Impact of non-perturbative Effects in t-Channel Simplified Dark Matter Models

in collaboration with

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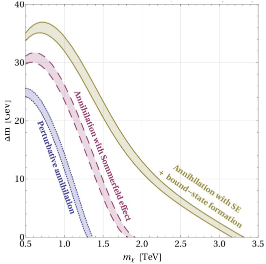
supported by DFG Emmy Noether Grant No. HA 8555/1-1.

Mathias Becker

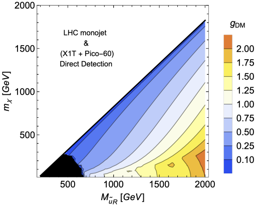
Motivation I

Figure from [Harz,Petraki (2018)]



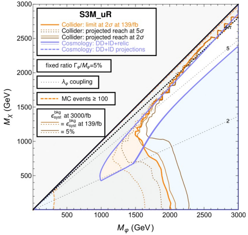
BSF in the early universe

Figure from [Mohan et. al (2019)]



RGE improved Direct Detection (DD)

Figure from [Arina et. al (2020)]



Experimental constraints from the LHC t-channel DM working group

⇒ How impactful are non-perturbative Effects for experimental exclusion limits

What is new?

$\rightarrow$ A flat correction factor for non-perturbative effects is unapplicable

$\rightarrow$ Corrections on the exclusion limits can be as large as $\mathcal{O}\,(100\%)$

$\rightarrow$ Bound State searches close gap between prompt and long-lived searches!

Simplified t-Channel Dark Matter

Universal framework for t-channel DM models [Arina,Fuks,Mantani (2020)]

S3M-uR t-channel Dark Matter

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{kin,BSM}} + g_{\mathrm{DM}}\overline{\chi}(u_R)_i(X^\dagger)_i + h.c.$$

$$\chi = (\mathbf{1},\mathbf{1})_0 \qquad X_i = (\mathbf{3},\mathbf{1})_{2/3}$$

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$$\chi = (\mathbf{1}, \mathbf{1})_0 \qquad X_i = (\mathbf{3}, \mathbf{1})_{2/3}$$

- Discrete $\mathcal{Z}_2$: SM fields even, dark sector fields odd
- Majorana fermion DM χ
- 3 generations of mediators X_i

Parameters: $(m_\chi = m_{\text{DM}}, \Delta m = m_X - m_{\text{DM}}, g_{\text{DM}})$

Dark Matter Freeze-Out

Assumptions during DM freeze-out:

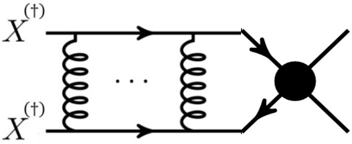
- Dark sector in *kinetic* eq. with the SM.
- Dark sector particles in *chemical* eq. with themselves.

Coannihilation

$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle \left(n^2 - \left(n^{\text{eq}} \right)^2 \right)$$

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}}$$

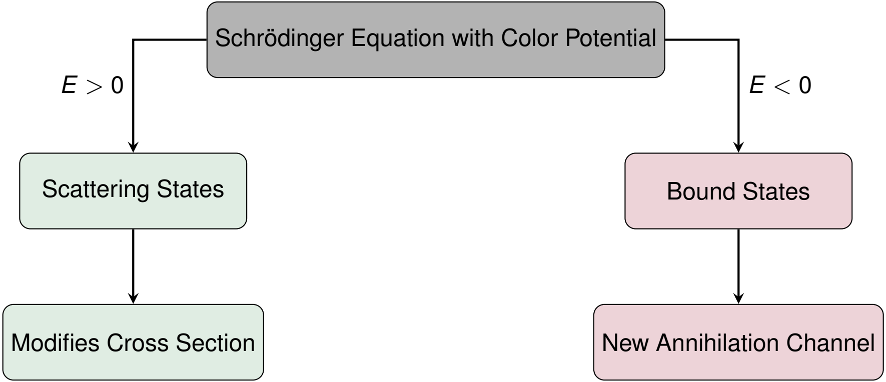
$$n = \sum_i n_i \quad \text{and} \quad i,j = \{\chi, X_1, X_2, X_3\}$$

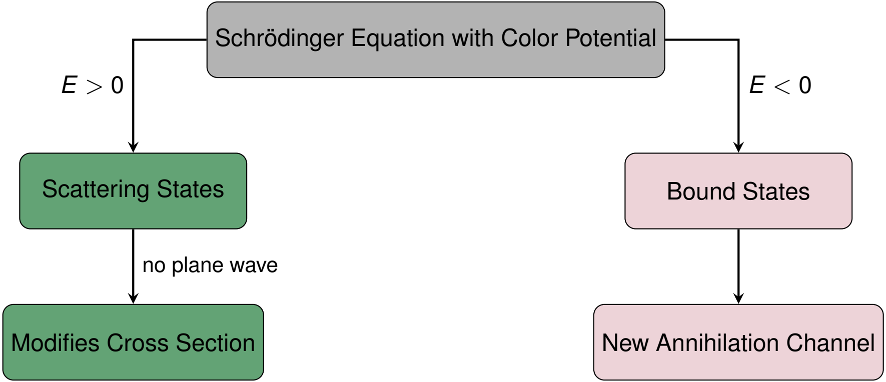


n-gluon exchanges contribute with $\left(\frac{\alpha}{v}\right)^n$ for $\alpha \sim v$

$\rightarrow$ Resummation required since $\alpha_s \sim v \sim 0.1$

$\rightarrow$ Reduces to Schrödinger Equation for $v \ll 1$. For details [Petraki,Postma,Wiechers(2015)]


Schrödinger Equation with Color Potential
$E > 0$
$E < 0$
Scattering States
Bound States
Modifies Cross Section
New Annihilation Channel


Schrödinger Equation with Color Potential
$E > 0$
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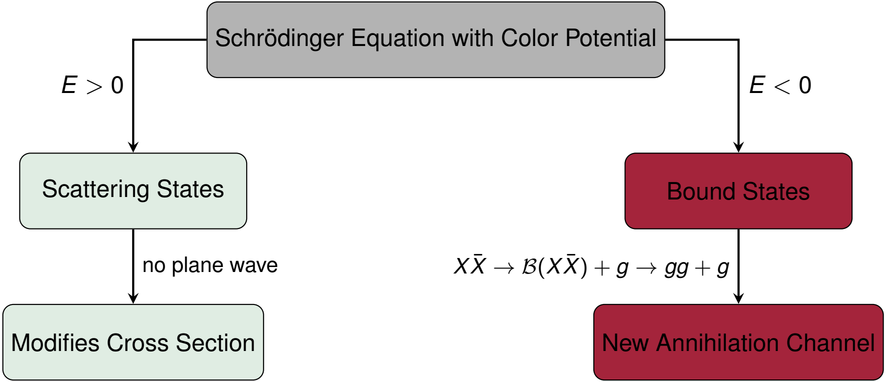
Scattering States
Bound States
no plane wave
Modifies Cross Section
New Annihilation Channel



Schrödinger Equation with Color Potential
$E > 0$
$E < 0$
Scattering States
Bound States
no plane wave
$X\bar{X} \to \mathcal{B}(X\bar{X}) + g \to gg + g$
Modifies Cross Section
New Annihilation Channel

SE vs BSF

Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\rm eff} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle S(\alpha/v_{ij}) \cdot \sigma_{ij} v_{ij} \rangle \frac{n_i^{\rm eq}}{n^{\rm eq}} \frac{n_j^{\rm eq}}{n^{\rm eq}} + \langle \sigma_{\rm BSF} v \rangle_{\rm eff} \left(\frac{n_X^{\rm eq}}{n^{\rm eq}} \right)^2$$

SE vs BSF

Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle\sigma_{\text{eff}}v\rangle = \sum_{i,j\in\{\chi,X\}} \langle S(\alpha/v_{ij})\cdot\sigma_{ij}v_{ij}\rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}}\frac{n_j^{\text{eq}}}{n^{\text{eq}}} + \langle\sigma_{\text{BSF}}v\rangle_{\text{eff}}\left(\frac{n_X^{\text{eq}}}{n^{\text{eq}}}\right)^2$$

$\langle\sigma_{\text{eff}}v\rangle$	Sommerfeld Effect	Bound State Formation
$g_{\text{DM}} \gg g_s$	$-$	0
$g_{\text{DM}} \ll g_s$	$+$	$++$

$\rightarrow$ No flat factor

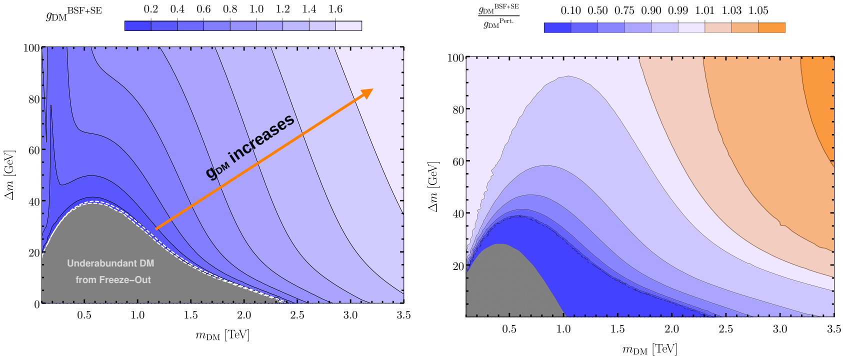
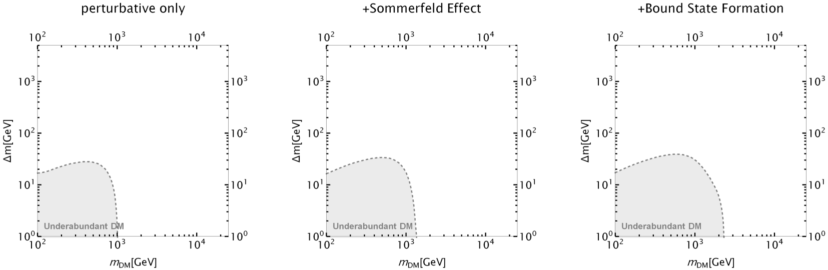


Figure from [MB,Copello,Harz,Mohan,Sengupta(2022)]

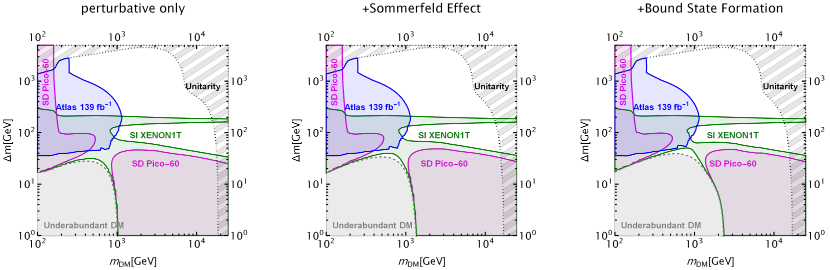


→ Bound State Formation increases the area where the strong interaction depletes the relic density significantly!

Direct Detection and Prompt Collider Searches

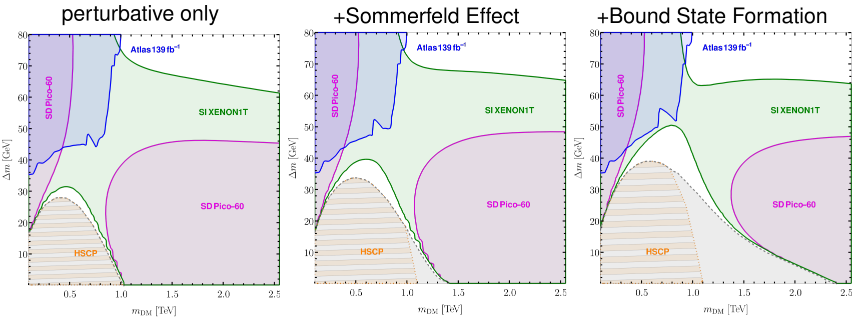
mono-jet + ETmiss search by ATLAS [arXiv:1711.03301]

multi-jets + ETmiss search by CMS [arXiv:1704.07781]



- **Direct Detection**: Inclusion of BSF opens up parameter space in this region
- **Prompt Collider Searches**: Non-Perturbative effects are mild/absent.

Long-Lived-Particle Searches



→ Region can not be fully tested when including non-perturbative effects

→ A precise treatment, assuming *conversion driven freeze-out*, has been analyzed in [Garny,Heisig (2021)]

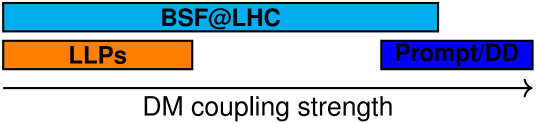
Bound State Formation at the LHC

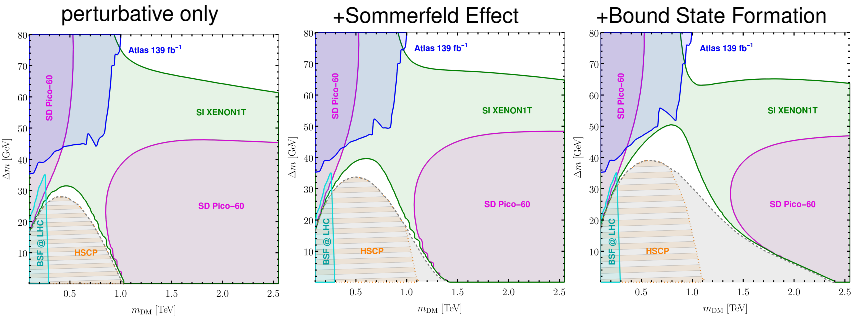
Production Cross Section[Martin(2008)]

$$\sigma\left(pp \to \mathcal{B}(XX^\dagger)\right) = \frac{\pi^2}{8m_\mathcal{B}^3}\Gamma\left(\mathcal{B}(XX^\dagger) \to gg\right)\mathcal{P}_{gg}\left(\frac{m_\mathcal{B}}{13\text{ TeV}}\right)$$

$\to$ try to observe the bound state resonance in $\gamma\gamma$ final state. ATLAS (2017)

Efficient for all g_{DM} small enough such that $\Gamma_X < E_B$, roughly speaking $g_{\text{DM}} \lesssim g_s$.



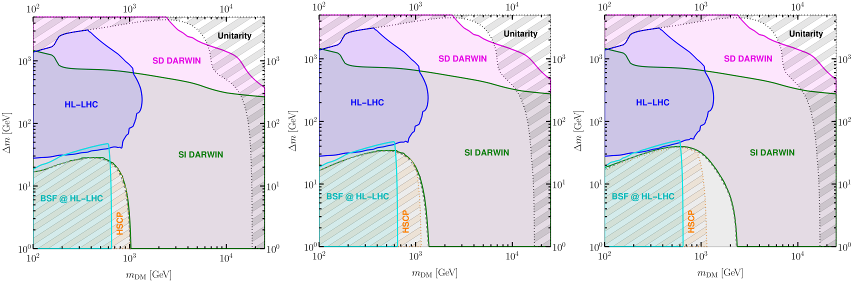


Limits at $37\,\mathrm{fb}^{-1}$ relatively weak in mass ($\sim 300\,\mathrm{GeV}$)

But huge potential: Closes the gap between prompt and LLP searches

Expected Future Limits

perturbative only | +Sommerfeld Effect | +Bound State Formation



- Highly testable: Parameter space almost completely probed
- Remember: HSCP not a strict exclusion here (BSF@LHC is!)
- Bound State effects enlarge the area still necessary to test

Conclusion

- Non-perturbative Effects can increase or decrease the annihilation cross section of DM
  $\rightarrow$ Cannot be handled by a flat correction factor!

- Non-perturbative Effects are non-neglible in scenarios of colored coannihilation and open up small mass parameter space:

  Viable Parameter space shifts from $(m_{\text{DM}}, \Delta m) < (1\,TeV, 30\,GeV)$ to $(1.4\,TeV, 40\,GeV)$ (Sommerfeld Effect) and $(2.4\,TeV, 50\,GeV)$ (Bound State Formation)
  $\rightarrow$ Sommerfeld Effect alone not a good approximation!

- Bound State searches at colliders close the gap in "coupling space" between prompt and long-lived-particle searches

When is BSF relevant?

No coannihilation required!
$\rightarrow$ Expect potentially large non-perturbative effects for $\alpha \sim v \sim 0.1$

Case I: Massless/light mediator (for instance colored annihilation)

$$\sigma_{\text{ann}} \sim \frac{\alpha^2}{m^2} \xrightarrow{\Omega_{\text{DM}} \sim 1/\sigma_{\text{ann}}} \alpha \sim 0.1 \frac{m}{\text{TeV}}$$

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$$\sigma_{\text{ann}} \sim \frac{\alpha^2}{m^2} \qquad \xrightarrow{\Omega_{\text{DM}} \sim 1/\sigma_{\text{ann}}} \qquad \alpha \sim 0.1 \frac{m}{\mathrm{TeV}}$$

Case II: Massive mediator with mass M (Yukawa potential)

$$\sigma_{\text{ann}} \sim \alpha^2 \frac{m^2}{(m^2 + M^2)^2} \qquad \xrightarrow{\Omega_{\text{DM}} \sim 1/\sigma_{\text{ann}}} \qquad \alpha \sim 0.1 \left(\frac{m}{\mathrm{TeV}}\right) \left(1 + \frac{M^2}{m^2}\right)$$

But Yukawa suppression sizable if $\alpha m \lesssim M$

How to constrain the "gray" area ?

Freeze-out leads to underabundant DM $\rightarrow$ correct abundance requires alternative production

Out-of-chemical equilibrium estimate

$$\frac{\Gamma_{X\leftrightarrow\chi}(\tilde{g}_{\rm DM})}{H} \lesssim 1\,,\ \text{at freeze-out}$$

$$\rightarrow \tilde{g}_{\rm DM} \lesssim \sqrt{\frac{m_{\rm DM}}{GeV}\left(10^{-9} + 6.8\cdot 10^{-11}\,\frac{\Delta m}{m_{\rm DM}}\right)}$$

For $g_{\rm DM} < \tilde{g}_{\rm DM}$ DM production is non-thermal

Long-Lived-Particle (LLP) searches constrain large lifetimes $\rightarrow g_{\rm DM} \geq g_{\rm DM}^{\rm LLP}$

Calculation of the Relic Density

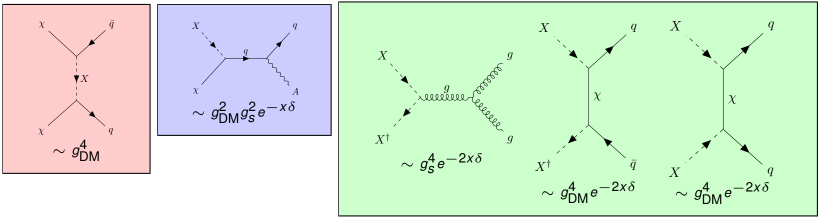
We adjusted micrOMEGAs 5.2.7 such that

- the Sommerfeld Effect is included for colored scalars up to the adjoint representation
- Bound State effects are included for colored scalars up to the adjoint representation

Determine $g_{DM,0}$ for each data point $(m_{\mathrm{DM}}, \Delta m)$ such that DM is *not* overproduced.

Annihilation Channels

NPE = Non-Perturbative Effects



⇒ No NPE ⇒ No NPE ⇒ Subject to NPE

Color Decomposition

Process: $(X_1)_{\mathbf{R_1}} + (X_2)_{\mathbf{R_2}} \to SM + SM$

Color Potential

$$V(r) = -\frac{\alpha_s}{2r}\left[C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\mathbf{R})\right] = -\frac{\alpha_{\mathrm{eff},[\mathbf{R}]}}{r}$$

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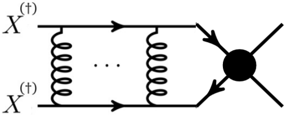
Color Configurations

$$\mathbf{3} \times \overline{\mathbf{3}} = \mathbf{1} + \mathbf{8} \to \alpha_{\text{eff},[\mathbf{1}]} = \frac{4}{3},\ \alpha_{\text{eff},[\mathbf{8}]} = -\frac{1}{6}$$

$$\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} + \mathbf{6} \to \alpha_{\text{eff},[\overline{\mathbf{3}}]} = \frac{2}{3},\ \alpha_{\text{eff},[\mathbf{6}]} = -\frac{1}{3}$$



n-gluon exchanges contribute with $\left(\frac{\alpha}{v}\right)^n$ for $\alpha \sim v$

Sommerfeld Effect

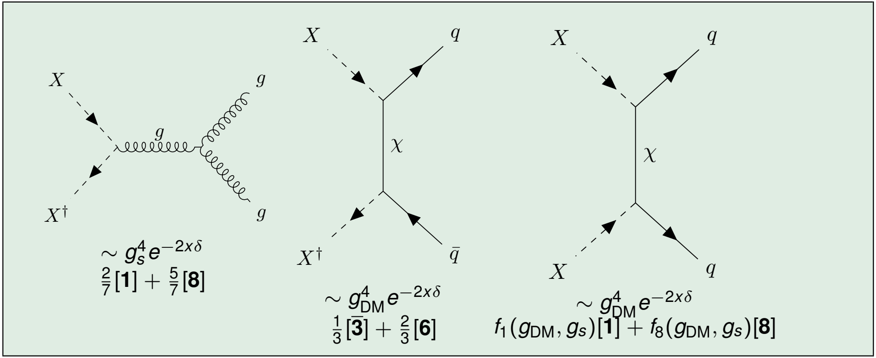
$$\sigma(X_1 X_2 \to SM\, SM) = S\left(\frac{\alpha_{\text{eff}}}{v}\right)\sigma_{\text{pert.}}$$

Sommerfeld Factor

$$S\left(\frac{\alpha_{\text{eff}}}{v}\right) = \begin{cases} 1 & \text{, if } \left|\frac{\alpha_{\text{eff}}}{v}\right| \ll 1, \\ \frac{\alpha_{\text{eff}}}{v} & \text{, if } \left|\frac{\alpha_{\text{eff}}}{v}\right| \gg 1 \wedge \alpha_{\text{eff}} > 0, \\ \exp\left(2\pi\frac{\alpha_{\text{eff}}}{v}\right) & \text{, if } \left|\frac{\alpha_{\text{eff}}}{v}\right| \gg 1 \wedge \alpha_{\text{eff}} < 0 \end{cases}$$

Figure from Talk by J.Harz @ DM Working Group

Annihilation Channels II



As a rule of thumb, we find:

$g_{\mathrm{DM}} > g_s \rightarrow$ Sommerfeld effect reduces annihilation cross section

$g_{\mathrm{DM}} < g_s \rightarrow$ Sommerfeld effect increases annihilation cross section

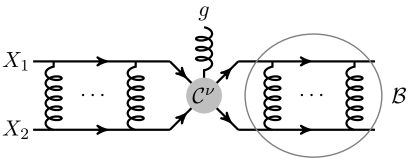
SE vs BSF

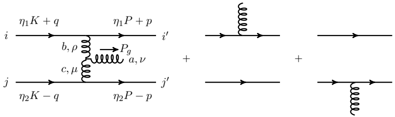
Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle S\left(\alpha / v_{ij}\right) \cdot \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}}$$

$\langle \sigma_{\text{eff}} v \rangle$	Sommerfeld Effect	Bound State Formation
$g_{\text{DM}} \gg g_s$	$-$	
$g_{\text{DM}} \ll g_s$	$+$	

Bound State Formation





Bound State Formation (BSF)

$$\sigma(X_1 X_2 \to \mathcal{B}(X_1 X_2)\ g) = \sigma_{\text{BSF}} \sim \frac{\alpha_s^2}{m_X^2} S_{\text{BSF}}\left(\frac{\alpha}{v}\right)$$

Bound state as an additional particle in the thermal bath.
$\Rightarrow$ Boltzmann Equation needs to be modified

Figures from [Harz,Petraki (2018)]

Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}} + \langle \sigma_{\text{BSF}} v \rangle_{\text{eff}} \frac{n_X^{\text{eq}}}{n^{\text{eq}}} \frac{n_X^{\text{eq}}}{n^{\text{eq}}}$$

Bound states effectively provide an additional annihilation channel.

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Bound states effectively provide an additional annihilation channel.

Bound State contribution to $\langle \sigma_{\text{eff}} v \rangle$

$$\langle \sigma_{\text{BSF}} v \rangle_{\text{eff}} = \langle \sigma_{\text{BSF}} v \rangle \frac{\Gamma_{\mathcal{B} \to SM}}{\Gamma_{\mathcal{B},\text{ion}} + \Gamma_{\mathcal{B} \to SM}}$$

$\rightarrow$ BSF only contributes to the annihilation cross section of DM if the bound states decay into SM particles!