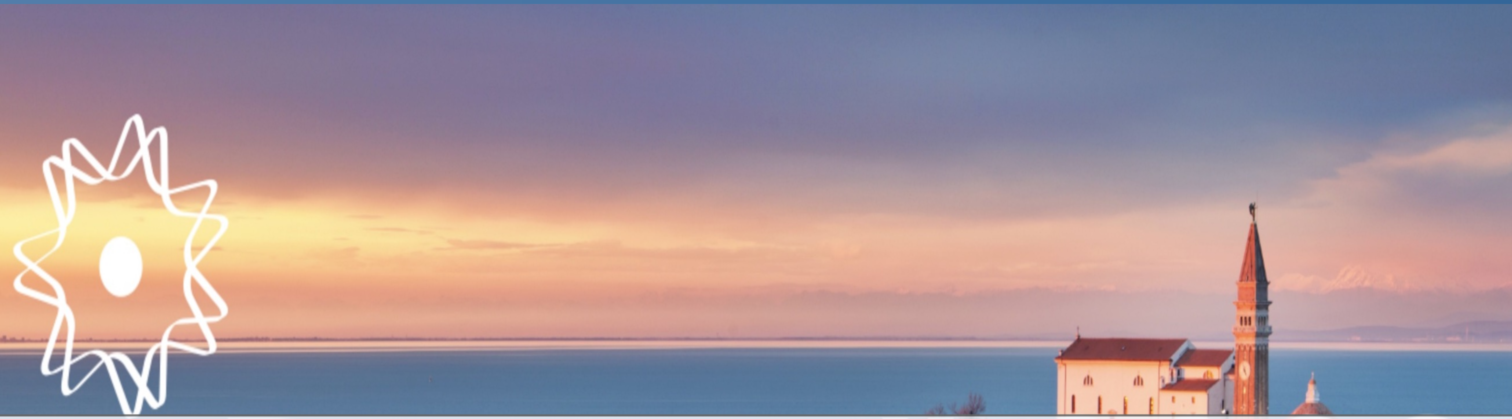

Hadronic contributions to the muon $g-2$ from lattice QCD

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Particle Physics from Early Universe to Future Colliders

April 11, 2023

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anr agence nationale de la recherche

JGU JOHANNES GUTENBERG UNIVERSITÄT MAINZ

- Muon $g-2$ Theory Initiative (White Paper, status in 2020)

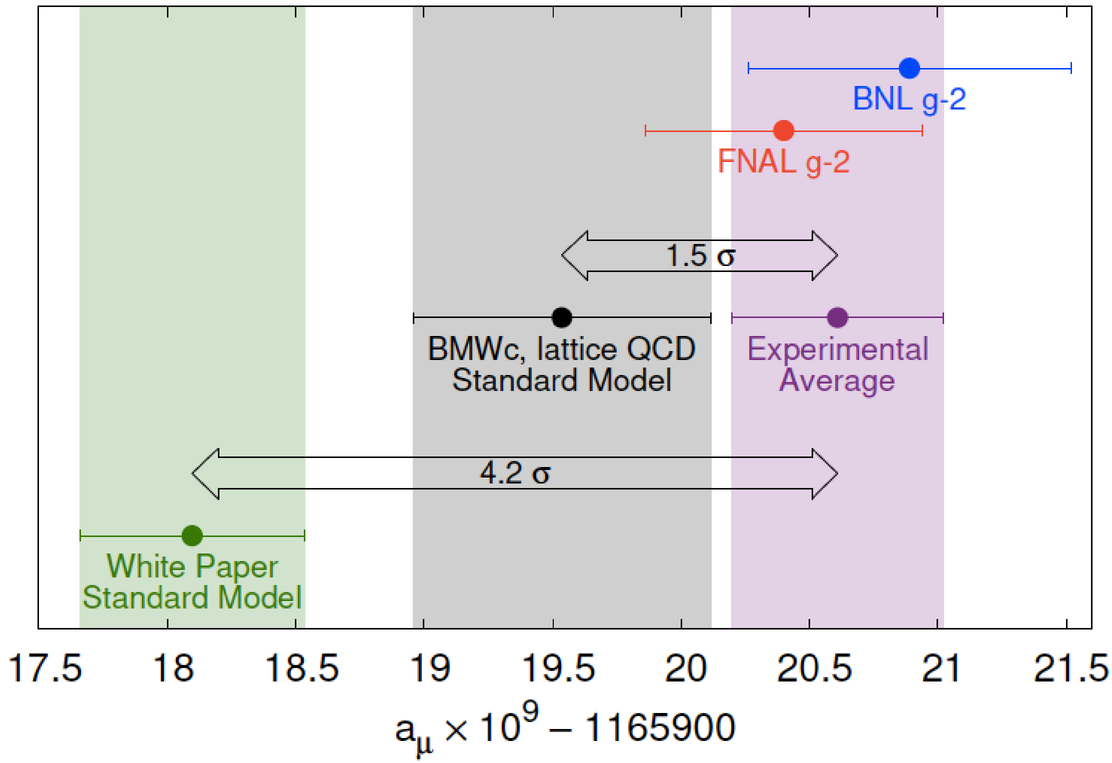
  The anomalous magnetic moment of the muon in the Standard Model [Phys.Rept. 887 (2020) 1-166]

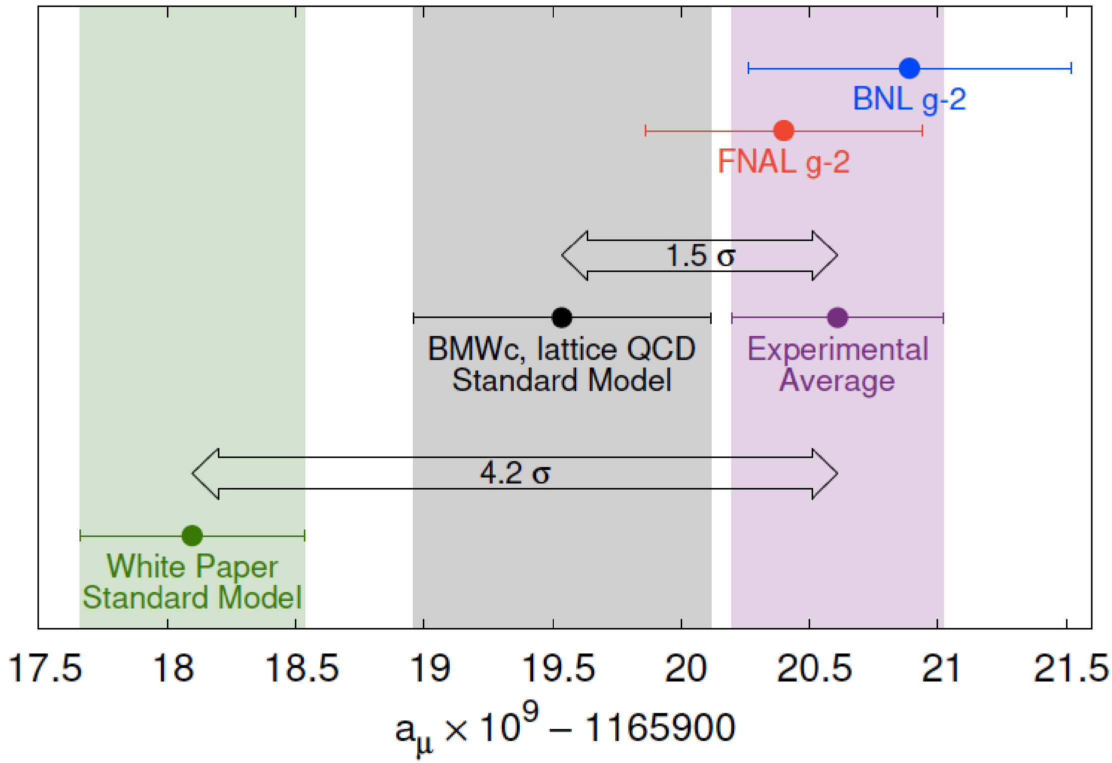
Contribution	$a_\mu \times 10^{11}$	
- QED (leptons, 10^{th} order)	$116\ 584\ 718.931 \pm 0.104$	[Aoyama et al. '12 '19]
- Electroweak	153.6 ± 1.0	[Gnendiger et al. '13]
- Strong contributions		
HVP (LO)	$6\ 931 \pm 40$	[DHMZ '19, KNT '20]
HVP (NLO)	-98.3 ± 0.7	[Hagiwara et al. '11]
HVP (NNLO)	12.4 ± 0.1	[Kurtz et al. "14]
HLbL	92 ± 18	[See WP]
Total (theory)	$116\ 591\ 810 \pm 43$	
Experiment	$116\ 592\ 061 \pm 41$	

→ Error budget **dominated by hadronic contributions**

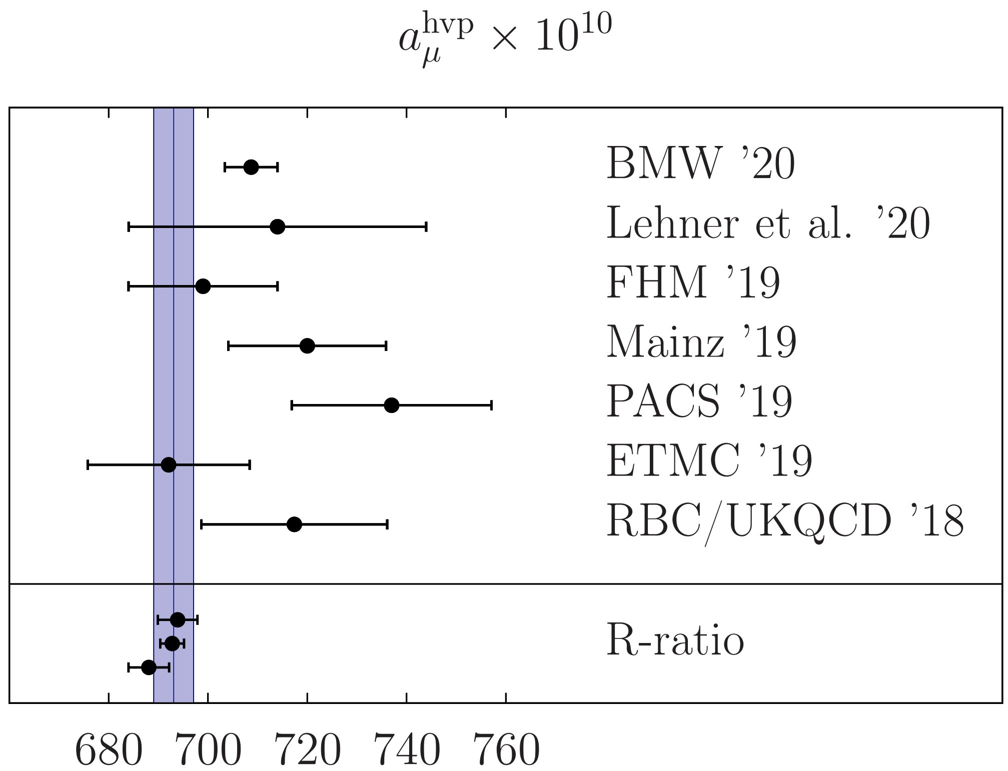
→ **Goal** - few permille on the HVP contribution

- below 10% on the HLbL contribution


BNL g-2
FNAL g-2
1.5 σ
BMWc, lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$



- BMW published their result on the same day (but on the ArXiv ~ 1 year earlier)

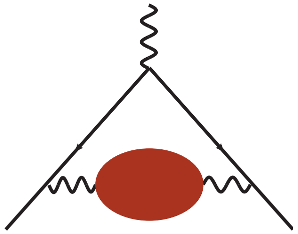


- other lattice calculations tend to be higher too ...
- ... but with larger errors

- Time-momentum representation [Bernecker, Meyer '11 '13]

$$a_\mu^{\mathrm{hvp}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t \; \widetilde{K}(t) \; G(t)$$

$$G(t) = -\frac{1}{3} \sum_{k=1}^{3} \sum_{\vec{x}} \langle V_k(x) V_k(0) \rangle$$



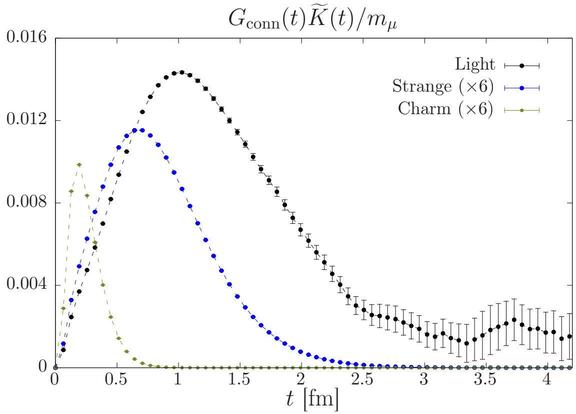
- $\widetilde{K}(t)$: known kernel function

- Electromagnetic current :

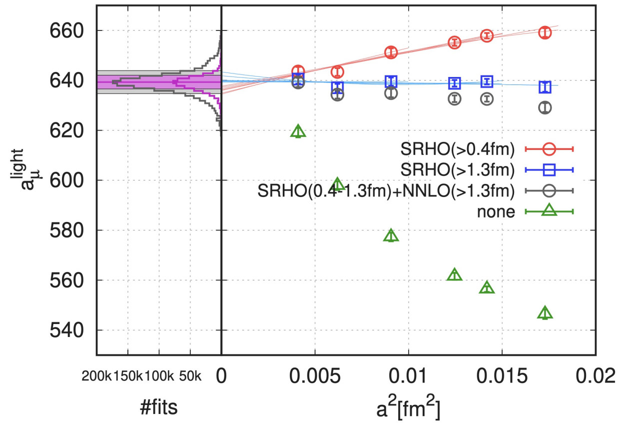
$$V_\mu(x) = \frac{2}{3}\overline{u}(x)\gamma_\mu u(x) - \frac{1}{3}\overline{d}(x)\gamma_\mu d(x) - \frac{1}{3}\overline{s}(x)\gamma_\mu s(x) + \cdots$$

Challenges for sub-percent precision

▶ Noise problem (light-quark contribution)



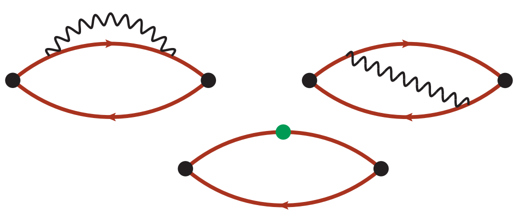
▶ Continuum extrapolation [BMW '20]



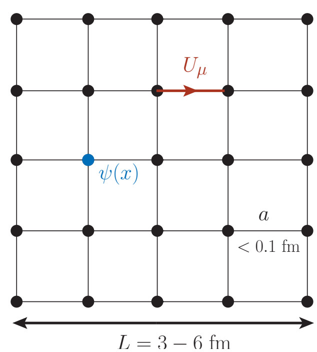
▶ QED / strong isospin breaking corrections

$m_u \neq m_d$: $\mathrm{O}(\frac{m_u - m_d}{\Lambda_{\rm QCD}}) \approx 1/100$

$Q_u \neq Q_d$: $\mathrm{O}(\alpha_{\rm em}) \approx 1/100$



▶ Finite-volume effects $\mathcal{O}(3\%)$

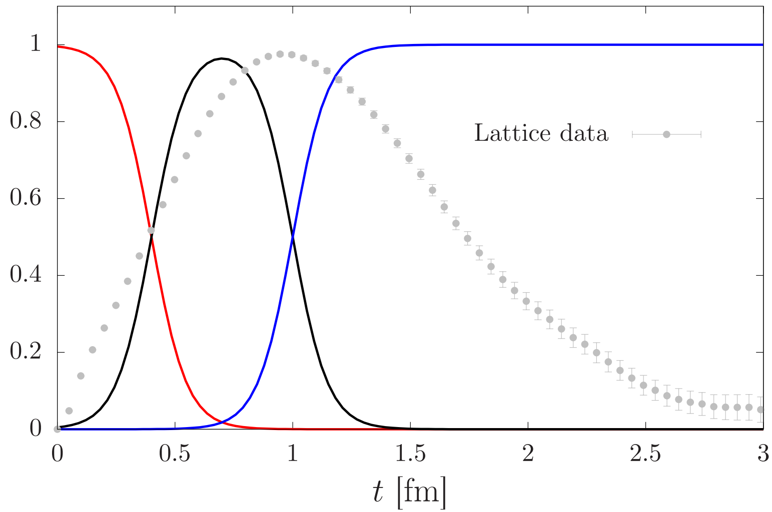


$$a_\mu^{\text{win}} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t)\, \widetilde{K}(t)\; W(t; t_0, t_1)$$

- Short distances : $W^{\text{SD}}(t; t_0) = [1 - \Theta(t, t_0, \Delta)]$
- Intermediate distances : $W^{\text{ID}}(t; t_0, t_1) = [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]$
- Long distances : $W^{\text{LD}}(t; , t_1) = \Theta(t, t_1, \Delta)$

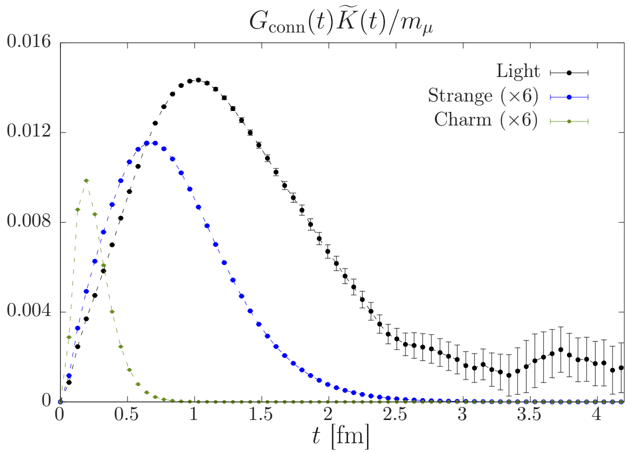
Smooth step function : $\Theta(t, t', \Delta) = [1 + \tanh[(t - t')/\Delta]]\, /2$

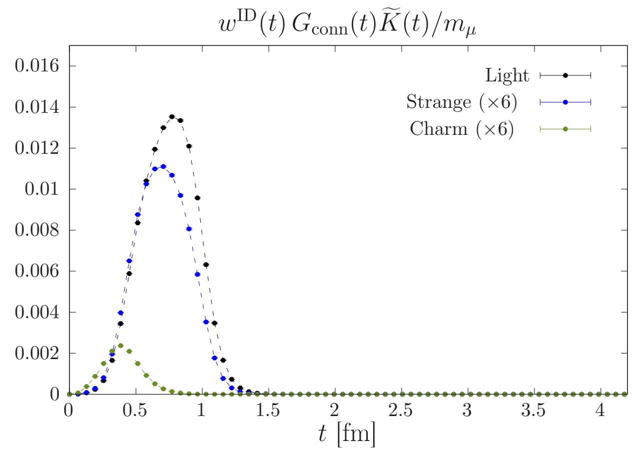
Standard parameters : $t_0 = 0.4 \text{ fm},\ t_1 = 1.0 \text{ fm},\ \Delta = 0.15 \text{ fm}$



▶ **Intermediate window** : $\sim 30\%$ of the total contribution.

- much easier to compute on the lattice (and accessible from R-ratio data !)
  - $\rightarrow$ 1-2 permille statistical precision can be reached on the integrand
  - $\rightarrow$ finite-volume effects are strongly suppressed





- still need to have a good control over the continuum extrapolation
- Region $600\ \mathrm{MeV} \leq \sqrt{s} \leq 900\ \mathrm{MeV}$ (around the rho-resonance) :
  - $\rightarrow$ 55%-60% on both full hvp and window observables !
  - $\rightarrow$ $\sqrt{s} \leq 600\ \mathrm{MeV}$ slightly suppressed, $\sqrt{s} \geq 900\ \mathrm{MeV}$ slightly enhanced.

figure taken from [2301.08696 [hep-lat]]

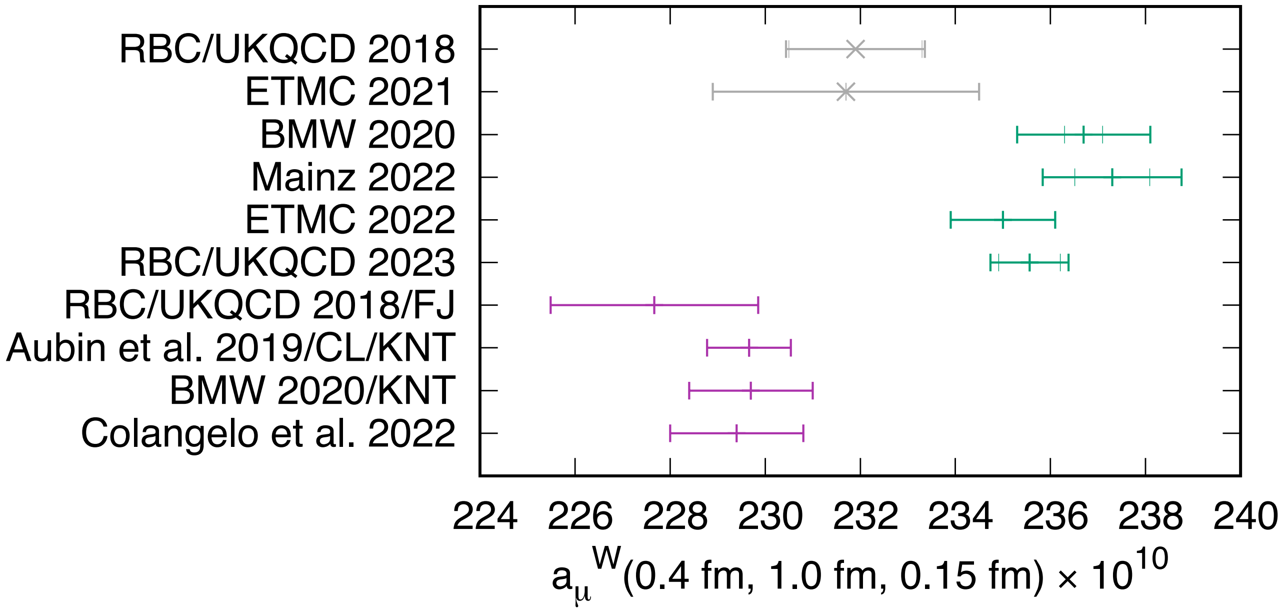
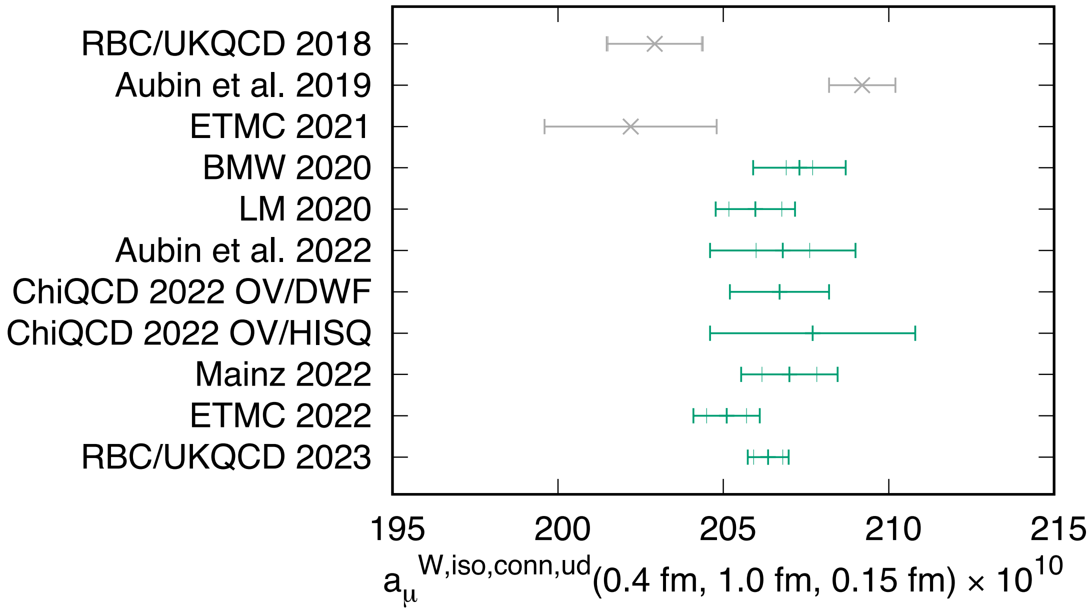
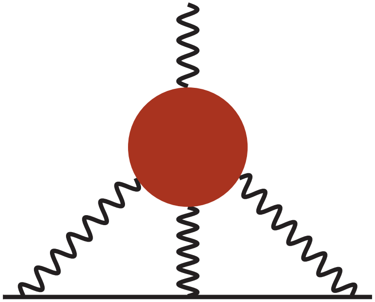


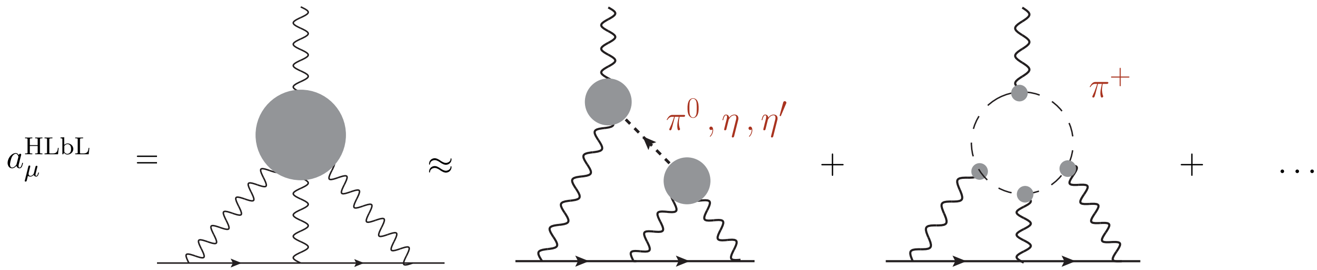
figure taken from [2301.08696 [hep-lat]]



- light quark contribution : ∼88% of the total !
- good agreement among many collaborations (using different discretizations)

Hadronic light-by-light scattering contribution





Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
HLbL (data-driven) '20	92 ± 19
LO HVP	6931 ± 40
HLbL (lattice) Mainz '22	109.6 ± 15.9

- $\Delta a_\mu^{\rm exp/SM} = 251(59) \times 10^{-11} \approx 3 \times a_\mu^{\rm hlbl}$

Two approaches :

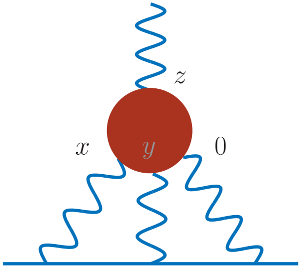
- Complete lattice calculation
- π^0, η, η' : accessible on the Lattice

- Compute the QED part in position space (continuum + infinite volume)

[J. Green et al. '16] [N. Asmussen et al. '16 '17]

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int \mathrm{d}^4y \int \mathrm{d}^4x \; \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

$$i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y) = -\int \mathrm{d}^4z \, z_\rho \, \langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \rangle$$

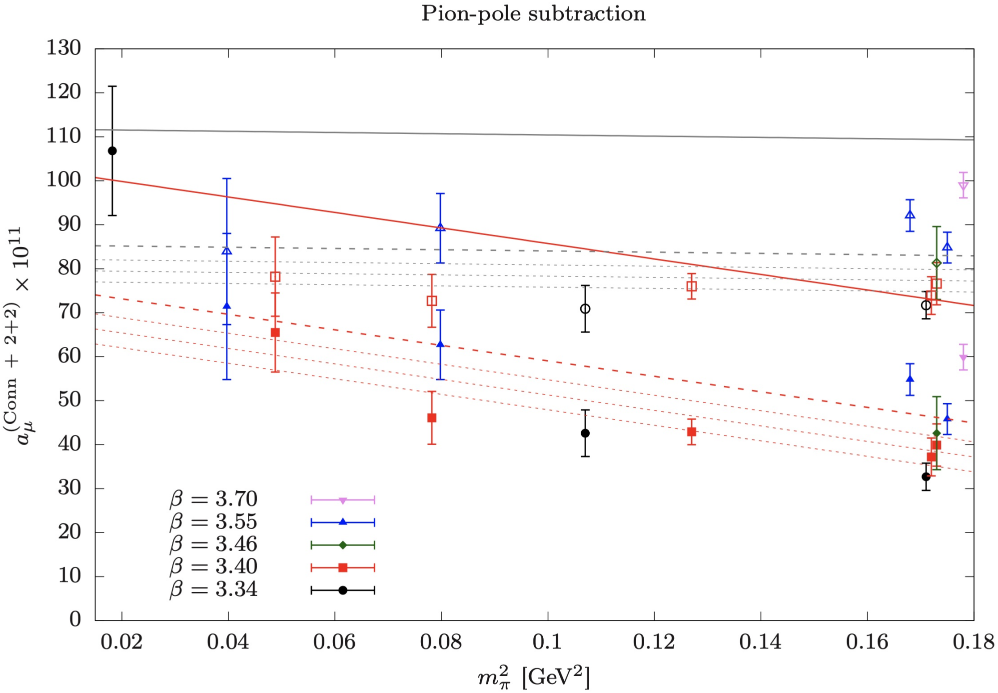


→ $\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$: four-point correlation function computed on the lattice

→ $\mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$: QED kernel, computed semi-analytically in position-space

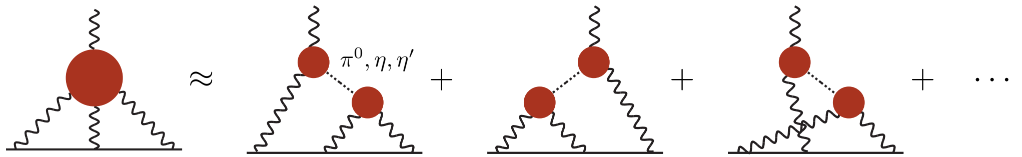
→ Avoid $1/L^2$ finite-volume effects from the massless photons $\Rightarrow \sim e^{-m_\pi L}$

[Eur.Phys.J.C 80 (2020) 9, 869]
[Eur.Phys.J.C 81 (2021) 7, 651]
[Eur.Phys.J.C 82 (2022) 8, 664]
2210.12263 [hep-lat]

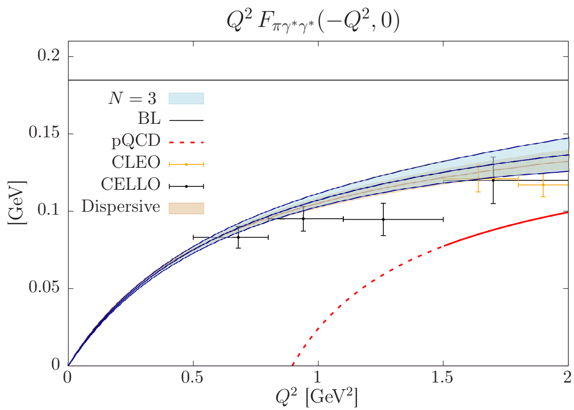


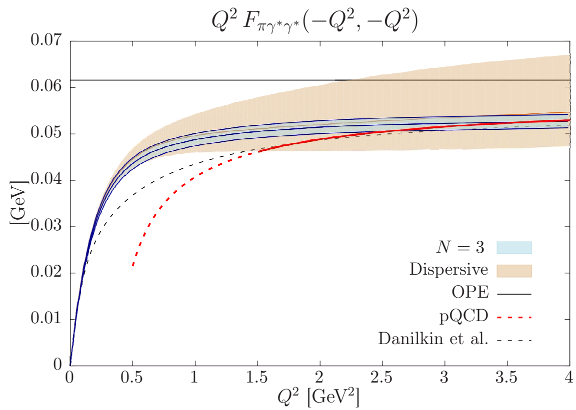
- Filled symbols : original data
- Open symbols : $a_\mu^{\mathrm{hlbl,cor}}(a, m_\pi) = a_\mu^{\mathrm{hlbl,data}}(a, m_\pi) + \left(a_\mu^{\pi^0,\mathrm{phys}(a,m_\pi)} - a_\mu^{\pi^0}(a, m_\pi)\right)$
- 15% precision close to the target precision ($\sim 10\%$)

$$a_\mu^{\mathrm{hlbl}} = 109.6(15.0) \times 10^{-11}$$



- Pion transition form factor [In collaboration with Mainz group]





- Fully model independant

$$a_\mu^{\mathrm{HLbL};\pi^0} = (59.9 \pm 3.6) \times 10^{-11}$$

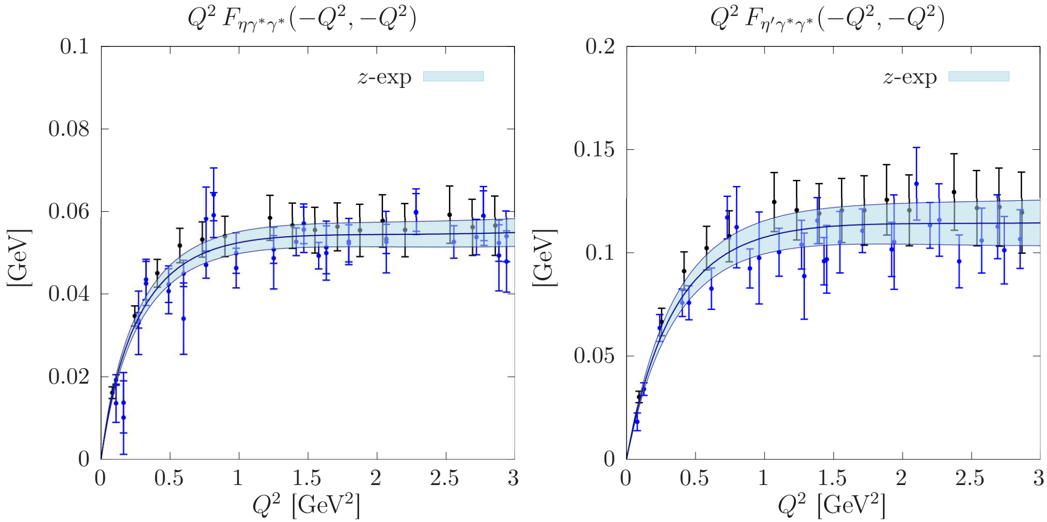
[A. G et al, Phys.Rev. D100 (2019)]

$\rightarrow$ Compatible with the dispersive result

$$a_\mu^{\mathrm{HLbL};\pi^0} = 62.6^{+3.0}_{-2.5} \times 10^{-11}$$

[Hoferichter et al. '18]

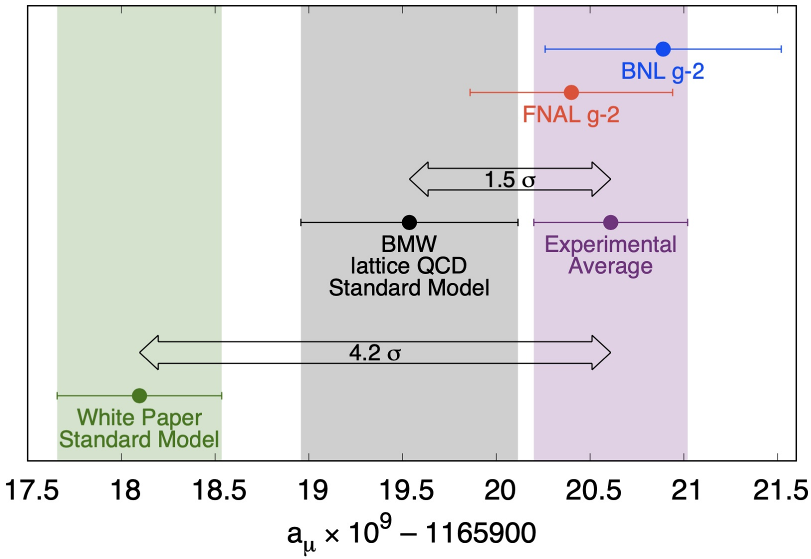
- Work done in collaboration with the BMW collaboration
- More challenging : mixing between η and η', , quark disconnected contributions



- preliminary continuum extrapolation (statistical errors only) :

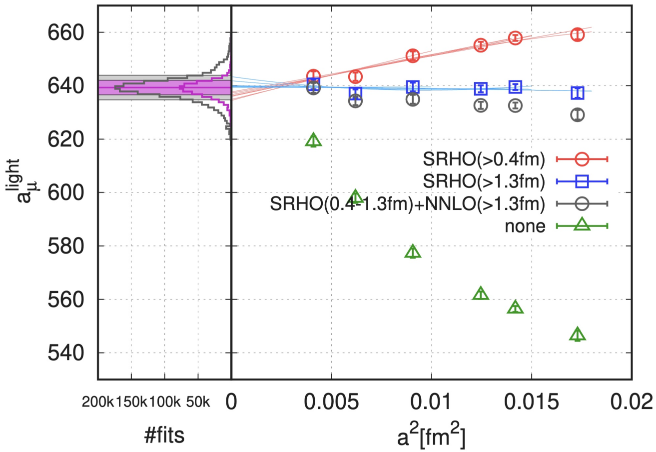
$$a_\mu^{\mathrm{HLbL};\eta} = (11.6 \pm\ 2 \pm ??) \times 10^{-11}$$

$$a_\mu^{\mathrm{HLbL};\eta'} = (15.7\ \pm\ 4 \pm ??) \times 10^{-11}$$

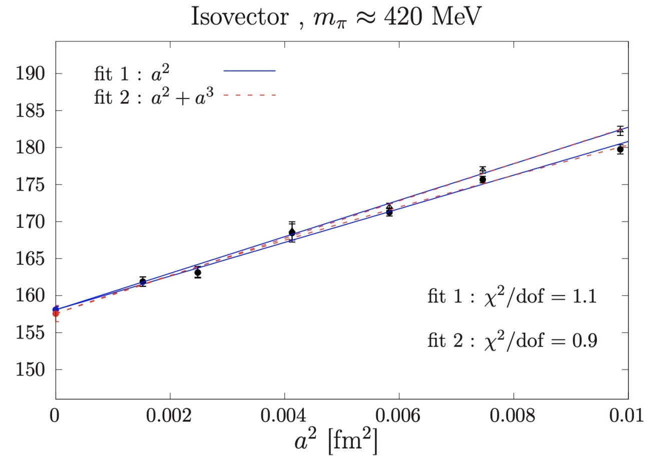


- First run $\approx 1/20$ of the expected total statistics
  - → are we close to NP discovery ?
  - → non-perturbative hadronic contributions dominate the error
  - → recent lattice results tend to reduce the tension ...
  - → ... but there is now a tension with the dispersive framework.
- Rapid progress on the lattice
  - → first sub-percent lattice HVP calculation by BMW, intermediate window ...
  - → first complete lattice HLBL calculation with Mainz

- Is the continuum extrapolation under control ?
  - → Based on Symanzik Effective Field Theory : one expects a^2 scaling ...
  - → ... up to logarithms $a^2/\log(a)^\Gamma$!
  - → For pure Yang-Mills one has $\Gamma > 0$. Might also be true for QCD [Husung et al '19]
  - → Often logarithms are neglected (2 reasons : Γ are not known + lack of data)



[BMW collaboration]



[Mainz group]