

# Probing Minimal Grand Unification through Gravitational Waves

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arXiv: 2212.05291, Saad (to appear in JHEP)

# Outline

- Yukawa Sector of Minimal  $SO(10)$  Unification
- Symmetry breaking and Gravitational Waves
- Proton decay and Coupling Unification

# Searching for a Minimal Yukawa Sector

- Most constructions: complex  $10_H$  Babu, Mohapatra 1992
- Reducing number of parameters:  $SO(10) \times U(1)_{PQ}$
- \* Our proposal: only  $SO(10)$  gauge symmetry Babu, Bajc, Saad 2016
  - No new fermions beyond the three families of chiral 16s.
  - Non-supersymmetric framework.

# Proposal: Minimal Yukawa Sector

- Fermion bilinear:  $16 \times 16 = 10_s + 120_a + 126_s$
- 10 and 120 are real representations of  $SO(10)$
- 126 is complex representation of  $SO(10)$
- The most general Yukawa sector

$$\mathcal{L}_{yuk} = 16_F (Y_{10}^i 10_H^i + Y_{120}^j 120_H^j + Y_{126}^k \overline{126}_H^k) 16_F$$

$$i = 1, 2, \dots n_{10}, j = 1, 2, \dots n_{120} \text{ and } k = 1, 2, \dots n_{126}$$

\*  $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$

Babu, Bajc, Saad 2016

# Fermion masses

$$M_U = \overbrace{D}^{10_H} + \overbrace{S}^{\overline{126}_H} + \overbrace{A}^{120_H}$$

$$M_D = D + r_1 S + e^{i\phi} A$$

$$M_E = D - 3r_1 S + r_2 A$$

$$M_{\nu_D} = D - 3S + r_2^* e^{i\phi} A$$

$$M_{\nu_R} = \underbrace{c_R S}_{\text{diag}(M_1, M_2, M_3)}$$

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}.$$

# Fermion Fit

Observables $(\Delta m_{ij}^2 \text{ in } eV^2)$	Values at $M_Z$ scale			Values at $M_{GUT}$ scale	
	Input	Best Fit: NO	Best Fit: IO	NO	IO
$y_u/10^{-6}$	$6.65 \pm 2.25$	6.65	6.71	2.85	2.87
$y_c/10^{-3}$	$3.60 \pm 0.11$	3.60	3.60	1.54	1.54
$y_t$	$0.986 \pm 0.0086$	0.986	0.986	0.48	0.48
$y_d/10^{-5}$	$1.645 \pm 0.165$	1.645	1.675	0.73	0.74
$y_s/10^{-4}$	$3.125 \pm 0.165$	3.125	3.146	1.38	1.39
$y_b/10^{-2}$	$1.639 \pm 0.015$	1.639	1.639	0.637	0.637
$y_e/10^{-6}$	$2.7947 \pm 0.02794$	2.7947	2.7899	2.8873	2.8817
$y_\mu/10^{-4}$	$5.8998 \pm 0.05899$	5.8998	5.9021	5.924	5.894
$y_\tau/10^{-2}$	$1.0029 \pm 0.01002$	1.0029	1.0012	0.985	0.989
$\theta_{12}^{\text{CKM}}/10^{-2}$	$22.735 \pm 0.072$	22.735	22.739	22.73	22.74
$\theta_{23}^{\text{CKM}}/10^{-2}$	$4.208 \pm 0.064$	4.208	4.204	4.79	4.79
$\theta_{13}^{\text{CKM}}/10^{-3}$	$3.64 \pm 0.13$	3.64	3.64	4.15	4.15
$\delta^{\text{CKM}}$	$1.208 \pm 0.054$	1.208	1.204	1.207	1.204
$\Delta m_{21}^2/10^{-5}$	$7.425 \pm 0.205$	7.425	7.433	9.714	950.84
$\Delta m_{31}^2/10^{-3}$ (NO)	$2.515 \pm 0.028$	2.515	-	12.909	-
$\Delta m_{32}^2/10^{-3}$ (IO)	$-2.498 \pm 0.028$	-	-2.497	-	-12.515
$\sin^2 \theta_{12}$	$0.3045 \pm 0.0125$	0.3045	0.3053	0.308	0.177
$\sin^2 \theta_{23}$ (NO)*	$0.5705 \pm 0.0205$	0.5726	-	0.484	-
$\sin^2 \theta_{23}$ (IO)*	$0.576 \pm 0.019$	-	0.5819	-	0.542
$\sin^2 \theta_{13}$ (NO)	$0.02223 \pm 0.00065$	0.02223	-	0.007	-
$\sin^2 \theta_{13}$ (IO)	$0.02239 \pm 0.00063$	-	0.02238	-	0.0223
$\chi^2$	-	$3 \times 10^{-8}$	$2.77^\dagger$	-	-

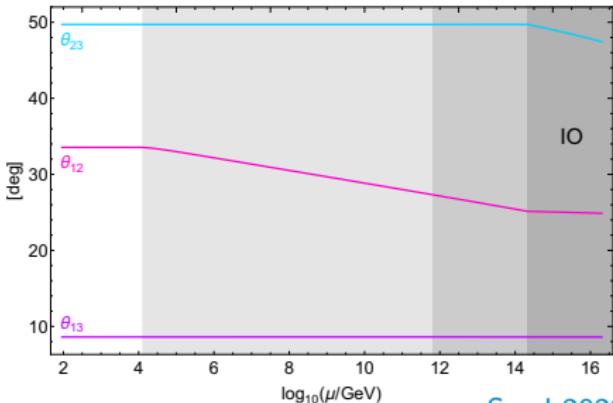
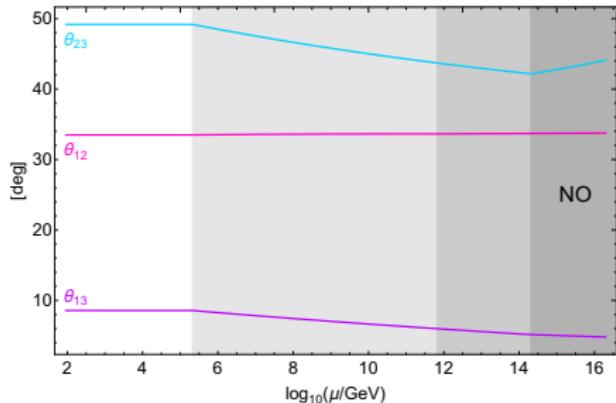
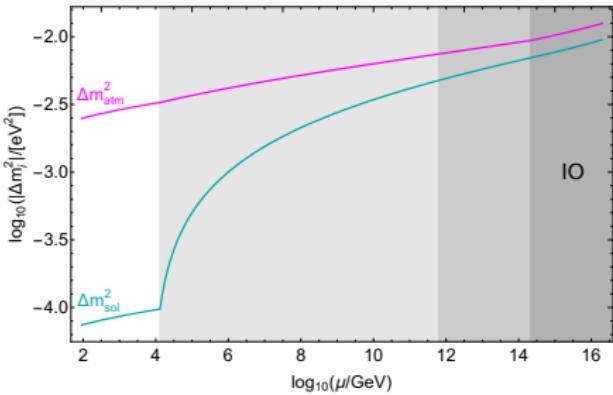
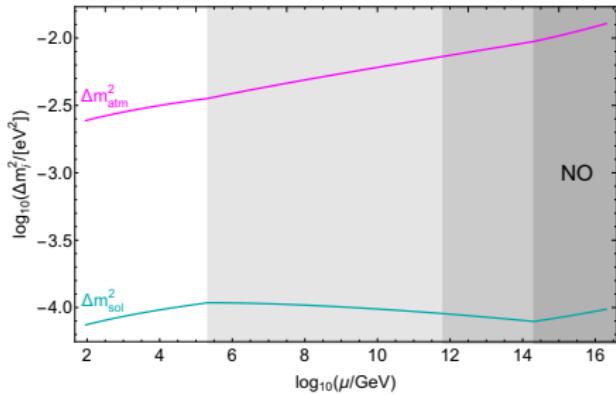
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# Features: $M_R$

Quantity	Best fit prediction	
	NO	IO
$(m_1, m_2, m_3)$	$(0.00014, 0.0086, 0.0501)$ eV	$(0.04922, 0.04997, 0.00038)$ eV
$(\sum_i m_i, m_\beta, m_{\beta\beta})$	$(0.0589, 0.0088, 0.0016)$ eV	$(0.099, 0.041, 0.033)$ eV
$(\delta, \varphi_1, \varphi_2)$	$(326.4, 109.0, 94.7)^\circ$	$(209.5, 164.4, 343.4)^\circ$
$(M_1, M_2, M_3)$	$(2.13 \times 10^5, 6.46 \times 10^{11}, 2.28 \times 10^{14})$ GeV	$(1.31 \times 10^4, 6.42 \times 10^{11}, 2.37 \times 10^{14})$ GeV

- $M_3 \sim 10^{14}$  GeV (seesaw scale)
- $M_2 \sim \frac{m_c}{m_t} M_3 \sim 10^{11}$  GeV
- $M_{2,3} \gg M_1 \sim 10^5$  GeV
- $M_1^{\text{IO}} \sim M^{\text{NO}} / 10$

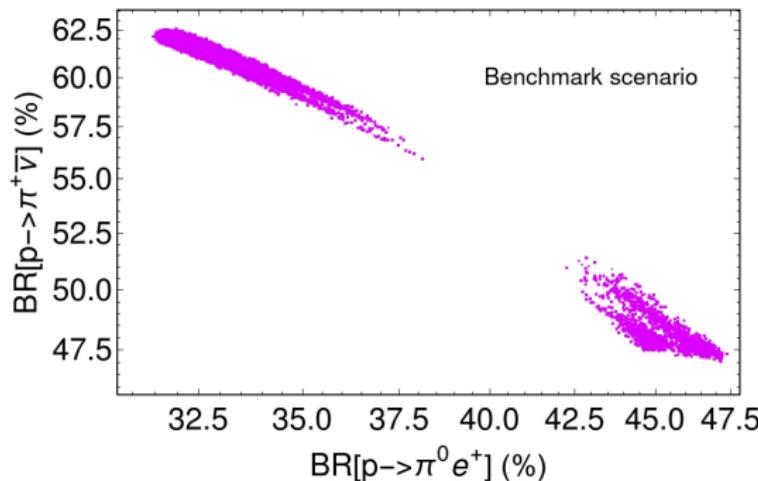
# Importance of RGE running



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# Features: Proton decay correlations

$p$ decay modes	BR[%] predictions
$(p \rightarrow e^+ \pi^0, p \rightarrow e^+ K^0, p \rightarrow e^+ \eta)$	(32.4, 0.52, 0.03)
$(p \rightarrow \mu^+ \pi^0, p \rightarrow \mu^+ K^0, p \rightarrow \mu^+ \eta)$	(0.42, 4.3, 0.002)
$(p \rightarrow \bar{\nu} \pi^+, p \rightarrow \bar{\nu} K^+)$	(61.5, 0.81)



# Pulsar timing data

- NANOGrav : 12.5 yrs of pulsar timing data 2020
  - strong evidence for a stochastic common-spectrum process
  - Interpreted as a GW signal:  $A \sim \mathcal{O}(10^{-15})$  @  $f \sim 1/\text{yr}$
- ★ Cosmic string model gives an excellent fit Ellis, Lewicki 2020
  - $G\mu = (2 \times 10^{-11} - 3 \times 10^{-10})$  @ 95% C.L.
- \* Similar hints from PPTA 2021, EPTA 2021, and IPTA 2022
  - \* GW energy density Fu et. al. 2022

$$(\Omega_{\text{GW}}(f) h^2)_{\text{PTA}} \approx 2.02 \cdot 10^{-10} \left( \frac{A}{10^{-15}} \right)^2 \times \left( \frac{f}{f_{\text{yr}}} \right)^{5-\gamma}$$

# Higgs sector

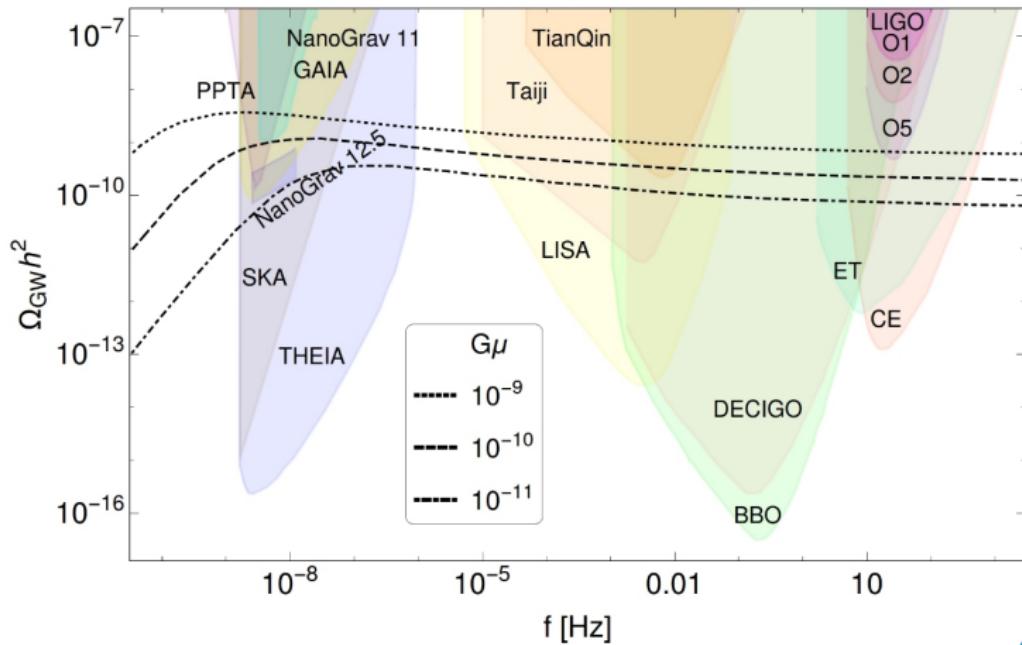
- Minimal scenario to provide cosmic string network

$$\begin{aligned} SO(10) &\xrightarrow[54_H]{M_X} SU(4)_C \times SU(2)_L \times SU(2)_R \times D \\ &\xrightarrow[45_H]{M_I} \boxed{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \\ &\xrightarrow[126_H]{M_{II}} \boxed{SU(3)_C \times SU(2)_L \times U(1)_Y} \\ &\xrightarrow[10_H+126_H]{M_{EW}} SU(3)_C \times U(1)_{em} \end{aligned}$$

the breaking by  $126_H$  leaves a remnant  $Z_2$  symmetry (not broken by tensor representations)

# GW from stable cosmic string network

$$G\mu = 4.22 \times 10^{-38} v_R^2$$



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# String scale and fermion mass fit

- CMB:  $G\mu < 1.1 \times 10^{-7}$  Charnock et. al. 2016  $\Rightarrow M_{II} \lesssim 2 \times 10^{15}$  GeV
- LIGO: 2021  $G\mu \lesssim 10^{-8}$  ( $f \sim \mathcal{O}(10)$  Hz)  $\Rightarrow M_{II} \lesssim 5 \times 10^{14}$  GeV
- \* Neutrino mass fit:  $M_3 \lesssim 10^{15}$  GeV
- \* Up-quark mass fit:  $M_3 \gtrsim 2 \times 10^{13}$  GeV
- \* Valid:  $M_3 = [2 \times 10^{13} - 10^{15}]$  GeV

# String scale and fermion mass fit

- $S = v_{126}^u \underbrace{Y_{126}}_{\text{diagonal}}$
  - $10_H$ :  $M_U, M_D \propto \underbrace{D}_{\text{no extra factor}}$
  - $m_b \sim D_{33} + r_1 S_{33}$
  - $m_t \sim S_{33} \gg D_{33} + r_1 S_{33}$
  - $M_3 = v_R (Y_{126})_{33}$
- $$v_R = \begin{cases} v_R^{\min} = 0.5 \times M_3 , \\ v_R^{\max} = 2.05 \times M_3 , \end{cases} \quad \begin{aligned} (Y_{126})_{33}^{\max} &= 2 \\ (Y_{126})_{33}^{\min} &= 0.48 \end{aligned}$$

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# Consistent $SO(10)$ and GW fits

- excellent fit for:

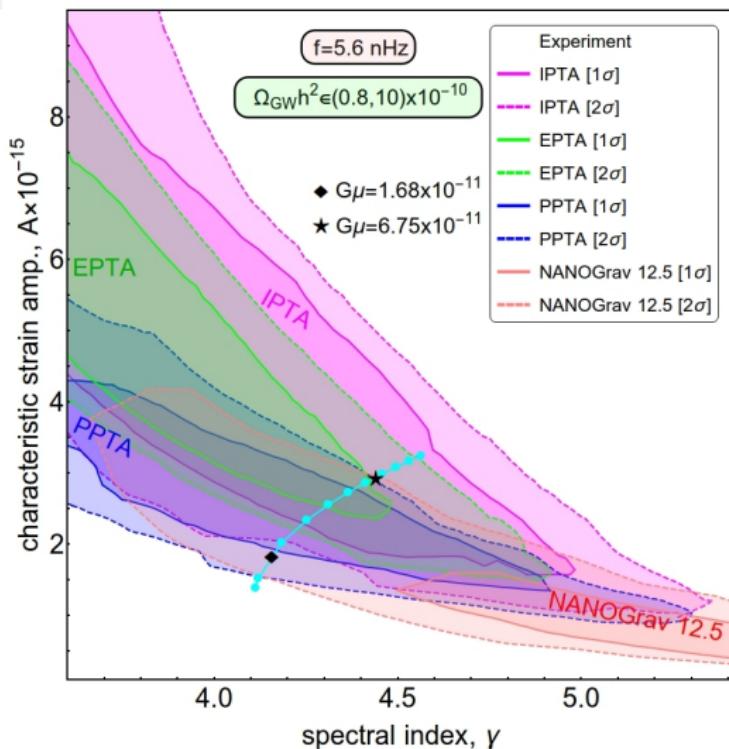
$$\rightarrow \Omega_{\text{GW}} h^2 \in (2, 6) \times 10^{-10} @ 2\sigma \text{ CL}$$

- Corresponds to

$$\rightarrow G\mu \in (4.9, 6.9) \times 10^{-11}$$

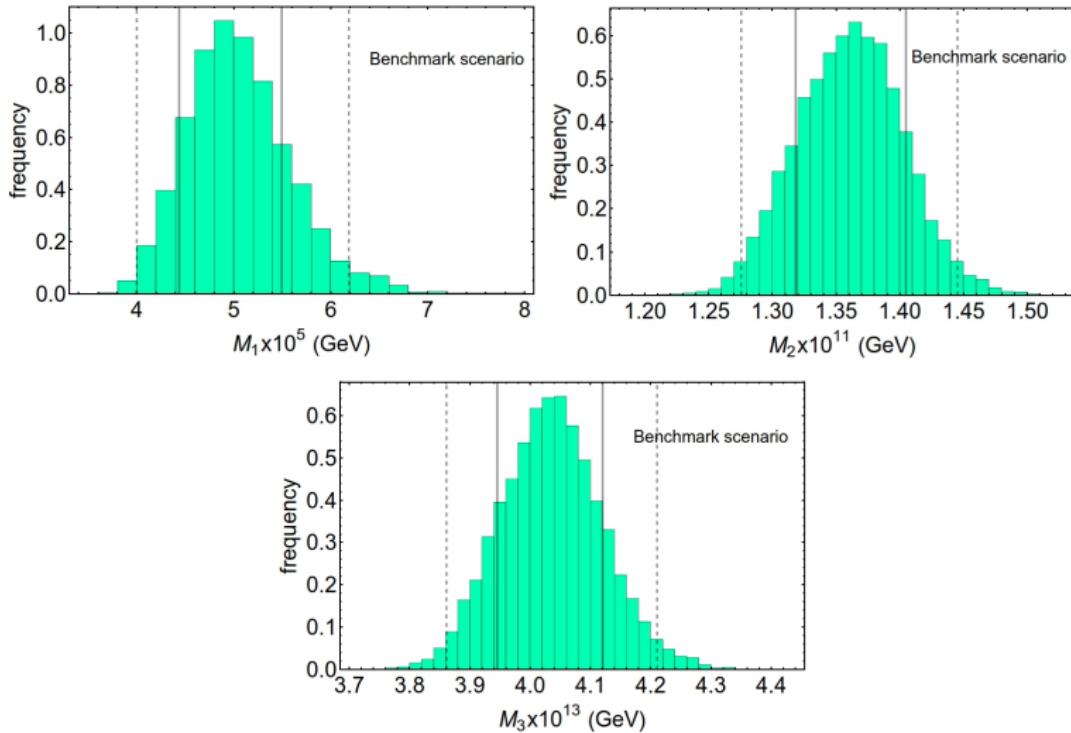
- Restricted seesaw scale:

$$\rightarrow v_R \in (3.4, 4.1) \times 10^{13} \text{ GeV}$$

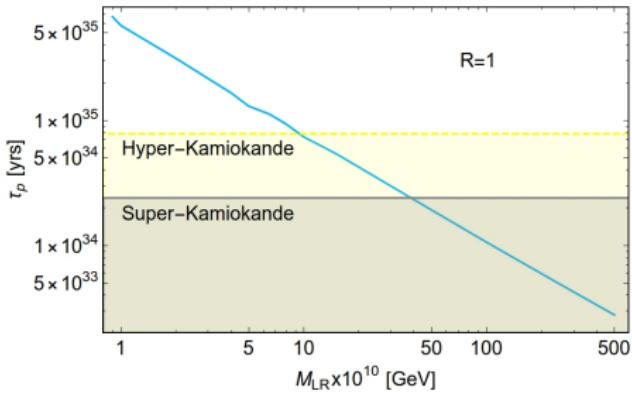
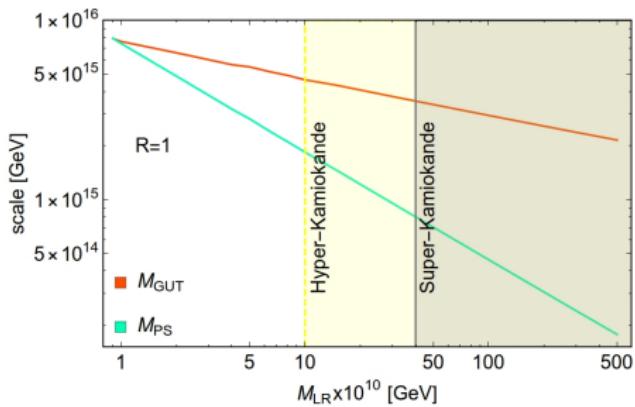


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# Consistent $SO(10)$ and GW fits

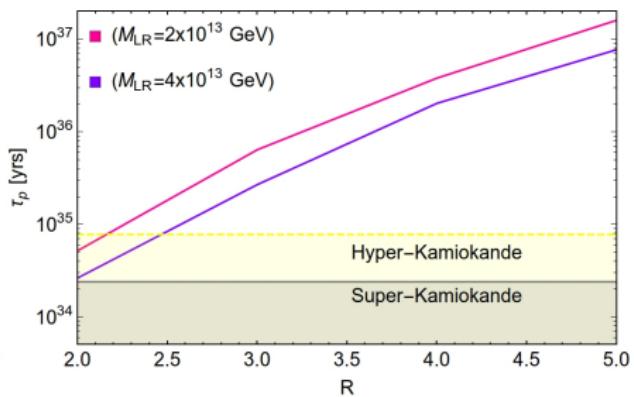
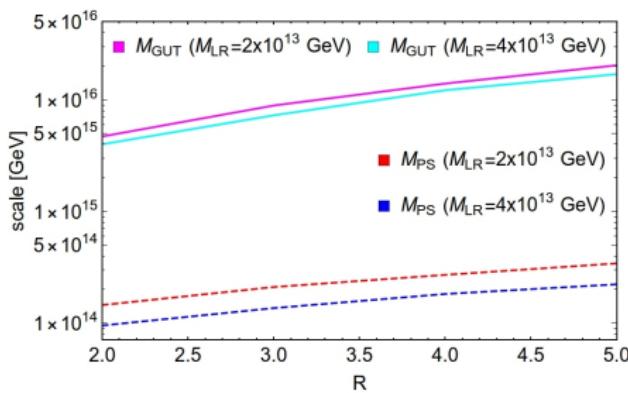


# GW from $SO(10)$ : consistent with proton decay?



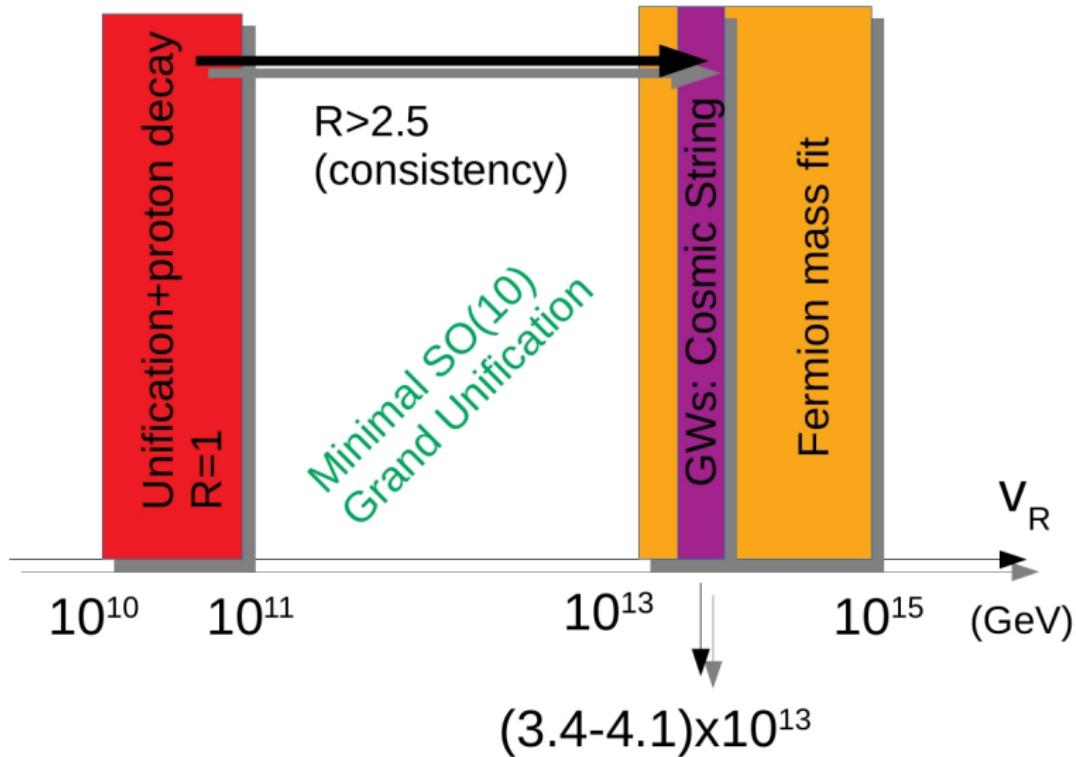
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# Compatibility with a small threshold correction



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# Upshot



# Summary

- ✿ Minimal Yukawa sector:  $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$
- ✿ Minimal SSB sector (cosmic string):  $45_H + 54_H$
- ✿ Dominant modes:  $p \rightarrow \bar{\nu}\pi^+$  and  $p \rightarrow e^+\pi^0$
- ✿ Fermion mass fit:  $M_3 = [2 \times 10^{13} - 10^{15}] \text{ GeV}$
- ✿  $M_2 \sim \frac{m_c}{m_t} M_3 \sim 10^{11} \text{ GeV}$
- ✿  $M_{2,3} \gg M_1 \sim 10^5 \text{ GeV}$ ,  $M_1^{\text{IO}} \sim M^{\text{NO}}/10$
- ✿ GW/PTAs:  $v_R \in (3.4, 4.1) \times 10^{13} \text{ GeV}$
- ✿ Fully testable in a number of gravitational wave observatories

THANK YOU!

# Explaining features

$$\frac{|S|}{v} \sim \begin{pmatrix} 4.5 \times 10^{-10} & 0. & 0. \\ 0. & 1.3 \times 10^{-3} & 0. \\ 0. & 0. & 4.8 \times 10^{-1} \end{pmatrix}$$

$$\frac{|D|}{v} \sim \begin{pmatrix} 3.0 \times 10^{-6} & 2.8 \times 10^{-5} & 1.7 \times 10^{-4} \\ 2.8 \times 10^{-5} & 2.4 \times 10^{-4} & 2.7 \times 10^{-3} \\ 17 \times 10^{-4} & 2.7 \times 10^{-3} & 2.6 \times 10^{-3} \end{pmatrix}$$

$$\frac{|A|}{v} \sim \begin{pmatrix} 0 & 2.3 \times 10^{-5} & 1.2 \times 10^{-4} \\ 2.3 \times 10^{-5} & 0 & 2.5 \times 10^{-3} \\ 1.2 \times 10^{-4} & 2.5 \times 10^{-3} & 0 \end{pmatrix}$$

# Explaining features

$$\frac{|S|}{v} \sim \begin{pmatrix} 4.5 \times 10^{-10} & 0. & 0. \\ 0. & 1.3 \times 10^{-3} & 0. \\ 0. & 0. & 4.8 \times 10^{-1} \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$\frac{|D|}{v} \sim \begin{pmatrix} 3.0 \times 10^{-6} & & \\ & 2.4 \times 10^{-4} & \\ & & 2.6 \times 10^{-3} \end{pmatrix} \sim \begin{pmatrix} y_{d,e,u} & 0 & 0 \\ 0 & y_{s,\mu} & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

$$M_U \sim S, \quad M_{D,E} \sim D, \quad M_2 \sim \frac{y_c}{y_t} M_3, \quad M_1 \ll M_{2,3}$$

# Explaining features

- At the GUT scale  $m_t/m_b \sim 75$
- $y_t \sim S_{33}/v$
- $y_b \sim (D_{33} + r_1 S_{33})/v$
- $(D_{33} + r_1 S_{33})/S_{33} \sim 10^{-2}$
- $D_{33}, r_1 S_{33} \ll S_{33} \sim m_t$
- $M_3 = v_R (Y_{126})_{33}$  determines String scale

# Explaining features

$$\begin{aligned} \frac{|M_{\nu_D}|}{\nu} &\sim (D - 3S)/\nu \\ &= \begin{pmatrix} 3.1 \times 10^{-6} & 2.8 \times 10^{-5} & 1.7 \times 10^{-4} \\ 2.8 \times 10^{-5} & & 2.73348 \times 10^{-3} \\ 1.7 \times 10^{-4} & 2.73348 \times 10^{-3} & \end{pmatrix} \\ &- \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 \times 10^{-3} & 0 \\ 0 & 0 & 1.4 \end{pmatrix} \end{aligned}$$

# Explaining features

$$M_N = -M_{\nu_D} \begin{pmatrix} M_1^{-1} & & \\ & M_2^{-1} & \\ & & M_3^{-1} \end{pmatrix} M_{\nu_D}^T$$
$$\sim \begin{pmatrix} \frac{10^{-7}}{M_1} + 10^{-15} & \frac{10^{-7}}{M_1} + 10^{-14} & \frac{10^{-6}}{M_1} + 10^{-13} \\ \frac{10^{-7}}{M_1} + 10^{-14} & \frac{10^{-7}}{M_1} + 10^{-12} & \frac{10^{-6}}{M_1} + 10^{-12} \\ \frac{10^{-6}}{M_1} + 10^{-13} & \frac{10^{-6}}{M_1} + 10^{-12} & \frac{10^{-5}}{M_1} + 10^{-10} \end{pmatrix}$$

Works if  $M_1 \sim 10^5$  GeV !

(IO works if first column and row have significant entries:  
 $M^{\text{IO}} \sim M^{\text{NO}}/10$ )