Resistive wall impedance for pipes with arbitrary cross-section

VACI Suite

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FUTURE CIRCULAR COLLIDER

DESY.

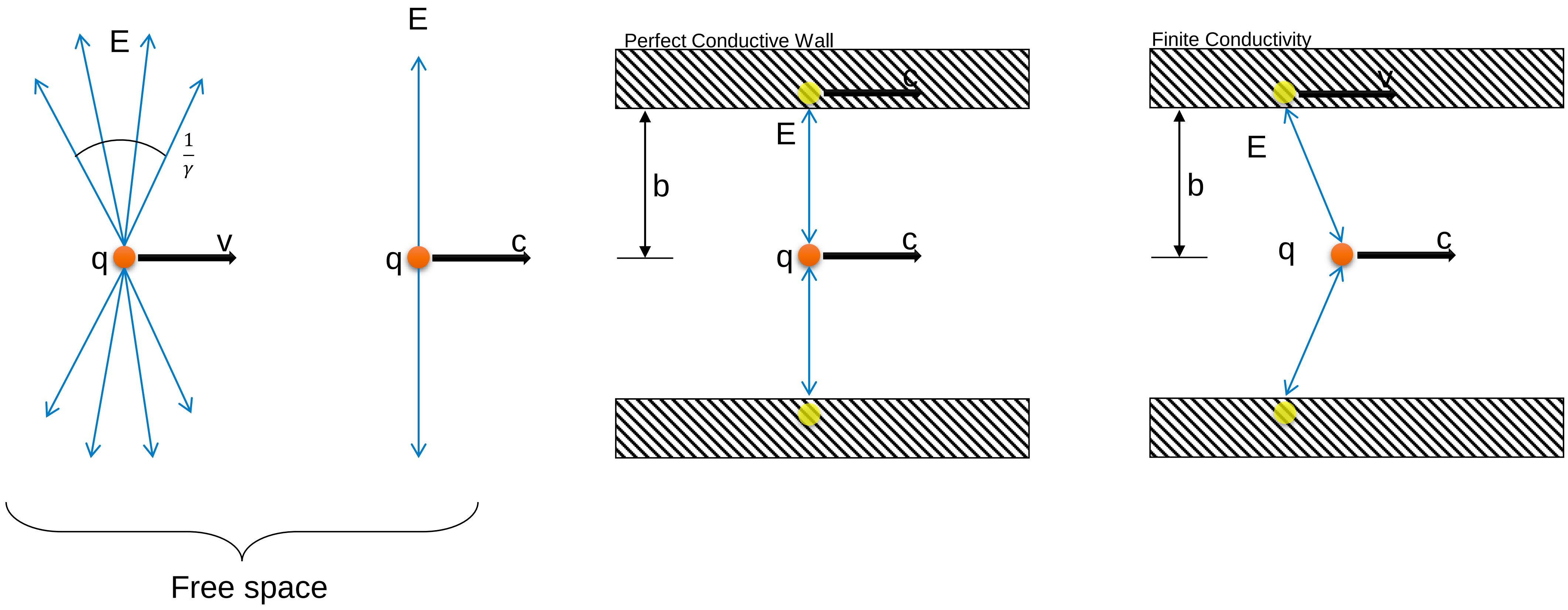
HELMHOLTZ

Contents

Resistive Wall Impedance

Electromagnetic field carried by an ultra-relativistic point charge

A simplified concept of resistive wall wake field



Chao, Alexander Wu. "Physics of collective beam instabilities in high energy accelerators." *Wiley series in beam physics and accelerator technology* (1993)

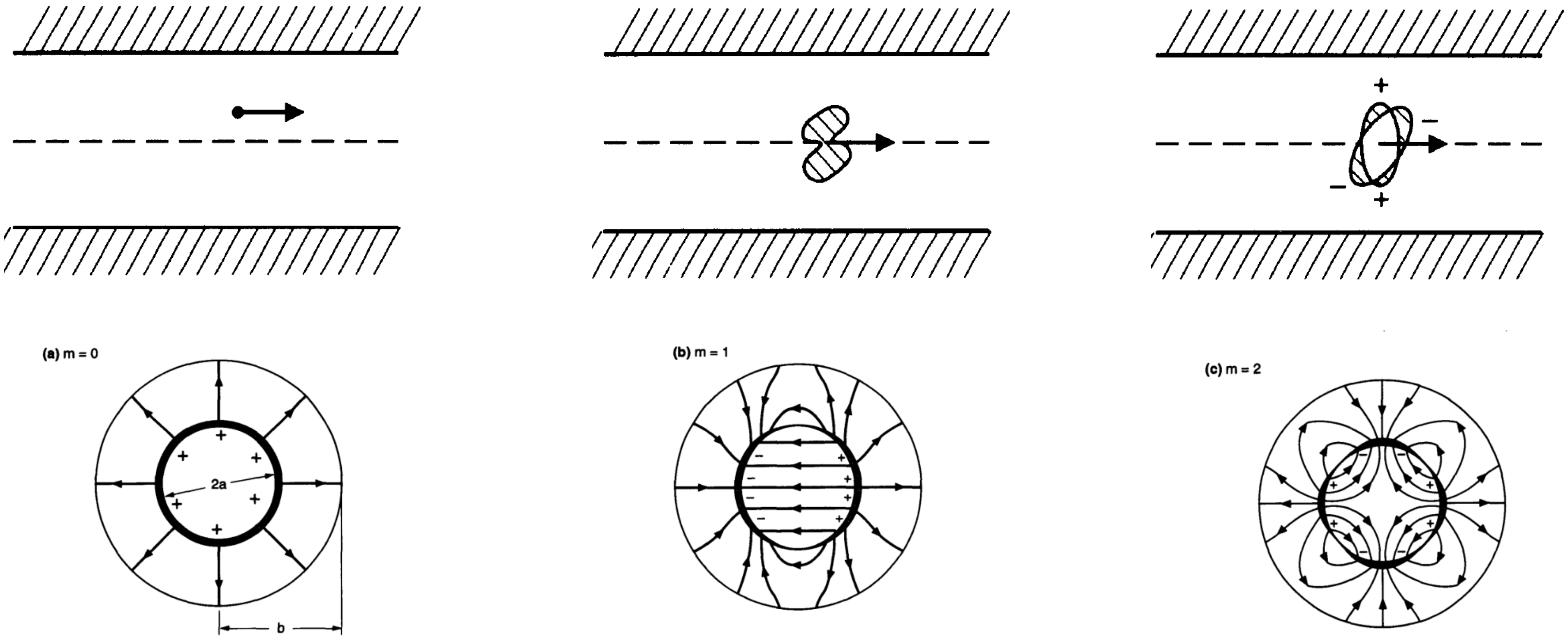
NEG coating

Why NEG is important

- To achieve ultra high vacuum (UHV) in accelerators people usually use Non-Evaporable Getter (NEG) coating on the inner side of vacuum chambers. Many accelerators such as CERN LHC, ESRF, etc. utilize this method and they successfully reached to UHV.
- One of the NEG coating that I know is TiZrV ternary alloy (such as 30% Titanium, 30% Zirconium and 40% Vanadium). A typical conductivity of such chamber materials is around $\sigma = 1.098e6$ which is around ~50 times less than copper conductivity of $\sigma = 5.87e7$.
- NEG coating may increase the resistive-wall impedance of the machine significantly.

Electric Field patterns

Due to Monopole, Dipole and Quadrupole electron distribution



A mathematical approach to RW impedance

Solving maxwell's equations

Wake field (for uniform ring)

$$E_s = -\frac{16q}{b^2}\left(\frac{1}{3}e^u \cos\sqrt{3}\,u - \frac{\sqrt{2}}{\pi}\int_0^\infty dx \frac{x^2 e^{ux^2}}{x^6+8}\right),$$

$$E_r = B_\theta = \frac{8qr}{(2\chi)^{1/3} b^3}\left(\frac{1}{3}e^u \cos\sqrt{3}\,u - \frac{1}{\sqrt{3}}e^u \sin\sqrt{3}\,u - \frac{\sqrt{2}}{\pi}\int_0^\infty dx \frac{x^4 e^{ux^2}}{x^6+8}\right)$$

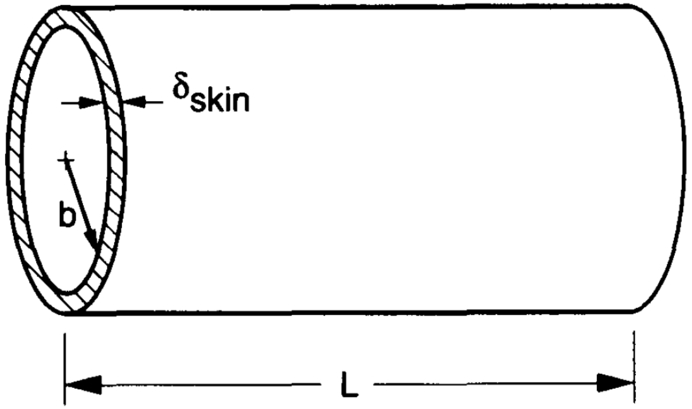
Wake Function

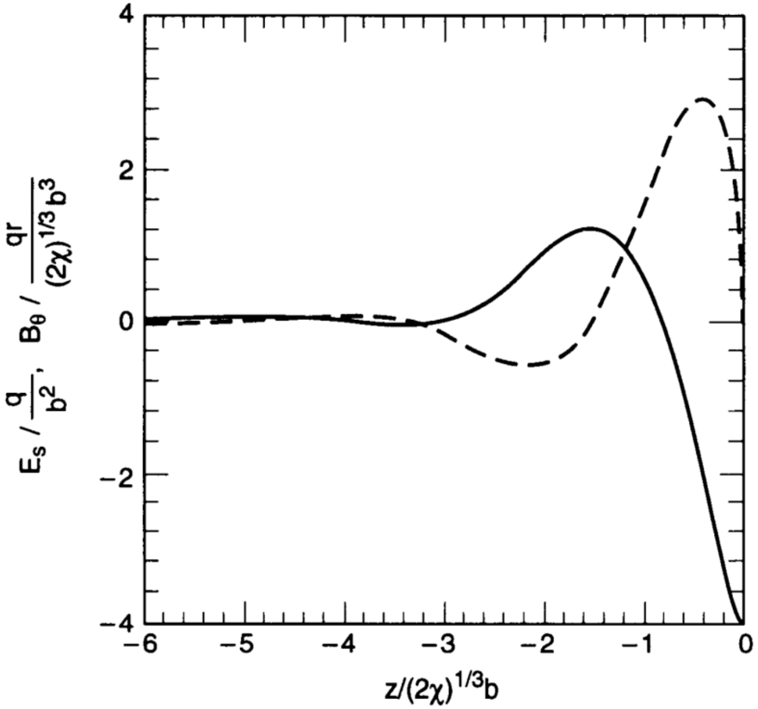
FFT

Resistive Wall Impedance (general form)

$$\frac{Z_m^{\parallel}(\omega)}{L} = \frac{\omega}{c}\frac{Z_m^{\perp}(\omega)}{L} = -\frac{1}{cI_m}A\left(\frac{\omega}{c}\right)$$

$$= \frac{4/b^{2m}}{(1+\delta_{m0})bc\sqrt{\frac{2\pi\sigma}{|\omega|}}\,[1+\mathrm{sgn}(\omega)i] - \frac{ib^2}{m+1}\omega + \frac{imc^2}{\omega}}$$





Resistive Wall Impedance

Simulation

Existing Simulation Codes

Maxwell Solvers vs Analytical Solvers

Maxwell's Equations solvers:

- ImpedanceWake2D by Mounet * (free)
- BeamImpedance2D by Niedermayer ** (free)
- Yokoya's Code *** (free)
- ECHO -1 / 2 / 3D code by Zagorodnov **** (free)
- CST Microwave Studio (commercially available)
- GDFIDL (commercially available)
- VACI Suite

Analytical formulas solvers:

- ReWall developed by Mounet et al CERN
- Numerical impedance calculations by Doliwa et al and Niedermayer
- Mathematica code developed in DESY
- CETA by Chao Li @ DESY for RW Impedance
- And …

* https://twiki.cern.ch/twiki/bin/view/ABPComputing/ImpedanceWake2D

** Niedermayer, Uwe, Oliver Boine-Frankenheim, and Herbert De Gersem. "Space charge and resistive wall impedance computation in the frequency domain using the finite element method." *Physical Review Special Topics-Accelerators and Beams* 18.3 (2015): 032001.

*** Yokoya, Kaoru. "Resistive wall impedance of beam pipes of general cross section." Part. Accel. 41.KEK-Preprint-92-196 (1993): 221-248.

**** https://echo4d.de/

VACI suite a versatile tool to calculate RW impedance

Introduction

VACI (VAcuum Chamber Impedance) suite

Creating Geometry → Specifying Physical Groups → Creating Mesh → Converting Mesh to a readable format → Applying the Finite Element Method → Post Processing the results → Plotting the Results

Resistive Wall impedance

Maxwell's Equations

$$\begin{aligned} \mathrm{div}\vec{D} &= \rho_m, \\ \vec{\mathrm{curl}}\vec{H} - j\omega\vec{D} &= \vec{J}_m, \\ \vec{\mathrm{curl}}\vec{E} + j\omega\vec{B} &= 0, \\ \mathrm{div}\vec{B} &= 0, \end{aligned}$$

$$\rho(r,z;\omega) = J_z(r,z;\omega)$$

$$J_n = \frac{Q_n}{A}\,\sigma(r;\ a,b)e^{in\theta}e^{-iks}$$

where:

$\sigma(r;\ a,b)$ means particles are in a ring with a thicknes of (b−a)

A is the ring area

θ is the angle distribution of electrons around the ring

$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial}{\partial t}\vec{A}$ And $\vec{B} = \vec{\nabla}\times\vec{A}$

$\vec{\nabla}.\vec{A} = 0$ Coulomb gauge

$\partial_t \Longrightarrow -i\omega$ Fourier Transform

$\partial_z \Longrightarrow -i\,\omega/v$ Long pipe Appr.

$$\begin{cases} \vec{\nabla}.\left(\varepsilon\vec{\nabla}\varphi\right) = \rho_m \\ \vec{\nabla}\times\left({}^{1}\!/_{\mu}\,\vec{\nabla}\times\vec{A}\right) - \varepsilon\omega^2\vec{A} = -J_n\hat{e}_z + i\varepsilon\omega\vec{\nabla}\varphi \end{cases}$$

Resistive Wall impedance

Based on: *Robert L. Gluckstern, Elias Métral, and Uwe Niedermayer*

$$W_0'(z) = -\frac{1}{q\,Q}\int_0^L F_s\,ds = -\frac{1}{Q}\int_0^L E_s\,ds$$

$$Z_m^{//}(\omega) = -\int_{-\infty}^{+\infty} W_m'(z)\,e^{jkz}\,\frac{dz}{\upsilon} = \int_{-\infty}^{+\infty} W_m'(t)\,e^{jks}e^{-j\omega t}\,dt$$

$$W_1(z) = -\frac{1}{q\,Q\,a}\int_0^L F_x\,ds = -\frac{1}{Q\,a}\int_0^L \left(E_x - \upsilon\,B_y\right)ds$$

$$Z_m^{\perp}(\omega) = j\int_{-\infty}^{+\infty} W_m(z)\,e^{jkz}\,\frac{dz}{\upsilon} = -j\int_{-\infty}^{+\infty} W_m(t)\,e^{jks}e^{-j\omega t}\,dt$$

Resistive Wall impedance

Based on: *Robert L. Gluckstern, Elias Métral, and Uwe Niedermayer*

$$\vec{\underline{Z}}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, \omega) = -\int_{-\infty}^{\infty} \vec{W}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, s) e^{-i\omega s/v} \frac{\mathrm{d}s}{v}.$$

Single particle

$$\vec{\underline{Z}}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, \omega) = -\frac{1}{q_1 q_2} \int_{-\infty}^{\infty} \vec{\underline{F}}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, \omega) e^{+i\omega z/v} \mathrm{d}z,$$

one should note that the integral is not a Fourier transform, but the wake integration in the frequency domain.

$$\vec{J}_s(\vec{r}_{\perp}, z, t) = q_1 \sigma(\vec{r}_{\perp}) \delta(z - vt) \vec{v}$$

$$\underline{\vec{J}}_s(\vec{r}_{\perp}, z, \omega) = q_1 \sigma(\vec{r}_{\perp}) e^{-i\omega z/v} \vec{e}_z.$$

Integrating over the beam in FD
Like Convolution in TD

$$\vec{\underline{Z}}(\omega, \vec{r}_2^{\perp}) = -\frac{1}{q_1 q_2} \int_{\text{beam}} \vec{\underline{F}}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, \omega) e^{i\omega z/v} \sigma(\vec{r}_1^{\perp}) \mathrm{d}r_1^{\perp} \mathrm{d}z.$$

$$\underline{Z}_{\|}(\omega) = -\frac{1}{q^2} \int_{\text{beam}} \vec{\underline{E}} \cdot \vec{\underline{J}}_s^{*} \mathrm{d}V.$$

Resistive Wall impedance

Based on: *Robert L. Gluckstern, Elias Métral, and Uwe Niedermayer*

Non axis-symmetric structures => A current density with some azimuthal Fourier component may create an electromagnetic field with various different azimuthal Fourier components => A more general beam coupling impedance is defined in order to treat coupling of different azimuthal Fourier components

$$Z_{m,n}(\omega) = \int dv\, E_m * J_n^*$$

over the beam area

$$J_n = \frac{Q}{2\pi a^{|n|+1}} \delta(r-a)\, e^{jn\vartheta}\, e^{-jks}$$

and $m, n = 0, \pm 1, \pm 2, \dots$

assuming the principle of superposition:

$$\left.\begin{aligned} \bar{J}_m &= J_m + J_{-m} \\ \bar{E}_m &= E_m + E_{-m} \end{aligned}\right\} \Rightarrow \quad \bar{Z}_m(\omega) = -\frac{1}{Q^2} \int dV \left(E_m + E_{-m}\right)\left(J_m^* + J_{-m}^*\right)$$

$$\bar{Z}_m = Z_{m,m} + Z_{m,-m} + Z_{-m,m} + Z_{-m,-m}$$

Resistive Wall impedance

Based on: *Robert L. Gluckstern, Elias Métral, and Uwe Niedermayer*

Applying Panofksy-Wenzel theorem, one can obtain transverse impedance based on Longitudinal one:

$$k\,Z^{\perp} = \nabla_2^{\perp}\,Z$$

Here, Indice of 2 means witness particle

$$k\,Z_x = \left(Z_{0,1} + Z_{0,-1}\right) + x_1\,\bar{Z}_x + j\,y_1\left(-Z_{1,-1} - Z_{1,1} + Z_{-1,-1} + Z_{-1,1}\right) + 2\left(Z_{0,2} + Z_{0,-2}\right)x_2 + 2\left(Z_{0,2} - Z_{0,-2}\right)j\,y_2$$

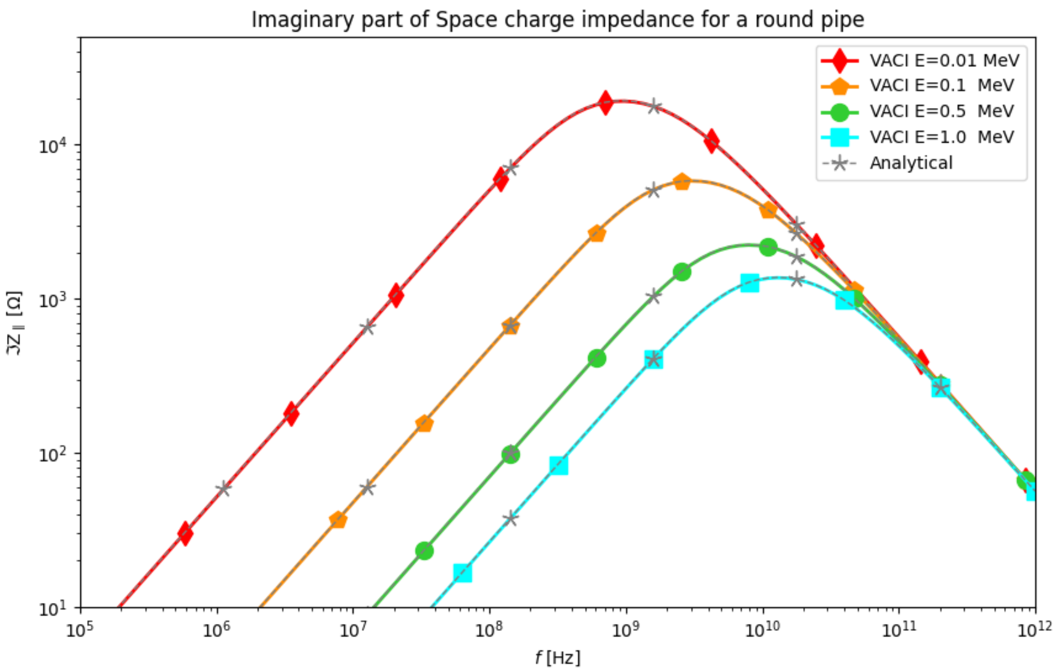
$$k\,Z_y = j\left(Z_{0,1} - Z_{0,-1}\right) + y_1\,\bar{Z}_y + j\,x_1\left(-Z_{1,-1} + Z_{1,1} - Z_{-1,-1} + Z_{-1,1}\right) - 2\left(Z_{0,2} + Z_{0,-2}\right)y_2 + 2\left(Z_{0,2} - Z_{0,-2}\right)j\,x_2$$

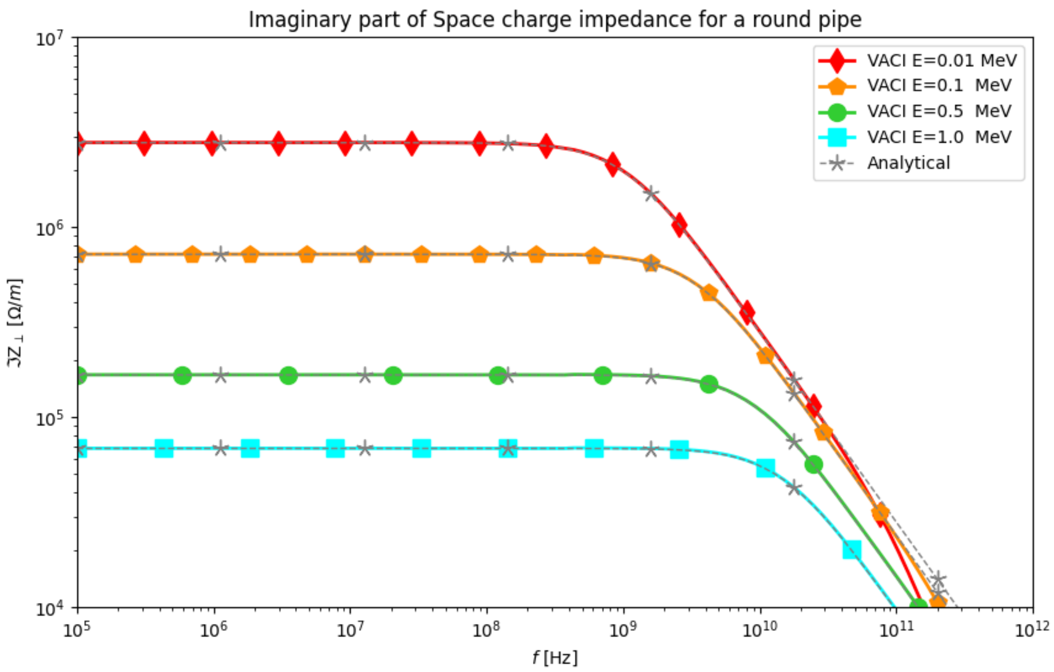
$$Z_x^{\text{driving}} = \bar{Z}_x / k \qquad Z_y^{\text{driving}} = \bar{Z}_y / k \qquad Z^{\text{detuning}} = -2\left(Z_{0,2} + Z_{0,-2}\right) / k$$

VACI Results

VACI results for Space-Charge

Round pipe

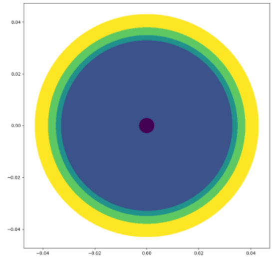


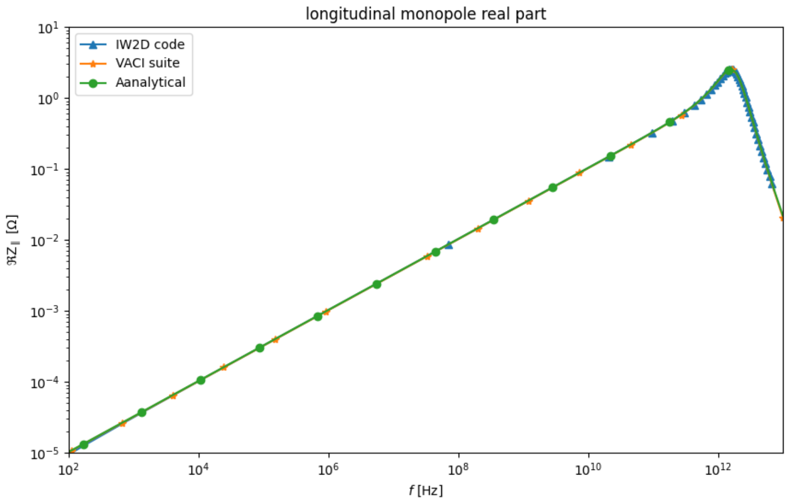


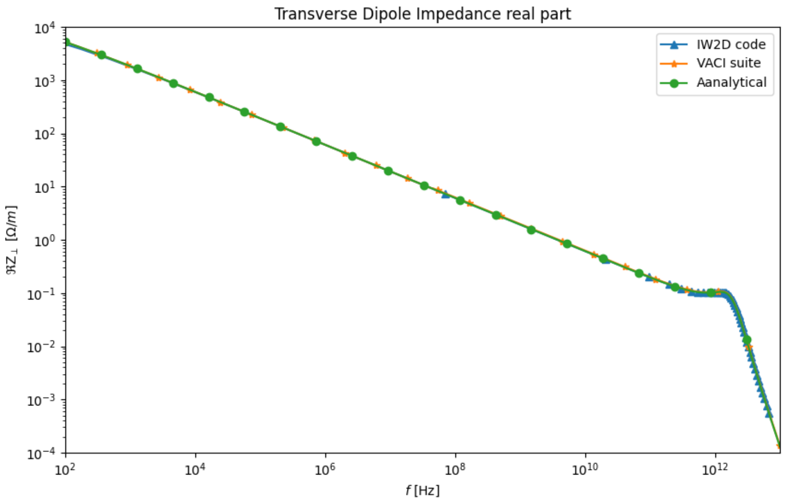
Gluckstern, Robert L. "Analytic methods for calculating coupling impedances." (2000).

VACI results for Round pipe

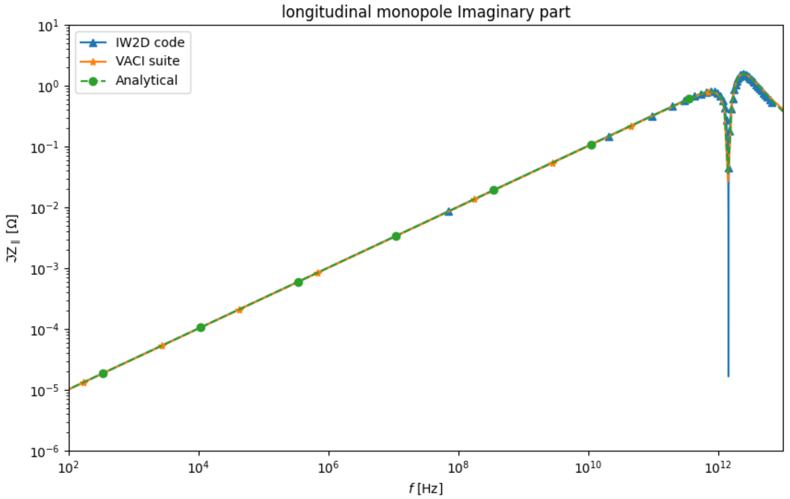
Impedance calculation

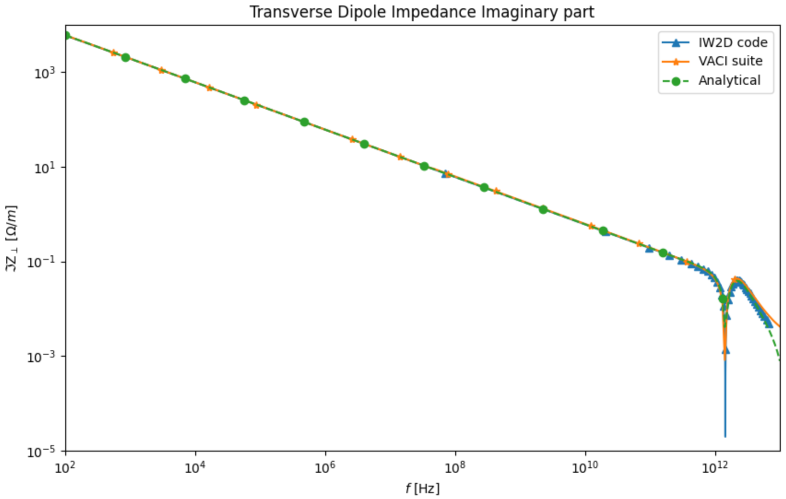






Energy: 15 GeV,
Round Pipe: r = 35 mm
Length = 1 m





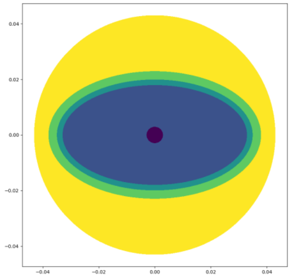
VACI results for Oval pipe

Impedance calculation

Energy: 15 GeV,
Ellipese pipe: r1 =35 mm− r2=20 mm,
Round Pipe: r = 20 mm

Yokoya's Form Factors:

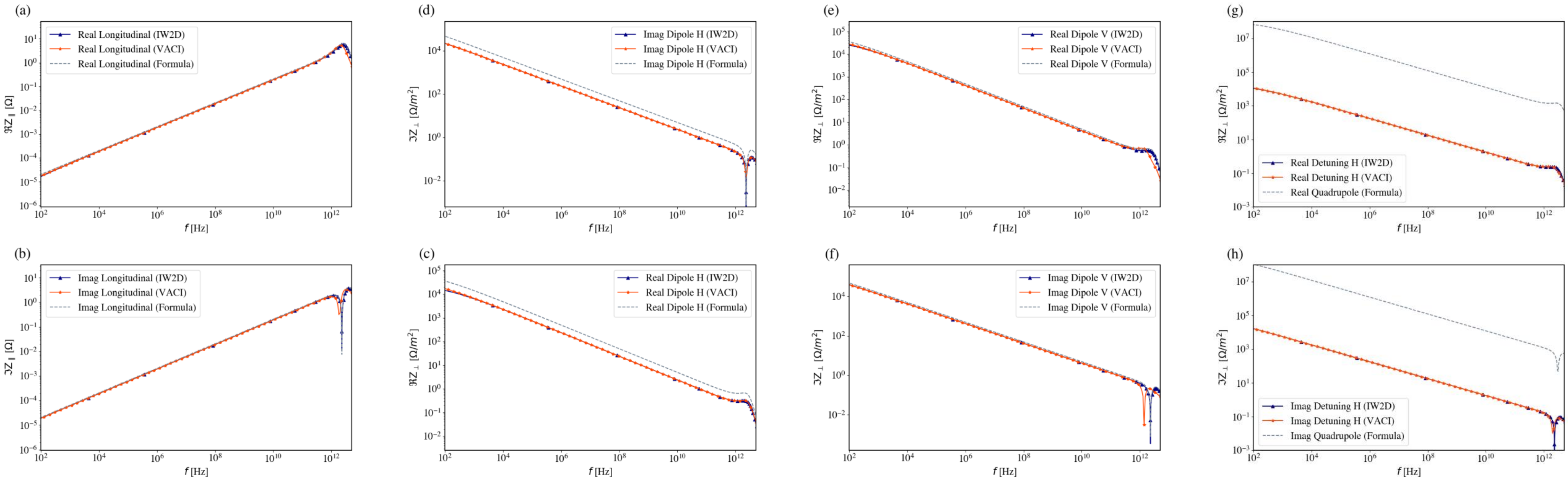
R [mm]	Long	X dip	Y dip	X quad	Y quad
20	1.0	0.46323	0.84038	-0.37701	0.38219



Monopole

Dipole

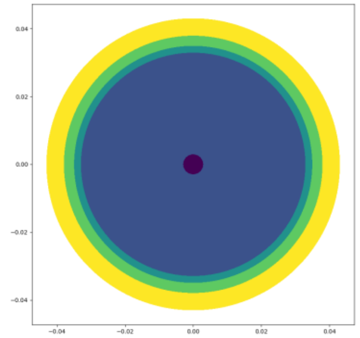
Detuning

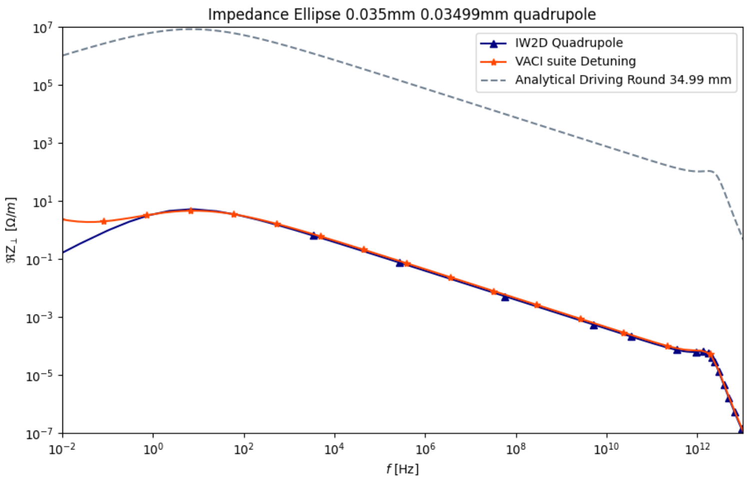


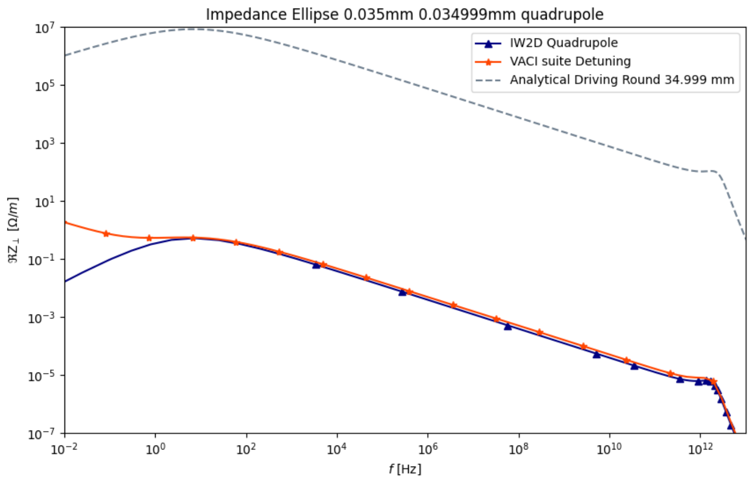
VACI results for Semi-Round pipe

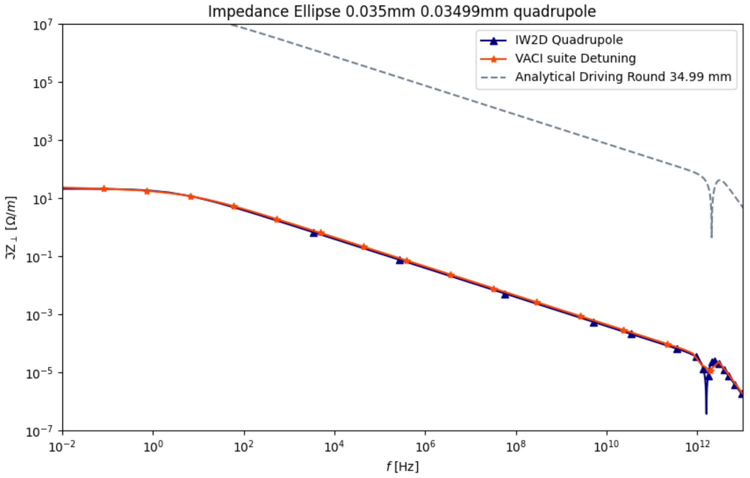
Detuning Impedance calculation

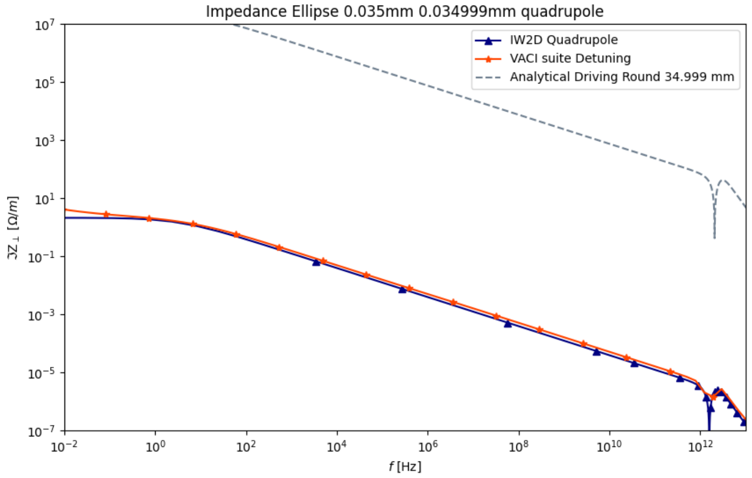
Energy: 15 GeV,
Ellipese pipe: r1 =35.00 mm– r2=34.99 & 34.999 mm
Round Pipe: r = 35.00 mm









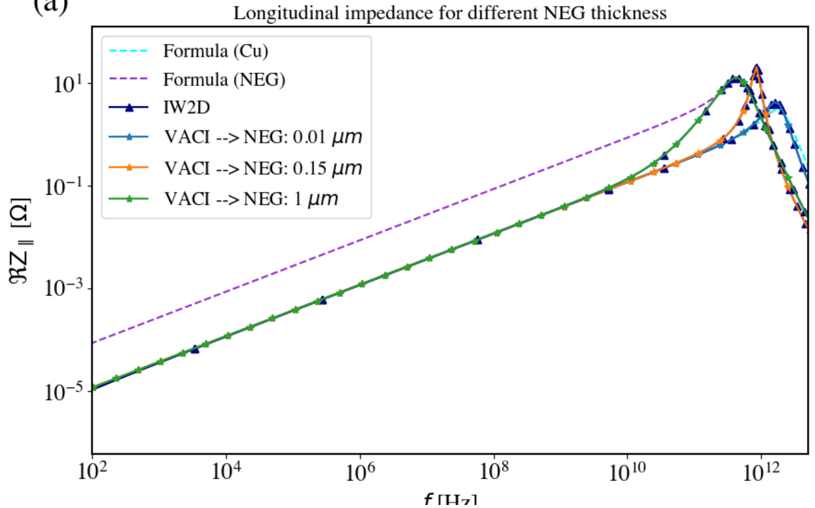


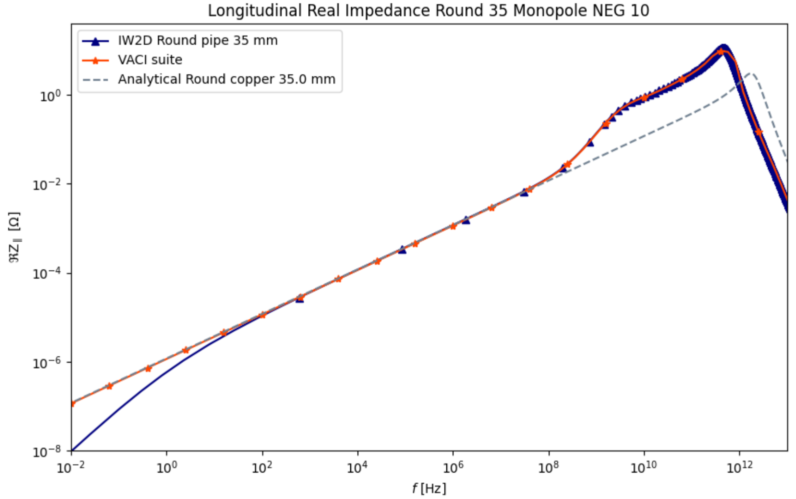
VACI results for Multi-Layer vacuum chamber

Impedance calculation of Round pipe With NEG coating

NEG properties:
$\sigma = 1.098e6$,
Thichness: 10 μm
Material: TiZrV alloy

(a)





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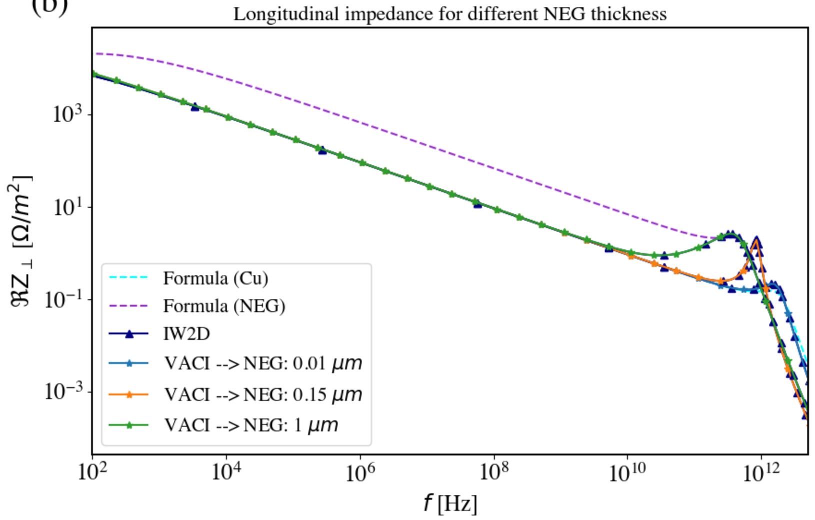
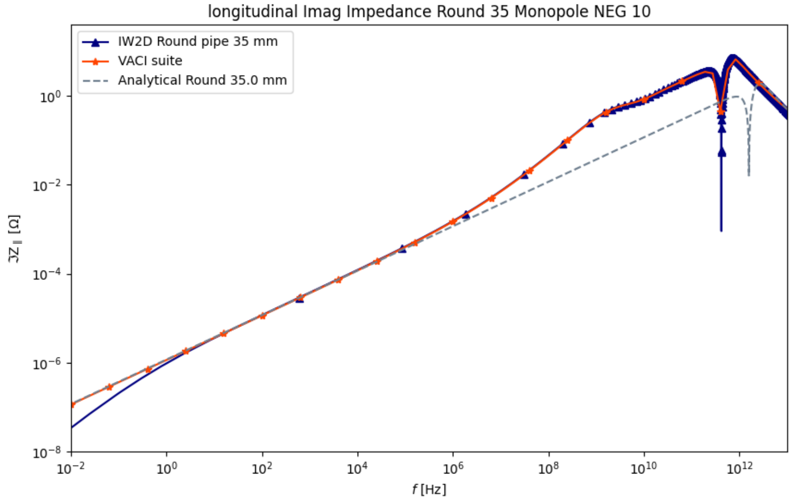




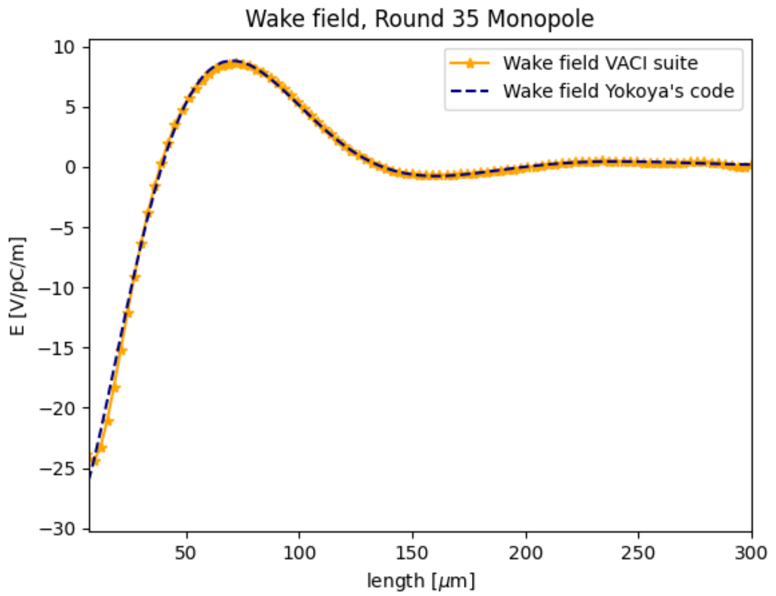
VACI also can give results in time domain

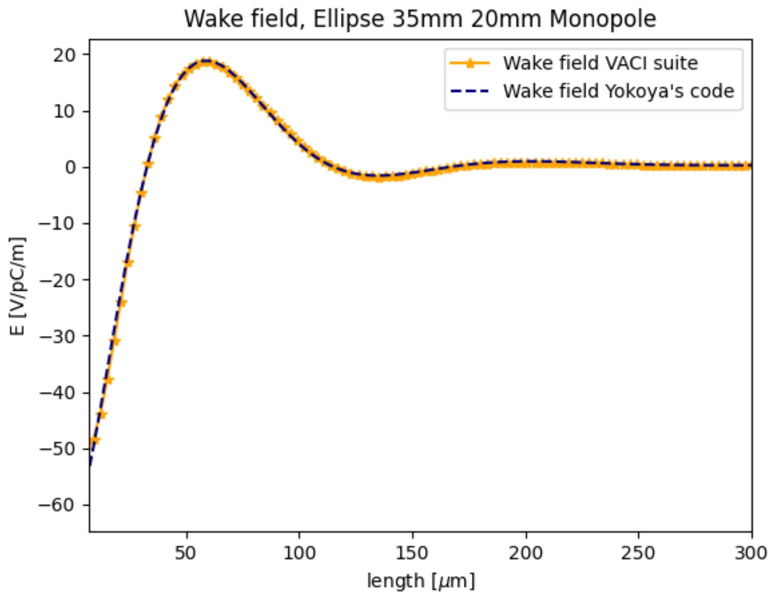
Longitudinal Wake field (E_z)

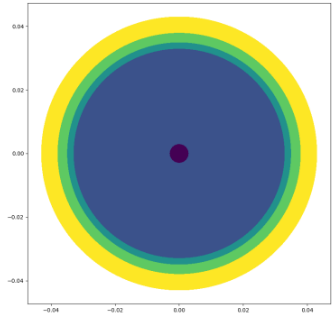
iFFT method:

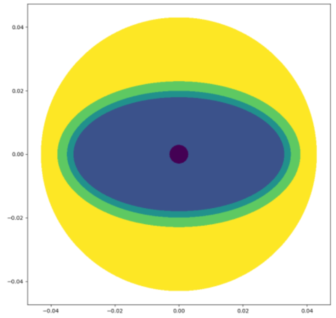
uneven sampling and a piecewise polynomial interpolation (cubic Hermite interpolation)

{Based on *Nicolas Mounet* Ph.D. thesis + some small upgrades}



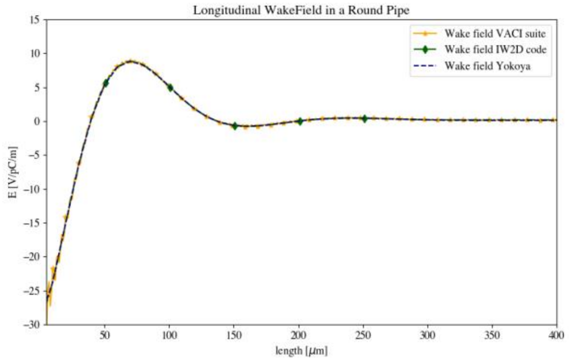


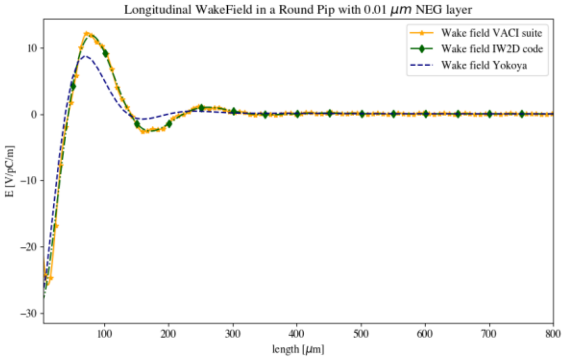


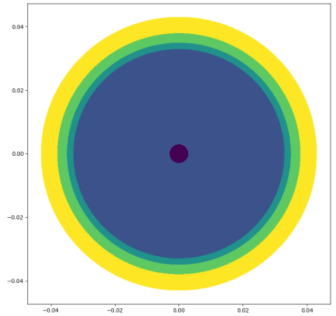


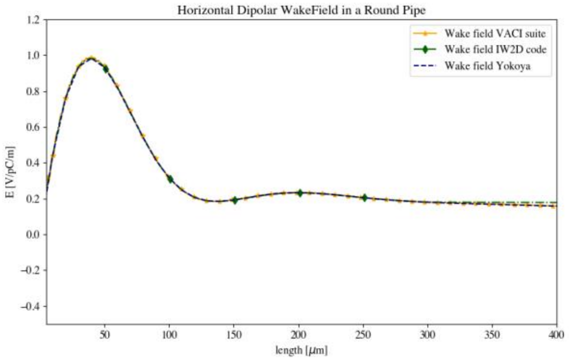
Mounet, Nicolas. *The LHC transverse coupled-bunch instability*. No. THESIS. EPFL, 2012.

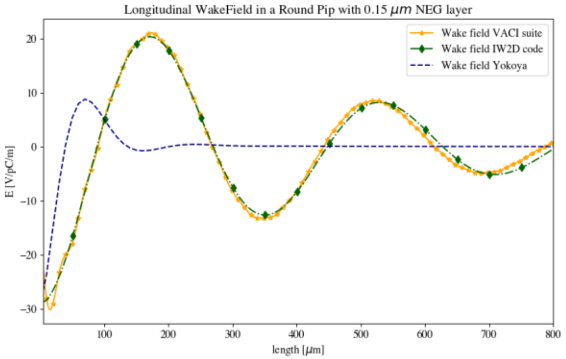
VACI also → Wakefields with and without NEG

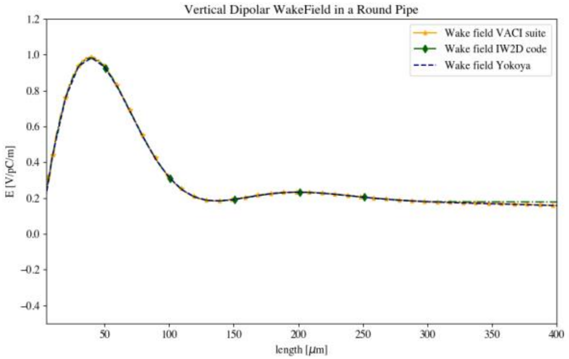


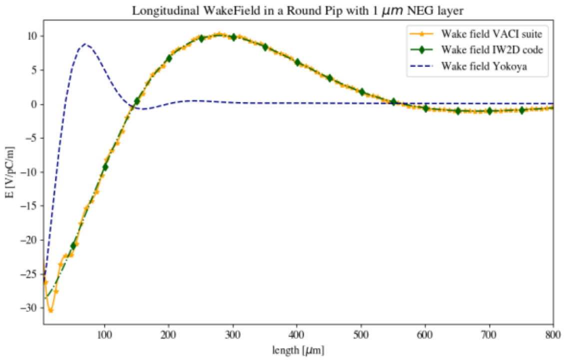












VACI also → Wakefields with and without NEG

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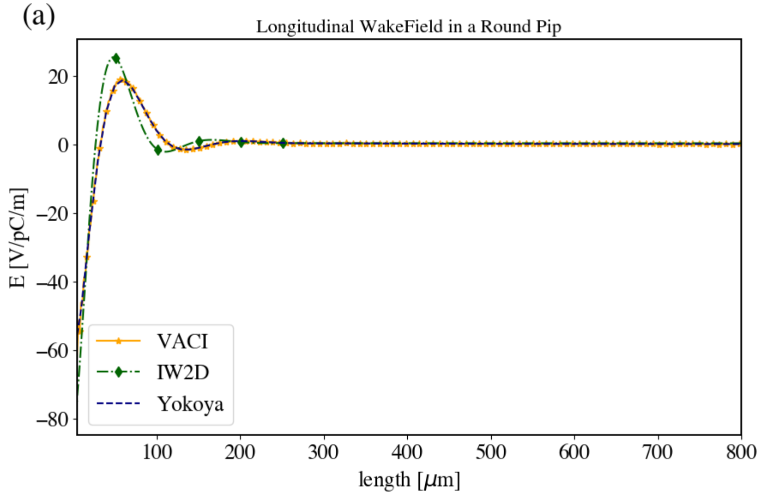

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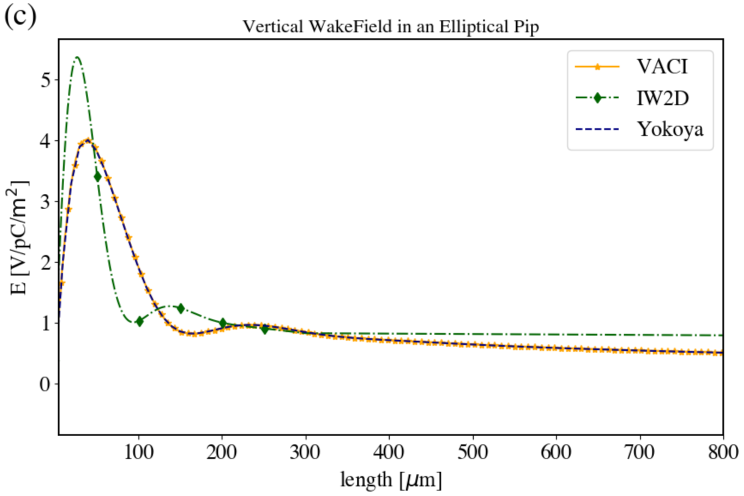

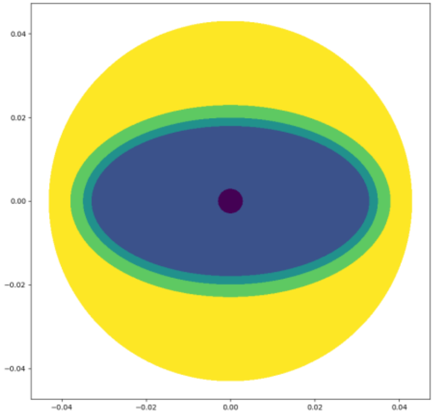

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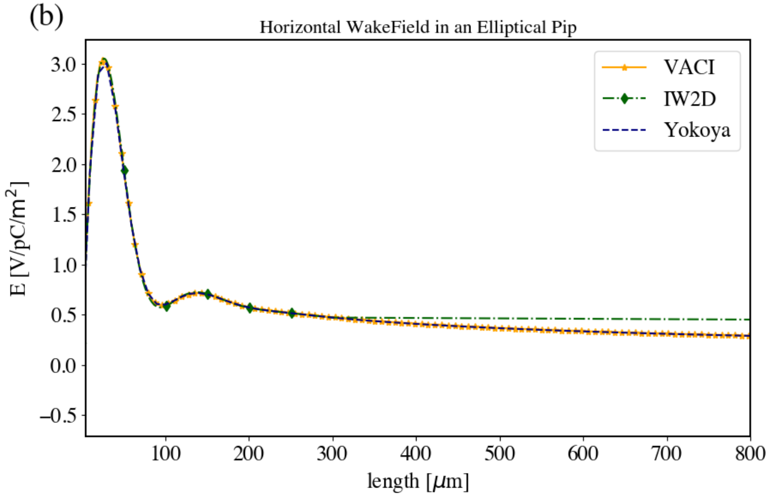

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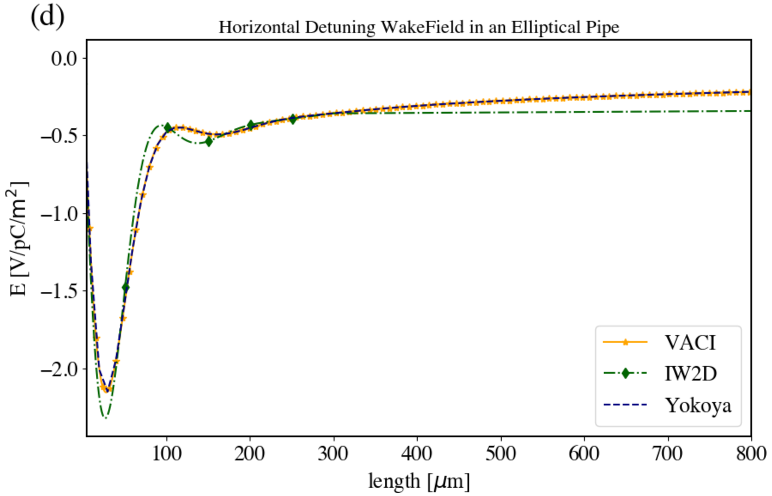

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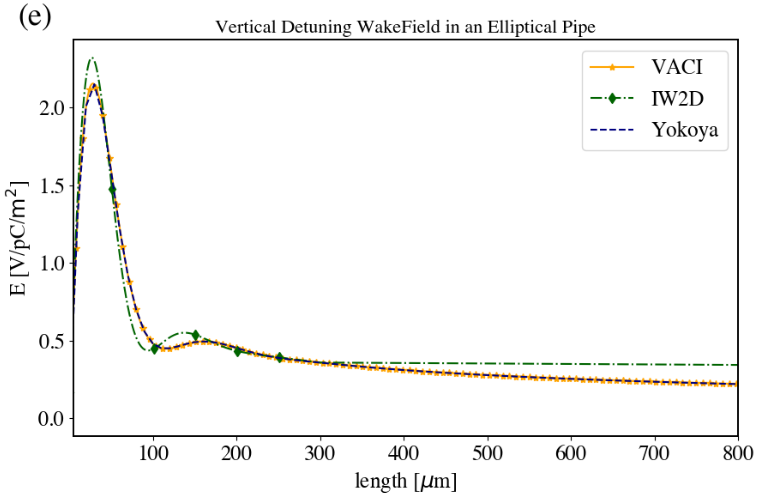
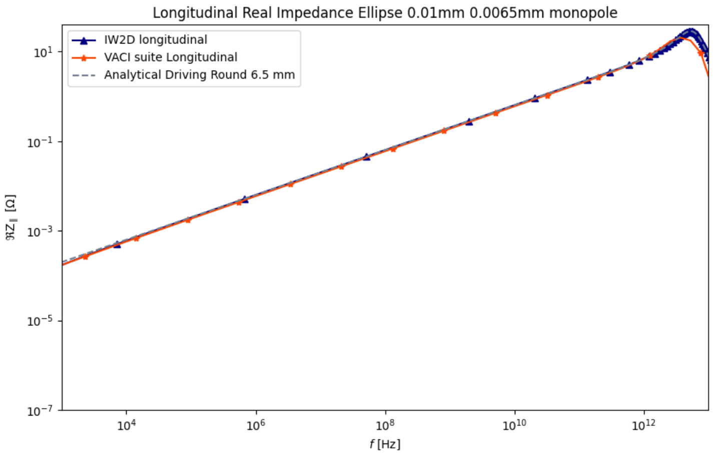


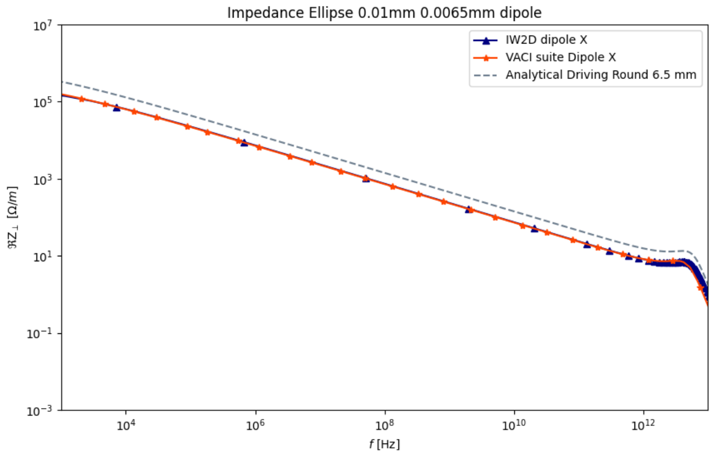
Undulator chamber

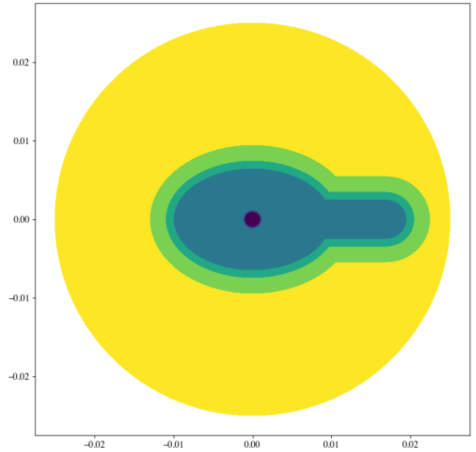
PETRA IV

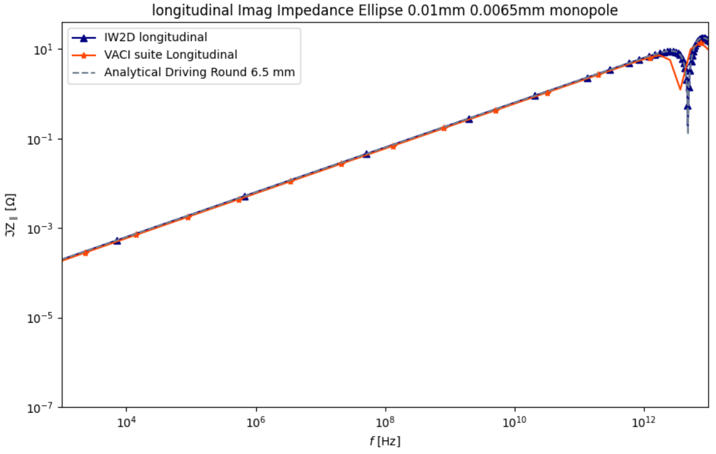
VACI results for PETRA IV

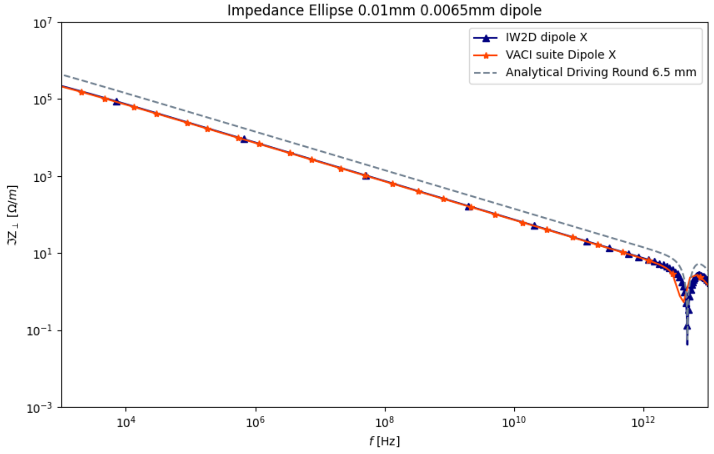
Monopole and Dipole impedances







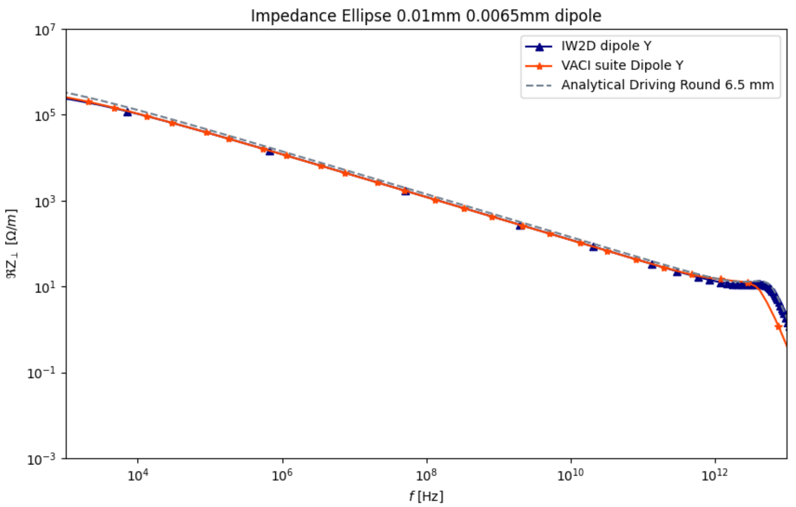


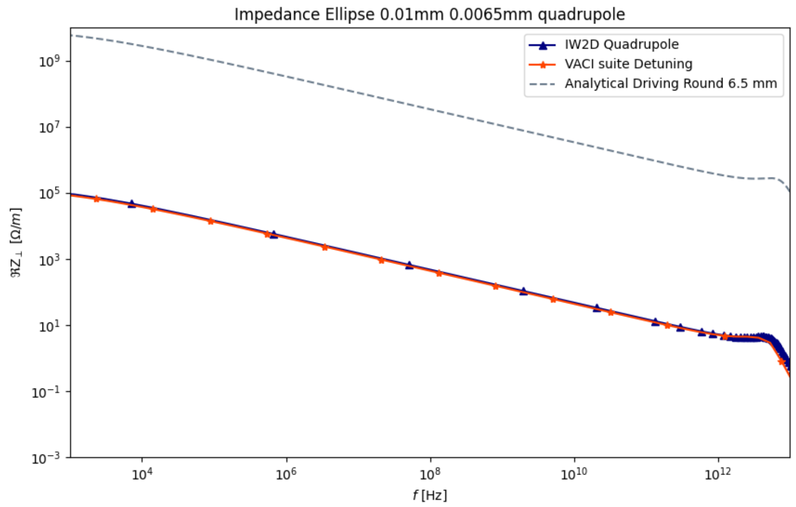


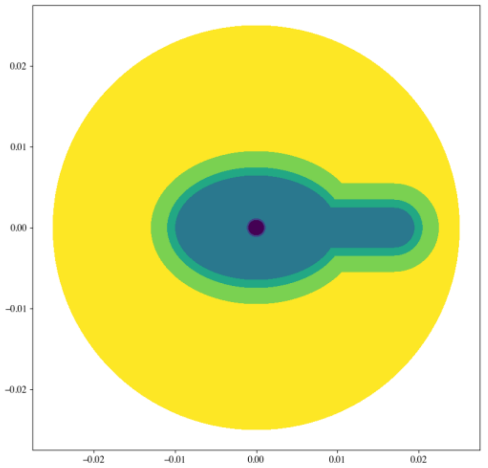
Copper Vacuum chamber
Length: 1m
No NEG considered.

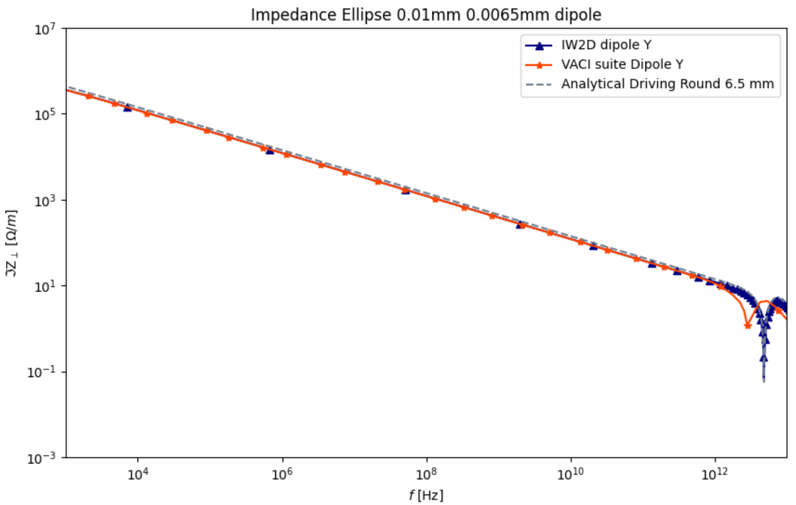
VACI results for PETRA IV

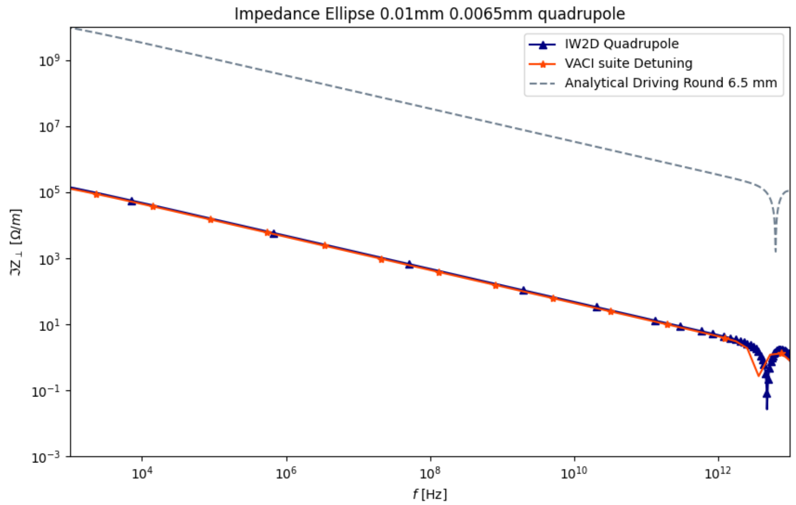
Dipole and Detuning impedances







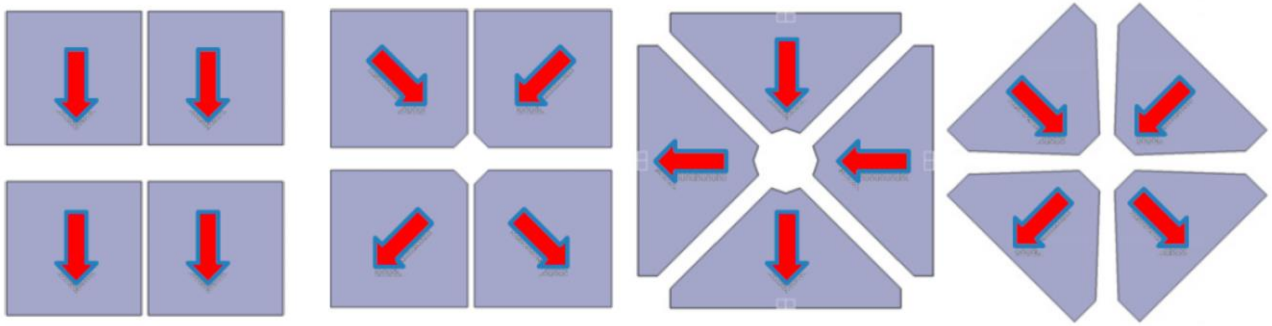




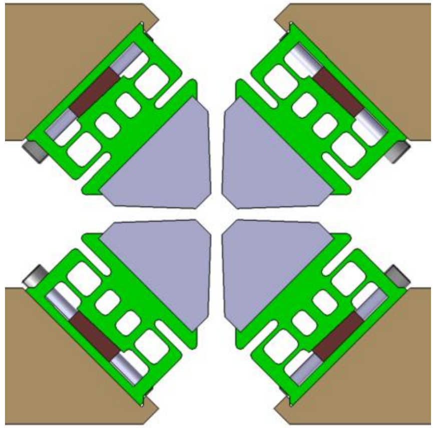
Undulator APPLE III
PETRA IV

Apple III

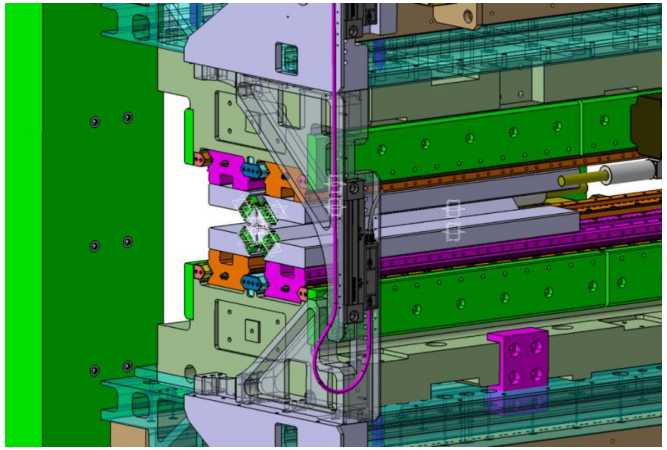
APPLE Designs



APPLE II, APPLE III [1], DELTA [2], proposed SwissFEL UE40 [3].



UE40

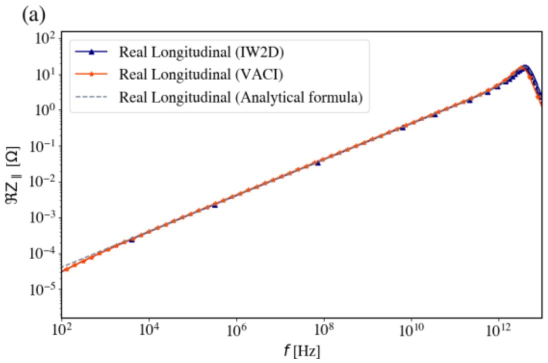


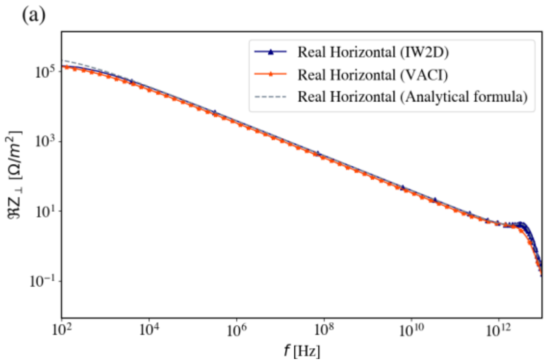
[1] *J. Bahrdt et al., "Undulators for the BESSY Soft XRay-FEL", FEL2004, Trieste, 2004 , pp.610-613.*
[2] *A. Temnykh, "Delta undulator for Cornell energy recovery linac", Physical Review Special Topics Accelerators and Beams 11, 120702 (2008).*
[3] *Schmidt, Th, et al. "Magnetic design of an APPLE III undulator for SwissFEL." Proceedings of FEL. 2014.*

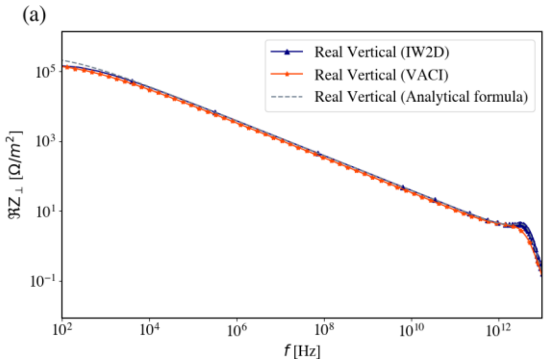
VACI results for Star shape pipe

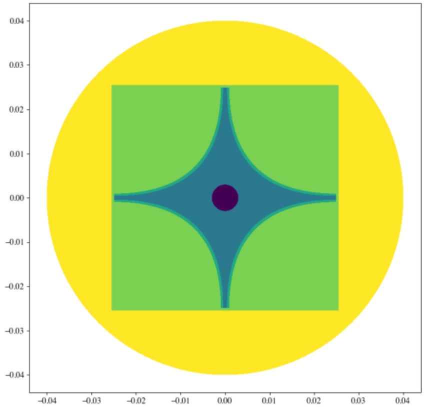
Monopole and dipole impedance

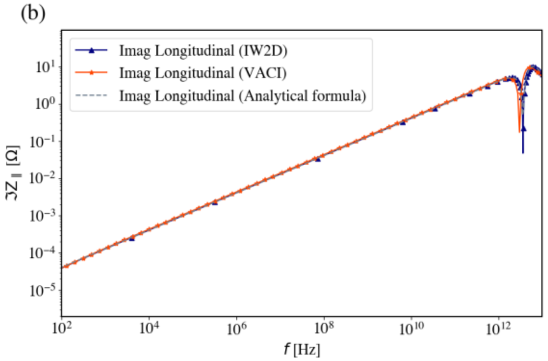
Copper Vacuum chamber
Length: 1m
No NEG considered.

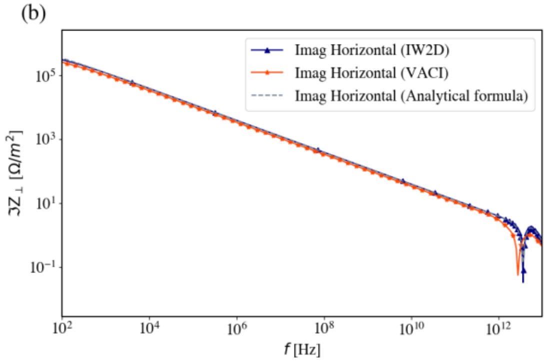


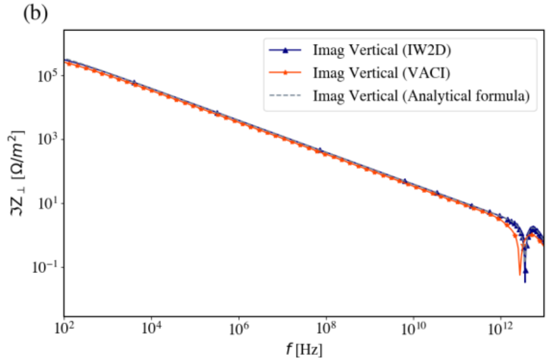


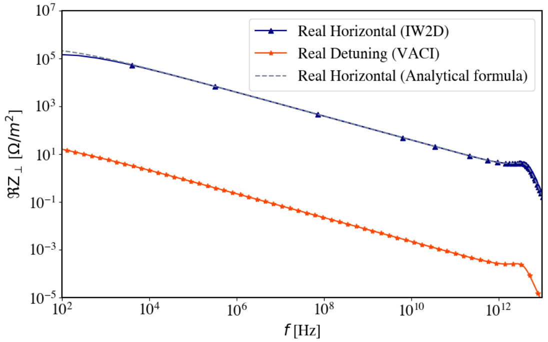






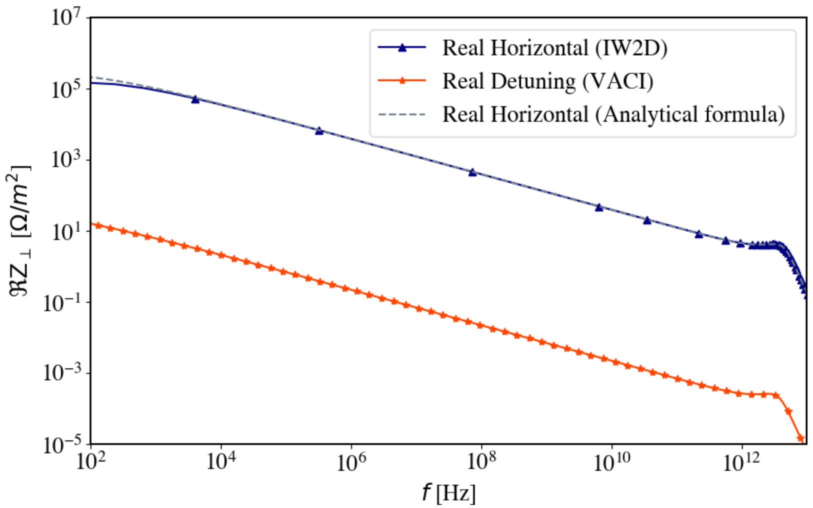


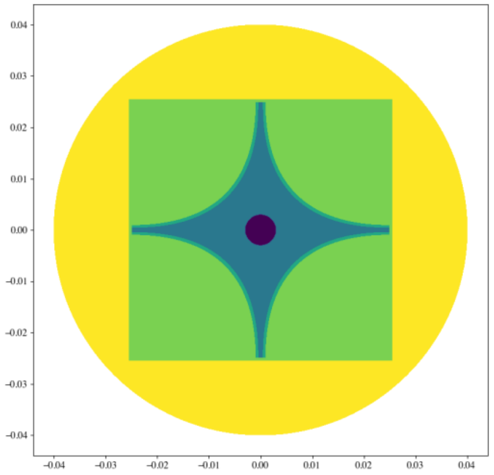


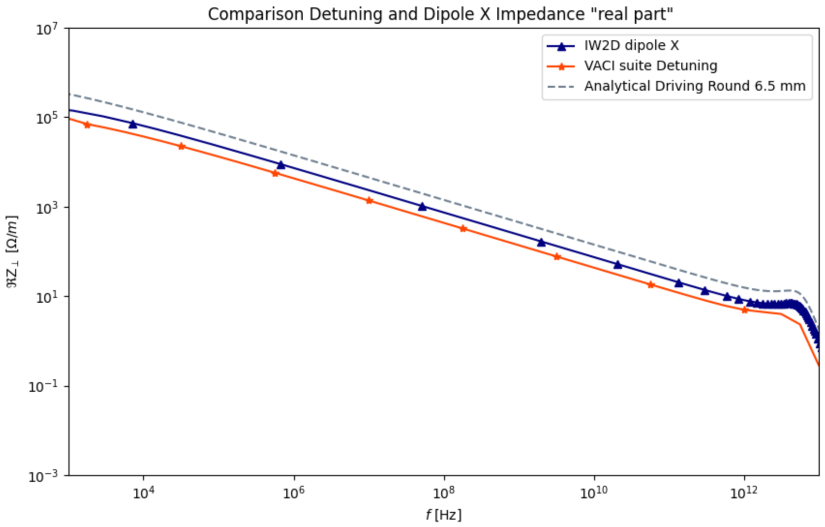


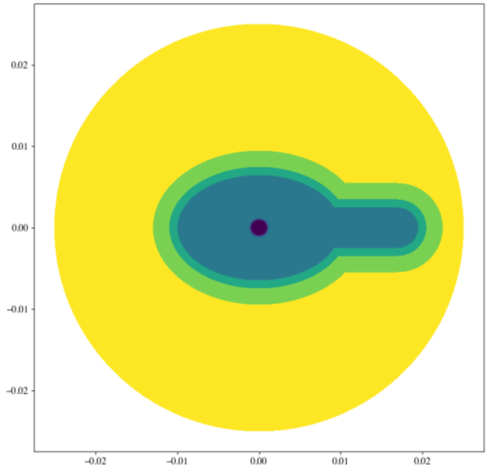
VACI results for star shape vs Petra IV

Dipole and Detuning impedance









FCC Booster elements
and vacuum chamber

FCC booster and main rings Geometries

Impedance sources

I. Beam pipes and Resistive Wall Impedance

II. RF Cavities (No. 56 in a 4-cell array)

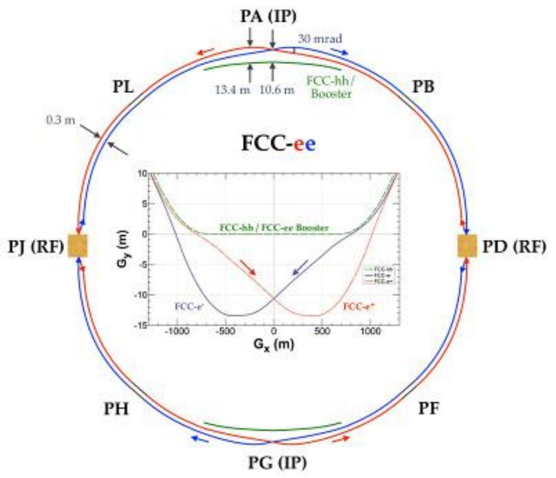
III. RF Cavity Tapers (No. 14 double tapers)

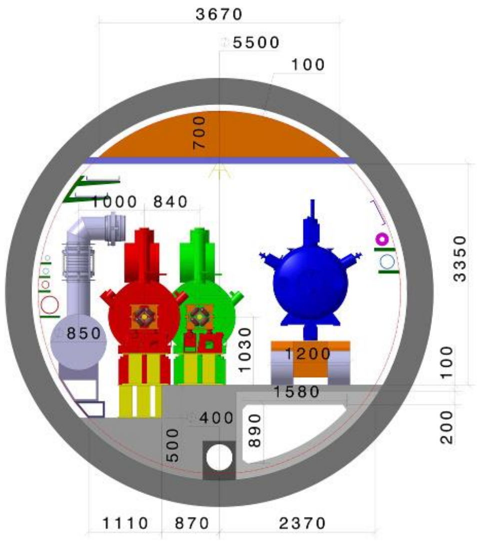
IV. Synchrotron Radiation (SR) absorbers

V. Collimators (No. 20)

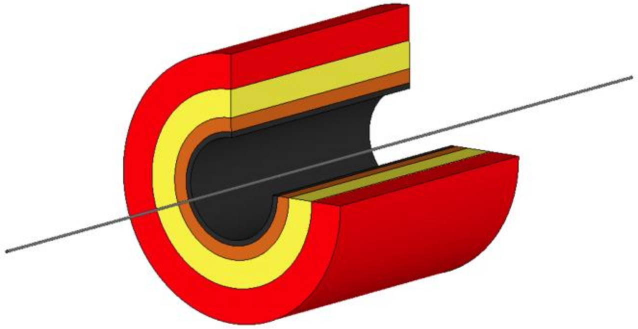
VI. Beam Position Monitors (No. 4000)

VII. Comb-Type RF shielding for bellows (No. 8000)

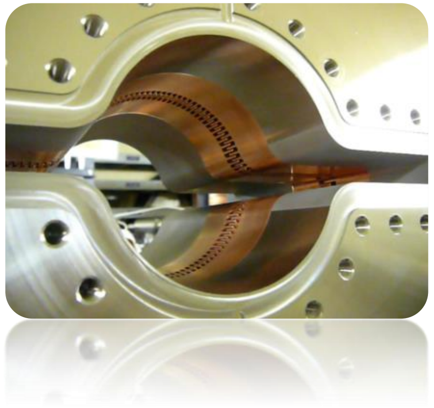




Booster ring



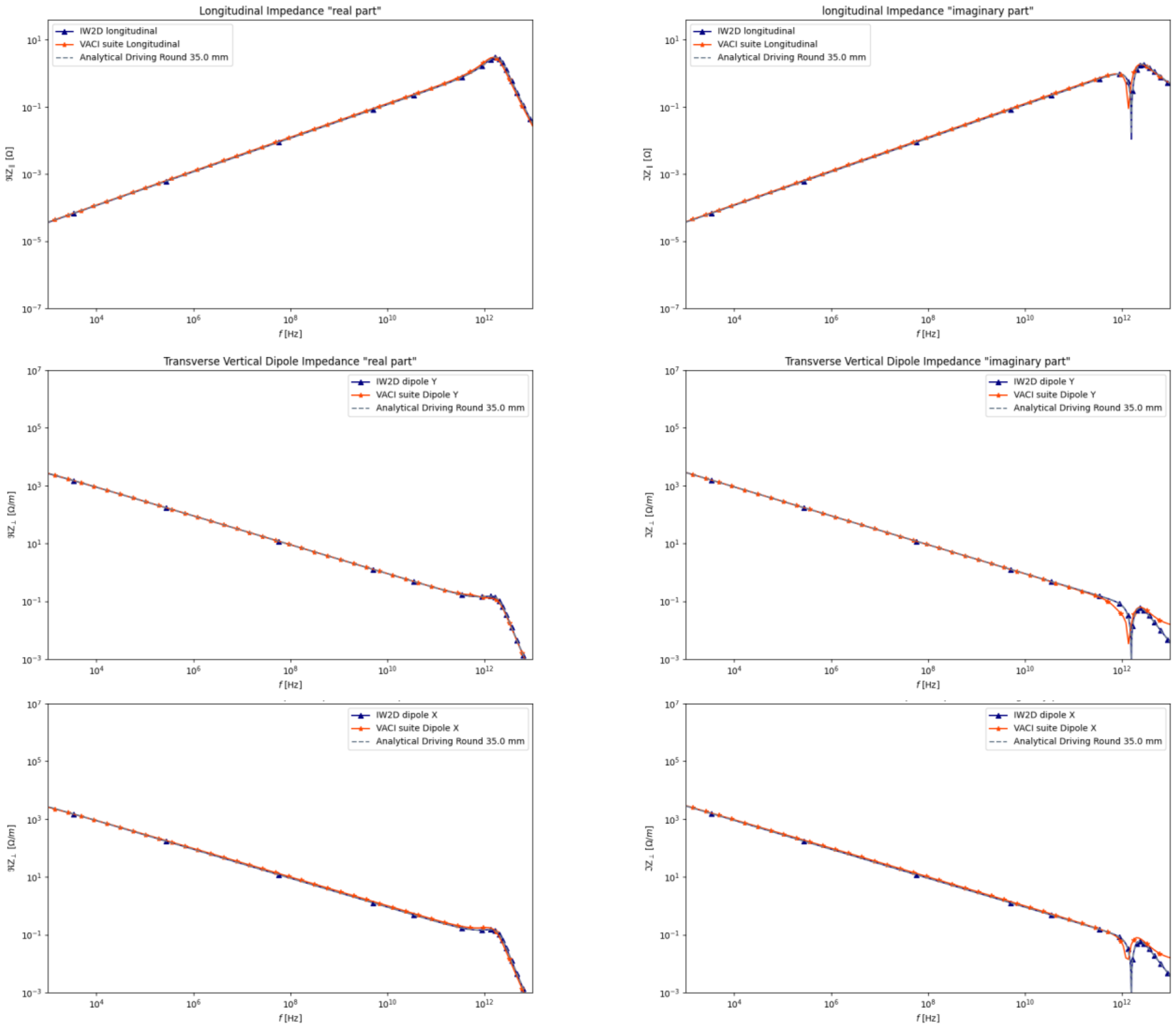
Main ring

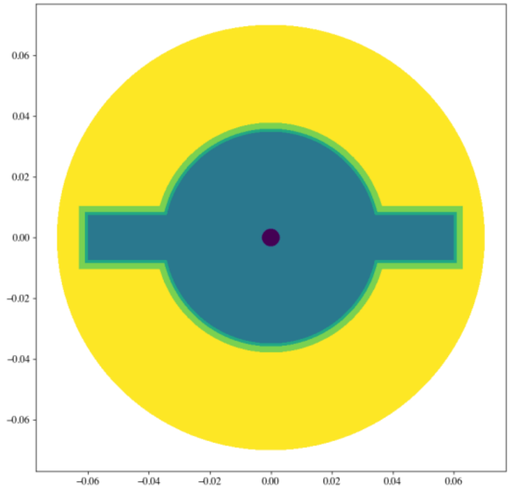


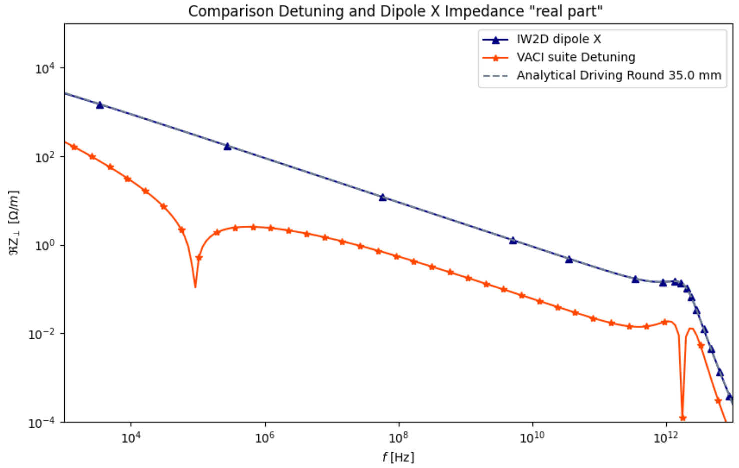
VACI results for FCC main ring

RW impedance

Copper Vacuum chamber
Length: 1m
No NEG considered.



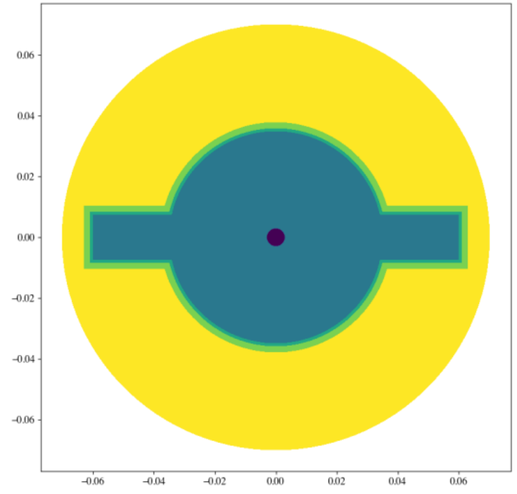


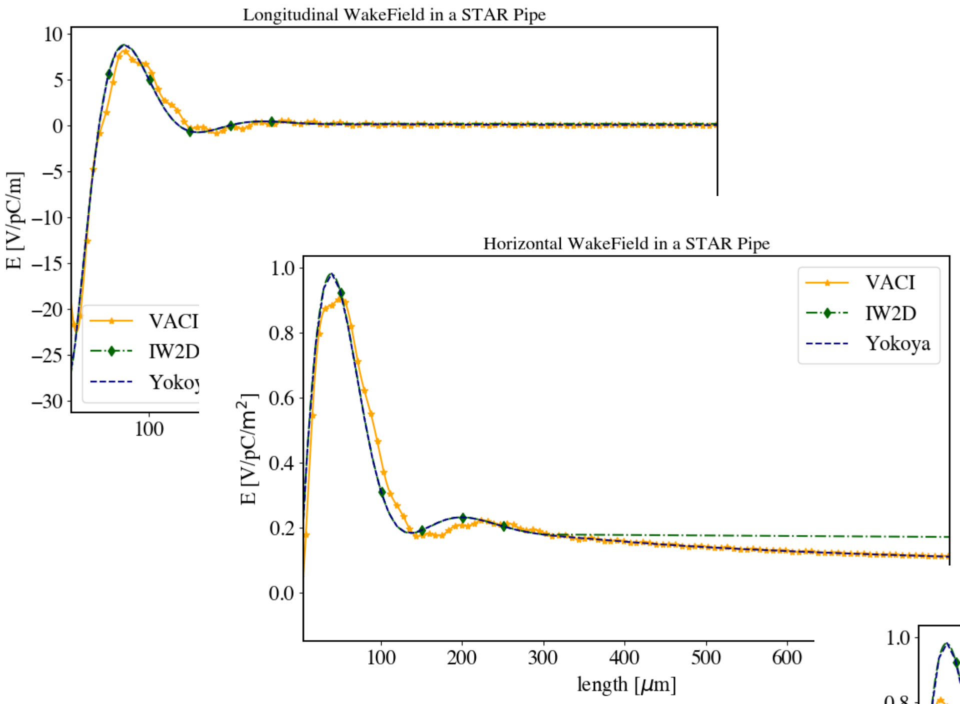


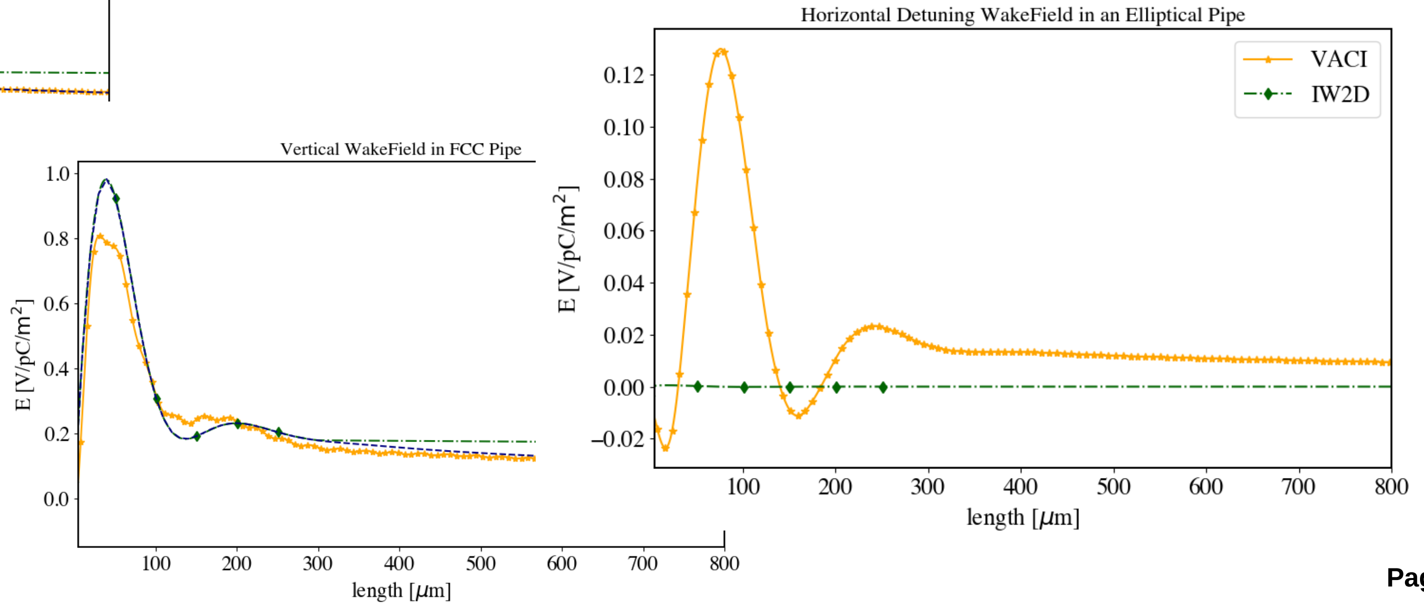
VACI results for FCC main ring

RW wakefield

Copper Vacuum chamber
Length: 1m
No NEG considered.



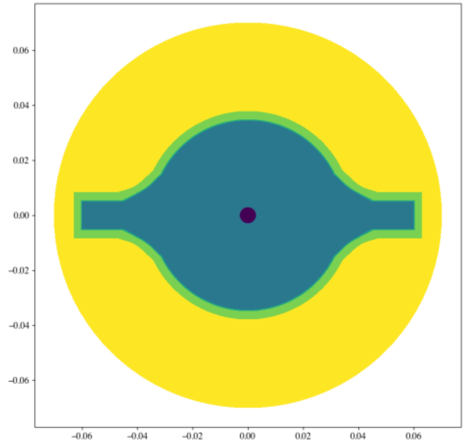


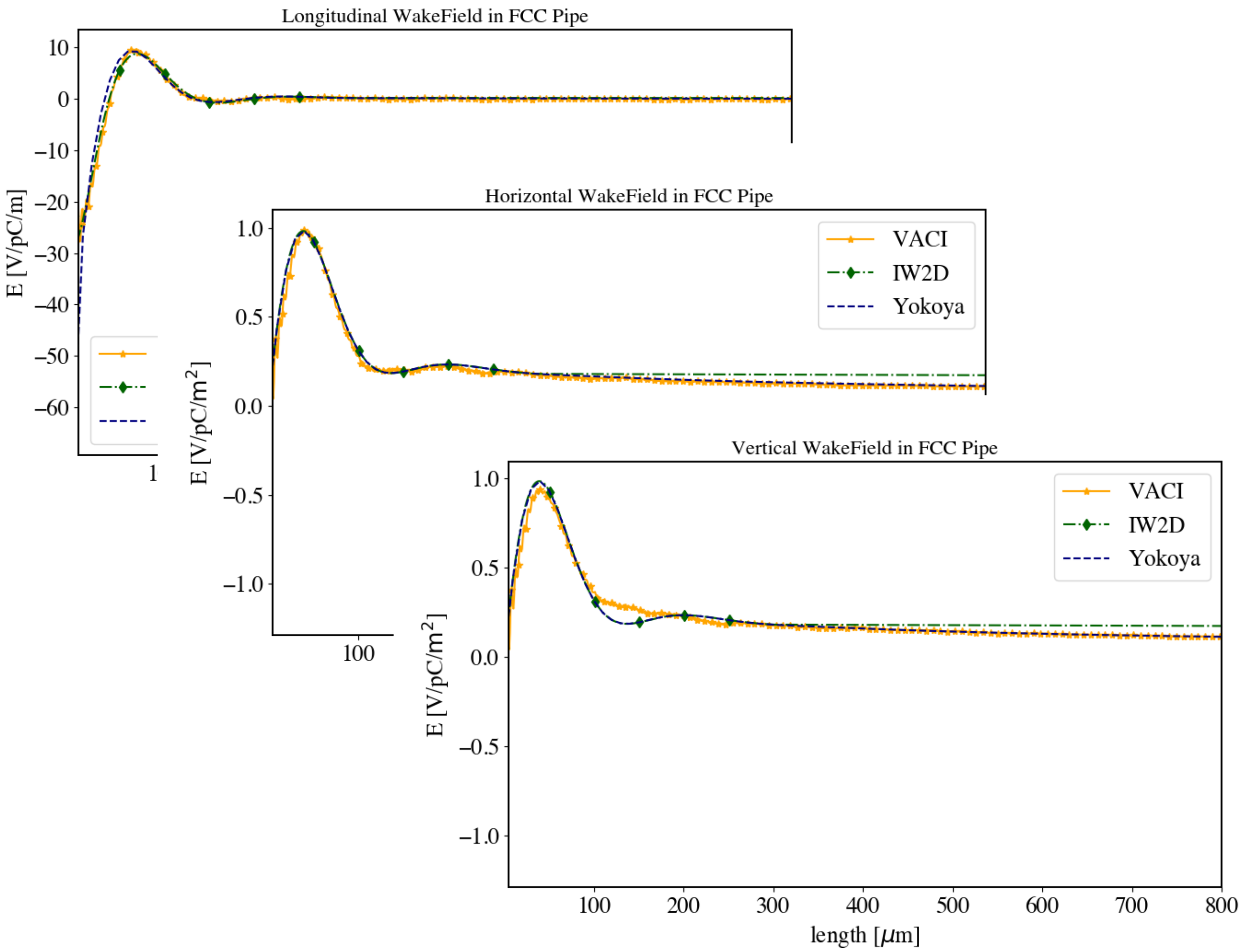


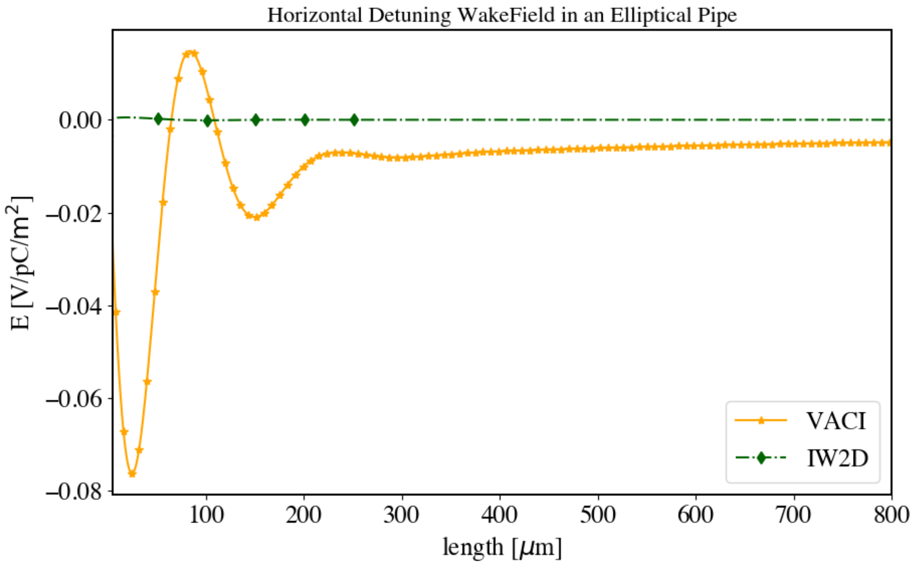
VACI results for FCC main ring

RW wakefield

Copper Vacuum chamber
Length: 1m
No NEG considered.



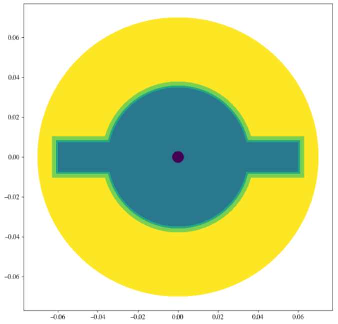


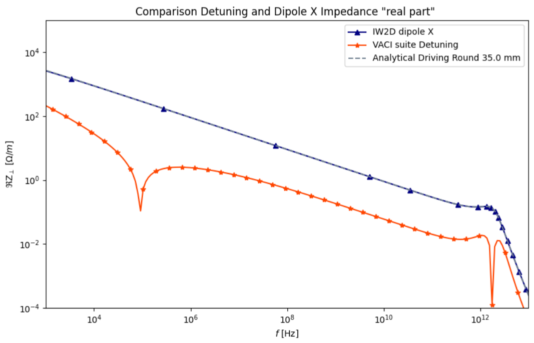


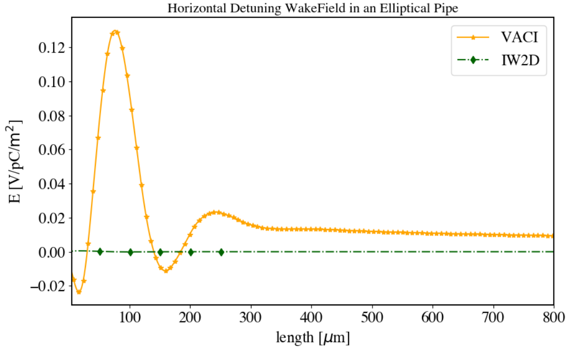
VACI results for FCC main ring

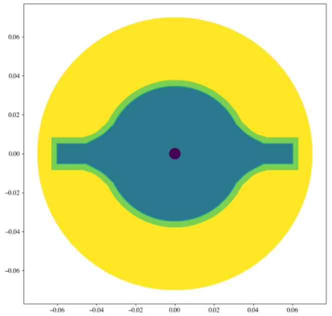
RW compare

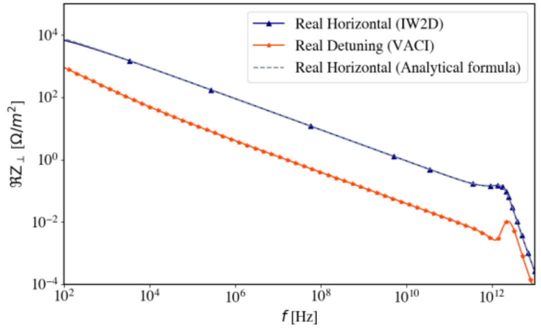
Copper Vacuum chamber
Length: 1m
No NEG considered.

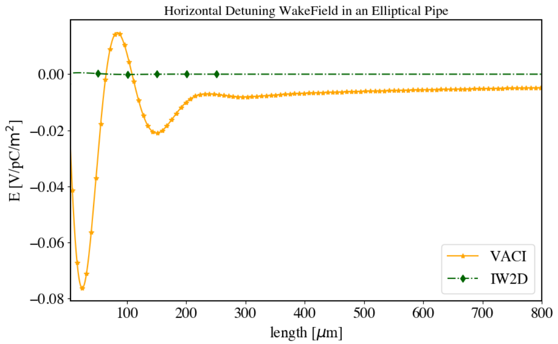








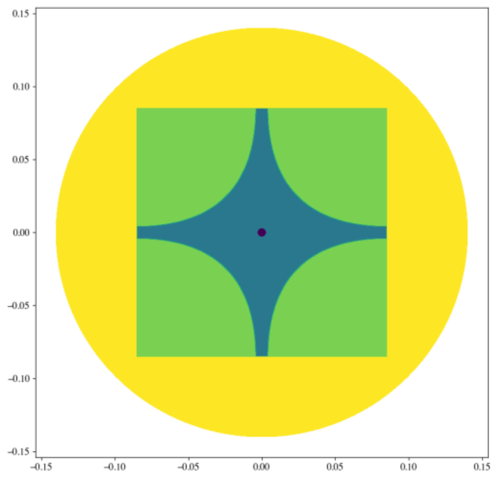


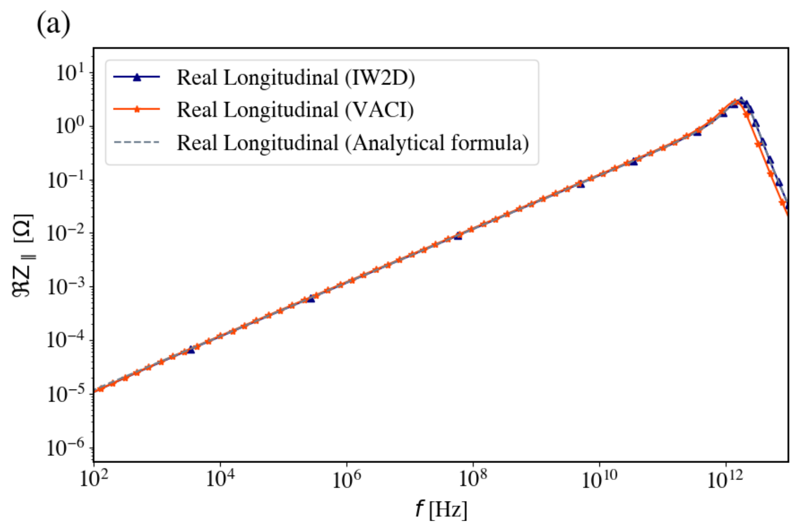


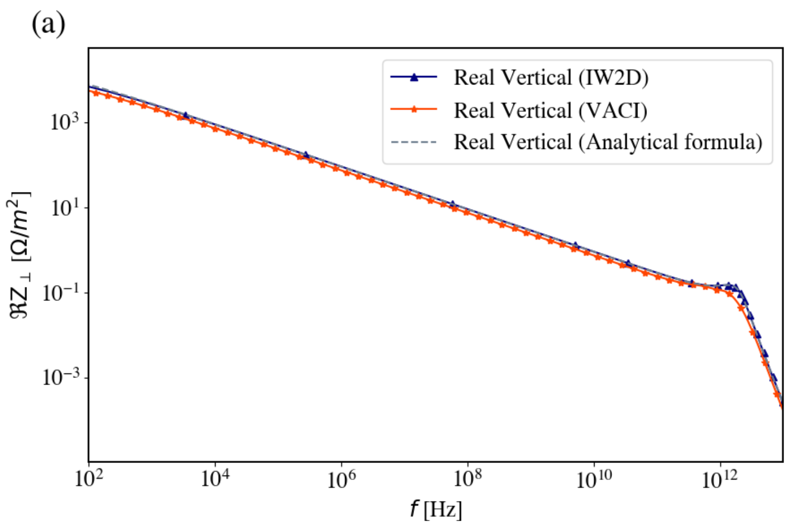
VACI results for FCC main ring

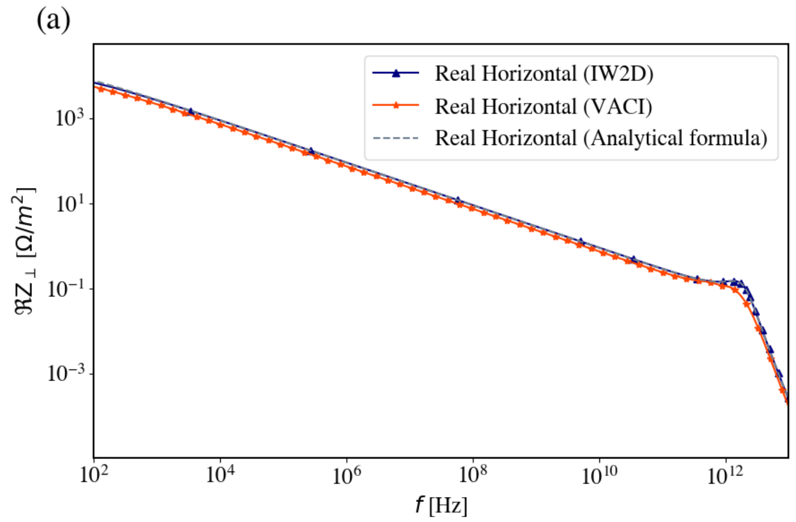
Star shape

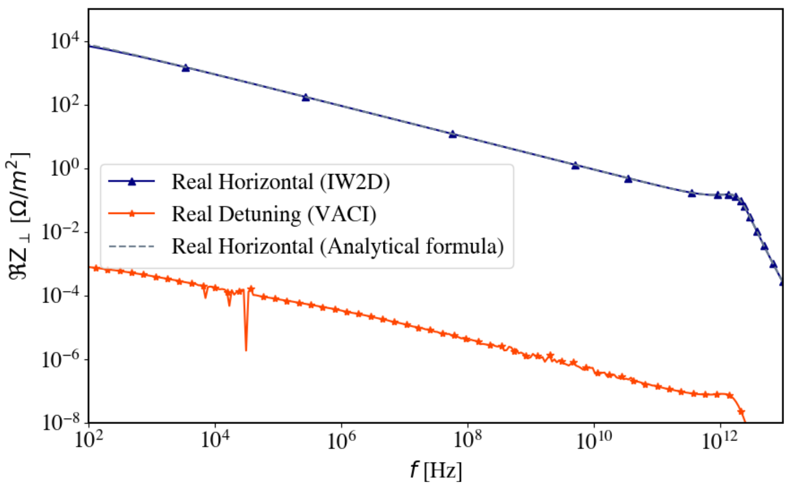
Copper Vacuum chamber
Length: 1m
No NEG considered.











Conclusion

And Future plans

I. Code can calculate 2D Impedance and wakefield in General Cross Section

II. 3D solver is under development.

III. GPU acceleration is going to be added to the code for 3D version.

IV. For FCC main ring and booster:

1. Obtaining wakefields for the correct geometry
2. Studying the NEG thickness in main ring and copper thickness in booster ring
3. Using X-suite for beam dynamic simulation based previous step.

Thank you for your Attention

Please feel free to share any Ideas, discussion, suggestions?

Contact

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Synchrotron DESY

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