



Revisiting $B \rightarrow K^{(*)}\nu\bar{\nu}$ decays in the SM (and beyond)

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IJCLab (Orsay)

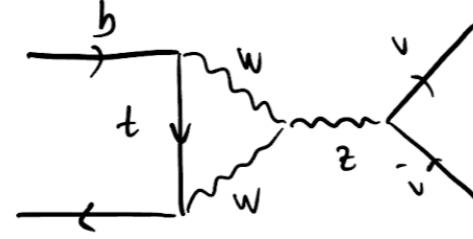
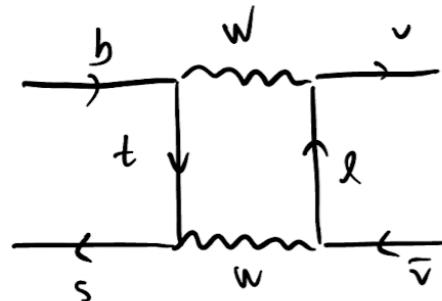
*Based on [2301.06990],
in collaboration with Damir Becirevic and Gioacchino Piazza*

Portorož: Particle Physics from Early Universe to Colliders, 11 March, 2023

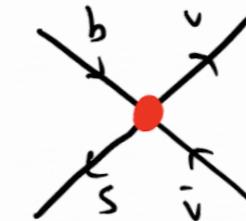
Motivation

See talks by Buras and Schwanda

- Flavor **C**hanging **N**eutral **C**urrent (**FCNC**) processes are powerful indirect probes of New Physics (NP) effects since they are **loop-** and **CKM suppressed**.



...



$$O_{\text{exp}} = O_{\text{SM}} \left(1 + \# \frac{C}{R^2} + \dots \right)$$

$\propto \frac{|V_{tb} V_{ts}^*|^2}{(16\pi^2)^2}$

- The **main obstacle** to probe NP at **low-energies** is the careful assessment of **hadronic uncertainties**:
 - Decays based on the $b \rightarrow s\nu\bar{\nu}$ transition are **theoretically cleaner** than those based on $b \rightarrow s\ell\ell$, since they are **not affected** by **problematic** long-distance contributions from **$c\bar{c}$ loops**.
- Further motivation** to revisit these decays:
 - Progress** in the **lattice QCD** determinations of the $B \rightarrow K$ **form-factor** [HPQCD, '22].
 - Upcoming measurement at **Belle-II** (this year?).

This talk:

- Revisit the **SM predictions** for $B \rightarrow K^{(*)}\nu\bar{\nu}$.
- What can we learn from these **measurements** (in the SM and beyond)?

Revisiting $B \rightarrow K^{(*)}\nu\bar{\nu}$ in the SM

SM predictions

See talk by Buras

- **Effective Hamiltonian** within the SM:

see e.g. [Buras et al. '14]

$$\mathcal{L}_{\text{eff}}^{\text{b} \rightarrow s\nu\nu} = \frac{4G_F \lambda_t}{\sqrt{2}} \frac{\alpha_{\text{em}}}{2\pi} \sum_i C_L^{\text{SM}} (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_{Li} \gamma^\mu \nu_{Li}) + \text{h.c.},$$

$\lambda_t = V_{tb} V_{ts}^*$

- **Short-distance** contributions known to **good precision**:

$$C_L^{\text{SM}} = -X_t / \sin^2 \theta_W$$

$$= -6.32(7)$$

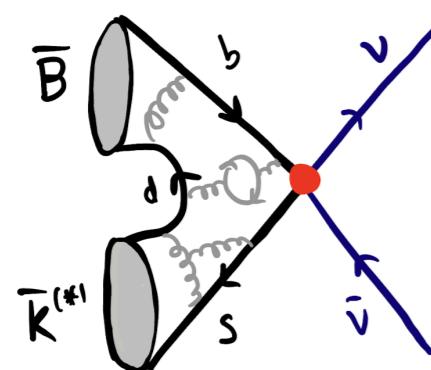
Including NLO QCD and two-loop EW contributions:

$$X_t = 1.462(17)(2)$$

[Buchala et al. '93, '99], [Misiak et al. '99], [Brod et al. '10]

Two main source of uncertainties:

i) Hadronic matrix-element:



Known Lorentz factors

$$\langle K^{(*)} | \bar{s}_L \gamma^\mu b_L | B \rangle = \sum_a K_a^\mu \mathcal{F}_a(q^2)$$

Form-factors (e.g., LQCD)

ii) CKM matrix:

From CKM unitarity:

$$|V_{tb} V_{ts}^*| = |V_{cb}| (1 + \mathcal{O}(\lambda^2))$$

Which value to take (incl. vs. excl.)?

Form-factors: $B \rightarrow K\nu\bar{\nu}$

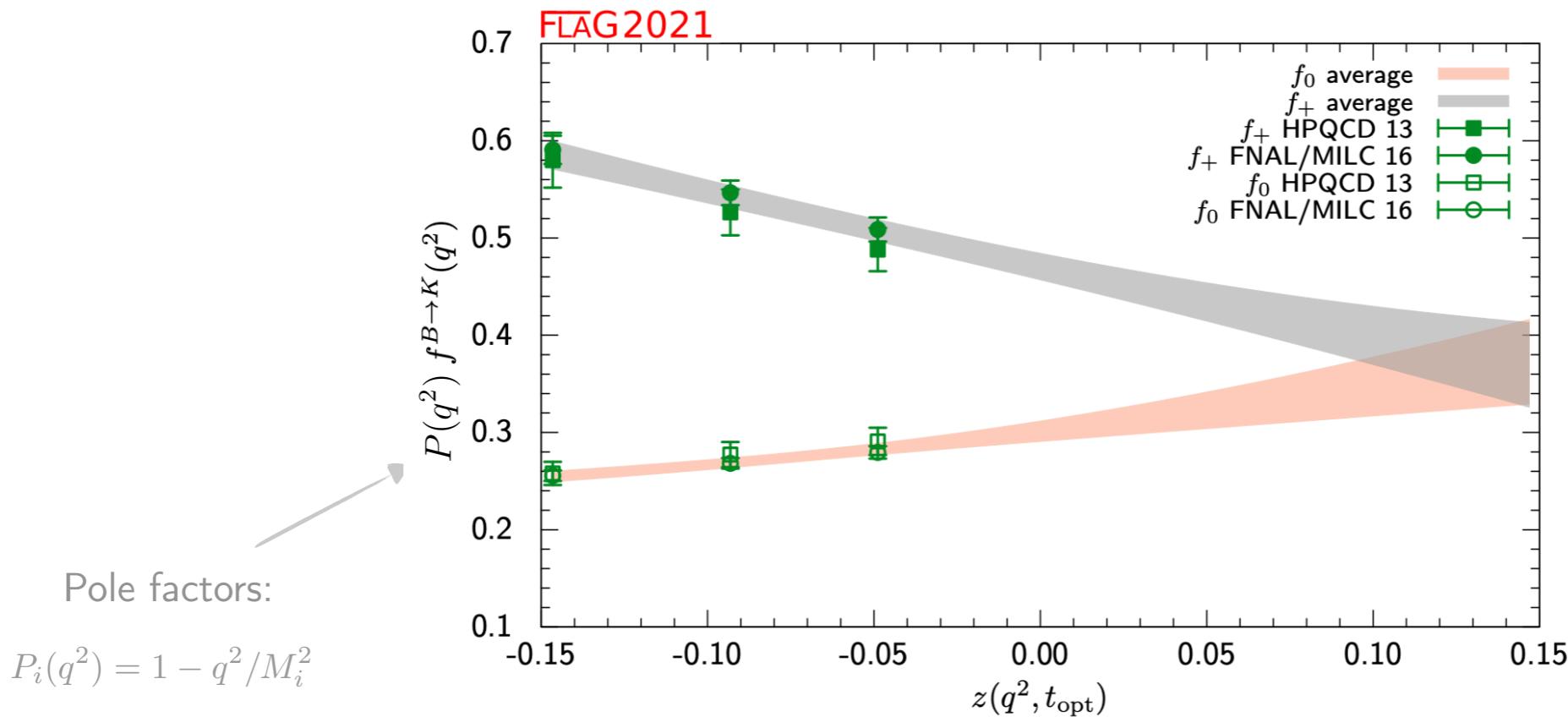
- Lattice QCD data available at **nonzero recoil** ($q^2 \neq q_{\max}^2$) for all form-factors:

$$\langle K(k) | \bar{s}_L \gamma^\mu b_L | B(p) \rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu \right] f_+(q^2) + q^\mu \frac{m_B^2 - m_K^2}{q^2} f_0(q^2)$$

with $f_+(0) = f_0(0)$.

Only form-factor needed for $B \rightarrow K\nu\bar{\nu}$!

- Lattice results — **circa 2021**:



$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

FLAG average of [HPQCD '13, FNAL/MILC '16]

Form-factors: $B \rightarrow K\nu\bar{\nu}$

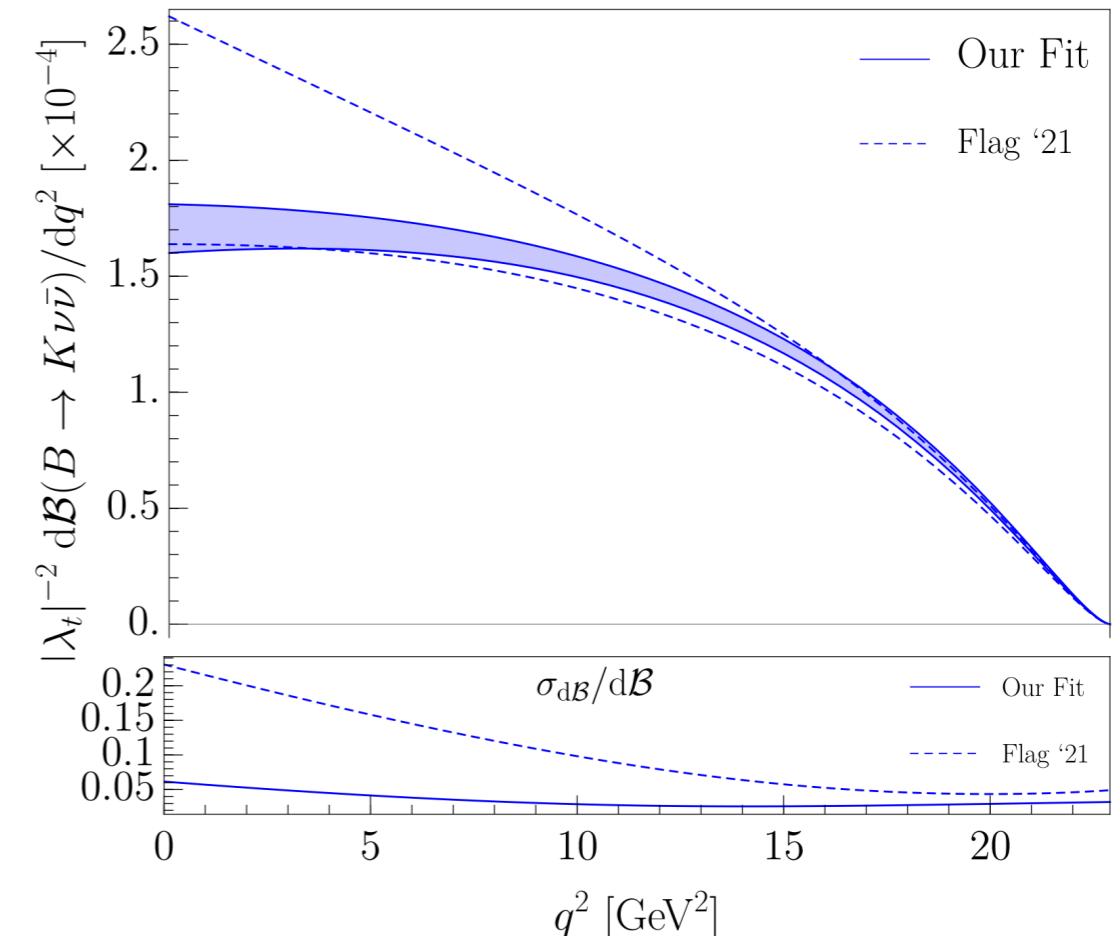
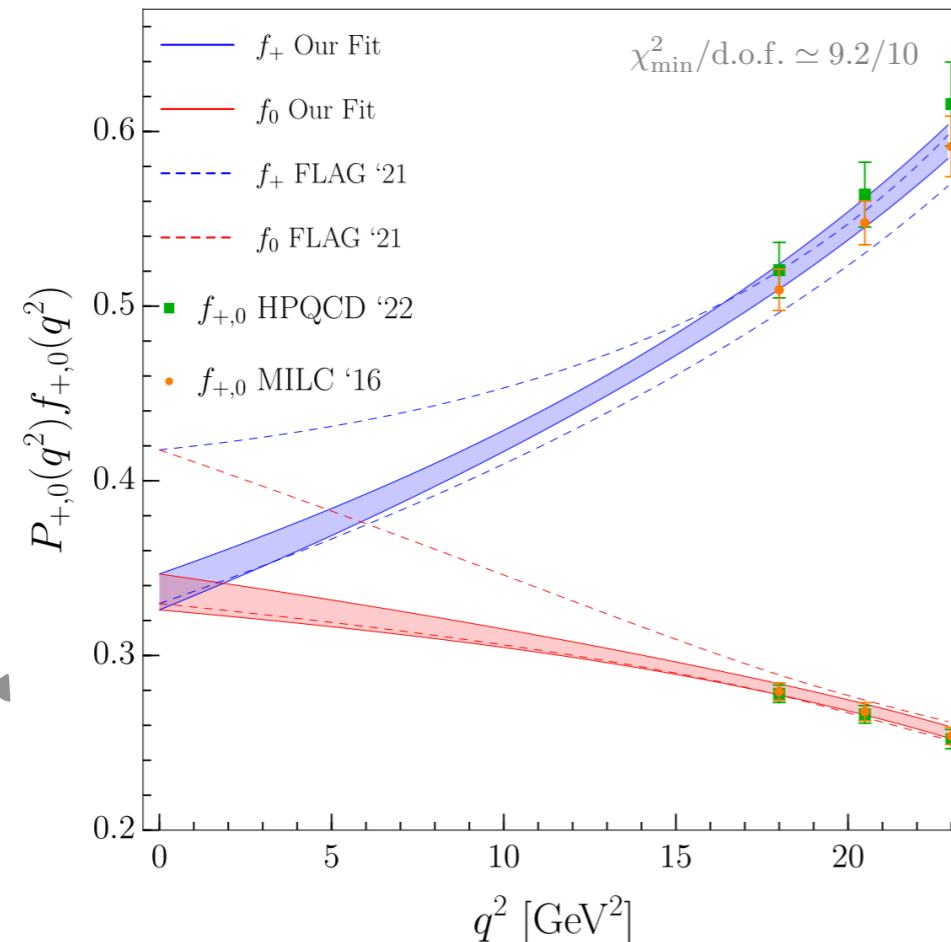
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with $f_+(0) = f_0(0)$.

Only form-factor needed for $B \rightarrow K\nu\bar{\nu}$!

- [NEW]** We update the FLAG average by combining [HPQCD '22] results with [FNAL/MILC '16]:

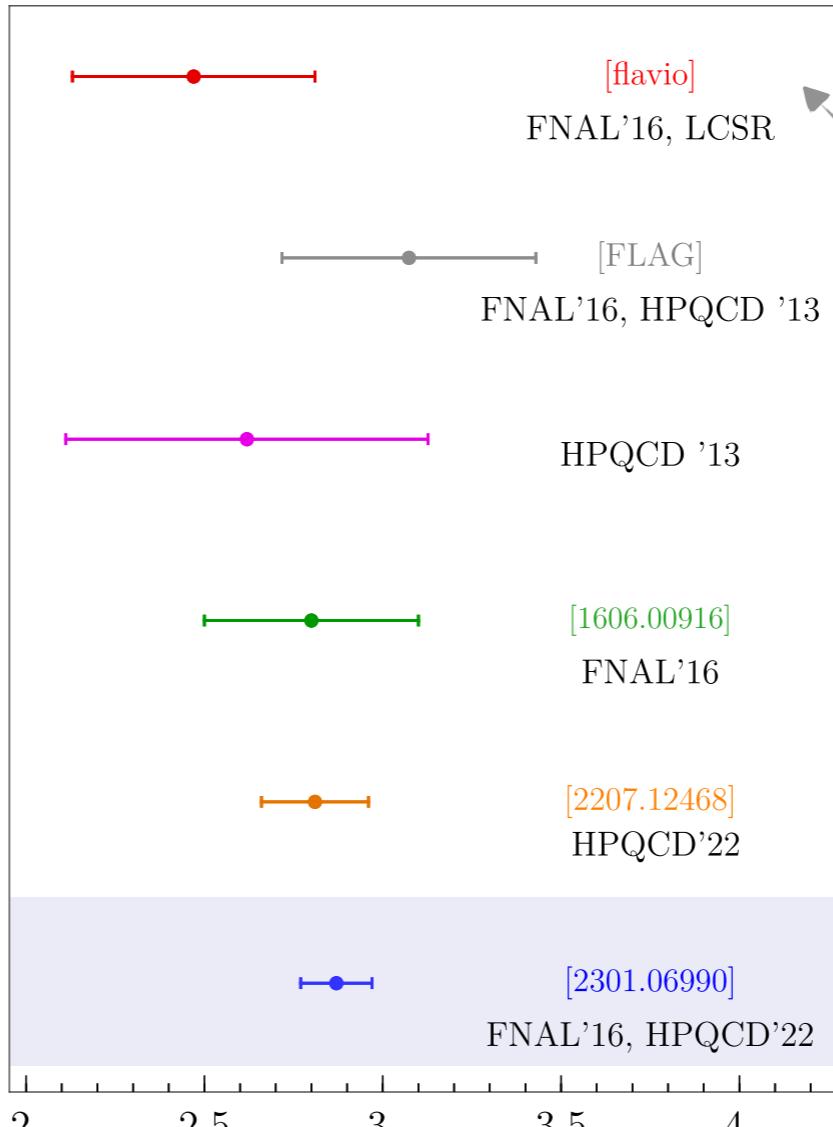


[Becirevic, Piazza, OS. 2301.06990]

Form-factors: $B \rightarrow K\nu\bar{\nu}$

*Annihilation contributions not included below (see next slides)!

- Our final predictions:



$$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{loop}}^{\text{SM}}/|\lambda_t|^2$$

$$\mathcal{B}(B \rightarrow K\nu\bar{\nu})^{\text{SM}}/|\lambda_t|^2 = \begin{cases} (1.33 \pm 0.04)_{K_S} \times 10^{-3} \\ (2.87 \pm 0.10)_{K^+} \times 10^{-3} \end{cases} \quad [\approx 3\% \text{ uncertainty}]$$

[Becirevic, Piazza, OS. 2301.06990]

[Intermezzo]: Cross-check of $f_+^{B \rightarrow K}(q^2)$

- SM predictions depend on the **extrapolation** of the LQCD **form-factors to low q^2** values — **parameterisation dependent?**

⇒ How can we **test the shape** of the **extrapolated LQCD form-factors?**

- We propose to measure:

[Becirevic, Piazza, OS. 2301.06990]

$$r_{\text{low/high}} = \frac{\mathcal{B}(B \rightarrow K\nu\bar{\nu})_{\text{low-}q^2}}{\mathcal{B}(B \rightarrow K\nu\bar{\nu})_{\text{high-}q^2}}$$

⇒ Independent of λ_t and the *form-factor normalisation*, as well as of *NP contributions*.

NB. w/o ν_R

- Using the bins $(0, q_{\max}^2/2)$ vs. $(q_{\max}^2/2, q_{\max}^2)$:

e.g, using (old) FLAG average:

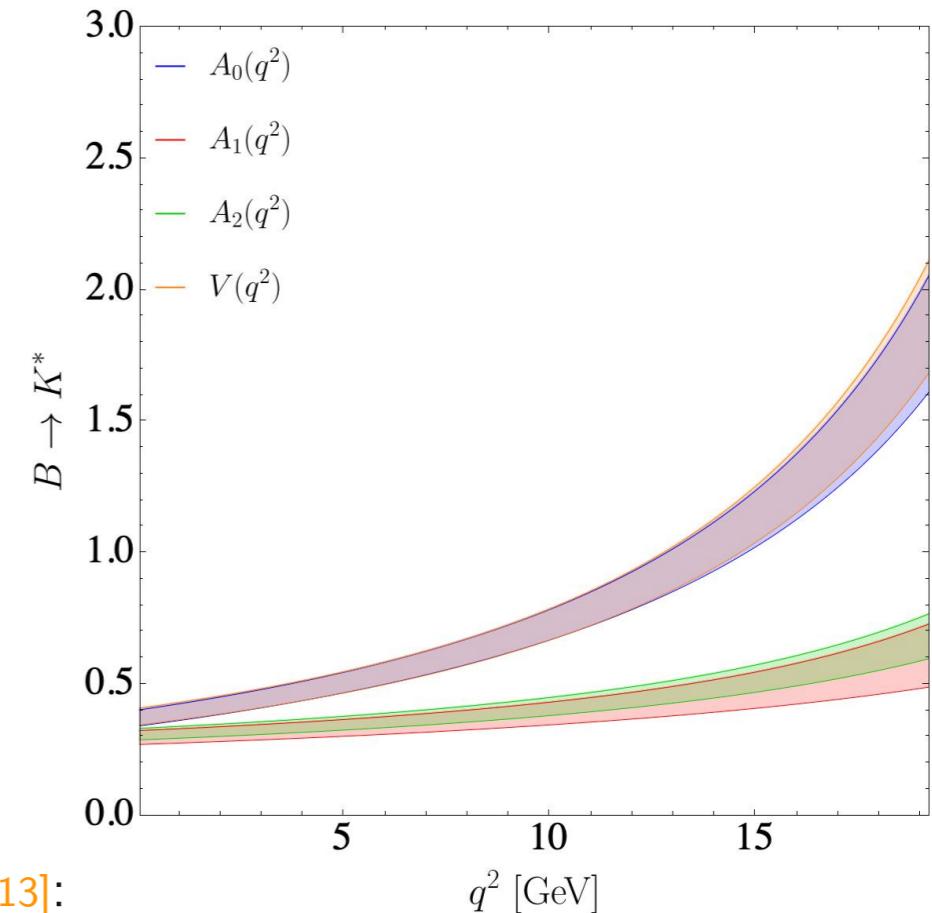
$$r_{\text{low/high}} = 1.91 \pm 0.06$$

$$r_{\text{low/high}} = 2.15 \pm 0.26$$

Form-factors: $B \rightarrow K^*\bar{\nu}\bar{\nu}$

- $B \rightarrow K^*\bar{\nu}\bar{\nu}$ decays are **more challenging** for several reasons:

$$\begin{aligned} \langle \bar{K}^*(k) | \bar{s} \gamma_\mu (1 - \gamma_5) b | \bar{B}(p) \rangle = & \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p^\rho k^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} \\ & - i\varepsilon_\mu^* (m_B + m_{K^*}) A_1(q^2) \\ & + i(p+k)_\mu (\varepsilon^* \cdot q) \frac{A_2(q^2)}{m_B + m_{K^*}} \\ & + iq_\mu (\varepsilon^* \cdot q) \frac{2m_{K^*}}{q^2} [A_3(q^2) - A_0(q^2)], \end{aligned}$$

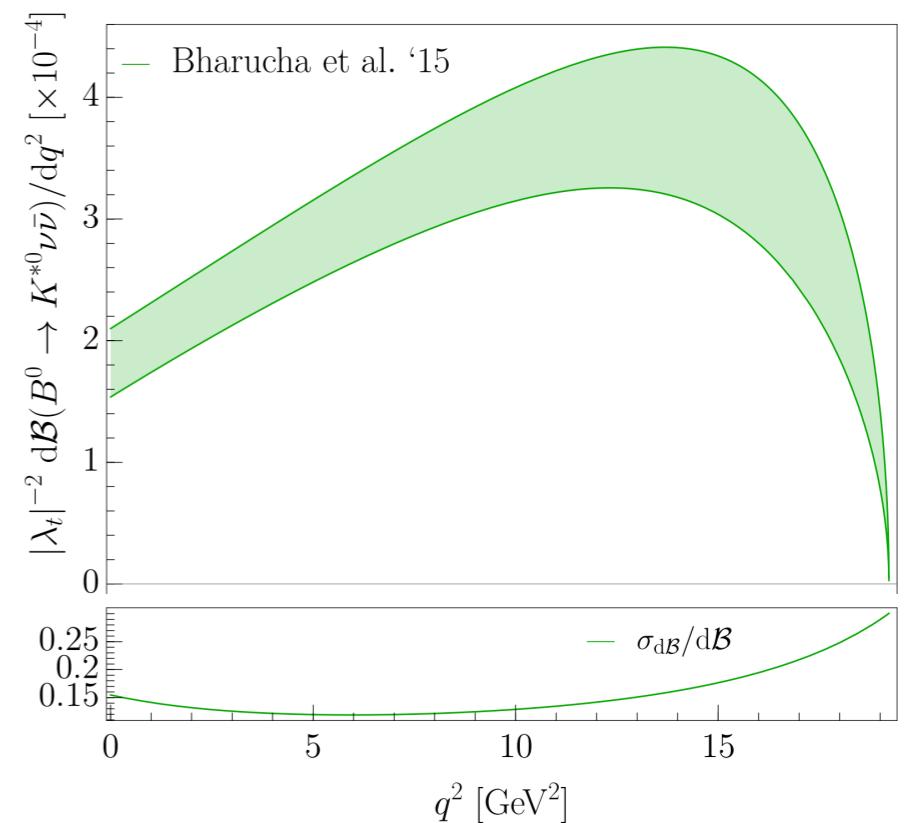


- We use LCSR (+LQCD) results from [Bharucha et al. '15, Horgan et al. '13]:

$$\mathcal{B}(B \rightarrow K^*\bar{\nu}\bar{\nu})^{\text{SM}} / |\lambda_t|^2 = \begin{cases} (5.9 \pm 0.8)_{K^{*0}} \times 10^{-3} \\ (6.4 \pm 0.9)_{K^{*+}} \times 10^{-3} \end{cases}$$

[$\approx 15\%$ uncertainty]

\Rightarrow Relatively small uncertainties, but are they accurate?



Which CKM value?

See talks by Buras and Becirevic

- Using available $b \rightarrow c\ell\bar{\nu}$ data:

$$|\lambda_t| \times 10^3 = \begin{cases} 41.4 \pm 0.8, & (B \rightarrow X_c l \bar{\nu}) \\ 39.3 \pm 1.0, & (B \rightarrow D l \bar{\nu}) \\ 37.8 \pm 0.7, & (B \rightarrow D^* l \bar{\nu}) \end{cases}$$

[HFLAV, '22]
[FLAG, '21]
[HFLAV, '22]

... to be compared to CKM global fits:

cf. also [Martinelli et al. '21]

$$|\lambda_t|_{\text{UTfit}} = (41.4 \pm 0.5) \times 10^{-2}$$

$$|\lambda_t|_{\text{CKMfitter}} = (40.5 \pm 0.3) \times 10^{-2}$$

- Alternative strategy: to use $\Delta m_{B_s} \propto f_{B_s}^2 \hat{B}_{B_s} |\lambda_t|^2$

[Buras, Venturini. '21, '22]

$$|\lambda_t| \times 10^3 = \begin{cases} 41.9 \pm 1.0, & (N_f = 2 + 1 + 1) \\ 39.2 \pm 1.1, & (N_f = 2 + 1) \end{cases}$$

$$f_{B_s} \sqrt{\hat{B}_{B_s}} = 256 \pm 6 \text{ MeV} \quad (N_f = 2 + 1 + 1)$$

[HPQCD '19]

$$f_{B_s} \sqrt{\hat{B}_{B_s}} = 274 \pm 8 \text{ MeV} \quad (N_f = 2 + 1)$$

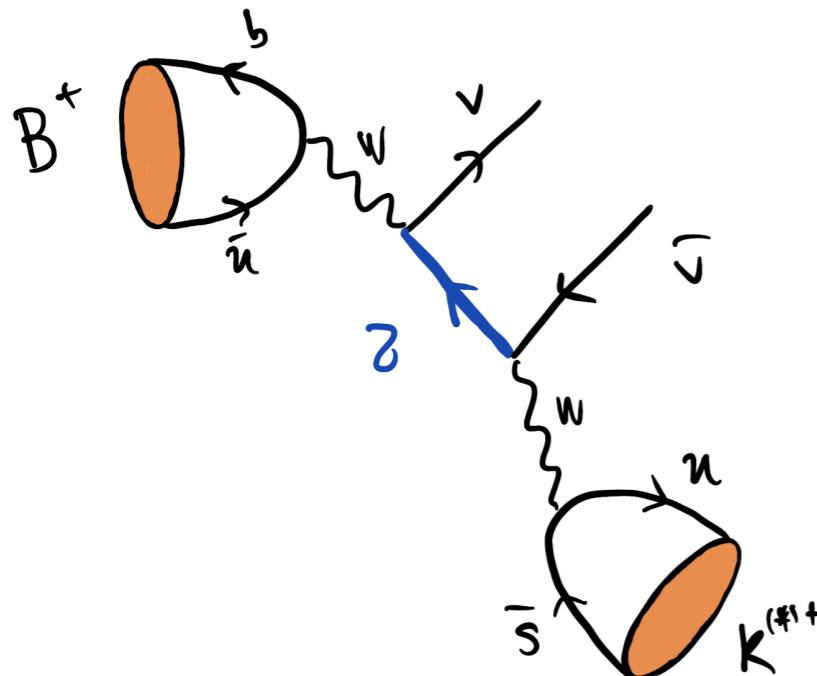
[FLAG '21]

There is **not a clear answer** to this **ambiguity** so far.

Weak-annihilation contributions

[Kamenik, Smith. '09]

- To keep in mind: decay modes with **charged mesons** are affected by **tree-level weak annihilation contributions**.



- Using *narrow-width approximation*:

$$\begin{aligned}\mathcal{B}(B^+ \rightarrow K^{(*)+} \nu \bar{\nu}) \\ \simeq \mathcal{B}(B^+ \rightarrow \tau^+ \bar{\nu}) \mathcal{B}(\tau^+ \rightarrow K^{(*)+} \nu)\end{aligned}$$

- *Non-negligible contributions*:

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{tree}}}{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}))_{\text{loop}}} \simeq 14 \%$$

$$\frac{\mathcal{B}(B^+ \rightarrow K^{*+} \nu \bar{\nu})_{\text{tree}}}{\mathcal{B}(B^+ \rightarrow K^{*+} \nu \bar{\nu}))_{\text{loop}}} \simeq 11 \%$$

$$m_{K^{(*)+}} \leq m_\tau \leq m_B$$

⇒ They *cannot be removed by a simple kinematical cut...*

Belle-II: Can these contributions be **disentangled** by exploiting the τ lifetime?

Summary

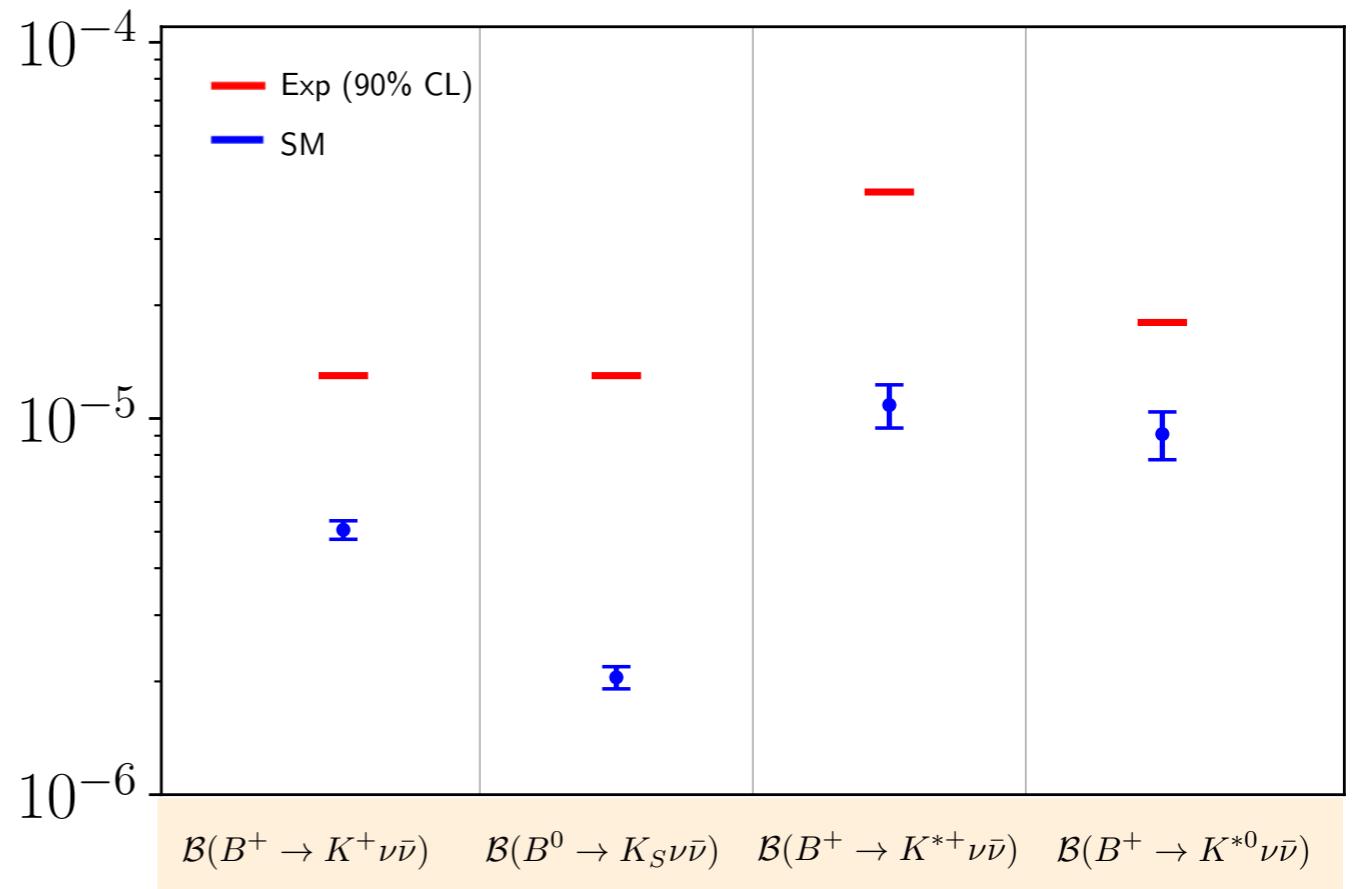
[Becirevic, Piazza, OS. 2301.06990]

[Belle 1303.3719, 1702.03224]

[BaBar 1009.1529, 1303.7465]

*Using V_{cb} from $B \rightarrow D\ell\bar{\nu}$ for illustration

Decay	Branching ratio
$B^+ \rightarrow K^+\nu\bar{\nu}$	$(5.06 \pm 0.14 \pm 0.25) \times 10^{-6}$
$B^0 \rightarrow K_S\nu\bar{\nu}$	$(2.05 \pm 0.07 \pm 0.12) \times 10^{-6}$
$B^+ \rightarrow K^{*+}\nu\bar{\nu}$	$(10.86 \pm 1.30 \pm 0.59) \times 10^{-6}$
$B^0 \rightarrow K^{*0}\nu\bar{\nu}$	$(9.09 \pm 1.20 \pm 0.55) \times 10^{-6}$



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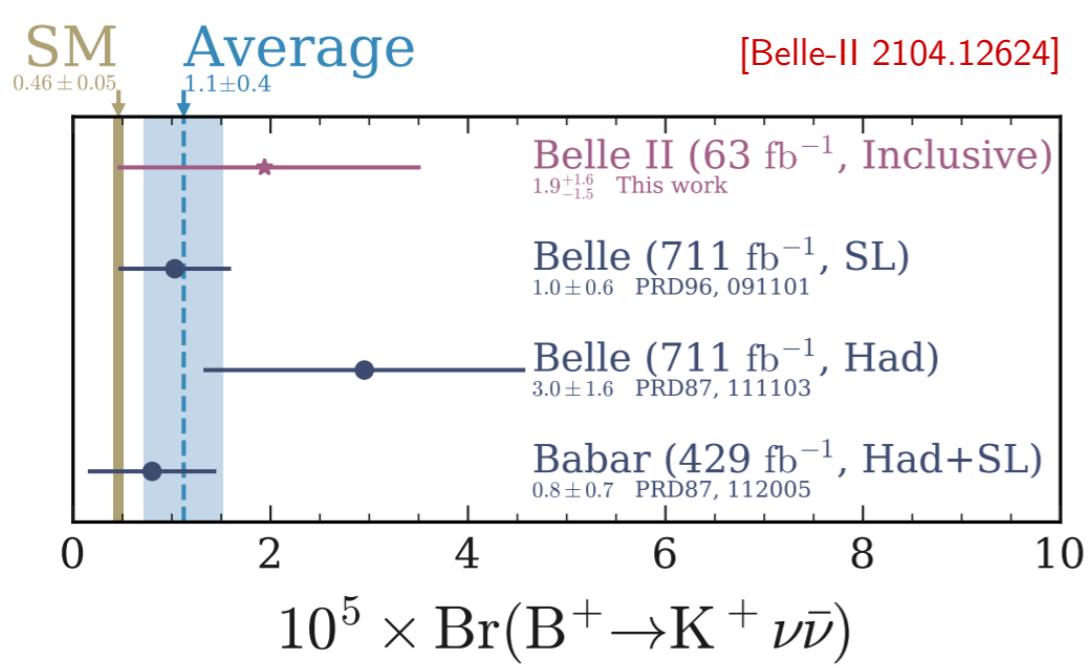
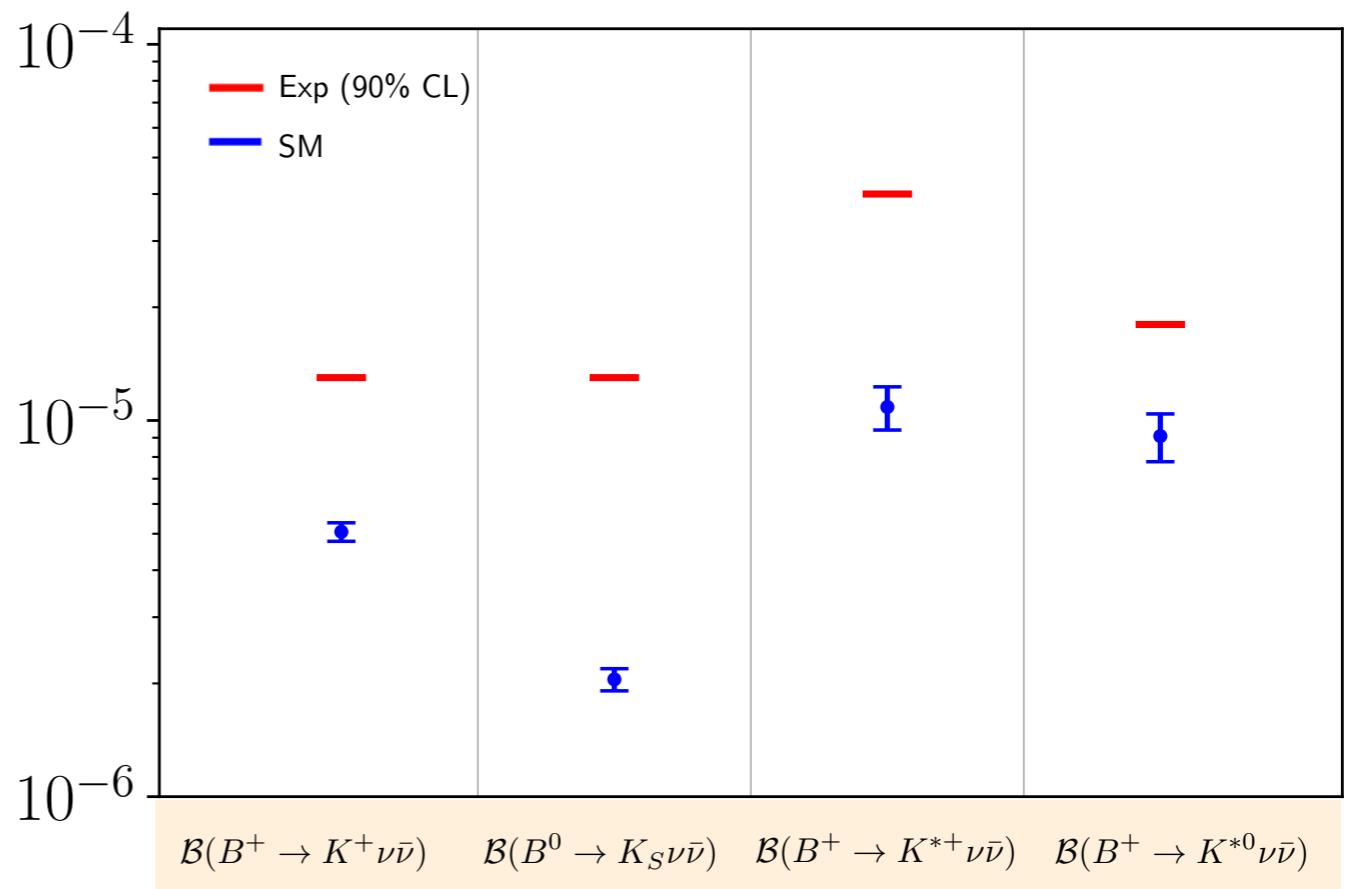
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Belle-II: New results expected soon?

See talk by Schwanda

Summary

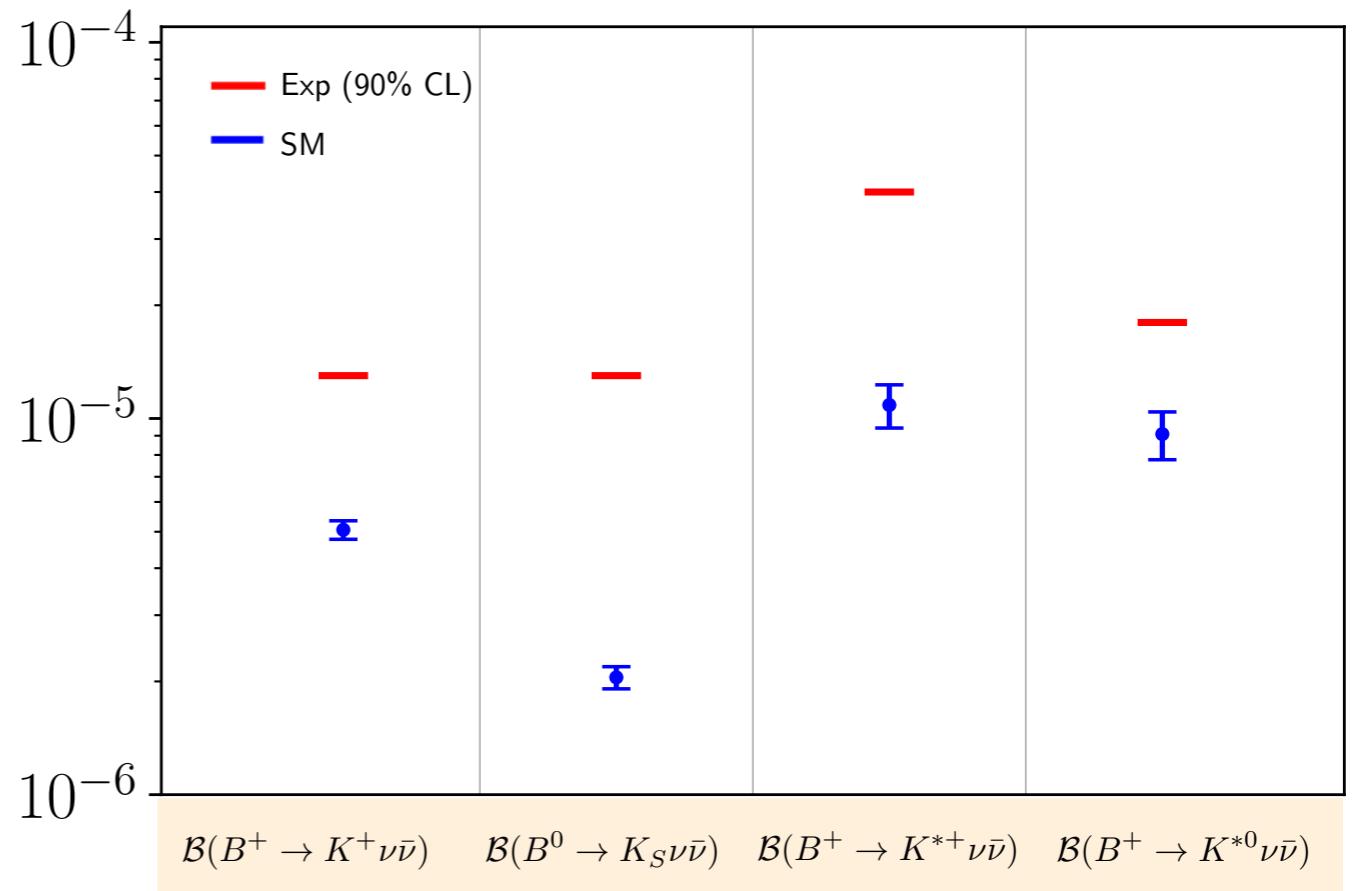
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Take-home:

- To remain **cautions** about **hadronic uncertainties** associated to the **form-factors** and the extraction of **CKM** matrix-elements — non-negligible given the projected Belle-II sensitivity.
- **Binned measurements** at Belle-II would be a **valuable piece of information** to **test the consistency the SM predictions**.

What can we learn from $B \rightarrow K^{(*)}\nu\bar{\nu}$?

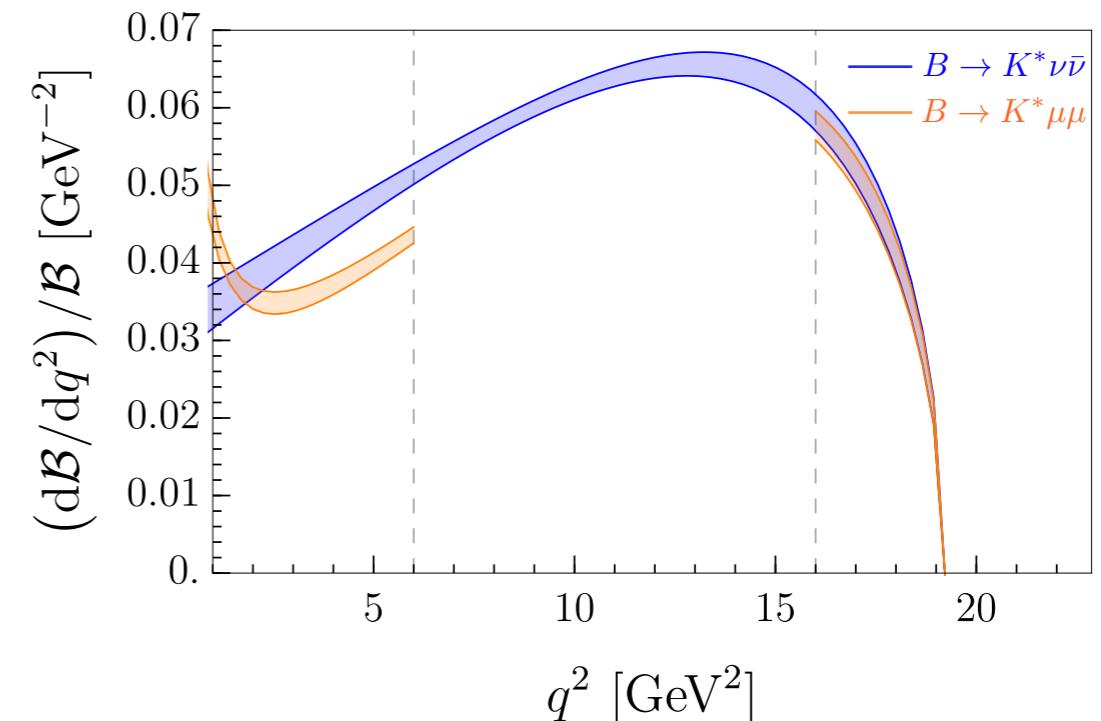
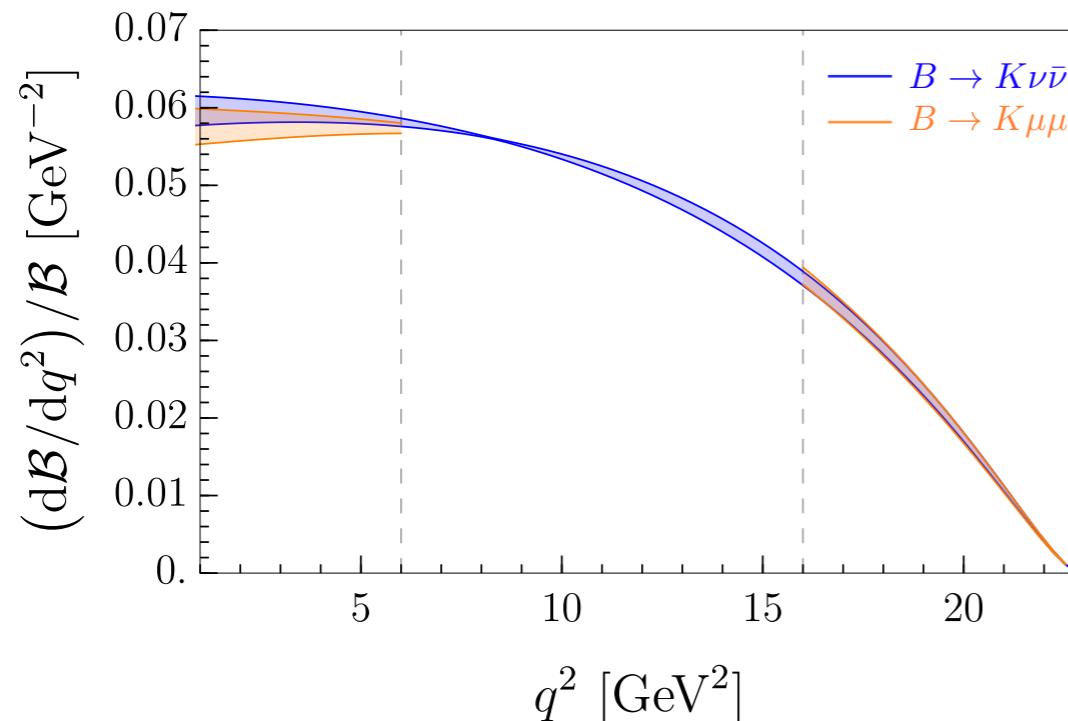
- **Remarks on $B \rightarrow K^{(*)}\nu\bar{\nu}/B \rightarrow K^{(*)}\mu\mu$**
- **Implications beyond the SM**

$B \rightarrow K^{(*)}\nu\bar{\nu}$ / $B \rightarrow K^{(*)}\mu\mu$

[Bartsch et al. '09]

[Becirevic, Piazza, OS. 2301.06990]

- $B \rightarrow K^{(*)}\nu\bar{\nu}$ and $B \rightarrow K^{(*)}\mu\mu$ have a similar decay spectrum away from the narrow $c\bar{c}$ resonances:



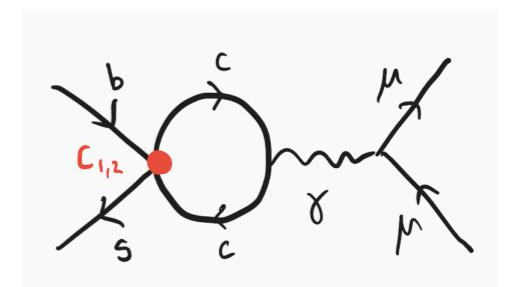
- We can define the **CKM-free ratio**:

*using 2-loop results for $c\bar{c}$ loops from [Asatryan et al. '09]

$$\mathcal{R}_{K^{(*)}}^{(\nu/l)}[q_0^2, q_1^2] \equiv \left. \frac{\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B \rightarrow K^{(*)}l\bar{l})} \right|_{[q_0^2, q_1^2]}$$

Ratio of partial branching fractions integrated in the same q^2 -bin.

- ⇒ **Form-factor** uncertainties **cancel out** to a good extent for $q^2 \gg m_\ell^2$.
- ⇒ Neglecting NP contributions, this ratio can be used to **extract** $C_9^{\mu\mu}$!



- Predictions using perturbative calculation of $c\bar{c}$ loops:

[Becirevic, Piazza, OS. 2301.06990]

$$\mathcal{R}_K^{(\nu/l)}[1.1, 6] \Big|_{\text{SM}} = 7.58 \pm 0.04$$

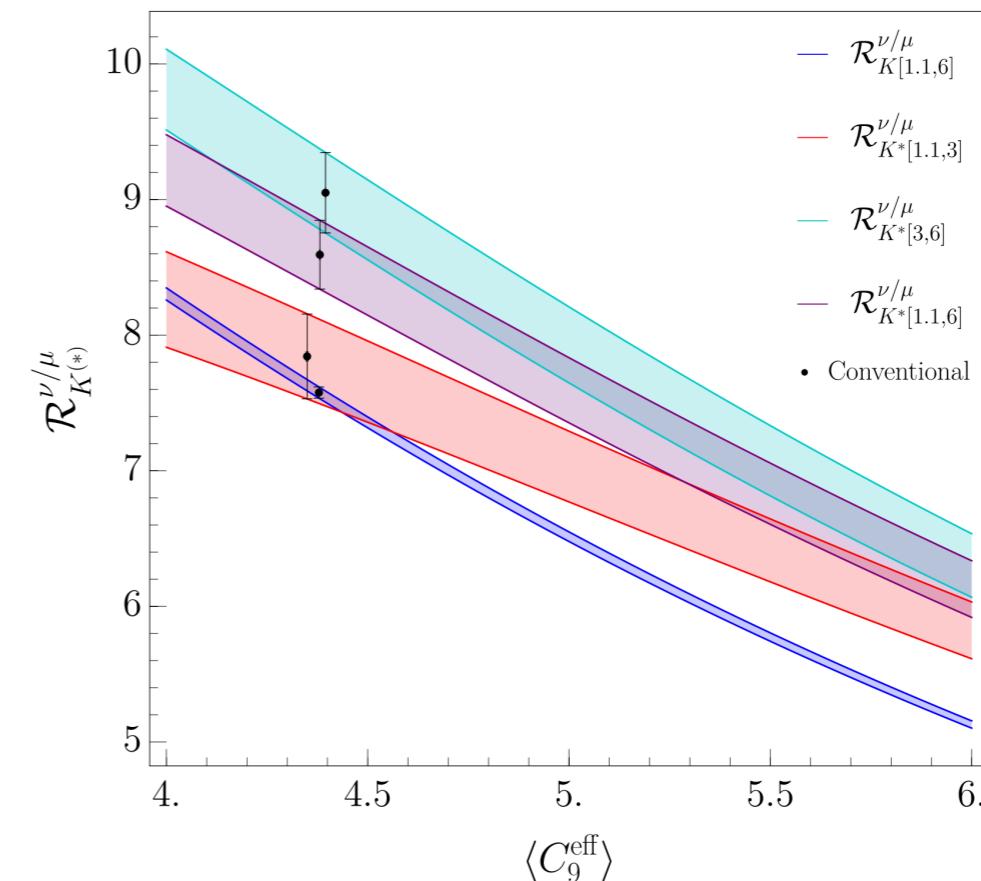
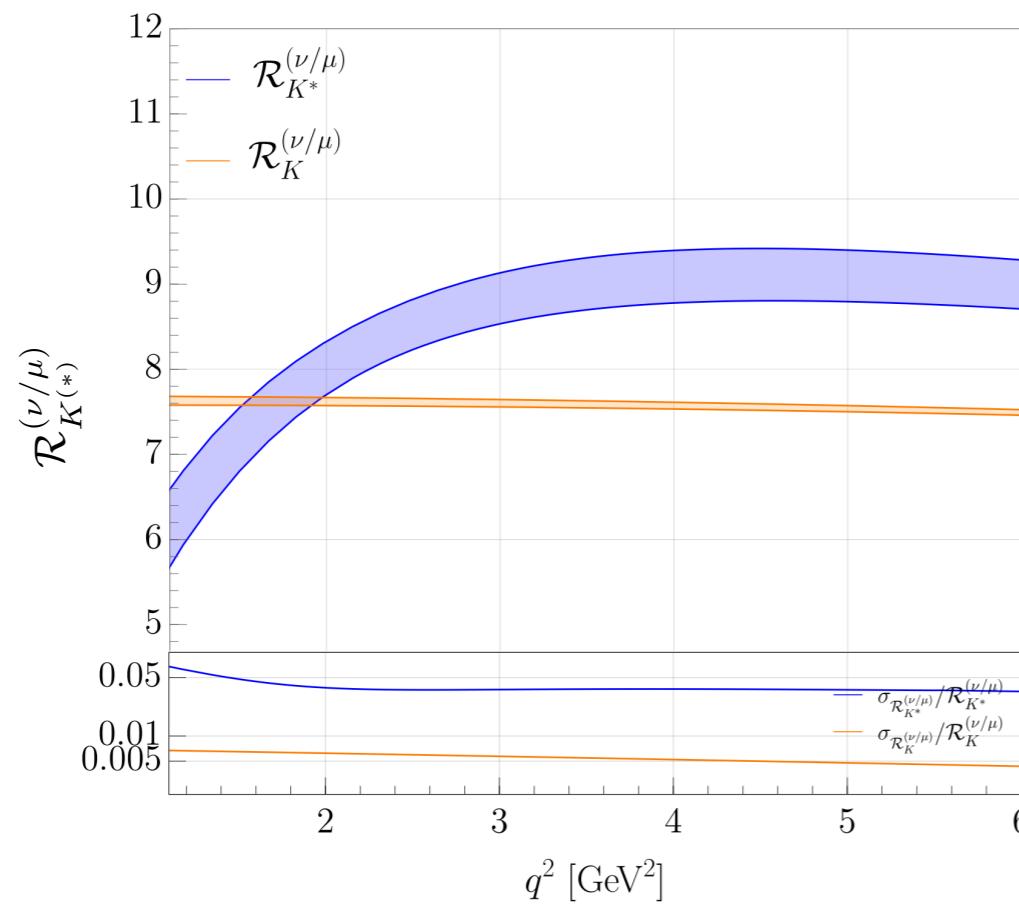
$$\mathcal{R}_{K^*}^{(\nu/l)}[1.1, 6] \Big|_{\text{SM}} = 8.6 \pm 0.3$$

with the following dependence on C_9^{eff} :

using [Asatryan et al. '09]

$$\frac{1}{\mathcal{R}_K^{(\nu/l)}[1.1, 6]} \Big|_{\text{SM}} \approx \\ [7.15 - 0.45 \cdot C_9^{\text{eff}} + 0.42 \cdot (C_9^{\text{eff}})^2] \times 10^{-2}$$

$$\frac{1}{\mathcal{R}_{K^*}^{(\nu/l)}[1.1, 6]} \Big|_{\text{SM}} \approx \\ [9.98 - 1.45 \cdot C_9^{\text{eff}} + 0.42 \cdot (C_9^{\text{eff}})^2] \times 10^{-2}$$



This measurement could help us to understand the various anomalies in $b \rightarrow s\mu\mu$ data.

Implications beyond the SM

- Low-energy EFT:

$$\mathcal{L}_{\text{eff}}^{\text{b} \rightarrow \text{s}\nu\nu} = \frac{4G_F\lambda_t}{\sqrt{2}} \frac{\alpha_{\text{em}}}{2\pi} \sum_{ij} \left[C_L^{\nu_i\nu_j} (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_{Li} \gamma^\mu \nu_{Lj}) + C_R^{\nu_i\nu_j} (\bar{s}_R \gamma_\mu b_R)(\bar{\nu}_{Li} \gamma^\mu \nu_{Lj}) \right] + \text{h.c.},$$

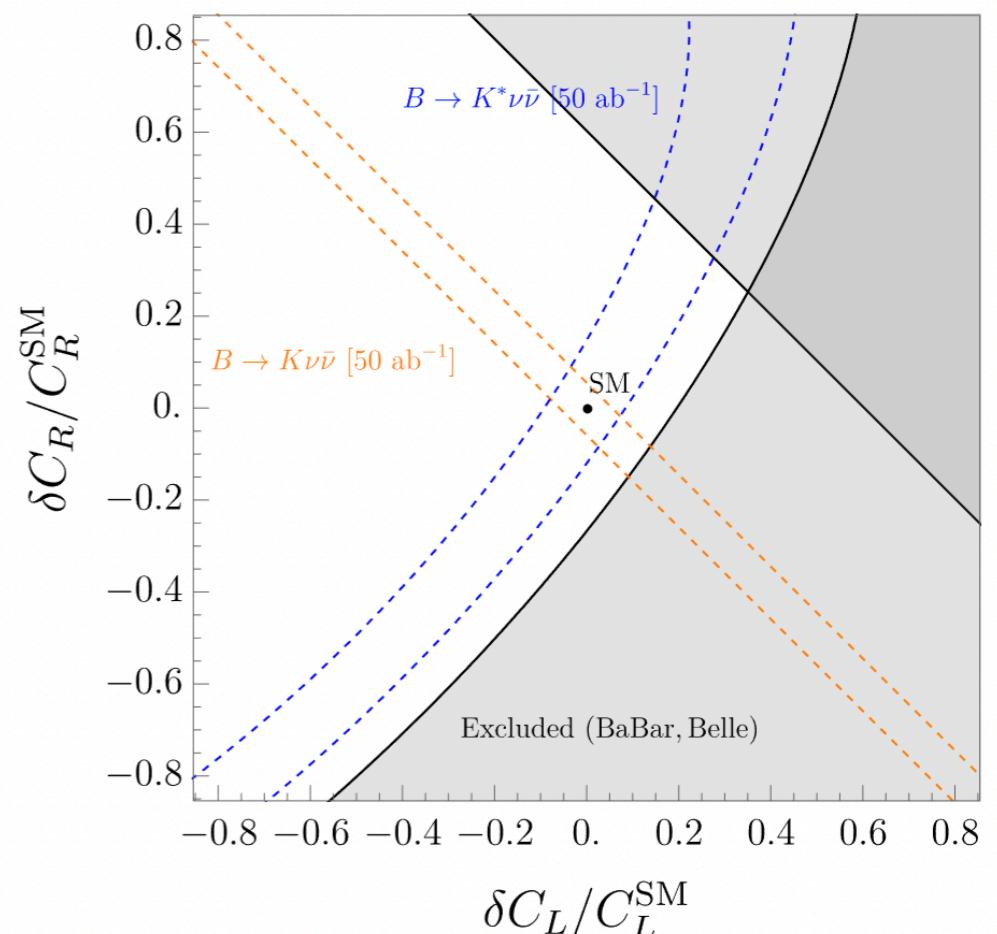
- Complementarity of $B \rightarrow K\nu\bar{\nu}$ and $B \rightarrow K^*\nu\bar{\nu}$:

$$\begin{aligned} \frac{\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})^{\text{SM}}} &= 1 + \sum_i \frac{2\text{Re}[C_L^{\text{SM}} (\delta C_L^{\nu_i\nu_i} + \delta C_R^{\nu_i\nu_i})]}{3|C_L^{\text{SM}}|^2} \\ &\quad + \sum_{i,j} \frac{|\delta C_L^{\nu_i\nu_j} + \delta C_R^{\nu_i\nu_j}|^2}{3|C_L^{\text{SM}}|^2} \\ &\quad - \eta_{K^{(*)}} \sum_{i,j} \frac{\text{Re}[\delta C_R^{\nu_i\nu_j} (C_L^{\text{SM}} \delta_{ij} + \delta C_L^{\nu_i\nu_j})]}{3|C_L^{\text{SM}}|^2}, \end{aligned}$$

$$\begin{aligned} \eta_K &= 0 \\ \eta_{K^*} &= 3.5(1) \end{aligned}$$

cf. [Buras et al. '14]

Example: $\delta C_{L(R)}^{\nu_i\nu_j} = \delta_{ij} \delta C_{L(R)}$



NB. $F_L(B \rightarrow K^*\nu\bar{\nu}) \leftrightarrow \mathcal{B}(B \rightarrow K\nu\bar{\nu}), \mathcal{B}(B \rightarrow K^*\nu\bar{\nu})$

SMEFT

- EFT invariant under $SU(2) \times U(1)_Y$ [ψ^4]:

$$b \rightarrow s\ell\ell$$

$$b \rightarrow s\nu\bar{\nu}$$

$$\begin{aligned} [\mathcal{O}_{lq}^{(1)}]_{ijkl} &= (\bar{L}_i \gamma^\mu L_j)(\bar{Q}_k \gamma_\mu Q_l) \\ [\mathcal{O}_{lq}^{(3)}]_{ijkl} &= (\bar{L}_i \gamma^\mu \tau^I L_j)(\bar{Q}_k \tau^I \gamma_\mu Q_l) \\ [\mathcal{O}_{ld}]_{ijkl} &= (\bar{L}_i \gamma^\mu L_j)(\bar{d}_k \gamma_\mu d_l) \\ [\mathcal{O}_{eq}]_{ijkl} &= (\bar{e}_i \gamma^\mu e_j)(\bar{Q}_k \gamma_\mu Q_l) \\ [\mathcal{O}_{ed}]_{ijkl} &= (\bar{e}_i \gamma^\mu e_j)(\bar{d}_k \gamma_\mu d_l) \end{aligned}$$

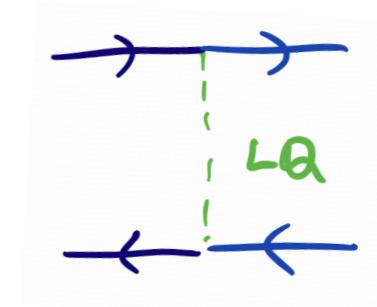
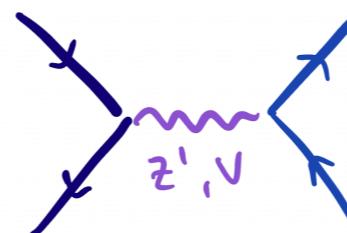
$$\begin{aligned} [\mathcal{O}_{lq}^{(1)}]_{ijkl} &= (\bar{L}_i \gamma^\mu L_j)(\bar{Q}_k \gamma_\mu Q_l) \\ &= (\bar{\ell}_{Li} \gamma^\mu \ell_{Lj})(\bar{d}_{Lk} \gamma_\mu d_{Ll}) + (\bar{\nu}_{Li} \gamma^\mu \nu_{Lj})(\bar{d}_{Lk} \gamma_\mu d_{Ll}) + \dots \\ [\mathcal{O}_{lq}^{(3)}]_{ijkl} &= (\bar{L}_i \gamma^\mu \tau^I L_j)(\bar{Q}_k \gamma_\mu \tau^I Q_l) \\ &= (\bar{\ell}_{Li} \gamma^\mu \ell_{Lj})(\bar{d}_{Lk} \gamma_\mu d_{Ll}) - (\bar{\nu}_{Li} \gamma^\mu \nu_{Lj})(\bar{d}_{Lk} \gamma_\mu d_{Ll}) + \dots \\ [\mathcal{O}_{ld}]_{ijkl} &= (\bar{L}_i \gamma^\mu L_j)(\bar{d}_k \gamma_\mu d_l) \\ &= (\bar{\ell}_{Li} \gamma^\mu \ell_{Lj})(\bar{d}_{Rk} \gamma_\mu d_{Rl}) + (\bar{\nu}_{Li} \gamma^\mu \nu_{Lj})(\bar{d}_{Rk} \gamma_\mu d_{Rl}) \end{aligned}$$

- Correlations for concrete mediators:

-	$Z' \sim (\mathbf{1}, \mathbf{1}, 0)$:	$\mathcal{C}_{lq}^{(1)} \neq 0, \quad \mathcal{C}_{lq}^{(3)} = 0$
-	$V \sim (\mathbf{1}, \mathbf{3}, 0)$:	$\mathcal{C}_{lq}^{(1)} = 0, \quad \mathcal{C}_{lq}^{(3)} \neq 0$
-	$U_1 \sim (\mathbf{3}, \mathbf{1}, 2/3)$:		$\mathcal{C}_{lq}^{(1)} = \mathcal{C}_{lq}^{(3)}$
-	$S_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$:		$\mathcal{C}_{lq}^{(1)} = 3 \mathcal{C}_{lq}^{(3)}$

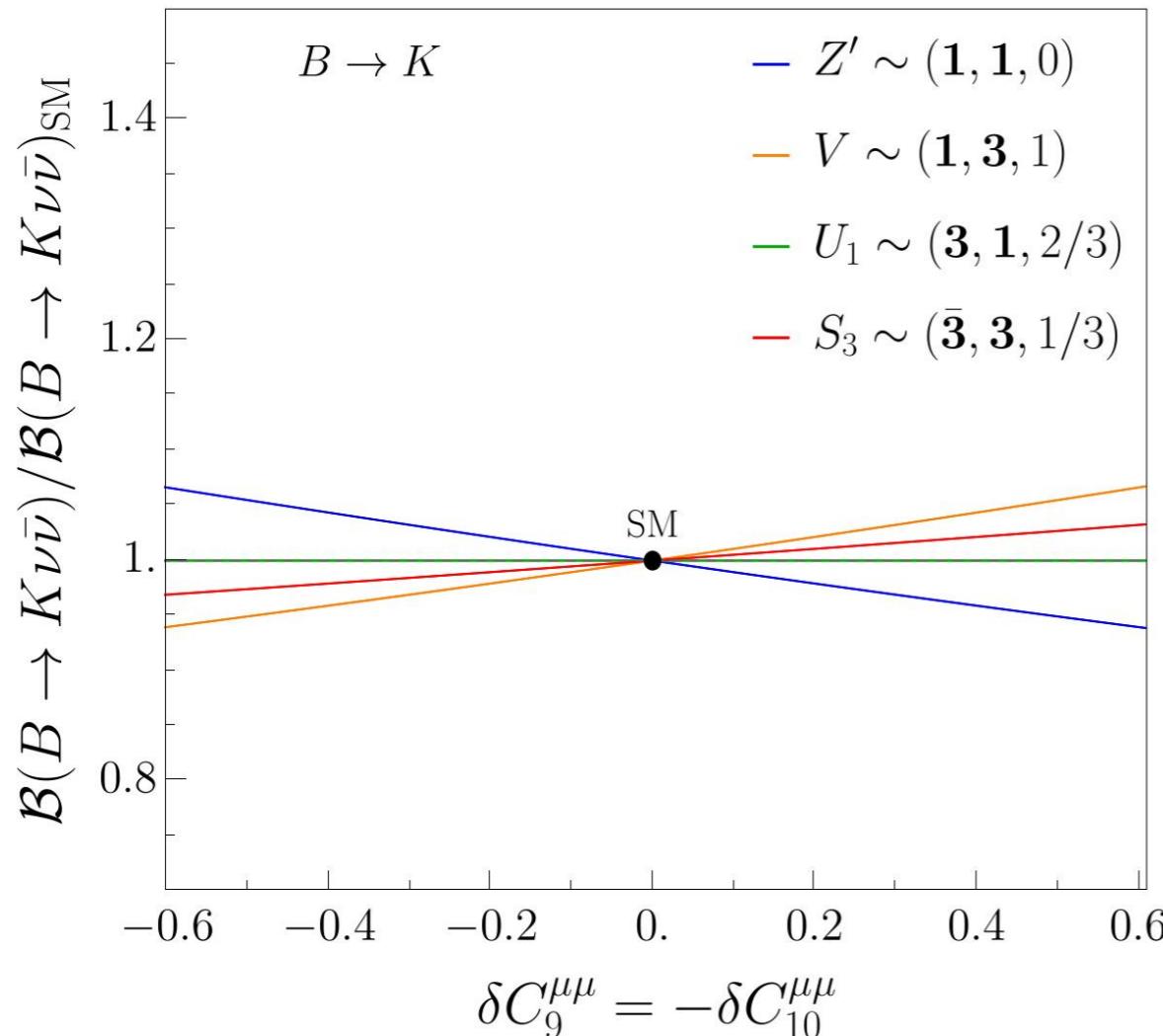
...

$$(SU(3)_c, SU(2)_L, U(1)_Y)$$

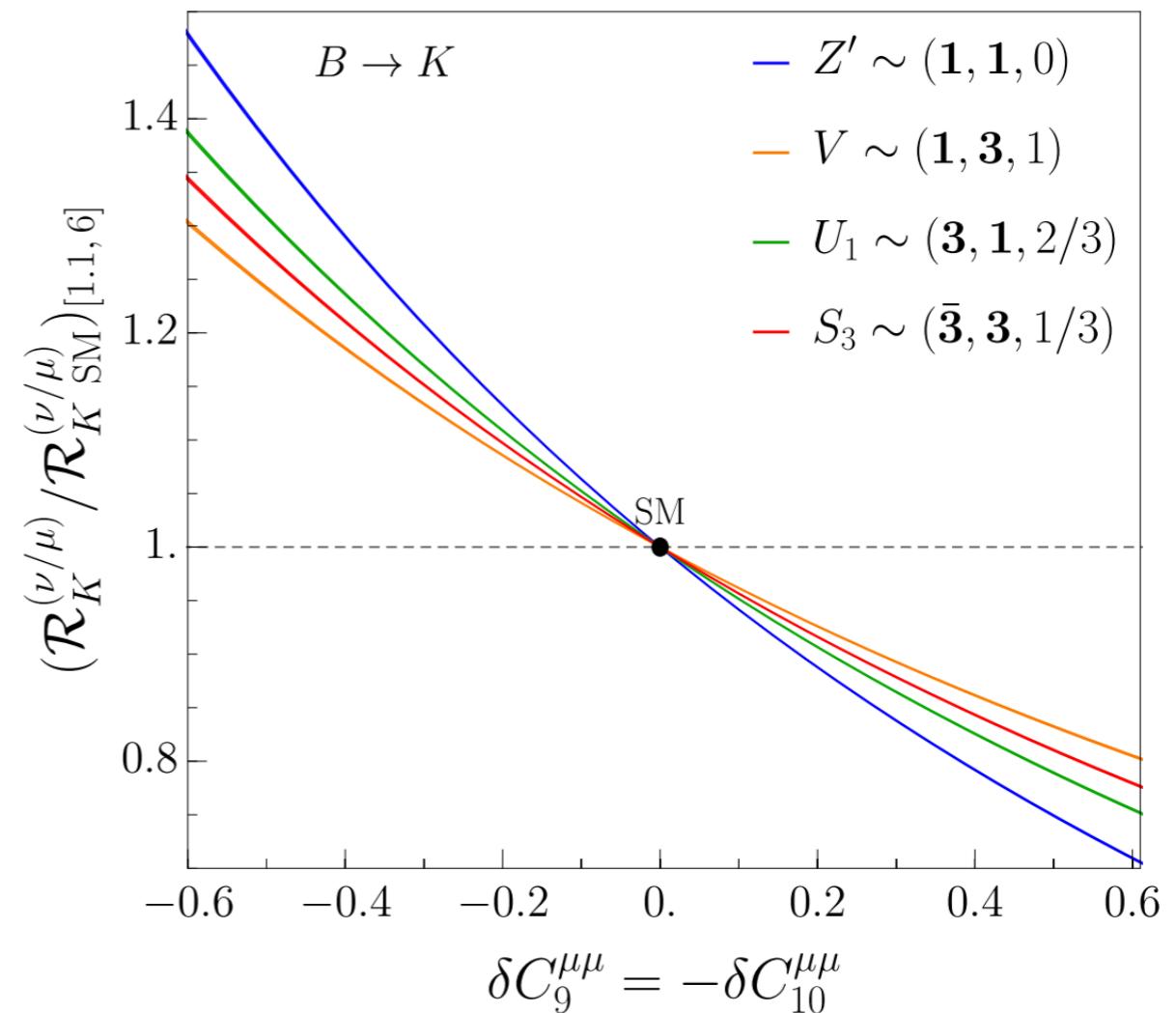


Example: couplings to μ_L

$b \rightarrow s\nu\nu$



$b \rightarrow s\nu\nu/b \rightarrow s\mu\mu$



Example: couplings to τ_L

[Allwicher, Faroughy, Jaffredo, OS, Wilsch. 2207.10756]

See talks by Jaffredo and Smolkovic

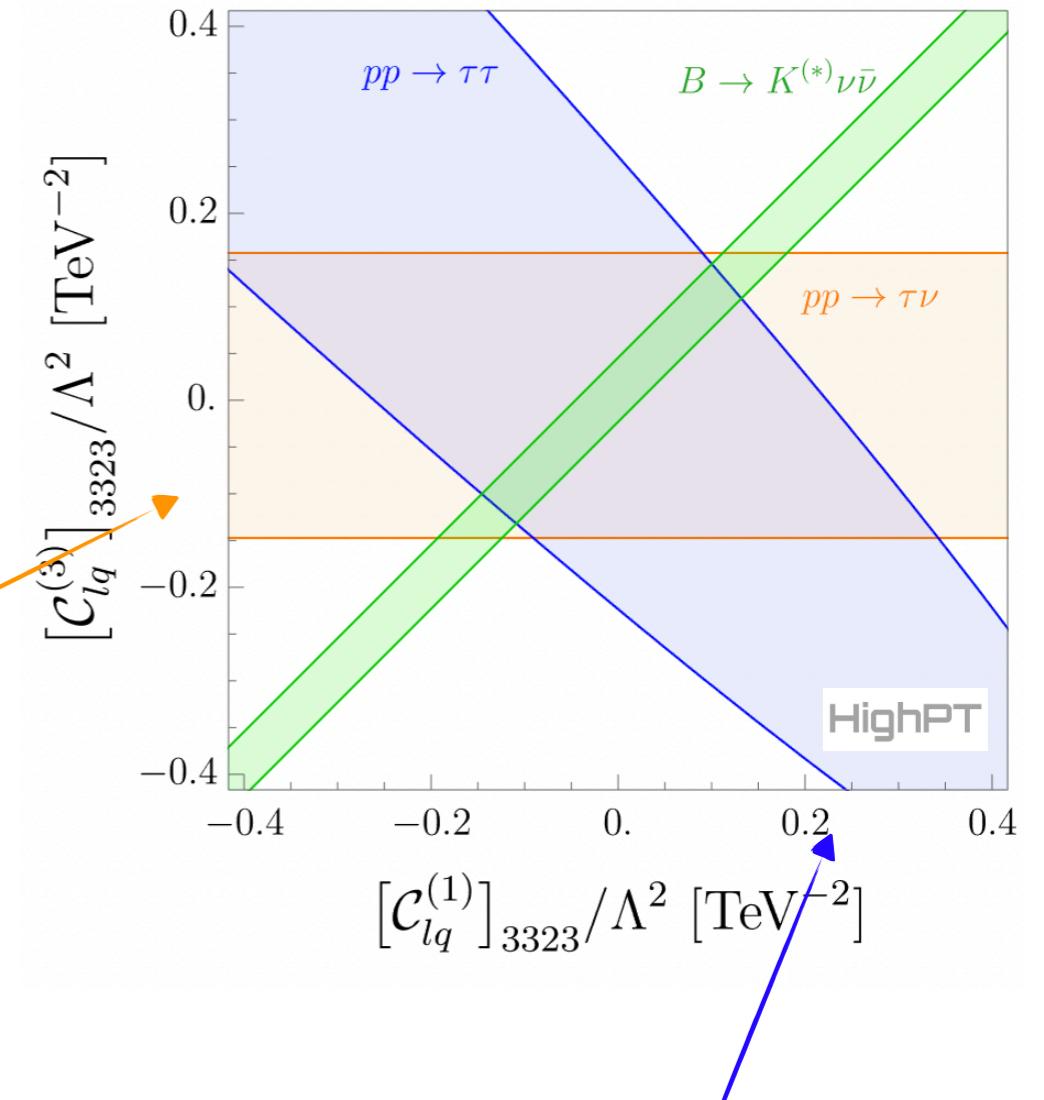
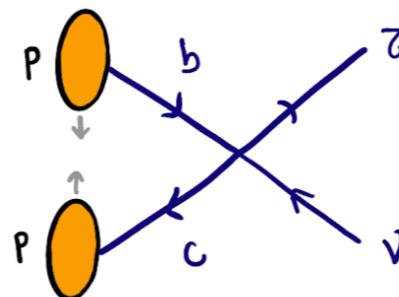
- Unique access to operators with **left-handed τ 's** :

$$[\mathcal{O}_{lq}^{(1)}]_{3323} = (\bar{L}_3 \gamma^\mu L_3) (\bar{Q}_2 \gamma_\mu Q_3)$$

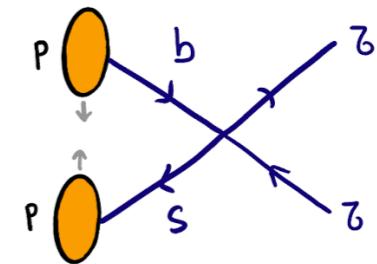
$$[\mathcal{O}_{lq}^{(3)}]_{3323} = (\bar{L}_3 \gamma^\mu \tau^I L_3) (\bar{Q}_2 \tau^I \gamma_\mu Q_3)$$

with

$$\frac{\Lambda}{|C_{lq}^{(1,3)}|} \gtrsim 16 \text{ TeV}$$



Complementarity with other observables!



Summary

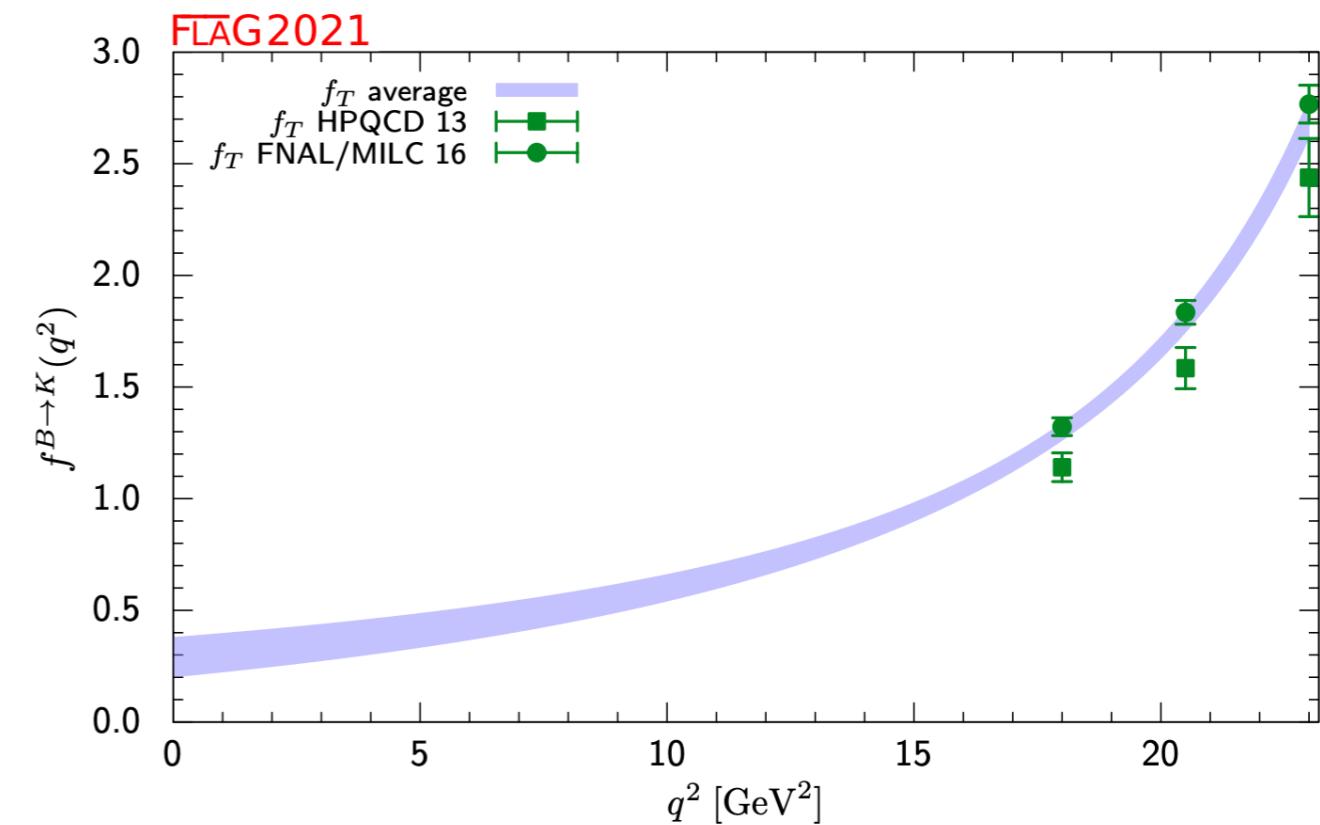
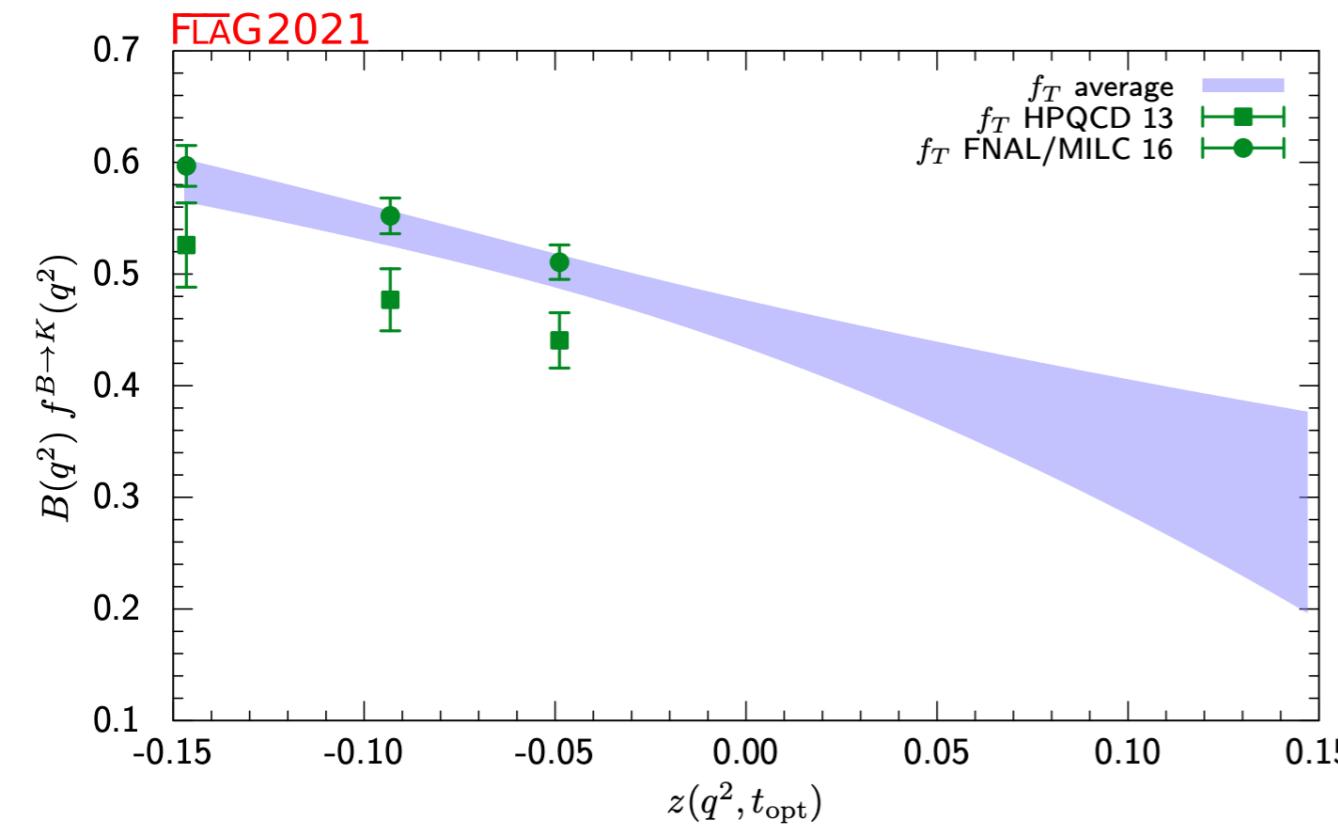
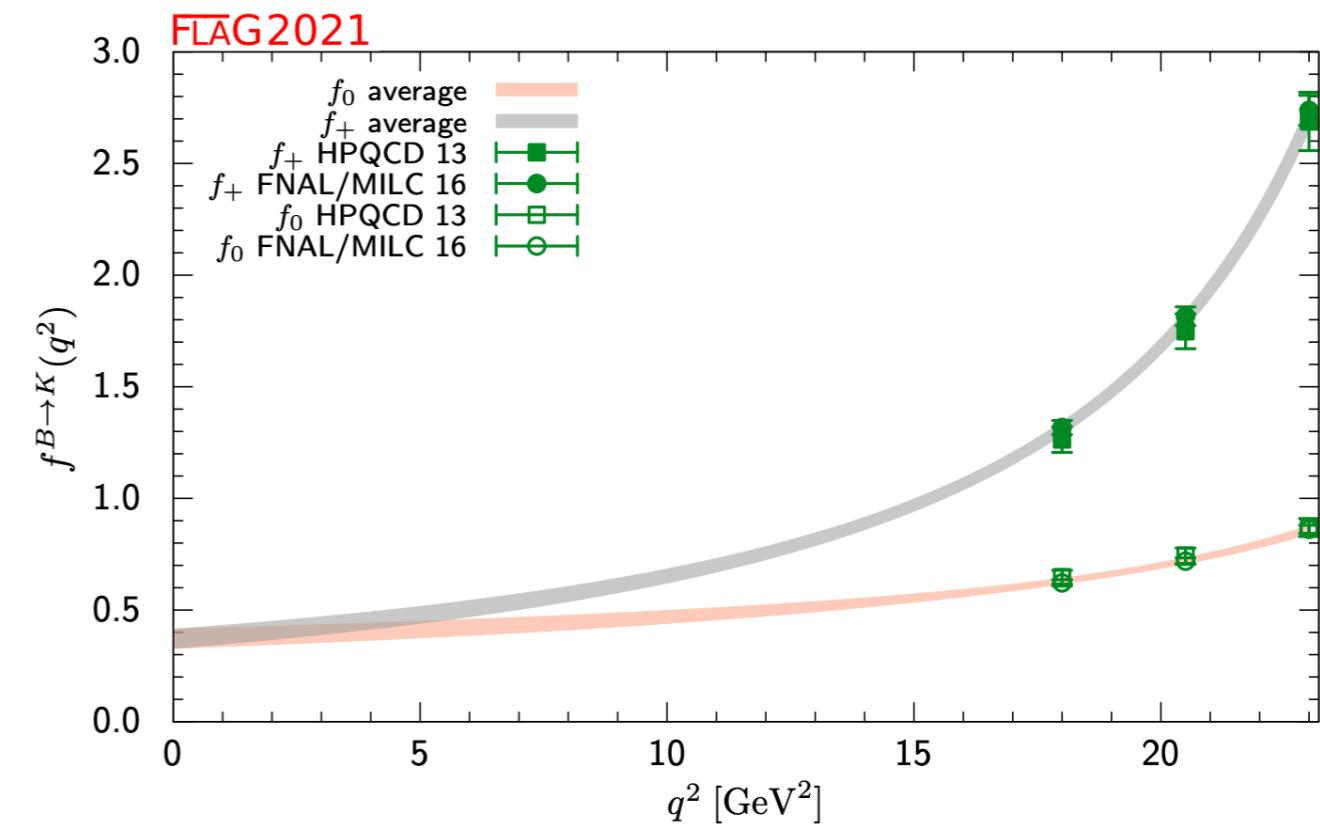
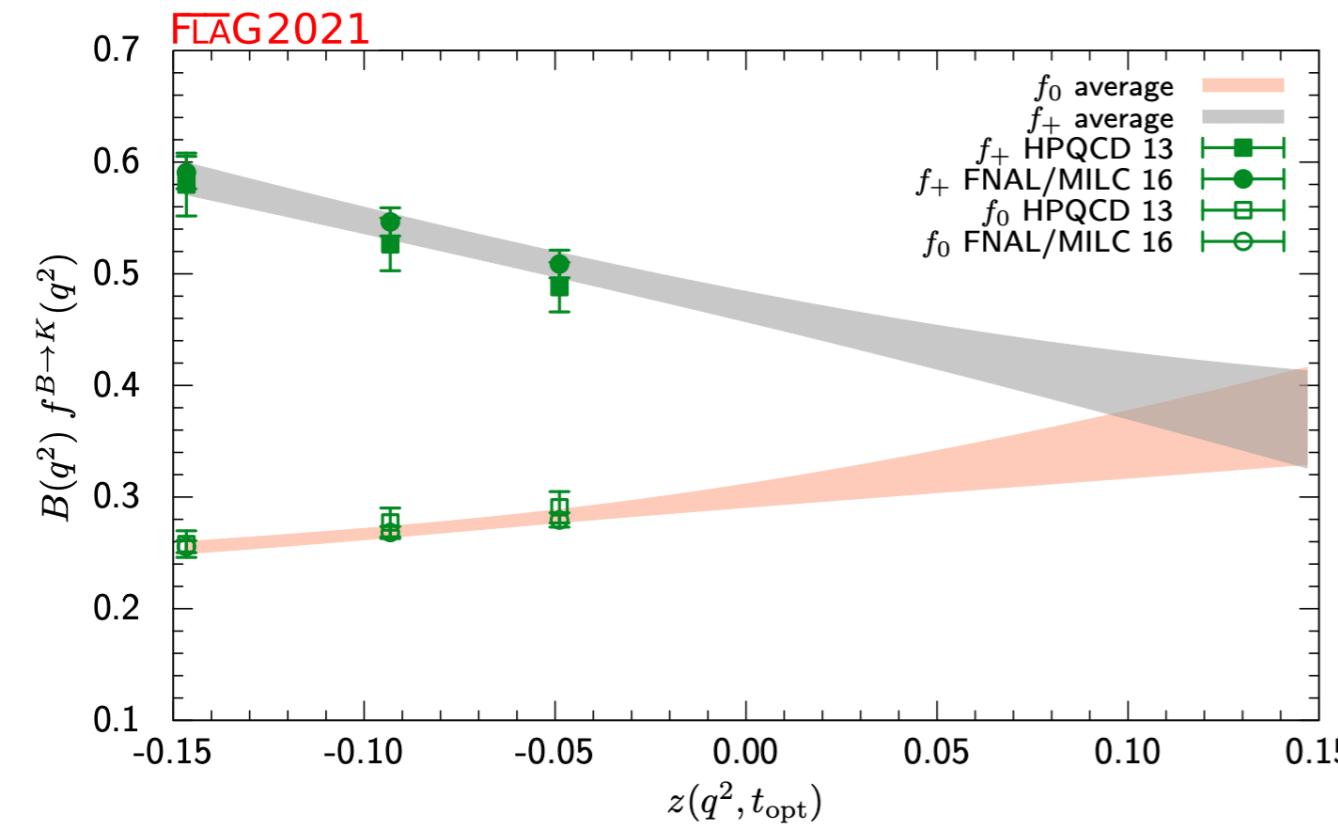
Summary

- $B \rightarrow K^{(*)}\nu\bar{\nu}$ decays are cleaner than $B \rightarrow K^{(*)}\mu\mu$, but one should still be **cautious** about the **hadronic uncertainties** associated to the **form-factors** and **CKM** matrix-elements.
- Recent LQCD calculation of $B \rightarrow K$ form-factors by HPQCD makes $\text{Br}(B \rightarrow K\nu\bar{\nu})$ rather precise, but it should be cross-checked independently:
 - ⇒ Binned data could be useful to test these predictions — which are **more reliable at high- q^2** .
 - ⇒ We propose to measure the ratio $\text{Br}(B \rightarrow K\nu\bar{\nu})_{\text{low}}/\text{Br}(B \rightarrow K\nu\bar{\nu})_{\text{high}}$, which **only depends** on the **q^2 -shape** of the (extrapolated) **vector form-factor**.
- The **ambiguity** in the **CKM matrix-element** determination is the dominant uncertainty for $B \rightarrow K\nu\bar{\nu}$ decays and it remains an **open problem** — *which value to take?*
- The ratio of $B \rightarrow K^{(*)}\nu\bar{\nu}/B \rightarrow K^{(*)}\mu\mu$ is independent of the CKM and only mildly dependent on the form-factors — **opportunity** to **extract** the **$c\bar{c}$ -contributions** to $C_9^{\mu\mu}$ (i.e., for $b \rightarrow s\mu\mu$).

We are looking forward to the **Belle-II** results! **Many opportunities** to test **(B)SM physics!**

Hvala!

Back-up



$$f_+(q^2) = \frac{1}{P_+(q^2)} \sum_{n=0}^{N-1} a_n^+ \left[z^n - (-1)^{n-N} \frac{n}{N} z^N \right]$$

$$f_0(q^2) = \frac{1}{P_0(q^2)} \sum_{n=0}^{N-1} a_n^0 z^n$$

Pole factors:

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$P_i(q^2) = 1 - q^2/M_i^2$$

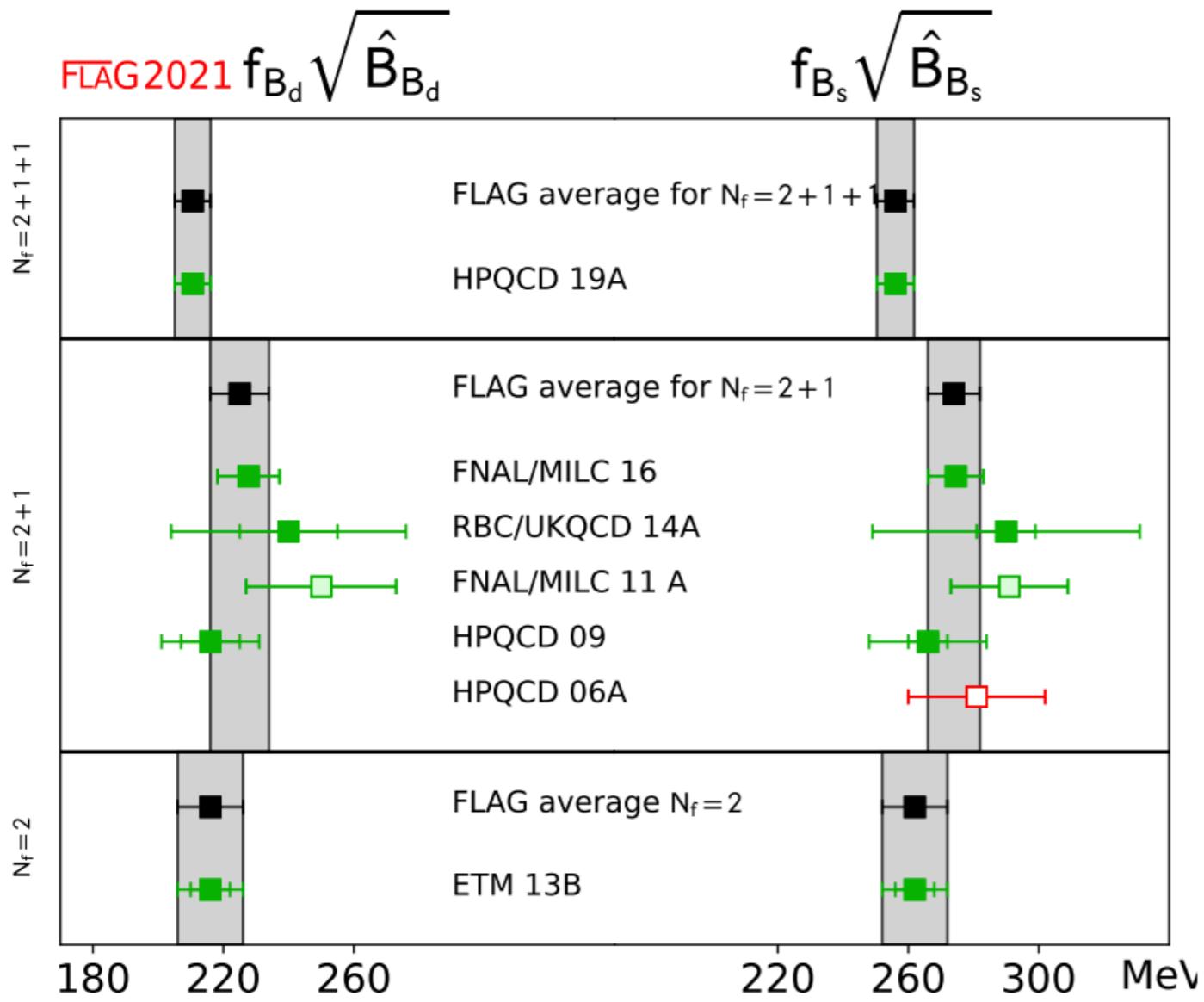
Numerical coefficients to be determined non-perturbatively.

a_0^+	a_1^+	a_2^+	a_0^0	a_1^0	a_0^T	a_1^T	a_2^T
0.4742(62)	-0.894(51)	-0.44(14)	0.2939(36)	0.277(40)	0.4752(92)	-0.921(82)	-0.53(35)
1	-0.2904	-0.0347	0.7480	0.1844	0.6558	-0.2193	-0.0751
.	1	0.7757	0.2291	0.8527	-0.2569	0.5371	0.2574
.	.	1	0.1690	0.8455	-0.1029	0.3700	0.2653
.	.	.	1	0.4568	0.5232	0.0314	0.0257
.	.	.	.	1	0.0182	0.4501	0.2372
.	1	-0.0255	-0.0535
.	1	0.6920
.	1

TABLE VI. Values of the z -expansion coefficients and correlation matrix obtained by our combined fit to the $B \rightarrow K$ form factors computed by HPQCD [27] and FNAL/MILC [28] (with $\chi^2_{\text{min}}/\text{d.o.f} \simeq 9.2/10$). We consider the parameterization from Eq. (A2)–(A4) with $N = 3$, and we remove the a_2^0 coefficient by imposing the relation $f_+(0) = f_0(0)$. The covariance matrix is provided with more digits in an ancillary file. See text for more details.

q^2 -bin [GeV 2]	$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K^+}} / \mathcal{B}_{K^+}$	$\mathcal{B}(B^0 \rightarrow K_S \nu \bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K_S}} / \mathcal{B}_{K_S}$
[0, 4]	(1.206 \pm 0.055 \pm 0.066)	0.07	(0.490 \pm 0.026 \pm 0.030)	0.08
[4, 8]	(1.161 \pm 0.039 \pm 0.064)	0.06	(0.477 \pm 0.018 \pm 0.029)	0.07
[8, 12]	(1.064 \pm 0.027 \pm 0.059)	0.06	(0.439 \pm 0.013 \pm 0.027)	0.07
[12, 16]	(0.889 \pm 0.020 \pm 0.049)	0.06	(0.365 \pm 0.009 \pm 0.022)	0.07
[16, q_{\max}^2]	(0.744 \pm 0.017 \pm 0.039)	0.06	(0.282 \pm 0.008 \pm 0.017)	0.07
[0, q_{\max}^2]	(5.06 \pm 0.14 \pm 0.28)	0.06	(2.05 \pm 0.07 \pm 0.12)	0.07

q^2 -bin [GeV 2]	$\mathcal{B}(B^+ \rightarrow K^{*+} \nu \bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K^{*+}}} / \mathcal{B}_{K^{*+}}$	$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K^{*0}}} / \mathcal{B}_{K^{*0}}$
[0, 4]	(1.77 \pm 0.20 \pm 0.09)	0.12	(1.38 \pm 0.18 \pm 0.08)	0.15
[4, 8]	(2.25 \pm 0.23 \pm 0.12)	0.10	(1.85 \pm 0.22 \pm 0.11)	0.13
[8, 12]	(2.63 \pm 0.30 \pm 0.15)	0.13	(2.23 \pm 0.28 \pm 0.14)	0.14
[12, 16]	(2.71 \pm 0.39 \pm 0.15)	0.15	(2.32 \pm 0.37 \pm 0.14)	0.17
[16, q_{\max}^2]	(1.50 \pm 0.30 \pm 0.09)	0.21	(1.27 \pm 0.29 \pm 0.08)	0.23
[0, q_{\max}^2]	(10.86 \pm 1.30 \pm 0.59)	0.12	(9.05 \pm 1.25 \pm 0.55)	0.15

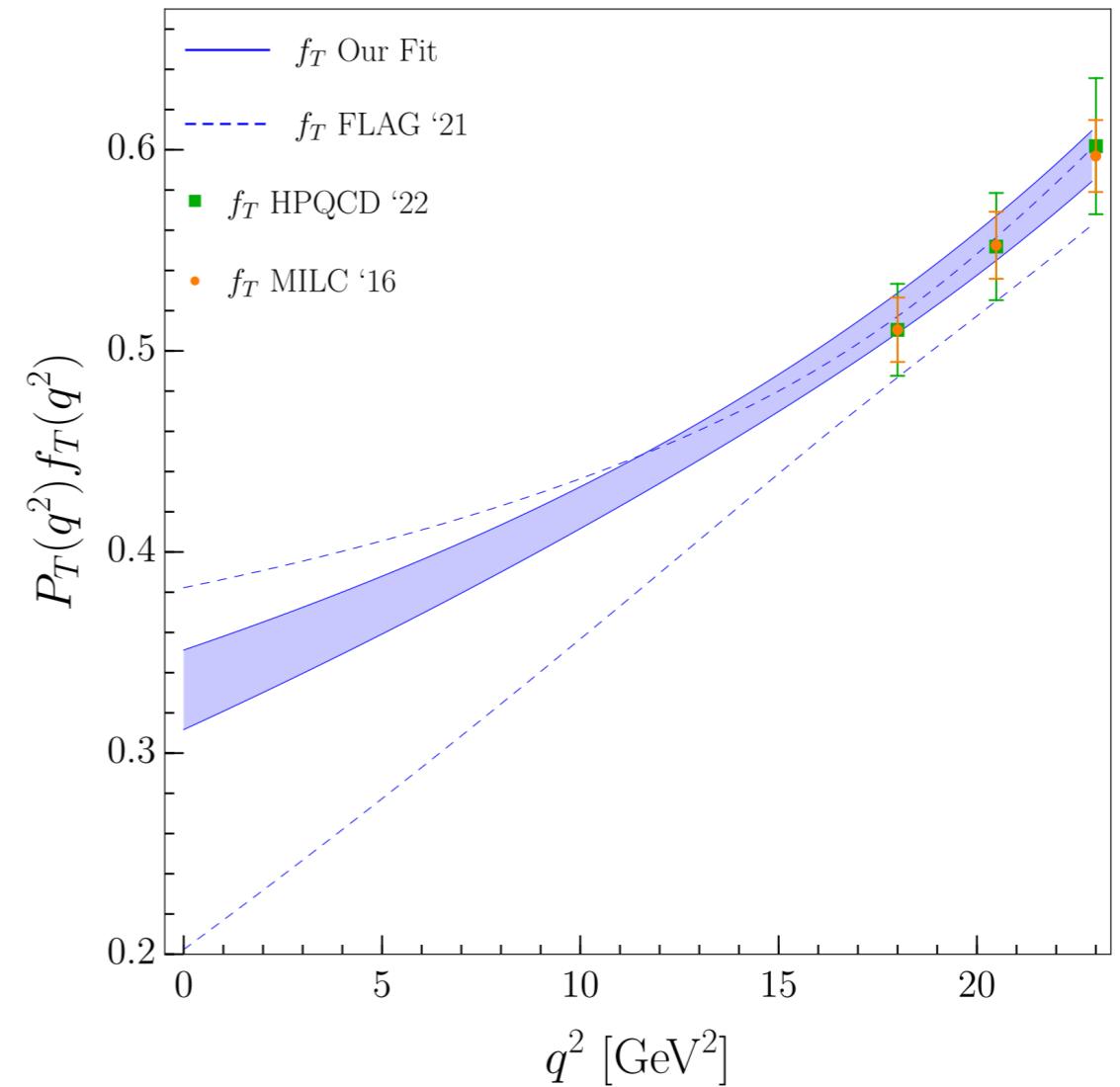
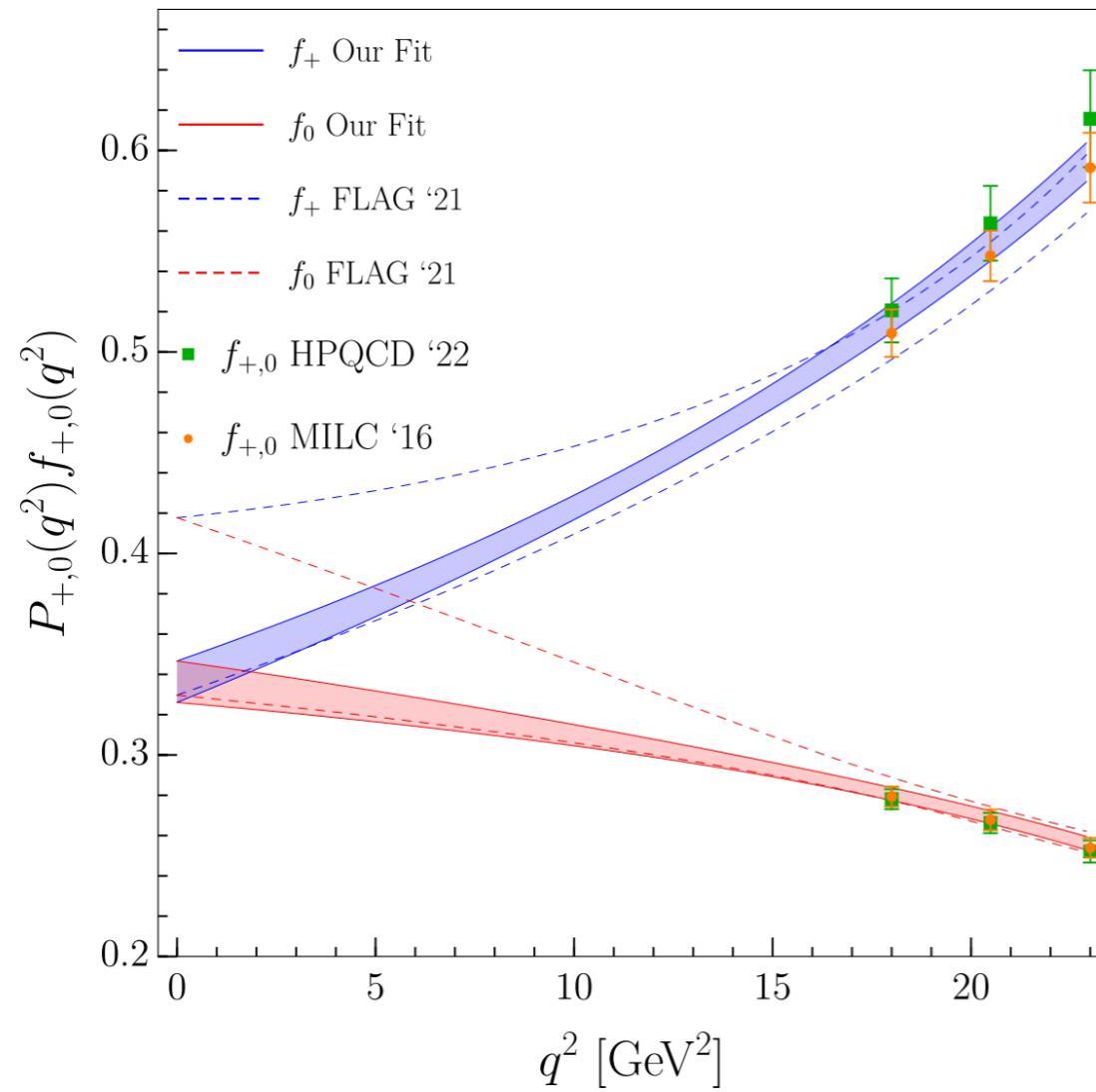


$$N_f = 2 + 1 :$$

$$\begin{aligned} f_{B_d} \sqrt{\hat{B}_{B_d}} &= 225(9) \text{ MeV} & f_{B_s} \sqrt{\hat{B}_{B_s}} &= 274(8) \text{ MeV} \\ \hat{B}_{B_d} &= 1.30(10) & \hat{B}_{B_s} &= 1.35(6) \\ \xi &= 1.206(17) & B_{B_s}/B_{B_d} &= 1.032(38) \end{aligned}$$

$$N_f = 2 + 1 + 1 :$$

$$\begin{aligned} f_{B_d} \sqrt{\hat{B}_{B_d}} &= 210.6(5.5) \text{ MeV} & f_{B_s} \sqrt{\hat{B}_{B_s}} &= 256.1(5.7) \text{ MeV} \\ \hat{B}_{B_d} &= 1.222(61) & \hat{B}_{B_s} &= 1.232(53) \\ \xi &= 1.216(16) & B_{B_s}/B_{B_d} &= 1.008(25) \end{aligned}$$



[Kamenik, Smith, 0908.1174]

