

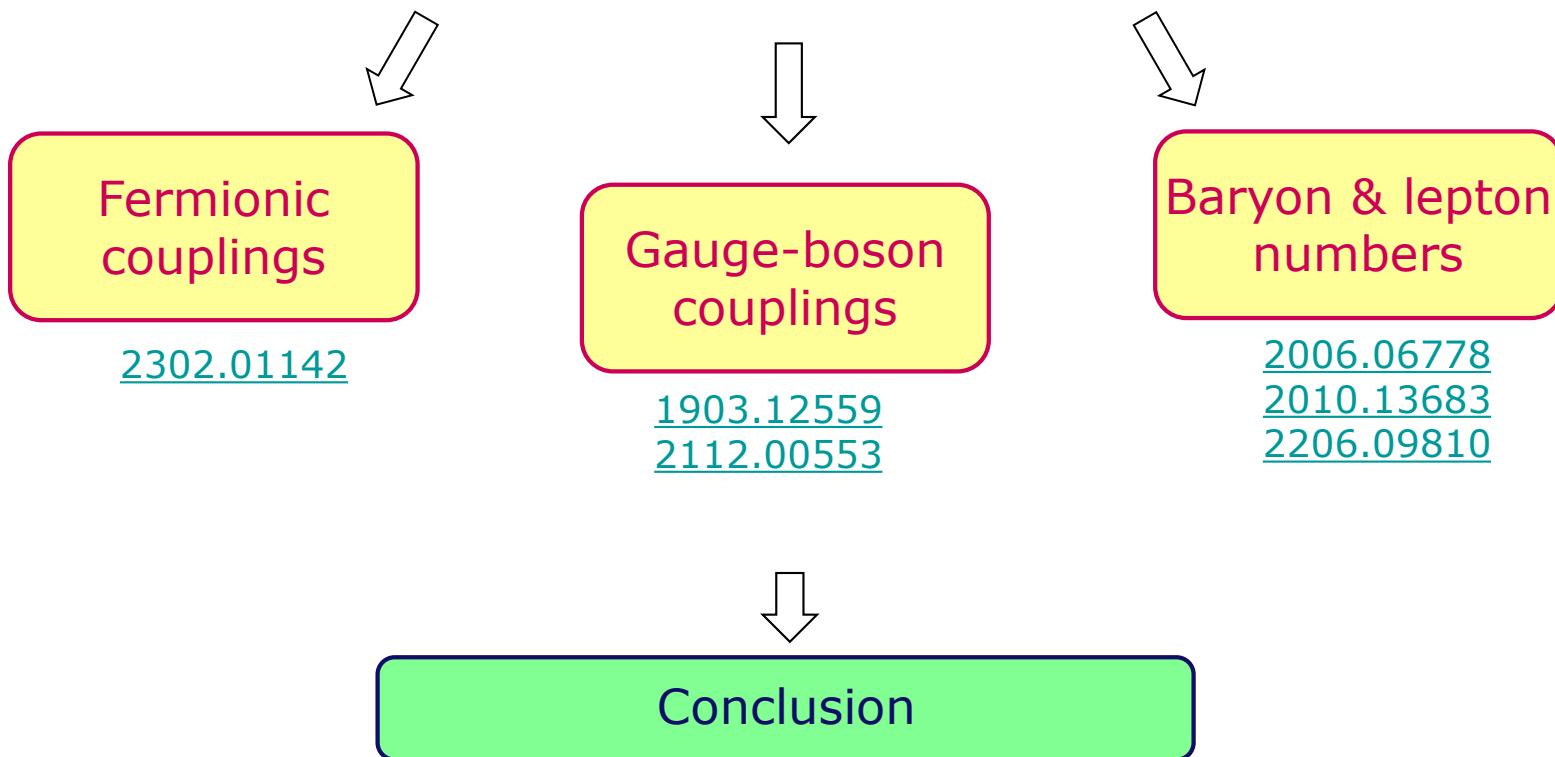
# Axion interactions



Christopher Smith



## Strong CP puzzle and axions



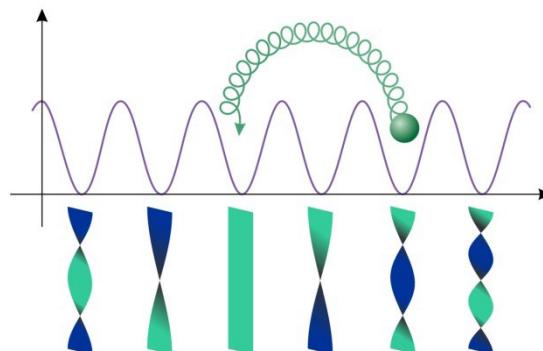
Strong CP puzzle and axions

## Origin of the strong CP puzzle - in a few words

Neutron EDM implies:  $\theta \equiv \theta_C - \arg \det Y_u - \arg \det Y_d < 10^{-10}$

$$\mathcal{L}_{CP} = (\theta_C - \arg \det Y_u - \arg \det Y_d) \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Non-trivial QCD topology:



Explains  
the large  
 $\eta'$  mass

Quark-Higgs Yukawa couplings:

We know they are complex.

$\delta_{CKM} \neq 0$   
from K and B physics

## Toy model for a “QED axion”

Add to the usual massless QED a **neutral complex scalar field**:

$$\mathcal{L}_{\text{axion}} = \frac{e^2}{16\pi^2} \theta F_{\mu\nu} \tilde{F}^{\mu\nu} + \bar{\psi}_{L,R} i \cancel{D} \psi_{L,R} - (y \bar{\psi}_L \psi_R \phi + h.c.) + \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

The  $U(1)_{PQ}$  symmetry is anomalous:

$$\phi \rightarrow \exp(-i\theta)\phi, \quad \begin{cases} \psi_L \rightarrow \exp(-i\alpha\theta)\psi_L \\ \psi_R \rightarrow \exp(-i(\alpha+1)\theta)\psi_R \end{cases}$$

Free parameter  $\alpha$  due to the conserved fermion number.

The  $U(1)_{PQ}$  symmetry is spontaneously broken:

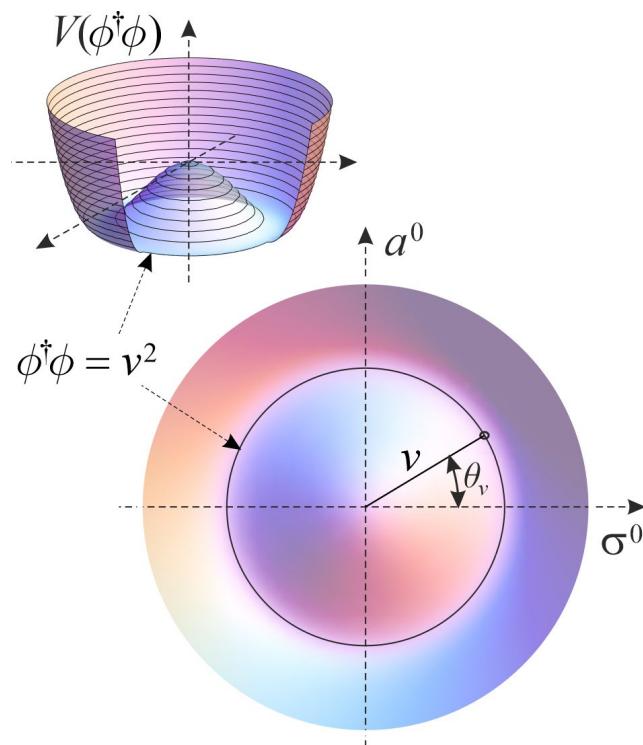
$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad \mu^2 < 0.$$

⇒ The Goldstone boson shift symmetry permits to rotate  $\theta$  away.

## First representation: Linear

Linear representation for the scalar field:  $\phi = \frac{1}{\sqrt{2}}(\sigma + i\alpha + v)$

$$\begin{aligned} \mathcal{L}_{linear} = & \frac{\alpha}{4\pi} \theta F_{\mu\nu} \tilde{F}^{\mu\nu} + \bar{\psi} i \not{D} \psi - m \bar{\psi} \psi \left( 1 + \frac{\sigma}{v} \right) - m \frac{\alpha}{v} \bar{\psi} i \gamma_5 \psi \\ & + \frac{\partial_\mu \alpha \partial^\mu \alpha}{2} + \frac{\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2}{2} \\ & - \lambda v \sigma (\sigma^2 + \alpha^2) - \frac{\lambda}{4} (\sigma^2 + \alpha^2)^2 \end{aligned}$$



With as usual  $\begin{cases} m = yv / \sqrt{2} \\ v^2 = -\mu^2 / \lambda \\ m_\sigma^2 = 2\lambda v^2 \end{cases}$

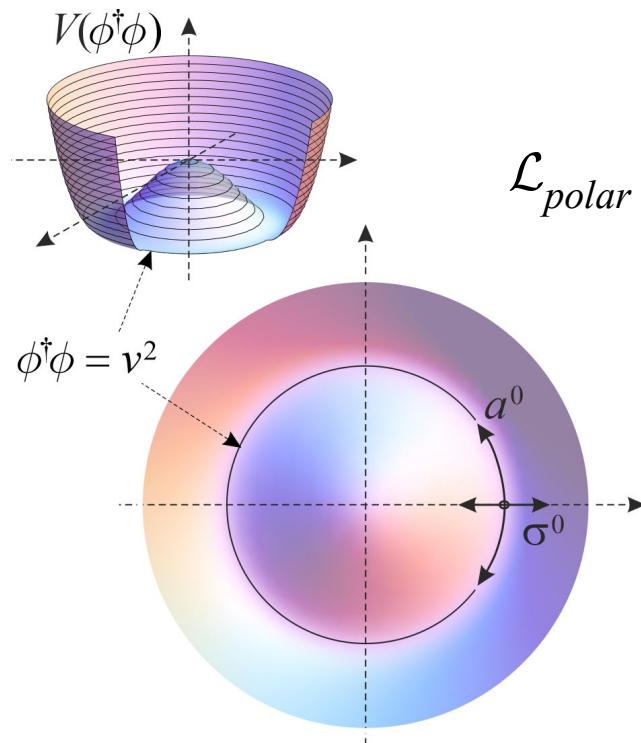
## Second representation: Polar

Polar representation for the scalar field:  $\phi = \frac{1}{\sqrt{2}}(\sigma + v) \exp(i\alpha/v)$

Non-linear representation of  $U(1)_{PQ}$ :

$\sigma \rightarrow \sigma$  ,  $\alpha \rightarrow \alpha + v\theta$  spans the vacuum.

No longer manifestly renormalizable:



$$\begin{aligned} \mathcal{L}_{polar} = & \frac{\alpha}{4\pi} \theta F_{\mu\nu} \tilde{F}^{\mu\nu} + \bar{\psi} i \not{D} \psi - m \left( 1 + \frac{\sigma}{v} \right) \bar{\psi} e^{i a y^5/v} \psi \\ & + \frac{\partial_\mu a \partial^\mu a}{2} \left( 1 + \frac{\sigma}{v} \right)^2 + \frac{\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2}{2} - \lambda v \sigma^3 - \frac{\lambda}{4} \sigma^4 \end{aligned}$$

## Second representation: Polar

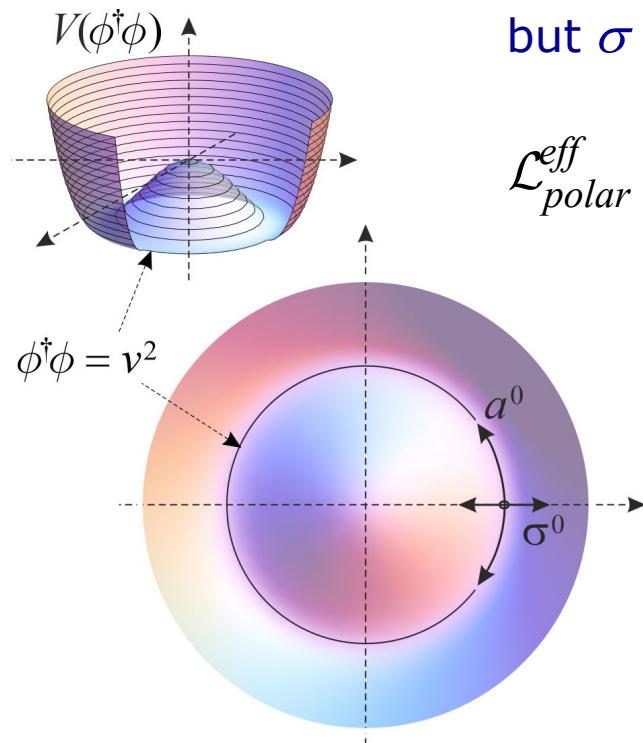
Polar representation for the scalar field:  $\phi = \frac{1}{\sqrt{2}}(\sigma + v) \exp(i\alpha/v)$

Non-linear representation of  $U(1)_{PQ}$ :

$\sigma \rightarrow \sigma$  ,  $\alpha \rightarrow \alpha + v\theta$  spans the vacuum.

No longer manifestly renormalizable,  
but  $\sigma$  can easily be integrated out:

$$\mathcal{L}_{polar}^{eff} = \frac{\alpha}{4\pi} \theta F_{\mu\nu} \tilde{F}^{\mu\nu} + \bar{\psi} \left( i\cancel{D} - m e^{i\alpha v^5/v} \right) \psi + \frac{\partial_\mu a \partial^\mu a}{2}$$



### Third representation: Derivative

Polar representation for the scalar field:  $\phi = \frac{1}{\sqrt{2}}(\sigma + v) \exp(i\alpha/v)$

Non-linear representation of  $U(1)_{PQ}$ :

$\sigma \rightarrow \sigma$  ,  $\alpha \rightarrow \alpha + v\theta$  spans the vacuum.

No longer manifestly renormalizable,  
but  $\sigma$  can easily be integrated out:

$$\mathcal{L}_{polar}^{eff} = \frac{\alpha}{4\pi} \theta F_{\mu\nu} \tilde{F}^{\mu\nu} + \bar{\psi} \left( i\cancel{D} - m e^{i\alpha y^5/v} \right) \psi + \frac{\partial_\mu a \partial^\mu a}{2}$$

Derivative representation:

Make  $\psi$  invariant under  $U(1)_{PQ}$  via  $\psi \rightarrow \exp(-i\alpha y^5/2v)\psi$ :

$$\mathcal{L}_{der}^{eff} = \frac{\alpha}{4\pi} \left( \theta - \frac{a}{v} \right) F_{\mu\nu} \tilde{F}^{\mu\nu} + \bar{\psi} \left( i\cancel{D} - m + \frac{\partial_\mu a \gamma^\mu \gamma^5}{2v} \right) \psi + \frac{\partial_\mu a \partial^\mu a}{2}$$

# Fermionic couplings

CS, '23

## Axion-fermion interactions in the non-relativistic limit

Derivative representation:

$$\mathcal{L}_{der} = \bar{\psi} \left( i \cancel{D} - m + \frac{\gamma^\mu \gamma^5 \partial_\mu a}{m} \right) \psi$$

(rem.:  $a \rightarrow g a \equiv \frac{m}{f_a} a$ )

*Foldy-Wouthuysen unitary rotations* 

$$\mathcal{H}_{der}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla a}{m} \right) + \frac{\gamma^5 \{ \boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t a \}}{2m^2} + \mathcal{O}(m^{-3})$$


Axion wind

Axioelectric

$$\mathcal{H}_{wind} = a(t) \mathbf{S}_\psi \cdot \mathbf{p}_a$$

→ Spin-precession experiments.

$$\mathcal{H}_{axioelec} = \partial_t a(t) \mathbf{S}_\psi \cdot \mathbf{p}_\psi$$

→ Ionization by solar axions.

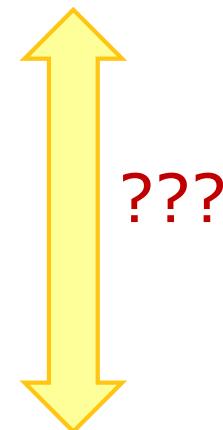
## Axion-fermion interactions in the non-relativistic limit

Derivative representation:

$$\mathcal{L}_{der} = \bar{\psi} \left( i \cancel{D} - m + \frac{\gamma^\mu \gamma^5 \partial_\mu a}{m} \right) \psi$$

Foldy-Wouthuysen  
unitary rotations

$$\mathcal{H}_{der}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla a}{m} \right) + \frac{\gamma^5 \{ \boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t a \}}{2m^2} + \mathcal{O}(m^{-3})$$



Polar representation:

$$\mathcal{L}_{pol} = \bar{\psi} \left( i \cancel{D} - m e^{2ia\gamma^5/m} \right) \psi$$

Foldy-Wouthuysen  
unitary rotations

$$\mathcal{H}_{pol}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla a}{m} \right) + \frac{\gamma^5 \{ \boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t a \}}{4m^2} + \mathcal{O}(m^{-3})$$

## Pion-nucleon interactions in the 60s

Dyson '48  
Case '49  
Berger,Foldy,Osborn '52  
Wentzel '52  
...

Derivative representation:

$$\mathcal{L}_{der} = \bar{\psi} \left( i \cancel{D} - m + \frac{\gamma^\mu \gamma^5 \partial_\mu \pi}{m} \right) \psi$$



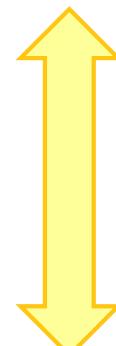
$$\mathcal{H}_{der}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla \pi}{m} \right) + \frac{\gamma^5 \{ \boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t \pi \}}{2m^2} + \mathcal{O}(m^{-3})$$

Linear representation:

$$\mathcal{L}_{lin} = \bar{\psi} \left( i \cancel{D} - m - 2i\pi\gamma^5 \right) \psi$$



$$\mathcal{H}_{lin}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla \pi}{m} - \frac{2\pi^2}{m} \right) + \frac{\gamma^5 \{ \boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t \pi \}}{4m^2} + \mathcal{O}(m^{-3})$$



Equivalence could not be achieved...

## Axion-fermion interactions in the non-relativistic limit

Derivative representation:

$$\mathcal{L}_{der} = \bar{\psi} \left( i \cancel{D} - m + \frac{\gamma^\mu \gamma^5 \partial_\mu a}{m} \right) \psi$$



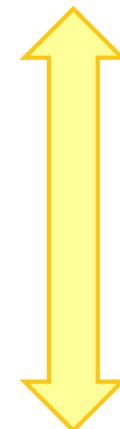
$$\mathcal{H}_{der}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla a}{m} \right) + \frac{\gamma^5 \{ \boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t a \}}{2m^2} + \mathcal{O}(m^{-3})$$

Polar representation:

$$\mathcal{L}_{pol} = \bar{\psi} \left( i \cancel{D} - m e^{2ia\gamma^5/m} \right) \psi$$



$$\mathcal{H}_{pol}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla a}{m} \right) + \frac{\gamma^5 \{ \boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t a \}}{4m^2} + \mathcal{O}(m^{-3})$$



For Goldstone bosons,  
equivalence must hold!

## Reparametrization invariance via unitary rotations

NR expansions are defined up to unitary transformations!

Consider  $|\psi\rangle \rightarrow e^{iS} |\psi\rangle$  with  $S = \mu \frac{\gamma^5 \{\boldsymbol{\gamma} \cdot \mathbf{p}, \mathbf{a}\}}{4m^2}$

Then,  $\mathcal{H}^{NR} \rightarrow \mathcal{H}^{NR} + [iS, \mathcal{H}^{NR}] - \partial_t S + \dots$

Higher order since

$$[iS, \gamma^0 m] = 0$$

Kills the total time derivative operator

$$-\partial_t S = -\mu \frac{\gamma^5 \{\boldsymbol{\gamma} \cdot \mathbf{p}, \partial_t \mathbf{a}\}}{4m^2}$$

Thus, we can choose:  $\mathcal{H}_{pol}^{NR} = \mathcal{H}_{der}^{NR} = \gamma^0 \left( m + \frac{\mathbf{p}^2}{2m} + \frac{\gamma^5 \boldsymbol{\gamma} \cdot \nabla \mathbf{a}}{m} \right) + \mathcal{O}(m^{-3})$

For neutral fermions: Screening of the axioelectric operator!

## Schiff's theorem

Dirac equation with EM fields in the non-relativistic limit:

$$\mathcal{L}_{EM} = \bar{\psi} (i\cancel{D} - m) \psi$$

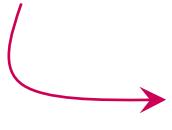


$$\mathcal{H}_{EM}^{NR} = \gamma^0 \left( m + \frac{\mathbf{P}^2}{2m} - \frac{e}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} \right) + e\phi + \mathcal{O}(m^{-2})$$

$$(\mathbf{P} \equiv \mathbf{p} - e\mathbf{A})$$

## Schiff's theorem

Dirac equation with EM fields in the non-relativistic limit:

$$\mathcal{L}_{EM} = \bar{\psi} \left( i\cancel{D} - m - \frac{ea}{4m} \sigma^{\mu\nu} F_{\mu\nu} + i \frac{d}{2} \sigma^{\mu\nu} \tilde{F}_{\mu\nu} \right) \psi$$


$$\mathcal{H}_{EM}^{NR} = \gamma^0 \left( m + \frac{\mathbf{P}^2}{2m} - \frac{e(1+a)}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} + d \boldsymbol{\sigma} \cdot \mathbf{E} \right) + e\phi + \mathcal{O}(m^{-2})$$

With the unitary transformation  $iS = -i(d/e)\gamma^5 \boldsymbol{\gamma} \cdot \mathbf{P}$ :

$$\begin{aligned} \mathcal{H}_{EM}^{NR} &\rightarrow \mathcal{H}_{EM}^{NR} + [iS, \mathcal{H}_{EM}^{NR}] - \partial_t S + \dots = \mathcal{H}_{EM}^{NR} - id[\gamma^5 \boldsymbol{\gamma} \cdot \mathbf{p}, \phi] - d\gamma^5 \boldsymbol{\gamma} \cdot \partial_t \mathbf{A} + \dots \\ &= \mathcal{H}_{EM}^{NR} - d\gamma^5 \boldsymbol{\gamma} \cdot (\nabla \phi + \partial_t \mathbf{A}) + \dots \\ &= \mathcal{H}_{EM}^{NR} - d\gamma^0 \boldsymbol{\sigma} \cdot \mathbf{E} + \dots \end{aligned} \quad (\gamma^5 \boldsymbol{\gamma} = -\gamma^0 \boldsymbol{\sigma})$$

For charged fermions: Screening of the EDM coupling.

## Axion-charged fermion interactions in the non-relativistic limit

Derivative representation:

$$\mathcal{L}_{der} = \bar{\psi} \left( i\cancel{D} - m + \frac{\gamma^\mu \gamma^5 \partial_\mu a}{m} \right) \psi$$

↷

$$\mathcal{H}_{der}^{NR} = \mathcal{H}_{EM}^{NR} + \frac{\gamma^0 \gamma^5 \gamma \cdot \nabla a}{m} + \frac{\gamma^5 \{ \gamma \cdot \mathbf{P}, \partial_t a \}}{2m^2} + \mathcal{O}(m^{-3})$$

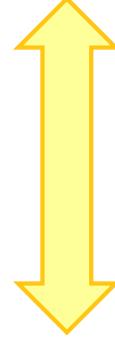
Polar representation:

$$\mathcal{L}_{pol} = \bar{\psi} \left( i\cancel{D} - m e^{2ia\gamma^5/m} \right) \psi$$

↷

$$\mathcal{H}_{pol}^{NR} = \mathcal{H}_{EM}^{NR} + \frac{\gamma^0 \gamma^5 \gamma \cdot \nabla a}{m} + \frac{\gamma^5 \{ \gamma \cdot \mathbf{P}, \partial_t a \}}{4m^2} - \frac{ea\gamma^5 \gamma \cdot \mathbf{E}}{2m^2} + \mathcal{O}(m^{-3})$$

Again, equivalence must hold!



Axion-induced EDMs encode all the effects!

Unitary rotation with  $iS = \mu \frac{\gamma^5 \{\gamma \cdot \mathbf{P}, \mathbf{a}\}}{4m^2}$

Barnhill '69  
Lee, Pittel '76  
Fiar '74, '77

translates into the equivalence  $\frac{\gamma^5 \{\gamma \cdot \mathbf{P}, \partial_t \mathbf{a}\}}{2m^2} \Leftrightarrow -\frac{e \mathbf{a} \gamma^5 \gamma \cdot \mathbf{E}}{m^2}$

**Doubly-screened:** Both vanish if either  $\mathbf{E} = 0$  or  $\partial_t \mathbf{a} = 0$ .

See the neutral case

See Schiff theorem

Time-dependent perturbation theory, with  $\phi = 0$  :

$$\frac{\gamma^5 \{\gamma \cdot \mathbf{P}, \partial_t \mathbf{a}\}}{2m^2} = \frac{\gamma^5 \{\gamma \cdot \mathbf{p}, \partial_t \mathbf{a}\}}{2m^2} - e \frac{\partial_t \mathbf{a} \gamma^5 \gamma \cdot \mathbf{A}}{m^2} \quad \Leftrightarrow \quad -\frac{e \mathbf{a} \gamma^5 \gamma \cdot \mathbf{E}}{m^2}$$

$\uparrow$

$0$

Axion-induced EDMs encode all the effects!

Unitary rotation with  $iS = \mu \frac{\gamma^5 \{\gamma \cdot \mathbf{P}, \mathbf{a}\}}{4m^2}$

Barnhill '69  
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The final NR Hamiltonian can be chosen as:

$$\mathcal{H}_{der}^{NR} = \mathcal{H}_{pol}^{NR} = \mathcal{H}_{EM}^{NR} + \frac{\gamma^0 \gamma^5 \gamma \cdot \nabla \mathbf{a}}{m} - \frac{e \mathbf{a} \gamma^5 \gamma \cdot \mathbf{E}}{m^2} + \mathcal{O}(m^{-3})$$

Dirac equation dual predictions:

$$m \bar{\psi} \psi \rightarrow \frac{e}{2m} \gamma^0 \boldsymbol{\sigma} \cdot \mathbf{B} \quad \rightarrow g_\psi = 2$$

$$1 \leftrightarrow i \gamma^5 \Leftrightarrow \mathbf{B} \leftrightarrow \mathbf{E}$$

$$m(2a/m) \bar{\psi} i \gamma^5 \psi \rightarrow \frac{e}{2m} (2a/m) \gamma^0 \boldsymbol{\sigma} \cdot \mathbf{E} \quad \rightarrow d_\psi = \frac{e \mathbf{a}}{m_\psi^2}$$

## Phenomenological consequences of the axionic EDM

Axioelectric effect originates from the EDM operator:

For bound electrons,  $\frac{ea\gamma^5\gamma \cdot \mathbf{E}}{m^2}$  with  $\mathbf{E} = -\nabla\phi$  gives the usual result.

Pospelov,Ritz,Voloshin '08

$$\frac{\gamma^5 \{\gamma \cdot \mathbf{P}, \partial_t \mathbf{a}\}}{2m^2} = \frac{\gamma^5 \{\gamma \cdot \mathbf{p}, \partial_t \mathbf{a}\}}{2m^2} - e \frac{\partial_t \mathbf{a} \gamma^5 \gamma \cdot \mathbf{A}}{m^2} \quad \Leftrightarrow \quad -\frac{e \mathbf{a} \gamma^5 \gamma \cdot \mathbf{E}}{m^2}$$

$$-\frac{e \mathbf{a} \gamma^5 \gamma \cdot \int \partial_t \mathbf{a} \mathbf{A} dt}{m^2} = \frac{e \mathbf{a} \gamma^5 \gamma \cdot \int \mathbf{a} \partial_t \mathbf{A} dt}{m^2}$$

$$-\frac{e \mathbf{a} \gamma^5 \{\gamma \cdot \mathbf{p}, \partial_t\}}{2m^2} \stackrel{EOM}{=} \frac{e \mathbf{a} \gamma^5 \gamma \cdot \nabla \phi}{m^2}$$

## Phenomenological consequences of the axionic EDM

Axioelectric effect originates from the EDM operator:

For bound electrons,  $\frac{ea\gamma^5 \mathbf{\gamma} \cdot \mathbf{E}}{m^2}$  with  $\mathbf{E} = -\nabla\phi$  gives the usual result.

Pospelov,Ritz,Voloshin '08

Leading EDM for charged leptons and quarks:

$d_\psi [e\text{cm}]$	$a(t)\bar{\psi}\gamma^5\psi$	$a(t)G\tilde{G}$ (*)	$d_\psi^{cst} \text{ exp.}$
$d_e$	$10^{-30} \cos m_a t$	$\approx 0$	$< 1.1 \times 10^{-29}$
$d_{n/p}$	$10^{-33} \cos m_a t$	$10^{-35} \cos m_a t$	$< 1.8_n / 21_p \times 10^{-26}$

Similar to  $g = 2 + 2a$ .

(\*) Graham,Rajendran '13

Limit on  $d_e^{cst}$  close to the QCD axion, holds for  $m_a \lesssim 10^{-15} \text{ eV} \approx 1 \text{ Hz}$ .

Future limits on  $d_{n,p}$  could reach the QCD axion up to  $m_a \lesssim 10^{-7} \text{ eV}$ .

# Gauge-boson couplings

Quevillon, CS, '19

Quevillon, CS, Vuong '21

## Typical Axion effective Lagrangian

Georgi,Kaplan,Randal, '86

Anomalous couplings to gauge bosons:

$$\mathcal{L}_{Jac} = \frac{a^0}{16\pi^2 f_a} (g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu})$$

...with  $\mathcal{N}_X = \sum_\psi Q_\psi C_X(\psi)$ .

Derivative couplings to fermions (and other PQ-charged fields):

$$\mathcal{L}_{Der} = -\frac{1}{f_a} \partial_\mu a J_{PQ}^\mu ,$$

with  $J_{PQ}^\mu = \sum_{\psi=L,R} Q_\psi \bar{\psi} \gamma^\mu \psi + \dots$

$\curvearrowright 0$  (partial integration + CVC)

$$= \sum_{\psi=u,d,e,v} \left[ (Q_{\psi_R} + Q_{\psi_L}) \bar{\psi} \gamma^\mu \psi + (Q_{\psi_R} - Q_{\psi_L}) \bar{\psi} \gamma^\mu \gamma_5 \psi \right] + \dots$$

All this seems ok... but actually there is a serious ambiguity issue!

Problem: The fermion PQ charges are ill-defined

Scalars have well defined PQ charges, but fermions do not:

KSVZ:  $\phi \bar{\Psi}_L \Psi_R$

	$\Psi_L$	$\Psi_R$	$q_L$	$u_R$	$d_R$	$\ell_L$	$e_R$	$\nu_R$
$U(1)_{PQ}$	$\alpha$	$\alpha - 1$	$\beta$	$\beta$	$\beta$	$\gamma$	$\gamma$	$\gamma$
$U(1)_Y$	$Y$	$Y$	$1/3$	$4/3$	$-2/3$	$-1$	$-2$	$0$

DFSZ:  $\phi H_u^\dagger H_d$

$$x \equiv v_u / v_d$$

	$q_L$	$u_R$	$d_R$	$\ell_L$	$e_R$	$\nu_R$
$U(1)_{PQ}$	$\beta$	$\beta + x$	$\beta - 1/x$	$\gamma$	$\gamma - 1/x$	$\gamma + x$
$U(1)_Y$	$1/3$	$4/3$	$-2/3$	$-1$	$-2$	$0$

Free parameters reflects the conservation of  $\Psi$ ,  $\mathcal{B}$ ,  $\mathcal{L}$  numbers.

Consequence: Ambiguous DFSZ Axion couplings for SM gauge bosons

Anomalous couplings to gauge bosons:

$$\begin{aligned}\mathcal{L}_{Jac} = & \frac{\textcolor{red}{a}}{16\pi^2 f_a} g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \\ & + \frac{\textcolor{red}{a}}{16\pi^2 f_a} g^2 \mathcal{N}_L W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} \\ & + \frac{\textcolor{red}{a}}{16\pi^2 f_a} g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu}\end{aligned}$$

$$\begin{aligned}\mathcal{N}_C &= \frac{1}{2} \left( x + \frac{1}{x} \right) \\ \mathcal{N}_L &= -\frac{1}{2} (3\beta + \gamma) \\ \mathcal{N}_Y &= \frac{1}{2} (3\beta + \gamma) + \frac{4}{3} \left( x + \frac{1}{x} \right)\end{aligned}$$

Derivative couplings to SM fermions:

$$\mathcal{L}_{Der} = -\frac{1}{2f_a} \partial_\mu a^0 \sum_{u,d,e,v} \chi_V^f \bar{\psi}_f \gamma^\mu \psi_f + \chi_A^f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$$

	$u$	$d$	$e$	$\nu$
$\chi_V$	$2\beta + x$	$2\beta + \frac{1}{x}$	$2\gamma + \frac{1}{x}$	$\gamma$
$\chi_A$	$x$	$\frac{1}{x}$	$\frac{1}{x}$	$-\gamma$

Both manifestly  $SU(2)_L \otimes U(1)_Y$  symmetric, both ambiguous!

Consequence: Ambiguous DFSZ Axion couplings for SM gauge bosons

Axion couplings in the polar/linear representation (= THDM!!!)

$$\begin{aligned}
 \mathcal{L}_{polar}^{eff} = & \frac{\textcolor{red}{a}}{16\pi^2 f_a} g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \\
 & + \frac{\textcolor{red}{a}}{16\pi^2 f_a} e^2 \mathcal{N}_{em} F_{\mu\nu} \tilde{F}^{\mu\nu} \\
 & + \frac{\textcolor{red}{a}}{16\pi^2 f_a} \frac{2e^2}{c_W s_W} (\mathcal{N}_0 - s_W^2 \mathcal{N}_{em}) \textcolor{green}{Z}_{\mu\nu} \tilde{F}^{\mu\nu} \\
 & + \frac{\textcolor{red}{a}}{16\pi^2 f_a} \frac{e^2}{c_W^2 s_W^2} (\mathcal{N}_1 - 2s_W^2 \mathcal{N}_0 + s_W^4 \mathcal{N}_{em}) \textcolor{green}{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} \\
 & + \frac{\textcolor{red}{a}}{16\pi^2 f_a} 2g^2 \mathcal{N}_2 W_{\mu\nu}^+ \tilde{W}^{-,\mu\nu}
 \end{aligned}$$

$$\mathcal{N}_C = \frac{1}{2} \left( x + \frac{1}{x} \right)$$

$$\mathcal{N}_{em} = \frac{4}{3} \left( x + \frac{1}{x} \right)$$

$$\mathcal{N}_0 = \frac{1}{2} \left( x + \frac{1}{x} \right)$$

$$\mathcal{N}_1 = \frac{1}{12} \left( 3x + \frac{4}{x} \right)$$

$$\mathcal{N}_2 = \frac{1}{4} \left( x + \frac{3}{2x} \right)$$

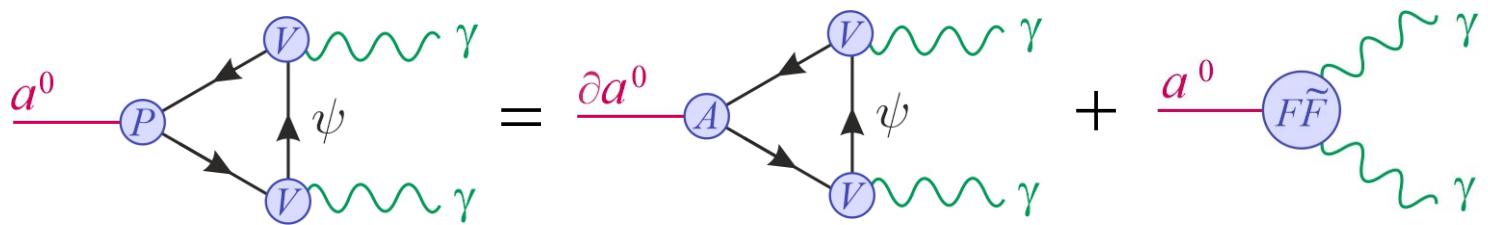
Gunion, Haber, Kao '91

Not ambiguous, but **does not match  $\mathcal{L}_{Jac}$** :

$$\mathcal{N}_{em} = \mathcal{N}_L + \mathcal{N}_Y \quad \text{but} \quad \mathcal{N}_0 \neq \mathcal{N}_1 \neq \mathcal{N}_2 \neq \mathcal{N}_L = -\frac{1}{2} (3\beta + \gamma)$$

## Solution: Violations of Sutherland-Veltman theorem

Axial current anomalies:  $2imP = \partial_\mu A^\mu - \frac{1}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$



Not anomalous

Anomalous

Anomaly

$$\partial_\mu A^\mu \rightarrow 0 \text{ when } m \rightarrow \infty$$

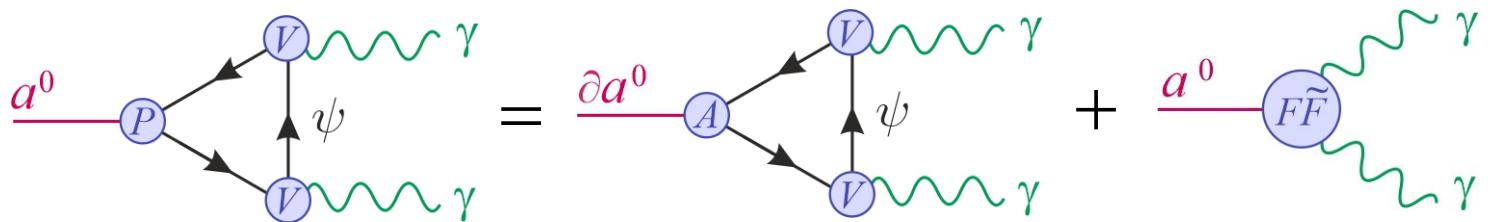
True coupling

$\mathcal{L}_{Der}$

$\mathcal{L}_{Jac}$

## Solution: Violations of Sutherland-Veltman theorem

Axial current anomalies:  $2imP = \partial_\mu A^\mu - \frac{1}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$



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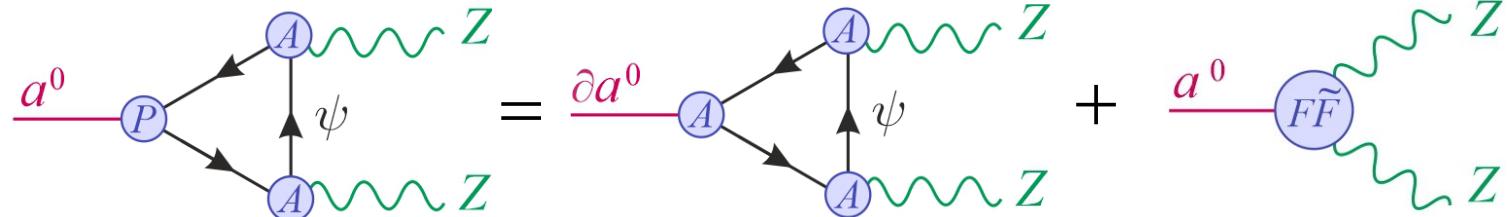
Anomaly

$$\partial_\mu A^\mu \rightarrow 0 \text{ when } m \rightarrow \infty$$

True coupling

$\mathcal{L}_{Der}$

$\mathcal{L}_{Jac}$



Not anomalous

Anomalous

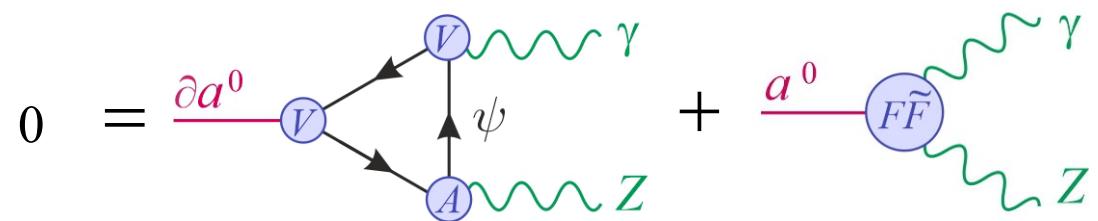
Anomaly

$$\partial_\mu A^\mu \not\rightarrow 0 \text{ when } m \rightarrow \infty$$

## Solution: Violations of Sutherland-Veltman theorem

Axial current anomalies:  $2imP = \partial_\mu A^\mu - \frac{1}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$

Vector current anomalies:  $0 = \partial_\mu V^\mu - \frac{1}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$



$$\partial_\mu V^\mu \not\rightarrow 0 \text{ when } m \rightarrow \infty$$

$\mathcal{L}_{Der}$

$\mathcal{L}_{Jac}$

Ambiguities due to  $\mathcal{B}$  and  $\mathcal{L}$  cancel out between  $\mathcal{L}_{Jac}$  and  $\mathcal{L}_{Der}$ .

SV holds for  $a^0 + \gamma\gamma, gg$  but  $a^0 + \gamma Z, ZZ, WW$  are not given by  $\mathcal{L}_{Jac}$ .

# $\mathcal{B}$ and $\mathcal{L}$ violations

Quevillon, CS, '20

Arias-Aragón, CS, '22

## Using the ambiguities to entangle PQ with $\mathcal{B}$ and $\mathcal{L}$

- Example: KSVZ axion as majoron

Langacker et al. '86  
Shin '87  
Clarke,Volkas '16

$$\phi \bar{\Psi}_L \Psi_R + \phi \bar{\nu}_R^C \nu_R \begin{cases} \xrightarrow{f_a} f_a \bar{\nu}_R^C \nu_R \rightarrow m_\nu \sim \frac{v^2}{f_a} Y_\nu^T Y_\nu \\ \xrightarrow{} a \bar{\nu}_R^C \nu_R \leftarrow \text{no } f_a \text{ suppression.} \end{cases}$$

The PQ current eats up the lepton current (sets  $\gamma = -1/2$  ):

	$\phi$	$H$	$\Psi_L$	$\Psi_R$	$q_L$	$u_R$	$d_R$	$\ell_L$	$e_R$	$\nu_R$
$U(1)_{PQ}$	1	0	$\alpha$	$\alpha - 1$	$\beta$	$\beta$	$\beta$	$-1/2$	$-1/2$	$-1/2$

- In general, SSB along two combinations of  $\mathcal{B}$  and  $\mathcal{L}$  only.

Beware though that EW instantons  $\Rightarrow \mathcal{L}_{eff} \sim (\ell_L q_L^3)^3 \Rightarrow 3\beta + \gamma = 0$ .

- PQ symmetry itself can stabilize  $\mathcal{B}$  and  $\mathcal{L}$  violating models.

For example,  $M \bar{\nu}_R^C \nu_R$  is incompatible with  $\phi \bar{\nu}_R^C \nu_R$ .

## Rich phenomenology with leptoquarks & diquarks

- Spontaneous proton decay

$$S_1^{8/3} \bar{d}_R e_R^C + \tilde{S}_1^{8/3} \bar{u}_R^C u_R + \phi^2 S_1^{8/3\dagger} \tilde{S}_1^{8/3}$$

(Very similar to Reig, Srivastava, '18)

- Spontaneous neutron-antineutron oscillation

$$S_1^{4/3} \bar{d}_R^C d_R + S_1^{8/3} \bar{u}_R^C u_R + \phi S_1^{4/3} S_1^{4/3} S_1^{8/3}$$

(Kind of similar to Barbieri, Mohapatra '81)

- ALP and the neutron lifetime puzzle

$$S_1^{2/3} \bar{d}_R^C u_R + V_{1,\mu}^{2/3} \bar{d}_R \gamma^\mu v_R + \partial^\mu \phi S_1^{2/3\dagger} V_{1,\mu}^{2/3}$$

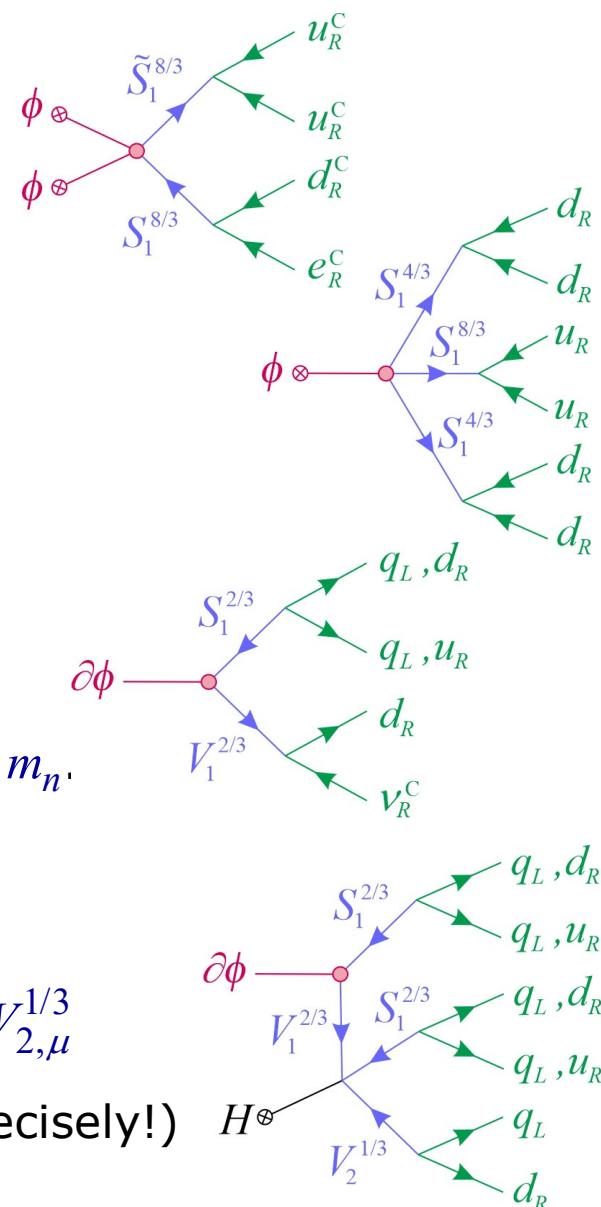
$$B(n \rightarrow \nu a) \sim 1\% \text{ with } p \rightarrow e \gamma \gamma > 10^{34} \text{ yr if } m_p < m_a < m_n.$$

(see Fornal, Grinstein, '18)

- Intense antimatter production?

$$S_1^{2/3} \bar{d}_R^C u_R + V_{2,\mu}^{1/3} \bar{d}_R^C \gamma^\mu q_L + \partial^\mu \phi V_{1,\mu}^{2/3\dagger} S_1^{2/3} + H V_{1,\mu}^{2/3} S_1^{2/3} V_{2,\mu}^{1/3}$$

Resonant  $n \rightarrow a^0 \bar{n}$  if  $\delta m_{n-\bar{n}} \approx B \times 10^{-7} \text{ eV} = m_a$  (precisely!)



# Conclusion

The axion mechanism is currently the best solution to the strong CP puzzle!

But its theoretical description still brings some surprises:

Fermion couplings: Axions induce EDMs for electrons, neutrons,...

Gauge couplings: Not driven by the anomaly.

$\mathcal{B}$  and  $\mathcal{L}$  violation: Unified and spontaneously broken along with PQ?

Book the date!

2018:



14–16 mai 2018

LPSC

Fuseau horaire Europe/Zurich

The strong CP puzzle and axions

Workshop + minischool with the lectures:

Symmetries and anomalies (F. Delduc, ENS Lyon),  
Strong CP puzzle (M. Goodsell, LPTHE),  
EDM searches (Chen-Yu Liu, Indiana),  
Axion physics at low-energy (G. Villadoro, ICTP)  
Axion cosmology (A. Ringwald, DESY)

2020: Follow-up meeting unfortunately cancelled...

2023: Axions++ in LAPTH, Annecy, September 25-28.