§ IV Electromagnetic transitions and Form factors (FFs)

$$\mathcal{L}_{QCD} \to P^-_{QCD}$$

$$\text{phen.} \to P^-_{\text{eff}}$$

$$\Big\} \; (P^+\hat{P}^- - P_\perp^2)\underline{|\psi_h\rangle} = \underline{\underline{m_h^2}}|\psi_h\rangle$$

LFWFs

$$\underline{\langle \hat{O} \rangle = \langle \psi | \hat{O} | \psi \rangle}$$

$|q\bar{q}\rangle$

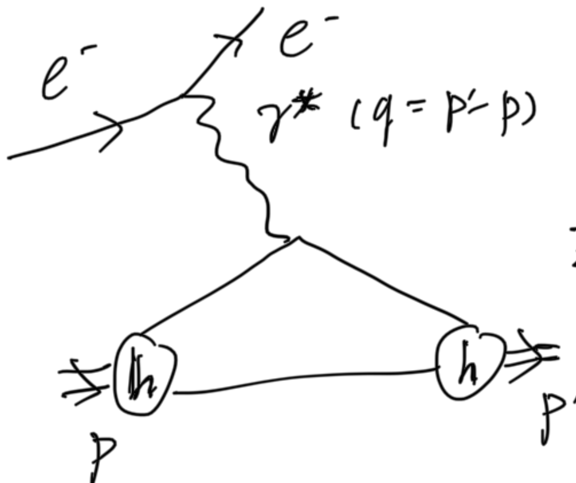
$$\underline{|\psi_h(P, j, m_j)\rangle} = \sum_{s\bar{s}} \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \boxed{\psi^{(m_j)}_{s\bar{s}/h}(\vec{k}_\perp, x)} \quad \text{LFWF}$$

(P1. Eq54)

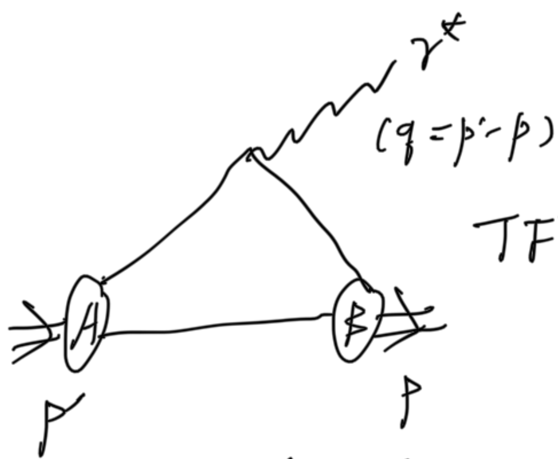
$$\frac{1}{\sqrt{N_c}} \sum_{i=1}^{N_c} b^\dagger_{si}(xP^+, \vec{k}_\perp + x\vec{P}_\perp)\, d^\dagger_{\bar{s}i}((1-x)P^+, -\vec{k}_\perp + (1-x)\vec{P}_\perp)|0\rangle$$

$$\sum_{s\bar{s}} \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \psi^{(m_j')*}_{s\bar{s}/h'}(\vec{k}_\perp, x)\, \psi^{(m_j)}_{s\bar{s}/h}(\vec{k}_\perp, x) = \delta_{hh'}\delta_{m_j m_j'}\delta_{jj'}$$

$$\langle \psi_h(P, j, m_j) | \psi_{h'}(P', j', m_j') \rangle = 2P^+(2\pi)^3 \delta^{(3)}(P - P')\delta_{hh'}\delta_{m_j m_j'}\delta_{jj'}$$



EFFs



TFFs

π^+ $\quad$ π^+ $\quad$ $J/\psi \to \eta_c + \gamma$

LFWF $\quad$ $j=1$ $\quad$ $j=0$

$$I^\mu_{m_j, m_j'} \equiv \underbrace{\langle \psi_B(P, j, m_j) | J^\mu | \psi_A(P', j', m_j') \rangle}_{\text{hadron matrix element}} = \underline{\sum F\, V^\mu}$$

$$\mathcal{M}_{\underset{\uparrow m_j'}{A} \to \underset{\uparrow m_j}{B} + \underset{\uparrow \lambda}{\gamma^*}}(q) = I^\mu_{m_j, m_j'} \cdot \varepsilon^*_{\mu,\lambda}(q)$$

(PDG)

$$\Gamma_{A \to B + \gamma} = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{m_A^2} \frac{1}{2j_A + 1} \sum_{m_j, m_j', \lambda} |\mathcal{M}_{A \to B\gamma}(q)|^2$$

↑ A rest frame

- frame independent
- $q^2 \neq 0$

$\Big\} \Rightarrow$ FFs

key Qs ① What are FFs?

② How to calculate FFs from LFWFs?

IV.A Lorentz structure decomposition of $I^\mu_{m_j, m_j'}$

<1> spacetime translation

$$\langle \psi'_h(p', j', m_j') | \underset{\sim}{J^\mu(x)} | \psi_h(p, j, m_j) \rangle$$

$$= \bar{\psi}(x) \gamma^\mu \psi(x)$$

$$= e^{-i\hat{p}x} \bar{\psi}(0) e^{i\hat{p}x} \gamma^\mu \underbrace{e^{-i\hat{p}x} \psi(0) e^{i\hat{p}x}}_{I}$$

$$= e^{i(p-p')x} \underbrace{\langle \psi'_h(p', j', m_j') | J^\mu(0) | \psi_h(p, j, m_j) \rangle}$$

<2> $\partial_\mu J^\mu = 0$

$\boxed{C.C.}$ $\Rightarrow \underbrace{(p'-p)_\mu}_{q_\mu} \langle \psi'_h | J^\mu(0) | \psi_h \rangle = 0$

<3> charge

$\boxed{N.C.}$ $Q \equiv \int dx_+ d^2x_\perp J^+(x)$

$Q | \psi_h(p, j, m_j) \rangle = e_h | \psi_h(p, j, m_j) \rangle$

$\langle \psi_h(p', j, m_j) | \times$ $\downarrow$

$$\underbrace{\int dx_+ d^2x_\perp \langle \psi_h(p', j, m_j) | J^+(x) | \psi_h(p, j, m_j) \rangle}_{(2\pi)^3 \delta^{(3)}(p-p') \langle \psi_h(p', j, m_j) | J^+(0) | \psi_h(p, j, m_j) \rangle} = e_h 2p^+ (2\pi)^3 \delta^{(3)}(p-p')$$

$$\Rightarrow \langle \psi_h(p, j, m_j) | J^+(0) | \psi_h(p, j, m_j) \rangle = 2 e_h p^+$$

$$\Rightarrow I^\mu_{m_j, m_j'} = \sum_i^n F_i V_i^\mu$$

F_i: FFs; V_i^μ: Lorentz vector

§1. spin-0 mesons , spin-1, $0 \leftrightarrow 1$

j^{PC}, $j=0$, $m_j = 0$, 0^+ scalar

0^- pseudoscalar

for quarkonium $C = 1$

$$\langle \psi_h(p') | J^\mu(0) | \psi_h(p) \rangle = e_h X^\mu$$

X^μ: p^μ, p'^μ

Scalars: $|p'|$, $|p|$, $p' \cdot p$ $\qquad F(q^2)$

$p'^\mu p'_\mu = m_{h'}^2$, $p^\mu p_\mu = m_h^2$

$q^\mu = p'^\mu - p^\mu$, $q^2 = m_{h'}^2 + m_h^2 - 2p' \cdot p$

$\bar{P}^\mu = p'^\mu + p^\mu$, $\bar{P}^2 = m_{h'}^2 + m_h^2 + 2p' \cdot p$

[illegible]

$\rightarrow \quad \chi^\mu = q^\mu H(q^2) + \bar{p}^\mu F(q^2)$

$\boxed{C.C}$ $\quad \partial^\mu J_\mu = 0$

$$q_\mu q^\mu H(q^2) + q_\mu \bar{p}^\mu F(q^2) = 0$$

$$H(q^2) = -\frac{m_{h'}^2 - m_h^2}{q^2} F(q^2)$$

(1.1) $\quad \langle \psi_{h'}(p') | J^\mu(0) | \psi_h(p) \rangle = e_h \left[\bar{p}^\mu - \frac{m_{h'}^2 - m_h^2}{q^2} q^\mu \right] F(q^2)$

$$\langle \psi_{h'}(p') | J^\mu(0) | \psi_h(p) \rangle = \langle \psi_h(p) | J^\mu(0) | \psi_{h'}(p') \rangle^*$$

$\Rightarrow$ $F(q^2)$ is real

EFF, $\quad h' = h, \quad m_{h'}^2 = m_h^2$

$$\langle \psi_h(p') | J^\mu(0) | \psi_h(p) \rangle = e_h \bar{p}^\mu F(q^2)$$

$\boxed{N.C.}$ $\quad \Rightarrow \quad F(0) = 1$

- Parity $\hat{P}$

$$(1.1)\text{LHS} = \langle \psi_{h'}(p', P_2) | \underbrace{\hat{P}\hat{P}^{-1}} J^\mu(0) \underbrace{\hat{P}\hat{P}^{-1}} | \psi_h(p, P_1) \rangle$$

$$\hat{P} | \phi(p^\mu, P) \rangle = P | \phi(\mathcal{P}^\mu{}_\nu p^\nu, P) \rangle \qquad \mathcal{P}\cdot p$$

$$\hat{P}^{-1} J^\mu \hat{P} = \mathcal{P}^\mu{}_\nu J^\nu$$

$$= P_2 P_1 \mathcal{P}^\mu{}_\nu \underbrace{\langle \psi_{h'}(\mathcal{P}\cdot p', P_2) | J^\nu(0) | \psi_h(\mathcal{P}\cdot p, P_1) \rangle}_{(1.1)}$$

$$= e_h (P_2 P_1) \mathcal{P}^\mu{}_\nu \mathcal{P}^\nu{}_\rho \left[\bar{p}^\rho - \frac{m_{h'}^2 - m_h^2}{q^2} q^\rho \right] F(q^2)$$

$$(\mathcal{P}^{-1})^\mu{}_\nu = \mathcal{P}^\mu{}_\nu \qquad \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\Downarrow$$

$$\delta^\mu{}_\rho$$

$= (1.1)\text{ RHS}$

$\Rightarrow \quad P_2 P_1 = 1$

$0^+ \to 0^+, \quad 0^- \to 0^-, \quad \cancel{0^\pm \to 0^\mp}$

- Charge conjugation $\hat{C}$

$$(1.1)\text{LHS} = \langle \psi_{h'}(p', C_2) | \hat{C}\hat{C}^{-1} J^\mu(0) \hat{C}\hat{C}^{-1} | \psi_h(p, C_1) \rangle$$

$$\hat{C} | \phi(p, C) \rangle = C | \phi(p, C) \rangle$$

$$\mathcal{C}^{-1} J^\mu \mathcal{C} = - J^\mu$$

$$= - \mathcal{C}_1 \mathcal{C}_2 \langle \psi_{h'}(\,) | J^\mu | \psi_h(\,) \rangle$$

$$= (1.1)\ RHS$$

$$\Rightarrow \quad \underline{\mathcal{C}_1} \underline{\mathcal{C}_2} = -1 \qquad \times$$

$$\cancel{0^{\pm +} \to 0^{\pm +}}$$

EFF $\quad j=0$ quarkonium $= 0$

$h = h'$

$\eta_c,\ \eta_c',\ \pi^0$

IV.B LFWF repr. of hadron matrix element

$$\langle \psi_B(P, j, m_j) | J^\mu(x) | \psi_A(P', j', m_j') \rangle \quad \checkmark$$

$$J^\mu(x) = \bar\psi(x) \gamma^\mu \psi(x)$$

(p1. Eq 39) $\to$

$$= \sum_{c_1 c_2} \sum_{\lambda_1 \lambda_2} \int \frac{d^2 p_{1\perp}}{(2\pi)^3} \frac{d p_1^+}{2 p_1^+} \int \frac{d^2 p_{2\perp}\, d p_2^+}{(2\pi)^3\, 2 p_2^+}$$

$$[\underbrace{b^+_{\lambda_2 c_2}(p_2)}_{b_2^+} \underbrace{\bar u_{\lambda_2}(p_2)}_{\bar u(2)} e^{i p_2 \cdot x} + \underbrace{d_{\lambda_2 c_2}(p_2)}_{d_2} \underbrace{\bar v_{\lambda_2}(p_2)}_{\bar v(2)} e^{-i p_2 \cdot x}]$$

$$\gamma^\mu [\underbrace{b_{\lambda_1 c_1}(p_1)}_{b_1} \underbrace{u_{\lambda_1}(p_1)}_{u(1)} e^{-i p_1 \cdot x} + \underbrace{d^+_{\lambda_1 c_1}(p_1)}_{d_1^+} \underbrace{v_{\lambda_1}(p_1)}_{v(1)} e^{i p_1 \cdot x}]$$

$$\boxed{P^-_{KE(q\bar q)} = \int dx^- d^2 x_\perp\ \frac{1}{2} \bar\psi \gamma^+ \frac{m^2 - \nabla_\perp^2}{i \partial^+} \psi}$$

$$\langle \psi_{q\bar q/B}(P, j, m_j) | J^\mu(0) | \psi_{q\bar q/A}(P', j', m_j') \rangle$$

$$= \frac{1}{N_c} \sum_{i,j=1}^{N_c} \langle 0 | \sum_{s\bar s} \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2 k_\perp}{(2\pi)^3} \psi^{(m_j)*}_{s\bar s/B}(\vec k_\perp, x)$$

$$d_{j\bar s}((1-x)P^+, -\vec k_\perp + (1-x)\vec P_\perp)\, b_{js}(xP^+, \vec k_\perp + x\vec P_\perp)$$

$$\sum_{c_1 c_2} \sum_{\lambda_1 \lambda_2} \int \frac{d^2 p_{1\perp}}{(2\pi)^3} \frac{d p_1^+}{2 p_1^+} \int \frac{d^2 p_{2\perp}\, d p_2^+}{(2\pi)^3\, 2 p_2^+} \{ b_2^+ \bar u(2) \gamma^\mu b_1 u(1)$$

J_q^μ

$$+ b_2^+ \bar u(2) \gamma^\mu d_1^+ v(1)$$

$$+ d_2 \bar v(2) \gamma^\mu b_1 u(1)$$

$$- d_1^+ d_2 \bar v(2) \gamma^\mu v(1) \}$$

$J_{\bar q}^\mu$

$$\sum_{s'\bar s'} \int_0^1 \frac{dx'}{2x'(1-x')} \int \frac{d^2 k_\perp'}{(2\pi)^3} \psi^{(m_j')}_{s'\bar s'/A}(\vec k_\perp', x)$$

$$b^+_{is'}(x' P'^+, \vec k_\perp' + x' \vec P_\perp')\, d^+_{i\bar s'}((1-x)P'^+, -\vec k_\perp' + (1-x)\vec P_\perp') | 0 \rangle$$

∘ J_q^μ

(p1. Eq42) $\to$

$$b_{js} b_2^+ = 2 p_2^+ \theta(p_2^+)(2\pi)^3 \delta(p_2^+ - xP^+) \quad - \quad \cancel{b_2^+ b_{js}} \to 0$$

$$\delta^2(\vec p_{2,\perp} - \vec k_\perp - x\vec P_\perp)\, \delta_{j c_2}\, \delta_{s \lambda_2}$$

$$b_1 b^+_{is'} \to p_1$$

$$d_{j\bar s}\, d^+_{i\bar s'} = 2(1-x)p^+\,\theta(p^+)\,\delta\big((1-x')p'^+ - (1-x)p^+\big)$$
$$\delta^2\big(-\vec k_\perp' + (1-x)\vec p_\perp' + \vec k_\perp - (1-x)\vec p_\perp\big)\,\delta_{ji}\,\delta_{\bar s'\bar s}$$

$\downarrow$

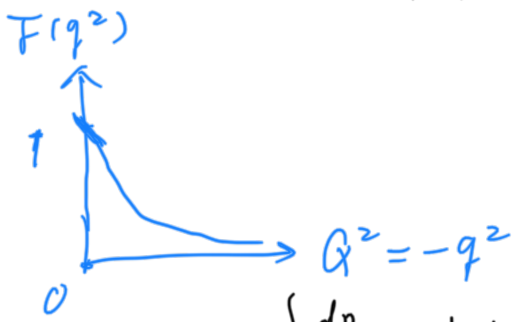
$$\sum_j \sum_{\bar s'} \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2 k_\perp}{(2\pi)^3}\, d\, d^+ \Rightarrow \frac{1}{x}$$

$x = 1-(1-x')\frac{p'^+}{p^+}$ $\leftarrow$

$\vec k_\perp = \vec k_\perp' - (1-x')\vec p_\perp' + (1-x)\vec p_\perp$

$j = i$

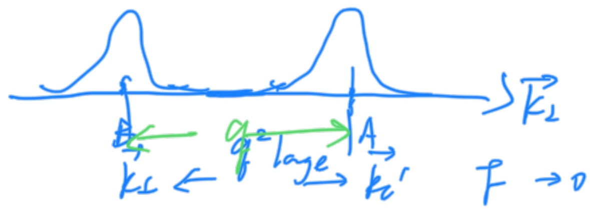
$\bar s' = \bar s$



$\int dp_1 \quad b_i b^+_{is'} \to 1$

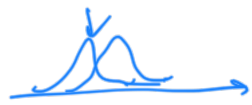
$\int dp_2 \quad b_{js} b_2^+ \to 1$

ψ



EFF

$\langle \psi_{q\bar q/B}(p, j, m_j) | J^\mu_q(0) | \psi_{q\bar q/A}(p', j', m_j') \rangle$ ✓



$$\frac{F(q^2)}{\cancel{2p^+}} = \sum_{s\bar s} \int_{\max(0,\,1-\frac{p^+}{p'^+})}^1 \frac{dx'}{2x'(1-x')} \int \frac{d^2 k_\perp'}{(2\pi)^3} \frac{1}{x} \sum_{s'} \psi^{(m_j')}_{s'\bar s/A}(\vec k_\perp', x')$$

$F \to 1$

$p'^\mu + p^\mu$

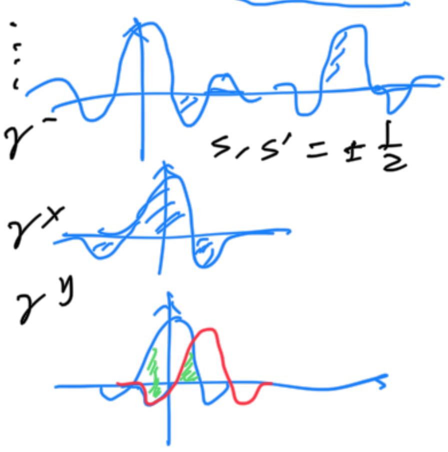
$$\psi^{(m_j)*}_{s\bar s/B}(\vec k_\perp, x)\, \underbrace{\bar u_s(xp^+, \vec k_\perp + x\vec p_\perp)\, \gamma^\mu\, u_{s'}(x'p'^+, \vec k_\perp' + x'\vec p_\perp')}$$

$x = \frac{p_q^+}{p^+}$, $\quad x \in (0, 1)$

$\bar u_s(p)\gamma^+ u_{s'}(p') = 2\sqrt{p^+ p'^+}\,\delta_{ss'}$

$0 < x' < 1$

$0 < x < 1 \quad \Rightarrow \quad x' > 1 - \frac{p^+}{p'^+}$



γ^- $\quad s, s' = \pm\frac{1}{2}$

γ^x

γ^y

$A(p') \to B(p) + \gamma$

$p'^+ > p^+$

$A(p') + \gamma \to B(p)$ ✓

$p'^+ < p^+$

IV.C Frames & kinematics

$\swarrow \gamma$

$\psi_A(p') \to \psi_B(p) + X(q = p' - p)$

$F(q^2)$
$\uparrow$

$$q^2 = (p' - p)^2 = \zeta m_A^2 - \frac{\zeta}{1-\zeta} m_B^2 - \frac{1}{1-\zeta}\Delta_\perp^2$$

$$\begin{cases} \zeta \equiv \dfrac{p'^+ - p^+}{p'^+} \\ \vec\Delta_\perp \equiv \vec q_\perp - \zeta \vec p_\perp' = (1-\zeta)\vec p_\perp' - \vec p_\perp \end{cases}$$

$v^+ \to v^+$, $\quad \vec v_\perp \to \vec v_\perp + v^+ \vec\beta_\perp$

- Drell-Yan frame $\quad \xi = 0$, $\Delta_\perp = q_\perp$, $q^2 = -\Delta_\perp^2 < 0$

  space like

- Longitudinal frame $\quad \vec{\Delta}_\perp = 0 \quad \underline{\underline{q^2}} = \xi m_A^2 - \frac{\xi m_B^2}{1-\xi} \le (\underline{m_A} - \underline{m_B})^2$

  $q^2 > 0$