FUTURE CIRCULAR COLLIDER
Innovation Study

Beam-Beam Simulations for FCC-ee

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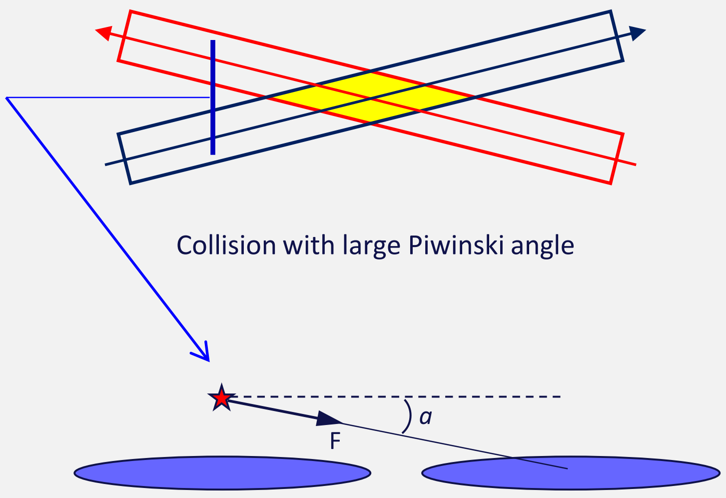
Disruption Parameter

At IPs, both bunches act on each other. Is it necessary to take into account the change in their distribution functions *during collision*?

- Beam-beam kicks depend on the distribution of transverse coordinates of the oncoming beam, and [almost] does not depend on the distribution of transverse momenta.
- The kicks change the transverse momenta, not the coordinates. However, during the interaction, $\Delta p_{x,y}$ will have time to transfer into Δx, Δy.
- The magnitude of change in the transverse coordinates during collision is described by the disruption parameter (here $\xi_{x,y}$ refers to one IP):

$$D_{x,y} = 4\pi\xi_{x,y}\frac{\sigma_z}{\beta^*_{x,y}}$$

- In crab waist collision, we have D_x << 1, but large ξ_y and β_y^* << σ_z. Does it mean that D_y >> 1? No, **σ_z in the above formula should be replaced by $L_i \approx \beta_y^*$, so we have $D_y \sim 1$.**
- Relatively small disruption parameter ($D_{x,y} \leq 1$) means that the distribution of coordinates remains almost unaffected during interaction.
- Examples of $D_{x,y}$ >> 1: linear colliders (ILC, CLIC).



Collision with large Piwinski angle

Vertical plane: kick from the opposite bunch (slice)

If the horizontal distance between slices is greater than several σ_x, then the force is directed to the center of the opposite slice and falls as 1/r, and $\alpha \propto \sigma_y/\sigma_x$ << 1. The vertical kick: $F_y = F \cdot \sin(\alpha)$ << F.

Therefore, the vertical kick is localized in the overlap region of the two beams (marked in yellow).

Simulation Models

Interaction with the opposite bunch

1) **Weak-strong (WS)**

   The opposite (strong) bunch is not affected during long-term (many turns) tracking. This is a simple and fast model. It is always recommended to start with it.

2) **Strong-strong (SS)**

   Both bunches are affected and updated during each collision. This is a complex and time-consuming model, but we must use it when $D_{x,y} >> 1$. Simplified variant (to avoid solving Poisson equation): take into account the barycenter of each slice (transverse displacements) and fit the transverse distribution to Gaussian.

3) **Quasi-strong-strong (QSS)**

   Swap the "weak" and the "strong" bunches every *n-th* turn, and thus update the parameters of the opposite bunch. More realistic and more complex option: simulate two beams simultaneously (in parallel) and exchange data every turn. The opposite bunch is *frozen* (not affected by beam-beam) during collision. This is much faster than SS, but cannot be used when $D_{x,y} >> 1$.

Particle tracking between IP(s)

1) **Linear lattice (constant transport matrix, can be with coupling)**

   A simplified model for chromaticity, impedance, etc. can also be included. This model is simple, fast, and most flexible. If beam-beam is considered as the major nonlinearity, it is recommended to start with this approach.

2) **Realistic nonlinear lattice**

   This is more time-consuming, but correctly accounts chromaticity, DA and momentum acceptance, interference between beam-beam and lattice-driven resonances (especially when considering misalignments and errors).

Plus space charge, IBS, impedance, etc.

Simulation Codes

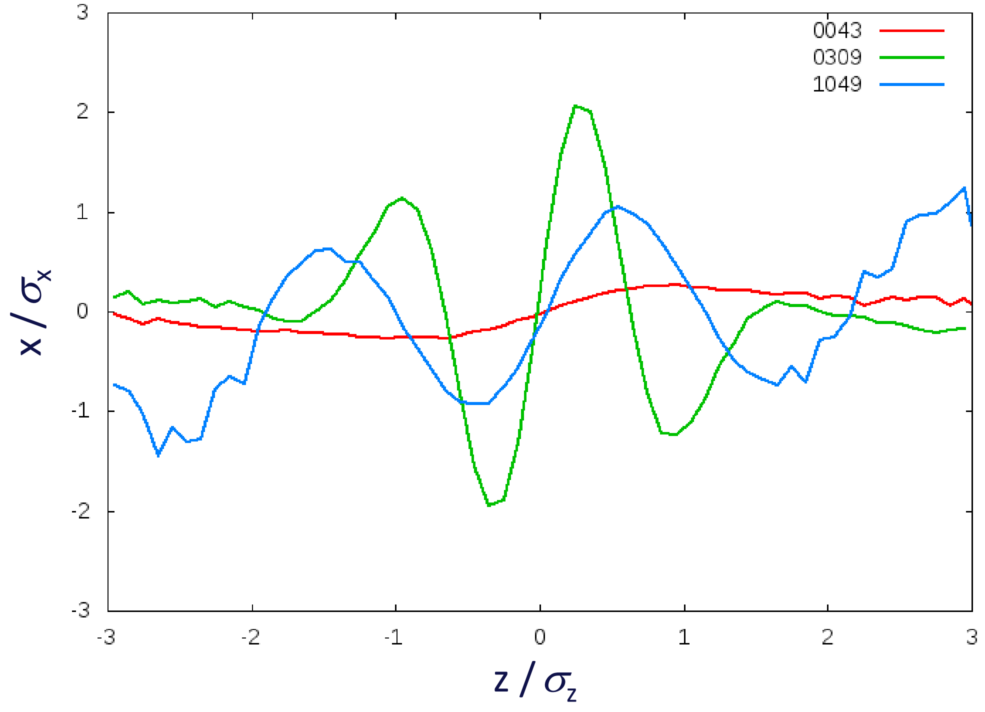
1. Lifetrac (D. Shatilov)
   - WS and QSS simulations
   - Realistic lattice with errors, misalignments and corrections
   - Upcoming updates: tapering, realistic SR in all magnets
2. BBWS, BBSS (K. Ohmi)
   - WS and SS simulations
   - Linear lattice with possible consideration of chromaticity, impedance, etc.
3. SAD (K. Oide et al.) + BBWS
   - Realistic lattice with errors, misalignments and corrections
   - Tapering, realistic SR in all magnets, spin tracking, etc.
   - Beam-beam (WS) is provided by BBWS code
4. IBB (Y. Zhang)
   - WS, SS and QSS simulations
   - Linear lattice with possible consideration of chromaticity, impedance, etc.
   - Next steps: realistic lattice with errors, misalignments, SR in all magnets
5. Xsuite (P. Kicsiny, X. Buffat et al. – for BB module)
   - WS, SS and QSS simulations (now testing, work in progress)
   - Realistic lattice with all effects included

The functionality of different codes is not completely the same, but there are large areas of overlap where cross-validation can be done.

Example: Coherent Beam-Beam Instability (TMCI type)

discovered by K. Ohmi in SS simulations

QSS (Lifetrac): bunch shape in the horizontal plane at some turns



The shape (barycenter of slices) changes every turn due to betatron and synchrotron oscillations.

QSS simulations:

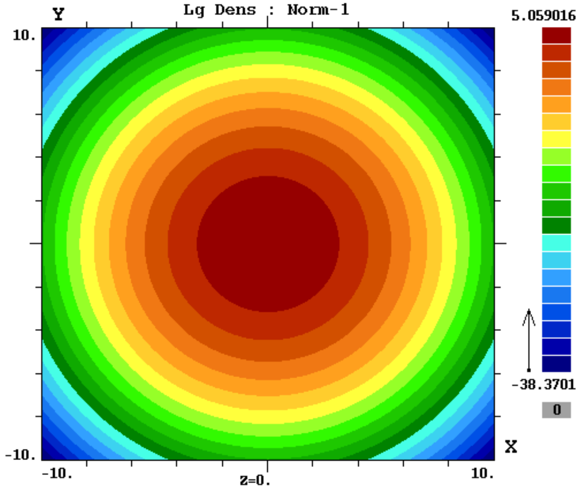
- Opposite bunch is represented as a sequence of several hundred slices with individual transverse displacements.
- Two colliding bunches are tracked simultaneously in parallel, and their shapes (transverse emittances and shifts of slices) are updated every turn.
- Particles collide with slices (not slices with slices!) for both bunches. Since D_x is small, the bunch shape in X-Z plane does not change during collision.
- The transverse distribution of slices is assumed to be Gaussian, but $\sigma_{x,y}$ depend on the azimuth.

Very good agreement was obtained between SS and QSS simulations.

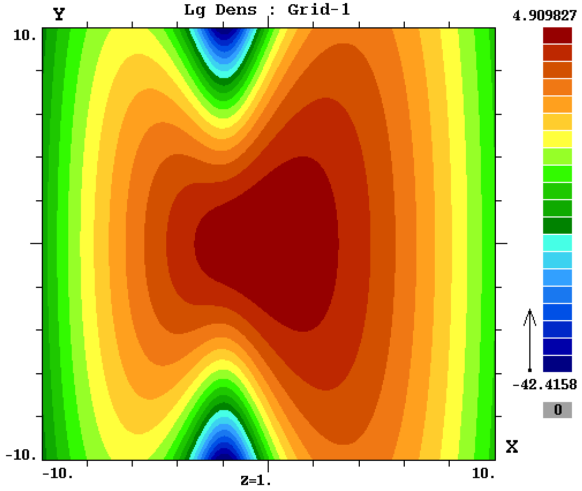
Non-Gaussian Distributions

- Non-Gaussian longitudinal beam profile can be easily implemented by changing the "weights" of slices.
- For non-Gaussian transverse distributions, we have to use grids. There are several options here:
  1) Uniform rectangular grid and PIC => we find the potential, its derivatives give us the kicks, and this is symplectic.
  2) Non-uniform and non-rectangular grid that follows the shape of hour-glass and can be much larger with moderate number of nodes. It is not suitable for solving differential equations. Instead, the beam-beam kicks are "stored" at all nodes, and high-order approximation is used between nodes. It may not be entirely symplectic, but this can be controlled.

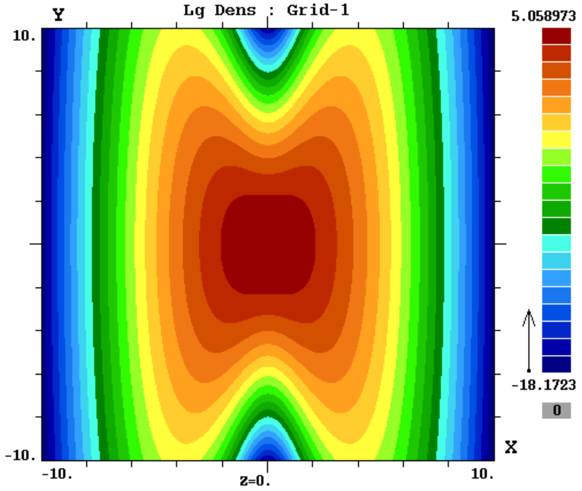
Transverse Distribution for CW Bunches



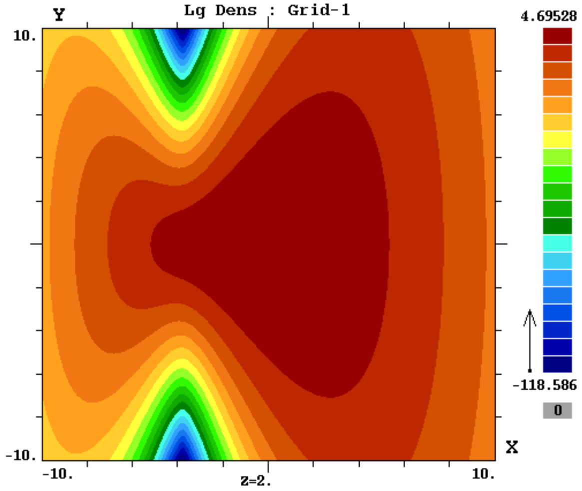
Gaussian, Z=0



Crabbed, Z=1 cm



Crabbed, Z=0



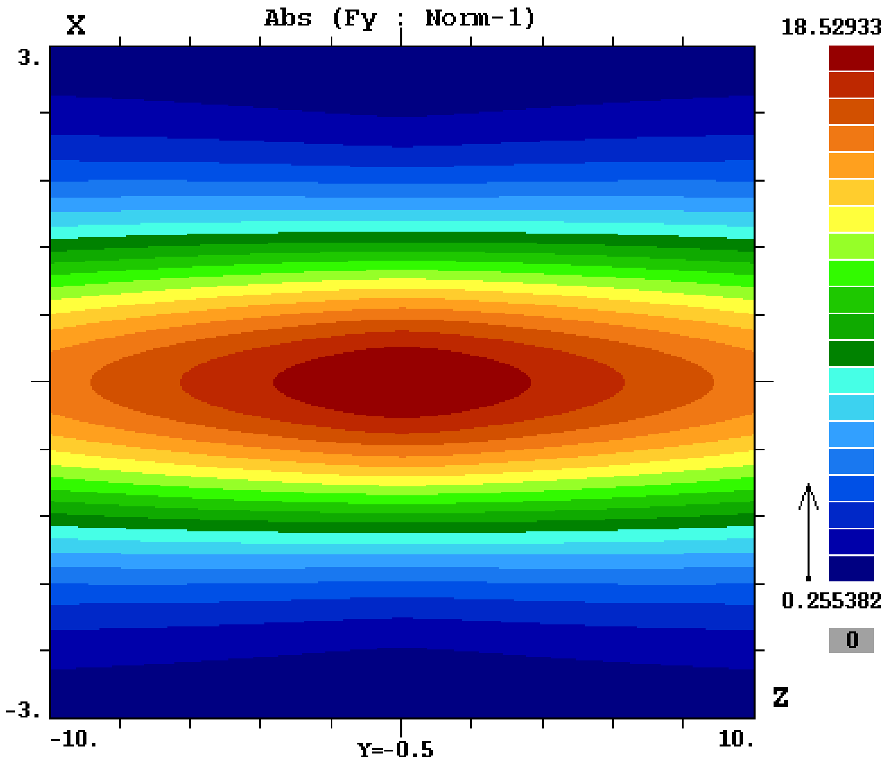
Crabbed, Z=2 cm

Log (density)

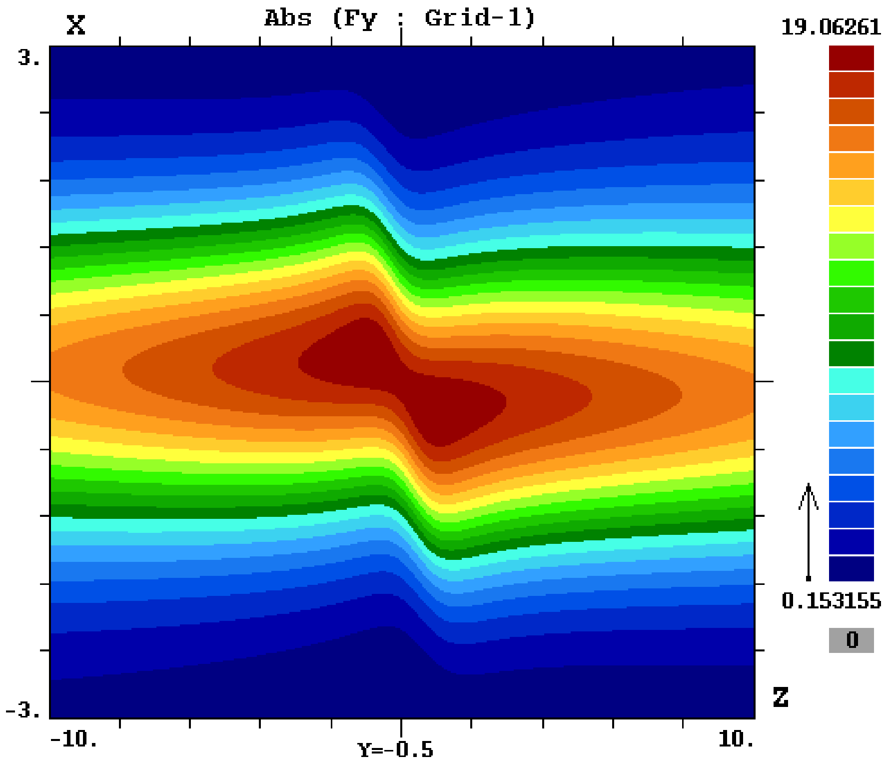
The axes are x/σ_x and y/σ_y

Z is the distance to IP

Beam-Beam Kick: Gaussian vs. CW



F_y (Gaussian, Y=-0.5 σ_y)



F_y (Crabbed, Y=-0.5 σ_y)

Next Steps

Chromatic waist to obtain monochromatization (P. Raimondi)

- The beam distribution will be more complicated. We need to build it from a realistic tracking in nonlinear lattice, all sextupoles included. This is like SS model, but we don't need to update the grid it every turn. With large statistics (many turns), the "grid noise" will be much smaller.
- At each grid point, we need to collect not only the density and average transverse momentum, but also the energy and energy spread.
- Then for every elementary particle-slice collision, we will know not only the kicks and luminosity, but also the c.m. energy and the energy spread. Finally, we will be able to produce the luminosity vs. $E_{c.m.}$ histogram, thus obtaining a realistic mono-chromatization parameter.

Thank you
for your attention.