INTENSITY ASYMMETRY IN COLLIDERS WITH STRONG BEAMSTRAHLUNG

K. Le Nguyen Nguyen[1, 2, 3] P. Kicsiny[1, 2] X. Buffat[1] T. Pieloni[2]

[1] European Organisation for Nuclear Research (CERN)

[2] Ecole Polytechnique Fédérale de Lausanne (EPFL)

[3] Cavendish Laboratory, University of Cambridge

Motivations and contents

- Analytical method of determining equilibrium **lengths** for *symmetric* collisions of flat Gaussian beams with strong beamstrahlung have been found [1].
  - Non-symmetric parameter sets?

  → Start with asymmetry in *intensity:* $N_{s/w} = N_0(1 \pm \Delta N)$
- Luminosity of asymmetric collisions vs. symmetric collisions
- Stability of resultant configurations: longitudinal flip-flop? (ie. only longitudinal blow-up)

  → Linear stability analysis
- Extending the model to include variable width?
- Beamstrahlung lifetime.

Beamstrahlung equilibrium length: assumptions

- Dispersive effects at the interaction point (IP) are ignored.
- Corrections due to the hourglass effect and the crab-waist collision scheme are either taken to be negligible or weak in various models.
- Other effects of beamstrahlung not directly related to the equilibrium bunch lengths, such as the reduction of particle number, are assumed to be absent.

Beamstrahlung equilibrium length: assumptions [1]

- For typical crossing angles (30mrad), differences in beam and photon energies between the boosted and lab frames are negligible.
- Ignore disruption effects and horizontal rms angular beam divergence compared with crossing angle, but account for vertical hourglass effect.
- Flat beam shape ($\sigma_y \ll \sigma_x^*$) in the collision region of interest.

Beamstrahlung equilibrium length: hourglass effect, crab waist optics [1]

Hourglass/ crab waist factor:

$$\sigma_{y,w/s}(x,s) = \sigma^*_{y,w/s} \times G(x,s)_{w/s}$$

Hourglass effect

Crab waist optics

$$G(x,s)_{w/s} = \sqrt{1 + \left(\frac{s \mp \frac{x}{\theta_c}}{\beta_y^*}\right)^2}$$

Twiss y-parameter at IP

Beamstrahlung equilibrium length: hourglass effect, crab waist optics [1]

- In terms of normalised coordinates $\bar{y} = y/\sigma_y^*, \bar{x} = x/\sigma_x^*, \bar{z} = z/\sigma_z \;, \bar{s} = s/\sigma_z \;,$

$$G(\bar{s}, \bar{x}; \Phi) = \sqrt{1 + \frac{\sigma_z^2}{\beta_y^{*2}} \left(\bar{s} \mp \frac{\bar{x}}{2\Phi} \right)^2} \;,$$

where $\Phi = \theta_c \sigma_z / (2\sigma_x^*)$ is the Piwinski angle.

Beamstrahlung equilibrium length: assumptions [1]

- The inverse local bending radius at normalised spacetime coordinates can be approximated as

$$\frac{1}{\rho(\bar{x},\bar{y},\bar{s},\bar{z};\Phi)} = |F(\bar{x}-2\Phi\bar{s}+\Phi\bar{z},\bar{y},\bar{s};\Phi)|\frac{2N_b r_e}{\gamma\sigma_z}\exp\left[-\frac{(2s-z)^2}{2\sigma_z^2}\right],$$

and F can be expressed in terms of the Faddeeva function $w(z) = \mathrm{exp}(-\mathrm{z}^2)\mathrm{erfc}(-\mathrm{iz})$.

Beamstrahlung equilibrium length: assumptions [1]

- In the flat beam approximation, the function F can be expanded to $O\left(\sigma_y/\sigma_x^*\right)$ as

$$F(\bar{x},\bar{y},\bar{s}) \approx \frac{\exp(-\bar{x}^2/2)}{\sigma_x^*}\int_{-i\bar{x}}^{\bar{y}/G(\bar{x},\bar{s};\Phi)} \exp\left(-\frac{1}{2}\tau^2\right) d\tau$$

- Then, assuming $|\bar{x}| < \sim 1$ and $|\bar{y}/G| < \sim 1$, Taylor-expanding the integrand gives

$$|F(\bar{x},\bar{y},\bar{s})| \approx \frac{\exp(-\bar{x}^2/2)}{\sigma_x^*}\left|\frac{\bar{y}}{G(\bar{x},\bar{s};\Phi)} + i\bar{x}\right|$$

Beamstrahlung equilibrium length: local curvature

$$G(x,s)_{w/s} = \sqrt{1 + \left(\frac{s \mp \frac{x}{\theta_c}}{\beta_y^*}\right)^2}$$

Hourglass effect

Crab waist optics

Twiss y-parameter at IP

- Local curvature of a particle in the weak bunch, flat beam approximation, due to transverse electromagnetic field from strong bunch:

$$\frac{1}{\rho(x,y,z,s;t)_w} = \sqrt{\frac{2}{\pi}}\,\frac{2r_eN_s}{\gamma_w\sigma_{z,s}}\,\frac{\exp\left[-\frac{\left(x-\left(s-\frac{z}{2}\right)\theta_c\right)^2}{2\sigma_{x,s}^{*2}}\right]}{\sigma_{x,s}^*}\exp\left[-\frac{(2s-z)^2}{2\sigma_{z,s}^2}\right]$$

$$\times\left[\left(\frac{y}{\sigma_{y,s}^*\,G\left(x-\left(s-\frac{z}{2}\right)\theta_c, s\right)_s}\right)^2 + \left(\frac{x-\left(s-\frac{z}{2}\right)\theta_c}{\sigma_{x,s}^*}\right)^2\right]^{\frac{1}{2}}$$

Beamstrahlung equilibrium length: central integral

- Central integral: average n[th] power of the curvature of a particle in the weak bunch

$$I_{n,w} \equiv \int_{-\infty}^{+\infty} ds \left\langle \frac{1}{\rho_w^n}(s,t) \right\rangle$$

Averaged over Gaussian distribution of particles in the weak bunch

Complicate Gaussian integrals → Approximations required!

$$= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} ds \iiint_{x,y,z \in R^3} \frac{dxdydz}{\sigma_{x,w}^{*}\, \sigma_{y,w}^{*}\, G(x,s)_w \sigma_{z,w}} \rho(x,y,z,s;t)_w^{-n}$$

$$\times \exp\left[-\frac{\left(x + \frac{z\theta_c}{2}\right)^2}{2\sigma_{x,w}^{*2}} - \frac{y^2}{2\sigma_{y,w}^{*2} G(x,s)_w^2} - \frac{z^2}{2\sigma_{z,w}^2} \right]$$

Beamstrahlung equilibrium length: central integral [1]

- n=1: Average number of photons *per collision* emitted by the weak beam,

$$N_{ph,w} \approx \frac{5}{2\sqrt{3}} \alpha\gamma I_{1,w}$$

- n=2: Average energy loss *per collision* due to beamstrahlung of the weak beam relative to the beam energy,

$$\delta_{BS,w} \equiv \frac{1}{\tau_{z,BS,w}} \approx \frac{2}{3} r_e \gamma_w^3 I_{2,w}$$

- n=3: Quantum excitation from beamstrahlung emitted by the weak bunch *per collision,*

$$\left\{N_{ph,w}\langle u^2\rangle\right\}_{z,w,BS} \approx \frac{55}{24\sqrt{3}} \frac{r_e^2 \gamma_w^5}{\alpha} I_{3,w}$$

Beamstrahlung equilibrium length: approaching equilibrium

$$\delta_{BS,w} \equiv \frac{1}{\tau_{z,BS,w}} \approx \frac{2}{3} r_e \gamma_w^3 I_{2,w}$$

$$\{N_{ph,w}\langle u^2\rangle\}_{z,w,BS} \approx \frac{55}{24\sqrt{3}} \frac{r_e^2 \gamma_w^5}{\alpha} I_{3,w}$$

- Bunch length increases due to quantum excitation from synchrotron (SR) and beamstrahlung (BS) emission but decreases due to SR and BS damping with characteristic damping times $\tau_{z,SR/BS}$:

$$\frac{d\sigma_{z,w/s}^2}{dt} = \frac{1}{2}\{\dot{N}_{ph}\langle u^2\rangle\}_{z,w/s,SR} + \frac{1}{2}\{\dot{N}_{ph}\langle u^2\rangle\}_{z,w/s,BS} - \left(\frac{2}{\tau_{z,SR}} + \frac{2}{\tau_{z,w/s,BS}}\right)\sigma_{z,w/s}^2$$

- This can be rewritten as

$$\frac{d\sigma_{z,w/s}^2}{dt} \equiv f_{w/s}(\sigma_{z,w}^2, \sigma_{z,s}^2) = \frac{2}{\tau_{z,SR}}\left(\sigma_{z,w/s,SR}^2 + A_{w/s} I_{3,w/s}\right) - \left(\frac{2}{\tau_{z,SR}} + B_{w/s} I_{2,w/s}\right)\sigma_{z,w/s}^2,$$

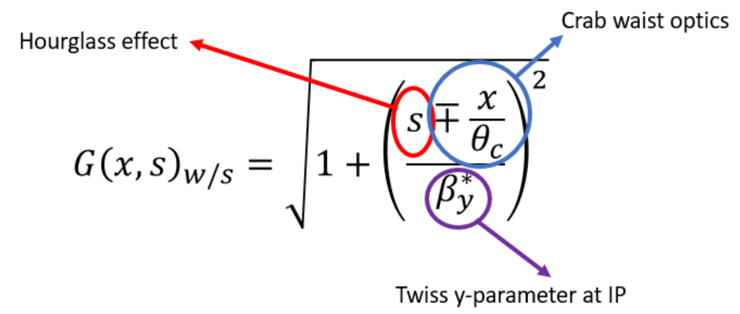
where

$$A_{w/s} \equiv \frac{n_{IP}\tau_{z,w/s,SR}}{4T_{rev}}\left(\frac{\alpha_p C}{2\pi Q_{sync}}\right)^2 \frac{55}{24\sqrt{3}} \frac{r_e^2 \gamma_{w/s}^5}{\alpha}, \qquad B_{w/s} \equiv n_{IP}\frac{4}{3} r_e \gamma_{w/s}^3.$$

- Equilibrium bunch lengths imply two simultaneous equations:

$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma_{z,w/s,SR}^2 + A_{w/s} I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s} I_{2,w/s}\right)\sigma_{z,w/s,eqm}^2$$

Beamstrahlung equilibrium length: simplifying assumptions



- **No hourglass, no crab waist →** $G(s,x)_{w/s} \approx 1.$
- Result: $I_{n,w}^{\text{no hg, no CW}} \approx \frac{R_{n,w}}{M_{n,w}^{\frac{n}{2}+1}} F\left(\frac{n}{2}+1, \frac{1}{2}, 1; P_{n,w}\right) \ (n > 0)$

Hypergeometric function

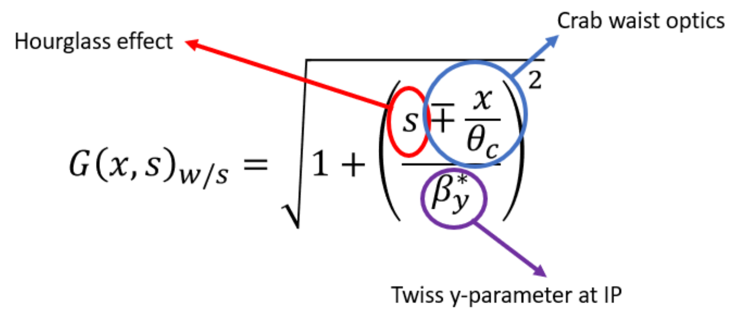
where

$$M_{n,w} = \frac{n}{2} + \frac{2n\sigma_{x,s}^{*2}}{\theta_c^2\left(n\sigma_{z,w}^2 + \sigma_{z,s}^2\right) + 4n\sigma_{x,w}^{*2}} \equiv \frac{n}{2} + 2n\Lambda_{n,w}^2, \qquad \Theta_w = \frac{\sigma_{y,s}^*}{\sigma_{y,w}^*}, \qquad P_{n,w} = 1 - \frac{\Theta_w^2}{M_{n,w}}$$

$$R_{n,w} = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \Gamma\left(\frac{n}{2}+1\right) \frac{\sigma_{z,s}\sigma_{y,s}^*}{\sigma_{y,w}^*\sigma_{x,s}^{*n-1}} \left(\frac{2r_e N_s}{\gamma_w \sigma_{z,s}} \sqrt{\frac{2}{\pi}}\right)^n \times \frac{1}{\sqrt{\theta_c^2\left(n\sigma_{z,w}^2 + \sigma_{z,s}^2\right) + 4n\sigma_{x,w}^{*2}}}$$

- Reduces to special cases presented (n=1, 3) in [1] for *symmetric* collisions.

Beamstrahlung equilibrium length: simplifying assumptions



- **Weak hourglass, no crab waist**

$$\rightarrow G(s,x)_{w/s} \rightarrow \sqrt{1+\left(\frac{s}{\beta_y^*}\right)^2} \approx 1+\frac{1}{2}\left(\frac{s}{\beta_y^*}\right)^2.$$

- **Weak hourglass, weak crab waist**

$$\rightarrow G(x,s)_{w/s} = \sqrt{1+\left(\frac{s\mp\frac{x}{\theta_c}}{\beta_y^*}\right)^2} \approx 1+\frac{1}{2}\left(\frac{s\mp\frac{x}{\theta_c}}{\beta_y^*}\right)^2.$$

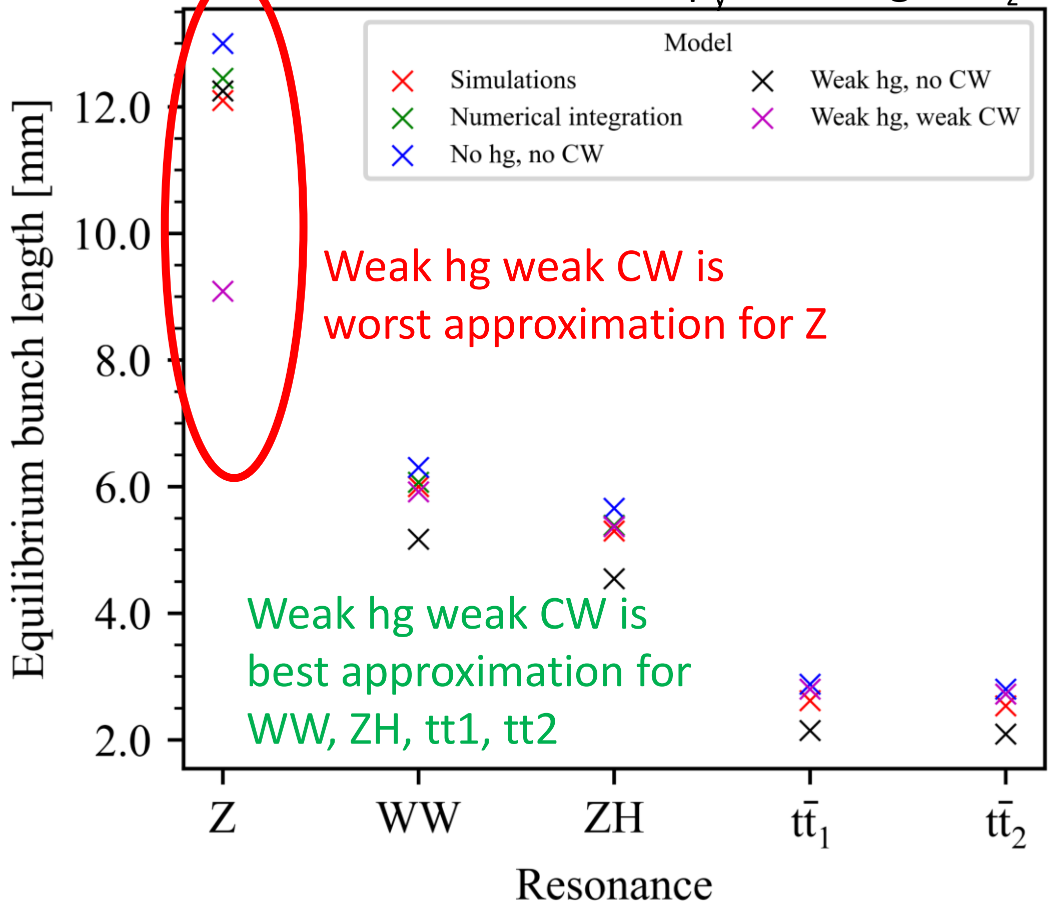
Analytical expressions for $I_{n,w}$ obtained for both assumptions.

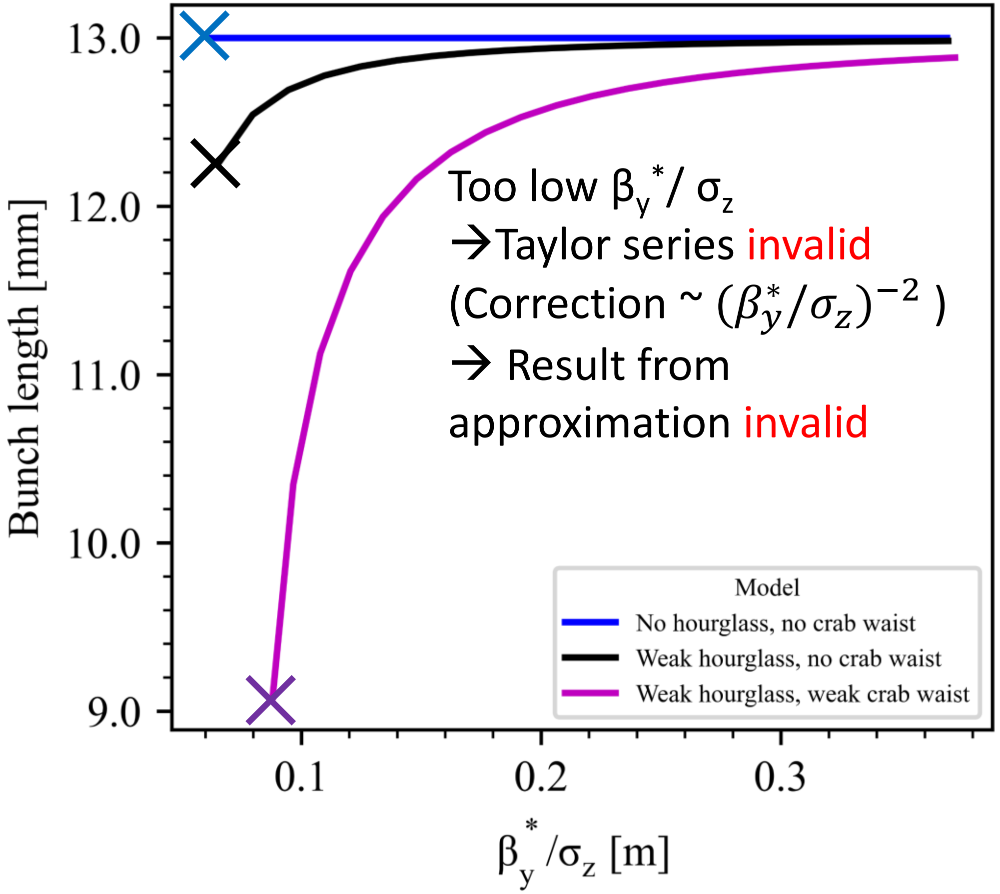
Beamstrahlung equilibrium lengths: symmetric intensities

$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma^2_{z,w/s,SR} + A_{w/s} I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s} I_{2,w/s}\right)\sigma^2_{z,w/s,eqm}$$

Calculations [1], Simulations [2]

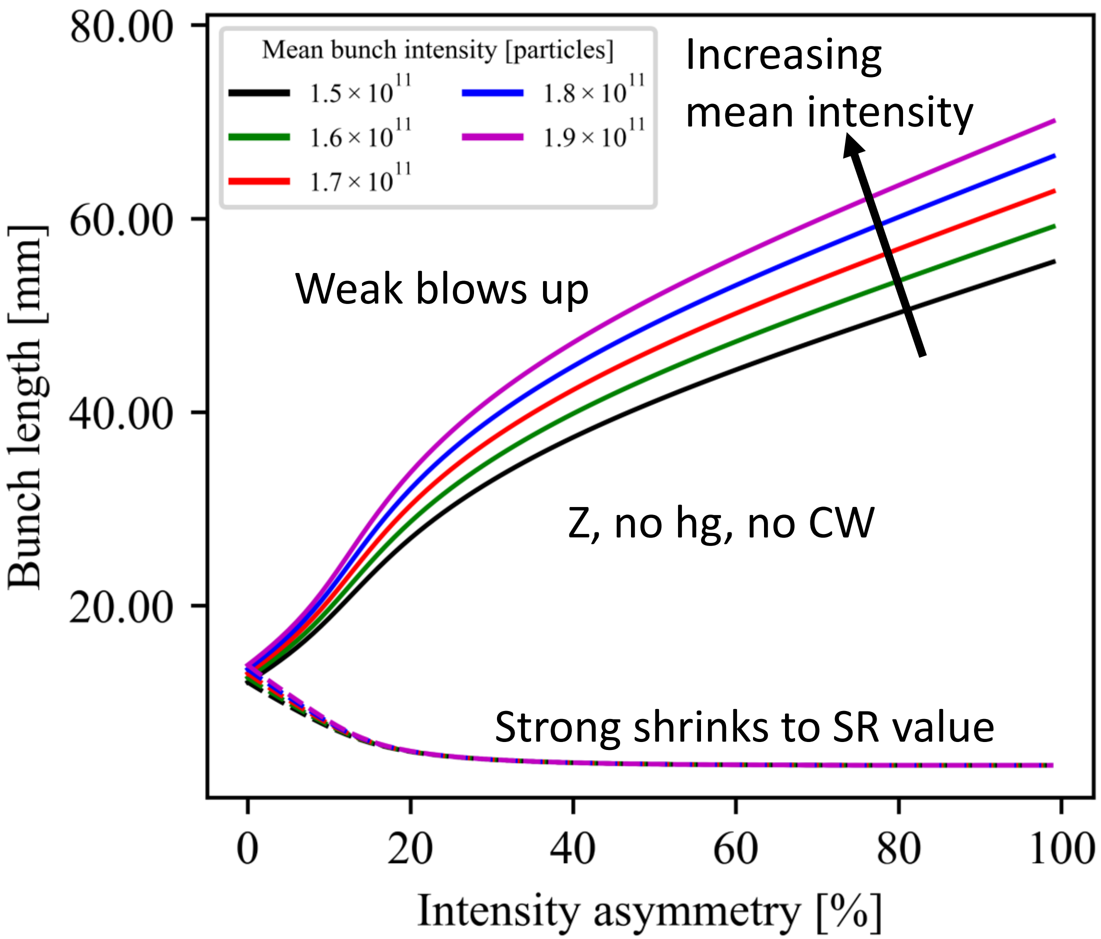
Z has smallest β_y^* and largest σ_z: vary β_y^* to investigate





Beamstrahlung equilibrium lengths: asymmetric intensities

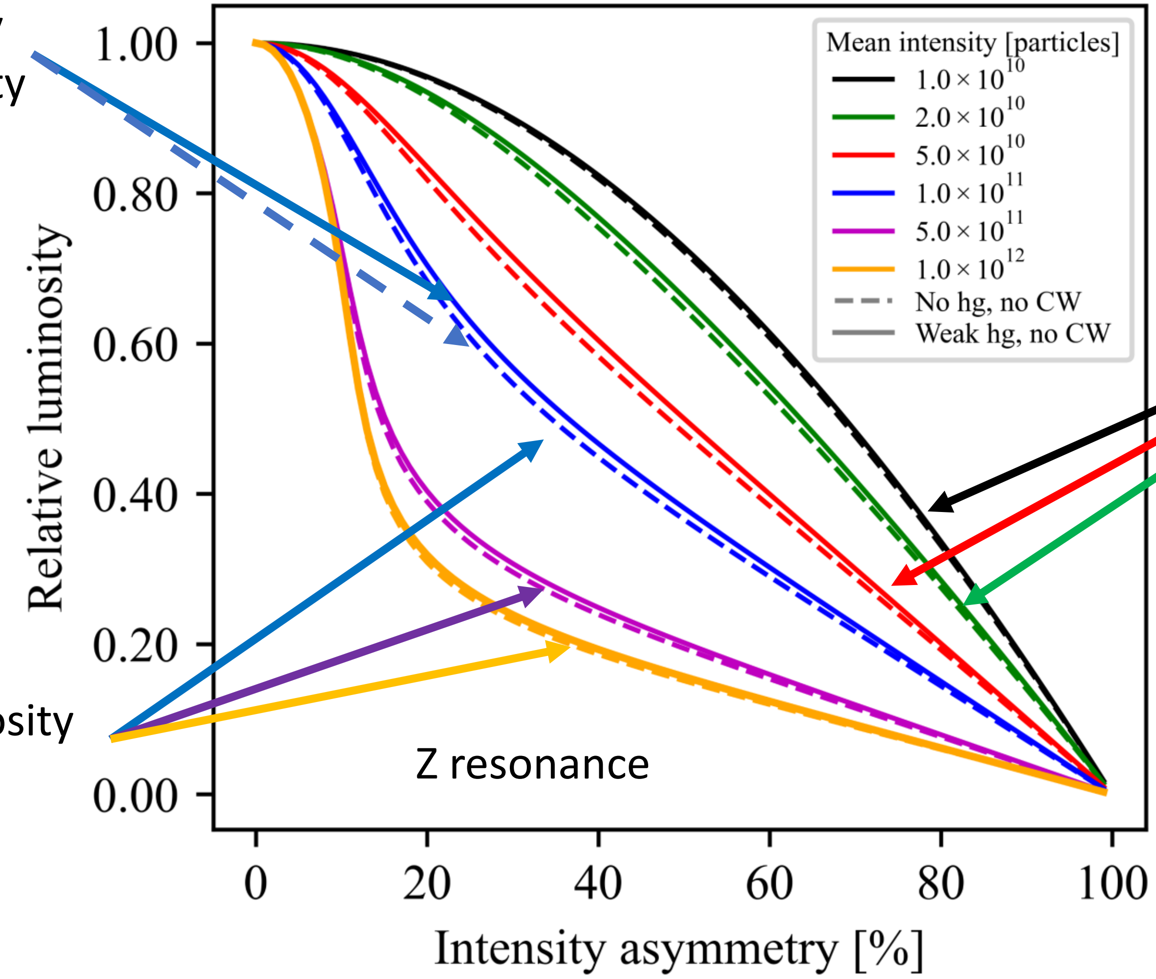
$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma^2_{z,w/s,SR} + A_{w/s}I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s}I_{2,w/s}\right)\sigma^2_{z,w/s,eqm}$$



Luminosity per interaction point [3]

Accounting for weak hg/cw only slightly affects relative luminosity

SR-dominated: Relative luminosity drops at an increasing rate with increasing intensity asymmetry.



BS-dominated: Relative luminosity drops rapidly at low intensity asymmetry.

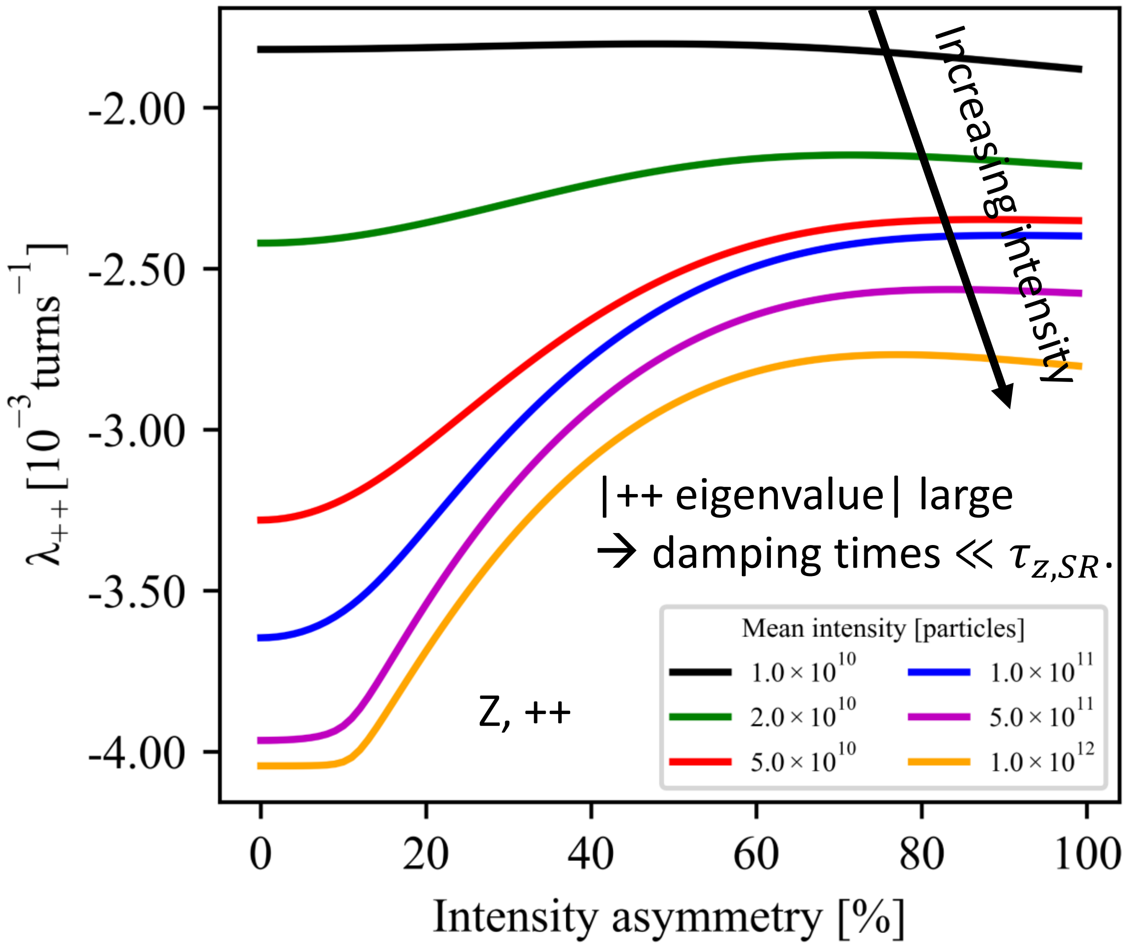
Stability analysis of equilibrium solutions

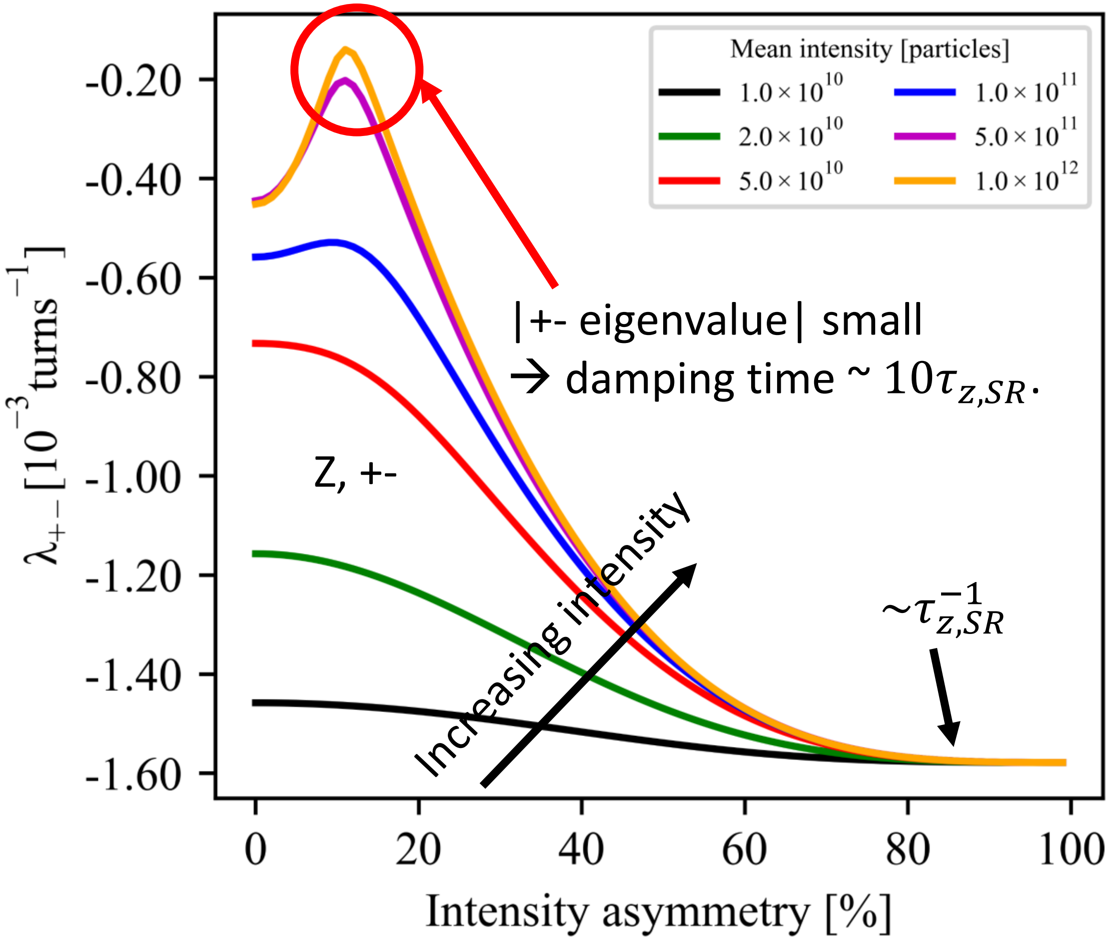
- Evolution of bunch lengths: $\frac{d\sigma_{z,w/s}^2}{dt} = f_{w/s}(\sigma_{z,w}^2, \sigma_{z,s}^2)$.
- Stability of equilibrium solutions assessed by eigenvalues of Jacobian matrix

$$J\left(\sigma_{z,w,eqm}^2, \sigma_{z,s,eqm}^2\right) = \begin{pmatrix} \partial f_w/\partial\sigma_{z,w}^2 & \partial f_w/\partial\sigma_{z,s}^2 \\ \partial f_s/\partial\sigma_{z,w}^2 & \partial f_s/\partial\sigma_{z,s}^2 \end{pmatrix}.$$

- Derivatives can be evaluated analytically for the “no hourglass, no crab waist” scenario for different intensity asymmetries.
- Eigenvectors found to always be +- (one bunch grows, the other shrinks) and ++ (both bunches grow or both bunches shrink).
- Both eigenvalues always negative → All equilibrium configurations are stable → **No longitudinal flip-flop seen.**

Stability analysis of equilibrium solutions



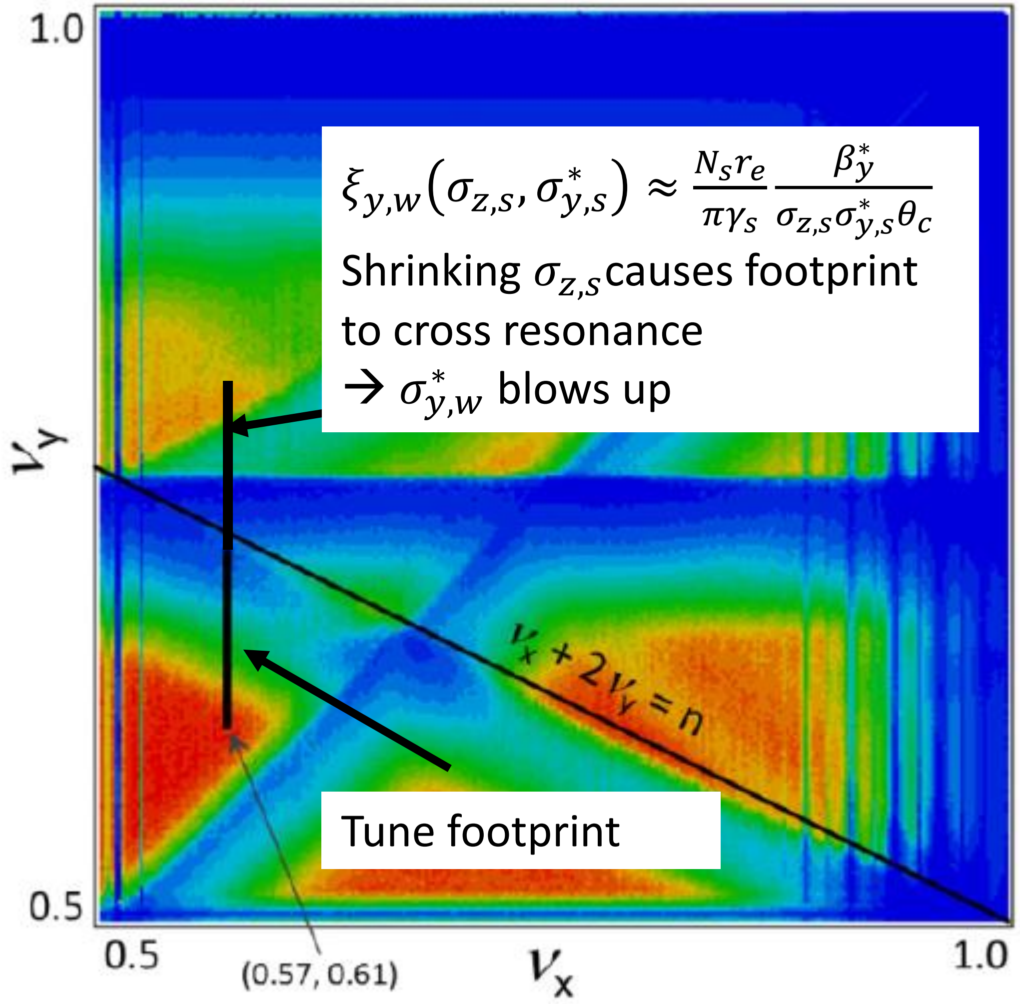


+- perturbation mode slow to dissipate in these parameter ranges, affecting top-up injection mechanism at high mean intensity.

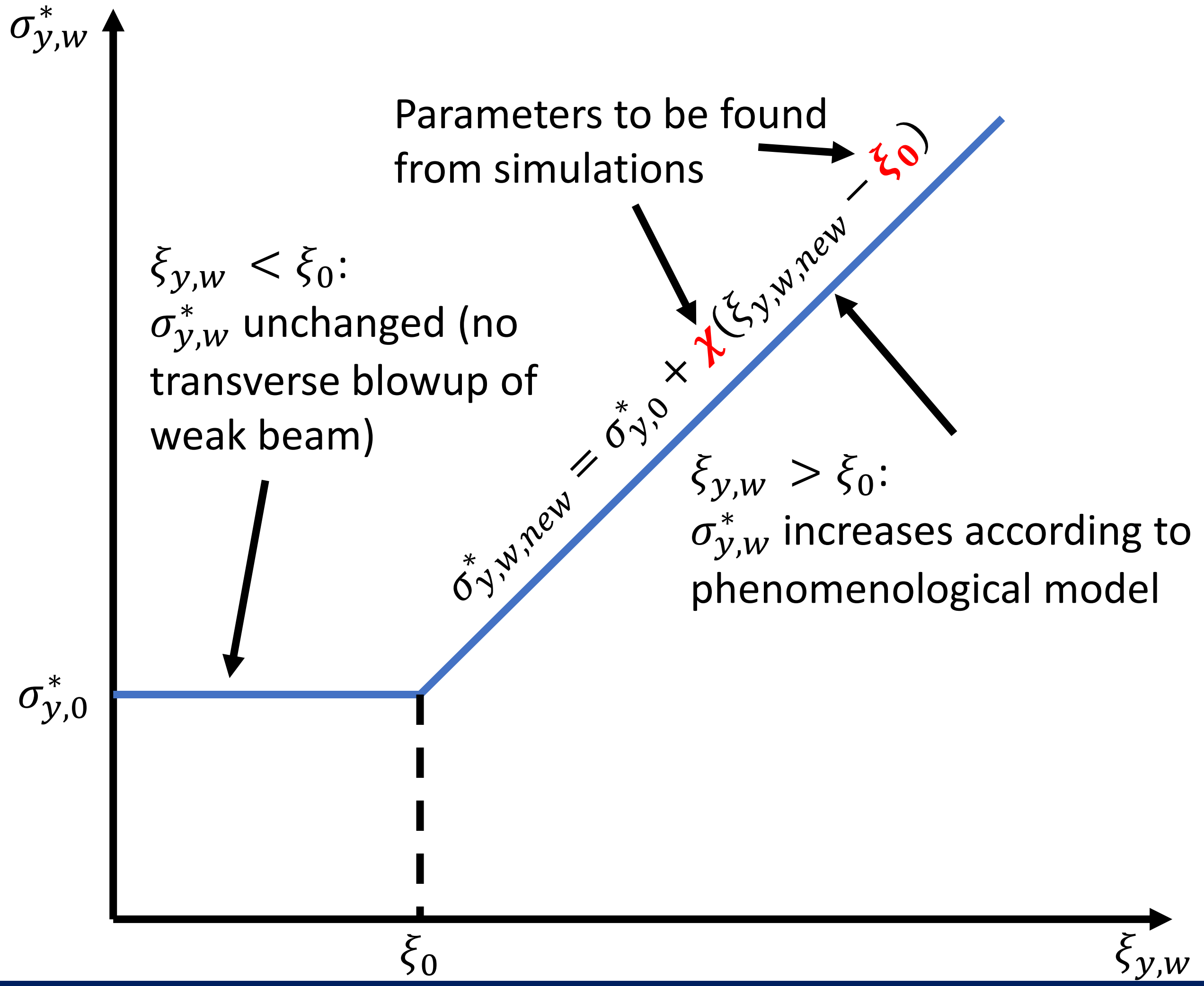
Transverse blow-up: a phenomenological model

$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma^2_{z,w/s,SR} + A_{w/s} I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s} I_{2,w/s}\right)\sigma^2_{z,w/s,eqm}$$

$$\sigma^*_{y,w,new} = \sigma^*_{y,0} + \chi[\xi_{y,w,new}\left(\sigma_{z,s,eqm,new}, \sigma^*_{y,s} = \sigma^*_{y,0}\right) - \xi_0]$$



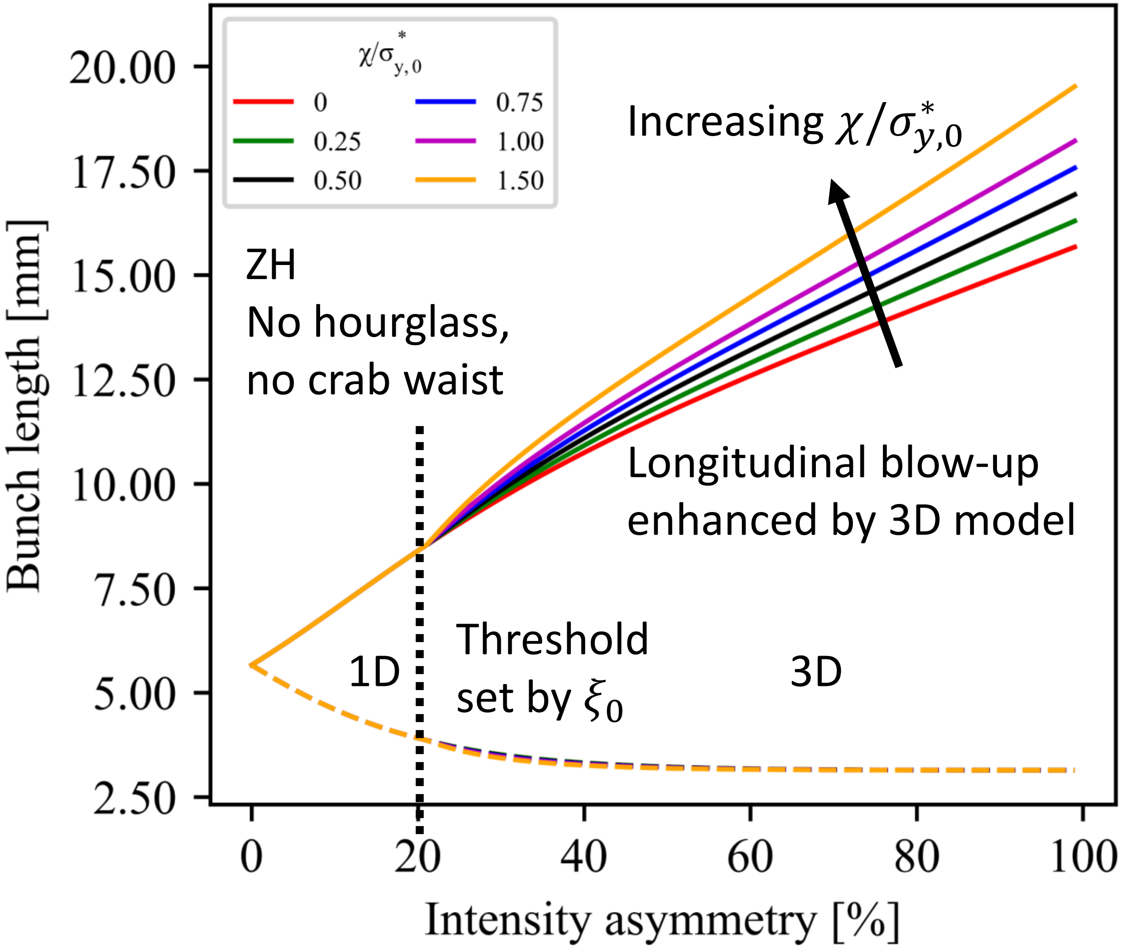
Luminosity as a function of betatron tunes [2]



Transverse blow-up: bunch lengths

$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma_{z,w/s,SR}^2 + A_{w/s}I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s}I_{2,w/s}\right)\sigma_{z,w/s,eqm}^2$$

$$\sigma^*_{y,w,new} = \sigma^*_{y,0} + \chi\left[\xi_{y,w,new}\left(\sigma_{z,s,eqm,new}, \sigma^*_{y,s} = \sigma^*_{y,0}\right) - \xi_0\right]$$



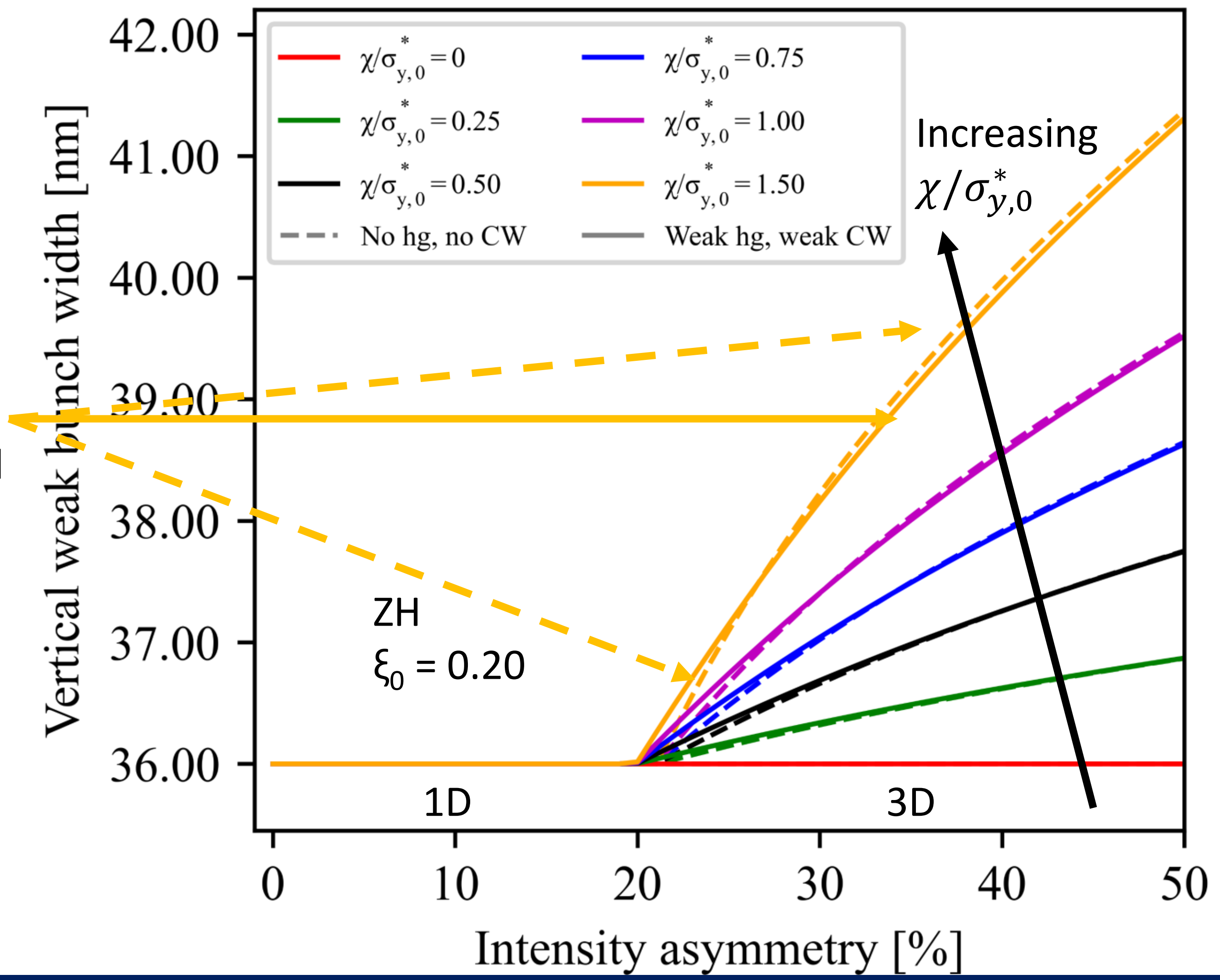
Hg/CW slightly reduces longitudinal blowup.

Transverse blow-up: weak y-width

$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma_{z,w/s,SR}^2 + A_{w/s}I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s}I_{2,w/s}\right)\sigma_{z,w/s,eqm}^2$$

$$\sigma^*_{y,w,new} = \sigma^*_{y,0} + \chi\left[\xi_{y,w,new}\left(\sigma_{z,s,eqm,new}, \sigma^*_{y,s} = \sigma^*_{y,0}\right) - \xi_0\right]$$

Effect of hg+CW on y-width is complicated



Transversal blow-up caused by 3D model

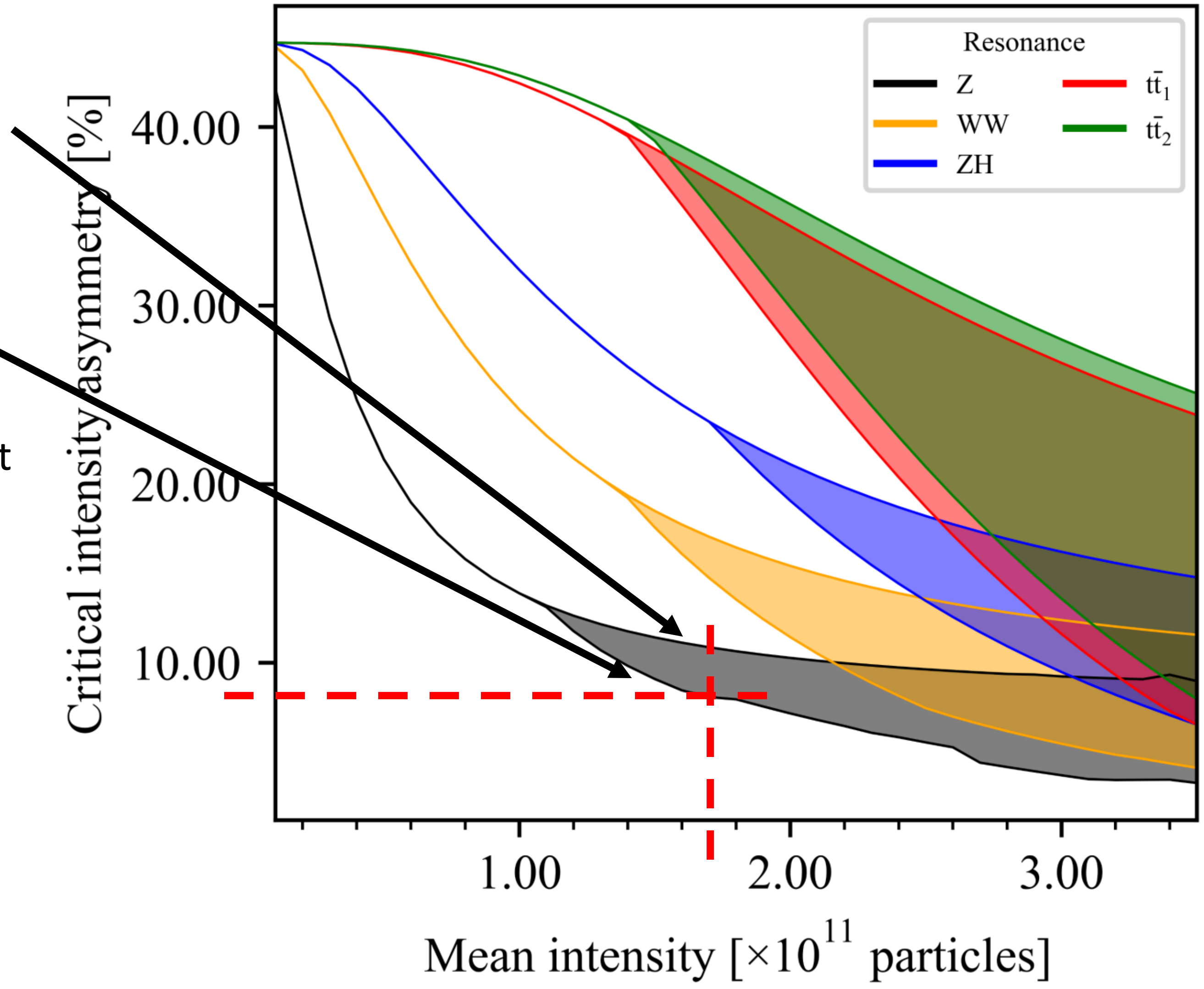
Impact of transverse blow-up on luminosity

Lower bound:
$\chi/\sigma^*_{y,0} = 0$ (effectively 1D)

Upper bound:
$\chi/\sigma^*_{y,0} = 1.50$

Model:
No hourglass, no crab waist

$L_{rel,\ thres} = 80\%$
$\xi_0 = 0.20$



Relative luminosity sets different intensity asymmetry tolerances for different resonances. eg. Z resonance has critical asymmetry of **~8%** for largest $\chi/\sigma^*_{y,0}$.

Beamstrahlung lifetime

- The “beamstrahlung lifetime” has been measured in simulations [4], giving intensity asymmetry thresholds of **±5%** (Z) and **±3%** (other resonances).

→ Almost the same for Z resonance luminosity restriction with current range of parameters, but much more stringent for other resonances.

- Analytical formulae [5] are for symmetric collisions only and would only include strong parameters for asymmetric collisions → No additional formula found.
- Analytical results greatly overestimates lifetime [4].

Summary and conclusions

- Developed analytical methods to evaluate equilibrium bunch lengths for collisions between Gaussian beams with arbitrary parameters, including crossing angle, hourglass effect and crab waist optics.

→ Step up from previous work.

- Expressions used to calculate luminosity and to show via linear stability analysis that longitudinal flip-flop does not exist.

→However, eigenvalues corresponding to long damping times may still impact top-up injection scheme.

- Extended the model to account for 3D flip-flop in transversal blow-up of the weak beam; verify models and find phenomenological parameters by comparing with future simulations.
- Explored constraints on allowed intensity asymmetry through luminosity degradation limit, comparing with beamstrahlung lifetime requirements.

→Thresholds are almost the same for Z resonance but BS lifetime is more stringent for others.

References

[1] M. A. V. Garcia and F. Zimmermann, Beam blow up due to beamstrahlung in circular e^+e^- colliders, The European Physical Journal Plus **136**, 501 (2021).

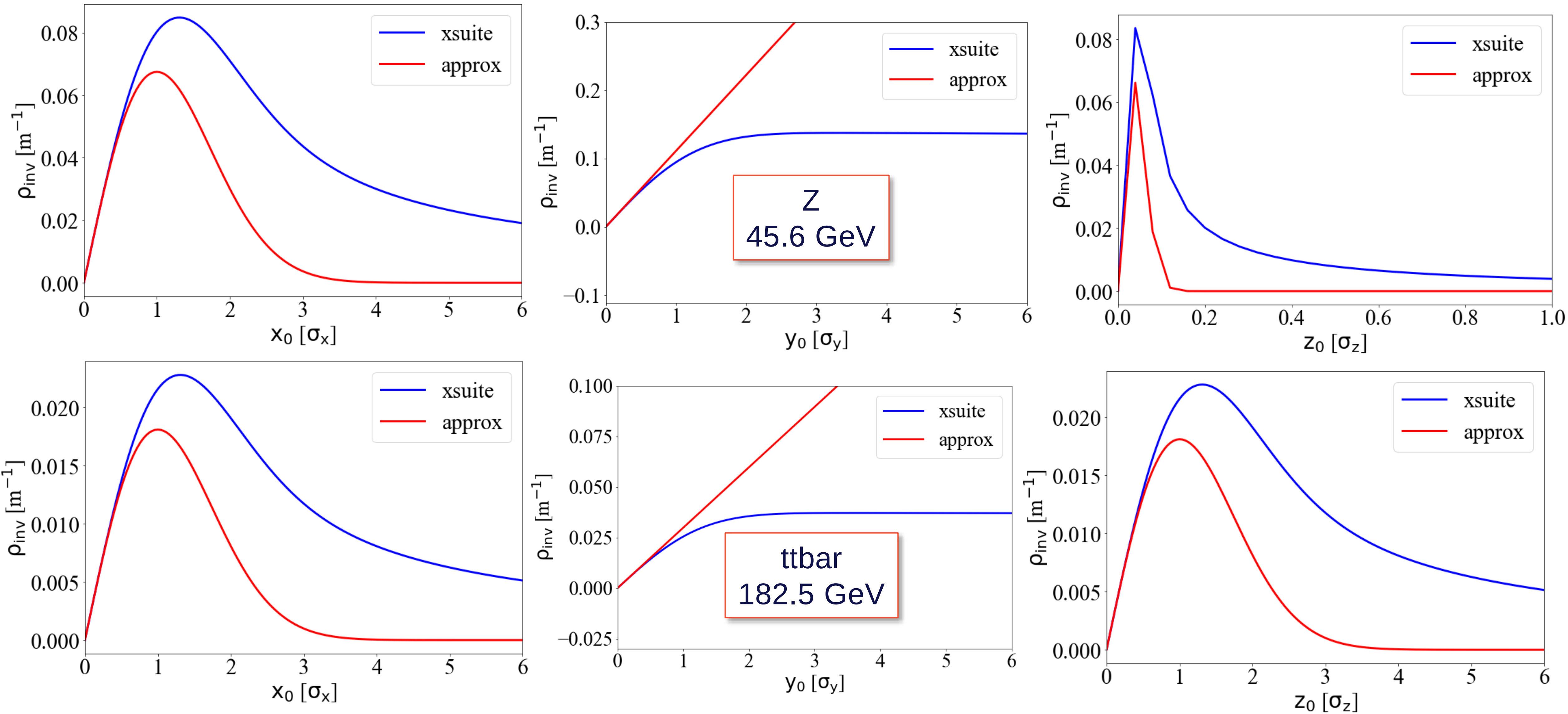
[2] D. Shatilov, FCC-ee parameter optimisation, ICFA Beam Dynamics Newsletter **72**, 30 (2017).

[3] H. Damerau, *Creation and storage of long and flat bunches in the LHC*, Ph.D. thesis, Technische Universität Darmstadt (2005).

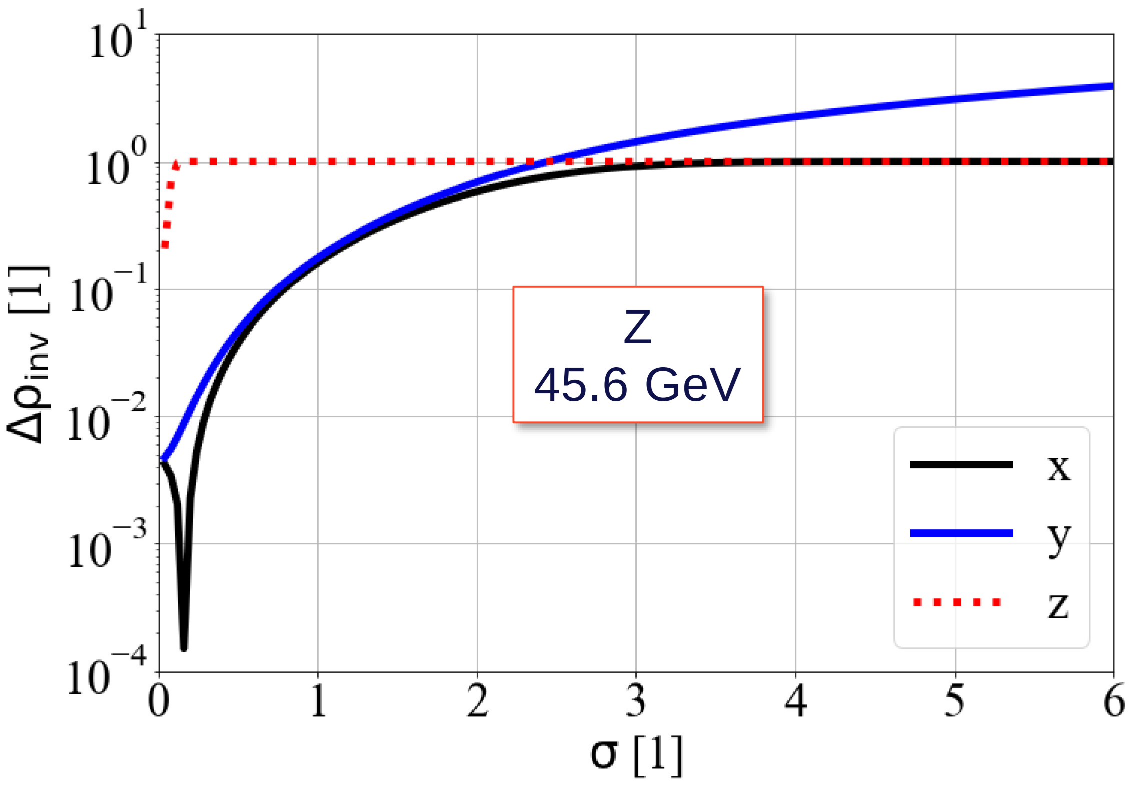
[4] A. Abada, M. Abbrescia, S. AbdusSalam, *et al.*, FCC-ee: The lepton collider, The European Physical Journal Special Topics **228**, 261 (2019).

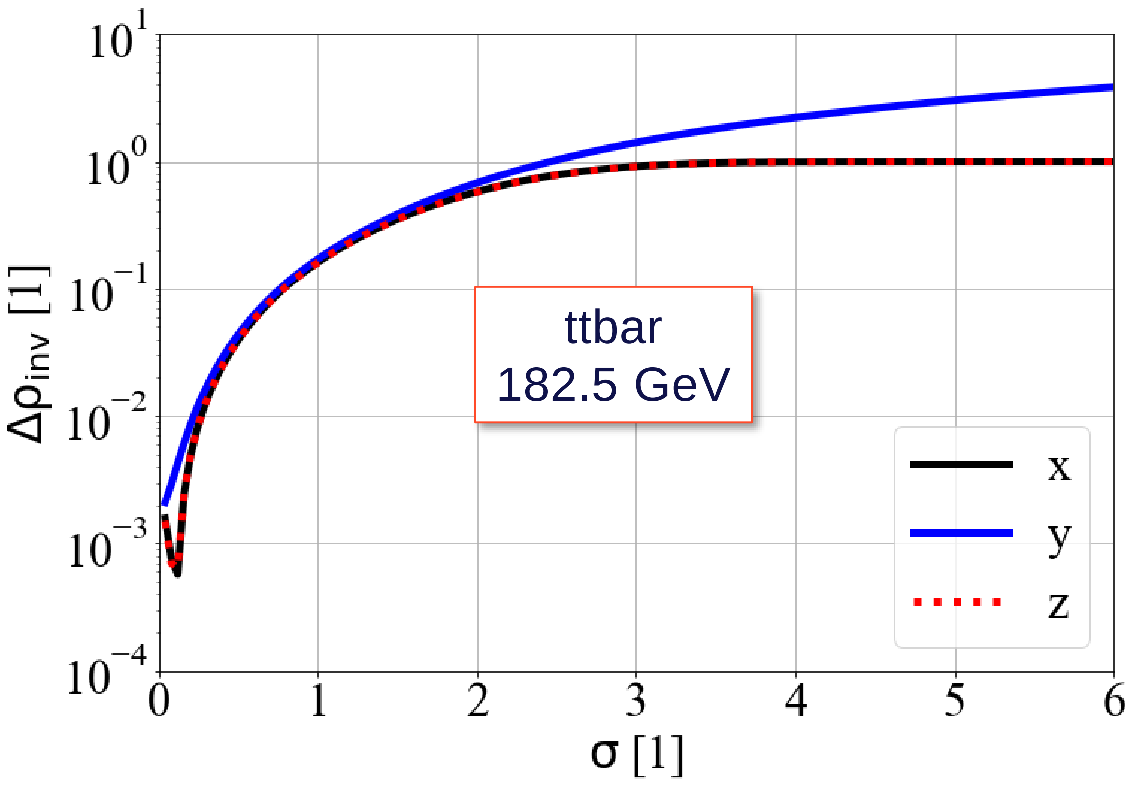
[5] A. Bogomyagkov, E. Levichev, and D. Shatilov, Beam-beam effects investigation and parameters optimization for a circular e^+e^- collider at very high energies, Physical Review Special Topics – Accelerators and Beams **17** (2014).

Validity of analytical approximation



Validity of analytical approximation





- Comparison to xsuite
- Within +/- 1σ up to 10% rel. abs. error between simulation and theoretical approximation (|rho_approx-rho_xsuite| / rho_xsuite)

Beamstrahlung equilibrium length: simplifying assumptions

$$G(x,s)_{w/s} = \sqrt{1+\left(\frac{s+\frac{x}{\theta_c}}{\beta_y^*}\right)^2}$$

Hourglass effect

Crab waist optics

Twiss y-parameter at IP

- **Weak hourglass, no crab waist →** $G(s,x)_{w/s} \to \left[1+\left(\frac{s}{\beta_y^*}\right)^2\right]^{-\frac{1}{2}} \approx 1-\frac{1}{2}\left(\frac{x}{\beta_y^*}\right)^2.$
- Result:

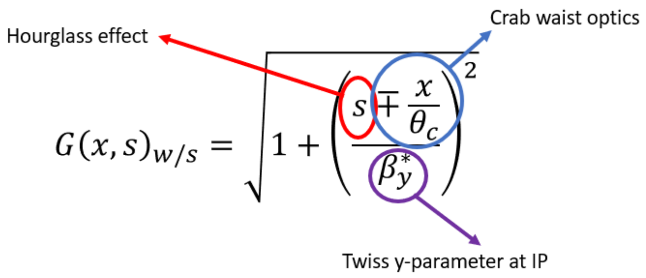
$$I_{n,w}^{\text{weak hg, weak CW}} = I_{n,w}^{\text{no hg, no CW}} - \frac{R_{n,w}}{2G\beta_y^{*2}M_{n,w}^{\frac{n}{2}+1}}\left[F_1\left(\frac{1}{2};-1,\frac{n}{2}+1;1;\Theta_w^2-n,P_{n,w}\right) + \frac{B^2\sigma_{x,s}^{*2}}{4GM_{n,w}}\left(\frac{n}{2}+1\right)F_1\left(\frac{1}{2};-1,\frac{n}{2}+2;2;\Theta_w^2-n,P_{n,w}\right)\right]$$

Correction terms

where

$$B = \frac{\theta_c}{\sigma_{x,w}^{*2}}, \qquad G = \frac{\theta_c^2}{2\sigma_{x,w}^{*2}} + \frac{2n}{n\sigma_{z,w}^2+\sigma_{z,s}^2}$$

Beamstrahlung equilibrium length: simplifying assumptions



- **Weak hourglass, weak crab waist →** $G(x,s)_{w/s} = \sqrt{1+\left(\frac{s\mp\frac{x}{\theta_c}}{\beta_y^*}\right)^2} \approx 1+\frac{1}{2}\left(\frac{s\mp\frac{x}{\theta_c}}{\beta_y^*}\right)^2$
- Result:

$$I_{n,w}^{\text{weak hg, weak CW}} \approx I_{n,w}^{\text{no hg, no CW}} - \frac{R_{n,w}}{16\beta_y^{*2} M_{n,w}^{\frac{n}{2}+1}} \times \left\{ C_1 F_1\left(\frac{1}{2}; -1, \frac{n}{2}+2; 2; \Theta_w^2, P_{n,w}\right) + C_2 F_1\left(\frac{1}{2}; -1, \frac{n}{2}+1; 1; \Theta_w^2, P_{n,w}\right) + n\left[D_1 F\left(\frac{n}{2}+2, \frac{3}{2}, 3; P_{n,w}\right) + D_2 F\left(\frac{n}{2}+1, \frac{3}{2}, 2; P_{n,w}\right)\right]\right\}$$

Correction terms

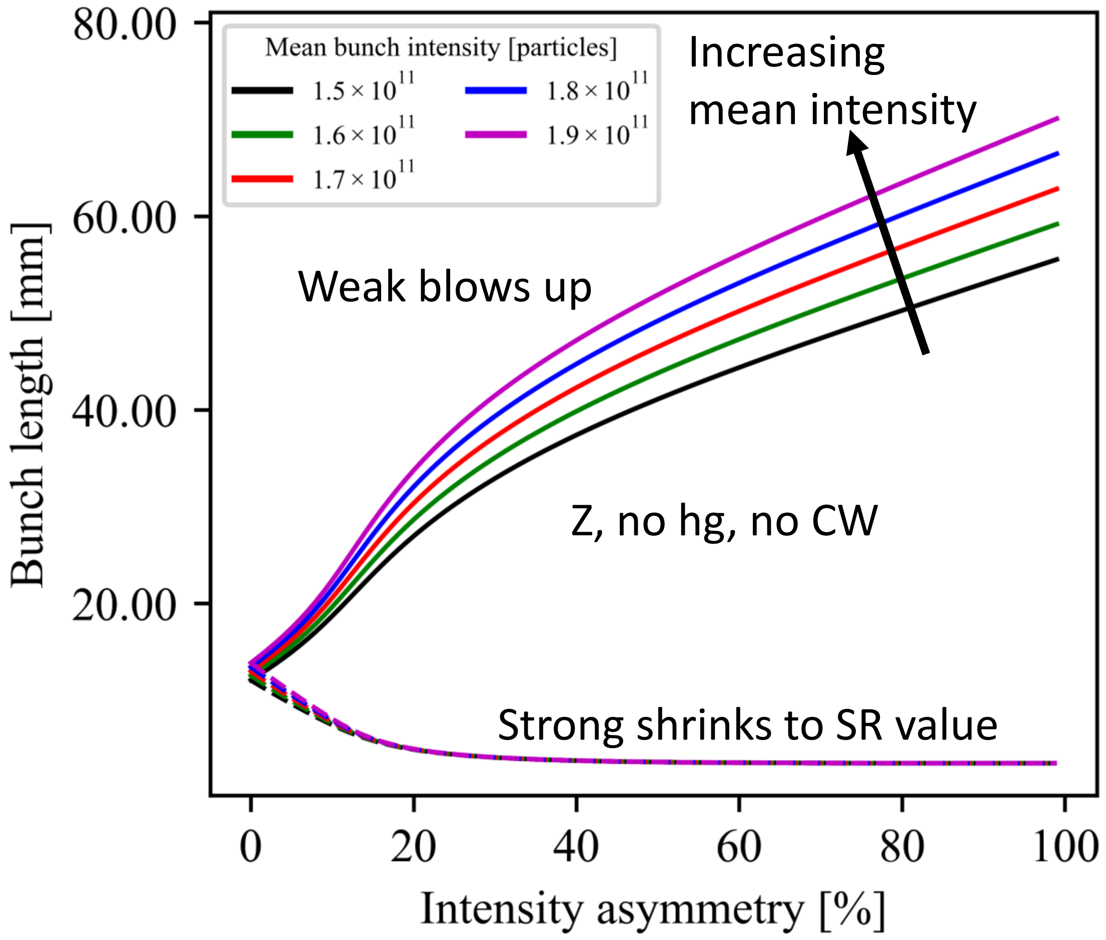
where:

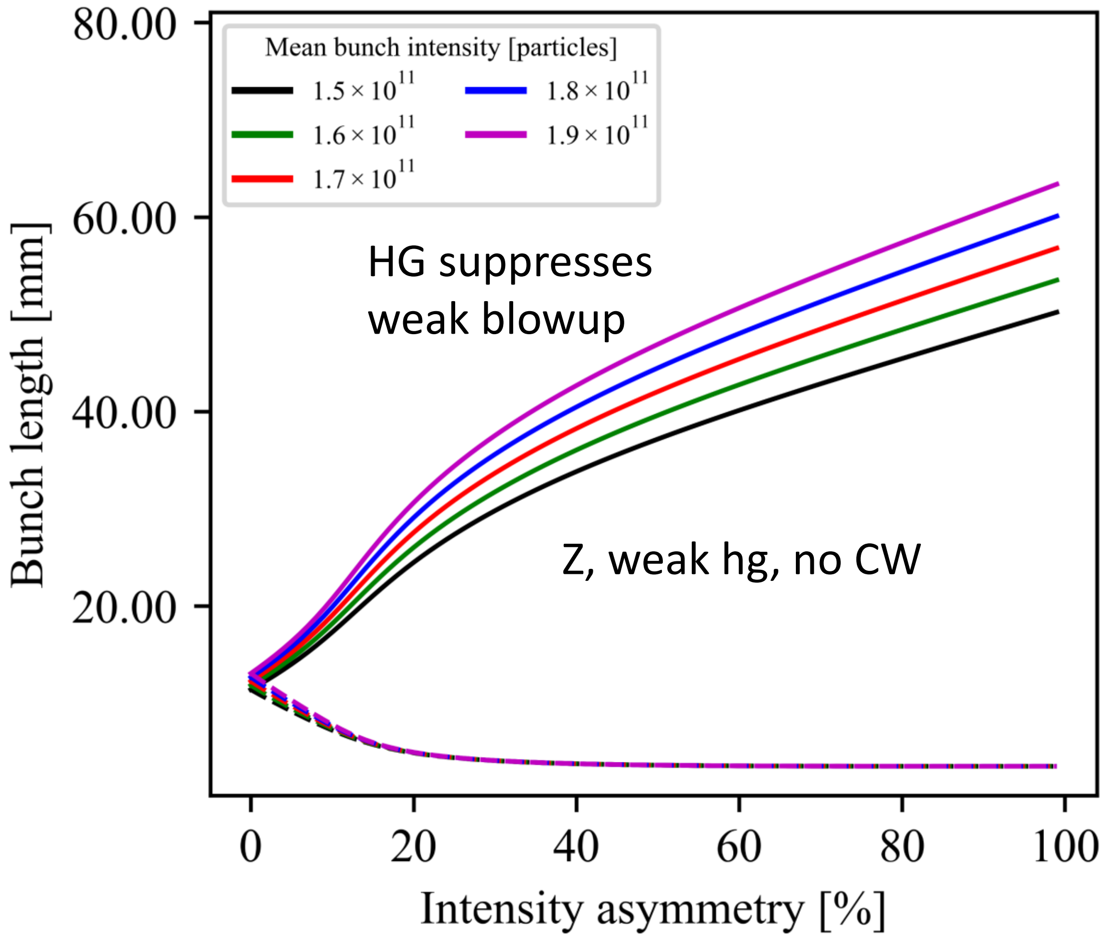
$$C_1 \equiv \left(\frac{n}{2}+1\right)\frac{\sigma_{x,s}^{*2}}{M_{n,w}}\left(\frac{2}{\theta_c}+\frac{2n\theta_c\sigma_{z,w}^2}{\sigma_{x,s}^{*2}}\Lambda_{n,w}^2\right)^2, \qquad C_2 \equiv \frac{1}{D}\left[1+4n^2\frac{\sigma_{z,w}^2\sigma_{x,w}}{\sigma_{z,s}^2\sigma_{x,s}^{*2}}\Lambda_{n,w}^2\right], \qquad D \equiv \frac{n}{2\sigma_{z,s}^2}+\frac{1}{2\sigma_{z,w}},$$

$$D_1 \equiv \left(\frac{n}{2}+1\right)\frac{\sigma_{x,s}^{*2}}{M_{n,w}}\left[\frac{1}{\theta_c}-\frac{\theta_c}{\sigma_{x,s}^{*2}}\left(n\sigma_{z,w}^2+\sigma_{z,s}^2\right)\Lambda_{n,w}^2\right]^2, \qquad D_2 \equiv \frac{2}{G}.$$

Beamstrahlung equilibrium lengths: asymmetric intensities

$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma^2_{z,w/s,SR} + A_{w/s} I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s} I_{2,w/s}\right)\sigma^2_{z,w/s,eqm}$$





Luminosity per interaction point

Luminosity formula for general bunch parameters with negligible hourglass and crab waist:

$$\text{Luminosity/IP} = \frac{N_s N_w}{T_{rev}} \frac{\cos(\theta_c/2)}{2\pi} \frac{1}{\sqrt{\sigma_{y,w}^{*2}+\sigma_{y,s}^{*2}}} \frac{1}{\sqrt{\left(\sigma_{x,w}^{*2}+\sigma_{x,s}^{*2}\right)\cos^2(\theta_c/2)+\left(\sigma_{z,w}^{2}+\sigma_{z,s}^{2}\right)\sin^2(\theta_c/2)}}$$ [5]

Modelling transverse blow-up

- Beam-beam parameters for flat bunches with large Piwinski angle [3]

$$\xi_{x,w/s}\left(\sigma_{z,s/w}\right) \approx \frac{N_{s/w} r_e}{\pi \gamma_{s/w}} \times \frac{2\beta_x^*}{\left(\sigma_{z,s/w}\,\theta_c\right)^2}$$

$$\xi_{y,w/s}\left(\sigma_{z,s/w}, \sigma_{y,s/w}^*\right) \approx \frac{N_{s/w} r_e}{\pi \gamma_{s/w}} \times \frac{\beta_y^*}{\sigma_{z,s/w}\sigma_{y,s/w}^*\theta_c}$$

- Dominating parameter is $\boldsymbol{\xi_{y,w}}$.
- Simulations: beamstrahlung has greatest effect in blowing up the y-width of the weaker beam.

Modelling transverse blow-up

1. A "threshold" beam-beam parameter ξ_0 is chosen. y-widths of both bunches set to $\sigma^*_{y,w} = \sigma^*_{y,s} = \sigma^*_{y,0}$.
2. A 1D model to calculate the equilibrium bunch lengths is chosen → Equilibrium bunch lengths $\sigma_{z,w/s,eqm}$ obtained by solving $f_{w/s}\left(\sigma^2_{z,w}, \sigma^2_{z,s}\right) = 0$.
3. $\xi_{y,w}$ is calculated for this configuration.
   a) If $\xi_{y,w} < \xi_0$: bunch lengths are registered.
   b) If $\xi_{y,w} \geq \xi_0$: bunch lengths are discarded.
      i. $\sigma_{z,w/s,eqm}$ are recalculated using $f_{w/s}\left(\sigma^2_{z,w}, \sigma^2_{z,s}\right) = 0$ and the condition $\sigma^*_{y,w,new} = \sigma^*_{y,0} + \chi\left[\xi_{y,w,new}\left(\sigma_{z,s,eqm,new}, \sigma^*_{y,s} = \sigma^*_{y,0}\right) - \xi_0\right]$
      ii. χ = phenomenological "transversal blow-up factor".
      iii. New bunch lengths are registered.

Transverse blow-up: bunch lengths

$$\frac{2}{\tau_{z,w/s,SR}}\left(\sigma^2_{z,w/s,SR} + A_{w/s}I_{3,w/s}\right) = \left(\frac{2}{\tau_{z,SR}} + B_{w/s}I_{2,w/s}\right)\sigma^2_{z,w/s,eqm}$$

$$\sigma^*_{y,w,new} = \sigma^*_{y,0} + \chi\left[\xi_{y,w,new}\left(\sigma_{z,s,eqm,new}, \sigma^*_{y,s} = \sigma^*_{y,0}\right) - \xi_0\right]$$