

Afflictions of the “minimal” SO(10) GUT

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Outline

The latest (final?!) chapter in the story of “minimal” SO(10) GUT ...

- Introduction to the model $45 + 126 + 10\mathbb{C}$ (Breaking and Yukawa sector)
 - An obstruction to the usual SM Higgs doublet fine-tuning
 - Fine-tuning... ultimately can be done, but only in non-perturbative parameter regions
- This affliction of the model does not seem to be curable

The model: $45 + 126 + 10_{\mathbb{C}}$

- non-SUSY renormalizable SO(10) GUT
- Field content:
 - fermions: $3 \times 16_F$
 - scalars: $45 + 126 + 10_{\mathbb{C}}$

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 \rightarrow scalars: $45 + 126 + 10_{\mathbb{C}}$
- Motivation: “**minimality**” and “**calculability**” of proton decay:
no $(45_G \cdot 45_G \cdot S)/M_{Pl}$ operator, GUT scale more robust

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- **Breaking sector:** 2-stage breaking (GUT & seesaw scale)

$$\text{SO}(10) \xrightarrow{\langle 45 \rangle} G \xrightarrow{\langle 126 \rangle} \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$$

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- **Yukawa sector:** realistic, $Y_{10}, \tilde{Y}_{10}, Y_{126}$ symmetric

$$\mathcal{L}_{Yuk} = 16_F 16_F (Y_{10} \textcolor{blue}{10} + \tilde{Y}_{10} \textcolor{blue}{10}^* + Y_{126} \textcolor{blue}{126}^*)$$

Symmetry breaking I

- The $45 + 126$ Higgs model

$$\text{SO}(10) \xrightarrow{\langle 45 \rangle} G \xrightarrow{\langle 126 \rangle} \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$$

- 3 SM-singlet VEVs: $\omega_{BL}, \omega_R \in 45$ and $\sigma \in 126$
- What are the possibilities for G ?

$\langle 45 \rangle$ direction

$\omega_{BL} \approx \omega_R$

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$\omega_{BL} \sim \omega_R$

$\omega_R \ll \omega_{BL}$

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$\omega_{BL} \sim \omega_R$ $3_c 2_L 1_R 1_{B-L}$

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$\omega_{BL} \ll \omega_R$ $4_C 2_L 1_R$

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$\langle 45 \rangle$ direction	G	tree-level
$\omega_{BL} \approx \omega_R$	$5_Y 1$	✗ (unification)
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$\omega_{BL} \sim \omega_R$	$3_c 2_L 1_R 1_{B-L}$	✗ (tachyonicity)
$\omega_R \ll \omega_{BL}$	$3_c 2_L 2_R 1_{B-L}$	✗ (tachyonicity)
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$\langle 45 \rangle$ direction	G	one-loop
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$\omega_{BL} \approx -\omega_R$	$5' 1'$	Ø (unification)
$\omega_{BL} \sim \omega_R$	$3_c 2_L 1_R 1_{B-L}$	Ø (non-perturbative $\frac{\omega_{BL}\omega_R}{ \sigma ^2}$)
$\omega_R \ll \omega_{BL}$	$3_c 2_L 2_R 1_{B-L}$	$\checkmark \textcircled{S}$ ($M_{GUT} \approx 10^{18}$ GeV)
$\omega_{BL} \ll \omega_R$	$4_C 2_L 1_R$	$\checkmark \textcircled{S}$ ($M_{GUT} \approx 10^{15}$ GeV)

Symmetry breaking II

- Focus now on the feasible scenario: $\omega_{BL} \ll |\sigma| \ll \omega_R$
- Tachyonic instabilities:

$$M_{(8,1,0)}^2 = -2 a_2 \omega_R^2$$

$$M_{(1,3,0)}^2 = +4 a_2 \omega_R^2$$

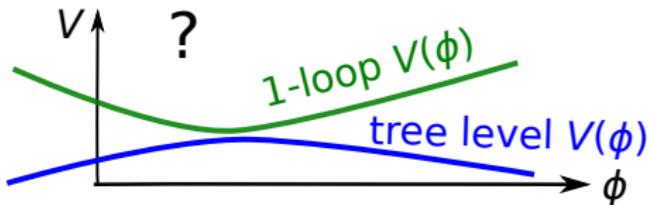
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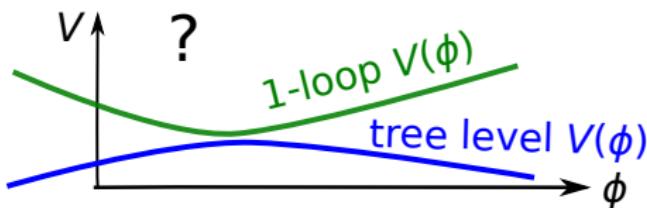
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Yes! Model first consistent at 1-loop.

Higgs model: $45 + 126$ see 2109.06784.

Full model: $45 + 126 + 10_{\mathbb{C}}$ shown here

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leads to

$$M_U = Y_{10} v_{10}^u + \tilde{Y}_{10} v_{10}^d{}^* + Y_{126} v_{126}^u,$$

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- (a) SM mass fit possible already with just Y_{10} , Y_{126}
- (b) \tilde{Y}_{10} present; Peccei-Quinn would forbid cure for tachyonicity
- (c) SM Higgs: must be **admxiture** of $\mathbf{10}$ - and $\mathbf{126}$ -doublets

Doublet fine-tuning

- Doublet mass matrix:

$$M_{(1,2,\pm 1/2)}^2 = \begin{pmatrix} M_{126}^2 & M_{\text{mix}}^2 \\ M_{\text{mix}}^{2\dagger} & M_{10}^2 \end{pmatrix}$$

SM Higgs obtained by fine-tuning: $\det M_{(1,2,\pm 1/2)}^2 = 0$.

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- Way out: tune down **only to next-order corrections**

→ What happens at 1-loop?

→ If 2-loop order not computed: necessary but not sufficient condition

Numerical analysis — setup

For a point in parameter space:

(1) Compute **1-loop** and **tree-level** spectrum

[interesting problem on its own, but technical; will not discuss further]

(2) **Viability** considerations:

(2a) **non-tachyonicity** for all states

(2b) **unification** of gauge couplings g_i @ **2-loop**

(2c) **perturbativity** (degree of arbitrariness in definition)

→ corrections to masses under control: “ $|\delta m^2/m^2| < 1$ ”

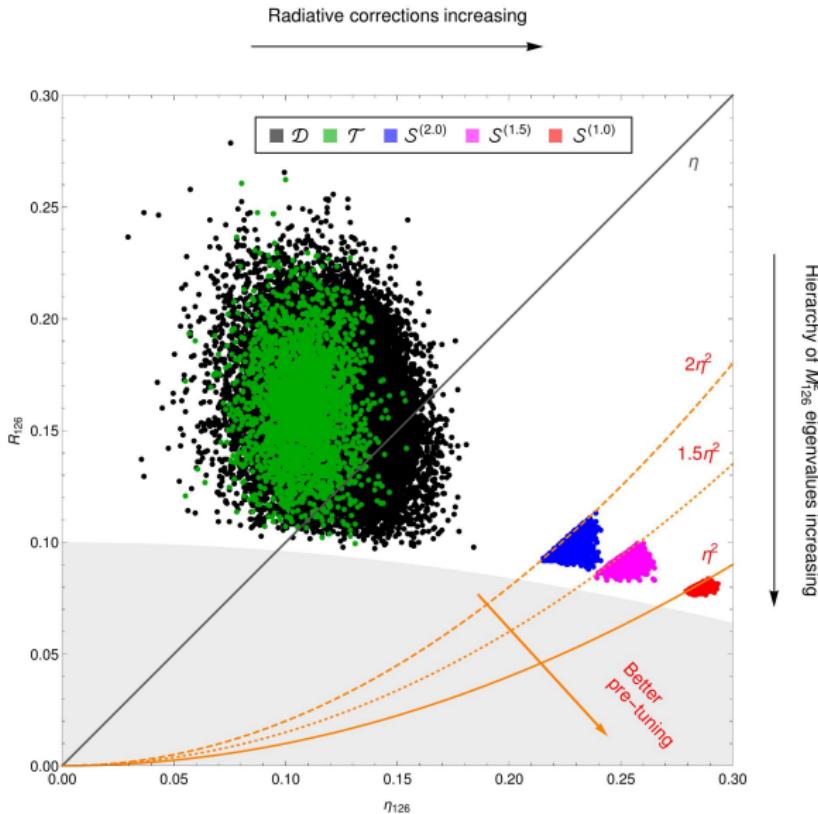
→ RGE for λ under control: up to $M_{GUT} \cdot 10^{+0.5}$

(3) Doublet **fine-tuning**: considered through quantities

$$R := \frac{m_{(1)}^2}{m_0^2}, \quad \eta := \frac{\delta m_{(1)}^2}{m_0^2}, \quad \text{where } m_0^2 \text{ "typical" tree-level mass.}$$

(use heavy doublet in 126 as proxy)

Fine-tuning vs perturbativity I



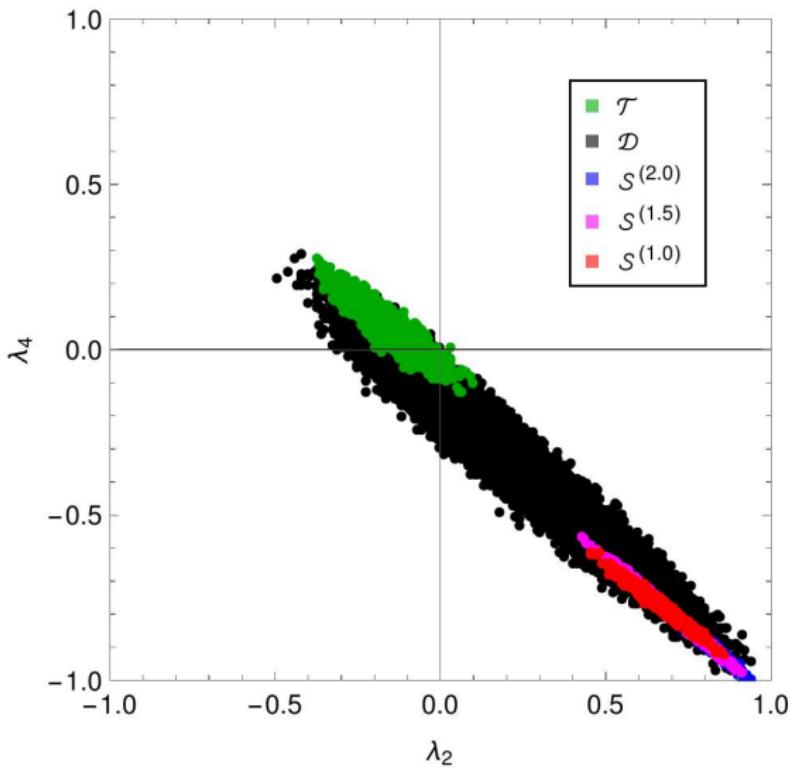
Datasets of viable points:

- \mathcal{D} : generic point
- \mathcal{T} : enhanced RGE perturbativity $10^{\pm 1} M_{GUT}$
- \mathcal{S} : M_{126}^2 tuned to 2-loop expectation

Plot: η vs R

Tuning happens at the expense of perturbativity!

Fine-tuning vs perturbativity II



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 $10^{\pm 1} M_{GUT}$
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Regions with
enhanced perturbativity
and doublet tuning
incompatible!

(best demonstrated in the
 λ_2 - λ_4 parameter plane)

SO(10) GUT model with scalar sector $\underline{45 + 126 + 10_{\mathbb{C}}}$:

- Tricky: **1-loop** is first consistent perturbative order
 - Suitable vacuum only at 1-loop
 - Small patch of parameter space viable
 - Doublet fine-tuning (1-loop): clashes with perturbativity
- Model **perturbatively not viable**
- A more general message:
**large # of fields ⇒ perturbativity vs other demands
(possible clash)**

Summary

SO(10) GUT model with scalar sector $45 + 126 + 10_C$

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→ A more general message:

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Thank you for your attention!

Backup: technical challenges

- A lot of particles: scalar mass matrix $M_S^2(\phi)$ in V_1 is 297×297 in Higgs model, 317×317 in full theory
- A lot of parameters: the scalar potential written schematically is

$$\begin{aligned} V(45, 126) = & \mu^2 45^2 + a 45^4 + \\ & + \nu^2 |126|^2 + \lambda |126|^4 + \eta 126^4 + \tau 45 \cdot |126|^2 + \\ & + (\alpha, \beta) 45^2 \cdot |126|^2 + \gamma 45^2 \cdot 126^2 + h.c., \end{aligned}$$

$$\begin{aligned} V(45, 126, 10) = & V(45, 126) + \xi^2 10^2 + h 10^4 \\ & + \kappa 10^2 45^2 + \zeta 45^2 \cdot 126 \cdot 10 + \rho 10^2 |126|^2 + \\ & + \rho' 10^2 126^2 + \varphi |126|^2 \cdot 126 \cdot 10 + h.c. \end{aligned}$$

(possibly >1 independent contraction, for brevity 10^* was written as 10)

- Parameters in full theory:
 $(15 \mathbb{R} + 14 \mathbb{C})$ dimensionless, $(5 \mathbb{R} + 1 \mathbb{C})$ massive