 055207 (2020)
(on quantum computers, W. Qian, R. Basili, S. Pal, G. Luecke, J. P. Vary, arXiv: 2112.01927)

pion, rho, ...

$$H_{\gamma_5} = \int dx^- \int d\mathbf{x}^\perp P^+ \lambda\ \bar{\psi}(x)\gamma^5\psi(x)\bar{\psi}(x)\gamma^5\psi(x)$$



Nucleon, C. Mondal,S. Xu, C. Mondal, J. Lan, X. Zhao, Y. Li, and J. P. Vary (BLFQ), Phys. Rev. D104, 094036 (2021)

proton, neutron

Light meson with one dynamical gluon, J. Lan, K. Fu, C. Mondal, X. Zhao, and j. P. Vary (BLFQ), Phys. Lett. B 825, 136890 (2022)

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

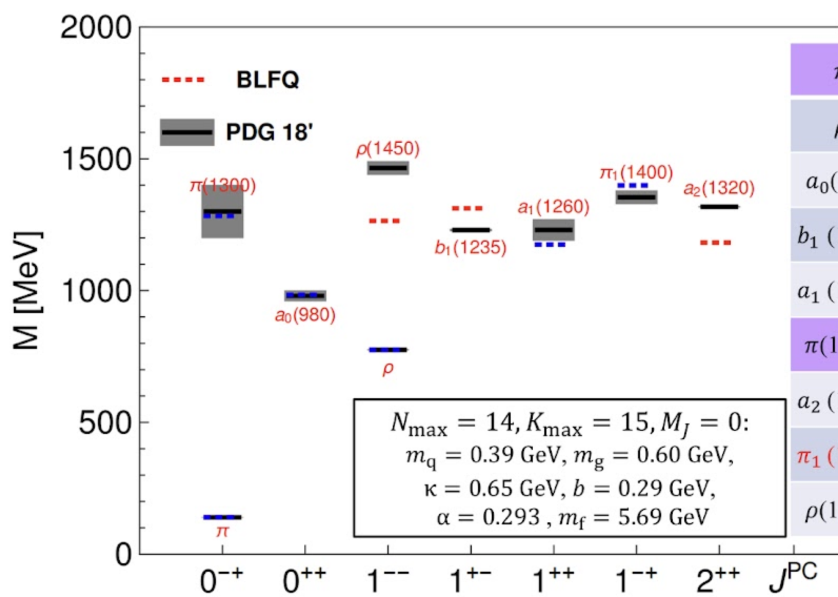
$H_{trans} \sim \kappa_T^4 r^2$ -- Brodsky, Teramond arXiv: 1203.4025

$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+ \frac{1}{(i\partial^+)^2} j^+$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Light Meson Mass Spectrum



	Norm$q\bar{q}$	DC [MeV]
π	0.492	151
ρ	0.486	141
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	13
$\pi(1300)$	0.284	59
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	50

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Fix the parameters by fitting six blue states
- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

[Lan, Fu, Mondal, Zhao, Vary, Phys. L[redacted]

2. Small-basis Light-Front Wavefunction (sLFWF) by design

M. Li, Y. Li, G. Chen, T. Lappi, and J. P. Vary, Eur. Phys. J. C 82, 1045 (2022), arXiv:2111.07087 [hep-ph]

I. Write the LFWF as a basis expansion

Consider a meson state h consisting of a quark (q) and an anti-quark $(\bar{q})$, with momentum $(P^+, \vec{P}_\perp)$, and expand its wavefunction on orthonormal basis $\{\beta_1, \beta_2, \dots, \beta_N\}$ as

$$\psi(\vec{k}_\perp, \text{x}) = \sum_{i=1}^{N} C_{h,i} \beta_i(\vec{k}_\perp, \text{x}; a, b, \dots)$$

$x = p_q^+/P^+$, the longitudinal momentum fraction of q

$\vec{k}_\perp = \vec{p}_{q,\perp} - x\vec{P}_\perp$, the relative transverse momentum between q and $\bar{q}$

Parameters to be determined:

- $C_{h,i}$, basis coefficients of h
- a,b, ..., parameters in the functional forms

Choose suitable basis functions:

- have a simple functional form
- retain physical interpretation of the system

II. Write conditions and observables in terms of parameters

① The wavefunctions satisfy the orthonormal relation as

$$\sum_{i=1}^{N} C_{h,i} C^*_{h',i} = \delta_{h,h'}$$

② Physical quantities and observables (O) such as decay widths read as functions (f_O) of those parameters,

$$O_h = f_O(C_{h,i}; a, b, \dots)$$
$$\bar{O}_{h,h'} = f_{\bar{O}}(C_{h,i}, C_{h',i}; a, b, \dots)$$

III. Obtain LFWFs by determining unknowns according to the constraints

UNKNOWNS	CONSTRAINTS
$C_{h,i}$, a,b, ...	①,② $(f_O = f_{O,Exp})$, other relevant conditions

Note that the number of constraints and the number of unknowns may not be equal. The procedure is solving a system of constraint equations or doing an optimization by parameter fitting.

examples:

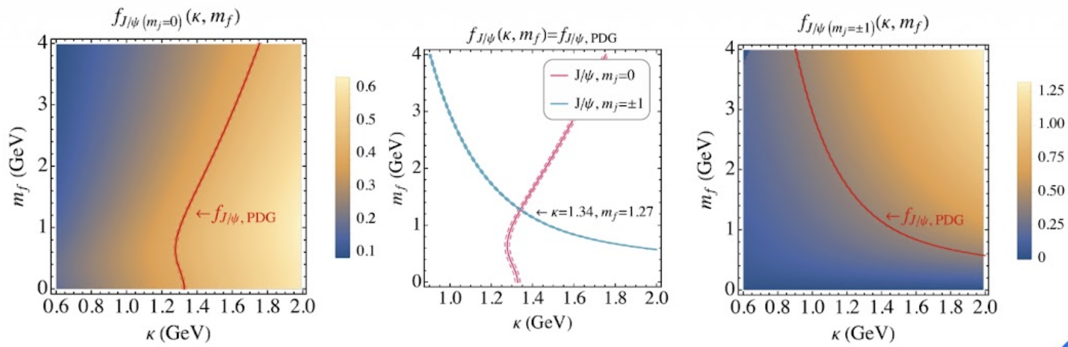
$f_{J/\psi}$, $\Gamma_{\eta_c \to \gamma\gamma}$ as constraints

1) J/ψ is constructed as a LF-1S, 1^{--} state

$$\psi^{(m_j=0)}_{J/\psi} = \psi^{(m_j=0)}_{\mathrm{LF-1}S,1--} ,$$
$$\psi^{(m_j=1)}_{J/\psi} = \psi^{(m_j=1)}_{\mathrm{LF-1}S,1--} ,$$
$$\psi^{(m_j=-1)}_{J/\psi} = \psi^{(m_j=-1)}_{\mathrm{LF-1}S,1--} .$$

The values of κ, m_f are determined by matching J/ψ decay constant to the PDG derived value, rotational symmetry is also enforced,

$$f_{J/\psi\ (m_j=0)}(\kappa, m_f) = f_{J/\psi\ (m_j=\pm 1)}(\kappa, m_f) = f_{J/\psi, PDG}$$



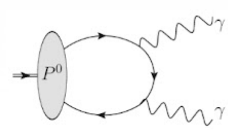
2) η_c is constructed as a LF-1S dominated 0^{-+} state

$$\psi_{\eta_c} = C_{\eta_c,1S}\,\psi_{\mathrm{LF-1}S,0-+} + C_{\eta_c,2S}\,\psi_{\mathrm{LF-2}S,0-+} + C_{\eta_c,1P}\,\psi_{\mathrm{LF-1}P,0-+} .$$

The values of $C_{\eta_c,nl}$ are determined by normalization and matching η_c diphoton decay width to the PDG value

The pseudoscalar-photon transition form factor $F_{P\gamma}$ is defined with the electromagnetic transition matrix element,

$$I^{\mu}_{\lambda_1}(P, q_1) \equiv \langle \gamma^*(q_1, \lambda_1) |\, J^{\mu}(0)\, | \mathcal{P}(P) \rangle = -ie^2 F_{\mathcal{P}\gamma}(Q_1^2, Q_2^2)\,\epsilon^{\mu\alpha\beta\sigma} P_\alpha q_{1\beta} \epsilon^*_{\sigma,\lambda_1}(q_1)$$



- Calculated from LFWF:

$$F_{\mathcal{P}\gamma}(0,0)|_{J^+} = 2Q_f^2\sqrt{N_c}\int_0^\infty \frac{k_\perp\,\mathrm{d}k_\perp}{(2\pi)^2}\int_0^1 \frac{\mathrm{d}x}{\sqrt{2x(1-x)}} \frac{-m_f^2\phi_{0/\mathcal{P}}(k_\perp, x) + \sqrt{2}m_f k_\perp \phi_{1/\mathcal{P}}(k_\perp, x)}{[k_\perp^2 + m_f^2]^2}$$

$$F_{\mathcal{P}\gamma}(0,0)|_{J^\perp} = -2Q_f^2\sqrt{N_c}\int_0^\infty \frac{k_\perp\,\mathrm{d}k_\perp}{(2\pi)^2}\int_0^1 \frac{\mathrm{d}x}{\sqrt{2x(1-x)}} \frac{\phi_{0/\mathcal{P}}(k_\perp, x)}{k_\perp^2 + m_f^2} .$$

- Measured from experiment: $\Gamma_{\mathcal{P}\to\gamma\gamma} = \dfrac{e^4 m_{\mathcal{P}}^3}{64\pi}|F_{\mathcal{P}\gamma}(0,0)|^2$